Istituto Nazionale di Fisica Nucleare
Sezione di Bologna

INFN/AE-67/11
9 Ottobre 1967

A. Bertin, A. Placci, A. Vitale and E. Zavattini(x): TRANSFER RATES OF NEGATIVE MUONS FROM DEUTERIUM MESOATOMS TO HELIUM, NEON, ARGON, KRYPTON, XENON, AND NITROGEN ATOMS. -

1. INTRODUCTION. -

A negative muon slowing down in deuterium is soon captured to form a μd mesoatom[1]. If the pure deuterium is contaminated by a small amount of an element Y_Z (Z being the atomic number of Y_Z), the μd mesoatom may undergo the reactions

(1) $$\mu d + d_2 \rightarrow \mu d + d_2$$

(2) $$\mu d + d_2 \rightarrow \mu dd + d$$

(3) $$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$ (o)

besides the transfer process[2]

(4) $$\mu d + Y_Z \rightarrow (\mu Y_Z)^x + d.$$

The excited $(\mu Y_Z)^x$ mesoatom de-excites to its lowest level within 10^{-10} sec[1]. Afterwards, the μ^- may either decay according to

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(x) - CERN, Geneva, Switzerland.

(o) - The decay rate of the free muon at rest for process (3) is assumed to be

$$\lambda_o = 4.55 \times 10^5 \text{ sec}^{-1}.$$

process (3), or be captured by the nucleus Y_Z at a rate λ_{cY}.

We wish to report here the results of a measurement of the transfer rate for process (4), obtained by looking at the yield of the decay electrons coming from process (3) when negative muons are stopped in gaseous pure deuterium to which a known small amount of a gas Y_Z has been added. The gaseous element Y_Z (here Y_Z stands for Xe, Kr, Ar, Ne; the cases of helium and nitrogen were treated in a somewhat special way, as will be described in the following) was mixed at a partial pressure p_Y with gaseous deuterium at a pressure of 6 abs atm, and at a temperature of 293°K. It has been possible to work at such a low pressure without being overwhelmed by the accidental counts, mainly by putting inside the target a fast grid proportional counter; this counter, which worked using the deuterium itself as a detecting gas, was required to define the incoming beam (see Section 3).

It is interesting to compare the rate for process (4) with the corresponding rate obtained when hydrogen mesoatoms are involved, i.e. for the process

(5) $$\mu p + Y_Z \rightarrow (\mu Y_Z)^x + p$$

for which some experimental results are already available[3,4]. Such a comparison is sought for in order to get a wider test of the existing calculations on processes of the types (4) and (5)[5,3].

A theoretical study of the inelastic scattering processes (4) and (5) has been done by Gershtejn, using the two-state approximation[6]. In this case, quite generally, the solution can be given in semi-classical approximation, provided that the kinetic energy of the μd (μp) mesoatom is much larger than a limiting energy $\overline{T}$. More quantitatively, assuming between the neutral μd mesoatom and the Y_Z nucleus (at large distance R) the potential obtained in second-order perturbation theory approximation (such a potential results to be attractive, and goes like R^{-4}), the forementioned condition can be stated, for the case of reaction (4)[5]

(6) $$T_{\mu d} \gg \overline{T}_{\mu d} = \frac{2}{Z^2} \text{ eV} .$$

Moreover, Gershtejn has shown that, if E_o and E_i are the energies of the isolated mesic atoms of the proton and of the Y_Z nucleus, and $V_{oo}(R)$ and $V_{ii}(R)$ are the potential energies of the terms corresponding as $R \rightarrow \infty$ to the meson situated on the K orbit of the proton, and in a definite i-state at the Y_Z nucleus, respectively, a characteristic feature of this inelastic process is the existence of a crossing point R_o at which

(7) $$E_o + V_{oo}(R_o) = E_i + V_{ii}(R_o)$$

This fact is responsible for the large cross-sections obtained for the transfer reactions (4) and (5). When condition (6) is verified, the cross-section for processes (4) and (5) can, in good approximation, be written

(8) $$\sigma_Y = \text{const} \ \frac{f(Z)}{vMT}$$

where

v and T are the velocity and the kinetic energy of the mesoatom respectively;
M is the reduced mass of the (mesoatom - Y_Z) system;
f(Z) is an unknown function of Z.

Using expression (8), the corresponding rate can be written

(9) $$\lambda_Y = \text{const} \ \frac{f(Z)}{MT}$$

In our experimental conditions (6 abs atm pressure, and 293°K), the initially formed μd mesoatoms are completely thermalized within a time of the order of 100 nanosec[7]; hence we will assume

(10) $$T_{\mu d} = 3/2 \ K\mathcal{T} = 0.038 \ \text{eV}$$

($\mathcal{T}$ being the absolute temperature in Kelvin).

Condition (6) is therefore certainly satisfied for xenon, krypton, and argon(x). According to Gershtejn, in the limit

(11) $$T_{\mu d} \ll \bar{T}_{\mu d}$$

the quasi-classical approximation is still valid, and σ_Y in this case can be written[5] (in mesic atom units) for reactions (4) and (5)

(12) $$\sigma_Y = 3\pi (2/M)^{1/2} \ \frac{Z w_t}{v}$$

(x) - Condition (6) appears to be verified for these elements even if Z is substituted by a proper Z_{eff} value, to take into account the screening of the nuclear charge Z by the very inner orbital electrons.

where w_t is a dimensionless transition probability which depends on M as $M^{1/2}$. From equation (12), the rate for process (4) can then be written under condition (11)

$$\lambda_Y = \text{const } Z\, g(Z) , \tag{13}$$

where g(Z) is an unknown function of Z.

As for the case of helium, Gershtejn foresees an exceptionally small transfer probability for process (5) (at least five orders of magnitude smaller than the probability of transition to carbon and oxygen); this is due to the fact that the crossing point R_o lies very far away, and that at these large distances the μp and μHe wave functions overlap very little. Our measurements are intended to check experimentally whether this is also the case for process (4).

As a closing remark to this section, we would like to mention that we have studied process (4) also in view of a measurement of the μ^- nuclear capture rate in deuterium at low density, in relation to which it is necessary to establish which is the limit of purity required for the deuterium.

2. THE METHOD. -

In the experimental conditions stated in Section 1, the expected differential time distribution dn_e/dt of the electrons coming from process (3) can be written in the form

$$\frac{dn_e}{dt} = \text{const} \cdot \left\{ \lambda_o \exp\left[-\left(\lambda_o + \varphi \frac{p_Y}{p_o} \right) t \right] + \right.$$

$$\left. + \frac{\varphi \frac{p_Y}{p_o} \lambda_Y \lambda_o}{\lambda_{cY} - \varphi \frac{p_Y}{p_o} \lambda_Y} \left[\exp\left(-\left(\lambda_o + \varphi \frac{p_Y}{p_o} \lambda_Y \right) t \right) - \exp\left(-\left(\lambda_o + \lambda_{cY} \right) t \right) \right] \right\} \tag{14}$$

where

p_o is the total pressure of the $(d_2 + Y_Z)$ mixture;

$\varphi = \rho(p_o)/\rho_{liq}$ is the ratio between the density of deuterium at pressure p_o and the density of liquid hydrogen;

p_Y is the partial pressure of the added element Y_Z;

λ_Y is the transfer rate of process (4) referred to a density of molecules of the element Y_Z equal to the assumed density of molecules for liquid hydrogen(x);

λ_{cY} is the nuclear capture rate of negative muons by the element Y_Z.

Formula (14) is valid for both conditions specified by inequalities (6) and (11) for times t at which the kinetic energy of the μd mesoatom is constant (i.e. $t > 100$ nanosec). Furthermore, in deriving this equation we neglected the mesomolecular formation processes, the nuclear capture of a μ^- by a deuteron, and the transfer of the μ^-'s from the μd mesoatoms to the iron nuclei of the wall of the container(7).

The transfer rates λ_Y were then derived from the observed dn_e/dt distributions after a time $t > 100$ nanosec, the values of λ_o, p_Y and λ_{cY} being known.

Most of the chosen elements Y_Z were monoatomic ones, with a view to producing and observing processes as free as possible from molecular effects.

As for the cases of helium and nitrogen, for which the nuclear capture rate is very small compared to λ_o, the transfer rates in process (4) were measured by comparing the dn_e/dt distribution of the decay electrons, obtained from a given deuterium-xenon mixture, with the corresponding figure obtained when a small known amount of helium or nitrogen had been added to the previous mixture. In this case the expected electron yield as a function of time is (the index y now referring to xenon):

$$\frac{dn_e}{dt} = \text{cost} \times \left\{ \lambda_o \exp\left[-\left(\lambda_o + \varphi \left(\frac{p_Y}{p_o} \lambda_Y + \frac{p}{p_o} \lambda \right) \right) t \right] + \frac{\varphi \frac{p_Y}{p_o} \lambda_Y \lambda_o}{\lambda_{cY} - \varphi \left(\frac{p_Y}{p_o} \lambda_Y + \frac{p}{p_o} \lambda \right)} \times \right.$$

$$\times \left[\exp\left(-\left(\lambda_o + \varphi \left(\frac{p_Y}{p_o} \lambda_Y + \frac{p}{p_o} \lambda \right) \right) t \right) - \exp\left(-\left(\lambda_o + \lambda_{cY} \right) t \right) \right] + \frac{\varphi \frac{p_Y}{p_o} \lambda \lambda_o}{\lambda_c - \varphi \left(\frac{p_Y}{p_o} \lambda_Y + \frac{p}{p_o} \lambda \right)} \times \tag{15}$$

$$\left. \times \left[\exp\left(-\left(\lambda_o + \varphi \left(\frac{p_Y}{p_o} \lambda_Y + \frac{p}{p_o} \lambda \right) \right) t \right) - \exp\left(-\left(\lambda_o + \lambda_c \right) t \right) \right] \right\}$$

(x) - The density of liquid hydrogen has been assumed to correspond to 2.11×10^{22} molecules/cm^3.

where

p is the partial pressure of the helium or nitrogen gas;

λ is the transfer rate of a negative muon from a μd mesoatom to a helium or nitrogen atom;

λ_c is the nuclear capture rate of a negative muon by helium or nitrogen.

3. THE EXPERIMENTAL APPARATUS. -

The measurements were carried out at the CERN 600 MeV Synchrocyclotron, using part of an already existing apparatus built up to measure the μ^- nuclear capture rate by a proton[8]. A schematic drawing of the experimental set-up, as well as a simplified block diagram of the electronics, are given in Fig. 1.

Two plastic scintillation counters(x) in coincidence monitored the incoming muon beam, which was collimated by a cylindrical iron collimator (ϕ = 7 cm). The back and side walls of the target were faced by five plastic scintillator counters (counter A_5, and counters A_1, A_2, A_3, A_4, respectively, all of them 2 x 50 x 40 cm) the signals of which were put in fast anticoincidence with the signal coming from the (1, 2) counter telescope.

The (deuterium + Y_Z) gas mixture was contained in a stainless steel cylindrical tank T (130 cm long, 26 cm ϕ), the wall and the entrance window of which were 2.5 mm and 1 mm thick respectively. The entering particles were first moderated by a 2 cm thick beryllium moderator (which was placed inside the gas container to minimize losses due to multiple scattering), and has then to pass a grid proportional counter (counter α in Fig. 1) before entering the useful volume of gas. The structure of counter α (see Fig. 2) was the same as the one described by Alberigi Quaranta et al.[9]. The distance between the planes of the grids limiting the counting gap was 7 mm; at this condition the working voltage was 14.5 kV.

The MUSTOP coincidence $(1, 2, \overline{A_1 A_2 A_3 A_4 A_5}, \alpha)$ defined the stopping of negative muons in the useful volume of gas; furthermore, in order to reduce the accidental counts, the pulses from counter α, as shown in Fig. 1, were accepted only if they were bigger than a properly chosen threshold.

The electrons coming from the decay of the negative muons and detected by the four plastic scintillators A_i (i = 1, 2, 3, 4) were scaled within a gate 10 microsec long, which was delayed by about 1/2 microsec with

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(x) - Counters 1 and 2 were 1 cm and 0.3 cm thick, respectively.

respect to the MUSTOP signal. The differential time distribution of these electrons was measured by recording in a pulse-height analyser (PHA) the output pulses of a time-to-pulse-height converter (TPHC), which was triggered by the MUSTOP signal, and by a delayed pulse coming from any of the A_i counters (as START and STOP pulses, respectively).

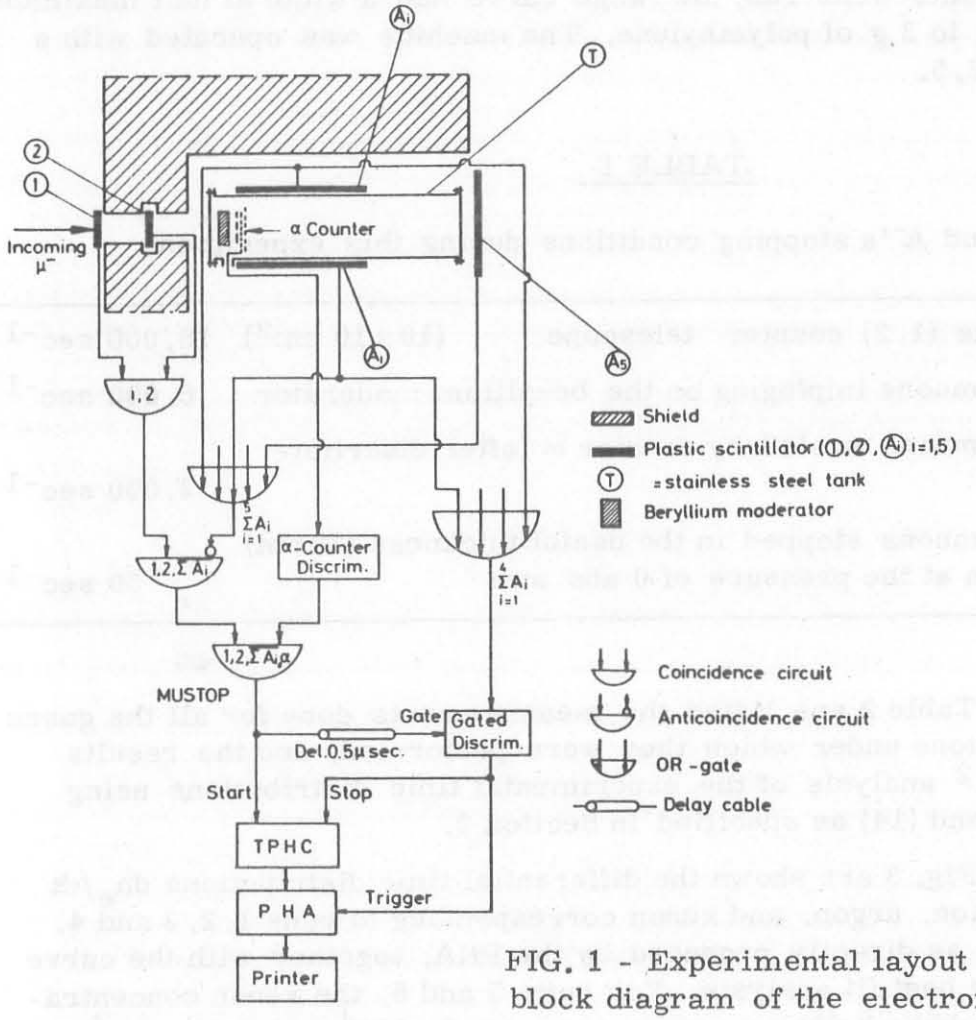


FIG. 1 - Experimental layout and simplified block diagram of the electronics.

The filling system used to prepare the mixtures of gases in the target was actually the same as the one described in a previous paper[4]; before beginning the measurements, the target was kept for several days at a vacuum of 10^{-6} mm Hg in order to obtain a satisfactory degassing of its walls. High purity for the deuterium, before its admission into the tank, was achieved by filtering it through a palladium purifier(*). All the other gases injected were pure to more than $1/10^4$, except the

(*) - Supplied by Engelhard Industries, Newark, N.J., U.S.A.

xenon, which was supplied as pure as $1/10^{3(\varkappa)}$.

4. THE MEASUREMENTS AND THE ANALYSIS OF THE DATA. -

In Table1 are listed the beam characteristics under which the measurements were run; the range curve had a width at half maximum corresponding to 3 g of polyethylene. The machine was operated with a duty cycle of 2.5.

TABLE 1

Beam and μ^-'s stopping conditions during this experiment

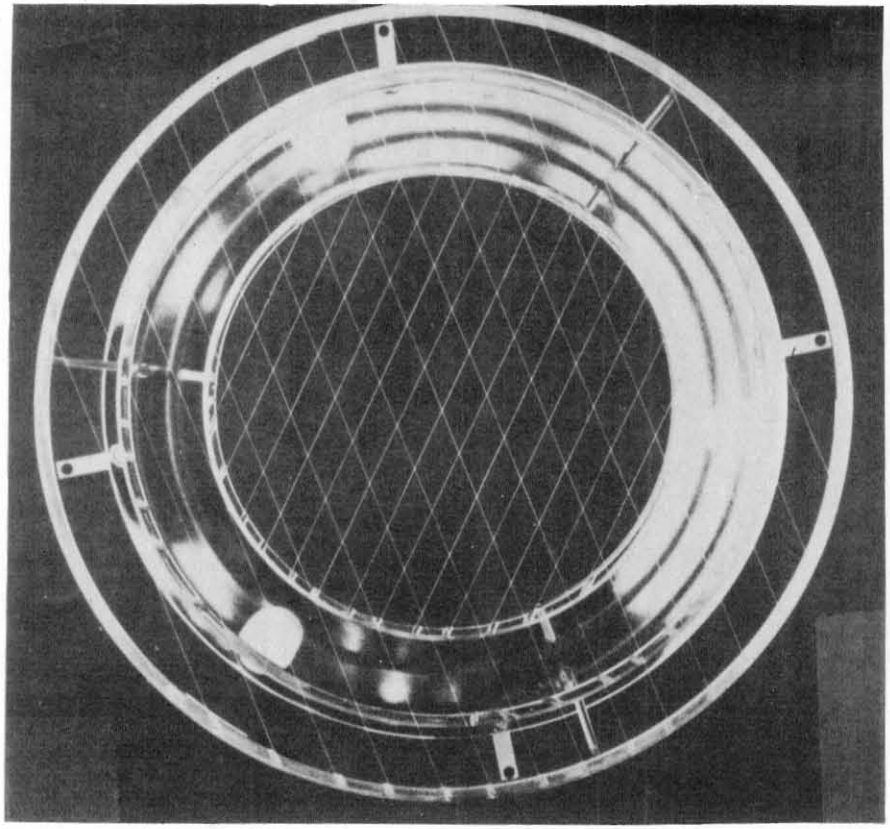
Counts of the (1,2) counter telescope (10×10 cm^2)	15,000 sec^{-1}
Number of muons impinging on the beryllium moderator	6,000 sec^{-1}
Number of muons scaled by counter α (after discrimination)	2,000 sec^{-1}
Number of muons stopped in the useful thickness (70 cm) of deuterium at the pressure of 6 abs atm	50 sec^{-1}

In Table 2 are listed the measurements done for all the gases Y_Z, the conditions under which they were performed, and the results obtained by χ^2 analysis of the experimental time distributions using formulae (13) and (14) as specified in Section 2.

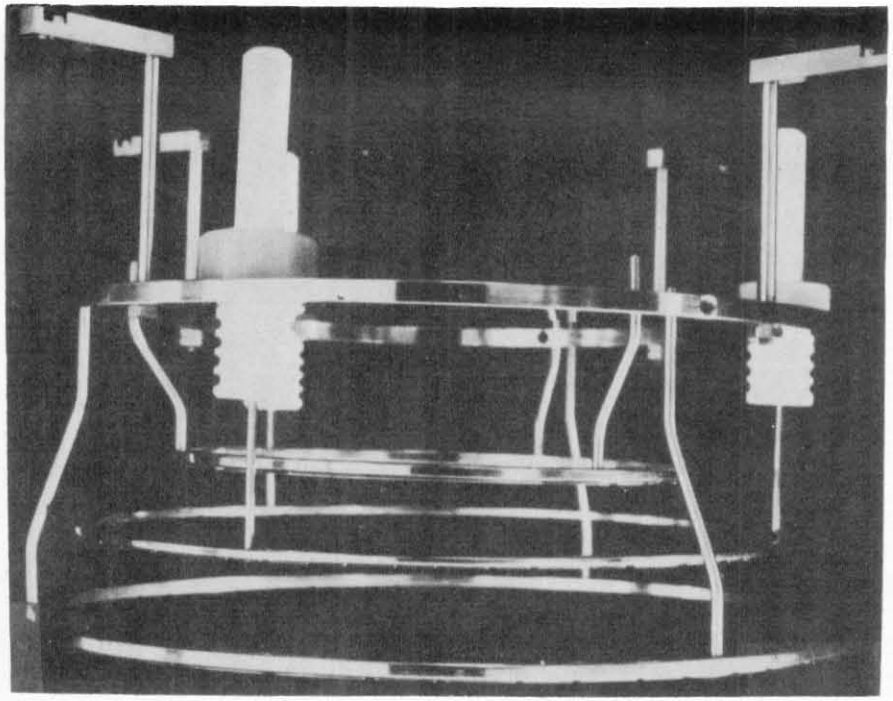
In Fig. 3 are shown the differential time distributions dn_e/dt for neon, krypton, argon, and xenon corresponding to runs 1, 2, 3 and 4, (respectively), as directly produced by the PHA, together with the curve obtained by the best fit analysis. For runs 5 and 6, the xenon concentrations, before adding helium or nitrogen, were 5×10^{-4} and 7.33×10^{-5}, respectively.

After each of the main measurements listed in Table 2, a supplementary run was effected, in which a further large amount of xenon (about 1%) was added to the Y_Z + deuterium mixture; this was done to control in the various measurements the behaviour of the background at early times. The accidental counts were about 10% of those of first considered time-channel (see Fig. 2a, 2b, 2c, 2d).

($\varkappa$) - The supplying firms were in turn Air Liquide, Paris; G.L. Loos, Amsterdam, Holland; and Carba, Bern, Switzerland.



a)



b)

FIG. 2 a) Side view of the α-grid proportional counter (beam entrance from the right). The supporting stainless steel rings have, from left, 15 cm, 13 cm, and 10 cm ϕ. The central ring is the H. V. electrode. The gaps between the electrodes are 14 mm (left gap) and 7 mm (right gap). The biggest electrode acts merely as a shielding cathode, owing to the larger value of its distance from the counting central electrode.
b) Front view of the α counter; the stainless steel wires constituting the grids had 100 micron (H. V. electrode) and 200 micron (earth electrodes) diameters.

TABLE 2

Summary of the experimental conditions and of the results of the experimental (deuterium pressure 6 abs atm, temperature = 293°K).

Added element Y_Z	Atomic number Z	Partial pressure $p_Y \times 10^3$ (abs atom)	Transfer velocity(x) $\lambda_Y \times 10^{11}$ (sec^{-1})	Run No.	Fitt. form:	Fig.	χ^2	No. of points
Ne	10	10.5 ± 0.5	0.71 ± 0.10	1	(13)	2a	75	75
Ar	18	2.57 ± 0.1	0.47 ± 0.04	2	(13)	2b	73	86
Kr	36	1.33 ± 0.085	1.29 ± 0.12	3	(13)	2c	81	86
Xe	54	0.44 ± 0.03	2.36 ± 0.23	4	(13)	2d	76	86
N	7	15.6 ± 0.7	0.50 ± 0.15	5	(14)	--	89	80
He	2	600 ± 30	$(8 \pm 10) \times 10^{-5}$	6	(14)	--	76	86

(x) - Referred to a density of Y_Z molecules equal to the density of molecules assumed for liquid hydrogen.

5. DISCUSSION AND CONCLUSIONS. -

5.1. High Z elements (Ar, Kr, Xe). -

Since in this case one expects that λ_Y behaves as $(TM)^{-1}$, [see formula (9)], we can check this point by comparing our results with those obtained by Alberigi-Quaranta et al.[4] for process (5), which we have reported in Table 3.

It has been experimentally shown[7] that after some tens of nanoseconds following its formation, the μp mesoatom (at 25 atm and 300°K), contrary to what happens in the case of the μd mesoatom, has still a rather high kinetic energy (about 0.16 eV), and that the elastic scattering cross-section σ against the atoms of the surrounding hydrogen molecules is quite small, namely about 10^{-20} cm^2. From this value one can show that the time required for the μp mesoatom to undergo two collisions (after which the residual kinetic energy will be very near to the thermal value) is of the order of 400 nanosec. On the other hand, the results listed in Table 3 were obtained by fitting data centred around 0.8 microsec after the formation on the μp mesoatom; they can therefore be considered as corresponding to a condition in which the μp mesoatom has attained thermal energy. We will, therefore, compare the results

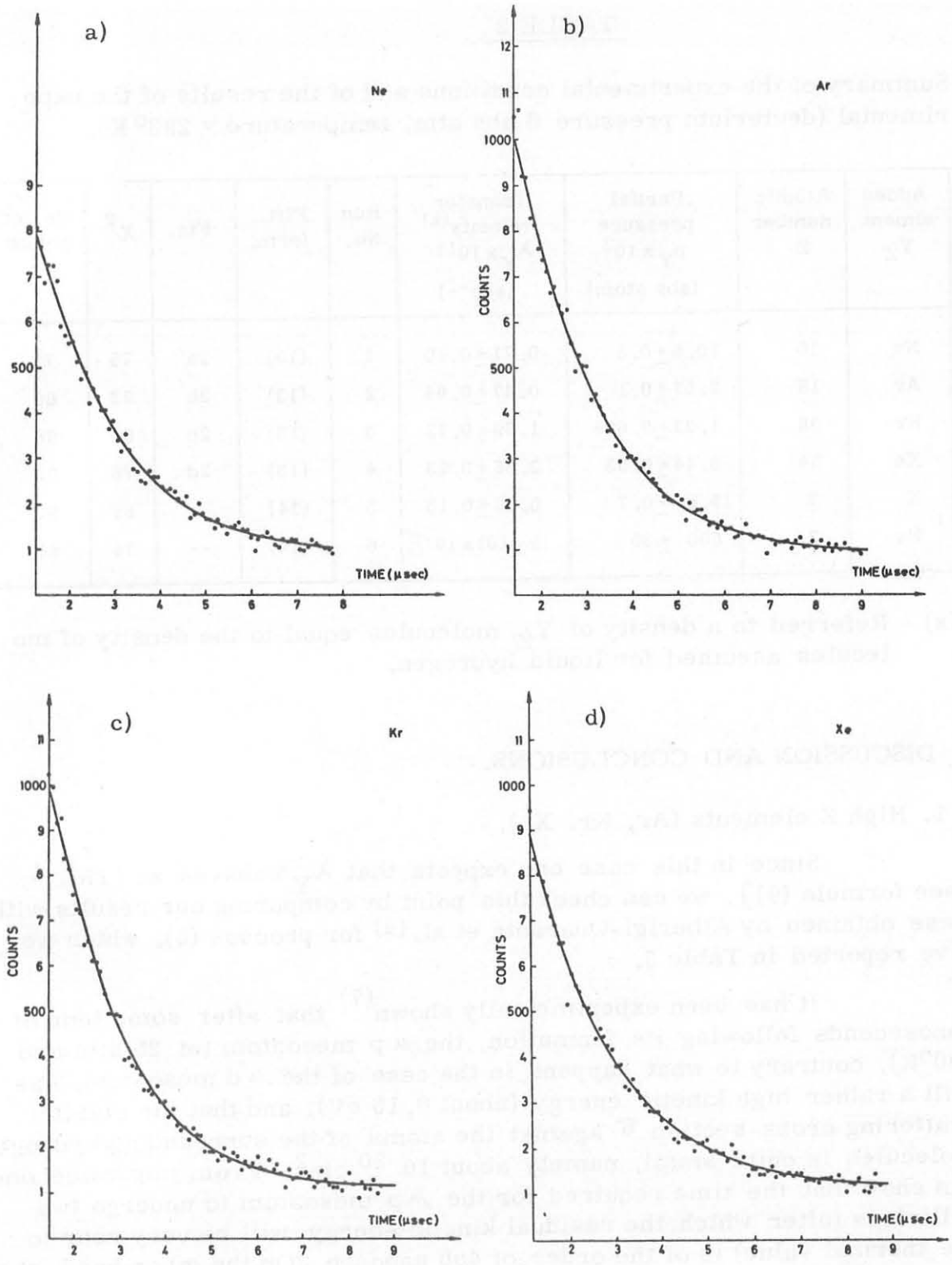


FIG. 3 - Differential time distribution dn_e/dt obtained in the measurements corresponding to neon (a), argon (b), krypton (c) and xenon (d). Background counts are not subtracted in the figure.

of Table 3 with the results obtained in the present experiment assuming $T_{\mu p} = T_{\mu d} \simeq 0.04$ eV.

TABLE 3

Y_Z	Z	H_2 pressure	Temper.	$\lambda_Y^{(x)}$ reaction (5)	B
Xe	54	25 atm	300°K	4.96 ± 0.46	2.10 ± 0.30
Kr	36	25 atm	300°K	3.42 ± 0.63	2.60 ± 0.50
Ar	18	25 atm	300°K	1.74 ± 0.30	3.70 ± 0.80
Ne	10	25 atm	300°K	0.58 ± 0.14	0.82 ± 0.22

(x) - Data taken from ref. 4; as usual they refer to a density of the Y_Z element equal to the density of molecules of hydrogen for liquid hydrogen (2.11×10^{22} mol/cm^3).

In the last column of Table 3 are given, for each element, the ratio B between the transfer speed for process (5) and the transfer speed for process (4); these ratios are in fair agreement with the value one expects applying formula (9).

In this case the values of λ_Y obtained suggest for the function f(Z) [see formula (8)] that a linear behaviour upon Z, at least for atomic number Z between 18 and 54, is an adequate approximation (see Fig. 4).

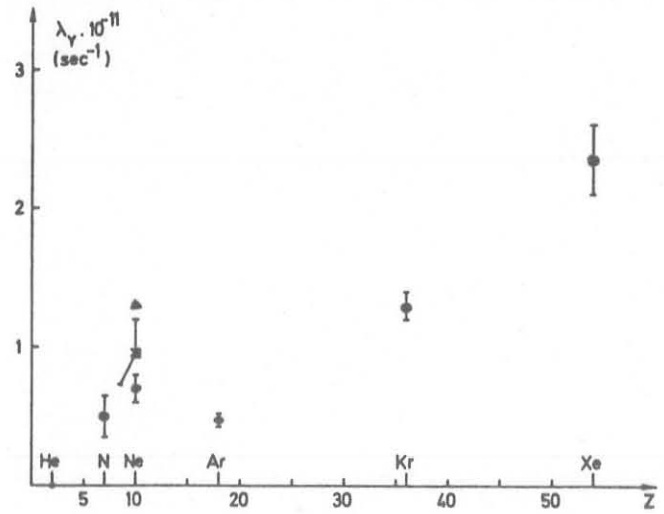


FIG. 4 - Experimental results on the transfer rates λ_{Y_Z} for reaction (4).

- ● present experiment;
- ▲ reference 2;
- ■ reference 10.

5.2. Low Z elements (Ne, N, He). -

For these elements probably neither conditions (6) or (11) are sufficiently well satisfied. However, from the results listed in Table 3 and shown in Fig. 4 we can make the following remarks:

i) The transfer rate of a negative muon meson from a μd and a μp mesoatom to neon are very similar in magnitude [as would be the case if formula (13) would apply].

ii) The transfer rate λ_{Ne} to neon for process (4) turns out to be dependent on $T_{\mu d}$ (although weakly) since the values obtained at liquid hydrogen temperature[2, 10] are higher than those obtained at 293°K by us.

iii) The transfer rate λ_{He} to helium for process (4) is four orders of magnitude, or more, smaller than the transfer rate to neon. This confirms rather well the theoretical prediction of Gershtejn and supports the belief that for the transfer processes (4) and (5) the crossing of the molecular therms is responsible for the high cross-sections experimentally found for the other elements.

We wish to express our sincere gratitude to our technicians Messrs. R. Schillsott and G. Sicher, to the scanning-girl R. Mincioni, for their skillful assistance, and to Mr. F. Dydak for helping us during the measurements. In particular we would like to thank Prof. A. Alberigi-Quaranta for the interest he has shown in this work and his continuous encouragement, and also Prof. P. Bassi and Prof. A. Zichichi for their support.

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