Istituto Nazionale di Fisica Nucleare
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V. Berzi[x] and E. Recami[o]: A MODEL FOR $p\bar{p}$ ANNIHILATION IN FLIGHT WITH MULTIPION PRODUCTION[+]. -

SUMMARY -

In this work we deal with a "nucleon-exchange" model for $p\bar{p}$ annihilation in flight into pions. The aim of the model is to explain the observed fact that, in the C.M. system, charged mesons seem to prefer the direction of the nucleon of equal charge.

As suggested by S. Minami[9], it is assumed that a virtual annihilation, with multipion production (treated in the spirit of the statistical model), be preceded by a peripheral emission of one pion both by nucleon and antinucleon.

We have taken into proper account the conditions that Lorentz and isospin invariances impose on the structure both of the contributions of the peripheral-emission vertices, and of the virtual-annihilation amplitude.

A final formula for π^+ (or π^-) angular distribution is given. With the help of some physical simplifying considerations, this formula is reduced to a numerically evaluable one (by using some phase-space techniques), and the results are compared with the available experimental data at the entering laboratory-momenta of 1.6 Gev/c, 3.3 GeV/c and 5.7 GeV/c.

Despite of the fact that our model neglects resonance production, a satisfactory enough accord has been found. Some Appendices conclude this work.

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1 - INTRODUCTION -

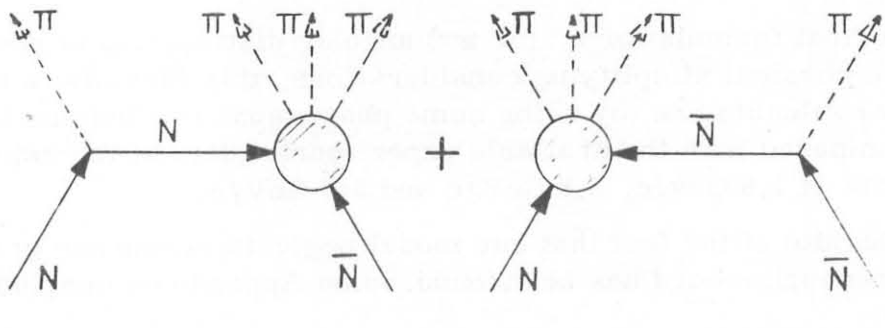
Recently it has been observed that the charged mesons emitted from annihilation in flight of antiprotons, with a laboratory momentum of a few GeV/c, have a rather definite orientation with respect to the incoming particles[1, 2]: negatively charged mesons prefer small CM angles with the antiproton momentum direction; positively charged mesons prefer large ones[3]. That is to say, charged mesons from $p\bar{p}$ annihilation in flight seem to prefer the direction of the nucleon of equal charge, in contrast with a purely statistical model.

A mechanism for producing angular asymmetries of annihilation mesons is easily estabilished in the Koba-Takeda model[5, 6]. In this model, the $p\bar{p}$ annihilation is considered to proceed via a "core" annihilation (treated for instance according to the statistical theory), coupled to the dispersion of the pion clouds without further interactions.

Thus, in $p\bar{p}$ annihilation, the proton cloud, in which positive charge dominates[7], continues its forward flight in the global center-of-mass, as the antiproton cloud does, in which negative charge dominates.

For a quantitative treatment, one might assume that cloud mesons are emitted isotropically in the rest-frame of their mother nucleon. But Pilkuhn[8] observed that the statistical "core"-annihilation probability becomes then a complicated function of the CM angles and momenta of the cloud mesons, which is difficult to calculate numerically.

Therefore Pilkuhn tried[8] to obtain the asymmetries working with a "pole model", one pole being associated to a <u>peripheral</u> emission of one meson (see fig. 1, where it is illustrated the case with <u>one</u> "peripheral" meson). His conclusions were against the assumption of exactly one pole



<u>FIG. 1</u> - Graphs of the "pole model", as considered by H. Pilkuhn[8].

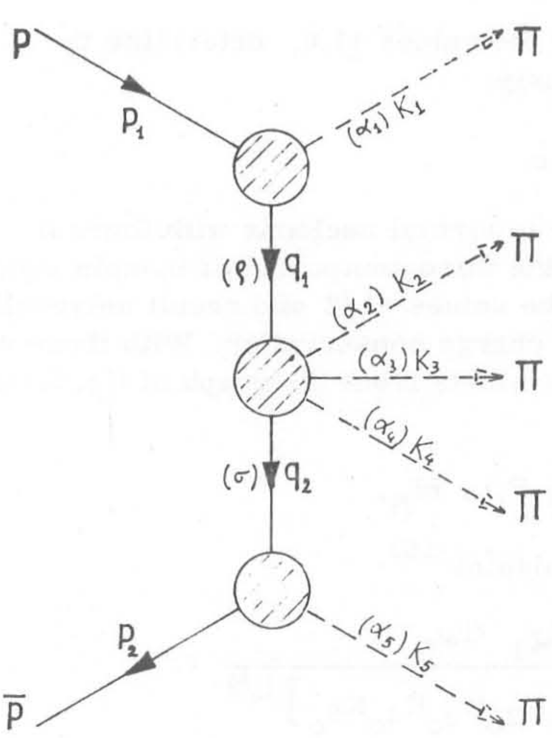


FIG.2 - Our model for $p\overline{p} \to 5\pi$.

for all π-multiplicities, and they were doubtful about the case of postulating exactly two poles for all multiplicities.

Recently Minami[9] proposed that the $p\overline{p}$ annihilation be dominated by a graph with two internal nucleon-lines (as in fig. 2). Thus, here we consider a model, in which a virtual annihilation, with statistical[10] multipion production, is preceded by a peripheral symmetric emission of one pion by both nucleon and antinucleon. Roughly, *a priori* one expects that this "peripheroid" model explain qualitatively the main physical characteristics of the pionic CM angular distributions in $p\overline{p}$ annihilation, expecially if one bears in mind the experimental observation that the F/B asymmetry increases as the total energy increases and as the pion multiplicity decreases. Moreover, for every multiplicity, both the anisotropy and the asymmetry in the angular distribution of the charged pions are experimentally due mainly to the pions emitted with greater impulse.

In particular[11], for $pp \to 5\pi$, the model assumes the diagram of fig. 2.

It was tempting to analize carefully the consequence of the model, taking into account all kinematic coefficients and spin and isospin factors. Obviously our model, in this version, does not consider resonance production.

2 - GENERAL FORMULATION OF THE MODEL -

We consider in this work the particular process:

$$p\,\overline{p} \to \pi^+\pi^+\pi^-\pi^-\pi^o, \tag{1}$$

for which good experimental information (i. e. a good statistics) is available[1, 12, 13, 14, 15]. We want to study the contributions to the transition matrix elements ($T \equiv S-1$), for this process, arising from graphs with the same structure of the one of fig. 2.

In fig. 2 the tetravectors $p_1, p_2, K_1, K_2, K_3, K_4, K_5$ are the fourmomenta of the corresponding (entering or outgoing) particles, while the

$\alpha_i (i=1,\dots,5)$ indices, which can assume the values $\pm 1, 0$, determine the charge state of the outgoing pions. Obviously:

$$\sum_{i=1}^{5} \alpha_i = 0. \tag{2}$$

The two internal lines of the graphs refer to virtual nucleons with fourmomenta $q_1 = p_1 - K_1$ and $q_2 = K_5 - p_2$ and with the third component of isospin equal to ρ and σ. The ρ and σ may assume the values $\pm 1/2$ and result univocally determined if we fix α_1 and α_5, owing to charge conservation. With those notations, the contribution to the T-matrix element from the graph of fig. 2 can be written:

$$\langle f | T | i \rangle = \delta^{(4)}(P_f - P_i) \cdot M_{fi}, \tag{3}$$

where, applying the standard rules, one obtains[16]:

$$M_{fi} = -\frac{m}{(2\pi)^{21/2}} \cdot \frac{G_{\alpha_1} \, G_{\alpha_5}}{\left[32 \, p_{1_0} p_{2_0} K_{1_0} K_{2_0} K_{3_0} K_{4_0} K_{5_0}\right]^{1/2}} \cdot \tilde{v}(\vec{p}_2) \, \gamma_5 \, \frac{\gamma \cdot q_2 + m}{q_2^2 - m^2} \, \mathcal{A}^{\alpha/\rho\sigma}(q_1 q_2 K) \, \frac{\gamma \cdot q_1 + m}{q_1^2 - m^2} \, \gamma_5 \, w(\vec{p}_1). \tag{4}$$

In formula (4):

a) - $G_{\pm 1} = \sqrt{2}\, G$; $G_o = G$, G being the $(pp\pi^o)$ coupling constant[17].

b) - m is the nucleon mass.

c) - $w(\vec{p}_1)$ is a positive-energy spinor with momentum $\vec{p}_1$ and $v(\vec{p}_2)$ is a negative-energy spinor with momentum $-\vec{p}_2$, satisfying the equations: $(\gamma \cdot p_1 - m) w(\vec{p}_1) = (\gamma \cdot p_2) + m) v(\vec{p}_2) = 0$; the adopted normalization is: $\tilde{w}(\vec{p}_1) w(\vec{p}_1) = 1$, and $\tilde{v}(\vec{p}_2) v(\vec{p}_2) = -1$; the elicity indices are understood.

d) - $\alpha \equiv (\alpha_2, \alpha_3, \alpha_4)$ and $K \equiv (K_2, K_3, K_4)$.

e) - $\mathcal{A}^{\alpha/\rho\sigma}(q_1 q_2 K)$ is a 4x4 matrix in the Dirac-spinor space, and has the proper Lorentz and isospace transformation properties.

Bearing in mind that the intrinsic parity of a π is -1, the Lorentz structure of $\mathcal{A}$ is assumed to be the following one:

$$\mathcal{A}^{\alpha/\rho\sigma}(q_1 q_2 K) = \gamma_5 \, A^{\alpha/\rho\sigma}(q_1 q_2 K), \tag{5}$$

where $A^{\alpha/\rho\sigma}(q_1 q_2 K)$ is a Lorentz scalar.

Keeping into account its transformation properties for isorotation, we can write (see fig. 3):

(6) $$A^{\alpha/\rho\sigma}(q_1q_2K) = \sum_{0}^{1}{}_{\nu,T}\langle\alpha|T,\nu\rangle A^{T,\nu}(q_1q_2K)\langle T|\rho,-\sigma\rangle$$

The $\langle\rho,\sigma|T\rangle$ and $\langle\alpha|T,\nu\rangle$ are the coefficients of the decomposition into eighenstates of total isospin T and its third component (which is not explicitly written), respectively for a state formed by two particles with isospin 1/2 and third components ρ, $-\sigma$, and for a state formed by three particles with isospin 1 and third components α_2, α_3, α_4.

It is well known that in the second case the total isotopic spin and its 3rd component are not enough to single out the decomposition terms, and it is necessary to introduce a third quantum number ν, that appears in formula (6).

At this point, as more detailed dynamic informations are lacking, we set down the "statistical"(18, 23) hypothesis that $A^{T,\nu}(q_1q_2K)$ be independent of all those variables on which a priori it should depend, writing:

(7) $$|A^{T,\nu}(q_1q_2K)|^2 = \Lambda^4,$$

where Λ is a constant, with the dimensions of a lenght, which -if one takes the model seriously- will result to be, e.g., about 8 fm for an entering laboratory momentum of 5.7 GeV/c. (Actually, as we do not concern ourselves with different-multiplicity processes, the Λ-parameter introduction is not strictly necessary). With our assumptions, we get:

(8) $$\left|A^{\alpha/\rho\sigma}(q_1q_2K)\right|^2 = \Lambda^4 I^{\alpha}_{\rho,-\sigma} + \text{interferential terms},$$

having set:

(9) $$I^{\alpha}_{\rho,-\sigma} = \sum_{\nu,T}|\langle\alpha|T,\nu\rangle|^2|\langle T|\rho,-\sigma\rangle|^2.$$

Let us consider the reaction:

(10) $$p(p_1)+\bar{p}(p_2)\rightarrow\pi(K_1\alpha_1)+\pi(K_2\alpha_2)+\pi(K_3\alpha_3)+\pi(K_4\alpha_4)+\pi(K_5\alpha_5);$$

the contribution of the graph of fig. 2 to the differential cross-sections, averaged on the entering nucleon helicities, is given, if we neglect the interference terms in (8), by

$$d\sigma = \Lambda^4 \cdot \frac{G_{\alpha_1}^2 \; G_{\alpha_5}^2}{(2\pi)^{19}} \cdot \frac{m^2\, \delta^{(4)}(p_1+p_2 - \sum_{i=1}^{5} K_i)}{\left[(p_1 \cdot p_2)^2 - m^4\right]^{1/2}} \cdot$$

(11) $$\cdot \frac{I^{\alpha}_{\beta,-\sigma}}{(q_1^2-m^2)^2 (q_2^2-m^2)^2} \cdot \frac{1}{4} \sum_{(\text{helicities})} \left| \tilde{v}(\vec{p}_2)\, \gamma_5 (\gamma \cdot q_2 + m) \cdot \right.$$

$$\left. \cdot\, \gamma_5 (\gamma \cdot q_1 + m)\, \gamma_5 w(\vec{p}_1) \right|^2 \frac{d\vec{K}_1}{2K_{1_0}} \frac{d\vec{K}_2}{2K_{2_0}} \frac{d\vec{K}_3}{2K_{3_0}} \frac{d\vec{K}_4}{2K_{4_0}} \frac{d\vec{K}_5}{2K_{5_0}} .$$

We find easily[19]:

$$\frac{1}{4} \sum_{(\text{helicities})} \left| \hat{v}(\vec{p}_2)\, \gamma_5 (\gamma \cdot q_2 + m)\, \gamma_5 (\gamma \cdot q_1 + m)\, \gamma_5 w(\vec{p}_1) \right|^2 =$$

(12) $$= \frac{1}{16\, m^2} \cdot \mathrm{Tr} \left\{ (\gamma \cdot q_2 - m)(\gamma \cdot q_1 + m)(\gamma \cdot p_2 + m)(\gamma \cdot q_2 + m) \cdot \right.$$

$$\left. \cdot\, (\gamma \cdot q_1 - m)(\gamma \cdot p_1 + m) \right\} = \frac{1}{16\, m^2} \cdot F(p_1 p_2 K_1 K_5).$$

The explicit trace expression is:

$$F(p_1 p_2 K_1 K_5) = 4 \left\{ m^6 + m^4 (p_1 \cdot p_2 - q_1^2 - q_2^2) + m^2 \left[2(q_1 \cdot q_2) \cdot (q_1 q_2 - 2p_1 \cdot p_2 - p_1 \cdot q_1 - \right. \right.$$

$$- p_1 \cdot q_2 + p_2 \cdot q_1 + p_2 \cdot q_2) - 2\,(p_1 \cdot q_1 - p_1 \cdot q_2)(p_2 \cdot q_1 - p_2 \cdot q_2) + q_1^2 (p_1 \cdot p_2 -$$

$$\left. - 2p_2 \cdot q_2) + q_2^2 (p_1 \cdot p_2 + 2p_1 \cdot q_1 - 2p_2 \cdot q_1 - q_1^2) \right] - 2\,(p_1 \cdot q_1)(q_1 \cdot q_2)(p_1 \cdot q_2 +$$

$$+ p_2 \cdot q_2) + 2q_1^2 (p_1 \cdot q_2)(p_2 \cdot q_2) + 2q_2^2 (p_1 \cdot q_1)(p_2 \cdot q_1) + 2(p_1 \cdot p_2)(q_1 \cdot q_2)^2 -$$

$$\left. - q_1^2\, q_2^2\, (p_1 \cdot p_2) \right\}.$$

Some details of this evaluation are given in APPENDIX B, while in APPENDIX A we give the meaning of the invariants one meets with in this trace calculation.

If we put:

$$G(1,5) \equiv G(p_1 p_2\, K_1 K_5) \equiv \frac{F(p_1 p_2\, K_1 K_5)}{(q_1^2 - m^2)^2 (q_2^2 - m^2)^2},$$

where $q_1 = p_1 - K_1$ and $q_2 = K_5 - p_2$, we get:

$$(13) \qquad d\sigma = \Lambda^4 \frac{G^2_{\alpha_1} G^2_{\alpha_5}}{16\,(2\pi)^{19}} \cdot \frac{G(1,5)\, I^{\alpha}_{\varrho,-\sigma}}{\sqrt{(p_1 \cdot p_2)^2 - m^4}} \cdot \delta^{(4)}(p_1 + p_2 - \sum_{i=1}^{5} K_i)\, \frac{d\vec{K}_1}{2K_{1_o}} \cdots \frac{d\vec{K}_5}{2K_{5_o}} .$$

Let us now restrict ourselves to reaction (1), that is:

$$(14) \qquad p(p_1) + \bar{p}(p_2) \rightarrow \pi^+(K_1) + \pi^+(K_2) + \pi^o(K_3) + \pi^-(K_4) + \pi^-(K_5),$$

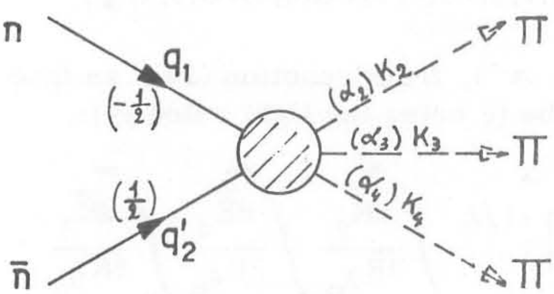


FIG. 3 - The statistical "core" annihilation here considered. Note that with our conventions: $\sigma' = -\sigma$; $q'_2 = -q_2$.

and consider for that process the contributions of the twelve graphs that one can obtain by exchanging the identical-particle momenta one another in the three diagrams of fig. 4.

With the same procedure used for the diagram of fig. 2 (and with the same approximation), we will evaluate the contribution of those graphs to the transition rate and to the cross-section.

Then, if we neglect the intereference terms between the various graph contributions, we get, for the differential cross-section of reaction (14), the expression:

$$d\sigma = \frac{\Lambda^4 G^4}{16\,(2\pi)^{19}} \Big\{ 4\, I^{1,0,-1}_{-\frac{1}{2},\frac{1}{2}} \Big[G(1,5) + G(2,5) + G(1,4) + G(2,4) \Big] + $$

$$+ 2\, I^{1,-1,-1}_{-\frac{1}{2},-\frac{1}{2}} \Big[2\,G(1,3) + 2\,G(2,3) \Big] + 2\, I^{1,1,-1}_{\frac{1}{2},\frac{1}{2}} \Big[2\,G(3,5) + $$

$$+ 2\,G(3,4) \Big] \Big\} \cdot \frac{\delta^{(4)}(p_1 + p_2 - \sum_{i=1}^{5} K_i)}{\sqrt{(p_1 \cdot p_2)^2 - m^4}} \cdot \frac{d\vec{K}_1}{2K_{1_o}} \cdots \frac{d\vec{K}_5}{2K_{5_o}} ,$$

with, in general:

$$(16) \qquad G(j,h) \equiv G(p_1 p_2 K_j K_h) \equiv \frac{F(p_1 p_2\, K_j\, K_h)}{(q_1^2 - m^2)(q_2^2 - m^2)} ,$$

where now $q_1 = p_1 - K_j$ and $q_2 = K_h - p_2$, and with:[20]

$$I^{1,0,-1}_{-\frac{1}{2},\frac{1}{2}} = \frac{1}{6}\cdot\frac{1}{2} + \frac{2}{5}\cdot\frac{1}{2} = \frac{17}{60}; \quad I^{1,-1,-1}_{-\frac{1}{2},-\frac{1}{2}} = I^{1,1,-1}_{\frac{1}{2},\frac{1}{2}} = \frac{3}{5}. \tag{16'}$$

Therefore, with our assumptions, if we define:

$$\begin{aligned} H(p_1 p_2\, K_1 \ldots K_5) \equiv\ & \frac{17}{15}\Big[G(1,5)+G(2,5)+G(1,4)+G(2,4)\Big] + \\ & + \frac{12}{5}\Big[G(1,3)+G(2,3)+G(3,5)+G(3,4)\Big], \end{aligned} \tag{17}$$

the C.M. angular distribution of π^+ (or π^-), from reaction (14), as function of the scattering-angle cosine, will be (c being the light velocity):

$$\begin{aligned} \frac{1}{c}\cdot\frac{d\sigma^+}{d\cos\theta_1} = \frac{\Lambda^4 G^4}{8(2\pi)^{18}}\cdot\Big[s\Big(\frac{s}{4}-m^2\Big)\Big]^{-1/2}\cdot \int_{-\infty}^{\infty}\frac{d\vec{K}_2}{2K_{2_o}}\int_{-\infty}^{\infty}\frac{d\vec{K}_4}{2K_{4_o}}\int_{-\infty}^{\infty}\frac{d\vec{K}_5}{2K_{5_o}}\cdot \\ \int_0^{\infty}\frac{d|\vec{K}_1|\,|\vec{K}_1|^2}{2K_{1_o}}\cdot\frac{\delta(\sqrt{s}-\sum_{i=1}^{5}K_{i_o})}{2K_{3_o}}\cdot H(p_1p_2K_1\ldots K_5)\Big|_{\vec{K}_3=-\vec{K}_1-\vec{K}_2-\vec{K}_4-\vec{K}_5} \end{aligned} \tag{18}$$

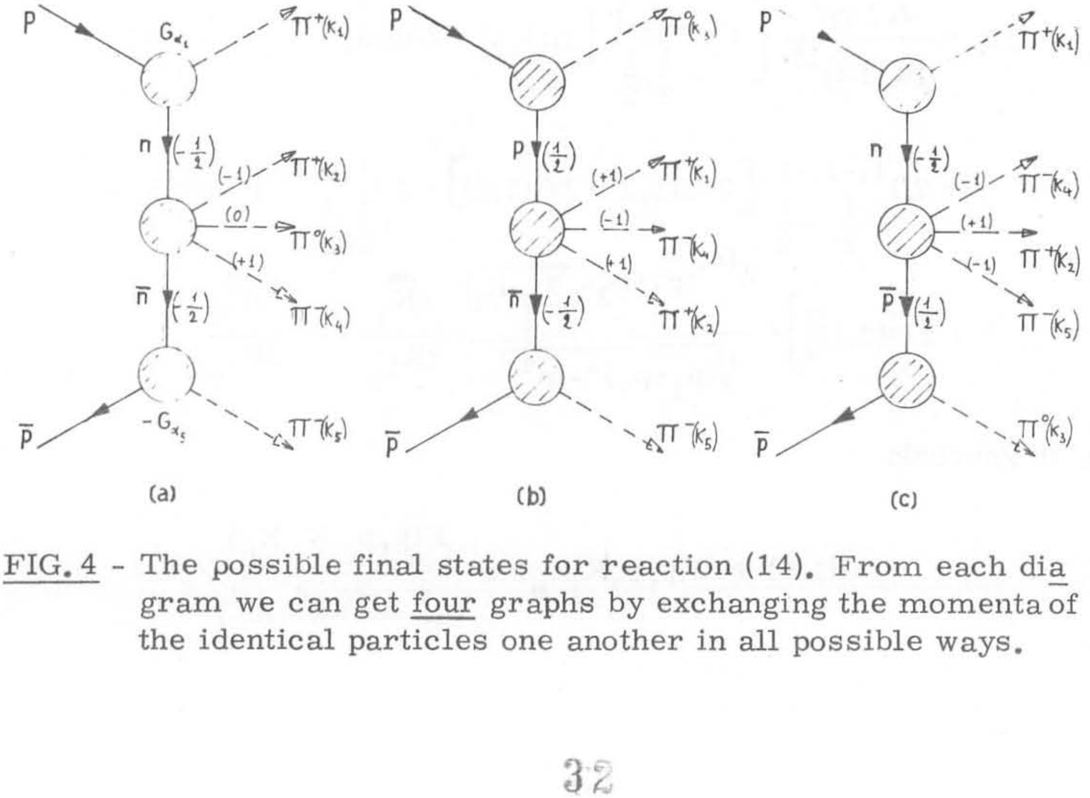


FIG. 4 - The possible final states for reaction (14). From each diagram we can get four graphs by exchanging the momenta of the identical particles one another in all possible ways.

3 - NUMERICAL EVALUATION AND COMPARISON WITH EXPERIENCE -

If we put, for every function G(j, h) entering in formula (17):

$$\mathcal{J}_{j,h} = \int \frac{d\vec{K}_2}{2K_{2_0}} \int \frac{d\vec{K}_3}{2K_{3_0}} \int \frac{d\vec{K}_4}{2K_{4_0}} \int \frac{d\vec{K}_5}{2K_{5_0}} G(j,h)\, \delta(\sqrt{s} - \sum_{i=1}^{5} K_{i_0})\, \delta^{(3)}(\sum_{i=1}^{5} \vec{K}_i), \tag{19}$$

it is immediate to see that:

$$\mathcal{J}_{1,5} = \mathcal{J}_{1,4} = \mathcal{J}_{1,3} \equiv \mathcal{J}_1(s, |\vec{K}_1|, \cos\theta_1);$$

and

$$\mathcal{J}_{2,5} = \mathcal{J}_{2,4} = \mathcal{J}_{2,3} = \mathcal{J}_{3,5} = \mathcal{J}_{3,4} \equiv \mathcal{J}_2(s, |\vec{K}_1|, \cos\theta_1).$$

Therefore, formula (18) may be rewritten as follows:

$$\frac{d\sigma^+}{d\cos\theta_1} = \Lambda^{(4)} \cdot \alpha(s) \cdot \int_0^\infty \frac{d|\vec{K}_1|\, |\vec{K}_1|^2}{2x} \cdot \mathcal{J}(s, |\vec{K}_1|, \cos\theta), \tag{20}$$

where:

$$\begin{cases} x \equiv K_{1_0};\ \cos\theta = \cos\theta_1; \\ \alpha(s) = \dfrac{G^4}{8(2\pi)^{18}} \cdot \left[s\left(\dfrac{s}{4} - m^2\right)\right]^{-1/2}; \\ \mathcal{J}(s, |\vec{K}_1|, \cos\theta) = \dfrac{14}{3}\mathcal{J}_1 + \dfrac{142}{15}\mathcal{J}_2 . \end{cases} \tag{21}$$

Thus the problem has been reduced to the evaluation of the two integrals $\mathcal{J}_j$(j=1,2). One may write, applying the <u>generalized mean-value theorem</u>, that:

$$\mathcal{J}_j = \int \frac{d\vec{K}_2}{2K_{2_0}} \int \frac{d\vec{K}_3}{2K_{3_0}} \int \frac{d\vec{K}_4}{2K_{4_0}} \left[G(j,5)\right]\Bigg|_{\vec{K}_5 = \vec{K}_5^{(m)}} \int \frac{d\vec{K}_5}{2K_{5_0}} \cdot \tag{22}$$

$$\cdot\, \delta(\sqrt{s} - \sum_{i=1}^{5} K_{i_0})\, \delta^{(3)}(\sum_{i=1}^{5} \vec{K}_i), \qquad (j = 1,2)$$

where m stays for <u>mean</u>.

Let us consider firstly the integral $\mathcal{J}_1$. In order to be able to evaluate it numerically, we make the assumption, apparently reasonable on physical basis, that (for every fixed K_1):

$$\vec{K}_5^{(m)} = -\vec{K}_1 ; \tag{23}$$

which is equivalent, more in general, to substitute

$$\vec{q}_2 = \vec{q}_2^{(m)} = \vec{q}_1 ; \qquad q_{2_o} = q_{2_o}^{(m)} = -q_{1_o} \tag{24}$$

into the function G(1, 5) (see also the (1A)).
If we put:

$$\left\{\begin{aligned} & G(s, x, \cos\theta) = G(1,5)\Big|_{\vec{K}_5 = -\vec{K}_1} ; \\ & \mathcal{E} = \sqrt{s} - x ; \\ & R_4(\vec{Q},\mathcal{E}) = \int \frac{d\vec{K}_2}{2K_{2_o}} \int \frac{d\vec{K}_3}{2K_{3_o}} \int \frac{d\vec{K}_4}{2K_{4_o}} \int \frac{d\vec{K}_5}{2K_{5_o}} \, \delta\Big(\sum_{i=2}^{5} K_{i_o} - \mathcal{E}\Big) \, \delta^{(3)}\Big(\sum_{i=2}^{5} \vec{K}_i - \vec{Q}\Big), \end{aligned}\right. \tag{25}$$

then we have:

$$\mathcal{J}_1 = G(s, x, \cos\theta) \, R_4(-\vec{K}_1, \mathcal{E}). \tag{26}$$

The four-body "phase-space" (for equal mass particles), R_4, can be easily calculated[21, 22]. Let us set:

$$\begin{aligned} & \mathcal{E}_5 \equiv \sqrt{s} ; \quad \mathcal{E}_n = \sqrt{\mathcal{E}^2_{n+1} - 2x_{n+1}\mathcal{E}_{n+1} + \mu^2} ; \qquad (n = 2,3,4) \\ & x_5 \equiv x \equiv K_{1_o} ; \quad x_{n+1} = \frac{\mathcal{E}^2_{n+1} - (n^2-1)\mu^2}{2\mathcal{E}_{n+1}} . \end{aligned} \tag{27}$$

As R_4 is <u>Lorentz-invariant</u>, it will be:

$$R_4(-\vec{K}_1, \mathcal{E}) = R_4(\vec{0}, \mathcal{E}_4). \tag{28}$$

And, using a simple recurrence relation[23], we have (μ = pion mass):

$$(29)\quad R_4(0,\mathcal{E}_4) = \int \frac{d\vec{K}}{2K_o} \cdot \theta(\mathcal{E}_3 - 3\mu) \cdot R_3(0,\mathcal{E}_3) = 2\pi \int_{\mu}^{X_4} dx_4 \sqrt{x_4^2 - \mu^2}\; R_3(0,\mathcal{E}_3),$$

where the explicit expression of the Lorentz-invariant three-body "phase-space" (for equal mass particles), R_3, is well-known[21]:

$$(30)\quad R_3(0,\mathcal{E}_3) = \pi^2 \int_{\mu}^{X_3} dx_3 \sqrt{x_3^2 - \mu^2} \left[\frac{\mathcal{E}_2^2 - 4\mu^2}{\mathcal{E}_2^2} \right]^{1/2} .$$

Therefore, we may rewrite the (26) as follows:

$$(31)\quad \mathfrak{I}_1 = 2\pi^3 G(s,x,\cos\theta) \int_{\mu}^{X_4} dx_4 \sqrt{x_4^2 - \mu^2} \int_{\mu}^{X_3} dx_3 \sqrt{x_3^2 - \mu^2} \left[\frac{\mathcal{E}_2^2 - 4\mu^2}{\mathcal{E}_2^2} \right]^{1/2}$$

and we get in conclusion:

$$I_1(s,\cos\theta) = \frac{7}{3} \cdot \alpha(s) \cdot \int_0^{\infty} dx_5 \sqrt{x_5^2 - \mu^2} \cdot \theta(x_5 - \mu) \cdot \theta(\mathcal{E}_4 - 4\mu) \cdot \mathfrak{I}_1(s,x,\cos\theta) =$$

$$(32)\quad = \frac{7}{3} \cdot \alpha(s) \cdot \int_{\mu}^{X_5} dx_5 \sqrt{x_5^2 - \mu^2}\; \mathfrak{I}_1(s,x,\cos\theta) .$$

Considering now the integral $\mathfrak{I}_2$ of formula (22), we could proceed as we did for $\mathfrak{I}_1$, assuming in this case:

$$(33)\quad \vec{K}_5^{(m)} = -\vec{K}_2,$$

that is to say, more in general, effecting the substitution (24), for every fixed q_1, into the function G(2,5).
But, as $\mathfrak{I}_2$ relates to the charged pions emitted in the virtual "core" annihilation, one may reasonably assume that it depend only weakly on the direction of $\vec{K}_1$, thus supplying a quasi-isotropic contribution to the charged-pion distribution. We do not make any attempt to evaluate such a "background", but we keep it as an <u>additive fitting-parameter</u>, depending only on the total energy $\sqrt{s}$.

In conclusion, one obtains the following final formula for the charged-pion distribution from reaction (14):

(35) $$\frac{d\sigma}{d\cos\theta} = \Lambda^4 \cdot I_1(s, \cos\theta) + Z(s).$$

The comparison with experimental data has been done for 1.6 GeV/c(1), 3.3 GeV/c(13) and 5.7 GeV/c(14) laboratory-momenta, using an IBM-7040 elaborator.

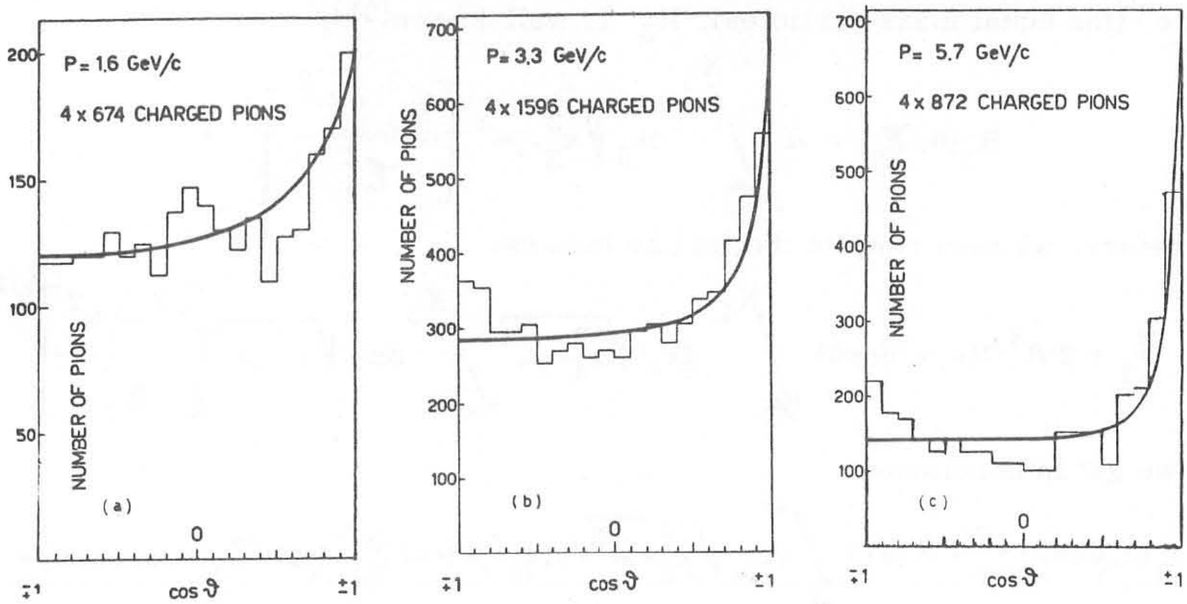


FIG. 5 - C.M. distributions of the (charged) $\pi^{\mp}$ from reaction (14), with respect to the direction of the incoming antiproton, at the three experimentally-available laboratory momenta. The continuous lines are the theoretical curves, yielded by our model. The experimental data are respectively taken:a) from ref.(1), for 1.6 GeV/c; b) from ref.(13),for 3.3 GeV/c; c) from ref.(14), for 5.7 GeV/c.

It is shown in fig. 5. The best fit has been obtained with quite reasonable(18, 23, 24) Λ-values: namely, e.g., Λ= (13.9 ± 0.5) fm for 3.3 GeV/c, and Λ= (7.8 ± 0.3) fm for 5.7 GeV/c. The accord between the theoretical lines, normalized to the charged-pion numbers, and the experimental hystograms(1, 13, 14) is satisfactory enough, except for the backward "tail", which appears at the higher momenta, i.e. at 3.3 and 5.7 GeV/c.

Our model does not keep into account the production of resonances, that seem to appear largely in the more recent data, for the pion-multiplicity here considered (expecially the ρ , which enters very abundantly). A natural modification of the model would be the one represented in fig. 6. But we believe - as it may be argued also a priori - that the CM distributions of the charged pions would not be substancially affected by this change. On the contrary, the aforementioned "backward tail" could possibly be obtained conside

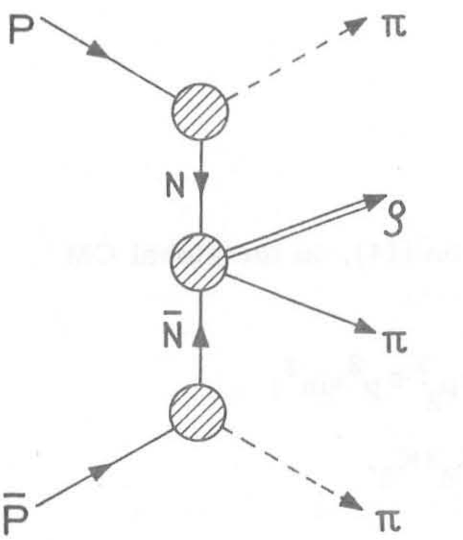


FIG. 6 - A natural modification of the model. We believe that the C. M. charged-pion distributions would not be affected substantially by this changement. Here N means Nucleon.

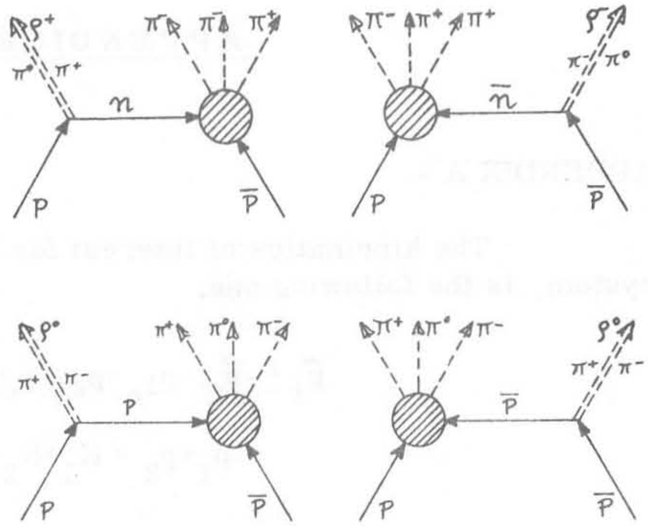


FIG. 7 - Another proposed "model", whose contribution at high energies could possibly explain the "backward tail" we can observe in the charged pion distributions (see in particular fig. 5).

ring also graphs of the type of the ones one gets from fig. 1, substituting a peripheral ρ -emission vertex to the one-pion vertex (see fig. 7). Finally another model, similar with the one shown in fig. 6 but with only one "peripheral" vertex, has been proposed very recently in ref. (14).

AKNOWLEDGEMENTS -

We are greatly indebted to prof. P. Caldirola, for his kind interest, and to prof. F. Duimio, prof. E. Montaldi and prof. S. Ratti for some helpful discussions. We express our gratitude also to Dr. A. Airoldi, for her help in numerical computations, and to Dr. V. Pelosi, who first suggested to us this model.

APPENDICES

APPENDIX A -

The kinematics of interest for reaction (14), in the global CM system, is the following one.

$$\vec{p}_1 = -\vec{p}_2;\ p1_o = p2_o \equiv p_o;\ p_1^2 = p_2^2 \equiv p^2 = m^2;$$

$$p_1 + p_2 = K_1 + K_2 + K_3 + K_4 + K_5.$$

By definition (see fig. 2):

$$q_1 = p_1 - K_1; \qquad q_2 = K_5 - p_2;$$

$$|\vec{p}_1| = |\vec{p}_2| \equiv |\vec{p}| \equiv P.$$

Limiting ourselves to the diagram of fig. 4a, we can choose the variables:

$$s \equiv (p_1 + p_2)^2 = 4p_o^2;$$

$$E_1 \equiv x \equiv K_{1_o};\ E_5 \equiv K_{5_o};$$

$$Q^2 = (K_1 + K_5)^2;$$

θ_1, θ_5: scattering angles of pions 1 and 5 relative to the entering antiproton direction.

Then we get:

$$\sqrt{s-4m^2} = 2P;\quad 2\sqrt{\vec{p}^2 + m^2} = \sqrt{s};$$

$$|\vec{K}_1| = \sqrt{E_1^2 - \mu^2};\ |\vec{K}_5| = \sqrt{E_5^2 - \mu^2};$$

$$p_1 \cdot p_2 = \frac{s}{2} - m^2;$$

$$K_1 \cdot p_1 = \frac{E_1\sqrt{s}}{2} + P|\vec{K}_1|\cos\theta_1;$$

$$K_1 \cdot p_2 = \frac{E_1\sqrt{s}}{2} - P|\vec{K}_1|\cos\theta_1;$$

$$K_5 \cdot p_1 = \frac{E_5\sqrt{s}}{2} + P|\vec{K}_5|\cos\theta_5;$$

$$K_5 \cdot p_2 = \frac{E_5\sqrt{s}}{2} - P|\vec{K}_5|\cos\theta_5;$$

$$p_1 \cdot q_1 = m^2 - K_1 \cdot p_1;$$
$$p_2 \cdot q_2 = K_5 \cdot p_2 - m^2;$$
$$p_1 \cdot q_2 = K_5 \cdot p_1 - p_1 \cdot p_2;$$
$$p_2 \cdot q_1 = p_1 \cdot p_2 - K_1 \cdot p_2;$$
$$q_1^2 = m^2 + \mu^2 - E_1 \sqrt{s} - 2P|\vec{K}_1| \cos\theta_1;$$
$$q_2^2 = m^2 + \mu^2 - E_5 \sqrt{s} + 2P|\vec{K}_5| \cos\theta_5;$$
$$q_1 \cdot q_2 = \mu^2 - \frac{Q^2}{2} - p_1 \cdot p_2 + K_5 \cdot p_1 + K_1 \cdot p_2.$$

The assumption:

(1A) $$\vec{q}_1 = \vec{q}_2; \qquad q_{1_o} = -q_{2_o},$$

which is equivalent to set in the present case (see the (23) of the text):

$$\vec{K}_5 = -\vec{K}_1; \quad E_5 = E_1;$$
$$\cos\theta_5 = -\cos\theta_1;$$
$$Q^2 = 4E_1^2 = 4x^2,$$

brings many simplifications.

APPENDIX B -

We want evaluate the spin factor for the first graph (fig. 4a), i. e.:

$$\frac{1}{4} \sum_{r,s}^{1,2} \left| \tilde{v}^r(p_2) O\, w^s(p_1) \right|^2 \equiv \frac{1}{16\, m^2} F(p_1 p_2 K_1 K_5),$$

with:

$$O = \gamma_5 (\not{q}_2 + m)\, \gamma_5 (\not{q}_1 + m)\, \gamma_5.$$

The proceeding is "classic". The explicit expression of the function F of formula (12) is:

$$F(p_1 p_2 K_1 K_5) = \mathrm{Tr} \left\{ (\not{q}_2 - m)(\not{q}_1 + m)(\not{p}_2 + m)(\not{q}_2 + m)(\not{q}_1 - m)(\not{p}_1 + m) \right\} =$$
$$= T_o + T_2 + T_4 + T_6;$$

where first of all:

$$T_o = 4m^6.$$

Observing that:

$$\mathrm{Tr}\, \not{A}\not{B} = 4\, A\cdot B,$$

one then finds:

$$T_2 = 4m^4(p_1\cdot p_2 - q_1^2 - q_2^2).$$

Besides, noting that:

$$\frac{1}{4}\mathrm{Tr}\, \not{A}\not{B}\not{C}\not{D} = (A\cdot B)(C\cdot D) - (A\cdot C)(B\cdot D) + (A\cdot D)(B\cdot C),$$

one gets:

$$T_4 = 4m^2\Big[2(q_1\cdot q_2)(q_1\cdot q_2 - 2p_1\cdot p_2 - p_1\cdot q_1 - p_1\cdot q_2 + p_2\cdot q_1 + p_2\cdot q_2) +$$
$$+ 2\,(q_1\cdot p_2)(p_1\cdot q_2 - p_1\cdot q_1) + 2(q_2\cdot p_2)(p_1\cdot q_2 - p_1\cdot q_2) +$$
$$+ q_1^2(p_1 p_2 - 2p_2\cdot q_2) + q_2^2(p_1\cdot p_2 + 2p_1\cdot q_1 - 2p_2\cdot q_1 - q_1^2)\Big].$$

Finally one has to evaluate:

$$T_6 = \mathrm{Tr}\,(\gamma_\lambda \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_\tau),$$

where we have put:

$$\gamma_\lambda = \gamma_\rho = \gamma\cdot q_2; \quad \gamma_\mu = \gamma_\sigma = \gamma\cdot q_1; \quad \gamma_\nu = \gamma\cdot p_2; \quad \gamma_\tau = \gamma\cdot p_1.$$

With an iterative procedure of the following type:

$$T_6 = 2\, g_{\mu\nu}\,\mathrm{Tr}\,(\gamma_\nu \gamma_\rho \gamma_\sigma \gamma_\tau) - \mathrm{Tr}\,(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_\tau) =$$
$$= 8\, g_{\lambda\mu}\,(g_{\nu\tau}\, g_{\rho\sigma} - g_{\nu\sigma}\, g_{\rho\tau} + g_{\nu\rho}\, g_{\sigma\tau}) -$$
$$- 2\, g_{\lambda\nu}\,\mathrm{Tr}\,(\gamma_\mu \gamma_\rho \gamma_\sigma \gamma_\tau) + \mathrm{Tr}\,(\gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\rho \gamma_\sigma \gamma_\tau) =$$
$$= \ldots,$$

one arrives to the expression:

$$T_6 = 4\Big\{2(q_1\cdot q_2)\Big[(p_1\cdot p_2)(q_1\cdot q_2) - (p_1\cdot q_1)(p_2\cdot q_2) - (p_1\cdot q_2)(p_2\cdot q_1)\Big] +$$
$$+ 2\, q_1^2(p_1\cdot q_2)(p_2\cdot q_2) + 2\, q_2^2(p_1\cdot q_1)(p_2\cdot q_1) - q_1^2 q_2^2(p_1\cdot p_2)\Big\}.$$

From a computative point of view, with the simplifying assumption (1A), one gets in the C.M. (see the (23) of the text):

$$\frac{1}{4}F(p_1 p_2 K_1 K_5) \equiv \frac{1}{4}F(s,x) = \mathcal{A} + \mathcal{B}\cos\theta_1 + \mathcal{C}\cos^2\theta_1 + \mathcal{D}\cos^3\theta_1,$$

where $x = E_1 = K_{1_0}$, and where:

$$\mathcal{A} \equiv m^6 + m^4(t-2d) + m^2(2w^2-2a^2-2c^2-d^2+2cd+2dt+4ac+4ad-4aw-4cw-4wt)+$$
$$+2a^2w+2c^2w+2w^2t-d^2t-4acd;$$

$$\mathcal{B} \equiv b\left[4m^4+m^2(8w-8a-4c-4t-2d)+4ad+4cd+4dt-4aw-4cw+8ac\right];$$

$$\mathcal{C} \equiv 4\,b^2\,(2m^2+w-2a-2c-d-t);$$

$$\mathcal{D} \equiv 8\,b^3,$$

being:

$$a \equiv m^2 - \frac{x\sqrt{s}}{2};$$

$$b \equiv \left[\left(\frac{s}{4}-m^2\right)(x^2-\mu^2)\right]^{1/2};$$

$$c \equiv \frac{\sqrt{s}}{2}(x-\sqrt{s}) + m^2;$$

$$d \equiv m^2 - e;$$

$$e \equiv x\sqrt{s} - \mu^2;$$

$$t \equiv p_1 \cdot p_2 = \frac{s}{2} - m^2$$

$$w \equiv \mu^2 - 2x^2 - t + x\sqrt{s} - 2b\cos\theta_1.$$

Besides, in the adopted approximation:

$$(q_1^2-m^2)(q_2^2-m^2) = (2b\cos\theta_1 + e)^2$$

APPENDIX C -

While considering the kinematics of our reaction with five equal-mass final bodies, we evaluated also the CM "volume", in the impulse-space, of the allowed kinematical region for the three-momentum $\vec{K}_5$ of a final particle, at fixed three-momentum $\vec{K}_1$ of one of the other four final particles.

Owing to its intrinsic interest, we report here that evaluation. We purpose calculating the integral:

$$(1C) \qquad g(s, \vec{K}_1) = \int_{C_1} d\vec{K}_5$$

where $C_1 = C_1(\vec{K}_1)$ is the set of the values of $\vec{K}_5$ for which, at fixed $\vec{K}_1$, the following system ($K_{i_o} = \sqrt{K_i^2 - \mu^2}$):

$$(2C) \qquad \begin{cases} \sqrt{s} - \sum_{i=1}^{5} K_{i_o} = 0, \\ \sum_{i=1}^{5} \vec{K}_i = 0. \end{cases}$$

can be satisfied. That is to say, we have to determine, for each fixed $\vec{K}_1$, the set of the values of $\vec{K}_5$, in corrispondence to which there exist vectors $\vec{K}_2$, $\vec{K}_3$ and $\vec{K}_4$ that satisfy the system (2C).
As we already did elsewhere, often the dependence on s is understood.

Let us firstly notice that, whatever $\vec{K}_5$ be, the second equation of the (2C) can be satisfied, provided that one choose: $\vec{K}_4 = -(\vec{K}_1 + \vec{K}_2 + \vec{K}_3 + \vec{K}_5)$. Thus one is driven to look for the values of $\vec{K}_5$, in corrispondence to which there exist some $\vec{K}_2$ and $\vec{K}_3$ that satisfy the:

$$(3C) \qquad \sqrt{s} - K_{1_o} - K_{2_o} - K_{3_o} - \sqrt{(K_1 + K_5 + K_2 + K_3)^2 + \mu^2} - K_{5_o} = 0.$$

We may undertake a gradual dealing. Firstly, one may look for what conditions we have to impose on $\vec{K}_1$, $\vec{K}_5$ and $\vec{K}_2$ in order that (3C) may be satisfied by some values of $\vec{K}_3$. Those conditions single out a certain region $C(\vec{K}_1 \vec{K}_5 \vec{K}_2)$. Next, one looks for what conditions on $\vec{K}_1$ and $\vec{K}_5$ are necessary to the existence of some values of $\vec{K}_3$, for which $C(\vec{K}_1 \vec{K}_5 \vec{K}_2)$ is not empty. Thus one obtains a new region $C(\vec{K}_1 \vec{K}_5)$; the set of the values of $\vec{K}_5$, for which $C(\vec{K}_1 \vec{K}_5)$ is not empty, will be the integration domain $C_1(\vec{K}_1)$ we are looking for.

To make this program progressing, let us put ($x \equiv K_{1_o}$):

$$
\text{(4C)} \quad \begin{cases}
\mathcal{E} = \sqrt{s} - K_{1_o} = \sqrt{s} - x; \\
A = \mathcal{E} - K_{5_o}; \\
B = A - K_{2_o}; \\
\vec{v} = \vec{K}_1 + \vec{K}_5; \quad v = |\vec{v}|; \\
u = \vec{v} + \vec{K}_2; \quad u = |\vec{u}|; \\
k_i = |\vec{K}_i|, \ (i = 1, \dots, 5).
\end{cases}
$$

The (3C) may be rewritten, setting $z'' = \cos\widehat{\vec{K}_3\vec{u}}$:

$$
\text{(5C)} \qquad B - \sqrt{k_3^2 + \mu^2} + \sqrt{u^2 + k_3^2 + 2uk_3z'' + \mu^2} = 0.
$$

The above-defined region $C(K_1K_5K_2)$ is determined by the condition that the (5C) may be satisfied by some values of k_3 and z'', with $k_3 \geq 0$, $|z''| \leq 1$. One obtains:

$$
\text{(6C)} \qquad C(\vec{K}_1\vec{K}_5\vec{K}_2): \qquad B \geq \sqrt{4\mu^2 + u^2}\ .
$$

More explicitly, if one sets $z' = \cos\widehat{\vec{K}_2\vec{v}}$, one has:

$$
\text{(7C)} \qquad C(\vec{K}_1\vec{K}_5\vec{K}_2): \quad A - \sqrt{k_2^2 - \mu^2} - \sqrt{4\mu^2 + v^2 + k_2^2 + 2vk_2z'} \geq 0.
$$

The request that the (7C) be satisfied by some values of k_2 and z', with $k_2 \geq 0$, $|z'| \leq 1$, picks out the region $C(\vec{K}_1\vec{K}_5)$:

$$
\text{(8C)} \qquad C(\vec{K}_1\vec{K}_5): \quad \begin{array}{l} (A \geq 0, \ \underline{\text{and}}\ A^2 - v^2 - 3\mu^2 \geq 0), \\ \underline{\text{or}}\ (A^2 - v^2 - 3\mu^2 \geq 2\mu\sqrt{A^2 - v^2}). \end{array}
$$

This condition (at fixed $\vec{K}_1$) depends only on k_5 and $z = \cos\widehat{\vec{K}_1\vec{K}_5}$. Let us now identify $\vec{K}_5$ by means of its polar cohordinates k_5, z, ϕ, being ϕ the azymuthal angle with respect to a reference polar-plane passing through $\vec{K}_1$. It is then clear that, for every allowed pair of values of k_5 and z, all the ϕ values are allowed too. Consequently, $C_1(\vec{K}_1)$ is the topological product of the interval $(0, 2\pi)$ and of the set $\overline{C}_1(\vec{K}_1)$, consisting of all the pairs k_5 and z for which at least one inequality (8C) may hold.

Thus one reaches this result: $\overline{C}_1(\vec{K}_1)$ is empty, unless (for $\sqrt{s} > 5\mu$):

(9C) $$x \leq \frac{s - 15\mu^2}{2\sqrt{s}}$$

If the (9C) is verified, $\overline{C}_1(K_1)$ results formed as follows ($x \geq \mu$):

$$\text{(i) if} \quad x \leq \frac{(\sqrt{s}-\mu)^2 - 8\mu^2}{2(\sqrt{s}-\mu)},$$

(10C) $$\overline{C}_1(\vec{K}_1): \quad -1 \leq z \leq z_1(s,x,K_{5_0}); \; \mu \leq K_{5_0} \leq \mathcal{E}_2(s,x),$$

where:

$$z_1(s,x,K_{5_0}) = \begin{cases} 1, \; \underline{\text{when}}: \; \mathcal{E}_1(s,x) \leq K_{5_0} \leq \mathcal{E}_2(s,x), \\ \dfrac{\mathcal{E}^2 - k_1^2 - 8\mu^2 - 2\mathcal{E}K_{5_0}}{2k_1\sqrt{K_{5_0}^2 - \mu^2}}, \; \underline{\text{when}}: \; \mu \leq K_{5_0} \leq \mathcal{E}_1(s,x); \end{cases}$$

$$\text{(ii) if} \quad x \geq \frac{(\sqrt{s}-\mu)^2 - 8\mu^2}{2(\sqrt{s}-\mu)},$$

(11C) $$\overline{C}_1(\vec{K}_1): \quad -1 \leq z \leq z_2(s,x,K_{5_0}); \quad \mathcal{E}_1(s,x) \leq K_{5_0} \leq \mathcal{E}_2(s,x),$$

where:

$$z_2(s,x,K_{5_0}) = \frac{\mathcal{E}^2 - k_1^2 - 8\mu^2 - 2\mathcal{E}K_{5_0}}{2\,k_1\,k_5}.$$

$\mathcal{E}_1(s,x)$ and $\mathcal{E}_2(s,x)$ are the two solutions of the equation:

(12C) $$4\left[(\mathcal{E}-\mu)^2 - k_1^2\right]\mathcal{E}^2 - 4(\mathcal{E}-\mu)\left[(\mathcal{E}-\mu)^2 - k_1^2 - 3\mu^2\right]\mathcal{E} + \left\{\left[(\mathcal{E}-\mu)^2 - k_1^2 - 3\mu^2\right] + 4\,k_1^2\mu^2\right\} = 0;$$

that is to say:

(13C) $$\mathcal{E}_{\substack{1\\2}}(s,x) = \frac{\mathcal{E}(\mathcal{E}^2 - k_1^2 - 8\mu^2)}{2(\mathcal{E}^2 - k_1^2)} \pm \frac{k_1}{2(\mathcal{E}^2 - k_1^2)}\left[(\mathcal{E}^2 - k_1^2 - 8\mu^2)^2 - 4\mu^2(\mathcal{E}^2 - k_1^2)\right]^{1/2}.$$

According to these results, $g(s,\vec{K}_1)$ depends - besides on the total energy-only on k_1:

$$g(s,\vec{K}_1) = \bar{g}(s,x), \tag{14C}$$

and finally we have:

$$\bar{g}(s,x) = 2\pi\,\theta\left(\frac{s-15\mu^2}{\sqrt{s}} - x\right)\left\{\theta\left(\frac{(\sqrt{s}-\mu)^2-8\mu^2}{2(\sqrt{s}-\mu)} - x\right) g_1(s,x) + \theta\left(x - \frac{(\sqrt{s}-\mu)^2-8\mu^2}{2(\sqrt{s}-\mu)}\right) g_2(s,x)\right\}, \tag{15C}$$

with:

$$\bar{g}_1(s,x) = \int_{\mu}^{\mathcal{E}_2(s,x)} dK_{5_o}\, K_{5_o}\sqrt{K_{5_o}^2-\mu^2}\left[z_1(s,x,K_{5_o})+1\right] = \frac{3}{2}(\mathcal{E}_2^2-\mu^2)^{3/2} - \frac{3}{4}(\mathcal{E}_1^2-\mu^2)^{3/2} + \frac{1}{4k_1}\left[(\mathcal{E}^2-x^2-7\mu^2)(\mathcal{E}_1^2-\mu^2) - \mathcal{E}(\mathcal{E}_1^3-\mu^3)\right]; \tag{16C}$$

$$\bar{g}_2(s,x) = \int_{\mathcal{E}_1(s,x)}^{\mathcal{E}_2(s,x)} dK_{5_o}\, K_{5_o}\sqrt{K_{5_o}^2-\mu^2}\left[z_2(s,x,K_{5_o})+1\right] = \frac{3}{4}(\mathcal{E}_2^2-\mu^2)^{3/2} - \frac{3}{4}(\mathcal{E}_1^2-\mu^2)^{3/2} + \frac{1}{4k_1}\left[(\mathcal{E}^2-x^2-7\mu^2)(\mathcal{E}_2^2-\mathcal{E}_1^2) - \mathcal{E}(\mathcal{E}_2^3-\mathcal{E}_1^3)\right]. \tag{16'C}$$

FOOTNOTES AND FOOT-REFERENCES -

(1) - For the first experimental evidence of this fact, see: B. Maglić, G. Kalbfleisch and M. Stevenson, Phys. Rev. Letters 7, 137 (1961).

(2) - For the subsequent experimental works, related to four-pronged anhilations, see e.g.: Ref.(13), Ref.(12) and Ref.(14), and Ref.(15).

(3) - It is to be remarked that, as pointed out by Pais[(4)], the charge-conjugation invariance implies that the π^+ and π^- CM angular distributions, with respect to the direction of the nucleon of equal charge, be the same if both p and $\overline{p}$ are unpolarized.

(4) - A. Pais, Phys. Rev. Letters 3, 242 (1959).

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(9) - S. Minami: "A possible Model for Nucleon-Antinucleon Annihilation", preprint (Osaka, 3 sept. 1965). Previously the model was suggested to us by V. Pelosi.

(10) - With the word "statistical" we mean that the off-mass-shell amplitude for the "core" annihilation is assumed to depend in the simplest way by all the variables on which it a priori must depend.

(11) - Actually, for momenta up to few GeV/c, the mean π-multiplicity is about five and the four-prong annihilation in $(5\pi)^o$ constitute a large amount of the annihilation processes. Without the statistic hypothesis for the virtual annihilation, the matrix element for the diagram of fig. 2 might be:

$$\langle f | S-1 | i \rangle = i\, G_{\alpha_1} G_{\alpha_5} \int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2 \int_{-\infty}^{\infty} d^4x_3 \cdot$$

$$\cdot \langle \pi | \gamma_5 \vec{\tau} \psi(x_1) \cdot \vec{\phi}(x_1) | p \rangle \; K_+(x_2 - x_1) \cdot$$

$$\cdot \langle \pi\pi\pi | T \{ g'(x_2) \gamma_5 \tau_2 \varepsilon^{hkm} \phi_h(x_2) \phi_h(x_2) \phi_m(x_2) +$$

$$+ g''(x_2) \gamma_5 \vec{\tau} \cdot \vec{\phi}(x_2) \phi_n(x_2) \phi^n(x_2) \} | p\overline{p} \rangle K_+(x_3 - x_2) \cdot$$

$$\cdot \langle \pi | \tilde{\psi}(x_3) \gamma_5 \vec{\tau} \cdot \vec{\phi}(x_3) | p \rangle \; .$$

(12) - K. Bockmann, B. Nellen, E. Paul, B. Wagini, I. Borecka, J. Diaz, V. Wolff; J. Kidd, L. Mandelli, L. Mosca, V. Pelosi, S. Ratti, L. Tallone, Nuovo Cimento 42, 954 (1966), and references therein; V. Russo, pri

vate communication; A. M. Rusconi:Degree Thesis (unpublished, 1965, Università di Milano).

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(16) - With the metric (+---), and in natural units.

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