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ISOTROPY IN $\pi - \mu$ DECAY

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ABSTRACT

An extensive analysis is made of all results on the angular distribution of π-μ decays in the nuclear emulsion stack used by Hulubei et al.

Contrary to their previous analyses, which favoured anisotropy for this distribution, it is shown that no strong indication of anisotropy subsists which is free from serious suspicion of residual uncorrected bias. The safe part of the scanning of that stack is in good agreement with isotropy.

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INTRODUCTION

The angular distribution of π-μ decays at rest in nuclear emulsions have been re-examined recently by HULUBEI et al. [1]. They obtained an isotropic distribution in contradistinction to their previous anisotropic results [2-5].

However they conclude that these different angular distributions are meaningful thus ascribing the variations to not well defined changes in previous pion history.

An extensive analysis of the previous results on the angular distribution was made by HULUBEI et al.[5] with the conclusion that the significant departure from isotropy in π-μ decay was a genuine physical result. However the fact that we had obtained [6] a distribution consistent with isotropy-using plates from the same stack where they found the anisotropic result-was minimized. They argue that the conclusions of that paper [6] were based on qualitative considerations, that scanning efficiencies have not been estimated nor were beam mouns investigated. They claim that results of reference [6] are not at variance with their more recent results [5]. Finally they state that the comparison of our results, uncorrected, with their old results [2] uncorrected for bias, seems of no pratical interest.

We think, however, that we should extract all possible informations from these different analysis of the stack where pion history is the same.

Actually this is exactly the aim of the present paper where not only the answers to the criticisms to reference [6] above mentioned are given, with the pertinent additional informations, but also a detailed analysis of all results obtained with that stack is made. The conclusion is that no strong indication of anisotropy in π-μ decay exists which is free from serious suspicion of residual uncorrected bias.

RESULTS FOR SEVERAL SCANNINGS

The angular distribution we are considering in $dN/d\theta$, θ being the angle between the initial direction of the μ meson projected on the emulsion's plane and the direction of π beam.

In [2] the method of area scanning was used, looking for π-μ vertex. A $\chi^2 = 181.4$, for three degrees of freedom, was found in the comparison of the results with isotropy (7526 decays).

After we obtained a few plates of that same stack thanks to the kindness of Professor Hulubei, a scanning by the same method was made finding a distribution also not compatible with isotropy [6]. A $\chi^2 = 22.9$ was obtained for three degrees of freedom (2594 decays), thus giving a probability $P \simeq 0.01\%$ for isotropy. However these results were also hardly compatible with those of reference [2] leading to $\chi^2 = 10.4$ (three d.of f.) or a probability of 1.5% for the two samples to correspond to the same distribution. To check some indications of observational bias unfavouring small π-μ angles a new scanning was made [6] using a different method. The scanners were instructed to look for all black tracks ending in the emulsion and follow them back (in the same plate) to see whether they correspond to a μ resulting from π decay at rest. The π-μ vertex must be found within the μ range and accepted even if it looks as a scattering, in which case it would have been lost in the π-μ vertex scanning. The distribution was then compatible with isotropy ($\chi^2 = 5.2$ for three d. of f., or P = 15,8%; 4132 decays). It was, however, incompatible with the results of (2) ($\chi^2 = 42.8$ for three d. of f., or P << 0.001%).

In (7) we increased the statistics using the same method with essentially the same results of (6) (total of 8669 decays).

The departures from isotropy in the angular distribution are not, however, only due to a forward-backward asymmetry, as a pole-equator asymmetry was also observed.

Thus the values of coefficients (5) b and d,

$$b = \frac{2\ (\text{forward-backward})}{\text{forward+backward}} = \frac{2\ (X_1+X_2-X_3-X_4)}{X_1+X_2+X_3+X_4}$$

$$d = \frac{2\ (\text{pole - equator})}{\text{equator + pole}} = \frac{2\ (X_1+X_4-X_2-X_3)}{X_1+X_2+X_3+X_4}$$

are useful to analyse these distributions. Here X_i are the numbers of observed decays with θ in the intervals as follows:

X_1 : 0° - 45° plus 315° - 360°

X_2 : 45° - 90° plus 270° - 315°

X_3 : 90° - 135° plus 225° - 270°

X_4 : 135° - 225°

Their values are given in Table I for the above mentioned results, all of them uncorrected for efficiency, the numbers corresponding to the number of reference, [(6) refers only to π-μ vertex scanning].

TABLE I

Exp.	b	d
(2)	-0.115 ± 0.023	-0.268 ± 0.023
(6)	-0.040 ± 0.039	-0.180 ± 0.039
(7)	-0.026 ± 0.022	-0.057 ± 0.022

It is also convenient to introduce the coefficient

$$a = \frac{2}{\sqrt{3}} \left(\frac{4X_1}{X_1+X_2+X_3+X_4} - 1 \right)$$

In the Table II the coefficients are given for all experiments made with Hulubei stack: L, H, T_1 and T_2 are the experiments quoted in references [5]. L is the same described in references [2] and [3] with a slight increase in statistics and with correction for bias. H is an experiment with higher scanning efficiency [5] using the same method as in [2].

In T, the scanning was made following the gray track in the beam until it ended [4]. If a positron track was present the μ-track was followed back for approximately 600 μ to look for the π-μ vertex.

In experiment T_2 the same method was used but the π-μ-e event had both vertices in the same plate where the gray tracks was picked up. In both case it was assumed that there was no bias.

Experiment E in Table II is part of reference [7] corrected for efficiencies as analised in the next section.

TABLE II

Exp.	$b\times10^3$	$d\times10^3$	$a\times10^3$
L	-124 ± 21	-131 ± 21	-208 ± 21
H	- 95 ± 38	-124 ± 38	-134 ± 36
T_1	-143 ± 48	- 88 ± 48	-108 ± 46
T_2	- 16 ± 59	+ 2 ± 59	+ 70 ± 60
E	+ 8 ± 38	- 51 ± 38	- 18 ± 40

CORRECTED RESULTS

The correction for scanning efficiency by double scanning could not be made for all scanners as we had to send back the plates used. However 1700 out of 2594 decays found in reference [6] using π-μ vertex scanning were in the same area scanned with the black track ending method.

They could be used to determine the efficiencies of three of the scanners (A, B and C) who had used the last method. The results are given in Table III. In Table IV the corrected values of X_1, of coefficients b, d and a and of χ^2 for isotropy with three degrees of freedom are given.

TABLE III

Scan.	Observed results				Efficiencies $\times 10^3$			
	X_1^0	X_2^0	X_3^0	X_4^0	ϵ_1	ϵ_2	ϵ_3	ϵ_4
A	683.5	705.5	742.5	690.5	75.0±3.3	78.1±2.7	80.2±2.7	75.7±3.0
B	706.5	800.0	778.0	717.5	66.1±3.4	66.8±3.0	66.2±3.0	62.7±3.3
C	332.5	329.0	337.0	309.5	72.7±13.4	77.8±13.9	83.3±10.8	93.3±6.4

TABLE IV

Scan.	X_1	X_2	X_3	X_4	$b\times10^3$	$d\times10^3$	$a\times10^3$	χ^2
A	911.3±53.1	903.3±46.2	925.8±46.1	912.2±50.1	-13±54	- 3±54	- 2±57	0.1
B	1068.8±68.9	1197.6±68.5	1175.2±67.9	1144.3±74.1	-23±61	-70±61	-78±60	2.0
C	457.2±88.1	423.0±78.9	404.4±56.7	331.6±29.7	+178±159	-48±166	+152±198	6.4

The combined corrected results of A, B and C and the corresponding errors were obtained using the following equations. Let us call $P_i^A = X_i^A / \sum_{j=1}^{4} X_j^A$ the relation of corrected number of cases found by observer A in interval i to the total corrected number of decays found by this observer (and similarly for the other observer) and $\sigma_i^A = \Delta X_i^A / \sum_{j=1}^{4} \Delta X_j^A$ where ΔX_i^A is the error in X_i^A. Then the combined value for P_i was found by the expression ([8]):

$$P_i = P_i^o + \delta P_i$$

where

$$\frac{P_i^o}{\sigma_i^2} = \frac{P_i^A}{(\sigma_i^A)^2} + \frac{P_i^B}{(\sigma_i^B)^2} + \frac{P_i^C}{(\sigma_i^C)^2} \qquad (1)$$

and

$$\delta P_i = (1 - \sum_{j=1}^{4} P_j^o)\, \sigma_i^2 / \sum_{k=1}^{4} \sigma_k^2 .$$

Here σ_i given by

$$\frac{1}{\sigma_i^2} = \frac{1}{(\sigma_i^A)^2} + \frac{1}{(\sigma_i^B)^2} + \frac{1}{(\sigma_i^C)^2} \qquad (2)$$

is the error in P_i and P_i satisfies

$$\sum_{i=1}^{4} P_i = 1 .$$

The value of χ^2 of the combination of two experiences A and B, which generalize Pearson's formula is in this case given by ([8]):

$$\chi^2_{AB} = \sum_{i=1}^{4} \frac{(P_i^A - P_i^B)^2}{(\sigma_i^A)^2 + (\sigma_i^B)^2} + \frac{1 - \sum_{k=1}^{4} P_k^{o\,AB}}{\sum_{j=1}^{4} (\sigma_j^{AB})^2}$$

where $P_k^{o\,AB}$, σ_j^{AB} are given by expressions (1) and (2) for the combined experiences A, B only.

It should be mentioned that for purely statistical distributions $\Sigma P_i^n = 1$ and the additional terms in $1 - \Sigma P_i^o$ disappear.

The values obtained for P_i were:

$$P_1 = 0.2461 \pm 0.0103 \qquad P_2 = 0.2560 \pm 0.0095$$

$$P_3 = 0.2568 \pm 0.0093 \qquad P_4 = 0.2411 \pm 0.0091$$

In no case δP_i was larger than 0.003. The values of χ^2 for the combination of the scannings two by two are (3 degrees of freedom):

$$\chi^2_{AB} = 1.1 \qquad \chi^2_{BC} = 4.5 \qquad \chi^2_{AC} = 4.5$$

which shows the consistency of these three scannings. The values of the coefficients a, b and d given in Table I (line E) and corresponding errors were obtained from the above values of P_i.

MUON BEAM CONTAMINATION

The possibility that μ meson scatterings where taken as π-μ decays is excluded in our experiments ([7]). Indeed:

1) Each event accepted was looked three times and examined for characteristic change of ionization and coulomb scattering. First with objectives 25X and eye pieces 15X and then with objective 100X and eye pieces 15X. In the second time depth and angles of positron and μ mesons with the direction of π beam were measured. In the third time the projected angle between π and μ was measured. Each event was looked in the second or third time by an experienced physicist. In no case a μ scattering was found to be taken as π-μ decay.

2) All μ lenghts were approximately measured. Thus if a μ-scattering was taken as a π-μ decay, the scattering had to occur at about 600 μ of the μ track's end. However, the estimated contamination of μ mesons in our stack was about 5%. Thus, as shown in reference ([5]) the fraction of muons scattered by angle great than 5° at approximately 600 μ is negligibly small and would lead to a negligible correction.

DISTORSION IN THE STACK

In ([5]), ([6]) and ([7]) the $dN/d\vartheta$ distribution was obtained for a total of 18993 particles from contamination stars.

The results give a probability of 24% (χ^2 with three degrees of freedom) for it to be isotropic. This may be taken as an indication that distorsion in the stack is not significant.

COMPARISON OF THE EXPERIMENTS

Table V gives the probabilities for compatibility of the several experiments among themselves, with isotropy and with the α stars distribution. They were obtained from the values of χ^2 for three degrees of freedom.

TABLE V

Exp.	E	L	H	T_1	T_2	α
Isot.	56%	0.001%	0.06%	0.46%	14.5%	24%
α	66%	0.001%	0.03%	0.16%	14.0%	
T_2	29%	0.008%	2.4%	12.5%		
T_1	8.5%	2.7%	61.5%			
H	12.5%	16%				
L	0.04%					

We see that experiment L is incompatible with T_2, E and hardly compatible with T_1, and thus must be discarded. We see also that experiments T_2 and E are the only ones which are compatible with isotropy, and with the α distribution.

As we wish to analise if the anisotropy is due to a genuine physical effect or if it may have been originated from some uncorrected bias in the experiments, we first combine the two experiments which are more sure to be free of such biases, that is, experiments T_2 and E.

As pointed out in reference ([6]) the greatest danger of loss concerns π-μ decays with small projected π-μ angles, leading to smaller X_1 value.

This loss may not be completely corrected for by the double scanning procedure and the uncorrected loss may be larger then the estimation made in reference ([5]). Such a loss is significant in the π-μ vertex scanning, say for the L and H experiments as in these cases if a π-μ vertex was taken as a μ meson it is lost. In the other experiments it may be found when we return 600 μ back from the μ-end. A strong indication that the double scanning procedure analised in reference ([5]) did not correct all bias losses in L and H comes from the fact that the "corrected" results of experiment L are, as indicated in Tavle V not compatible with experiment T_2 and hardly compatible with experiment T_1 of the same workers. In the same way H is not compatible ([9]) with T_2.

It was shown in ([5]) that in the T_2 experiment the loss was smaller than 1% of the total π-μ decays. This is also true for the E experiment, as in both only μ's completely contained in the same plate were accepted. However for the T_1 experiment, where μ's leaving the plate were accepted and followed, the analysis of ([5]) is not applicable as the grain counting results for flat μ-meson cannot be extrapolated to the steeper ones of this experiment.

Thus we find no justification for taking T_1 in the same foot as T_2 ([10]). On the contrary T_1 is more compatible with H, although the methods are completely different. Thus we separately combine the results of T_2 and E (29% probability of compatibility) and the results of H and T_1 (61.5% probability of compatibility). Table VI gives the probabilities for compatibility of those results among themselves, with isotropy and with α distribution, obtained from the values of χ^2 for three degrees of freedom.

TABLE VI

Exp.	Isotropy	α	H + T_1	H	T_1
E + T_2	31%	35%	1%	3%	7%
H + T_1	<0.001%	<0.001%			

Table VII gives the values of b, d and a coefficients for the several cases. We should mention that coefficient a was introduced to characterize the lack of events in X_1 interval; the factor $2/\sqrt{3}$ was chosen to make the error in a of the same order as those in b and d.

Δb, Δd and Δa given in Table VII are the differences of b, d and a of considered cases and those of E + T_2.

TABLE VII

Exp.	$b \times 10^3$	$d \times 10^3$	$a \times 10^3$	$\Delta b \times 10^3$	$\Delta d \times 10^3$	$\Delta a \times 10^3$
E + T_2	+2 ± 32	-34 ± 32	+10 ± 33	- -	- -	- -
H + T_1	-113 ± 29	-110 ± 29	-124 ± 28	115 ± 43	76 ± 43	134 ± 43
H	-95 ± 38	-124 ± 38	-134 ± 36	97 ± 50	90 ± 50	144 ± 49
T_1	-143 ± 48	-88 ± 48	-108 ± 46	145 ± 58	54 ± 58	118 ± 57

We see from Table VI and VII that the combination H + T_1 is not compatible with E + T_2, not only because the probability of compatibility obtained from χ^2 is $\simeq$ 1% but also because Δa and Δb are respectively 3.1 and 2.7 standard deviations.

As for H and T_1 separately which have a small probability of compatible with E + T_2 because Δa for H and Δb for T_1 are respectively 2.9 and 2.5 standard deviations.

We thus conclude that it is not satisfactory to combine either H or T_1 with E + T_2, T_2 and E being the only experiments on this stack which are surely free of bias.

CONCLUSIONS

If we accept the conclusion of the previous section we must use only $T_2 + E$ as the result of the analysis of π-μ decay angular distribution for the stack under consideration. Thus we come to the conclusion that these results ($T_2 + E$) are in good agreement with isotropy as seen from Table VI and from values of a, b and d in Table VII. The agreement of $T_2 + E$ with the α distribution in the same stack, which should be isotropic, is also shown in Table VI. These results ($E + T_2$) are now compatible with the μ- distribution obtained in reference (1) (40% probability from χ^2 with three degrees of freedom) which was in good agreement with isotropy. But now the question is raised of why the results of (1), with π-μ vertex scanning, corrected for scanning efficiency, should be more reliable then those of L and H. The losses with this method depend on the training of the scanners, the rapidity of the scanning, the conditions of the development of the stack, type of emulsion, optical equipment used and on the awareness of the scanners that they may loose a certain kind of events. Thus it is possible that some of these factors are responsible for the increasing isotropy in the succession of experiments L, H and of reference (1).

Summing up our conclusions we may state that:

1) The results of the Hulubei stacks (5) indicate there is some residual uncorrected bias for experiments L, H, T_1, that make them not compatible with experiment T_2+E which is free of bias.

2) The results of experiment E+ T_2, which seems to be safe part of the scanning of this stack, are in good agreement with isotropy and with the μ- distribution of reference (1).

Therefore there is no remaining indication of anisotropy in π-μ decay in such experiments and no need to appeal to unknown differences in the π-μ history to explain different forms of angular distributions.

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(9) Not only the probability for compatibility of H and T_2 from χ^2 is near 2.4% but the difference of their a-values (Table II) is 2.9 standard deviations.

(10) Experiment T of reference (1) is the combination of T_1 and T_2. These however, although having a probability for compatibility from χ^2 of 12.5%, have coefficients a differing by 2.3 standard deviations (Table III).

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