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DEEP INELASTIC PROCESSES AND THE EQUATIONS OF MOTION

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Abstract

We show that the Politzer theorem on the equations of motion implies approximate constraints on the quark correlator, restricting considerably, for sufficiently large Q^2, the number of independent distribution functions that characterize the internal structure of the nucleon, and of independent fragmentation functions. This result leads us to suggesting an alternative method for determining transversity. Moreover our approach implies predictions on the Q^2-dependence of some azimuthal asymmetries, like Sivers, Qiu-Sterman and Collins asymmetry. Lastly, we discuss some implications on the Burkhardt-Cottingham and Efremov-Leader-Teryaev sum rules.

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1 Introduction

The problem of calculating inclusive cross sections at high energies and high momentum transfers has become quite important in the last two decades, during which a lot of experimental data on deep inelastic processes have been accumulated. In particular we refer to deep inelastic scattering (DIS)[1-12], semi-inclusive DIS (SIDIS)[13-26], Drell-Yan (DY)[27-32], e^+e^- annihilation into two back-to-back jets[33], while analogous experiments have been planned recently[34-39]. One of the aims of high energy physicists is to extract from data distribution and/or fragmentation functions, especially if unknown. Among them, the transversity[40-43] is of particular interest, since it is the only twists-2 distribution function for which very poor information[44,45] is available till now. But also transverse momentum dependent (TMD) functions - especially the T-odd ones - are taken in great consideration; for instance, knowledge of the Collins fragmentation function[46] or of the Boer-Mulders function[47] could help extracting transversity, which is chiral-odd and therefore couples only with chiral-odd functions. Moreover, TMD functions are involved in several intriguing azimuthal asymmetries, like the already mentioned Collins[46] and Boer-Mulders[47] effects, or the Sivers[48,49], Qiu-Sterman[50–52] and Cahn[53,54] effects, which, in part, have found experimental confirmation[19–23,25,33] and, in any case, have stimulated a great deal of articles[55-64]. Lastly some questions remain open, among which the parton interpretation of the polarized structure function g_2[65,66]. Obviously, all of these data and kinds of problems are confronted with the QCD theory and in this comparison short and long distance scales are interested, so that the factorization theorems[67-70] play a quite important role in separating the two kinds of effects. Strong contributions in this sense have been given by Politzer[71], Ellis, Furmansky and Petronzio[72,73](EFP), Efremov, Radyushkin and Teryaev[74,75], Collins, Soper and Sterman[76,77,69], and Levelt and Mulders[78](LM).

In the present paper we propose an approach somewhat similar to EFP's and to LM's, but we use more extensively the Politzer theorem on equations of motion (EOM[71]). We consider in particular the hadronic tensor for SIDIS, DY and $e^+e^- \to \pi\pi X$. We also consider energies and momentum transfers high enough for assuming one photon approximation, but not so large that weak interactions be comparable with electromagnetic ones. As regards time-like photons, we assume to be far from masses of vector resonances, like J/Ψ, Υ or Z^0. Lastly, we do not consider the case of active (anti-)quarks originating from gluon annihilation.

Our starting point is the "Born" approximation[78] for the hadronic tensor, which reads, in the three above mentioned reactions, as

$$W_{\alpha\beta}(P_A, P_B, q) = C\sum_a e_a^2 \int dp^- d^2p_\perp Tr\left[\Phi_A^a(p)\gamma_\alpha \Phi_B^b(p')\gamma_\beta\right]. \tag{1}$$

Here C is due to color degree of freedom, $C = 1$ for SIDIS and $1/3$ for DY and e^+e^- annihilation. p and p' denote the four-momenta of the active partons, such that

$$p \mp p' = q, \tag{2}$$

q being the four-momentum of the virtual photon and the $-$ sign referring to SIDIS, the $+$ to DY or to e^+e^- annihilation. Φ_A and Φ_B are correlators, relating the active partons to the (initial or final) hadrons h_A and h_B, whose four-momenta are, respectively, P_A and P_B. We restrict ourselves to spinless and spin-1/2 hadrons. a and b are the flavors of the active partons, with $a = u, d, s, \bar{u}, \bar{d}, \bar{s}$ and $b = a$ in SIDIS, $b = \bar{a}$ in DY and e^+e^- annihilation; e_a is the fractional charge of flavor a. In DY Φ_A and Φ_B encode information on the active quark and antiquark distributions inside the initial hadrons. In SIDIS Φ_B is replaced by the fragmentation correlator Δ_B, describing the fragmentation of the struck quark into the final hadron h_B. In the case of e^+e^- annihilation, both correlators Φ_A and Φ_B have to be replaced by Δ_A and Δ_B respectively.

In the approximation considered we define the distribution correlator (commonly named correlator) as

$$\Phi_{ij}(p; P, S) = N\int \frac{d^4x}{(2\pi)^4} e^{ipx}\langle P, S|\bar{\psi}_j(0)\psi_i(x)|P, S\rangle. \tag{3}$$

Here N is a normalization constant, to be determined in sect. 4. ψ is the quark[1] field of a given flavor and $|P, S\rangle$ a state of a hadron (of spin 0 or 1/2) with a given four-momentum P and Pauli-Lubanski (PL) four-vector S, while p is the quark four-momentum. The color and flavor indices have been omitted in ψ for the sake of simplicity and from now on will be forgotten, unless differently stated. On the other hand, the fragmentation correlator is defined as

$$\Delta_{ij}(p; P, S) = N\int \frac{d^4x}{(2\pi)^4} e^{ipx}\langle 0|\bar{\psi}_j(0)a(P, S)a^\dagger(P, S)\psi_i(x)|0\rangle, \tag{4}$$

where $a(P, S)[a^\dagger(P, S)]$ is the destruction (creation) operator for the fragmented hadron, of given four-momentum and PL four-vector.

The hadronic tensor (1) is not color gauge invariant. Introducing a gauge link is not sufficient to fulfil this condition, but EOM suggest to add suitable contributions of

[1] For an antiquark eqs. (3) and (4) should be slightly modified, as we shall see in sects. 2 and 6.

higher correlators, involving two quarks and a number of gluons, so as to construct a gauge invariant hadronic tensor.

We adopt an axial gauge, obtaining for the correlator a gM/Q expansion, where g is the coupling, M the rest mass of the hadron and Q the QCD "hard" energy scale, generally assumed equal to $\sqrt{|q^2|}$. We examine in detail the first two terms of the expansion. The zero order term corresponds to the QCD parton model approximation. As regards the second term, it concerns the T-odd functions; in particular, we discuss an interesting approximation, already proposed by Collins[79]. In both cases we obtain several approximate relations among "soft" functions, which survive perturbative QCD evolution, as a consequence of EOM. Our approach allows also to determine the Q-dependence of some important azimuthal asymmetries and to draw conclusions about the Burkhardt-Cottingham[80] and Efremov-Leader-Teryaev[65] sum rules.

Section 2 is devoted to the gauge invariant correlator (more properly to the distribution correlator), whose properties are deduced with the help of EOM. In particular, we derive an expansion in powers of gM/Q, whose terms can be interpreted as Feynman-Cutkosky graphs. In section 3 we give a prescription for writing a gauge invariant sector of the hadronic tensor which is of interest for interactions at high Q. In sects. 4 and 5 we study in detail the zero order term and the first order correction of the expansion, deducing approximate relations among functions which appear in the usual parameterizations of the correlator[55,81]. Sect. 6 is dedicated to the fragmentation correlator. In sect. 7 we illustrate the azimuthal asymmetries involved in the three different deep inelastic processes. Lastly sect. 8 is reserved to a summary of the main results of the paper.

2 Gauge Invariant Correlator

The correlator (3) can be made gauge invariant, by inserting between the quark fields a link operator[76,77,55], in the following way:

$$\Phi_{ij}(p;P,S) = N\int \frac{d^4x}{(2\pi)^4} e^{ipx}\langle P,S|\bar{\psi}_j(0)\mathcal{L}(x)\psi_i(x)|P,S\rangle. \tag{5}$$

Here

$$\mathcal{L}(x) = \mathrm{P}exp\left[ig\Lambda_{\mathcal{I}}(x)\right], \qquad \text{with} \qquad \Lambda_{\mathcal{I}}(x) = \int_{0(\mathcal{I})}^{x} \lambda_a A_\mu^a(z)dz^\mu, \tag{6}$$

is the gauge link operator, "P" denotes the path-ordered product along a given integration contour $\mathcal{I}$, λ_a and A_μ^a being respectively the Gell-Mann matrices and the gluon fields. The link operator depends on the choice of $\mathcal{I}$, which has to be fixed so as to make a physical sense. According to previous treatments[55,79,82,83], we define two different contours, $\mathcal{I}_\pm$, as sets of three pieces of straight lines, from the origin to $x_{1\infty} \equiv (\pm\infty, 0, \mathbf{0}_\perp)$, from

$x_{1\infty}$ to $x_{2\infty} \equiv (\pm\infty, x^+, \mathbf{x}_\perp)$ and from $x_{2\infty}$ to $x \equiv (x^-, x^+, \mathbf{x}_\perp)$, having adopted a frame, whose z-axis is taken along the hadron momentum, with $x^\pm = 1/\sqrt{2}(t \pm z)$. We remark that the choice of the path is important for the so-called T-odd[2] functions[47]: the path $\mathcal{I}_+$ is suitable for DIS distribution functions, while $\mathcal{I}_-$ has to be employed in DY[82,83]. For an antiquark the signs of the correlator (5) and of the four-momentum p have to be changed.

In the following of the section we investigate some properties of the correlator.

2.1 T-even and T-odd correlator

We set[82]

$$\Phi_{E(O)} = \frac{1}{2}[\Phi_+ \pm \Phi_-], \tag{7}$$

where $\Phi_\pm$ corresponds to the contour $\mathcal{I}_\pm$ in eqs. (6), while Φ_E and Φ_O select respectively the T-even and the T-odd "soft" functions. These two correlators contain respectively the link operators $\mathcal{L}_E(x)$ and $\mathcal{L}_O(x)$, where

$$\mathcal{L}_{E(O)}(x) = \frac{1}{2}\mathrm{P}\left\{exp\left[ig\Lambda_{\mathcal{I}_+}(x)\right] \pm exp\left[ig\Lambda_{\mathcal{I}_-}(x)\right]\right\} \tag{8}$$

and $\Lambda_{\mathcal{I}_\pm}(x)$ are defined by the second eq. (6). Eqs. (7) and (8) imply that the T-even functions are independent of the contour ($\mathcal{I}_+$ or $\mathcal{I}_-$), while T-odd ones change sign according as to whether they are involved in DIS or in DY[79,82]. In this sense, such functions are not strictly universal[79], as already stressed. It is convenient to consider an axial gauge,

$$\mathbf{A}^- = \mathbf{A}^+ = 0, \tag{9}$$

with antisymmetric boundary conditions[55]. Here we have adopted the shorthand notation $\mathbf{A}^\mu$ for $\lambda^a A^\mu_a$. In this gauge - proposed for the first time by Kugut and Soper[87] and named KS gauge in the following - we have

$$\Lambda_{\mathcal{I}_+}(x) = -\Lambda_{\mathcal{I}_-}(x) = \int_{x_1}^{x_2} dz_\mu \mathbf{A}^\mu(z), \tag{10}$$

where x_i is a shorthand notation for $x_{i,+\infty}$, $i = 1, 2$. Therefore, in the KS gauge,

$$\mathcal{L}_E(x) = \mathrm{P}cos\left[g\Lambda_{\mathcal{I}_+}(x)\right], \qquad \mathcal{L}_O(x) = i\mathrm{P}sin\left[g\Lambda_{\mathcal{I}_+}(x)\right] \tag{11}$$

and the T-even (T-odd) part of the correlator consists of a series of even (odd) powers of g, each term being endowed with an even (odd) number of gluon legs. As a consequence,

[2] More precisely, one should speak of "naive T", consisting of reversing all momenta and angular momenta involved in the process, without interchanging initial and final states[84–86].

the zero order term is T-even, while the first order correction is T-odd. This confirms that no T-odd terms occur without interactions among partons, as claimed also by other authors[57–59,79]. Gauge invariance of the correlator implies that these conclusions hold true in any axial gauge, such that conditions (9) are fulfilled. From now on we shall work in such a type of gauge[88,89].

2.2 Power Expansion of the Correlator

We consider Φ_+, which, as explained before, refers to DIS. As regards DY, the T-odd terms will change sign, as follows from the choice of the path - $\mathcal{I}_-$ instead of $\mathcal{I}_+$ - and from the first eq. (10) and from the second eq. (11). We rewrite $\mathcal{L}(x)$ as

$$\mathcal{L}(x) = \sum_{n=0}^{\infty}(ig)^n \Lambda_n(x). \tag{12}$$

Here $\Lambda_0(x) = 1$, while for $n \geq 1$ one has, in the KS gauge,

$$\Lambda_n(x) = \int_{x_1}^{x_2} dz_1^{\mu_1} \int_{x_1}^{z_1} dz_2^{\mu_2} ... \int_{x_1}^{z_{n-1}} dz_n^{\mu_n} \left[\mathbf{A}_{\mu_n}(z_n)...\mathbf{A}_{\mu_2}(z_2)\mathbf{A}_{\mu_1}(z_1)\right], \tag{13}$$

where the $z_i \equiv (\infty, z_i^+, \mathbf{z}_{i\perp})$, $i = 1, 2, ...n$, are points in the space-time along the line through x_1 and x_2. Substituting eq. (12) into eq. (5), we have the following expansion of Φ in powers of g:

$$\Phi = \sum_{n=0}^{\infty}(ig)^n \Gamma_n, \tag{14}$$

with

$$(\Gamma_n)_{ij} = N \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle P, S|\bar{\psi}_j(0)\Lambda_n(x)\psi_i(x)|P, S\rangle. \tag{15}$$

As already noticed, Γ_n is T-even for even n and T-odd for odd n.

Now we invoke the Politzer theorem[71], concerning EOM. This states that, if we consider the matrix element between two hadronic states of a given composite operator, constituted by quark and/or gluon fields, each such field fulfils EOM, no matters if the parton is off-shell and/or renormalized. We show in Appendix A that, owing to the Politzer[71] theorem, the term Γ_0 fulfils the Dirac homogeneous equation, *i. e.*,

$$(\not{p} - m)\Gamma_0 = 0, \tag{16}$$

where m is the quark rest mass. The corresponding Feynman-Cutkosky graph is represented in fig. 1.

For $n \geq 1$ we have instead

$$(ig)^n \Gamma_n = N \int d\Omega_n S^{\mu_1...\mu_n} \Phi^{(n)}_{\mu_1...\mu_n}(p, k_1, k_2...k_n). \tag{17}$$

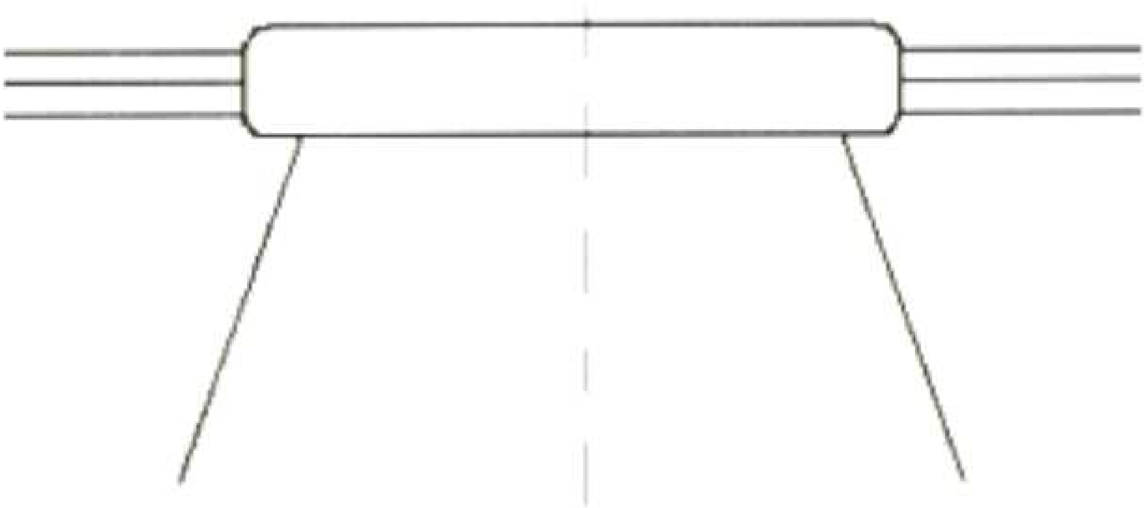

Figure 1: Feynman-Cutkosky graph for zero order term of expansion (14).

Here we have set

$$d\Omega_n = \prod_{l=1}^{n} \frac{d^4 k_l}{(2\pi)^4}, \tag{18}$$

$$\begin{aligned} S^{\mu_1 ... \mu_n} &= \frac{ig}{\not p - m + i\epsilon} i\gamma^{\mu_1} \frac{ig}{\not p - \bar{\not k}_1 - m + i\epsilon} i\gamma^{\mu_2} ... \\ &\times \frac{ig}{\not p - \bar{\not k}_{n-1} - m + i\epsilon} i\gamma^{\mu_n}, \end{aligned} \tag{19}$$

$$\bar{k}_l = \sum_{r=1}^{l} k_r. \tag{20}$$

The k_r ($r = 1, 2, ...n$) are the four-momenta of the n gluons involved in the quark-gluon correlator $\Phi^{(n)}_{\mu_1 ... \mu_n}$, defined as

$$\begin{aligned} \left[\Phi^{(n)}_{\mu_1 ... \mu_n}(p, k_1, k_2 ... k_n)\right]_{ij} &= N \int \frac{d^4 x}{(2\pi)^4} e^{i(p - \bar{k}_n)x} \\ \times \quad \langle P, S|\bar{\psi}_j(0) \mathrm{P}'[\mathbf{B}_{\mu_n}(k_n) ... \mathbf{B}_{\mu_1}(k_1)] \psi_i(x)|P, S\rangle, \end{aligned} \tag{21}$$

with

$$\mathbf{B}_\mu(k) = \hat{\mathbf{A}}_\mu(k) + \tilde{\mathbf{A}}_\mu(k), \tag{22}$$

$$\hat{\mathbf{A}}_\mu(k) = \int \frac{d^4 z}{(2\pi)^4} \mathbf{A}_\mu(z) e^{ikz}, \tag{23}$$

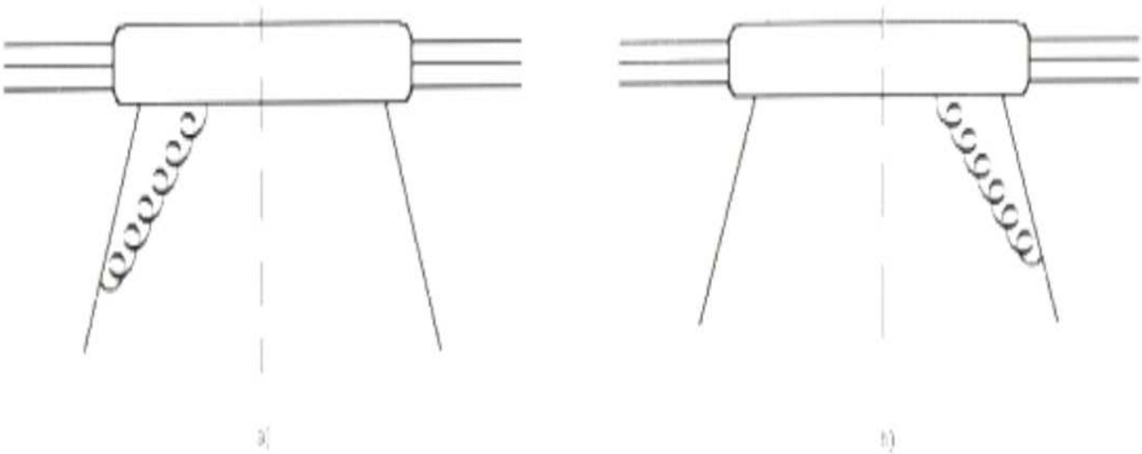

Figure 2: Same as fig. 1 for first order correction in the coupling.

$$\tilde{\mathbf{A}}_\mu(k) \quad = \quad \delta(k^+) \lim_{M\to\infty} \int d\kappa e^{-i\kappa M} \hat{\mathbf{A}}_\mu(k^-, \kappa, \mathbf{k}_\perp), \tag{24}$$

the reference frame being the one defined at the beginning of this section. Moreover the operator product P' is defined according to the following rules:

- any $\hat{\mathbf{A}}_\mu(k)$ is at the left of any $\tilde{\mathbf{A}}_\mu(k)$;
- the $\tilde{\mathbf{A}}_\mu(k)$ are ordered as $\tilde{\mathbf{A}}_{\mu_1}(k_1)\tilde{\mathbf{A}}_{\mu_2}(k_2)...\tilde{\mathbf{A}}_{\mu_l}(k_l)$;
- the $\hat{\mathbf{A}}_\mu(k)$ are ordered as $\hat{\mathbf{A}}_{\mu_m}(k_m)...\hat{\mathbf{A}}_{\mu_2}(k_2)\hat{\mathbf{A}}_{\mu_1}(k_1)$.

Lastly the quark-gluon correlators $\Phi^{(n)}_{\mu_1...\mu_n}$ fulfil the following homogeneous equation:

$$(\not{p} - \bar{\not{k}}_n - m)\Phi^{(n)}_{\mu_1...\mu_n}(p, k_1, k_2...k_n) = 0. \tag{25}$$

Each term of the expansion (14) - somewhat similar to the one obtained by Collins and Soper[76,77] - may be interpreted as a Feynman-Cutkosky graph. It corresponds to an interference term between the amplitude

$$\text{"nucleon} \rightarrow \text{quark} + \text{spectator partons"} \tag{26}$$

without any rescattering, and an analogous one, where n gluons are exchanged between the active quark and the spectator partons.

In particular, the interference term is such that the gluons (for $n > 0$) are attached to the left quark leg, see figs. 2a and 3a. An important result, deduced at the end of

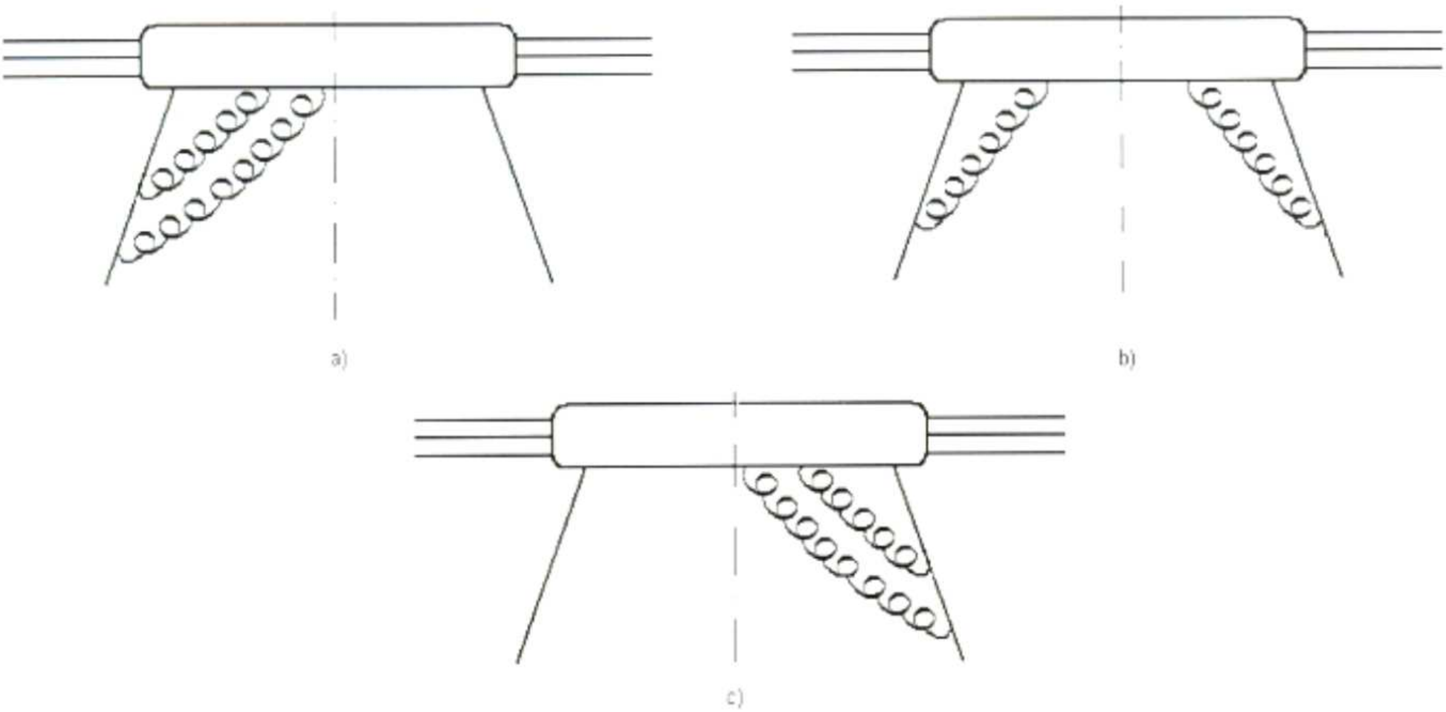

Figure 3: Same as fig. 2 for second order correction.

Appendix A, is that such a term turns out to correspond to any interference term between two amplitudes, such that k and $n-k$ gluons respectively are exchanged between the active quark and the spectator partons, with $0 \leq k \leq n$. The situation is illustrated in figs. 2 and 3 for n = 1 and 2.

It is worth noting that a radiation ordering similar to the one established here is found in semiinclusive processes at large x[90] and in totally inclusive DIS at small x[91]. Moreover the terms (21) consist of quark-gluon-quark correlations, analogous to the one introduced by Efremov and Teryaev[75] and by Qiu and Sterman[50–52].

As a consequence of the Politzer theorem, formulae (14) to (21) hold for renormalized fields, provided we take into account the scale dependence of the coupling g, of the quark mass m and of the correlators $\Phi^{(n)}_{\mu_1...\mu_n}(p, k_1, k_2...k_n)$[92]. Moreover one has to observe that the four-momenta appearing in the propagators are highly off-shell: p^2 and $(p - \bar{k}_r)^2$ are of order Q^2[77,78], because the uncertainty principle demands hard interactions to occur in a very limited space-time interval, corresponding to the condition

$$|p^2| \gg M^2. \tag{27}$$

Therefore we have $p^2 \approx 2p^+p^-$ and $p^+ = O(Q)$, whence

$$|p^-| = O(Q) \tag{28}$$

and it follows that the coefficients Γ_n are of order Q^{-n}, up to QCD corrections, consisting of terms of the type $g^{2k}(lnQ)^m$, with k and m integers and $k \geq m$[93]. For the same reason, the coupling g, which appears in expansion (14), assumes small values, corresponding to short distances and times.

To summarize, we have found that the T-even and the T-odd correlators, given by eqs. (7), may be written as expansions in gM/Q, *i. e.*,

$$\Phi_E(p) = \sum_{n=0}^{\infty} \left(\frac{igM}{Q}\right)^{2n} \bar{\Gamma}_{2n}, \qquad \Phi_O(p) = \sum_{n=0}^{\infty} \left(\frac{igM}{Q}\right)^{2n+1} \bar{\Gamma}_{2n+1}, \tag{29}$$

where $\bar{\Gamma}_n = \Gamma_n Q^n/M^n$ depend still on Q, as told above. As explained above, Φ_O changes sign when involved in DY. Stated differently, T-odd terms present an odd number of quark propagators, see eq. (19) for odd n: in the limit of negligible quark mass, quark four-momenta in DIS are spacelike, whereas in DY they are timelike[82].

The first two terms of expansion (14) will be studied in detail in sects. 4 and 5 respectively.

3 Hadronic Tensor

In the present section we refer indifferently to the hadronic tensor of the three processes introduced, which may involve one or two fragmentation correlators; in fact, as we shall see in sect. 6, this object requires only minor modifications with respect to the treatment of last section.

If we substitute the gauge invariant correlator (5) into the hadronic tensor (1), this latter does not fulfil the requirement of electromagnetic gauge invariance: only the term of zero order in the coupling satisfies this condition. In order to get a complete gauge invariance at any order, we have to recall the interpretation given above of the correlator. For example, at first order in the coupling in SIDIS, we see that the ”hard” scattering amplitude $q\gamma^* \to q'\tilde{g}$ - where we have denoted by q and q' the initial and final quark and by $\tilde{g}$ a gluon - consists not only of the graph of fig. 4a, encoded in the first order term of the correlator, but also of the one represented in fig. 4b, which interferes coherently with it. This guarantees electromagnetic gauge invariance for the first order graph[94].

Furthermore, convoluting ”hard” graphs with the ”soft” factors, these two amplitudes give rise, among other objects, to asymmetric Feynman-Cutkosky graphs (fig. 5), related to interference terms. These are observables - necessarily gauge invariant - and therefore assume real values. This procedure, already suggested in ref. [78], can be generalized to the three kinds of hadronic tensors considered in the present article, at any order in g, so as to obtain sets of graphs corresponding to observable, and therefore gauge

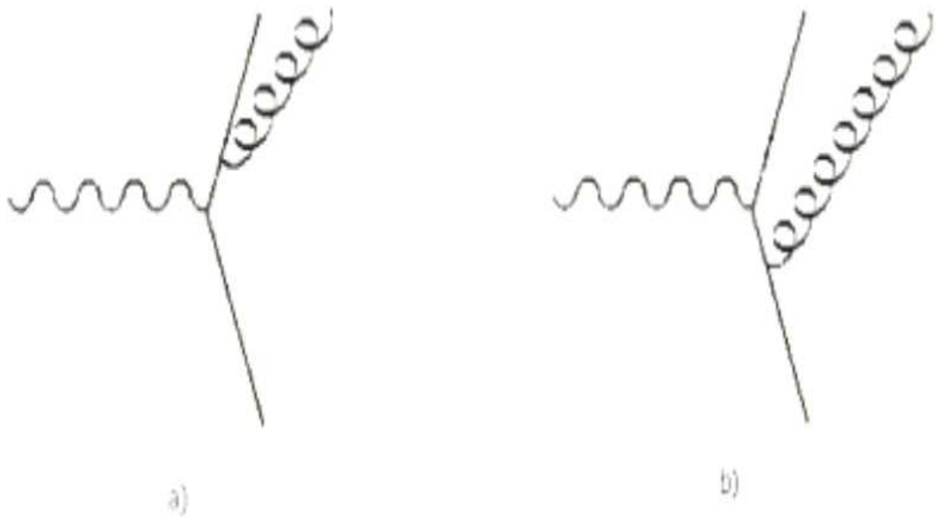

Figure 4: Graphs for ”hard” amplitudes interfering coherently, first order correction in the coupling.

invariant, quantities. We show how to construct them at any order n, corresponding to the overall number of gluons exchanged between active quarks and spectator partons. The procedure consists in following steps, for a given n:

- Consider the $n + 1$ possible combinations of gluons occurring in the hadronic tensor (1), say, s for hadron A and $n - s$ for hadron B, with s = 0, 1 ... n.

- For a given $s \ \ (n - s)$, consider all possible correlators, according to the definition given in subsect. 2.2: as seen at the end of last section, these amount to $s + 1 \ \ (n - s + 1)$ correlators equal to $\Gamma_s \ \ (\Gamma_{n-s})$.

- Add each such correlator all those graphs whose ”hard” parts interfere coherently with it, as shown in fig. 5. In practice, one has to do this for the correlator whose gluons are attached to the ”left” quark leg and to multiply by the number of gluons of each correlator.

Then we have, up to QCD corrections at each order of the expansion,

$$W_{\alpha\beta}(q) = \sum_{n=0}^{\infty} W_{\alpha\beta}^{(n)}(q), \tag{30}$$

with

$$W_{\alpha\beta}^{(n)} \quad = \quad C \int dp^- d^2p_\perp \int d\Omega_n \sum_{r=0}^{n} \sum_{s=0}^{n} Tr M_{\alpha\beta}^{(n)}, \tag{31}$$

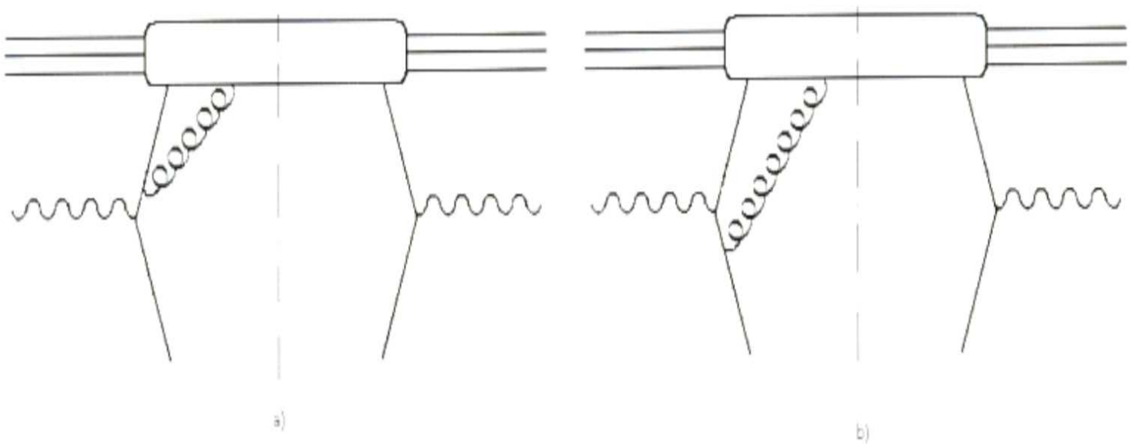

Figure 5: Feynman-Cutkosky graphs corresponding to "hard" amplitudes of fig. 4. Also the complex conjugate graphs, which amount to specular images of these two, contribute to first order corrections.

$$M_{\alpha\beta}^{(n)} = \sum_{s=0}^{n}(s+1)(n-s+1)\left[\tilde{\Gamma}_{\alpha}^{(s,0)}\Phi_A^{s,0}\tilde{\Gamma}_{\beta}^{(n,s)}\Phi_B^{n,s}\right], \tag{32}$$

$$\tilde{\Gamma}_{\rho}^{(l,r)} = \sum_{m=r}^{l} S_r^m \gamma_\rho S_m^l. \tag{33}$$

Here we have used the following shorthand notations:

$$S_r^m = S^{\mu_{r+1},\mu_{r+2},\ldots,\mu_m}, \qquad \Phi^{n,s} = \Phi^{n,s}_{\mu_{s+1},\mu_{s+2},\ldots,\mu_n}, \tag{34}$$

moreover we have set S_r^m = 1 for $m = r$.

For each term of expansion (30) we have to take into account three kinds of effects:

a) gluon radiation by scattered partons;

b) perturbative QCD corrections;

c) higher correlators, such that the active quarks exchange gluons with quark-antiquark pairs or gluon pairs or triplets belonging to spectator partons.

The first two effects may be calculated according to the algorithm suggested in refs. [76,77]. As to the contributions c), they can be included in the basic term of expansion (30), since they have the same (T-even or T-odd) behavior. Lastly we recall that, unless we integrate over some final transverse momentum [of the lepton pair in the case of DY, of a final hadron in SIDIS or e^+e^- annihilation], the phase space of the final gluons emitted undergoes a restriction[93], expressed by a doubly logarithmic form factor; this is more and more sizable at increasing energy, resulting in the well-known Sudakov-like damping[76,95].

4 Zero order term: the QCD parton model

In this section and in the following one we shall be concerned with the hadronic tensor for DY process, but our results may be trivially extended to the other two deep inelastic processes, with a slight difference for the fragmentation function, to be discussed in sect. 6.

Let us consider the hadronic tensor (31) at zero order, *i. e.*,

$$W^{(0)}_{\alpha\beta} = C \int d^2p_\perp dp^- Tr\left[\gamma_\alpha \Gamma_0^A(p)\gamma_\beta\Gamma_0^B(p')\right], \tag{35}$$

where the Γ_0's are given by eq. (15) for n = 0 and fulfil the homogeneous Dirac equation (16). The tensor (35), T-even, corresponds to the Born approximation considered in the introduction. As appears from eq. (35), the study of this tensor amounts to analyzing the correlator Γ_0, which is itself T-even and gauge invariant at zero order in g. In Appendix B we show that

$$\Gamma_0(p) = \frac{N}{4\mathcal{P}}(\not p + m)\left[f_1(p) + \gamma_5 \not S^q_\parallel g_{1L}(p) + \gamma_5 \not S^q_\perp h_{1T}(p)\right] 2p^+\delta(p^2 - m^2). \tag{36}$$

Here $f_1(p)$, $g_{1L}(p)$ and $h_{1T}(p)$ are functions of the four-momentum p of the active quark, which, in this case, is on shell, $p \equiv (E, \mathbf{p})$, with $E = \sqrt{m_q^2 + \mathbf{p}^2}$. $S^q_\parallel$ and $S^q_\perp$ are the components of the quark PL vector, respectively parallel and perpendicular to the hadron momentum. Moreover we have set

$$\mathcal{P} = \frac{1}{\sqrt{2}} p \cdot n_-, \tag{37}$$

having defined the dimensionless, light-like four-vectors $n_\pm$ in such a way that

$$n_+ \cdot n_- = 1 \tag{38}$$

and their spatial components are along (+) or opposite (-) to the hadron momentum. It is important to notice that, if integrated over p^-, the expression obtained for the zero order correlator turns out to be proportional to the density matrix of a quark confined in a finite volume, but free of interactions with other partons[96]. Therefore we fix the normalization constant N so as to obtain, after integration, just the density matrix *i. e.*,

$$N = 2\mathcal{P}. \tag{39}$$

Lastly, it is convenient to express $S^q_\parallel$ and $S^q_\perp$ in terms of the components of the PL vector of the hadron. As shown in Appendix B, one has

$$S^q_\parallel = \lambda\left(\frac{\bar{p}}{m}\right) - \bar{\eta}_\perp + O(\bar{\eta}^2_\perp), \qquad S^q_\perp = S_\perp + \bar{\lambda}_\perp \frac{\bar{p}}{m} + O(\bar{\eta}^2_\perp). \tag{40}$$

Here

$$\begin{aligned} \lambda &= -S\cdot\frac{n_+ + n_-}{\sqrt{2}}, & S_\perp = S - \lambda\frac{n_+ + n_-}{\sqrt{2}}, \quad (41)\\ \bar{p} &\equiv (|\mathbf{p}|, E\hat{\mathbf{p}}), \quad \hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|, & \bar{\eta}_\perp = p_\perp/\mathcal{P}, \quad (42)\\ \bar{\lambda}_\perp &= -S\cdot\bar{\eta}_\perp, & p_\perp \equiv (0,0,\mathbf{p}_\perp) \quad (43) \end{aligned}$$

and $\mathbf{p}_\perp$ is the transverse momentum of the active quark with respect to the hadron momentum.

Equation (36) has important consequences on TMD T-even functions, as we are going to illustrate in the two next subsections. To this end we compare that equation with the naive parameterization of the TMD correlator in terms of Dirac components, without introducing any dynamic conditions[55,56,81]. We give such a parameterization in Appendix C, up to and including twist-3 terms. The twist-2, T-even sector, which we study in subsect. 4.1, corresponds to quark distribution functions which survive when interactions with gluons are turned off. As regards the twist-3 functions, we distinguish among the T-even, the T-odd and the ”hybrid” ones, these lasts deriving contributions both from T-even and T-odd terms.

4.1 Twist-2, T-even Correlator

If quark-gluon interactions are neglected, the correlator is usually parameterized as[97,98]

$$\begin{aligned} \Phi^f_E &= \frac{\mathcal{P}}{\sqrt{2}}\{f_1\not{n}_+ + (\lambda g_{1L} + \lambda_\perp g_{1T})\gamma_5\not{n}_+ + \frac{1}{2}h_{1T}\gamma_5[\not{S}_\perp, \not{n}_+] \\ &+ \frac{1}{2}(\lambda h^\perp_{1L} + \lambda_\perp h^\perp_{1T})\gamma_5[\not{\eta}_\perp, \not{n}_+]\}2p^+\pi\delta(p^2 - m^2). \end{aligned} \tag{44}$$

Here the Dirac operators considered are purely T-even, as can be checked; moreover

$$\eta_\perp = p_\perp/\mu_0, \qquad \lambda_\perp = -S\cdot\eta_\perp \tag{45}$$

and μ_0 is an undetermined energy scale, introduced for dimensional reasons, in such a way that all functions embodied in the parameterization of Φ have the dimensions of a probability density. This scale[97] determines the normalization of the functions which depend on $\eta_\perp$. In particular, as is well-known, the 6 twist-2 functions, which appear in the parameterization (44), are interpreted as TMD probability densities: f_1 is the unpolarized quark density, g_{1L} the longitudinally polarized density in a longitudinally polarized (spin 1/2) hadron, g_{1T} the longitudinally polarized density in a transversely polarized hadron, $h_{1L}^{\perp}$ the transversity in a longitudinally polarized hadron and

$$h'_{1T} = h_{1T} + |\eta_{\perp}^2| h_{1T}^{\perp} \tag{46}$$

is the TMD transversity in a transversely polarized hadron.

Now we compare the parameterization (44) with the correlator (36). To this end we consider projections of both matrices over the various Dirac components, *i. e.*, for a given Dirac operator Γ,

$$\Phi^{\Gamma} = \frac{1}{2} Tr \Gamma \Phi, \tag{47}$$

possibly taking into account eqs. (40).

First of all, Γ = $\gamma_5 \gamma^+$ and $\gamma_5 \gamma^+ \gamma_i$ (i = 1, 2) yield, approximately in the limit of m = 0,

$$h_{1L}^{\perp} \approx -\frac{\mu_0}{\mathcal{P}} g_{1L}, \qquad g_{1T} \approx \frac{\mu_0}{\mathcal{P}} h_{1T}, \qquad h_{1T}^{\perp} \approx \frac{\mu_0^2}{\mathcal{P}^2} h_{1T}. \tag{48}$$

These relations hold up to terms of order $(gM/Q)^2$, since, as we have seen, the T-even Dirac components of Φ derive contributions only from even powers of gM/Q. Moreover, the Politzer theorem implies that the relations are not modified by renormalization effects, and therefore hold also taking into account QCD evolution.

In order to determine μ_0, we observe that the functions involved on both sides of eqs. (48) are independent of $\mathcal{P}$. Therefore we must set $\mu_0 = C_0 \mathcal{P}$, C_0 being a dimensionless numerical constant, independent of momentum. But since these functions are quark densities, they should be normalized adequately, setting C_0 = 1. Then, neglecting the quark mass,

$$\mu_0 = \mathcal{P} = \frac{1}{\sqrt{2}} p \cdot n_-. \tag{49}$$

This result differs from the treatments of previous authors[55,81], who assume $\mu_0 = M$.

By comparing CLAS[26] and HERMES[20] results, at not too high values of Q^2 (1.5 to 3 GeV) the first relation (48), together with eq. (49), is verified for $x < 0.35$[96], discrepancies at larger x being attributed to higher twist contributions.

4.2 Twist-3, ”Hybrid” Correlator

Now we consider a sector of the correlator which, as already explained, has both T-even and T-odd contributions. In particular, here we focus on that part of ”hybrid” correlator which comes from the so-called ”kinematic” twist-3 terms. In Appendix C we find, according to the usual notations[55,81],

$$\Phi_H^f = \{\frac{1}{2}(f^\perp + \lambda g_L^\perp\gamma_5 + \lambda_\perp g_T^\perp\gamma_5)\not{p}_\perp + \frac{1}{4}\lambda_\perp h_T^\perp\gamma_5[\not{S}_\perp, \not{p}_\perp]$$
$$+ \quad \frac{1}{2}xM\left(e + g_T'\gamma_5\not{S}_\perp + \frac{1}{2}(\lambda h_L + \lambda_\perp h_T)\gamma_5[\not{n}_-, \not{n}_+]\right)\}2p^+\delta(p^2 - m^2). \quad (50)$$

Comparing the projections of the operators (36) and (50) over $\Gamma = \gamma_i$ (i = 1, 2) yields the approximate relation

$$f^\perp \approx f_1, \quad (51)$$

which corresponds to the Cahn[53,54] effect and is approximately verified for sufficiently large Q^2 and small x[45]. Also this equation, like eq. (48), survives QCD evolution. As we shall se in the next section, eq. (51) holds up to terms of order gM/Q, since $f^\perp$ derives also (T-odd) contributions from one-gluon exchange.

The projections of the same operators over $\Gamma = \gamma_5\gamma_i$ (i = 1, 2) yield (after integration over $\mathbf{p}_\perp$)

$$g_T(x) \approx \frac{m}{xM}h_1(x), \quad (52)$$

where

$$g_T(x) = \int d^2p_\perp g_T'(x, \mathbf{p}_\perp^2) \quad (53)$$

and

$$h_1(x) = \int d^2p_\perp \left[h_{1T}(x, \mathbf{p}_\perp^2) + |\eta_\perp^2| h_{1T}^\perp(x, \mathbf{p}_\perp^2)\right], \quad (54)$$

as obtained by integrating eq. (46) over transverse momentum. In this case the contribution of the QCD parton model is very small: m is negligible for u- and d-quarks, while for s-quarks h_1 is presumably small, because the sea is produced mainly by annihilation of gluons, whose transversity is zero in a nucleon. Therefore the contribution of quark-gluon interactions, neglected in the approximation considered, becomes prevalent in this case, as well as for $\Gamma = 1$ and $\gamma_5\gamma_+\gamma_-$, corresponding respectively to e and h_L. The effect of such interactions will be discussed in sect. 5.

4.3 Remarks

To conclude this section, we sketch some consequences of our theoretical results.

A) In the expression (46) or (54) for transversity, the second term is due to a relativistic effect. To illustrate this, consider a transversely polarized hadron. The longitudinal polarization of the quark, due in this case to the transverse momentum, is magnified by the boost from the quark rest frame. This additional polarization, along the quark momentum, has again a transverse component with respect to the nucleon momentum.

B) Eq. (54), together with the last two eqs. (48), suggests a method for determining approximately the transversity of a hadron. Indeed, g_{1T} can be conveniently extracted from double spin asymmetry[99–101] in SIDIS with a transversely polarized target. This asymmetry is expressed as a convolution of the unknown function with the usual, well-known fragmentation function of the pion. Therefore the method appears complementary to the one usually proposed[16,102], based on the Collins effect[46] in single spin SIDIS asymmetry; in this latter case one is faced with the convolutive product of h_{1T} with the Collins function, which is poorly known[63,64].

C) Eq. (52) establishes a relation between transversity and transverse spin. Indeed, the two quantities are related to each other. But, unlike transversity, the transverse spin operator is chiral even and does not commute with the free hamiltonian of a quark[42]: in QCD parton model it is proportional to the quark rest mass, which causes chirality flip.

D) We note that g_{1T}, $h_{1L}^{\perp}$ and $h_{1T}^{\perp}$ are associated with "twist-2" Dirac operators[42, 43], and yet, in our treatment, they are multiplied by inverse powers of Q (Q^{-1} for the first two functions, Q^{-2} for the third one). This would be unacceptable for common distribution functions, but, when transverse momentum is involved, also the orbital angular momentum plays a role. To illustrate this point, we recall that the quark distribution functions may be regarded as the absorptive parts of u-channel quark-hadron amplitudes[44]. For example, g_{1T} corresponds to an amplitude of the type $\langle ++|-+\rangle$, denoting by $|\Lambda\lambda\rangle$ a state in which the nucleon and quark helicities are, respectively, Λ and λ. The amplitudes corresponding to the functions in question involve a change $\Delta L = 1$ (for g_{1T} and $h_{1L}^{\perp}$) or $\Delta L = 2$ (for $h_{1T}^{\perp}$) in the orbital angular momentum, therefore they are of the type

$$\mathcal{A} = A(sin\theta)^{\Delta L}, \tag{55}$$

where θ = $arcsin|\mathbf{p}_\perp|/|\mathbf{p}|$ is the angle between the nucleon momentum and the quark momentum, while A is weakly energy dependent. Since $|\mathbf{p}|$ is of order Q and $|\mathbf{p}_\perp|$ of order M, our result can be understood[3].

5 First Order Correction

The first order correction in g of the hadronic tensor reads [see eqs. (31) and (32)]

$$W^{(1)}_{\alpha\beta} = -2gC\int dp^- d^2p_\perp \int \frac{d^4k}{(2\pi)^4} Tr N_{\alpha\beta}, \tag{56}$$

with

$$N_{\alpha\beta} = 2[h^\mu_\alpha(p,p',k)\Phi^{(1)}_{A\mu}(p,k)\gamma_\beta\Gamma^B_0(p') + \gamma_\alpha\Gamma^A_0(p)h^\mu_\beta(p',p,k)\Phi^{(1)}_{B\mu}(p',k)] \tag{57}$$

[3] This observation is the fruit of a stimulating discussion with Nello Paver.

and

$$h^\mu_\alpha(p,p',k) = \gamma_\alpha \frac{1}{\not p - m + i\epsilon}\gamma^\mu + \gamma^\mu \frac{1}{\not p' - \not k - m + i\epsilon}\gamma_\alpha. \tag{58}$$

Moreover the $\Phi^{(1)}_\mu$'s are given by eq. (21) for n = 1 and fulfil the homogeneous Dirac equation

$$(\not p - \not k - m)\Phi^{(1)}_\mu(p,k) = 0. \tag{59}$$

In Appendix B we show that, according to the Politzer theorem, and adopting, as in the previous sections, an axial gauge, $\Phi^{(1)}_\mu(p,k)$ is parameterized as

$$\Phi^{(1)}_\mu(p,k) = \Psi_\mu(p,k)\delta\left(p_1^- - \frac{m^2 + \mathbf{p}^2_{1\perp}}{2p_1^+}\right). \tag{60}$$

Here

$$p_1 = p - k, \qquad \text{with} \qquad p_1^2 = m^2 \tag{61}$$

and

$$\Psi_\mu(p_1,k) \approx \frac{1}{2}(\not p_1 + m)\hat L[\mathcal{C}_\mu + \Delta\mathcal{C}_\mu\gamma_5\not S^q_\parallel + \Delta_T\mathcal{C}_\mu\gamma_5\not S^q_\perp + \Delta_T\mathcal{C}'_\mu\gamma_5\not{\bar S}_\perp]. \tag{62}$$

Moreover we have

$$\sqrt{|p^2_\perp|}\bar S_{\perp\alpha} = \epsilon_{\alpha\beta\rho\sigma}n^\beta_+ n^\rho_- p^\sigma_\perp, \tag{63}$$

$$\hat L = \sqrt{\frac{\mathcal{P}}{\mathcal{P}_1}}\left[cosh\varphi + \gamma_0\gamma_3 a\frac{sinh\varphi}{2\varphi}\right], \tag{64}$$

where $\mathcal{P}_1 = p_1^+/\sqrt{2}$, while φ and a are defined in Appendix B. Lastly

$$\mathcal{C}_\mu = p_{1\perp\mu}\mathcal{C}_1 + \epsilon_{\mu\nu\rho\sigma}n^\nu_-(\mathcal{C}_2 S^{q\rho}_\parallel p^\sigma_{1\perp} + \mathcal{C}_3 M S^{q\rho}_\perp n^\sigma_+), \tag{65}$$

$$\Delta\mathcal{C}_\mu = \Delta\mathcal{C}p_{1\perp\mu}, \tag{66}$$

$$\Delta_T\mathcal{C}_\mu = \Delta_T\mathcal{C}p_{1\perp\mu}, \tag{67}$$

$$\Delta_T\mathcal{C}'_\mu = \Delta_T\mathcal{C}'p_{1\perp\mu}. \tag{68}$$

Here $\mathcal{C}_i$ ($i = 1,2,3$), $\Delta\mathcal{C}$, $\Delta_T\mathcal{C}$ and $\Delta_T\mathcal{C}'$ are correlation functions, the $\mathcal{C}_i$ being unpolarized, while the others are polarized. More precisely, $\Delta\mathcal{C}$ and $\Delta_T\mathcal{C}$, which are, respectively, longitudinally and transversely polarized, are related to an overall nucleon polarization. On the contrary, $\Delta_T\mathcal{C}'$ is a transversely polarized correlation function connected to quark-gluon interaction, for example, to a spin-orbit coupling[57–59].

5.1 Approximate Factorization

The second term of eq. (58) is not factorizable, in agreement with the observations of various authors[57–59,103,104], who have shown failures of universality[103,104] at large tranverse momentum. However, for sufficiently large Q, and adopting an axial gauge, this term is negligibly small[94] in comparison with the first one, which instead is factorizable. In fact, the gluon corresponding to the first term has a smaller offshellness than the one involved in the second term. This approximation is especially acceptable, even for relatively small Q, provided we limit ourselves to small transverse momenta[79] of the initial hadrons with respect to the direction of the momentum of the virtual photon in the center of mass of the DY pair. However, as already explained in sect. 2, also in the case when factorization is approximately satisfied, the T-odd distribution functions change sign from SIDIS to DY. We shall illustrate phenomenological implications of this change of sign in sect. 7.

In this approximation eq. (57) simplifies to

$$N_{\alpha\beta} = 2\int dp^- d^2p_\perp \gamma_\alpha [\Gamma_1^A(p)\gamma_\beta \Gamma_0^B(p') + \Gamma_0^A(p)\gamma_\beta \Gamma_1^B(p')], \tag{69}$$

with

$$\Gamma_1(p) = \frac{1}{\not p - m + i\epsilon}\gamma^\mu \int \frac{d^4k}{(2\pi)^4}\Phi_\mu^{(1)}(p,k). \tag{70}$$

Then, in an axial gauge, under the kinematic conditions above described, the tensor $W_{\alpha\beta}^{(1)}$ can be written [see eqs. (56) and (69)] in a form similar to $W_{\alpha\beta}^{(0)}$, giving thus rise to an approximate[57] factorization of T-odd functions. Our conclusion is quite analogous to the one drawn by Collins[79] and presents some similarity with the Qiu-Sterman assumption about the quark-gluon-quark correlation functions[50].

Moreover, eqs. (65) to (68), together with eqs. (40), induce for Γ_1 the following parameterization, at twist-3 approximation:

$$\begin{aligned}\Gamma_1(p) \quad &\approx \quad \frac{2p^+}{\pi(p^2 - m^2 + i\epsilon)}\frac{1}{2}\gamma_-\gamma_+[\not p_\perp f_o^\perp + \gamma^i \epsilon_{i\nu\sigma\rho} n_-^\nu (p_\perp^\sigma S_\parallel^\rho g_L' \\ &+ \quad M n_+^\sigma S_\perp^\rho g_{T,o}') + \gamma_5 \not S_\perp \not p_\perp h_{T,o} + \gamma_5 \bar{\not S}_\perp \not p_\perp h + \lambda\gamma_5 \not p_\perp \tilde{f}_L^\perp].\end{aligned} \tag{71}$$

Here

$$\bar{p}_0 \equiv \left(|\mathbf{p}|, \mathbf{p}\frac{\sqrt{\mathbf{p}^2 + m^2}}{|\mathbf{p}|}\right). \tag{72}$$

Moreover

$$f_o^\perp \quad = \quad -\int d\tilde{\Omega}\mathcal{C}_1, \qquad g_{T,o}' = \int d\tilde{\Omega}\mathcal{C}_3, \qquad h_{T,o} = -\int d\tilde{\Omega}\Delta_T \mathcal{C}, \tag{73}$$

$$h' \quad = \quad \int d\tilde{\Omega}\Delta_T \mathcal{C}', \qquad\qquad f_L^\perp = \int d\tilde{\Omega}(\mathcal{C}_2 + r\Delta\mathcal{C}), \tag{74}$$

$$d\tilde{\Omega} \quad = \quad \pi\frac{d^3\tilde{k}}{(2\pi)^4}\frac{p^- p_1^+}{2p^+}L^{(-)}, \qquad\qquad r = \frac{k^- \bar{p}_0^+}{p_1^+ p^-}\frac{L^{(+)}}{L^{(-)}}, \tag{75}$$

$$L^{(\pm)} \quad = \quad \frac{\mathcal{P}}{\mathcal{P}_1}\left[cosh\varphi \pm a\frac{sinh\varphi}{2\varphi}\right] \quad \text{and} \quad d^3\tilde{k} = 2p_1^+ d^4p_1 \delta(p_1^2 - m^2). \tag{76}$$

The notations for the functions are somewhat similar to those introduced by refs. [55,81]. The suffix "o" in $f_o^\perp$, $g_{T,o}$ and $h_{T,o}$ denotes T-odd contribution to these three functions, classified as "hybrid" in sect. 4, since they have T-even counterparts, see eq. (50). These functions are normalized coherently with their counterparts: indeed, if the quark is on-shell,

$$[\pi(p^2 - m^2 + i\epsilon)]^{-1} \to -i\delta(p^2 - m^2), \tag{77}$$

the $(-i)$-factor being compensated by the $i-$factor in expansion (14). This constrains also the normalization of the other functions included in eq. (71). Moreover, as already noticed in connection with correlation functions, the function h' describes a quark transverse polarization induced by quark-gluon interactions: this polarization, present also in spinless or unpolarized hadrons, is somewhat similar to the Boer-Mulders function[47], although it is twist-3 and not twist-2.

5.2 Twist-3, T-odd correlator

Now we compare the parameterization (71) with the naive parameterization of the twist-3, interaction dependent correlator. This reads, according to Appendix C,

$$\Phi^i = \Phi^i_H + \Phi^i_O. \tag{78}$$

where $\Phi_H^{(i)}$ is given by eq. (50), substituting $\delta(p^2 - m^2)$ by $[\pi(p^2 - m^2 + i\epsilon)]^{-1}$. On the other hand

$$\begin{aligned}\Phi^i_O &= \frac{2p^+}{p^2 - m^2 + i\epsilon}\{\epsilon_{ij}S^i_\perp(p^j_\perp e^\perp_T + M\gamma^j f_T) + \epsilon_{ij}\bar{S}^i_\perp p^j_\perp e'^\perp_T + \gamma_5(xMe_L\lambda \\ &+ e_T p_\perp \cdot S_\perp + e'_T p_\perp \cdot \bar{S}_\perp) + \epsilon_{ij}\gamma_i p^j_\perp(f^\perp_L\lambda + f^\perp_T\lambda_\perp + \gamma_5 g^\perp) \\ &+ \gamma_5 \not{p}_\perp \bar{\not{S}}_\perp h' + \frac{1}{2}\gamma_5[\gamma_+, \gamma_-]p_\perp \cdot \bar{S}_\perp h'^\perp\}.\end{aligned} \tag{79}$$

Comparison between parameterization (78) and result (71), component by component, yields the following approximate relations:

$$g^\perp \approx f^\perp_o, \qquad f^\perp_L \approx g^\perp_L, \quad f_T \approx g'_{T,o}, \tag{80}$$

$$e_T \approx -e^\perp_T \approx h^\perp_{T,o} \approx h_{T,o}, \tag{81}$$

$$e'_T \approx -e'^\perp_T \approx h'^\perp \approx h', \tag{82}$$

$$e_L \approx f^\perp_L \approx g^\perp_{T,o} \approx e_o \approx h_{L,o} \approx 0. \tag{83}$$

Also these equations survive QCD evolution, like eqs. (48) and (51).

5.3 Remarks

A) From most of the above relations one can see that a unique function represents both unpolarized (f or e) and polarized (g or h) distribution functions, independently of the fact that the nucleon is polarized or not. This is a consequence of the spin-orbit coupling[57] in gluon-quark interactions. For the same reason, the parameterization of $\Phi_\mu^{(1)}$ includes 5 independent functions and not only 3, despite the fact that it fulfils the homogeneous Dirac equation like Γ_0.

B) Among eqs. (80) to (83), those which concern only T-odd functions hold up to terms of order $(gM/Q)^2$. On the contrary, those which involve "hybrid" functions - including eq. (51) - hold up to terms of order gM/Q. Analogous approximate relations of this latter type have been found in ref. [105].

C) By integrating the correlator (71) over the transverse momentum of the quark, we obtain interesting results as regards twist-3 common functions. First of all, the fourth eq. (83) implies that $e(x)$ derives just T-even contributions, and therefore, apart from the (negligible) term illustrated in the previous section, it is essentially of order $(gM/Q)^2$. On the contrary, the main contributions to g_T and h_L are of order gM/Q and are T-odd, therefore they change sign according as to whether they are involved in DIS or DY reaction. These last predictions could be tested by confronting the DIS double spin asymmetry[4,5,10] with the DY one[106,107]. In the case of DY one has to integrate over the transverse momentum of the virtual photon; moreover, if possible, it may be more promising to detect $\tau^+\tau^-$ pairs, whose polarization is perhaps less problematic to determine[108].

D) Lastly, it is worth noting that, unlike previous authors[47,56,81], we find that the functions related to longitudinal and transverse polarization are associated to the same inverse power of Q. For example, $g^\perp$ and h' describe, respectively, the longitudinal quark polarization in an unpolarized nucleon. Similarly, $f_L^\perp$ and f_T are unpolarized quark densities in a longitudinally and transversely polarized nucleon. Conversely, the twist-2 T-odd functions $h_1^\perp$, corresponding to transverse polarization in an unpolarized nucleon, and the unpolarized distribution function $f_{1T}^\perp$[47] in a tranversely polarized nucleon find no place in parameterization (71).

5.4 Consequences on g_1 and g_2

Now we examine some consequences of our results on the DIS structure functions $g_1(x)$ and $g_2(x)$, whose properties have been studied by various authors[65,66,109]. To this end, here, and only in this subsection, we re-introduce the flavor indices, dropped out in formula (1), in order to recover the usual definitions of those functions. Recalling that

$$g_T^a(x) = g_1^a(x) + g_2^a(x) \qquad (a = u, d, s), \tag{84}$$

and setting

$$g_i(x) = \sum_a e_a^2 [g_i^a(x) + \bar{g}_i^a(x)] \quad (i = 1, 2), \tag{85}$$

$$g_{T,o}(x) = \sum_a e_a^2 \int d^2 p_\perp [g'^a_{T,o}(x, \mathbf{p}_\perp^2) + \bar{g}'^a_{T,o}(x, \mathbf{p}_\perp^2)], \tag{86}$$

eq. (52) implies

$$g_1(x) + g_2(x) = g_{T,e}(x) + g_{T,o}(x) + O(M^2/Q^2), \tag{87}$$

where [cfr. eq. (52)]

$$g_{T,e}(x) = \sum_a e_a^2 \frac{m_a}{xM} [h_1^a(x) + \bar{h}_1^a(x)]. \tag{88}$$

Since, as discussed in subsect. 4.2, $g_{T,e}$ is negligibly small for a nucleon, result (87) is in contrast with the Burkhardt-Cottigham[80] (BC) sum rule, *i. e.*,

$$\int_0^1 g_2(x) dx = 0. \tag{89}$$

Indeed, integrating both sides of eq. (87) between 0 and 1, and assuming relation (89), implies

$$\int_0^1 g_1(x) dx \approx \int_0^1 g_{T,o}(x) dx, \tag{90}$$

which is impossible, since $g_1(x)$ is a T-even function, while $g_{T,o}$ is, by definition, T-odd. Furthermore eq. (89) implies, together with the operator product expansion[65],

$$g_1(x) + g_2(x) = \int_x^1 \frac{dy}{y} g_1(y) + g_T^{(3)}, \tag{91}$$

where $g_T^{(3)}$ is the twist-3 contribution to g_T[65], to be identified, according to our results, with $g_{T,o}$. Then eq. (87) would yield

$$\int_x^1 \frac{dy}{y} g_1(y) = g_{T,e}(x) + O(M^2/Q^2), \tag{92}$$

which appears in contrast with data of $g_1(x)$[1,2,9], enforcing arguments against the BC rule (See ref. [65] and articles cited therein). An experimental confirmation of the violation of the BC rule was found years ago in a precision measurement of $g_2(x)$[10]. Also the Efremov-Leader-Teryaev (ELT)[65] sum rule, *i. e.*,

$$\int_0^1 dx x [g_1(x) + 2 g_2(x)] = 0, \tag{93}$$

is in contrast with our result. Indeed, it gives rise, together with eq. (87), to the approximate relation

$$\int_0^1 dx x g_1(x) \approx \int_0^1 dx 2x g_{T,o}(x), \tag{94}$$

which, again, relates a T-even function to a T-odd one.

6 Fragmentation Correlator

The fragmentation correlator (4) can be made gauge invariant analogously to the distribution correlator, *i. e.*, for a quark,

$$\Delta_{ij}(p; P, S) = 2\mathcal{P} \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle 0|\mathcal{L}(x)\bar{\psi}_j(0)a(P,S)a^\dagger(P,S)\psi_i(x)|0\rangle, \tag{95}$$

where $\mathcal{L}(x)$ is given by eq. (6). Object (95) may be treated analogously to the distribution correlator, according to the previous sections. Indeed, also in this case, for an antiquark one has to change the four-momentum from p to $-p$ and to put a minus sign in front of the correlator. Moreover one has to choose the path $\mathcal{I}_+$ for quark fragmentation from e^+e^- annihilation, whereas the path $\mathcal{I}_-$ refers to fragmentation in SIDIS. The only important difference with the distribution correlator is that one has to take into account also the nonperturbative interactions among the final hadrons produced. However, as we shall see in a moment, this does not involve any change in the parameterization.

We treat only the case of pions, adopting for T-odd terms an approximation analogous to the one discussed in subsection 5.1, valid for small transverse momenta of the final hadron with respect to the fragmenting quark. Under this condition, we have

$$\begin{aligned}
\Delta(p) &= 2p^+\{\bar{\Delta}^{(f)}(p)\delta(p^2-m^2) + \bar{\Delta}^{(i)}(p)[\pi(p^2-m^2+i\epsilon)]^{-1}\}, &(96)\\
\bar{\Delta}^{(f)}(p) &= \frac{1}{2}(\not{p}+m)D_\pi, &(97)\\
\bar{\Delta}^{(i)}(p) &= \gamma_-\gamma_+[\not{p}_\perp D_\pi^\perp + \not{p}_\perp H']. &(98)
\end{aligned}$$

Here D_π is the common fragmentation function of the pion, $D_\pi^\perp$ is defined as in ref. [55] and H' assumes the role of the Collins[46] function, describing the asymmetry of a pion fragmented from a transversely polarized quark.

Final state interactions give rise to terms - for instance interference terms - which decrease as inverse powers of Q, independent of the nature of the interactions themselves. To show this, we observe that these interactions may produce an azimuthal asymmetry in a pion fragmented from a transversely polarized quark[46,110]. Analogously to the distribution functions illustrated in remark D at subsect. 4.3, such an asymmetry may be regarded as the absorptive part of an amplitude of the type $\langle +|-\rangle$, where $\pm$ denotes the helicity of the fragmenting quark. This kind of amplitude - a typical helicity flip one - behaves as

$$\langle +|-\rangle = Bsin\theta, \tag{99}$$

where B is a given function, weakly dependent on the quark momentum, due to perturbative QCD corrections. Analogously to eq. (55), we conclude that interference terms decrease at least as Q^{-1}, just like the second term in eq (98). Our result agrees with the approach by Collins and Soper[76], who do not include ”soft” final state interaction in the leading term of (almost) back-to-back fragmentation in e^+e^- annihilation.

7 Asymmetries

In this section we consider some important azimuthal and single spin asymmetries, which, as is well known, may be produced by coupling two chiral-even or two chiral-odd TMD distribution or fragmentation functions. More precisely, the terms of the hadronic tensor which give rise to asymmetries are written as convolutive products of two ”soft” functions and depend on some azimuthal angle ϕ, relative to the final hadron (for SIDIS and e^+e^- annihilation), or to the final muon pair (for DY). Some of these asymmetries arise from the first order correction of the hadronic tensor, while others belong to the second order one, whose complete parameterization is not considered in this paper.

A) Cahn effect

This effect, pointed out for the first time by Cahn[53], has been exhibited by ref. [45] examining some SIDIS data[13–15] (see also ref. [111]). We consider the asymmetry corresponding to the ”product”

$$A_C \propto f^{\perp} \otimes D_{\pi} + f_1 \otimes D_{\pi}^{\perp}. \tag{100}$$

This asymmetry is proportional to $cos\phi$ and decreases like Q^{-1}. To the extent that $f^{\perp}$ and $D_{\pi}^{\perp}$ can be approximated by f_1 and D_{π} respectively, one speaks properly of Cahn effect[45]: this amounts to neglecting quark-gluon interactions, see eq. (51) for distribution functions, an analogous equation holding for unpolarized fragmentation functions. This approximation is acceptable for relatively large Q and at small x, as shown by ref. [45]. However, one has to observe that both $f^{\perp}$ and $D_{\pi}^{\perp}$ are ”hybrid” functions and in general their T-odd contributions cannot be neglected.

It is worth considering also the ”product”

$$A_{C2} \propto f^{\perp} \otimes D_{\pi}^{\perp}, \tag{101}$$

which generates a $cos2\phi$ asymmetry decreasing like Q^{-2}, hardly distinguishable from another one, arising from the ”product” of two chiral-odd functions, as we shall see in a moment. Under the approximation just discussed, we predict a sort of ”second order” Cahn effect.

B) Qiu-Sterman effect

An important transverse single spin asymmetry is the one predicted by Qiu and Sterman[50–52] (QS) (see also refs. [75,112,113,82]). This can be observed both in

SIDIS and in DY, where one integrates over the transverse momentum, respectively, of the final hadron detected and of the final pair. This is described by the "products"

$$A_{QS} \propto g'_T \otimes D_\pi \quad \text{(in SIDIS) and} \qquad \propto g'_T \otimes \bar{f}_1 + c.c. \quad \text{(in DY)}, \tag{102}$$

the "bar" indicating the antiquark function and $c.c.$ "charge conjugated". A similar effect could be observed in e^+e^- annihilation, if one of the the hadrons observed were spinning. This asymmetry decreases like Q^{-1}. Moreover, since g'_T is prevalently T-odd, while f_1, $\bar{f}_1$ and D_π are T-even, the asymmetry is expected to assume an opposite sign in SIDIS and DY.

C) Sivers effect

According to our treatment, the Sivers asymmetry[48,49] is described by the "product"

$$A_{SIV} \propto g'_T \otimes D^\perp \quad \text{(in SIDIS) and} \qquad \propto g'_T \otimes \bar{f}^\perp + c.c. \text{ (in DY)}. \tag{103}$$

Therefore this asymmetry - detected by HERMES[20,21] and COMPASS[23] experiments - is described a bit differently than in current literature[47,64,45]; in particular it results to decrease as Q^{-2} [4]. Moreover, comparing eq. (103) with eq. (102) shows a relation between the Sivers asymmetry and the QS asymmetry, as already noticed by other authors[82,114–117]. This relation is especially close if one adopts the approximation $f^\perp \approx f_1$, or $D^\perp_\pi \approx D_\pi$ according to the Cahn effect[53,54]. In this approximation one would observe the already predicted change of sign[79,62] in the asymmetry, similar to the QS effect; but if quark-gluon interactions - and therefore T-odd components of such functions - are not negligible, the prediction is not true.

D) Collins effect and Boer-Mulders effect

In the framework of chiral-odd functions, an important single spin asymmetry is produced by combination of two transversities. In particular, single transverse polarization gives rise to an asymmetry described by the "product"

$$A_{COL} \propto h_{1T} \otimes H' \quad \text{(in SIDIS), or} \tag{104}$$

$$A_{BM} \propto h_{1T} \otimes \bar{h}' + c.c. \text{ (in DY)}. \tag{105}$$

This asymmetry - exhibited by HERMES[20,21] data - is predicted to decrease like Q^{-1}.

We have also the $cos2\phi$ asymmetries

$$A_{CL2} \propto h' \otimes H' \quad \text{(in SIDIS), or} \tag{106}$$

[4] We obtained a different result in a previous paper[61], since we had started from a parameterization which is usually assumed for the correlator, but which is not in agreement with the results of the present paper.

$$A_{BM2} \quad \propto \quad h' \otimes \bar{h}' \quad \text{(in DY), or} \tag{107}$$

$$A_{CL3} \quad \propto \quad H' \otimes \bar{H}' \quad \text{(in e}^+\text{e}^- \text{ annihilation),} \tag{108}$$

which decrease like Q^{-2}. Also the asymmetries (104) to (108) - of which A_{BM2} has been detected experimentally[27–29] - are described differently than in other articles[47, 63,118]. As regards the Q^2 dependence of the Boer-Mulders asymmetry, our prediction is supported[61] by DY data[27–29]. On the other hand, the Q^2 dependence of the Collins and Sivers asymmetries might be tested in new planned experiments at higher energies[38].

8 Summary

In the present paper we have studied the gauge invariant quark-quark correlator, which we have expanded in powers of the coupling and split into a T-even and a T-odd part. Working in an axial gauge, the Politzer theorem on EOM has allowed us to interpret each term of the expansion in terms of Feynman-Cutkosky graphs, involving higher correlators and corresponding to powers of gM/Q. We have also elaborated an algorithm for writing a gauge invariant sector of the hadronic tensor in deep inelastic processes, like SIDIS, DY and e^+e^- annihilation. This gives rise to a rather long and complicate sum of terms. However, in the gauge considered, and especially at small transverse momenta, the factorizable terms prevail over the remaining ones, as we have shown explicitly for first order correction in gM/Q.

The zero order term and the first order correction of the expansion have been examined in detail. In both cases the Politzer theorem produces a considerable reduction of independent functions with respect to the naive parameterization in terms of Dirac components, giving rise to approximate (up to powers of gM/Q) relations among ”soft” functions. These relations survive QCD evolution. One such relation has been approximately verified against experimental data[26,20], another one suggests a method for determining approximately transversity, while others could be checked in next experiments[34,35,37,15]. Also an energy scale, introduced in the naive parameterization for dimensional reasons, has been determined in our approach, leading to predictions on Q^2 dependence of various azimuthal asymmetries. One such prediction finds confirmation in unpolarized DY data[27–29]. The hierarchy of TMD functions in terms of inverse powers of Q is established taking into account not only the Dirac operators, as in the case of common functions[42,43], but also the $p_\perp$ dependence, since in this case the orbital angular momentum plays a role as well as spin.

Moreover a relation is found among g_T, the QS asymmetry and the Sivers asymmetry; in particular, both g_T and the QS asymmetry are found to change sign according as to whether they are observed in SIDIS or in DY. Some doubts are cast, instead, on the predicted analogous change in the Sivers effect. We draw also some conclusions about the structure function $g_2(x)$, in particular against the BC and the ELT sum rules.

Quark fragmentation involves ”soft” interactions among final hadrons, but this does not imply a substantial difference with the distribution correlator. Rather, a caveat should be kept in mind for timelike photons, in DY and e^+e^- annihilation, when Q approaches the energy of a vector boson resonance, like the Υ or the Z^0. Since such a resonance interferes with the photon, one has to take into account its offshellness, quite different than Q^2. A particular attention has to be paid also to the case when the active quark (or antiquark) comes from gluon annihilation, as occurs, for example, in DY from proton-proton collisions. In this case the antiquarks come necessarily from the sea, which may sensibly change the Q^2 dependence of the coefficients of the T-odd functions. These two situations deserve a separate treatment.

As a conclusion, we stress that, although other authors already proposed, years ago, a decomposition of the hadronic tensor in terms of Feynman-Cutkosky graphs[72,73,75, 78], our deduction, based on EOM, leads to strong constraints on the parametrization of the ”soft” parts of the graphs.

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A Appendix A

We deduce a recursion formula for the terms of the expansion of the correlator. Our starting point is the Politzer theorem[71], which implies

$$\langle P,S|\bar{\psi}_j(0)\mathcal{L}(x)(i\not{D}-m)_{il}\psi_l(x)|P,S\rangle = 0. \tag{109}$$

Here $|P,S\rangle$ denotes the state of a hadron (for instance, but not necessarily, a nucleon) with four-momentum P and PL four-vector S. ψ is the quark field, of which we omit the color and flavor index. $D_\mu = \partial_\mu - ig\mathbf{A}_\mu$ is the covariant derivative, adopting for the gluon field the shorthand notation $\mathbf{A}_\mu$ for $A^a_\mu\lambda_a$. For the sake of simplicity, color and flavor indices of the quark field have been omitted. Moreover

$$\mathcal{L}(x) = \sum_{n=0}^{\infty}(ig)^n\Lambda_n(x), \tag{110}$$

where g is the strong coupling, while $\Lambda_0(x) = 1$. For $n \geq 1$, in an axial gauge $\mathbf{A}^+ = \mathbf{A}^- = 0$, we have, according to the notations and definitions of sect. 2,

$$\Lambda_n(x) = \int_{x_1}^{x_2} dz_1^{\mu_1} \int_{x_1}^{z_1} dz_2^{\mu_2} ... \int_{x_1}^{z_{n-1}} dz_n^{\mu_n} \left[\mathbf{A}_{\mu_1}(z_1)\mathbf{A}_{\mu_2}(z_2)...\mathbf{A}_{\mu_n}(z_n)\right]. \tag{111}$$

It is worth observing that

$$\partial_\mu \Lambda_n = \mathbf{A}_\mu(x_2)\Lambda_{n-1}. \tag{112}$$

Substituting expansion (110) into eq. (109), we get

$$\sum_{n=0}^{\infty}(ig)^n \left\{\bar{\psi}_j(0)\Lambda_n(x)(i\partial\!\!\!/ - m)_{il}\psi_l(x) - i\bar{\psi}_j(0)\Lambda_{n-1}(x)[i\mathbf{A}\!\!\!/(x)]_{il}\psi_l(x)\right\} = 0, \tag{113}$$

with

$$\Lambda_{-1}(x) = 0 \quad \text{and} \quad \Lambda_0(x) = 1. \tag{114}$$

Eq. (113) is an operator equation, to be intended in weak sense: it holds when calculated between hadronic states. All equations of this Appendix will be of this type from now on.

Looking for a perturbative solution for the correlator in powers of g, we set each term of the series (113) equal to zero, *i. e.*,

$$(i\partial\!\!\!/ - m)\mathcal{O}_n(x) = i\mathbf{A}\!\!\!/(x)\mathcal{O}_{n-1}(x), \tag{115}$$

where

$$[\mathcal{O}_n(x)]_{ij} = \bar{\psi}_j(0)\Lambda_n(x)\psi_i(x). \tag{116}$$

By Fourier transforming both sides of eq. (115), and recalling relation (112), we get

$$(p\!\!\!/ - m)\tilde{\mathcal{O}}_n(p) = i\gamma_\mu \int \frac{d^4x}{2\pi^4} e^{ipx} \left[\mathbf{A}^\mu(x_2)\mathcal{O}_{n-1}(x) + \mathcal{O}_{n-1}(x)\mathbf{A}^\mu(x)\right], \tag{117}$$

where

$$\tilde{\mathcal{O}}_n(p) = \int \frac{d^4x}{2\pi^4} e^{ipx}\mathcal{O}_n(x). \tag{118}$$

Eq. (117) can be rewritten as

$$(p\!\!\!/ - m)\tilde{\mathcal{O}}_n(p) = i\gamma_\mu \int \frac{d^4k}{2\pi^4} \left[\tilde{\mathbf{A}}^\mu(k)\tilde{\mathcal{O}}_{n-1}(p-k) + \tilde{\mathcal{O}}_{n-1}(p-k)\hat{\mathbf{A}}^\mu(k)\right], \tag{119}$$

where

$$\hat{\mathbf{A}}^\mu(k) = \int \frac{d^4x}{2\pi^4} e^{ikx}\mathbf{A}^\mu(x), \tag{120}$$

$$\tilde{\mathbf{A}}^\mu(k) = \delta(k^+) \lim_{M\to\infty} \int d\kappa e^{-i\kappa M}\hat{\mathbf{A}}_\mu(k^-, \kappa, \mathbf{k}_\perp). \tag{121}$$

Eq. (119) is a recursion formula for $\tilde{\mathcal{O}}_n(p)$, eqs. (114) constituting the first steps. This formula implies eqs. (16) (for n = 0) and (17) (for $n \geq 1$) in the text. In particular, as regards eq. (17), the quantity Γ_n results in

$$\Gamma_n = N\langle P, S|\tilde{\mathcal{O}}_n(p)|P, S\rangle, \tag{122}$$

where N is a normalization constant. The operator $\tilde{\mathcal{O}}_n(p)$ in eq. (119) corresponds to a graph endowed with n gluons, such that the n-th gluon leg is attached to the quark leg on the left side of the graph (see figs. 2a and 3a).

Taking into account the hermitian character of $\hat{\mathbf{A}}^\mu(k)$k and the relation $[\tilde{\mathcal{O}}_n(p)]^\dagger = \gamma_0\tilde{\mathcal{O}}_n(p)\gamma_0$, eq. (119) implies

$$\tilde{\mathcal{O}}_n(p)(\not{p} - m) = -i\int\frac{d^4k}{2\pi^4}[\tilde{\mathcal{O}}_{n-1}(p-k)\tilde{\mathbf{A}}^\mu(k) + \hat{\mathbf{A}}^{\mu\dagger}(k)\tilde{\mathcal{O}}_{n-1}(p-k)]\gamma_\mu. \tag{123}$$

In this case $\tilde{\mathcal{O}}_n(p)$ corresponds again to a graph with n gluons, but such that the n-th gluon is attached to the quark leg on the right side of the graph. This last result implies that Γ_n represents any graph with n gluons, each gluon leg being attached to the left or right quark leg.

B Appendix B

Here we deduce the parameterizations of the quark-quark correlator at zero order and of the quark-gluon-quark correlation, arising from first order correction.

B.1 The Zero Order Quark-Quark Correlator

The matrix $\Gamma_0(p)$, defind by

$$(\Gamma_0)_{ij} = N\int\frac{d^4x}{(2\pi)^4}e^{ipx}\langle P, S|\bar{\psi}_j(0)\psi_i(x)|P, S\rangle, \tag{124}$$

fulfils the homogeneous Dirac equation

$$(\not{p} - m)\Gamma_0(p) = 0, \tag{125}$$

where m is the rest mass of the quark. As shown in Appendix A, this is a consequence of the Politzer theorem, which implies, at zero order in the coupling,

$$(\not{\partial} - m)\psi(x) = 0. \tag{126}$$

Therefore, in the approximation considered, the quark can be treated as if it were on shell (see also ref. [119]). Then, initially, we consider the Fourier expansion of the unrenormalized field of an on-shell quark, *i. e.*,

$$\psi(x) = \int \frac{d^3\tilde{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\mathcal{P}}} e^{-ipx} \sum_s u_s(p) c_s(p), \tag{127}$$

where $s = \pm 1/2$ is the spin component of the quark along a given direction in the quark rest frame, u its four-spinor, c the destruction operator for the flavor considered and

$$d^3\tilde{p} = d^4p \; \delta\left(p^- - \frac{m^2 + \mathbf{p}_\perp^2}{2p^+}\right), \qquad\qquad \mathcal{P} = p^+/\sqrt{2}. \tag{128}$$

As regards the normalization of u_s and c_s, we assume

$$\bar{u}_s u_s = 2m, \qquad \langle P, S| c_s^\dagger(p') c_s(p) |P, S\rangle = (2\pi)^3 \delta^3(\tilde{\mathbf{p}}' - \tilde{\mathbf{p}}) q_s(p), \tag{129}$$

where

$$\tilde{\mathbf{p}} \equiv (p^+, \mathbf{p}_\perp) \tag{130}$$

and $q_s(p)$ is the probability density to find a quark with spin component s and four-momentum $p \equiv (p^-, \tilde{\mathbf{p}})$, with $p^- = (m^2 + \mathbf{p}_\perp^2)/2p^+$. For an antiquark the definition is analogous, except that, in the Fourier expansion (127), we have to substitute the destruction operators c_s with the creation operators $d_s^\dagger$ and p with $-p$ in the exponential.

Choosing the quantization axis along the hadron momentum $\mathbf{P}$ in the laboratory frame, and substituting eq. (127) into eq. (124), we get

$$\begin{aligned} (\Gamma_0)_{ij}(p) &= \frac{N}{2\mathcal{P}} \sum_{s,s'} \int \frac{d^3\tilde{p}'}{(2\pi)^3} \langle P, S| c_s^\dagger(p) c_{s'}(p') |P, S\rangle \\ \times \quad & [u_{s'}(p')]_i [\bar{u}_s(p)]_j \; \delta\left(p^- - \frac{m^2 + \mathbf{p}_\perp^2}{2p^+}\right). \end{aligned} \tag{131}$$

But owing to the second eq. (129) we have

$$\Gamma_0(p) = [\Gamma_0^a(p) + \Gamma_0^b(p)] \; \delta\left(p^- - \frac{m^2 + \mathbf{p}_\perp^2}{2p^+}\right), \tag{132}$$

where

$$\Gamma_0^a(p) = \frac{N}{2\mathcal{P}} \sum_s \langle P, S| c_s^\dagger(p) c_s(p) |P, S\rangle u_s(p) \bar{u}_s(p), \tag{133}$$

$$\Gamma_0^b(p) = \frac{N}{2\mathcal{P}} \sum_s \langle P, S| c_{-s}^\dagger(p) c_s(p) |P, S\rangle u_{-s}(p) \bar{u}_s(p). \tag{134}$$

Firstly we elaborate Γ_0^a. We have

$$u_s(p)\bar{u}_s(p) = \frac{1}{2}(\not{p} + m)(1 + 2s\gamma_5 \not{S}_\parallel^a). \tag{135}$$

Here $S_\parallel^a$ is a four-vector such that, in the quark rest frame, $S_\parallel^a \equiv (0, \lambda/|\lambda|\hat{\mathbf{P}})$, $\lambda = \mathbf{S} \cdot \hat{\mathbf{P}}$, $\hat{\mathbf{P}} = \mathbf{P}/|\mathbf{P}|$ and $\mathbf{S}$ is the unit spin vector of the hadron in its rest frame. Therefore

$$\Gamma_0^a(p) = \frac{N}{2\mathcal{P}} \frac{1}{2}(\not{p} + m)\left[f_1(p) + \Delta' q(p)\gamma_5 \not{S}_\parallel^a\right], \tag{136}$$

where

$$f_1(p) = \sum_s \langle P, S|c_s^\dagger(p)c_s(p)|P, S\rangle \tag{137}$$

is the unpolarized transverse momentum distribution of the quark, while

$$\Delta' q(p) = \sum_s 2s\langle P, S|c_s^\dagger(p)c_s(p)|P, S\rangle. \tag{138}$$

According to transformation properties of one-particle states under rotations, one has

$$|P, S\rangle = cos\frac{\theta}{2}|P, +\rangle + i|P, -\rangle sin\frac{\theta}{2}, \tag{139}$$

where $\pm$ denotes the (positive or negative) helicity of the hadron and θ the angle between $\mathbf{P}$ and $\mathbf{S}$. Substituting eq. (139) into eq. (138), and taking into account parity conservation, we get

$$\Delta' q(p) = cos\theta g_{1L}(p). \tag{140}$$

Here

$$g_{1L}(p) = \sum_s 2s\langle P, +|c_s^\dagger(p)c_s(p)|P, +\rangle = -\sum_s 2s\langle P, -|c_s^\dagger(p)c_s(p)|P, -\rangle. \tag{141}$$

is the longitudinally polarized TMD distribution of the quark, the last equality following from parity conservation.

Now we consider Γ_0^b. Eq. (139) yields, for $\theta = \pi/2$,

$$|\uparrow(\downarrow)\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle), \tag{142}$$

where $|\pm\rangle$ and $|\uparrow(\downarrow)\rangle$ denote quark states with spin components, respectively, along $\hat{\mathbf{P}}$ and along

$$\mathbf{S}_\perp = \mathbf{S} - \lambda\hat{\mathbf{P}}. \tag{143}$$

Substituting eqs. (139) and (142) into eq. (134), and taking into account again parity conservation, we get

$$\Gamma_0^b(p) = \frac{N}{2\mathcal{P}} \frac{1}{2} sin\theta h_{1T}(p)(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|), \tag{144}$$

where

$$h_{1T}(p) = \langle P, -|c_+^\dagger(p)c_-(p)|P, +\rangle = \langle P, +|c_-^\dagger(p)c_+(p)|P, -\rangle \tag{145}$$

is the TMD transversity of the quark. Returning to the Dirac notation, we have

$$|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\not p + m)\left(1 + \gamma_5 \not S_\perp^b\right), \qquad |\downarrow\rangle\langle\downarrow| = \frac{1}{2}(\not p + m)\left(1 - \gamma_5 \not S_\perp^b\right), \tag{146}$$

where $S_\perp^b$ is such that $S_\perp^b \equiv (0, \hat{\mathbf{n}})$ in the quark rest frame and

$$\hat{\mathbf{n}} = \frac{\mathbf{S}_\perp}{|\mathbf{S}_\perp|}. \tag{147}$$

Then eq. (144) goes over into

$$\Gamma_0^b(p; P, S) = \frac{N}{2\mathcal{P}} \frac{1}{2} sin\theta \Delta_T q(p)(\not p + m)\gamma_5 \not S_\perp^b. \tag{148}$$

Substituting eqs. (136), (140) and (148) into eq. (132) yields

$$\Gamma_0 = \frac{N}{2\mathcal{P}} \frac{1}{2}(\not p + m)\left[f_1 + g_{1L}\gamma_5 \not S_\parallel^q + h_{1T}\gamma_5 \not S_\perp^q\right] \delta\left(p^- - \frac{m^2 + \mathbf{p}_\perp^2}{2p^+}\right), \tag{149}$$

having set $S_\parallel^q = S_\parallel^a cos\theta$ and $S_\perp^q = S_\perp^b sin\theta$. Eq. (149) is a solution to eq. (125), which is a consequence of the Politzer theorem at zero order in g. Since this equation survives renormalization - which generally implies only a weak Q-dependence[70,93] - the structure of Γ_0 is not changed by QCD evolution.

Lastly we deduce the expressions of the four-vectors $S_\parallel^q$ and $S_\perp^q$ in the frame where the quark momentum is $\mathbf{p}$. In the quark rest frame we have

$$S_\parallel^q \equiv (0, \lambda\hat{\mathbf{P}}), \qquad S_\perp^q \equiv (0, \mathbf{S}_\perp). \tag{150}$$

In view of the Lorentz boost, it is convenient to further decompose $\lambda\hat{\mathbf{P}}$ and $\mathbf{S}_\perp$ into components parallel and perpendicular to the quark momentum. We have

$$\lambda\hat{\mathbf{P}} = \lambda cos\alpha\hat{\mathbf{p}} + \mathbf{\Sigma}_\parallel, \qquad \mathbf{\Sigma}_\parallel = -cos\alpha\frac{\mathbf{p}_\perp}{|\mathbf{p}|} + sin^2\alpha\hat{\mathbf{P}}, \tag{151}$$

$$\mathbf{S}_\perp = \lambda_\perp\hat{\mathbf{p}} + \mathbf{\Sigma}_\perp, \qquad \mathbf{\Sigma}_\perp = |\mathbf{S}_\perp| cos\beta(cos\beta\hat{\mathbf{n}} - sin\beta\hat{\mathbf{k}}), \tag{152}$$

where

$$\hat{\mathbf{p}} = \frac{\mathbf{p}}{|\mathbf{p}|}, \qquad \hat{\mathbf{k}} = \hat{\mathbf{n}} \times \frac{\hat{\mathbf{p}} \times \hat{\mathbf{n}}}{|\hat{\mathbf{p}} \times \hat{\mathbf{n}}|}, \tag{153}$$

$$\alpha = arccos(\hat{\mathbf{P}} \cdot \hat{\mathbf{p}}) \quad \text{and} \quad \beta = arcsin(\hat{\mathbf{n}} \cdot \hat{\mathbf{p}}). \tag{154}$$

The boost which transforms the four-momentum of the quark from $(m, 0)$ to $(E, \mathbf{p})$, with $E = \sqrt{m^2 + \mathbf{p}^2}$, changes only the components along $\hat{\mathbf{p}}$ of $\lambda\hat{\mathbf{P}}$ and of $\mathbf{S}_\perp$. In particular, the boost transforms the four-vector $(0, \mathbf{p})$ to $\bar{p}/m$, with $\bar{p} \equiv (|\mathbf{p}|, E\hat{\mathbf{p}})$. Therefore, since α and β are $O(|\mathbf{p}_\perp|/|\mathbf{p}|)$ and $|\mathbf{p}|/\mathcal{P} = O(1)$, eqs. (150) go over into

$$S^q_\parallel = \lambda\left(\frac{\bar{p}}{m} - \bar{\eta}_\perp\right) + O(\bar{\eta}^2_\perp), \qquad S^q_\perp = S_\perp + \bar{\lambda}_\perp \frac{\bar{p}}{m_q} + O(\bar{\eta}^2_\perp), \tag{155}$$

where $\bar{\eta}_\perp = p_\perp/\mathcal{P}$ and $\bar{\lambda}_\perp = -S \cdot \bar{\eta}_\perp$.

B.2 The Quark-Gluon-Quark Correlator

Now we deduce a parameterization for the quark-gluon-quark correlator, defined by

$$\left[\Phi^{(1)}_\mu(p,k)\right]_{ij} = N \int \frac{d^4x}{(2\pi)^4} e^{i(p-k)x} \langle P, S|\bar{\psi}_j(0)[\hat{\mathbf{A}}_\mu(k) + \tilde{\mathbf{A}}_\mu(k)]\psi_i(x)|P, S\rangle. \tag{156}$$

As shown in Appendix A, the Politzer theorem implies, at order 1 in the coupling,

$$(\not{p} - \not{k} - m)\Phi^{(1)}_\mu(p,k) = 0, \tag{157}$$

which holds also after renormalization. Therefore our line of reasoning is the same as for Γ_0, that is, we start from unrenormalized fields and we take on-shell quarks, whose field satisfies expansion (127). Substituting this expansion into eq. (156), we get

$$\Phi^{(1)}_\mu(p,k) = \Psi_\mu(p,k)\delta\left(p^-_1 - \frac{m^2 + \mathbf{p}^2_{1\perp}}{2p^+_1}\right), \tag{158}$$

$$\Psi_\mu(p,k) = N \int \frac{d^3\tilde{p}'}{(2\pi)^3} \frac{1}{2\sqrt{\mathcal{P}_1\mathcal{P}'}} \sum_{s,s'} \mathcal{A}_{s,s',\mu}(p',k) u_s(p_1)\bar{u}_{s'}(p'). \tag{159}$$

Here $d^3\tilde{p}'$ and $\mathcal{P}'$ are defined analogously to eqs. (128),

$$p_1 = p - k, \qquad \mathcal{P}_1 = p^+_1/\sqrt{2} \tag{160}$$

and

$$\mathcal{A}_{s,s',\mu}(p',k) = \langle P, S|c^\dagger_s(p')[\hat{\mathbf{A}}_\mu(k) + \tilde{\mathbf{A}}_\mu(k)]c_{s'}(p_1)|P, S\rangle. \tag{161}$$

Moreover the matrix element (161) fulfils a relation of the type

$$\mathcal{A}_{s,s',\mu}(p',k) = (2\pi)^3 \mathcal{C}_{s,s',\mu}(p',k)\delta^3(\tilde{\mathbf{p}}' - \tilde{\mathbf{p}}_1 - \tilde{\mathbf{k}}), \tag{162}$$

where $\mathcal{C}_{s,s',\mu}(p',k)$ is a quark-gluon correlator and $\tilde{\mathbf{p}}'$, $\tilde{\mathbf{p}}_1$ and $\tilde{\mathbf{k}}$ are defined by eq. (130). Then eq. (159) yields

$$\Psi_\mu(p,k) = \frac{N}{2\sqrt{\mathcal{P}_1\mathcal{P}}} \sum_{s,s'} \mathcal{C}_{s,s',\mu}(p,k) u_s(p_1)\bar{u}_{s'}(p_0) \tag{163}$$

and

$$p_0 \equiv (p_0^-, \tilde{\mathbf{p}}), \qquad\qquad p_0^- = \frac{\mathbf{p}_\perp^2 + m^2}{2p^+}. \tag{164}$$

We rewrite eq. (163) as

$$\Psi_\mu(p,k) = \frac{N}{2\sqrt{\mathcal{P}_1\mathcal{P}}}(\Psi_\mu^a + \Psi_\mu^b), \tag{165}$$

where

$$\Psi_\mu^a = \sum_s \mathcal{C}_{s,s,\mu}(p_1,k)u_s(p_1)\bar{u}_s(p_0), \tag{166}$$

$$\Psi_\mu^b = \sum_s \mathcal{C}_{s,-s,\mu}(p_1,k)u_s(p_1)\bar{u}_{-s}(p_0). \tag{167}$$

Taking into account the appropriate Lorentz transformations for the spinors involved, we have

$$u_s(p_1)\bar{u}_s(p_0) = \frac{1}{2}(\not{p}_1 + m)U(p_1,p_0)(1 + 2s\gamma_5 \not{S}_{0\parallel}^q), \tag{168}$$

$$u_s(p_1)\bar{u}_{-s}(p_0) = \frac{1}{2}(\not{p}_1 + m)U(p_1,p_0)\gamma_5(cos\chi \not{S}_{0\perp}^q + sin\chi \bar{\not{S}}_\perp). \tag{169}$$

Here

$$U(p_1,p_0) = exp\left[\frac{1}{2}(\phi_1\hat{\mathbf{p}}_1 - \phi_0\hat{\mathbf{p}}_0)\cdot\vec{\alpha}\right], \tag{170}$$

$$\phi_1 = ln\frac{E_1 + |\mathbf{p}_1|}{m}, \qquad\qquad \hat{\mathbf{p}}_1 = \frac{\mathbf{p}_1}{|\mathbf{p}_1|}, \tag{171}$$

$$\mathbf{p}_1 \equiv (\mathbf{p}_{1\perp}, \frac{1}{\sqrt{2}}(p_1^+ - p_1^-)), \qquad\qquad E_1 = \sqrt{\mathbf{p}_1^2 + m^2}, \tag{172}$$

analogous definitions holding for ϕ_0 and $\hat{\mathbf{p}}_0$. Moreover $S_{0\parallel}^q$ and $S_{0\perp}^q$ refer to the PL vector of a quark with four-momentum p_0, directly connected with nucleon polarization; they can be related to the nucleon longitudinal and transverse PL vectors, using the formulae elaborated at the end of sect. B1. $\bar{S}_\perp$ refers to the spin caused by spin-orbit coupling,

$$\sqrt{|p_{0\perp}^2|}\bar{S}_{\perp\alpha} = \epsilon_{\alpha\beta\gamma\rho}n_+^\beta n_-^\gamma p_{0\perp}^\rho. \tag{173}$$

Lastly, χ is a real parameter.

We assume $\theta_0, \theta_1 << 1$, where θ_0 and θ_1 are, respectively, the angle between $\mathbf{p}_0$ and $\mathbf{P}$ and the one between $\mathbf{p}_1$ and $\mathbf{P}$. Then

$$U(p_1,p_0) \approx cosh\varphi + \frac{1}{2\varphi}\gamma_0(\gamma_3 a + \gamma_i r_\perp^i)sinh\varphi, \tag{174}$$

with

$$\varphi = \frac{1}{2}\sqrt{(\phi_0-\phi_1)^2+\theta^2\phi_0\phi_1}, \qquad \theta = \theta_1-\theta_0, \tag{175}$$

$$a = \phi_1-\phi_0-\frac{1}{2}(\phi_1\theta_1^2-\phi_0\theta_0^2), \qquad \mathbf{r}_\perp = \frac{\phi_1}{|\mathbf{p}_1|}\mathbf{p}_{1\perp}-\frac{\phi_0}{|\mathbf{p}_0|}\mathbf{p}_{0\perp}, \tag{176}$$

Then Ψ_μ results in

$$\Psi_\mu(p_1,k) \approx \frac{1}{2}(\not{p}_1+m)\mathcal{L}[\mathcal{C}_\mu+\Delta\mathcal{C}_\mu\gamma_5\not{S}_\parallel^q+\Delta_T\mathcal{C}_\mu\gamma_5\not{S}_\perp^q+\Delta_T\mathcal{C}'_\mu\gamma_5\not{S}_\perp^s], \tag{177}$$

where

$$\mathcal{L} = \frac{N}{2\sqrt{\mathcal{P}_1\mathcal{P}}}[cosh\varphi+\frac{1}{2\varphi}\gamma_0(\gamma_3 a+\gamma_i r_\perp^i)sinh\varphi] \tag{178}$$

and

$$\mathcal{C}_\mu(p_1,k) = \sum_s \mathcal{C}_{s,s,\mu}(p_1,k), \tag{179}$$

$$\Delta\mathcal{C}_\mu(p_1,k) = \sum_s 2s\mathcal{C}_{s,s,\mu}(p_1,k), \tag{180}$$

$$\Delta_T\mathcal{C}_\mu(p_1,k) = \sum_s cos\chi\mathcal{C}_{s,-s,\mu}(p_1,k), \tag{181}$$

$$\Delta_T\mathcal{C}'_\mu(p_1,k) = \sum_s sin\chi\mathcal{C}_{s,-s,\mu}(p_1,k) \tag{182}$$

are correlation functions. We parameterize these functions with the available transverse four-vectors, since we operate within an axial gauge:

$$\mathcal{C}_\mu = \mathcal{C}_1 p_{1\perp\mu}+\epsilon_{\mu\nu\rho\sigma}n_-^\nu(\mathcal{C}_2 S_\parallel^\rho p_{1\perp}^\sigma+\mathcal{C}_3 M S_\perp^\rho n_+^\sigma), \tag{183}$$

$$\Delta\mathcal{C}_\mu = \Delta\mathcal{C}p_{1\perp\mu}, \tag{184}$$

$$\Delta_T\mathcal{C}_\mu = \Delta_T\mathcal{C}p_{1\perp\mu}, \tag{185}$$

$$\Delta_T\mathcal{C}'_\mu = \Delta_T\mathcal{C}'p_{1\perp\mu}. \tag{186}$$

Here $\mathcal{C}_1$, $\mathcal{C}_2$, $\mathcal{C}_3$, $\Delta\mathcal{C}$, $\Delta_T\mathcal{C}$ and $\Delta_T\mathcal{C}'$ are ”soft” functions of p and k. The parameterization of $\Phi_\mu^{(1)}$ is obtained by inserting eqs. (177) and (183) to (186) into eq. (158). Again, as in the case of Γ_0, the Politzer theorem, of which eq. (157) is a consequence, implies that renormalization effects preserve the structure of that parameterization.

C Appendix C

Here we consider the parameterization of the correlator in terms of the Dirac components, up to and including twist-3 terms. This parameterization is similar to the usual ones[47,81], also as regards notations, except for an energy scale μ_0, which we leave undetermined here, and for the twist-2, T-odd sector, which we omit because it has no place in our procedure. The scale μ_0, usually set equal to the rest mass of the hadron[47,81], is determined in the sects. 4 and 5.

The parameterization reads

$$\Phi = 2p^+ \left[(\Psi_E^f + \Psi_H^f)\delta(p^2 - m^2) + (\Psi_O^i + \Psi_H^i)\frac{1}{\pi(p^2 - m^2 + i\epsilon})\right], \tag{187}$$

where

$$\begin{aligned}
\Psi_E^f &= \frac{\mathcal{P}}{\sqrt{2}}\{f_1\not{n}_+ + (\lambda g_{1L} + \lambda_\perp g_{1T})\gamma_5\not{n}_+ + \frac{1}{2}h_{1T}\gamma_5[\not{S}_\perp, \not{n}_+] \\
&+ \frac{1}{2}(\lambda h_{1L}^\perp + \lambda_\perp h_{1T}^\perp)\gamma_5[\not{\eta}_\perp, \not{n}_+]\}, \qquad (188) \\
\Psi_H &= \frac{1}{2}\{(f^\perp + \lambda g_L^\perp\gamma_5 + \lambda_\perp g_T^\perp\gamma_5)\not{p}_\perp + \frac{1}{4}\lambda_\perp h_T^\perp\gamma_5[\not{S}_\perp, \not{p}_\perp] \\
&+ \frac{1}{2}xM\left(e + g_T'\gamma_5\not{S}_\perp + \frac{1}{2}(\lambda h_L + \lambda_\perp h_T)\gamma_5[\not{n}_-, \not{n}_+]\right)\}, \qquad (189) \\
\Phi_O^i &= \frac{2p^+}{p^2 - m^2 + i\epsilon}\{\epsilon_{ij}S_\perp^i(p_\perp^j e_T^\perp + M\gamma^j f_T) + \epsilon_{ij}\bar{S}_\perp^i p_\perp^j e_T'^\perp + \gamma_5(xMe_L\lambda \\
&+ e_T p_\perp \cdot S_\perp + e_T' p_\perp \cdot \bar{S}_\perp) + \epsilon_{ij}\gamma_i p_\perp^j(f_L^\perp\lambda + f_T^\perp\lambda_\perp + \gamma_5 g^\perp) \\
&+ \gamma_5\not{p}_\perp\bar{\not{S}}_\perp h' + \frac{1}{2}\gamma_5[\gamma_+, \gamma_-]p_\perp \cdot \bar{S}_\perp h'^\perp\}. \qquad (190)
\end{aligned}$$

Here Ψ_H denotes the "hybrid" term, both interaction free (Ψ_H^f, T-even) and interaction dependent (Ψ_H^i, T-odd): the two terms have the same parameterization, but behave quite differently. For the "soft" functions we have adopted notations similar to those employed in ref. [81]. Note, however, that in the expression of Φ_O^i the functions f_T, $e_T'^\perp$, e_T', h' and $h'^\perp$ do not appear in the parameterization proposed by ref. [81]; on the contrary, we have not taken into account the functions h and $f_T'^\perp$, defined in that reference.

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