Q² DEPENDENCE OF AZIMUTHAL ASYMMETRIES IN SEMI-INCLUSIVE DEEP INELASTIC SCATTERING AND IN DRELL–YAN

Elvio Di Salvo

INFN, Sezione di Genova, Dip. Fisica, Univ. di Genova, I-16146 Genova, Italy

Abstract

We study several azimuthal asymmetries in semi-inclusive deep inelastic scattering and in Drell-Yan, interpreting them in the framework of the formalism of the quark correlator, with a particular reference to T-odd functions. The correlator contains an undetermined energy scale, which we fix on the basis of a simple and rather general argument. We find a different value than the one assumed in previous treatments of T-odd functions. This implies different predictions on the Q^2 dependence of the above mentioned asymmetries. Our result about the azimuthal asymmetry of unpolarized Drell-Yan agrees with presently available data, contrary to the alternative assumption on the scale. Predictions on other azimuthal asymmetries could be tested against data of planned experiments on Drell-Yan and semi-inclusive deep inelastic scattering.

1 Introduction

Azimuthal asymmetries of remarkable size have been observed in various high energy inclusive reactions, especially in unpolarized Drell-Yan[1], in singly polarized semi- inclusive deep inelastic scattering[2–5] (SIDIS) and in inclusive production of (anti-) hyperons[6] and pions[7] from singly polarized hadronic collisions. The interpretation of such asymmetries from basic principles of QCD is quite challenging and has stimulated the interest of high energy physicists. In particular, in the present paper we focus our attention on the SIDIS and Drell-Yan asymmetries, which are somewhat analogous, since the two reactions are kinematically isomorphic. The theoretical activity about this subject is quite intense and lively, as witnessed by the numerous articles dedicated to the topic[8-27] in the last 15 years.

An important element in the interpretation of such effects is the intrinsic transverse momentum of partons inside a hadron, whose crucial role in high energy reactions has been widely illustrated in the last years[10,11,28–31]. Indeed, the transverse momentum is connected to the T-odd quark densities[8,13,17], which provide a quite natural interpretation of the above mentioned asymmetries[13,17,21]. At the same time, the T-odd fuctions involve predictions of further azimuthal asymmetries in unpolarized and singly polarized inclusive reactions[15,12].

These functions explain simultaneously[21] the remarkable $cos2\phi$ asymmetry and the negligible $cos\phi$ Fourier component exhibited by unpolarized Drell-Yan data[1], where ϕ is the usual azimuthal angle adopted in the phenomenological fits[1]. The term $cos2\phi$ may be just interpreted as a signature[21] of the pair of chiral-odd (and T-odd) functions involved in this picture. However, the current treatment of the T-odd functions does not reproduce the dependence of this asymmetry on the effective mass of the Drell-Yan lepton pair. More generally, some doubts have been cast on the Q^2 dependence of the transverse momentum distribution functions[32–34], where Q is the QCD hard scale. This imposes a revision of the parametrization of the transverse momentum quark correlator, a fundamental theoretical tool for cross section calculations at high energies. This quantity - originally introduced by Ralston and Soper in 1979[35] and successively improved by Mulders and Tangerman[11,10] (see also the more recent contributions on the subject[16,36]) - consists of a 4×4 matrix. Therefore it may be parametrized according to the components of the Dirac algebra, taking into account the available vectors and hermiticity and Lorentz and parity invariance. The parametrization - whose coefficients are the quark distribution functions inside the hadron - includes an undetermined energy scale, μ_0[30], usually assumed[35,10,11] equal to the mass of the hadron related to the active quark. We shall see that this choice is not unique, perhaps not the most appropriate in normalizing some

"leading twist" functions. Alternatively, we propose $\mu_0 = Q/2$, which explains quite naturally the Q^2 dependence of the unpolarized Drell-Yan asymmetry. Moreover, concerning the SIDIS and other Drell-Yan azimuthal asymmetries, we get predictions which contrast with those given by previous authors, and which could be tested against present[2,4], forthcoming [5] and future[37–39] data.

Here we shall not study all azimuthal asymmetries considered in the literature[40, 41], we shall limit ourselves to SIDIS of unpolarized or longitudinally polarized lepton beams off unpolarized or transversely polarized targets, and to unpolarized or singly polarized (with transverse polarization) Drell-Yan; moreover, we shall consider just the asymmetries usually classified as leading twist[11,13,15].

The paper is organized as follows. In sect. 2 we give the general formulae for the SIDIS and Drell-Yan cross sections, introducing the formalism of the correlator; in particular we illustrate in detail the T-odd functions. Sect. 3 is dedicated to the theoretical formulae for azimuthal asymmetries. In sect. 4 we determine the parameter μ_0, by comparing the correlator with the density matrix of a free, on-shell quark. Such a determination leads to predictions on the Q^2 dependence of the asymmetries, which we illustrate in sect. 5. In sect. 6 we compare our results with experimental data, as regards unpolarized Drell-Yan. Lastly we draw a short conclusion in sect. 7.

2 SIDIS and Drell-Yan Cross Sections

2.1 General formulae

Consider the SIDIS and the Drell-Yan reactions, *i. e.*,

$$lh_A \to l'h_B X \quad \text{and} \quad h_A h_B \to l^+ l^- X, \tag{1}$$

where the l's are charged leptons and the h's are hadrons. Incidentally, these two reactions are topologically equivalent[9]. At not too high energies one can adopt one-photon exchange approximation, where the cross sections for such reactions have an expression of the type

$$\frac{d\sigma}{d\Gamma} = \frac{(4\pi\alpha)^2}{4\mathcal{F}Q^4} L^{\mu\nu} W_{\mu\nu}. \tag{2}$$

Here $d\Gamma$ is the phase space element, α the fine structure constant and

$$\mathcal{F} = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} \tag{3}$$

the flux factor, p_i and m_i ($i = 1, 2$) being the 4-momenta and the masses of the initial particles. Moreover $L^{\mu\nu}$ and $W^{\mu\nu}$ are respectively the leptonic and hadronic tensor. In

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particular, we have

$$L^{\mu\nu} = \ell^\mu \bar{\ell}^\nu + \ell^\nu \bar{\ell}^\mu - g^{\mu\nu} \ell \cdot \bar{\ell}, \qquad (4)$$

where ℓ and $\bar{\ell}$ are the four-momenta of the initial and final lepton (in SIDIS) or of the two final leptons (in Drell-Yan). As regards the hadronic tensor, one often adopts, in the framework of the factorization theorem[42,9,19], the so-called "handbag" approximation, where all information concerning the "soft" functions of the quark inside the hadrons is encoded in a parametrization of the quark-quark correlator, according to the various Dirac components[10,11]. In this approximation the hadronic tensor reads

$$W^{\mu\nu} = c \sum_a e_a^2 \int d^2 p_\perp Tr \left[\Phi_A^a(x_a, \mathbf{p}_\perp) \gamma^\mu \Phi_B^b(x_b, \mathbf{q}_\perp - \mathbf{p}_\perp) \gamma^\nu \right]. \qquad (5)$$

Here c is due to color degree of freedom, $c = 1$ for SIDIS and $c = 1/3$ for Drell-Yan. Φ_A and Φ_B are correlators, relating the active (anti-)quarks to the (initial or final) hadrons h_A and h_B. a and b are the flavors of the active partons, with $a = u, d, s, \bar{u}, \bar{d}, \bar{s}$ and $b = a$ in SIDIS, $b = \bar{a}$ in Drell-Yan; e_a is the fractional charge of flavor a. In Drell-Yan Φ_A and Φ_B encode information on (anti-)quark distributions inside the initial hadrons: the x's are the longitudinal fractional momenta of the active quark and antiquark, $\mathbf{p}_\perp$ is the transverse momentum of the activeparton of h_A and $\mathbf{q}_\perp$ is the transverse momentum of the lepton pair. In SIDIS Φ_B is replaced by the fragmentation correlator $\Delta[z, z(\mathbf{q}_\perp - \mathbf{p}_\perp)]$, describing the fragmentation of the struck quark into the final hadron h_B (see subsect. 2.4). Here z is the longitudinal fractional momentum of h_B with respect to the fragmenting quark and $z\mathbf{q}_\perp$ is the transverse momentum of h_B with respect to the virtual photon momentum. Approximation (5) holds for the hadronic tensor under the condition[21,18]

$$q_\perp << Q, \qquad (6)$$

where $q_\perp = |\mathbf{q}_\perp|$. Moreover we neglect the Sudakov suppression[43], as can be assumed at moderate Q^2.

2.2 Parametrization of the Correlator

The correlator for a nucleon may be parametrized according to the Dirac algebra, taking into account hermiticity and Lorentz and parity invariance. It is conveniently split into a T-even and a T-odd term, *i. e.*,

$$\Phi = \Phi_e + \Phi_o, \qquad (7)$$

where Φ_e is even under time reversal and Φ_o is odd under the same transformation. At leading twist one has[11,44,36]

$$\Phi_e \simeq \frac{\mathcal{P}}{\sqrt{2}} \left\{ f_1 \not{h}_+ + (\lambda g_{1L} + \lambda_\perp g_{1T}) \gamma_5 \not{h}_+ + \frac{1}{2} h_{1T} \gamma_5 [\not{S}_\perp, \not{h}_+] \right\}$$

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$$+ \quad \frac{\mathcal{P}}{\sqrt{2}} \left(\lambda h_{1L}^{\perp} + \lambda_{\perp} h_{1T}^{\perp} \right) \gamma_5 [\hbar_{\perp}, \hbar_{+}], \tag{8}$$

$$\Phi_o \quad \simeq \quad \frac{\mathcal{P}}{\sqrt{2}} \left\{ f_{1T}^{\perp} \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu} n_{+}^{\nu} \eta_{\perp}^{\rho} S_{\perp}^{\sigma} + i h_1^{\perp} \frac{1}{2} [\hbar_{\perp}, \hbar_{+}] \right\}. \tag{9}$$

In formulae (8) and (9) I have used the notations and the normalization of refs.[11,10] for the "soft" functions[1]. $n_{\pm}$ are lightlike vectors, such that $n_{+} \cdot n_{-} = 1$ and whose space components are directed along (+) or opposite (-) to the nucleon momentum. Moreover

$$S = \lambda \frac{P^+}{M} n_+ - \frac{M}{2P^+} n_- + S_{\perp} \tag{10}$$

is the Sudakov decomposition of the Pauli-Lubanski vector S of the nucleon, whose four-momentum is P, with $P^2 = M^2$, $P^+ = P \cdot n_-$ and $S^2 = -1$. Thirdly,

$$\mathcal{P} \quad = \quad \frac{1}{\sqrt{2}} p \cdot n_-, \qquad \lambda_{\perp} = -S \cdot \eta_{\perp}, \tag{11}$$

$$\eta_{\perp} \quad = \quad p_{\perp}/\mu_0, \qquad p_{\perp} = p - (p \cdot n_-)n_+ - (p \cdot n_+)n_- \tag{12}$$

and p is the quark four-momentum. Lastly, the energy scale μ_0, encoded in the dimensionless vector $\eta_{\perp}$, has been introduced in such a way that all functions involved in the parametrization of Φ have the dimensions of a probability density. This scale - defined for the first time in ref.[30], where it was denoted by m_D - determines the normalization of the functions which depend on $\eta_{\perp}$; therefore μ_0 has to be chosen in such a way that these functions may be interpreted just as probability densities. We shall see in sect. 4 that taking μ_0 equal to the rest mass of the hadron, as usually done[11,35], is not, perhaps, the most appropriate in this sense. Two observations are in order about μ_0. First of all, it is washed out by integration over $\mathbf{p}_{\perp}$ of the correlator, therefore it does not influence the common[45] distribution functions. Secondly, we can reasonably assume that this parameter is independent of the perturbative interactions among partons.

2.3   T-odd functions

As explained in the introduction, the T-odd functions deserve especial attention. In particular, the two functions introduced in formula (9) may be interpreted as quark densities: $h_1^{\perp}$ corresponds to the quark transversity in an unpolarized (or spinless) hadron, while $f_{1T}^{\perp}$ is the density of unpolarized quarks inside a transversely polarized spinning hadron[13,33].

A possible mechanism for generating these effects has been analyzed in detail, from different points of view, by various authors[18–20,23,24]. In particular, the function $f_{1T}^{\perp}$, known as the Sivers function, may give rise to a single spin asymmetry, as predicted for

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[1]The correlator (7) has a different normalization than in ref.[11].

the first time many years ago by Sivers[8] as a consequence of coherence among partons. Essential ingredients for producing the effect are[18]

a) two amplitudes with different quark helicities and different components ($\Delta L_z = 1$) of the orbital angular momentum;

b) a phase difference between such amplitudes, caused, for example, by one gluon exchange between the spectator partons and the active quark, either before or after the hard scattering: owing to the different orbital angular momenta, the gluon interaction causes a different phase shift in the two amplitudes.

Incidentally, a $\Delta L_z = 1$ is connected to the anomalous magnetic moment of the nucleon[18,24]; however, the difference in quark helicities could be attributed, in part, also to spontaneous chiral symmetry breaking[19]. In this connection, we think that the correct origin and the basic mechanisms for producing the Sivers asymmetry should be investigated more deeply.

The initial and final state interactions may be described by the so-called link operator, introduced in the definition of bilocal functions in order to assure gauge invariance[11,19,20]. Moreover they cause also a nonvanishing $h_1^\perp$[23]: in a scalar diquark model, this function is equal to $f_{1T}^\perp$[21] (see also last ref. [23]). In the mechanism which generates quark transversity in an unpolarized nucleon, angular momentum conservation implies a change by one or two units of orbital angular momentum of the quark; this change can be connected to a pseudovector particle exchange, while the above mentioned initial or final state interactions are interpreted as Regge (or absorptive) cuts[23].

From the above discussion it follows that quark-gluon interactions are essential for producing T-odd functions. Indeed, if such interactions are turned off, T-odd functions are forbidden by time reversal invariance[9] in transverse momentum space. On the contrary, they are allowed in the impact parameter space[24]: the Sivers asymmetry can be viewed as a left-right asymmetry with respect to the nucleon spin in that space, where final state interactions produce a chromodynamic lensing for the struck quark[24].

T-odd functions can be related[46,34] to the Qiu-Sterman[47] effect, which takes into account quark-gluon-quark correlations. For instance, in singly polarized Drell-Yan, these functions produce an asymmetry similar to the one described by gluon exchange between the spectator partons of the initial hadron h_A and the active parton of the hadron h_B, or *vice-versa*[48,49] [see reaction (1)]. Incidentally, analogous gluon exchanges may be used for describing, for example, the azimuthal asymmetries in unpolarized Drell-Yan (see sect. 6) and in singly polarized SIDIS. T-odd functions can be approximately factorized[19] - up to a sign, according as to whether the functions are involved in SIDIS or in Drell-Yan[19,46] - if condition (6) is fulfilled[21]; otherwise one is faced with serious difficulties as regards universality of the effect[50].

2.4 Fragmentation Correlator

The fragmentation correlator can be parametrized analogously to Φ, see subsect. 2.2. We have, in the case of quark fragmentation into a pion,

$$\Delta = \Delta_e + \Delta_o, \tag{13}$$

where, at leading twist, the T-even part is given by

$$\Delta_e \simeq \frac{1}{2} k \cdot n'_+ D h'_- \tag{14}$$

and the T-odd part reads

$$\Delta_o \simeq \frac{1}{4\mu_0^\pi} H_1^\perp [k_\perp, h'_-]. \tag{15}$$

Here k is the four-momentum of the quark, $k_\perp = k - (k \cdot n'_-)n'_+ - (k \cdot n'_+)n'_-$ and $n'_\pm$ are a pair of lightlike vectors, defined analogously to $n_\pm$, but such that the space component of n'_- is along the pion momentum. μ_0^π is an energy scale analogous to μ_0. Lastly D and $H_1^\perp$ are fragmentation functions, D is the usual one, chiral even, while $H_1^\perp$ - the Collins function[9] - is chiral odd. It is important to notice that the latter function is interaction dependent, as well as the T-odd distribution functions: indeed, it has been shown[51] that this function would vanish in absence of interactions among partons.

3 SIDIS and Drell-Yan Asymmetries

Now we deduce the expressions of the asymmetries involved in the two reactions considered, according to the formalism introduced in the previous section (see also, *e. g.,* refs.[11,15,52,53] for SIDIS and ref. [54] for Drell-Yan). As regards SIDIS, the initial lepton may be either unpolarized or longitudinally polarized, while the nucleon target may be either unpolarized or tranversely polarized. On the other hand, concerning Drell-Yan, we consider the cases where at most one of the two initial hadrons (typically a proton) is transversely polarized. For our aims, the most relevant kinematic variables are two azimuthal angles, denoted as ϕ and ϕ_S. In the case of Drell-Yan they are the azimuthal angles, respectively, of the momentum of the positive lepton and of the spin of the initial polarized hadron, in the Collins-Soper (CS) frame[55]. This is defined as the center-of-mass frame of the final lepton pair, such that the z-axis is along the bisector of the beam momentum and of the direction opposite to the target momentum, while the x-axis is along $\mathbf{q}_\perp$. As for SIDIS, ϕ and ϕ_S are respectively the azimuthal angles - defined in the Breit frame where the proton momentum is opposite to the photon momentum - of the final hadron momentum and of the target spin vector with respect to the production plane.

3.1 SIDIS Asymmetries

The doubly polarized SIDIS cross section, with a longitudinally polarized lepton and a tranversely polarized nucleon, may be written as a sum of 4 terms, *i. e.*,

$$\left(\frac{d\sigma}{d\Gamma}\right) = \left(\frac{d\sigma}{d\Gamma}\right)_{UU} + \left(\frac{d\sigma}{d\Gamma}\right)_{UT} + \left(\frac{d\sigma}{d\Gamma}\right)_{LU} + \left(\frac{d\sigma}{d\Gamma}\right)_{LT}. \tag{16}$$

Here we have singled out the unpolarized (UU), the singly polarized - either with a transversely polarized target, (UT), or with a longitudinally polarized beam, (LU) - and the doubly polarized (LT) contributions. According to the formalism introduced in sect. 2, we get, in leading twist approximation,

$$\left(\frac{d\sigma}{d\Gamma}\right)_{UU} \simeq \sum_a e_a^2 [U_0^a + U_1^a cos2\phi], \tag{17}$$

$$\left(\frac{d\sigma}{d\Gamma}\right)_{UT} \simeq \sum_a e_a^2 [S_1^a sin(\phi + \phi_S) + S_2^a sin(\phi - \phi_S)$$
$$+ \quad S_3^a sin(3\phi - \phi_S) + S_4^a sin2\phi], \tag{18}$$

$$\left(\frac{d\sigma}{d\Gamma}\right)_{LU} \simeq 0, \tag{19}$$

$$\left(\frac{d\sigma}{d\Gamma}\right)_{LT} \simeq \sum_a e_a^2 [D_1^a + D_2^a cos(\phi - \phi_S)]. \tag{20}$$

Here we have expressed the cross section in units $\alpha^2 x z^2 s / Q^4$, where s is the overall c.m. energy squared. Moreover, omitting the flavor indices of the functions involved, we have

$$U_0 = A(y)\mathcal{F}[w_{U_0}, f_1, D], \tag{21}$$

$$U_1 = -C(y)\frac{q_\perp^2}{\mu_0 \mu_0^\pi}\mathcal{F}[w_{U_1}, h_1^\perp, H_1^\perp], \tag{22}$$

$$S_1 = C(y)|\mathbf{S}_\perp|\frac{q_\perp}{\mu_0^\pi}\mathcal{F}[w_{S_1}, h_{1T}, H_1^\perp], \tag{23}$$

$$S_2 = A(y)|\mathbf{S}_\perp|\frac{q_\perp}{\mu_0}\mathcal{F}[w_{S_2}, f_{1T}^\perp, D], \tag{24}$$

$$S_3 = C(y)|\mathbf{S}_\perp|\frac{q_\perp^3}{\mu_0^2 \mu_0^\pi}\mathcal{F}[w_{S_3}, h_{1T}^\perp, H_1^\perp], \tag{25}$$

$$S_4 = \lambda C(y)\frac{q_\perp^2}{\mu_0 \mu_0^\pi}\mathcal{F}[w_{S_4}, h_{1L}^\perp, H_1^\perp], \tag{26}$$

$$D_1 = \lambda\lambda_\ell\frac{1}{2}E(y)\mathcal{F}[w_{D_1}, g_{1L}, D], \tag{27}$$

$$D_2 = \lambda_\ell\frac{1}{2}E(y)\frac{q_\perp}{\mu_0^\pi}\mathcal{F}[w_{D_2}, g_{1T}, D]. \tag{28}$$

We have denoted by λ_ℓ and $\mathbf{S}_\perp$ respectively the helicity of the initial lepton and the transverse component of the nucleon spin vector, with $|\mathbf{S}_\perp| = sin\phi_s$ and $\lambda = cos\phi_s$. Moreover

$$A(y) = 1 - y + 1/2y^2, \qquad C(y) = 1 - y, \qquad E(y) = y(2 - y), \qquad (29)$$

where $y \simeq q^-/\ell^-$ and q is the four-momentum of the virtual photon, such that $|q^2| = Q^2$. Lastly, $\mathcal{F}$ is a functional[15],

$$\mathcal{F}[w, f, D] = \int d^2p_\perp w(\mathbf{p}_\perp, \mathbf{q}_\perp) f(\mathbf{p}_\perp) D[z, z(\mathbf{q}_\perp - \mathbf{p}_\perp)], \qquad (30)$$

w, f and D being, respectively, a weight function, a distribution function and a fragmentation function. As to the weight functions, we have

$$w_{U_0} = w_{D_1} = 1, \qquad (31)$$
$$w_{U_1} = w_{S_4} = 2\hat{\mathbf{u}} \cdot \hat{\mathbf{p}}_\perp \hat{\mathbf{u}} \cdot \hat{\mathbf{k}}_\perp - \hat{\mathbf{k}}_\perp \cdot \hat{\mathbf{p}}_\perp, \qquad (32)$$
$$w_{S_1} = \hat{\mathbf{u}} \cdot \hat{\mathbf{k}}_\perp, \qquad w_{S_2} = w_{D_2} = \hat{\mathbf{u}} \cdot \hat{\mathbf{p}}_\perp, \qquad (33)$$
$$w_{S_3} = 4(\hat{\mathbf{u}} \cdot \hat{\mathbf{p}}_\perp)^2 \hat{\mathbf{u}} \cdot \hat{\mathbf{k}}_\perp - 2\hat{\mathbf{u}} \cdot \hat{\mathbf{p}}_\perp \hat{\mathbf{k}}_\perp \cdot \hat{\mathbf{p}}_\perp - \hat{\mathbf{u}} \cdot \hat{\mathbf{k}}_\perp \hat{\mathbf{p}}_\perp^2. \qquad (34)$$

Here we have set $\hat{\mathbf{u}} = \mathbf{q}_\perp/q_\perp$, $\hat{\mathbf{p}}_\perp = \mathbf{p}_\perp/q_\perp$ and $\hat{\mathbf{k}}_\perp = (\mathbf{q}_\perp - \mathbf{p}_\perp)/q_\perp$. Notice that the first two terms of the cross section (18) correspond respectively to the Collins and Sivers asymmetry[9,8].

3.2 Weighted Asymmetries in SIDIS

The weighted asymmetries are defined as

$$A_W = \frac{\langle W \rangle}{\langle 1 \rangle}. \qquad (35)$$

Here brackets denote integration of the weighted cross section over $q_\perp$ and over the azimuthal angles defined above. W is the weight function, consisting of the Fourier component we want to pick up [see eqs. (17) to (20)], times $(q_\perp/m_N)^{n_a}(q_\perp/m_\pi)^{n_b}$, where n_a and n_b are respectively the powers with which $\hat{\mathbf{p}}_\perp$ and $\hat{\mathbf{k}}_\perp$ appear in the functions w [see eqs. (31) to (34)]. For instance, the weight function corresponding to the Collins asymmetry is $W_{S_1} = (q_\perp/m_\pi)sin(\phi + \phi_S)$.

3.3 Drell-Yan Asymmetries

In the case of singly polarized Drell-Yan with a transversely polarized proton, we have (see also ref.[54])

$$\left(\frac{d\sigma}{d\Gamma'}\right)_{UU} = \sum_a e_a^2[U_0'^a + U_1'^a cos2\phi], \qquad (36)$$

$$\left(\frac{d\sigma}{d\Gamma'}\right)_{UT} = \sum_a e_a^2 [S_1'^a sin(\phi + \phi_S) + S_2'^a sin(\phi - \phi_S)$$
$$+ \quad S_3'^a sin(3\phi - \phi_S)]. \tag{37}$$

Here we have adopted the same approximation as before and have expressed the cross section in units $\alpha^2/3Q^2$. Moreover

$$U_0' = A'(y)\mathcal{F}[w_{U_0}', f_1, \bar{f}_1], \tag{38}$$

$$U_1' = C'(y)\frac{q_\perp^2}{\mu_0\mu_0'}\mathcal{F}[w_{U_2}', h_1^\perp, \bar{h}_1^\perp], \tag{39}$$

$$S_1' = -C'(y)\frac{q_\perp}{\mu_0'}\mathcal{F}[w_{S_1}', h_{1T}, \bar{h}_1^\perp], \tag{40}$$

$$S_2' = A'(y)\frac{q_\perp}{\mu_0}\mathcal{F}[w_{S_2}', f_{1T}^\perp, \bar{f}_1], \tag{41}$$

$$S_3' = -C'(y)\frac{q_\perp^3}{\mu_0^2\mu_0'}\mathcal{F}[w_{S_3}', h_{1T}^\perp, \bar{h}_1^\perp], \tag{42}$$

$$A'(y) = 1/2 - y + y^2, \qquad C'(y) = y(1-y) \tag{43}$$

and

$$y = 1/2(1 + cos\theta), \tag{44}$$

θ being the polar angle of the positive lepton in the CS frame. μ_0 and μ_0' are energy scales relative to the two initial hadrons in the Drell-Yan process. The functions w' are defined like the w's [see eqs. (31) to (34)], but now $\mathbf{p}_\perp$ and $\mathbf{k}_\perp$ denote, respectively, the transverse momenta of the active quark and antiquark in the two initial hadrons. The change of sign of the T-odd functions with respect to SIDIS has been taken into account in the coefficients S_1', S_2' and S_3', as already discussed at the end of subsect. 2.3.

4 Determining μ_0

Here we derive the appropriate value of the parameter μ_0, by comparing the correlator with the density matrix, to which Φ reduces for non-interacting, on-shell quarks. Indeed, in appendix we show that, in the limit of $g \to 0$ (g being the strong coupling constant), the correlator tends to the density matrix of a free, on-shell[56] quark. In particular, in the case of a transversely polarized nucleon, one has[57,58]

$$\Phi \to \rho = \sum_{T=\pm 1/2} q_T(x, \mathbf{p}_\perp^2)\frac{1}{2}(\not{p} + m)(1 + 2T\gamma_5\mathcal{S}_q). \tag{45}$$

Here m is the rest mass of the quark, such that $p^2 = m^2$. $2TS_q$ is the *quark* Pauli-Lubanski vector, where S_q is defined so as to coincide with S in the *quark* rest frame. The functions

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$q_T(x, \mathbf{p}_\perp^2)$ may be viewed as the probability densities of finding, in an infinite momentum frame, a quark with its spin aligned with ($T = 1/2$) or opposite to ($T = -1/2$) the proton spin. The density matrix can be conveniently rewritten as

$$\rho = \frac{1}{2}(\not{p} + m)[f_1(x, \mathbf{p}_\perp^2) + \gamma_5 \mathcal{S}_q h_{1T}(x, \mathbf{p}_\perp^2)], \tag{46}$$

where we have set, according to the definitions of the density functions,

$$f_1 = \sum_{T=\pm 1/2} q_T(x, \mathbf{p}_\perp^2), \qquad h_{1T} = \sum_{T=\pm 1/2} 2T q_T(x, \mathbf{p}_\perp^2). \tag{47}$$

We get, after some steps illustrated in appendix,

$$\begin{aligned} \rho &= \frac{1}{2} f_1(x, \mathbf{p}_\perp^2)(\not{p} + m) \\ &+ \frac{1}{2} h_{1T}(x, \mathbf{p}_\perp^2) \gamma_5 \left\{ \frac{1}{2}[\mathcal{S}, \not{p}] + \not{p}\bar{\lambda}_\perp - C_1 + m C_2 \right\} + O(\mathcal{P}^{-1}). \end{aligned} \tag{48}$$

Here $\mathcal{P}$ is defined by the first eq. (11), moreover

$$C_1 = \mathcal{P}\frac{1}{2}[\not{h}'_+, \not{h}'_-]\bar{\lambda}_\perp, \tag{49}$$

$$C_2 = \mathcal{S} + \frac{1}{\sqrt{2}} \left\{ \not{h}'_- \left(1 - \frac{m}{\mathcal{P}}\right) - \not{h}'_+ + \frac{1}{\sqrt{2}}[\not{h}'_+, \not{h}'_-] \right\} \bar{\lambda}_\perp \tag{50}$$

and

$$\bar{\lambda}_\perp = -p_\perp \cdot S/\mathcal{P} \tag{51}$$

is the light cone helicity of a quark in a transversely polarized nucleon.

Now we compare the various Dirac components of the density matrix (48) with those of the T-even correlator (8), taking into account the relation

$$p = \sqrt{2}\mathcal{P}n_+ + p_\perp + O\left(\mathcal{P}^{-1}\right). \tag{52}$$

As a result we get the following relations for a free, on-shell quark[58]:

$$\lambda_\perp h_{1T}^\perp = (1 - \epsilon_1)\bar{\lambda}_\perp h_{1T}, \tag{53}$$

$$\lambda_\perp g_{1T} = (1 - \epsilon_2)\bar{\lambda}_\perp h_{1T}. \tag{54}$$

Here $\epsilon_1 \simeq m/\mathcal{P}$ and $\epsilon_2 \simeq m/2\mathcal{P}$ are the correction terms due to the quark mass, which is small for light flavors. The terms of order $O\left[(m^2 + \mathbf{p}_\perp^2)/\mathcal{P}^2\right]$ have been neglected.

In order to determine μ_0, we observe that the functions g_{1T}, h_{1T} and $h_{1T}^\perp$, involved in formulae (53) and (54), are twist 2, therefore they may be interpreted as quark densities. For example, g_{1T} is the helicity density of a quark in a tranversely polarized nucleon.

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Therefore it is natural to fix μ_0 in such a way that g_{1T} and $h_{1T}^\perp$ are normalized like h_{1T}. This implies, neglecting the quark mass,

$$\lambda_\perp = \bar{\lambda}_\perp, \tag{55}$$

and, according to eq. (51) and to eqs. (11),

$$\mu_0 = \mathcal{P} = \frac{1}{\sqrt{2}} p \cdot n_-. \tag{56}$$

Now we take the space component of n_- along the direction of one of the two initial hadrons for Drell-Yan and along the direction of the virtual photon for SIDIS. In both cases we get $\mathcal{P} \simeq Q/2$. Therefore we assume

$$\mu_0 = \frac{Q}{2}, \tag{57}$$

the result being trivially extended to μ_0'.

Two remarks are in order.

a) It is worth comparing our approach to Kotzinian's[30], who starts from the approximate expression of the density matrix for a free ultrarelativistic fermion and adapts it to the case of a quark in the nucleon. He parametrizes the density matrix with the 6 twist-2, T-even functions that appear in the parametrization (8). Similar results are obtained by Tangerman and Mulders[10]. The difference with our approach is that those authors do not consider explicitly the limit for $g \to 0$, where relations of the type (53) and (54) hold.

b) Although deduced in the limiting case of noninteracting partons - where one has to do just with T-even functions -, result (57) cannot be significantly modified by the perturbative interactions among partons and is reasonably extended to the T-odd functions. Of course, μ_0 is modified by nonperturbative interactions: for example, in the case of the already cited quark-diquark model[18,23] (see subsect. 2.3), the interference term scales with Q^2, in agreement with the assumption $\mu_0 = M$[35,11]. However, the virtuality of the gluon exchanged between the active parton and the spectator partons is of the order of Q; therefore, in the scaling region, the gluon "sees" the single partons rather than the diquark as a whole, in which case μ_0 has to be identified with $Q/2$, as for noninteracting quarks. To summarize, if we take into account the intrinsic transverse momentum of quarks, we are faced with the *normalization* scale μ_0, which, for large Q, has to be identified with $Q/2$, while for smaller Q (such that nonperturbative interactions are not negligible) it is of the order of the hadron mass, in accord with a phase space restriction.

As regards the fragmentation correlator, we adapt our previous line of reasoning to the case of a quark fragmenting into a transversely polarized spin-1/2 particle, say a Λ.

For $g \to 0$ one has

$$\Delta \to \rho' = \frac{1}{2}(k + M')(D + H_1 \gamma_5 \mathcal{S}'). \tag{58}$$

Here M' and S' are, respectively, the mass and the Pauli-Lubanski vector of the final hadron, whereas H_1 is the transversely polarized fragmentation function; the other symbols have been introduced in subsect. 2.4. By comparing this limiting expression with a parametrization of Δ - analogous to eq. (8) as regards twist-2 terms - , we get $\mu_0^\pi = Q/2$.

5 Q^2 Dependence of Asymmetries

As a consequence of the results deduced in the previous section, we conclude that the azimuthal asymmetries illustrated in sect. 3 decrease with Q^2. In particular, as regards SIDIS, we predict

$$S_1, S_2, D_2 \propto \rho, \qquad U_1 \propto \rho^2, \qquad S_3 \propto \rho^3 \tag{59}$$

and

$$D_2 \propto \frac{M}{Q}, \qquad S_4 \propto \frac{\rho^2 M}{Q}, \tag{60}$$

where

$$\rho = q_\perp/Q. \tag{61}$$

Results (60) are consequences of the fact that λ [see eq. (10)] is proportional to Q^{-1} for a transversely polarized nucleon. Such predictions might be checked by comparing data of experiments which have been realized (HERMES[2] and COMPASS[4]) with those planned (CLAS[5]), which operate in different ranges of Q^2. A strategy could be, for instance, to isolate the various Fourier components in the cross section [see eqs. (21) to (28)] by means of the weighted asymmetries and to study their Q^2 dependence. A particular remark is in order as regards the unpolarized SIDIS asymmetry, which we predict to decrease as $1/Q^2$, just like the twist-4 $cos2\phi$ asymmetry arising as a consequence of the quark transverse momentum[59,30]. This makes the two asymmetries hardly distinguishable, but the last asymmetry can be parametrized, as well as the $cos\phi$ asymmetry (the Cahn effect[60]), by means of the unpolarized quark density.

Concerning Drell-Yan, the predictions are

$$S_1', S_2' \propto \rho, \qquad U_1' \propto \rho^2, \qquad S_3' \propto \rho^3. \tag{62}$$

As regards U_1', the result will be checked against unpolarized Drell-Yan data in the next section; the other three predictions could be verified, in principle, by comparison with data from experiments planned at various facilities, like RHIC[37], GSI[38] and FNAL[39].

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We conclude this section with some important remarks. Although commonly classified as twist 2, the asymmetries considered result to decrease with Q. As regards those asymmetries which involve one or more T-odd distribution or fragmentation functions, we stress once more that these functions depend crucially on interactions among partons and therefore on quark-gluon correlations. Therefore our result is not so surprising. In this connection it is worth recalling that the Drell-Yan single spin asymmetry, which according to our formalism could be attributed to the effects of T-odd functions (see the second term of eq. (37) and refs. [61,54]), was interpreted some years ago by means of a twist 3 correlation function and it was predicted to decrease as Q^{-1}[62,48,63,49,64] (see also refs. [47,65,66]). Moreover, also in the parametrization (8) of the T-even correlator, the function g_{1T}, twist 2, is multiplied by the factor $\lambda_\perp = \bar{\lambda}_\perp$ [see eq. (51)], therefore the second term of the cross section (20) decreases as Q^{-1}, although involving just twist 2 operators[12]. In this case the peculiar behavior of the asymmetry has a purely kinematical origin: indeed, $\bar{\lambda}_\perp g_{1T}$ is the average light cone helicity of a quark in a transversely polarized nucleon and therefore, according to the above mentioned equations, it decreases as Q^{-1}. All that casts some doubts on the correlation between the twist of an operator and the Q^2 dependence of the corresponding coefficient[32], as regards transverse momentum distributions. Indeed, unlike the case of common distribution functions, the quark is not collinear to the nucleon, therefore some Dirac operators, although commuting with the Hamiltonian[45] of free quarks, are proportional to the transverse momentum and therefore are suppressed in the infinite momentum frame.

6   Azimuthal Asymmetry in Unpolarized Drell-Yan

As is well-known, unpolarized Drell-Yan presents an azimuthal asymmetry. This has been seen, for example, in reactions of the type[1]

$$\pi^- N \to \mu^+ \mu^- X, \tag{63}$$

where N is an unpolarized tungsten or deuterium target, which scatters off a negative pion beam. The Drell-Yan angular differential cross section is conventionally expressed as

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1 + \lambda cos^2\theta + \mu sin2\theta cos\phi + \frac{1}{2}\nu sin^2\theta cos2\phi\right). \tag{64}$$

Here $\Omega = (\theta, \phi)$, θ and ϕ being respectively the polar and azimuthal angle of the μ^+ momentum in the CS frame. Moreover λ, μ and ν are parameters, which are functions of the overall center-of-mass energy squared s, of $q_\perp^2$, of Q and of the Feynman longitudinal fractional momentum x_F of the muon pair with respect to the initial beam. On the left-hand-side of eq. (64), $d\sigma/d\Omega$ is a shorthand notation for $d\sigma/d\Omega dQ^2 dx_F d^2 q_\perp$.
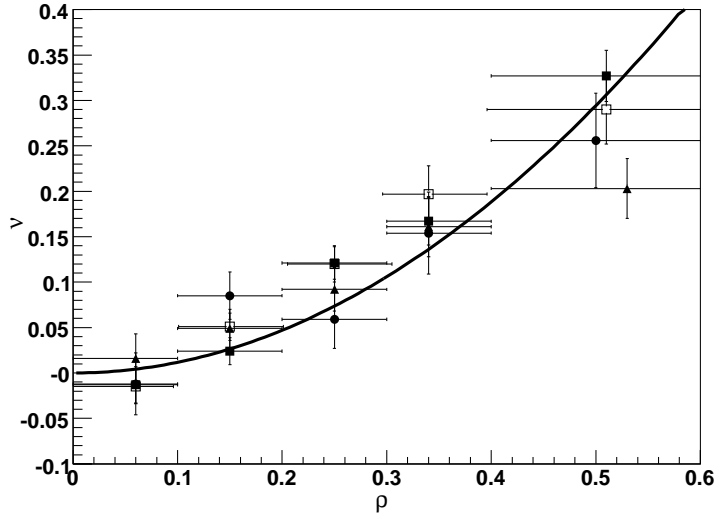
Figure 1: Behavior of the asymmetry parameter ν *vs* the dimensionless parameter ρ. Data are taken from the first two refs.[1]: circles correspond to $\sqrt{s}$ = 16.2 GeV, squares to $\sqrt{s}$ = 19.1 GeV and triangles to $\sqrt{s}$ = 23.2 GeV. The best fit is made with formula (65), A_0 = 2.34.

In the naive Drell-Yan model, where the parton transverse momentum and QCD corrections are neglected, one has $\lambda = 1$, $\mu = \nu = 0$. Therefore deviations of such parameters from the above predictions - observed experimentally both for λ and ν, while μ is consistent with 0[1] - can be attributed to transverse momentum or gluon effects, as illustrated in the first ref. [1]. We shall comment on λ in a moment. As regards μ and ν, their features are interpreted in terms of of T-odd functions[21]. Indeed, in the formalism of the correlator introduced in the previous sections, comparison of eq. (36) with eq. (64) yields $\mu = 0$ and

$$\nu = A_0 \frac{q_\perp^2}{Q^2} = A_0 \rho^2, \tag{65}$$

A_0 being a proportionality constant. Here eqs. (38), (39), (43) and the second eq. (62) have been taken into account.

We fit formula (65) to the experimental results of ν at different energies, both as a function of ρ (fig. 1) and as a function of Q at fixed $q_\perp$ (fig. 2), assuming A_0 as a free parameter. In particular, fig. 1 exhibits an approximate scaling law (see the first ref. [1]) for ν *versus* ρ, reproduced by our theoretical prediction. This behavior cannot be accounted for if we make the assumption $\mu_0 = M$[35,10,11], which would provide also a poor approximation to data of ν *versus* Q at fixed $q_\perp$, as can be seen from fig. 2.
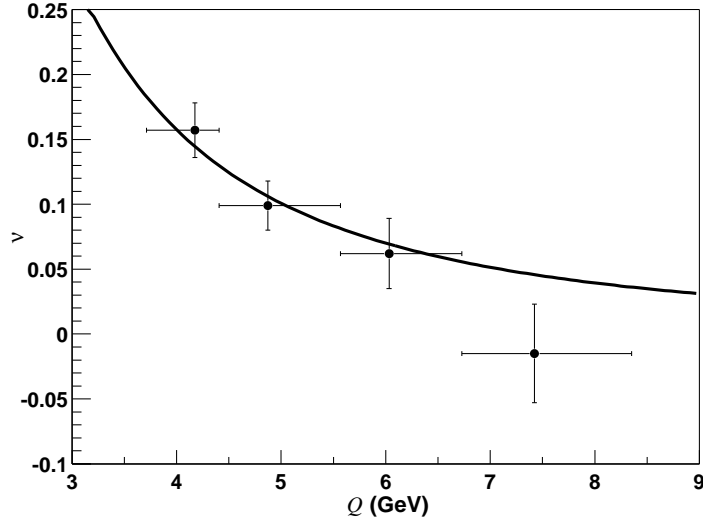
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Figure 2: Behavior of the asymmetry parameter ν *vs* the effective mass Q of the final lepton pair at fixed $q_\perp$. $\sqrt{s} = 23.2\ GeV$ Data from 2nd ref. [1] and fitted with formula (65), $A_0 \cdot q_\perp^2 = 2.52\ GeV^2$.

At this point it is worth recalling that the Drell-Yan cross section is very sensitive to higher order perturbative QCD corrections[67] and to power corrections[68,69] (see also ref.[70]). In particular, a $\lambda \neq 1$ is obtained by assuming for reaction (63) a simple model of initial state interactions[69], somewhat similar to the quark-gluon-quark correlations[47]. Here the Drell-Yan unpolarized cross section is of the type

$$d\sigma \propto |f_0 + f_1|^2, \tag{66}$$

where f_0 is the naive Drell-Yan amplitude and f_1 consists of two terms (due to gauge invariance), describing one gluon exchange between the spectator quark of the meson and each active parton. It results[69] $|f_0|^2 \propto (1-x)^2(1+cos^2\theta)$, $|f_1|^2 \propto \rho^2 cos^2\theta$ and $2\Re f_0 f_1^* \propto (1-x)\rho sin^2\theta cos\phi cos\psi_0$, where ψ_0 is the relative phase of the two amplitudes. This implies $\lambda - 1 \propto \rho^2/(1-x)^2$, in good agreement with data[1]. However μ depends crucially on ψ_0, moreover the third term of eq. (64) is absent. This could be recovered by inserting in eq. (66) a third amplitude, say f_1', describing one gluon exchange between each active parton and the spectator partons of the nucleon: the missing asymmetry is reproduced by the interference term $2\Re f_1' f_1^*$, as sketched at the end of subsect. 2.3.

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7 Conclusion

We have studied the parametrization of the transverse momentum dependent quark correlator, both for distributions inside the hadron and for fragmentation processes. We are faced with the energy scale μ_0, introduced in the parametrization for dimensional reasons, and determining the normalization of some of the quark densities (or fragmentation functions) involved. Comparison of the parametrization with the limiting expression of the correlator for noninteracting quarks yields $\mu_0 = Q/2$, contrary to the usual[35,10,11,21] assumption, $\mu_0 = M$, which appears more appropriate for situations where nonperturbative interactions among partons are present. The two different assumptions lead to different predictions on the Q^2 dependence of azimuthal asymmetries in SIDIS and Drell-Yan. Our result agrees with previous approaches to azimuthal asymmetries, in particular with the Q^2 dependence of quark-quark-gluon correlations[47], and also with data of azimuthal asymmetry in unpolarized Drell-Yan. These could not be explained with the usual assumption about μ_0. Further challenges to the two different theoretical predictions could come from future Drell-Yan experiments[37–39] and from comparison between present[2,4] and incoming[5] SIDIS data.

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Appendix

Here we derive formulae (45) and (48) for the quark density matrix in the limit of $g \to 0$, g being the coupling. To this end, we recall the definition of the correlator in quantum field theory, *i. e.*[11],

$$\Phi = \int \Phi'(p; P, S) dp^-, \tag{A. 1}$$

where the matrix elements of $\Phi'(p; P, S)$ are defined as

$$\Phi'_{ij}(p; P, S) = \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle P, S | \bar{\psi}_j(0) \mathcal{L}(x) \psi_i(x) | P, S \rangle. \tag{A. 2}$$

Here ψ is the quark field and $|P, S\rangle$ is a state of a nucleon with a given four-momentum P and Pauli-Lubanski four-vector S, while p is the quark four-momentum. Moreover

$$\mathcal{L}(x) = \mathrm{P} exp\left[-ig\Lambda_{\mathcal{P}}(x)\right], \qquad \text{with} \qquad \Lambda_{\mathcal{P}}(x) = \int_0^x \lambda_a A_\mu^a(z) dz^\mu, \tag{A. 3}$$

is the gauge link operator. Here "P" denotes the path-ordered product along the integration contour $\mathcal{P}$, λ_a and A_μ^a being respectively the Gell-Mann matrices and the gluon fields. The link operator depends on the choice of $\mathcal{P}$, which has to be fixed so as to make a physical sense. However, for our aims we can neglect the details of the contour. Indeed, in the limit for $g \to 0$, that is, for noninteracting quarks, $\mathcal{L}(x)$ tends to 1. Moreover, in that limit, the quark is on shell, as shown by Qiu[56] via equations of motion. Let us consider, for any flavor, the quark and antiquark field separately. The Fourier expansion of the field of a free, on-shell quark reads

$$\psi(x) = \int \frac{d^4p}{(2\pi)^{3/2}} \sqrt{\frac{m}{p^+}} \delta\left(p^- - \frac{\sqrt{m^2 + \mathbf{p}_\perp^2}}{2p^+}\right) e^{-ipx} \sum_s u_s(p) c_s(p), \tag{A. 4}$$

where m is the rest mass of the quark, $s = \pm 1/2$ its spin component along a given direction, u its four-spinor and c the destruction operator for the flavor considered. For an antiquark the definition is quite analogous. Substituting eq. (A. 4) into the definitions (A. 1) and (A. 2), we get

$$\Phi'_{ij}(\tilde{\mathbf{p}}; P, S) = \sum_{s,s'} \int \frac{d^3\tilde{p}'}{(2\pi)^3} \langle P, S | c_s^\dagger(\tilde{\mathbf{p}}) c_{s'}(\tilde{\mathbf{p}}') | P, S \rangle [u_{s'}(\tilde{p}')]_i [\bar{u}_s(\tilde{\mathbf{p}})]_j, \tag{A. 5}$$

where $\tilde{\mathbf{p}} \equiv (p^+, \mathbf{p}_\perp)$. But the matrix element in (A. 5) vanishes, unless $s' = s$ and $\tilde{\mathbf{p}}' = \tilde{\mathbf{p}}$. Then

$$\Phi'_{ij}(\tilde{\mathbf{p}}; P, S) = \sum_s \langle P, S | c_s^\dagger(\tilde{\mathbf{p}}) c_s(\tilde{\mathbf{p}}) | P, S \rangle [u_s(\tilde{\mathbf{p}})]_i [\bar{u}_s(\tilde{\mathbf{p}})]_j. \tag{A. 6}$$

But $[u_s(\tilde{\mathbf{p}})]_i[\bar{u}_s(\tilde{\mathbf{p}})]_j$ is nothing but the generic matrix element ρ_{ij} of the density matrix of the quark. In particular, if the nucleon is transversely polarized, we get

$$\rho = \sum_s q_s(x, \mathbf{p}_\perp^2)\frac{1}{2}(\not{p} + m)(1 + 2s\gamma_5\mathcal{S}_q), \qquad (A.7)$$

which corresponds to formula (45) in the text. Here $q_s(x, \mathbf{p}_\perp^2) = \langle P, S|c_s^\dagger(\tilde{\mathbf{p}})c_s(\tilde{\mathbf{p}})|P, S\rangle$ and $2sS_q$ is the Pauli-Lubanski vector of the quark in a transversely polarized nucleon. Eq. (A. 7) can be conveniently rewritten as

$$\rho = \frac{1}{2}(\not{p} + m)[f_1(x, \mathbf{p}_\perp^2) + \gamma_5\mathcal{S}_q h_{1T}(x, \mathbf{p}_\perp^2)], \qquad (A.8)$$

where we have set, according to the definitions of the density functions,

$$f_1 = \sum_{s=\pm1/2} q_s, \qquad h_{1T} = \sum_{s=\pm1/2} 2sq_s. \qquad (A.9)$$

Now we express S_q as a function of S. To this end we define a *quark* rest frame, whose y- and z-axes are taken, respectively, along the spin and along the momentum of the nucleon. In this frame the Pauli-Lubanski vector of the quark results to be $S_q^{(0)} = S \equiv (0, 0, 1, 0)$. Decomposing $S_q^{(0)}$ into a transverse and a longitudinal component with respect to the quark momentum, we get

$$S_q^{(0)} = \Sigma_\perp cos\theta' + \nu sin\theta'. \qquad (A.10)$$

Here we have, in the frame just defined,

$$sin\theta' = sin\theta cos\varphi, \qquad sin\theta = \frac{|\mathbf{p}_\perp|}{|\mathbf{p}|}, \qquad cos\varphi = \frac{-p_\perp \cdot S}{|\mathbf{p}_\perp|}, \qquad (A.11)$$

$$\mathbf{p} \equiv [\mathbf{p}_\perp, p \cdot \sqrt{2}(n_+ - n_-)], \qquad \nu \equiv (0, \mathbf{t}), \qquad \Sigma_\perp \equiv (0, \mathbf{n}), \qquad (A.12)$$

$$\mathbf{t} \equiv (sin\theta sin\varphi, sin\theta cos\varphi, cos\theta), \qquad p_\perp \equiv (0, \mathbf{p}_\perp, 0) \qquad (A.13)$$

and $\mathbf{n} \equiv (cos\theta sin\varphi, cos\theta cos\varphi, -sin\theta)$.

Here $\mathbf{p}$ denotes the momentum of the quark and $n_\pm$ are a pair of lightlike vectors, such that $n_+ \cdot n_- = 1$ and whose space direction is the one of the nucleon momentum. In order to find the expression of S_q as a function of S, we perform a boost along the quark momentum. This boost leaves $\Sigma_\perp$ invariant and transforms ν into $\bar{p}/m$, where

$$\bar{p} \equiv (|\mathbf{p}|, E_q\mathbf{t}), \qquad E_q = \sqrt{m^2 + \mathbf{p}^2}. \qquad (A.14)$$

As a result we get

$$S_q = S + \left[\frac{p}{m} - (\delta + \nu)\right] sin\theta cos\varphi, \qquad (A.15)$$

where

$$\delta = \frac{m}{\sqrt{2}|\mathbf{p}|} n'_- \left[1 + O(|\mathbf{p}|^{-2})\right] \tag{A. 16}$$

and n'_- is defined analogously to n_-, but with the space direction opposite to the quark momentum. But we observe that

$$|\mathbf{p}| = \mathcal{P} + O(\mathbf{p}_\perp^2/\mathcal{P}^2), \tag{A. 17}$$

where $\mathcal{P} = \frac{1}{\sqrt{2}} p \cdot n_-$. Therefore

$$sin\theta cos\varphi = \bar{\lambda}_\perp + O(\mathbf{p}_\perp^2/\mathcal{P}^2) \qquad \bar{\lambda}_\perp = -p_\perp \cdot S/\mathcal{P} \tag{A. 18}$$

and

$$S_q = S + \bar{\lambda}_\perp \left[\frac{p}{m} - (\delta + \nu)\right] + O(\mathbf{p}_\perp^2/\mathcal{P}^2). \tag{A. 19}$$

Now we substitute eq. (A. 19) into eq. (A. 7), taking into account the definitions (A. 12) and (A. 16) of ν and δ, and exploiting the relations $-\not{p}\mathcal{S} = 1/2[\mathcal{S}, \not{p}] - p \cdot S$, $p \cdot S = p_\perp \cdot S$ and $\nu = \frac{1}{\sqrt{2}}(n'_+ - n'_-)$. As a result we get

$$\begin{aligned}
\rho &= \frac{1}{2} f_1(x, \mathbf{p}_\perp^2)(\not{p} + m) \\
&+ \frac{1}{2} h_{1T}(x, \mathbf{p}_\perp)\gamma_5 \left\{\frac{1}{2}[\mathcal{S}, \not{p}] + \not{p}\bar{\lambda}_\perp - C_1 + mC_2\right\} + O(\mathcal{P}^{-1}),
\end{aligned} \tag{A. 20}$$

with

$$C_1 = \mathcal{P}\frac{1}{2}[\not{h}'_+, \not{h}'_-]\bar{\lambda}_\perp, \tag{A. 21}$$

$$C_2 = \mathcal{S} + \frac{1}{\sqrt{2}} \left\{\not{h}'_- \left(1 - \frac{m}{\mathcal{P}}\right) - \not{h}'_+ + \frac{1}{\sqrt{2}}[\not{h}'_+, \not{h}'_-]\right\} \bar{\lambda}_\perp. \tag{A. 22}$$

Eq. (A. 20) corresponds to eq. (48) in the text.

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