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ON THE RELATION BETWEEN THE ( $\omega$ - $\phi$ ) MIXING ANGLE AND THE  
RADIATIVE DECAY OF VECTOR MESONS (\*)

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ON THE RELATION BETWEEN THE ( $\omega - \varphi$ ) MIXING ANGLE  
AND THE RADIATIVE DECAY OF VECTOR MESONS

M. Fidecaro and M. Giorgi

1. In this note we wish to point out the possibilities of determining the  $\omega - \varphi$  mixing angle through the ratio of the  $R^2 = \Gamma(\omega \rightarrow \pi\gamma)/\Gamma(\rho \rightarrow \pi\gamma)^2$ ; here  $\omega$  and  $\varphi$  indicate the physical particles, and  $\omega_1$  and  $\omega_8$  the states which belong to the singlet and to the octet representation of  $SU(3)$ , respectively. The determination is based on the relation

$$R = \frac{\sqrt{3}}{\sin \vartheta} (1 \pm \alpha), \quad (1)$$

where  $|\alpha|$  is reasonably  $\ll 0.20$ , as will be evaluated in the following. However, it could be known exactly, if the width  $\Gamma(\varphi \rightarrow \pi\gamma)$  were measured.

2. While we are aware that Eq. (1) can be found in many papers and lecture notes<sup>2-4</sup>, for the benefit of experimentalists we think it is worth while to derive in a rather detailed way the expression for the decay rate of

$$V \rightarrow P + \gamma \quad (2)$$

where  $V, P$  are a vector meson and a pseudoscalar meson, respectively. Let us first show that the electromagnetic current transforms as an octet of  $SU(3)$ , or better as the member  $A_1^1$  of such a multiplet. The charge  $Q$  is defined as

$$Q = T_3 + Y/2 \quad (3)$$

where  $T_3$ , the third component of the isotopic spin, and  $Y$  the hypercharge, are  $SU(3)$  generators.

Let us call  $u_\alpha$ 's the basic triplet states of SU(3),  $u_1$  and  $u_2$  being the isospin doublet, and  $u_3$  the isospin singlet. For the mesons, that is for states of the type  $\bar{u}_\alpha u_\beta$ ,  $T_3$  and  $Y$  can be expressed in terms of the operators of creation and annihilation of the  $u$ 's :

$$T_3 = \frac{\bar{u}_1 u_1 - \bar{u}_2 u_2}{2}, \quad Y = \frac{\bar{u}_1 u_1 + \bar{u}_2 u_2 - 2\bar{u}_3 u_3}{3} \quad (4)$$

These expressions may be written in terms of the components of the octet  $A_\beta^\alpha$

$$A_\beta^\alpha = \bar{u}_\alpha u_\beta + \bar{u}_\beta u_\alpha - 2\delta_{\beta\alpha} \bar{u}_\gamma u_\gamma$$

as

$$T_3 = \frac{1}{2} (A_1^1 - A_2^2), \quad (5)$$

$$Y = A_3^3. \quad (6)$$

By combining Eqs.(5) and (6), one gets

$$Q = \frac{1}{2} (A_1^1 - A_2^2 - A_3^3). \quad (7)$$

$A_\alpha^\beta$  being a traceless tensor, the following relation holds

$$A_2^2 + A_3^3 = -A_1^1 \quad (8)$$

and from Eqs. (7) and (8)

$$Q = A_1^1. \quad (9)$$

But the charge is the integral of the fourth component of the four vector  $\vec{J}$ . Because of the fact that the Lorentz group commutes with the SU(3) group<sup>\*</sup>), the current itself must transform as the central component of the octet. As a consequence, the meson current coupled to the electromagnetic field should behave as the first component of an octet. By coupling the vector and the pseudoscalar mesons, four such octets can be constructed, because

$$\begin{aligned} \underline{1} \otimes \underline{8} &= \underline{8}; \underline{8} \otimes \underline{1} = \underline{8}; \\ \underline{8} \otimes \underline{8} &= \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10} \oplus \overline{\underline{10}} \oplus \underline{27}. \end{aligned}$$

The following currents can be written (the space-time index  $\mu$  being omitted on the vector field)

$$1. \quad (\bar{V}^0) P_\alpha^\beta$$

from a vector meson singlet and a pseudoscalar meson octet;

$$2. \quad (\bar{V}_\alpha^\beta) P^0$$

from a vector meson octet and a pseudoscalar meson singlet;

$$3. \quad (\bar{V}_\alpha^\beta) P_\gamma^\delta$$

from a vector meson octet and a pseudoscalar meson octet.

From the last current, two octets can be extracted, which will be indicated as  $F_\alpha^\beta$  and  $D_\alpha^\beta$ :

$$F_\alpha^\beta = \bar{V}_\alpha^\gamma P_\gamma^\beta - \bar{V}_\gamma^\alpha P_\beta^\gamma \quad (10)$$

$$D_\alpha^\beta = \bar{V}_\alpha^\gamma P_\gamma^\beta + \bar{V}_\gamma^\alpha P_\beta^\gamma - \frac{2}{3} \delta_\alpha^\beta \bar{V}_\gamma^\delta P_\delta^\gamma, \quad (11)$$

\*) They are independent because the Lorentz group refers to the degree of freedom of the particle in space time, while the SU(3) group refers to internal degrees of freedom.

where  $V_i^j$ ,  $P_\ell^k$  are the following matrices

$$V_i^j = \begin{pmatrix} \frac{\rho_0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho_0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & K^{*0} & -\frac{2\omega_8}{\sqrt{6}} \end{pmatrix} \quad (12)$$

$$P_\ell^k = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{2\eta}{\sqrt{6}} \end{pmatrix} \quad (13)$$

$$V^0 = \omega_1$$

$$P^0 = X^0$$

(No  $X^0 - \eta$  mixing is considered here).

The following should be observed:

1.  $V^0 = \omega_1$  is a u-spin singlet ( $u = 0$ ).

$P_1^1$  is the third component of a u-spin triplet ( $u = 1, u_3 = 0$ ).

In the frame of  $SU(3)$ , u is a good quantum number.

Therefore

$$\langle \omega_1 | \left( \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \right) \gamma \rangle = 0$$

or

$$\sqrt{3} \langle \omega_1 | \pi^0 \gamma \rangle = \langle \omega_1 | \eta \gamma \rangle \quad (14)$$

and the interaction takes the form:

$$\sim g \bar{\omega}_1 \pi^0 \quad (15a)$$

2. The interaction which involves  $(\bar{V}_\beta^\alpha) P^0$  does not exist because of energy conservation.

3.  $F_1^1$  and  $D_1^1$  are states of charge conjugation +1 and -1, respectively. Because the photon has charge conjugation -1, only  $(D_1^1)_\mu$  can interact with  $(A_1^1)_\mu$ . The interaction takes the form:

$$\sim \sqrt{3} f \left( \frac{1}{3} \bar{\rho}^0 \pi^0 + \frac{1}{\sqrt{3}} \bar{\omega}_8 \pi^0 + \frac{1}{\sqrt{3}} \bar{\rho}^0 \eta - \frac{1}{3} \bar{\omega}_8 \eta + \dots \right) \quad (15b)$$

By definition:

$$|\omega\rangle = |\omega_8\rangle \sin \vartheta + |\omega_1\rangle \cos \vartheta \quad (16)$$

$$|\varphi\rangle = -|\omega_8\rangle \cos \vartheta + |\omega_1\rangle \sin \vartheta \quad (17)$$

From Eqs. (15a) and (15b) one gets:

$$\langle \omega | \pi^0 \gamma \rangle \sim (g \cos \vartheta + f \sin \vartheta)$$

$$\langle \varphi | \pi^0 \gamma \rangle \sim (f \cos \vartheta - g \sin \vartheta)$$

$$\langle \rho | \pi^0 \gamma \rangle \sim \frac{1}{\sqrt{3}} f.$$

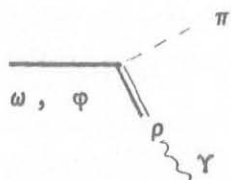
Therefore

$$R^2 = \frac{\Gamma(\omega | \pi^0 \gamma)}{\Gamma(\rho | \pi^0 \gamma)} = \frac{3}{\sin^2 \vartheta} \left[ 1 - \frac{\langle \varphi | \pi^0 \gamma \rangle}{\langle \rho | \pi^0 \gamma \rangle} \frac{\cos \vartheta}{\sqrt{3}} \right]^2 \quad (18)$$

Equation (18) is an equation in which  $\vartheta$  could be solved exactly, if the three amplitudes  $(\omega | \pi^0 \gamma)$ ,  $(\rho | \pi^0 \gamma)$ , and  $(\varphi | \pi^0 \gamma)$  were measured. Let us note, however, that the term

$$\alpha = \frac{\langle \varphi | \pi^0 \gamma \rangle}{\langle \rho | \pi^0 \gamma \rangle} \frac{\cos \vartheta}{\sqrt{3}} \text{ is small in comparison to } 1 .$$

The most safe evaluation of that term is probably the one which assumes a pole model for the decay of  $\omega$  and  $\varphi$ :



which gives 
$$\frac{\Gamma(\varphi \rightarrow \pi \gamma)}{\Gamma(\varphi \rightarrow 3 \pi)} \leq \frac{\Gamma(\omega \rightarrow \pi \gamma)}{\Gamma(\omega \rightarrow 3 \pi)}$$

(the diagram in which the  $\omega$  takes the place of the  $\rho$  is forbidden by G-parity).

From the following experimental numbers (see M. Ross tables)

$$\begin{aligned} \Gamma_{\text{tot}}(\omega) &= 12 \text{ MeV}, \Gamma_{\text{tot}}(\varphi) = 3.6 \text{ MeV} \\ \Gamma(\omega \rightarrow 3 \pi) &= 0.86 \times 12 = 10.3 \text{ MeV} \\ \Gamma(\varphi \rightarrow 3 \pi) &= 0.15 \times 3.6 = 0.54 \text{ MeV} \\ \Gamma(\omega \rightarrow \pi \gamma) &\leq 0.9 \text{ MeV} \end{aligned}$$

one gets:

$$\Gamma(\varphi \rightarrow \pi \gamma) < 0.045 \text{ MeV, and}$$

$$\frac{\langle \varphi | \pi^0 \gamma \rangle}{\langle \rho | \pi^0 \gamma \rangle} < \left( \frac{0.045}{0.1} \right)^{1/2} \left( \frac{M_\rho}{M_\varphi} \right)^{3/2} = 0.67 \times 0.65 = 0.435.$$

Therefore

$$|\alpha| < 0.20$$

where  $\vartheta$  has been taken  $\sim 35^\circ$ , the value predicted by Okubo, and Eq. (18) becomes



$$\sin \vartheta = \frac{\sqrt{3}}{R} (1 \pm |\alpha|) .$$

R.H. Socolow pointed out to us that in the frame of pole approximation (Gell-Mann, Sharp and Wagner model) the ratio 0.05 between the widths  $\Gamma(\phi \rightarrow 3\pi)/\Gamma(\omega \rightarrow 3\pi)$ , indicated above, should be depressed by a factor  $34 \pm 2$  to give the ratio  $\Gamma(\phi \rightarrow \pi\gamma)/\Gamma(\omega \rightarrow \pi\gamma)$ . This factor comes from a computation of Yellin [unpublished, quoted in A.J. McFarlane and R.H. Socolow, Phys.Rev. 144, 1194 (1966)] and takes into account the fact that the  $\rho$ , because of its mass and its width, is virtual in most of the decay  $\omega \rightarrow \rho\pi$ , and is real in most of the decay  $\phi \rightarrow \rho\pi$ .

In this context

$$|\alpha| = 0.03 .$$

In the quark model,  $|\alpha|$  becomes zero for a mixing angle of  $35^\circ$ , and 0.11 for an empirical mixing angle of  $40^\circ$ ; see, for instance, the formulae derived by Dalitz<sup>3)</sup> which give

$$\frac{\langle \phi | \pi\gamma \rangle}{\langle \rho | \pi\gamma \rangle} = \frac{\left( \sqrt{\frac{2}{3}} \sin \vartheta - \sqrt{\frac{1}{3}} \cos \vartheta \right) \mu_P}{\frac{1}{3} \mu_P} .$$

3. The above considerations would be invalidated if the current does not transform as an octet. For instance, it has been suggested that  $J = T_3 + Y/2 + c$ , where  $c$  is the "charm" quantum number. The present experimental evidence is against the existence of such a quantum number, as well as against the existence of the Bronzan-Low quantum number  $A$ , whose existence would give<sup>5)</sup>

$$\frac{\langle \rho | \pi\gamma \rangle}{\langle \omega | \pi\gamma \rangle} \ll 1 .$$

4. The theoretical facts presented in this note has been clarified with the help of W. Alles, J. Bell, S. Bergia, N. Cabibbo, B. Diu, S.L. Glashow, M. Pusterla and J.K. Zalewsky; their patience is hereby acknowledged.

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