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ELASTIC SCATTERING OF MESOATOMS ON HYDROGEN :
DETERMINATION OF THE TOTAL SPIN STATE OF THE $\mu$P
MESOATOM. -

## 1. - INTRODUCTION. -

It has been theoretically shown[1] that the nuclear capture
rate of a negative muon by a proton depends strongly on the total spin
state (F) of the $\mu$p atom in which the capture takes place. The me-
soatom can be either in a singlet state (F = 0) or in a triplet state
(F = 1), the $\mu^-$ being in a K orbit around a proton[xx]. To be able to
give a prediction of the rate of $\mu^-$ nuclear capture in $H_2$ (especially
if the phenomenon takes place in $H_2$ at low density) it is therefore
necessary to know which fraction of the $\mu$p mesoatoms are at a
given time in the singlet state.

We wish to report here the results of an experiment in which
we measured the cross section for the following processes[oo]:

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(x) - CERN visitor, on leave from Istituto di Fisica dell'Università di
Bologna.
(o) - Supported by a Consiglio Nazionale delle Ricerche fellowship.
(+) - CERN Fellow, on leave from Istituto di Fisica dell'Università di
Pisa, and Istituto Nazionale di Fisica Nucleare, Sezione di Pisa.
(^) - CERN staff member.
(xx) - The latest calculations give for the nuclear capture rate in the
F = 0 and F = 1 states respectively 626 sec$^{-1}$ and 12 sec$^{-1}$ (see
ref. (2) ).
(oo) - $F_1$ and $F_2$ being the two possible total spin states of the meso-
atoms. They can assume the values 0 or 1 in the case of process
(1) and 1/2 or 3/2 in the case of process (2).

2.

(1) $$(\mu p)_{F_1} + p = (\mu p)_{F_2} + p$$

(2) $$(\mu d)_{F_1} + d = (\mu d)_{F_2} + d$$

where the mesoatoms can have an energy up to 1 ev[3]. The experiment was performed by detecting the diffusion of the $\mu p$ ($\mu d$) atom through the hydrogen (or deuterium) contained in a vessel at a proper pressure.

As Gershtein has shown[4] the total cross-section for the particular reaction (1) is quite different for the statistical mixture of $F = 1$ and $F = 0$ states, and for the pure $F = 0$ state. A measurement of this cross-section can therefore give information on the total spin F of the $\mu p$ mesoatom formed in a hydrogeneous medium.

Measurements on processes (1) and (2) have been first performed by a group of the Dubna Sinchrocyclotron[5], using a diffusion chamber technique; our method has however the important feature that the $\mu p$ atoms are formed within a pure hydrogeneous medium, where no other elements are present.

The comparison of the ex perimental results for processes (1) and (2) with the theoretical predictions offers a very good check of the theory developed for the mesomolecular systems[6, 7]; in particular it is a control of the parameters which enter into the expression of the mesomolecular potentials for the systems $p\mu p$ and $d\mu d$[6, 2].

We have studied processes (1) and (2) mainly in view of a future measurement of the $\mu^-$ nuclear capture rate by protons (or deuterons) in hydrogen at low density; in relation to this we wish specifically to clarify the two following points:

a) - we are interested to know the time t after which practically all the $\mu p$ atoms present in the hydrogen gas are in $F = 0$ state; notice that to know this it is not sufficient to prove that the cross-section for process (1) with transition from $F = 1$ to $F = 0$ states is sufficiently high, but one has also to prove that after this time t the kinetic energy of the $\mu p$ systems is less than 0.18 eV to avoid the inverse reaction (see next paragraph).

b) - the $\mu p$ neutral atom (or $\mu d$) will diffuse throughout the hydrogen and may reach in its random motion the wall of the container; the $\mu^-$ from the $\mu p$ ($\mu d$) may there transfer to any atom of the material of the container and be captured by the nucleus. This process constitutes an unavoidable source of background of delayed neutrons, the amount of which depends on the value of the total cross-section for reaction (1) and (2).

## 2. SHORT SURVEY OF THE RESULTS OF THEORETICAL INVESTIGA-
   TIONS.

a) - Scattering of $\mu$p atoms against protons.

The energy difference between the $F = 1$ and $F = 0$ states of the mesoatoms $\mu$p is[8]

(3)
$$\Delta E = \frac{16}{3} \pi \beta_\mu \beta_N g_i \left| \psi(0) \right|^2 = 0.18 \quad eV$$

where $\beta_\mu$ is the mesonic and $\beta_N$ the nuclear Bohr magneton and $g_i = 2 \times 2.79$ is the gyromagnetic ratio of the proton.

This energy is much bigger than the thermal energy of a $\mu$p atom (0.03 eV for $T = 300^\circ K$), so that if this system has a thermal energy, and is in singlet state at some time, it will stay in this state forever.

As Cohen et al. and Gershtein have shown[7,4] the total elastic scattering cross-section of the $\mu$p atom, with energy E bigger than the transition energy $\Delta E$, on protons, may be written[x],

(4)
$$\sigma_{1,0} = 4\pi \left( \frac{3}{4} a_u^2 + \frac{1}{4} \frac{a_g^2}{1 + K^2 a_g^2} \right)$$

where $a_g$ and $a_u$ are the scattering lenghts for mesic atom potentials $V_g$ and $V_u$, corresponding to symmetric ($\Sigma_g$) and antisymmetric ($\Sigma_u$) states of the $\mu$p system with respect to the exchange of the space coordinates of the two protons, and

$$K = \left( \frac{2 M_p E}{\hbar^2} \right)^{1/2}$$

At these energies the $\mu$p atom is in a statistical mixture of the $F = 1$ and $F = 0$ states. The expression (4) is the sum of coherent and incoherent scattering; Gershtein[9] has shown that the scattering cross-section with the transition from $F = 1$ to $F = 0$ state for the $\mu$p system is given by the expression:

(5)
$$\sigma_{1 \to 0} = 4\pi \frac{3 (a_g - a_u)^2}{16 + K_o^2 (3a_u + a_g)^2}$$

$$K_o = \left( \frac{2 M_p \Delta E}{\hbar^2} \right)^{1/2}$$

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(x) - All scattering lenghts are in units of $\hbar^2 / m_\mu e^2$.

4.

When the $\mu$p atom has a kinetic energy E lower than the tran-
sition energy $\Delta E$ and is in the F = 0 state, the total elastic cross-section
becomes; [4]

$$(6) \qquad\qquad \sigma_o = 4\pi \left(\frac{3\,a_u + a_g}{4}\right)^2$$

The quantity $a_u$ and $a_g$ have been calculated by Cohen et al. [7] and Zel'do-
vich and Gershtein [6]. The results of their calculations are given in Ta-
ble I.

TABLE I

| Authors | $a_g$ | $a_u$ | $\sigma_o \cdot 10^{21} cm^2$ | $\sigma_{1 \to o} \cdot 10^{19} cm^2$ | $\sigma_{1,o} \cdot 10^{19} cm^2$ |
|---|---|---|---|---|---|
| Zel'dovich and Gershtein (6) | -17.3 | +5.25 | 1.2 | 7.8 | 7.9 |
| Cohen et al. (7) | -11 | +5 | 8.2 | 3.9 | 4.0 |

Theoretical values of the scattering lenghts and cross-sections relative to
the process: $\mu$ p + p $\longrightarrow \mu$p + p.

The value of $\sigma_o$ comes out to be so small mainly because $a_g$
(which has the opposite sign of $a_u$) is very large. This is due to the fact
that in the $\Sigma_g$ state the two protons, which are in an attractive potential
$V_g$, have a quasi real bound state of a bonding energy very near to ze-
ro[6]. Table I shows that the agreement between Zel'dovich et al., Ger-
shtein and Cohen et al. on $a_g$ is not as good as for $a_u$: the just mentioned
resonant character of $a_g$ may be the cause for such difference, since both
authors with different techniques perform the calculations by first order
perturbation method in $M\mu/M_p$.

As anticipated in the introduction, the expected values for $\sigma_o$
and $\sigma_{1,o}$ are very different between themselves; in particular, the theo-
ry predicts that when the kinetic energy of a slowing down $\mu$p atom be-
comes lower than the transition energy $\Delta E$, the total scattering cross
section of process (1) decreases by almost two orders of magnitude.

b) - Scattering of $\mu$d atoms on deuterons.

The mesoatoms $\mu$d can be either in an F = 3/2 or in an F = 1/2
total spin state, the $\mu^-$ being in a K orbit around the deuteron. Using the
Fermi's formula (3) one gets:

$$(7) \qquad\qquad \Delta E = 0.045 \ eV$$

In this case the transition energy $\Delta E$ is comparable with the
thermal energy (T = 300°K). As shown in[6, 7, 11] the total scattering cross-

-section of $\mu$d on deuteron when the kinetic energy of the incident $\mu$d is higher than the transition energy $\Delta E$ is given by:

$$(8) \qquad \sigma_{1/2,3/2} = 4\pi \, (\tfrac{2}{3} b_g^2 + \tfrac{1}{3} b_u^2)$$

where $b_g$ and $b_u$ are defined in an analogous way as $a_g$ and $a_u$.

At these energies, the $\mu$d atom is in a statistical mixture of $F = 3/2$ and $F = 1/2$ states. If the $\mu$d has an energy below the transition energy (7), and is in the state $F = 1/2$, then the value of the cross-section for process (2) is[11]:

$$(9) \qquad \sigma_{1/2} = 4\pi \left[ \frac{2}{3} (\frac{5b_g + b_u}{6})^2 + \frac{1}{3} (\frac{b_g + 2b_u}{3})^2 \right]$$

In Table II we present the results of the calculations done by Zel'dovich and Gershtein[6] and by Cohen et al. [7] for process (2). Notice that in this case $b_g$ and $b_u$

TABLE II

| Authors | $b_g$ | $b_u$ | $\sigma_{\frac{1}{2},\frac{3}{2}} \cdot 10^{19} cm^2$ |
|---|---|---|---|
| Zel'dovich and Gershtein[6] | 6.67 | 5.73 | 3.3 |
| Cohen et al. [7] | -- | -- | 3.4 |

Theoretical values of the scattering lenghts and cross-sections relative to the process: $\mu$d + d $\rightarrow$ $\mu$d + d.

have the same sign, and they are about equal in magnitude. For these reasons, the cross-section obtained applying (8) is very much equal in magnitude to the one obtained by (9). For the $\mu$d atom there is not, consequently, a critical energy below which the scattering cross-section decreases so strongly.

We close this paragraph with two remarks:

a) - In our experimental conditions the mesoatoms collide with molecules, not with atoms (p or d) as we will assume. At the beginning of the paragraph 6) we will discuss the eventual modifications of the results of our analysis due to this molecular effect.

b) - The experimental determination of the value of $\sigma_0$ does not yield a unique value of $a_g$, but gives two values, $a_{g1}$ and $a_{g2}$ since $\sigma_0$ is a quadratic function of $a_g$. We shall see however that the two admitted values $a_{g1}$ and $a_{g2}$ are not very different from one another.

6.

## 3. PRINCIPLE OF THE METHOD. -

In a gaseous target of isotopically pure hydrogen, negative muons are slowed down and captured in an atomic orbit around a proton, forming initially an excited $(\mu p)^x$ mesoatom; this system deexcites to its lowest orbital level within a time of the order of $10^{-10}$ sec, acquiring a kinetic energy of about 1 eV[3].

In a very short time afterwards the $\mu p$ atom is slowed down to the transition energy $\Delta E = 0.18$ eV and, according to Table I, the collision mean free path below this energy at 25 atmospheres becomes as big as $\sim 1$ mm.

If now a series of properly spaced foils of any material (gold for instance, as was the case in our experiment) are placed in the gas container, the mesoatom, in its wandering through the gas, may reach the surface of one of these foils. The $\mu$ (from the $\mu p$ system) is then quickly transferred to a gold atom, forming a $(\mu Au)^x$ excited system, which in a very short time ($10^{-10}$ sec) deexcites emitting a gamma ray characteristic of the $2P \rightarrow 1S$ transition for the gold mesic atom. The fast detection of these gamma rays (which was done in our case by mean of a NaI(Tl) crystal) gives then an indication of the time t of arrival of the $\mu p$ atom at the gold foil surface.

It can be shown that the yield of the gold mesic gamma rays $Q(\bar t)$ (being $\bar t = t - t_o$, where $t_o$ is the time of stopping of the muon in Hydrogen) is a unique function of the following parameters:

a) - the distance $d_o$ between the foils;
b) - the hydrogen density $\delta_o$ and its temperature T;
c) - the initial mean velocity $\bar v_{\mu p}$ of the $\mu p$ atom;
d) - the cross-section for process (1) (or (2));
e) - the total number N of gold mesic gamma rays which one
would count if all the muons stopped in hydrogen were transferred from the $\mu p$ mesic atoms to the gold atoms.

Purpose of this experiment is to measure Q(t) and N; information on the total cross-section for process (1) (or (2)) and on $v_{\mu p}$ (or $v_{\mu d}$) may be obtained from the known value of $d_o$, $\delta_o$, T, and by fitting the experimental data with the predictions deduced either by solving a diffusion equation or by a Monte Carlo calculation.

The processes in competition to reaction (1) are:

(10)               $\mu p + p_2 = p \mu p$

(11)               $\mu p + Y = \mu p + Y$

(12)               $\mu p + Y = (\mu Y)^x + p$

(13)               $\mu \rightarrow e + \nu_1 + \nu_2$

where Y is any gaseous impurity contained in the hydrogen of the target.

Reaction (11) and (12) have been avoided by putting only pure hydrogen (protonium) into the target, immediately after purification done by a palladium filter; no material which can easily degas impurities has moreover been placed inside the vessel.

A rough evaluation has shown that the contamination of the hydrogen (or deuterium) must have a concentration less than $10^{-5}$(x).

Reaction (10) is kept down by limiting the hydrogen pressure to 25 atms. From the known value of the rate of reaction (10) one obtains that at 25 atms. and in absence of diffusion to the gold foils, the formation of mesomolecules happens in 25% of the cases[12, 13].

Since the amount of hydrogen, with respect to other materials present in the target (Au and Fe) is forcibly much smaller, one expects that muons directly stopping and successively captured in these materials give rise to a rather high neutron background in the gamma rays detector. To overcome this difficulty the measurements of the yield $Q_H(\bar{t})$ of the $2P \rightarrow 1S$ gold gamma rays were frequently alternated (each hour) with the measurements of the corresponding yield $Q_{He}(\bar{t})$ when the target was filled with helium at the same pressure. Because of its positive charge, the helium mesic ions $(\mu He)^+$ cannot transfer the $\mu^-$ to a gold atom of the plates.

The experimental yield $Q(\bar{t})$ to be compared with the expected theoretical distribution is then given by the difference of the two distributions $Q_H(\bar{t})$ and $Q_{He}(\bar{t})$.

The value of N is obtained by counting the total number M of prompt gold mesic gamma rays, which are given by the muons stopping directly in the gold foils. In fact N can be deduced from M once the relative quantities of gold and hydrogen present inside the target are known, and by taking into account the different stopping powers of gold and of hydrogen.

We close this paragraph by giving an explanation of the reason why in the actual set-up (see next section) we have chosen $d_o = 1.5$ mm. It can be shown that the method which we have just described becomes poorly sensitive for the determination of the cross-section for process (1) when $d_o$ is much smaller than the diffusion length D of the $\mu$p system in the hydrogen gas (in this case the mesoatom would reach the surface of one gold plate without colliding with a $H_2$ molecule). On the other hand, to increase the counting rate one is tempted to put in the vessel as many plates as possible. A good compromise is $d_o \sim D$.

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(x) - Concentrations are all molecular concentrations.

8.

From Table I the expected theoretical value for D (at 25 atms.) is about 2.3 mm; on the other hand the corresponding experimental value found by the Dubna's group[5] is near to 0.6 mm. As a kind of compromise among all these values, we have put $d_o$ = 1.5 mm.

## 4. EXPERIMENTAL APPARATUS. -

The experimental arrangement is drawn in fig. 1. The $\mu^-$ beam (momentum 120 MeV/c) was supplied by the $\mu$-channel[14] of the CERN 600 MeV synchrocyclotron; its intensity was of about 14.000 $\mu^-$/sec; half of these particles could be stopped in 3 grams of polyethylene.

a) - <u>Target and counters arrangements</u>. - The target was a stain less steel cylinder, 46 cm long, 12.5 cm $\emptyset$, to be filled with pure hydrogen gas (or deuterium or helium) and containing 90 gold plates, each of them 10 microns thick, 11 cm $\emptyset$, 1.5 mm spaced, their surfaces being orthogonal to the cylinder axis.

The vessel was vacuum tight up to $10^{-6}$ mm Hg; its walls and flanges were respectively 2 and 1 mm thick. Before filling it with protonium (or deuterium) it was pumped for several days to degas its walls as well as the gold plates.

Pure hydrogen was obtained by letting protonium (isotopically pure hydrogen)[x] go through a special palladium purifier[+], immediately before being sent into the target. The gas circuitery provided also faci lities for filling the vessel with pure helium and pure deuterium gases.

A telescope of plastic scintillators (counters 1, 2, 3, 4, $\overline{5}$)[o] was used to define the beam; the target was completely surrounded by plastic counters in anticoincidence (counters ANTI 6, 7, 8, 9, 10, 11, all of them 1 cm thick), and viewed by a NaI crystal counter.

The NaI crystal had the function of selecting mesic gamma rays whose energy was corresponding to the 2P → 1S ($\mu$ Au) mesoatom transi tion (5.8 MeV)[15]; a typical resolution figure was of the order of 10%. Fig. 2 shows the spectrum amplitude obtained in the NaI crystal when $\mu^-$ stopping directly in gold are selected (prompt events). The visible peak corresponds to the 5.8 MeV 2P → 1S mesic gamma rays.

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(x) - Protonium was supplied by Air Liquide, Paris.
(+) - Engelhard Industries Inc. , Chemical Division, Newark, N. Y.
(o) - Counters 1, 2, 3, 4 were respectively 1.0, 0.5, 0.3, 2.0 cm
    thick; counter 5 was 1 cm thick, with a central hole of 10
    cm $\emptyset$.

b) - Electronics. - The diagram of the electronics is given in fig. 3. The beam intensity was scaled by the MONITOR coincidence. The time $t_o$ = 0 at which muons arrived and stopped in the tank was defined by the $\mu^-$ STOP signal coincidence ($\mu^-$STOP = 1, 2, 3 $\Sigma$ ANTI), whose output supplied the START signal to a time-to-pulse-height converter (TPHC). The time t of arrival of a $\mu$p atom at the surface of one of the gold plates was defined by the presence of a pulse corresponding to a 5.8 MeV gamma ray in the NaI crystal. The signal of the last dynode of the NaI crystal photomultiplier, properly delayed and amplified, gave the STOP command to the TPHC. The TPHC outputs were stored in a 1024 channels TMC Pulse Height Analyzer(PHA).

The PHA was triggered by the coincidence TRIGGER (see fig. 3) which insured us that:

a) - the pulse coming from the NaI crystal was sharply discriminated in amplitude, to select 5.8 MeV gamma rays. The limits of discrimination are shown in fig. 2 by the two vertical arrows;

b) - the pulse coming from the NaI crystal through the gated Discriminator was accepted only if contained between 1/2 $\mu$s before and 3 $\mu$s after the $\mu^-$STOP signal. Moreover the gate on signal was interrupted both by a second monitor pulse and by a signal coming from any of the anticoincidence counters.


## 5. MEASUREMENTS AND DATA ANALYSIS. -

### a) - The measurements.

The measurements of the yield of the delayed 5.8 MeV gamma rays were taken for two values of the protonium pressure, i. e. 26.2 and 5.4 abs. atms. In the case of deuterium, measurements were taken only at 12.1 abs. atms. As we already said every measurement done with protonium (or deuterium) in the target was followed by a measurement taken with the target filled with helium at the same pressure.

In fig. 4 are plotted as function of the gas pressure the differences between the total number of events obtained with protonium (or deuterium) in the target and the corresponding total number of events obtained with helium at the same pressure; the corresponding differential time distributions are shown in figs. 5, 6, 7.

The prompt events that is those events due to muons stopping directly in gold, were contained in an initial peak the width of which was defined by the time resolution of the TPHC ($\sim$10 ns). Their number per monitor was:

(14) $$M = 0.139 . 10^{-2}/\text{Monitor}$$

10.

As expected, we found M to be quite independent from the pressure of the gases filling the target.

In Table III the values of N, deduced from (14), relative to the different experimental conditions are given.

TABLE III

| Gas | Pressure (Atm. abs.) | $(N/Monitor) \cdot 10^8$ |
|---|---|---|
| $H_2$ | 26.2 | $14030 \pm 330$ |
| $H_2$ | 5.4 | $3050 \pm 83$ |
| $D_2$ | 12.1 | $6820 \pm 193$ |

Values of N, the total number of gold mesic $\gamma$-rays which one would count if all the muons stopped in hydrogen were transferred from the mesoatoms to the gold plates.

b) - Analysis of the data.

1) Filling gas: protonium.

The process is:

(1)                             $\mu p + p = \mu p + p$

Since the expected cross-section for this process is such that the corresponding mean free path $\lambda$ is comparable to the distance $d_o$ it is not possible to describe the random motion of the $\mu p$ system by means of a simple diffusion equation. The different expected yields to be compared with the experimental ones of fig. 5 and 6, were then obtained by a Monte Carlo calculation. The quantities in which we are interested, i.e. $\bar{v}_{\mu p}$ and the cross-section for process (1) were obtained using a $\chi^2$ analysis of the experimental distributions, fitted with the results of the Monte Carlo calculation. A first orientative analysis was done by performing the Monte Carlo calculation under the following simplifying assumptions:

a) - The $\mu p$ atom is slowed down from an average initial kinetic energy $\bar{E}_i$, and the cross-section for the process (1) was assumed to be unique and independent of the energy of the colliding systems.

b) - The angular distribution of the scattered $\mu p$ atom was assumed to be isotropic in the laboratory system.

c) - After each collision the residual kinetic energy E' of the system was taken to be:

(15)         $$E' = E \frac{(m^2 + M^2)}{(m + M)^2} + E_{H_2} \frac{2 m M}{(m + M)^2}$$

where :   m is the mass of the mesoatom
          M is the mass of the molecules of the gas (hydrogen)
          E is the kinetic energy of the mesoatom before the collision
          $E_{H_2}$ is the thermal energy of the molecules.

In the Monte Carlo calculation we have allowed the $\mu$p system to undergo the processes (1), (10) and (13), and the mesoatom history has been followed up to 2,5 microseconds.

The results obtained at different protonium pressures were consistent among them, and combining the two results together we got[x]:

(16)          $E_i = (0.16 \pm 0.03)$ eV;          $\sigma = (6.5 \pm 0.13) \cdot 10^{-21}$ cm$^2$

Whereas the run at 26.2 atm. contributes mostly to the determination of $\sigma$ (for the case at 5.4 atm. the corresponding $\lambda$ is too high compared to $d_o$) the value of $\overline{E}_i$ is sufficiently well determined also by the run at 5.4 Atms.

The calculations, moreover, show that the obtained value of $\overline{E}_i$ is practically independent from the value assumed by $\sigma$.

The values (16) obtained from this simplified analysis show that at least after a time t > 20 nsec practically all the $\mu$p are in a singlet state this result is valid without making any assumption on the value of $a_g$. In fact suppose that the above statement is not true, which means $\sigma_{1 \to 0}$ small, in any case one can write:

(17)                    $\sigma_o \lesssim 4\sigma = 2.6 \cdot 10^{-20}$ cm$^2$

since at least 1/4 of the total number of $\mu$p present in the $H_2$ gas are in the singlet state.

But from fig. 8 it is clear that the values of $\sigma_{1 \to 0}$ possible because of the limits (17) imposed on $\sigma_o$ are much too big to agree with the hypothesis that $\sigma_{1 \to 0}$ is small.

Therefore from this first analysis we conclude that the experimental data (under the assumption that $a_u = 5$) require the following picture: the $\mu$p atoms (which initially are in a statistical mixture of F = 1 and F = 0 states) from a certain initial energy E > $\Delta$E, in a very short time ($\lesssim$ 20 ns) are slowed down to an energy of about 0.16 + 0.03 eV (i. e. very near to the value of the transition energy $\Delta$E): from this moment the total scattering cross-section assumes the value (16), the $\mu$p mesoatom being mostly in a singlet state.

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(x) - The ratio $(\chi / \overline{\chi})^2$ was equal to 1.25 or better and the maximum number of points was 50.

A second more complete analysis was then done by taking the cross-section for process (1) to be given by the expressions (4) and (6) for a kinetic energy E of the mesoatom respectively bigger and smaller than the transition energy $\Delta E$. The value for $a_u$ was put equal to 5 (see paragraph 2). The simplifications b) and c) were retained for all collisions. Moreover the parameter, used in the Monte Carlo calculation.

(18)
$$\lambda = \frac{1}{g\sigma}$$

was substituted by,

(19)
$$\lambda' = \frac{\lambda}{1 - \overline{\cos \theta}}$$

with

(20)
$$\overline{\cos \theta} = \frac{2}{3}\frac{1}{A}.$$

and

(21)
$$A = \frac{v_{\mu p} + \overline{v}}{\frac{m}{M} v_{\mu p} - \overline{v}}$$

where :   $\overline{v}$ is the average velocity of the molecules of the gas and $v_{\mu p}$ is the velocity of the mesoatom.

From this further analysis we obtained that a mesoatom was quickly slowed down to the energy 0.16 eV, given in (16), starting from an initial energy $E_o$:

(22)
$$E_o = (0.55 \pm 0.2) \text{ eV}$$

The best fit analysis has given for $a_g$ two possible values which are listed in Table IV. The $\chi^2$ obtained in this second analysis were always better (for about 20%) than the ones obtained in the previous more simple analysis. The final results are listed in Table IV (see also figg. 5 and 6);

TABLE IV

| $H_2$ pressure T=300°K | $a_{g1}$ | $a_{g2}$ | $\sigma_o \cdot 10^{21}$ cm$^2$ | n° of points | $\chi^{2(x)}$ | $a_u$ |
|---|---|---|---|---|---|---|
| 26.2 | $-18.8 \pm 0.2$ | $-11.2 \pm 0.2$ | $7.4 \pm 0.8$ | 50 | 50 | +5 (o) |
| 5.4 | $-19.2 \begin{smallmatrix}+0.3\\-0.8\end{smallmatrix}$ | $-10.8 \begin{smallmatrix}+0.8\\-0.3\end{smallmatrix}$ | $8.9 \begin{smallmatrix}+1.5\\-3.0\end{smallmatrix}$ | 11 | 13 | +5 (o) |

Experimental results for the process: $\mu p + p \rightarrow \mu p + p$.

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(x) - The fit gave for $a_{g1}$ a $\chi^2 = 49$ whereas for $a_{g2}$ the resulting $\chi^2$ was $\chi^2 = 50$.

(o) - Assumed value.

By combining the two results on $\sigma_o$ one gets

(23) $$\sigma_o = (7.6 \pm 0.7) \cdot 10^{-21} \text{ cm}^2$$

In the errors of $a_{g1}$ and $a_{g2}$ that we have quoted we have not included the errors by which $a_u$ (which is the result of a calculation - see paragraph 2) may be affected.

By comparing the results of the calculations presented in ref. (7) and (6) one would put on $a_u$ an error of the order of 5%; with this included we get:

$$a_{g1} = 18.8 \pm 0.8 \quad , \quad a_{g2} = 11.2 \pm 0.8$$

2) - Filling gas: Deuterium.

The process is:

(2) $$\mu d + d = \mu d + d$$

In this case the expected value of the collision mean free path is much smaller than $d_o$ for all energies of the $\mu d$ mesoatom (see Table II); hence it is expected that the $\mu d$ atom is rather quickly thermalized.

In performing the Monte Carlo calculation we have assumed that the value of the cross section for the process (2) is independent of the energy of the mesoatom and that the $\mu d$ system initially had an energy $E_o = 0.55$ eV; i. e. the value (22) found for the $\mu p$ mesoatom formed in protonium.

All the other assumptions made in the Monte Carlo calculations performed to describe the motion of the $\mu p$ system were retained.

In Table V the results of this analysis are shown (see also fig. 7):

TABLE V

| Gas | pressure at 300°K | $\sigma \cdot 10^{19}$ cm$^2$ | $\chi^2$ | n° of points |
|-----|-------------------|-------------------------------|----------|--------------|
| $D_2$ | 12. 1 | 0. 5 $\pm$ 0. 2 | 37 | 29 |

Experimental results for the process: $\mu d + d \rightarrow \mu d + d$.

## 6. DISCUSSION AND CONCLUSIONS. -

As we have already said, in our experimental conditions the mesoatoms collide against bound and not free protons (deuterons). However from the results of the calculations of Dzhelepov et al.[5] it is possible to show that for a $\mu$-p atom in a singlet state the value of the parameter defined in (18) should be increased, because of the molecular binding of the protons of the gas, by not more than 10%. The correction, hence, seems quite small especially if compared to the experimental errors with which the scattering cross-section is determined. We will assume that also for the $\mu$-d mesoatom the effect of the molecular binding is negligible.

In Tables VI and VII our results as well as those obtained by Dzhelepov et al.[10] are presented together with the results of the calculations performed by Zel'dovich et Gershtein and Cohen et al.[6,7]:

TABLE VI

| Authors | $E_o$ eV | $\sigma_o \cdot 10^{21}$ cm$^2$ | $a_u$ | $a_g$ | $\sigma_{1 \to o} \cdot 10^{19}$ cm$^2$ | $\sigma_{1,o} \cdot 10^{19}$ cm$^2$ |
|---|---|---|---|---|---|---|
| Present experiment | $0.55 \pm 0.20$ | $7.6 \pm 0.7$ | $5^{(x)}$ | $-18.8 \pm 0.8$ $-11.2 \pm 0.8$ | $8.7 \pm 0.7$ $4.0 \pm 0.5$ | $8.8 \pm 0.7$ $4.1 \pm 0.5$ |
| Dzhelepov et al. (5) | few eV | $167 \pm 30$ | 5 | $-33. \pm 2.$ $+ 3. \pm 2.$ | $16.8 \pm 3.$ $0.049 \pm 0.013$ | $24. \pm 5.$ $1.74 \pm 0.3$ |
| Zel'dovich and Gershtein (6) | --- | 1.2 | 5.25 | $-17.3$ | 7.8 | 7.9 |
| Cohen et al. (7) | --- | 8.2 | 5. | $-11.$ | 3.9 | 4.0 |

Summary of theoretical and experimental results for the process: $\mu p + p \to \mu p + p$.

TABLE VII

| Authors | $b_u$ | $b_g$ | $\sigma \cdot 10^{19}$ cm$^2$ |
|---|---|---|---|
| Present experiment | ---- | ---- | $0.5 \pm 0.2$ |
| Ermolov et al. (16) | ---- | ---- | $1.5 \pm 0.5$ |
| Zel'dovich and Gershtein (6) | 5.76 | 6.67 | 3.3 |
| Cohen et al. (7) | ---- | ---- | 3.5 |

Summary of theoretical and experimental results for the process: $\mu d + d \to \mu d + d$.

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(x) - This value has been assumed equal to 5 as in (5) with an error of $\sim 5\%$.

In reference to these tables we can conclude:

i) - Our experimental results agree rather well with the pre
dictions resulting from the calculations of Zel'dovich and Gershtein and
Cohen et al. on reaction (1); the calculation of $a_g$ (hence of $\sigma_o$) is extre
mely difficult given the quasi resonant character of the $\Sigma_\varphi$ state (see
par. 2 and fig. 8). The agreement is less good for what is concerning the $\mu$d
mesoatom scattering on deuterium (reaction (2)).

ii) - The value that we have found for the initial energy $E_o$ of
the $\mu$p mesoatom is of the same order as the one found by Dzhelepov
et al. ; in any case it is definitively much higher than the mesoatom ther
mal energy (0.037 eV).

iii) - Our experimental results are in quantitative disagreement
with those obtained by Dzhelepov et al. for what is concerning the value of
the scattering cross-section, in the F = 0 state, of the $\mu$p mesoatom on
hydrogen. For what is concerning the scattering cross-section for reac-
tion (2), the agreement with the value obtained by the Dubna's[16] group
is fair.

We close this paper answering to the two questions, raised in
the introduction, relative to a future possible experiment to measure the
$\mu^-$ capture rate by protons in hydrogen gas.

a) - The value of $\sigma_{1 \to o}$ is in any case so high that even at few
atmospheres the transition of the   p mesoatom from the triplet to the
singlet state takes place within few nanoseconds.

However if the mesoatom has an initial energy higher than the
transition energy $\Delta E (= 0.18$ eV), as it is the case, then the number of
$\mu$p mesoatoms in the triplet state will decrease only when the mesoatoms
reach an energy $E < \Delta E$. If we take for $E_o$ and for $\sigma_{1, o}$ the values found
in this experiment (see Table VI) we find that the average time $\bar{t}$ necessa
ry for a $\mu$p mesoatom to reach the singlet state is, for a pressure of the
hydrogen equal to 10 Atms. (300°K), about 20  ns.

It is clear that $\bar{t}$ is inversely proportional to the hydrogen pres
sure.

b) - We can now evaluate the number of $\mu$p mesoatoms which
diffuse up to the surface of the hydrogen container. Let us call R the ra
tio between the number of muons transferred to the wall after a time $t_o$,
and the number of muons decaying after the same time. Assuming that
muons are uniformely stopped in a 30 cm $\emptyset$ vessel at a pressure of 10
atms. , the behaviour of R is shown in fig. 9. The minimum value (2.3%)
of R is big, and may represent a serious source of delayed background.

16.

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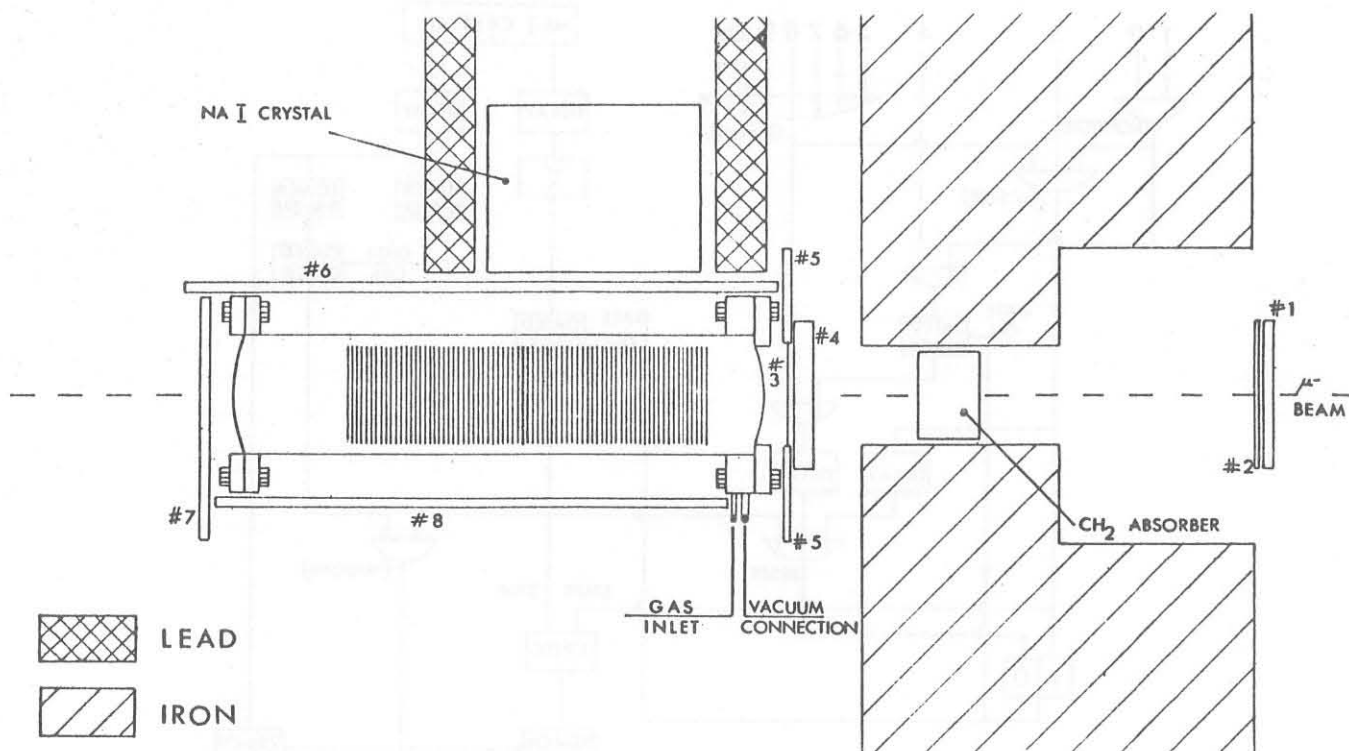(16)- P. F. Ermolov et al., JETP $\underline{50}$, 1235 (1966).

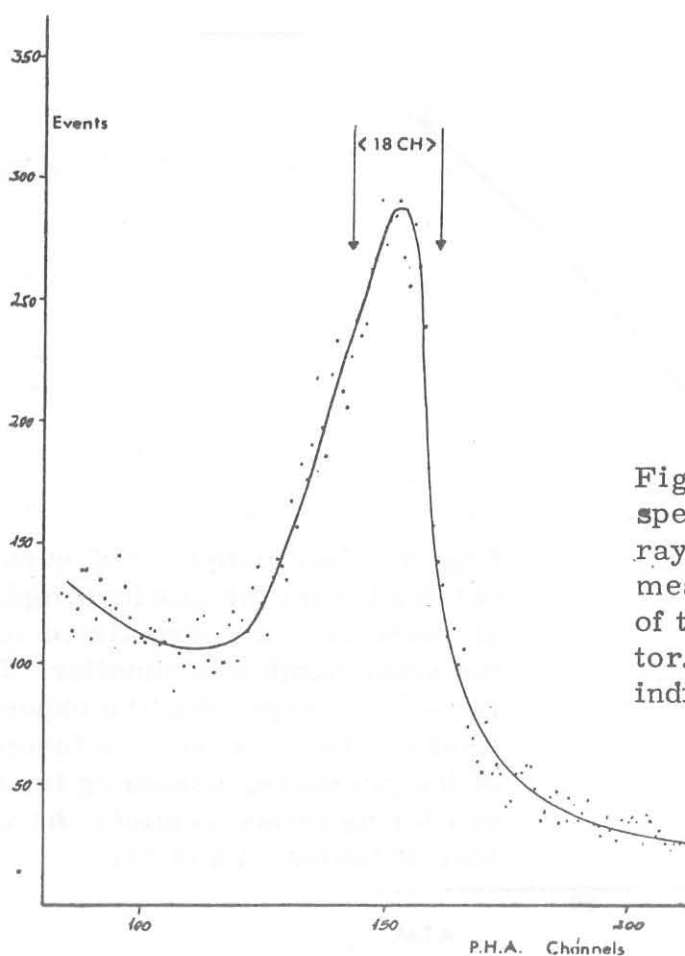Fig. 1 - Experimental layout - target and counters.



Fig. 2 - NaI pulse-amplitude spectrum of the gold mesic $\gamma$ rays. The spectrum has been measured to fix the threshold of the window slow discrimina̱tor. The chosen values are indicated by the arrows.
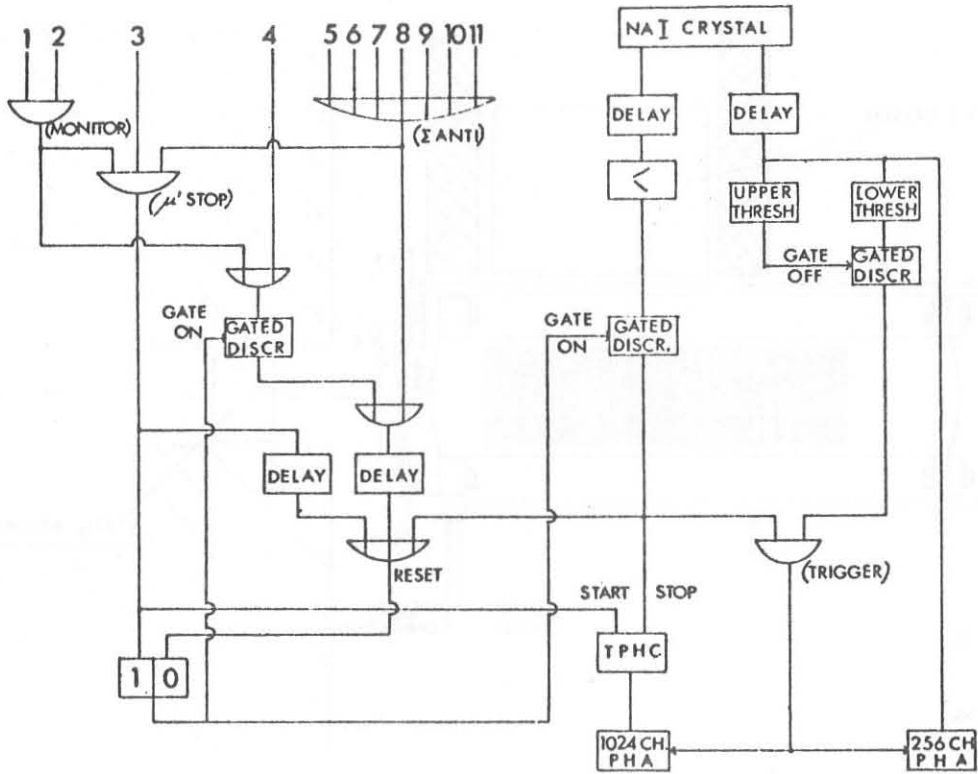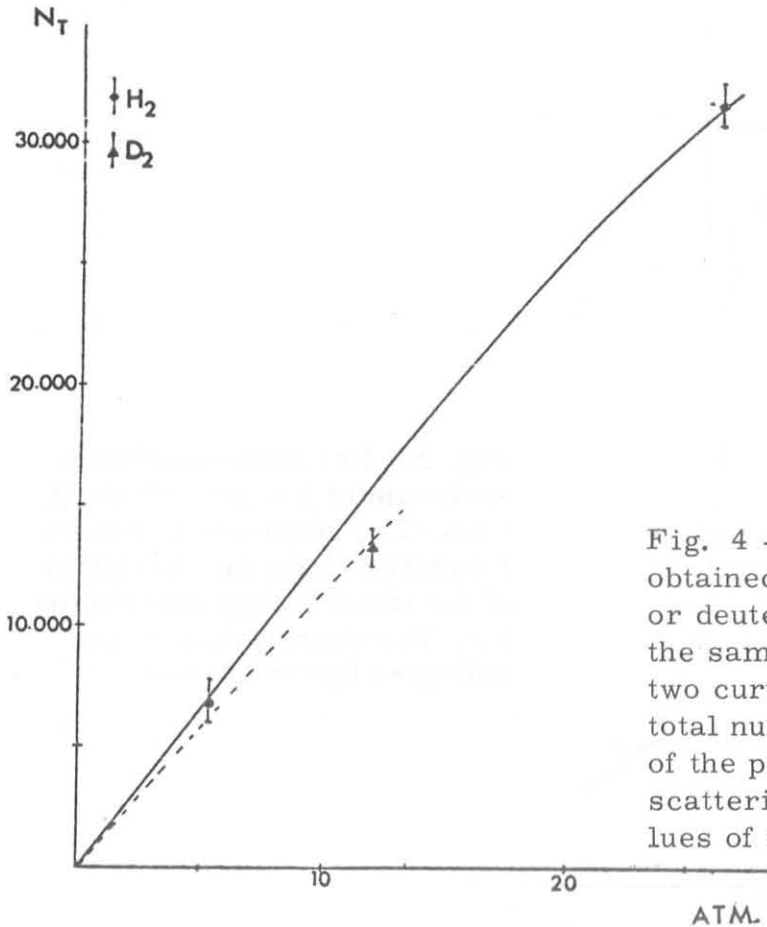
18.



Fig. 3 - Block diagram of the electronics.



Fig. 4 - Total number of events obtained from the gas (hydrogen ● or deuterium ▲ ) normalized to the same number of monitor. The two curves represent the expected total number of events as function of the pressure, assuming for the \ scattering cross-sections the values of tables VI and VII.
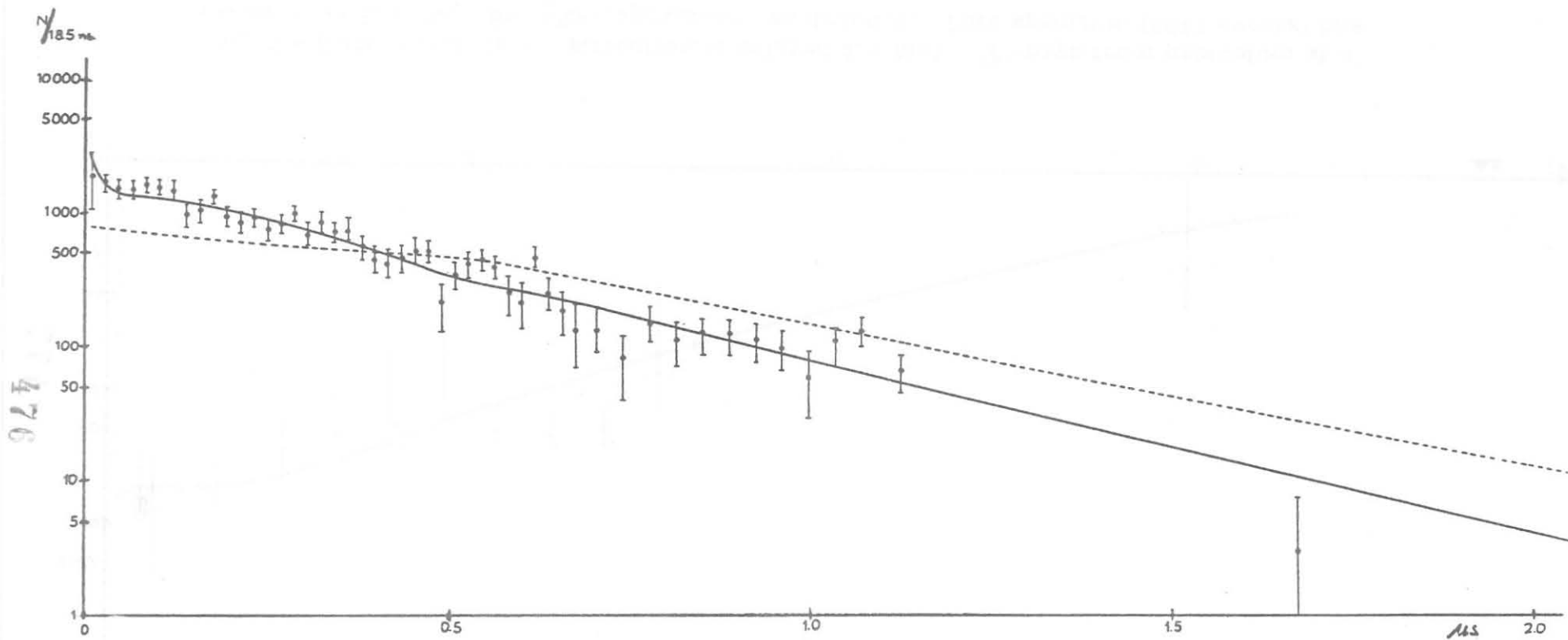
Fig. 5 - Differential time distribution of delayed 5. 8. MeV $\gamma$-rays from protonium at a pressure of 26. 2 atm. abs. after background subtraction. This spectrum (31705 events) has been obtained for 30. 6 x 10$^7$ Monitor.

The dashed line corresponds to the 5. 8 MeV $\gamma$-rays distribution in the case in which the velocities of the $\mu$p systems are thermal since time t = 0.

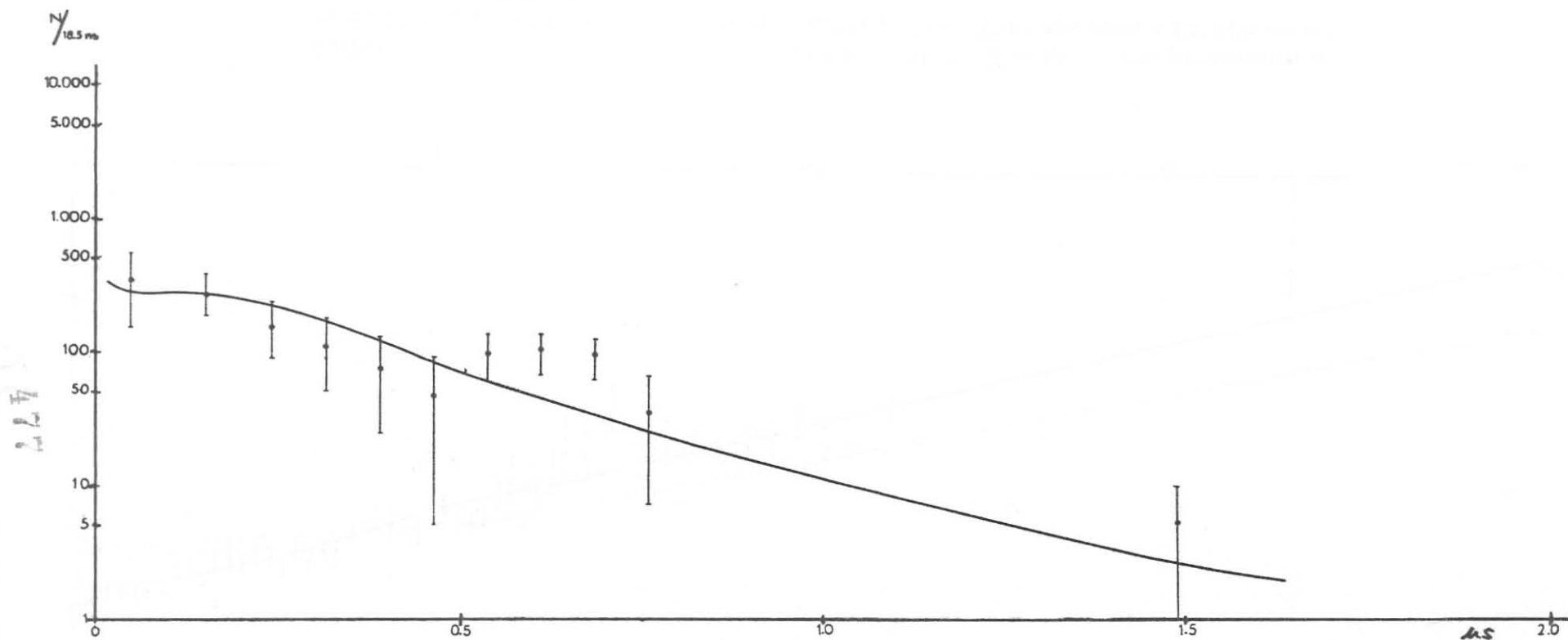Fig. 6 - Differential time distribution of delayed 5. 8 MeV $\gamma$-rays from protonium at a pressure of 5. 4 atm. abs. after background subtraction. This spectrum (6647 events) has been obtained for 29. 1 x 10⁷ Monitor.
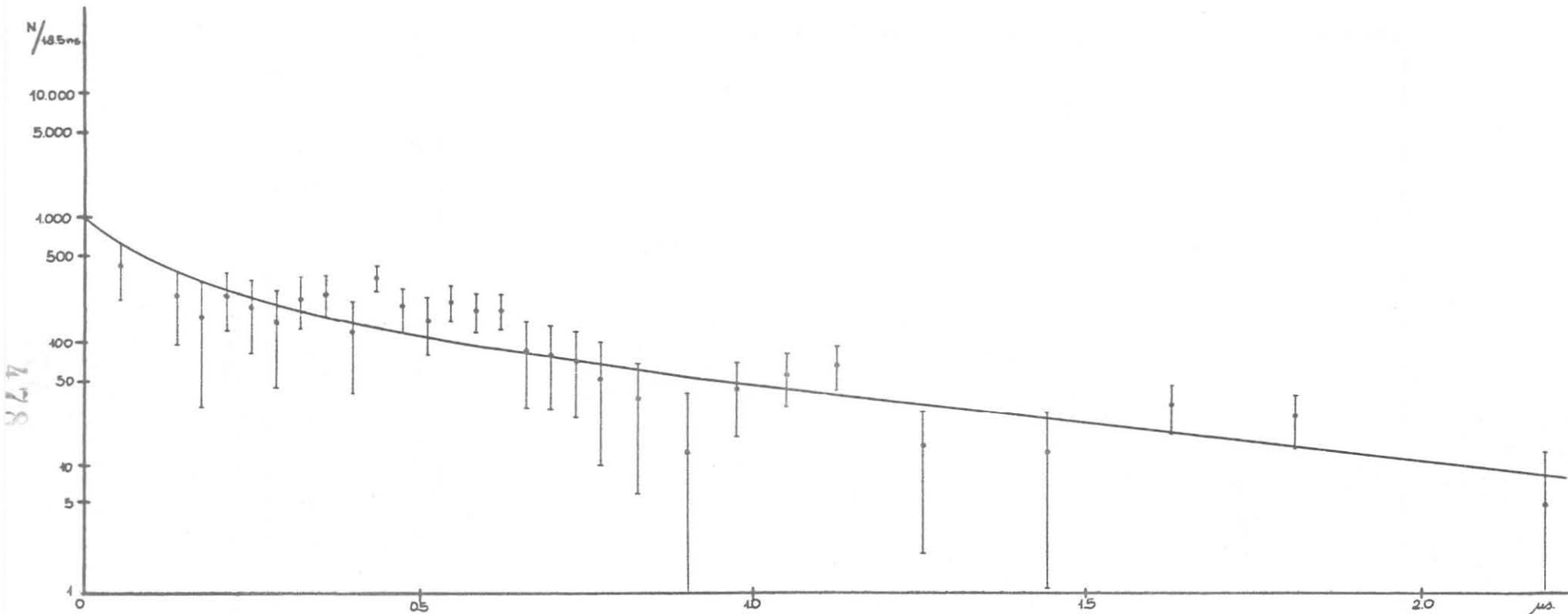
Fig. 7 - Differential time distribution of delayed 5.8 MeV $\gamma$-rays from deuterium at a pressure of 12.1 atm. abs. after background subtraction. This spectrum (11854 events) has been obtained for $27.0 \times 10^7$ Monitor.
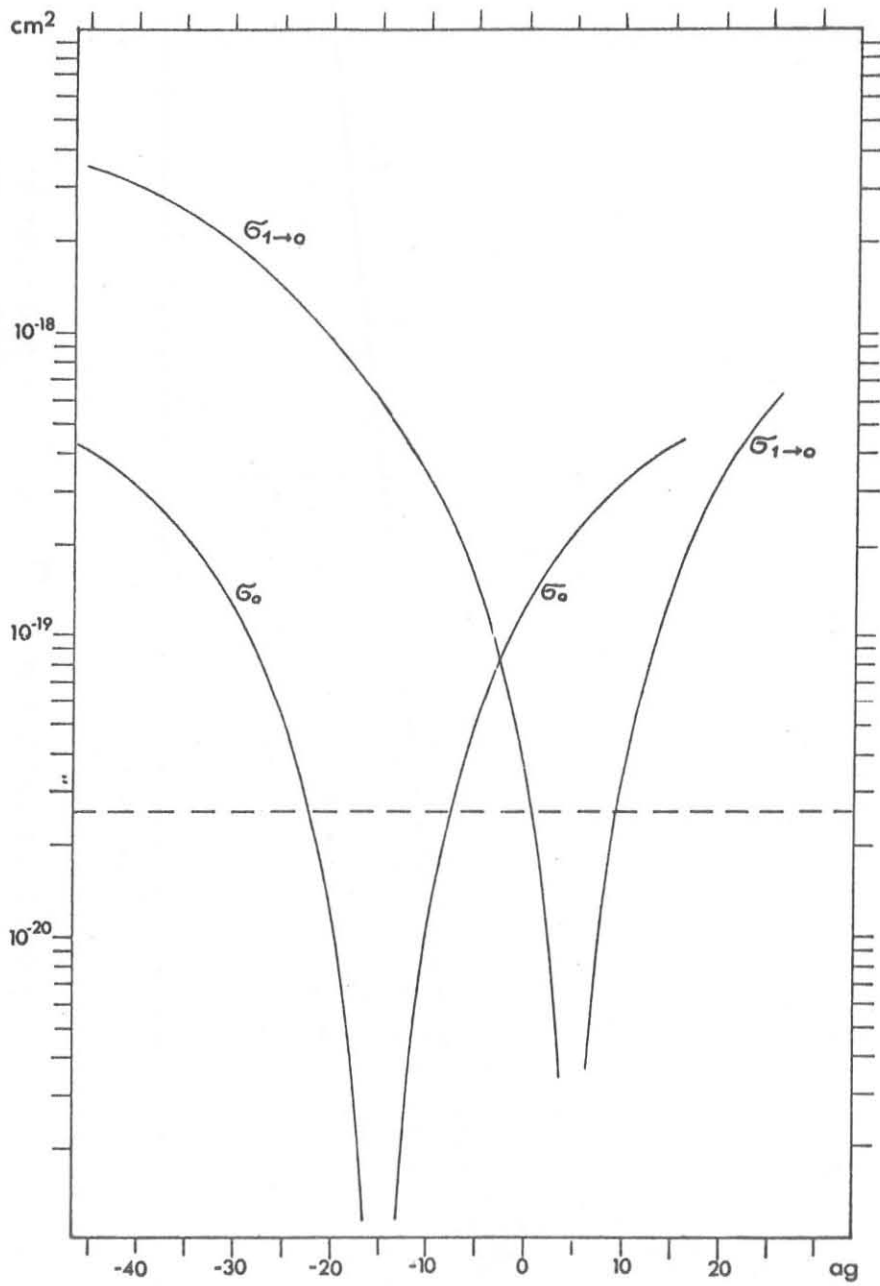
22.



Fig. 8 - Values of the cross sections $\sigma_{1 \to 0}$ and $\sigma_0$ as a function of $a_g$, assuming $a_u = 5$; $a_g$ and $a_u$ are in units of $\hbar^2 / M_\mu e^2$.

## MONTECARLO H$_2$   8 ATM.
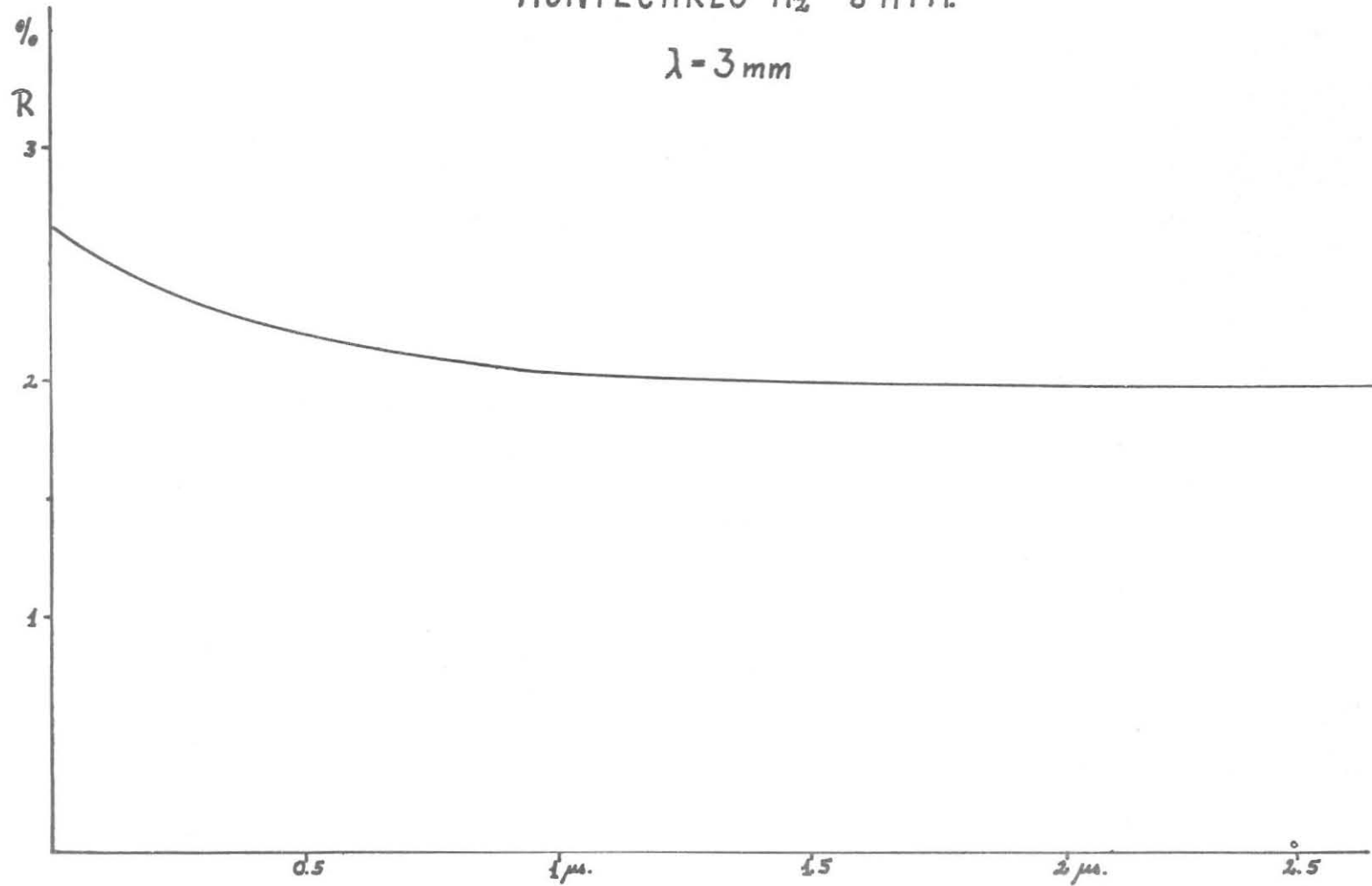### $\lambda$ = 3 mm

Fig. 9 - Time behaviour of

$$R = \frac{\text{number of muons transferred to the walls after a time to}}{\text{number of decaying muons after to}}$$