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MIR, a feasibility study for a measurement of the dynamical Casimir effect

C. Braggio[1], G. Bressi[2], G. Carugno[1], C. Del Noce[1], V. Dodonov[3],
A. Lombardi[4], A. Palmieri[4], G. Ruoso[4], D. Zanello[5]
[1] *INFN Sez di Padova, Via Marzolo 8, Padova*
[2] *INFN Sez di Pavia, Via U. Bassi 6, Pavia*
[3] *Departamento de Fisica, Universitade Federal de Sao Carlos*
[4] *INFN Laboratori Nazionali di Legnaro*
[5] *INFN Sez di Roma I, P.le Aldo Moro, Roma*

Abstract

Even at absolute zero, the vacuum is seething with activity and behaves as a fluid in which moving objects dissipate their kinetic energy. The dynamical Casimir effect foresees that a metallic mirror put in motion in quantum vacuum gives rise to "dissipated" energy in the form of real photons, called the *Motion Induced Radiation*. The final aim of this study is to present a solution to the main experimental difficulty, that is to realize the oscillating motion of the metallic wall at high frequency ($10^9 \div 10^{10}$ Hz). To obtain a fast moving wall we propose to switch an effective microwave mirror on and off in very short intervals of time changing the reflectivity of the semiconductor by shining pulsed laser beam on its surface. In this way a microwave mirror can be created and disappears at very high frequency, giving rise to a system equivalent to that of a fast moving metallic slab.

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Introduction

In this report we first make a general theoretical exposition concerning the effects of electromagnetic field fluctuations, devoting more concern on the dynamical Casimir effect for what we want to observe with our experiment is in effect the *Motion Induced Radiation*.
The final aim of this study is to present a solution to the main experimental difficulty, that is to realize the oscillating motion of the metallic wall at high frequency ($10^9 \div 10^{10}$ Hz), as purely mechanical excitation has limitations on the admissible velocity of acoustic waves in materials. To obtain a fast moving wall the idea is to produce an effective microwave mirror, changing the conductivity of a semiconductor layer, that is set as a wall of the cavity: with pulsed laser light over a direct semiconductor (GaAs) a microwave mirror is created and disappears in a very short time.
In the final part of this work the possible sources of noise are analyzed and the number of equivalent noise photons is estimated. This number has to be compared with the theoretical predictions of the number of dynamical Casimir effect photons.

Zero point oscillations and the Casimir effect

Let us consider boundaries for the electromagnetic field in free space, i.e. inserting metallic plates or external fields, the electromagnetic field is still fluctuating freely in infinite space, except for the constraints of vanishing of some components on the metallic plates. Each constraint by itself imposes restrictions on fluctuations of the field, increasing the free energy. In the best known configuration, two uncharged parallel plates separated by a distance a, the result is an attractive force which arises to bring the system back to minimum energy state:

$$F(a) = -\frac{\pi^2}{240}\frac{\hbar c}{a^4}S$$

a is the separation between the plates, S $\gg a^2$ is their area and $\hbar$ and c are the usual physical constants [2,3].

In fact, with two plates the space is partitioned into three domains, two with continuous modes and one with discrete modes and the system favours less separation between the plates so that the restrictive domain is as small as possible.

As the plates are assumed to be neutral, there is no force in classical electrodynamics, then it is only the vacuum which causes the plates to attract each other. It was Casimir [2] who was the first to extract the finite force acting between the two parallel neutral plates; to do this he had subtracted away the infinite vacuum energy of quantized electromagnetic field in free space from the infinite zero-point energy of the quantized electromagnetic field confined in between the plates[1]. The finite difference between the infinite vacuum energy densities in the presence of plates and in free space is observable and gives rise to Casimir force.

This effect has been studied theoretically in several configurations and the results obtained show the *boundary dependence of quantum vacuum*. The Casimir force switches from attractive to repulsive as a function of the size, geometry and topology of the boundary [3], [4].

A variety of measurable consequences of these quantum fluctuations under the influence of external conditions have been derived during the last decades [5]. If we focus our attention to the vacuum fluctuations of the electromagnetic field, they exert a mechanical action on any scatterer in vacuum. The Casimir force described before is a macroscopic quantum effect, but this zero-point energy has manifestations also with objects belonging to the microscopic world; with electrons in atoms vacuum oscillations lead to phenomena like spontaneous emission and the Lamb shift of energy levels for a single atom.

The dynamical Casimir effect

Another vacuum quantum effect, in addition to the Casimir force, is the creation of particles from vacuum when boundary conditions depend on time. The review of the several papers published on the dynamical Casimir effect is not one of the aims of the present report: what follows is to understand how theoretical physicists obtain the expression of the quantity we want to measure, that is, the number of the photons radiated.

In this effect, some part of the mechanical energy of moving bodies (mirrors) is transformed into the energy of quanta of electromagnetic (and, in principle, other) fields due to an interaction between the mirror and virtual particles, or vacuum fluctuations [6–11]. The back reaction of the emitted quanta results, in turn, in dissipation effects, because the vacuum behaves as a complex fluid that hinders and influences the bodies moving through it [12,1]. In an idealized case of a perfectly reflecting mirror moving in a single space dimension, the dissipative force is proportional to the third time derivative of the mirror's position [12,13]:

$$F_{\mathrm{diss}}(t) = \frac{\hbar}{6\pi c^2} q'''(t). \tag{1}$$

[1] Because the electric field must vanish at the boundaries, the normal modes of the cavity between the mirror are characterized by integer wave vectors $\vec{k} = (k_x, k_y, \pi n/d)$, with n integer and d distance between the plates. Once quantized, these normal modes are harmonic oscillators of frequencies $\omega(\vec{k}) = c|\vec{k}|$, each of which in its ground state has energy $\frac{\hbar\omega(\vec{k})}{2}$. The total sum of the ground state energies is formally infinite.

It is useful to rewrite this formula in the frequency domain:

$$F_{\rm diss}[\omega] = \frac{\hbar}{6\pi c^2} i\omega^3 q[\omega], \tag{2}$$

where $F_{\rm diss}[\omega]$ and $q[\omega]$ are the Fourier transforms of the force and the mirror's displacement. In the three-dimensional case, $F_{\rm diss}[\omega] \sim \omega^5 q[\omega]$ [14], so that the vacuum friction force becomes proportional to the fifth-order derivative of the coordinate. No dissipative force arises for a motion with uniform acceleration.

When a single mirror is oscillating at frequency Ω in vacuum, the number of radiated photons is [8]:

$$N = \frac{\Omega T}{6\pi} \left(\frac{v}{c}\right)^2 \tag{3}$$

Expression (3) is a product of two dimensionless factors, the number of mechanical oscillation periods during the time T, and the square of the ratio $\beta = \frac{v}{c}$ between the maximal velocity v of the mirror and the velocity of light *c*. The number obtained depends on Ω^3, which characterizes the motional susceptibility of vacuum, according to equation (2).

Since N scales as β^2, it remains very small for any macroscopic motion. In 10^{10} periods of oscillation and using v as the velocity of sound waves in quartz, we obtain only one photon.

Instead of a single mirror, the most favorable configuration in order to observe this motion-induced radiation is to study a cavity oscillating in vacuum. This allows one to enhance generation of particles by resonance effects, if the boundaries perform harmonic oscillations at twice the frequency of the cavity [6,7,15,16].

Let us consider a simple one-dimensional model, which roughly corresponds to the Fabry–Perot cavity with totally reflecting boundaries. Then the spectrum of electromagnetic modes is equidistant: $\omega_m = m\,\omega_1$, $\omega_1 = \pi\, c/L_0$ (L_0 is the equilibrium distance between mirrors), and the number of photons created from vacuum in the mth mode, under the condition of the strict parametric resonance $\Omega = 2\omega_1$ and *in the long-time limit* $\Omega T \gg 1$, can be written as [6,7,9,15,17]

$$N_m = \frac{v}{c}\frac{\Omega T}{\pi^3 m}\,, \tag{4}$$

provided m is an *odd* number (whereas no photons are produced in the modes with even numbers). The total number of photons in all the modes inside the cavity increases with time quadratically [9,17]:

$$N = \left(\frac{v}{c}\right)^2 \frac{(\Omega T)^2}{8\pi^4}\,. \tag{5}$$

However, such a growth cannot continue unlimitedly in a real cavity with finite quality factor Q. The stationary numbers of photons which could be created in a cavity with weakly transparent mirrors can be evaluated by replacing the product ΩT (the number of oscillations of the wall) by Q:

$$N_m^{(in)} \sim \frac{v}{c}\frac{Q}{\pi^3 m}\,, \qquad N^{(in)} \sim \left(\frac{v}{c}\right)^2 \frac{Q^2}{8\pi^4}\,. \tag{6}$$

The flux of the photons leaving the cavity *in the stationary regime* can be evaluated by multiplying each term of Eq. (6) by the factor Ω/Q:

$$\frac{dN_m^{(out)}}{dT} \sim \frac{v}{c}\frac{\Omega}{\pi^3 m}\,, \qquad \frac{dN^{(out)}}{dT} \sim Q\left(\frac{v}{c}\right)^2 \frac{\Omega}{8\pi^4}\,. \tag{7}$$

Formulae (7) should be compared with the results of studies [8], according to which, the number of photons emitted by the cavity at resonance is the product of the number N from equation (3) corresponding to a single mirror by the quality factor Q:

$$N^{(out)} = Q\frac{\Omega T}{6\pi}\left(\frac{v}{c}\right)^2\,. \tag{8}$$

We see that (7) and (8) differ only by numerical factors. It should be emphasized that equations (7) and (8) are obtained using quite different schemes of calculations and for different regimes. The formulae (4) and (5), which lead to (7), were derived in [6,7,9,15,17], where the *transient process* of creation of photons from vacuum (and other possible states of field) in *discrete* field modes *inside* the cavity was considered. On the other hand, in [8] was considered the *stationary* regime (when all transient processes have already been finished) of photon emission *outside* the cavity, for the continuous spectrum of emitted photons (with sharp peaks corresponding to the resonance field eigenfrequencies). Nonetheless, these different approaches lead to similar results (up to numerical constants) when their comparison has sense.

However, the rate of photon emission outside the cavity remains very small under realistic conditions even with account of the factor Q in (8), because the maximal velocity of material surface cannot exceed a few percent of sound velocity – otherwise the material will be destroyed due to immense internal deformations [9,18]. In the best case, $v/c \sim 10^{-7}$, and (8) gives the estimation of the flux outside the Fabry–Perot cavity whose mirrors oscillate at the frequency $\Omega/2\pi \sim 10$ GHz

$$N^{(out)}/T \le 10^{-4} Q\, \frac{\text{phot}}{\text{s}}$$

in the stationary regime. However, this stationary regime still should be attained.

It seems preferable to try to detect photons not outside, but *inside* the cavity, since in this case we gain an extra factor Q, counting the photons created during the transient period according to equation (6). At first glance, the numbers following from this equations under the same conditions as above, are also small:

$$N_1^{(in)} \sim 10^{-8}Q, \qquad N^{(in)} \sim 10^{-16}Q^2,$$

so that cavities with the quality factor $Q > 10^8$ are necessary to create at least one photon (note that superconducting cavities satisfying this condition are available for rather long time already). However, one should have in mind that Eq. (6) holds for the *one-dimensional* (Fabry–Perot) cavities. For more realistic *three-dimensional* cavities, whose different dimensions have the same order of magnitude, the spectrum is not equidistant. On the one hand, this fact makes calculations significantly more complicated in the case of arbitrary law of motion of the boundary. But, on the other hand, it extremely simplifies the treatment of the most interesting case of the *parametric resonance*, because only one mode (or, in some specific cases [11,19], two modes) can be in resonance with the oscillating boundary. In this case, the growth of the number of photons inside the cavity is *exponential* [7,9–11,18,19]. For the nondegenerate cavity with perfectly reflecting boundaries the mean number of created photons calculated in [7,9,18] can be expressed as

$$N = \sinh^2\left(\chi\beta\Omega T\right), \tag{9}$$

where $\chi \sim 1$ is some numerical factor, which depends on the cavity geometry, and $\beta \equiv v/c$. For $\beta \sim 10^{-7}$ one needs about 10^8 of full cycles of oscillations to generate several thousand of photons. A rigorous account of losses due to nonideality of boundaries is still an unsolved problem. According to a simplified model considered in [9,20], one can simply replace the factor $\chi\beta$ in Eq. (9) by $\chi\beta - 1/(2Q)$ (in the long-time limit $\Omega T \gg 1$). Therefore, the generation is possible only if the quality factor of the cavity Q exceeds the value $(2\chi\beta)^{-1}$. Of course, even in this case the number of photons cannot grow to infinity, because the solution (9) is valid only under the condition $\beta^2\Omega T \ll 1$. Nonetheless, it can be sufficiently high due to the exponential dependence on the parameters given by formula (9).

1 The boundary conditions and the experimental feasibility of the fast moving wall

As was shown in the Introduction, the critical point for the feasibility of the experimental verification of the nonstationary (dynamical) Casimir effect is the possibility to realize the oscillating motion of the boundary at the frequency of $10^9 \div 10^{10}$ Hz for a sufficiently long time and with sufficiently high maximal velocity. As a purely mechanical (acoustical) excitation is technically difficult and, moreover, has severe limitations on the admissible value of the ratio $v/c < 10^{-7}$, we have thought to alternative ways to change the boundary conditions of the electromagnetic field inside the cavity.

The reflection of microwaves depends on the concentration of electrons, so a good way to obtain a mirror is to create a high concentration of electrons, instead of moving at high frequency a macroscopic wall. In fact, this would require a huge amount of energy, necessary to overcome the inertia of all the nuclei.

1.1 Laser excitation of a semiconductor layer on the metallic wall

The idea is to produce an *effective* microwave mirror, changing the conductivity of a semiconductor layer covering one of the cavity walls with the aid of short powerful laser pulses. In a wide sense, this idea is not quite new, since it was already discussed, e.g., in Refs. [21,22]. However, a single-time excitation (ionization) of the medium considered in that papers imitates a *monotonous* motion of the mirror from one position to another and cannot result in a large number of photons. Possibilities of simulating the nonstationary Casimir effect by using time-dependent dielectric media or by instantaneous removing of a mirror dividing a cavity in two parts were discussed, e.g., in [23,24], but only as some conceivable ideas and without concrete proposals for experiments.

The principal difference of our idea is that we are planning to *realize periodic excitations* with many thousand pulses. The most important feature of our proposal is that the maximal velocity of displacement of an *effective mirror* is not subjected to strong limitations inherent to the acoustical excitation, since the velocity of the effective mirror is determined by

- the distance between the plasma layer in the semiconductor and the external metallic wall of the cavity;
- the duration of the laser light pulse

Furthermore, increasing the parameter β by several orders of magnitude immediately softens the requirements to the quality factor of the cavity, the number of pulses and admissible detuning from the strict resonance by the same orders. In principle, high (even relativistic) velocities could be achieved also for effective mirrors made of free electrons [25], but their creation requires tremendous power and hardly can be performed in the periodic regime.

In Fig. 1 we show a possible scheme for the resonant cavity. To clarify the idea and introduce the physical magnitudes to be used for describing the phenomenon, in the next section 1.1.1 we discuss how to obtain free charge carriers in semiconductors by laser irradiation and in section 1.1.2 we show how this increase in the number of electrons makes it possible to obtain a reflecting wall for microwaves, as if it were a metallic surface.

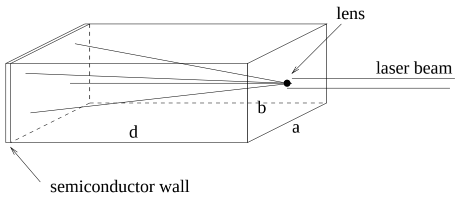

Figure 1: Laser excitation of a semiconductor layer: a laser beam enters the cavity through a lens (optical fiber) and incides on the wall covered by a semiconductor layer, thus producing a mirror for microwaves.

1.1.1 *Optical Absorption*

It is possible to enhance the concentration of free carriers in a semiconductor illuminating it with a laser beam, having a photon energy greater than the energy gap.

The light penetrates a distance into the semiconductor depending on the photon frequency and on the type of semiconductor: the thickness of the plasma formed is related to the frequency of the incident photon through the *absorption coefficient* α. In this paragraph are also inserted some figures showing α measured for Si and GaAs, that will be used in section 1.1.2 to calculate the concentration of free carriers in table 3.

If a light beam of intensity $I(0)$ enters into a semiconductor at $z = 0$, the intensity of the beam decreases with z according to [26]

$$I(x) = I(0)\exp(-\alpha z) \tag{10}$$

The constant α (in cm^{-1}) is called the *absorption coefficient*.

Measured values of α for silicon and GaAs are shown in figures 2, 3, 4, and in Tab.1.

Table 1: Optical absorption for three wavelengths in Si at room temperature.

λ (nm)	$h\nu$ (eV)	α (cm^{-1})	p = α^{-1} (μm)
1064	1.17	$2.5 \cdot 10^3$	400
750	1.65	$2 \cdot 10^3$	50
532	2.34	10^4	1

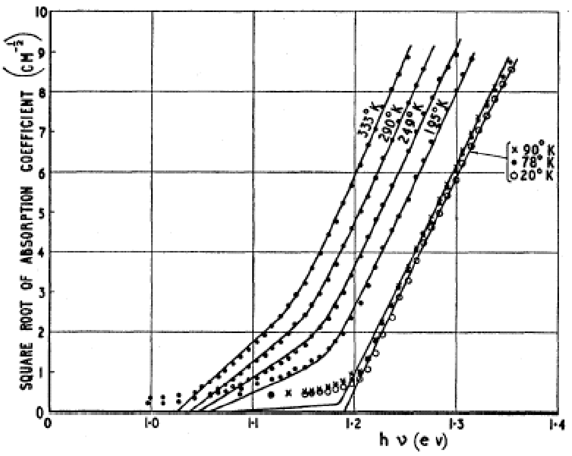

Figure 2: Dependence of absorption constant on photon energy for silicon at some values of temperature [27].

1.1.2 *Conductivity and Skin Depth*

The absorption of an optical pulse gives rise to an increase in free carrier density, which influences the reflection and transmission properties for electromagnetic waves in the semiconductor. To evaluate this change we will first study the semiconductor conductivity, related to the number of free charges. The conductivity in semiconductors is expressed in the following form:

$$\sigma = e(\mu_e n + \mu_h p) \tag{11}$$

where μ_e and μ_h are the mobility of electrons and holes and n, p are the electron and hole concentration.

Table 2: Mobility of electrons and holes at 300 K e 77 K in Si.[29]

	μ_e (cm^2/V $\cdot$ s)	μ_h (cm^2/V $\cdot$ s)
300 K	1350	480
77 K	2.1×10^4	1.1×10^4

The conductivity is involved in Maxwell equations, which describe the propagation of electromagnetic waves with generic frequency ω in the semiconductor:

$$\nabla \times \mathbf{E} = -iw\mu\mathbf{H} \tag{12}$$

$$\nabla \times \mathbf{H} = (\sigma + iw\epsilon)\mathbf{E} \tag{13}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon} \tag{14}$$

$$\nabla \cdot \mathbf{H} = 0 \tag{15}$$

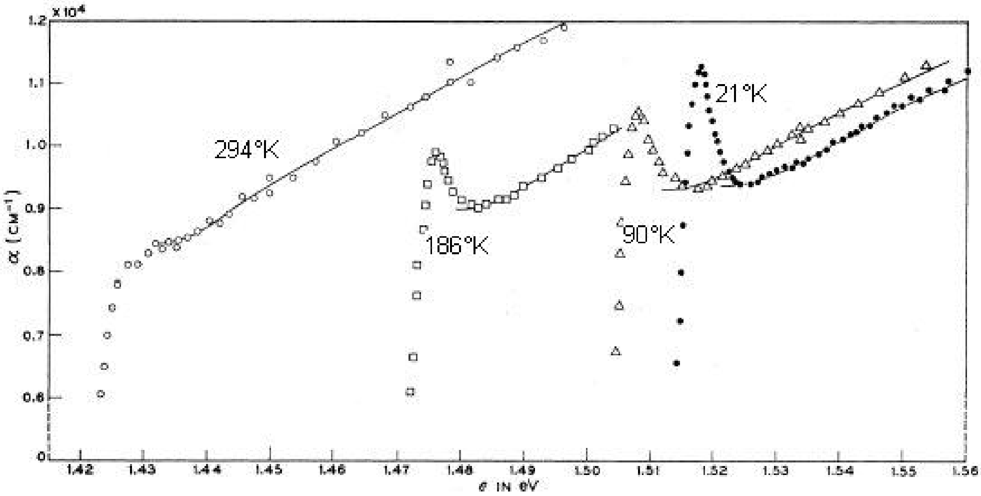

Figure 3: Upper portion of the GaAs intrinsic absorption edge, measured on a 6.5-μm-thick semi-insulating polished slice [28].

where ε is the dielectric constant, μ permeability, and ρ is the charge density. Solving the equations (12-15) for the magnetic field, we arrive at the Helmholtz equation for the monochromatic components of the electric field

$$\nabla^2\mathbf{E} = \gamma^2\mathbf{E} \tag{16}$$

where $\gamma = \sqrt{iw\mu(\sigma + iw\epsilon)} = \alpha + i\beta$ is called the *intrinsic propagation constant*; α the attenuation constant; β the phase constant.

A time-dependent inhomogeneous plane wave solution resulting from equation (16) is as follows:

$$E = E_0 e^{-\alpha z}\cos(wt - \beta z) \tag{17}$$

Lossy media are characterized by non-zero conductivity ($\sigma \neq 0$). There are three types of lossy media: good conductor, poor conductor and lossy dielectric; the presence of a loss in the medium introduces wave dispersion by conductivity.
A medium is a good conductor when $\sigma \gg w\epsilon$, so that the conduction current is much larger than the displacement current. The energy carried by the wave traveling through the medium will decrease continuously as the wave propagates because ohmic losses are present.

The propagation constant γ, expressed as

$$\gamma = iw\sqrt{\mu\epsilon}\sqrt{1 - i\frac{\sigma}{w\epsilon}}$$

becomes, for $\frac{\sigma}{w\epsilon} \gg 1$,

$$\gamma = (1 + i)\sqrt{\pi f \mu \sigma}$$

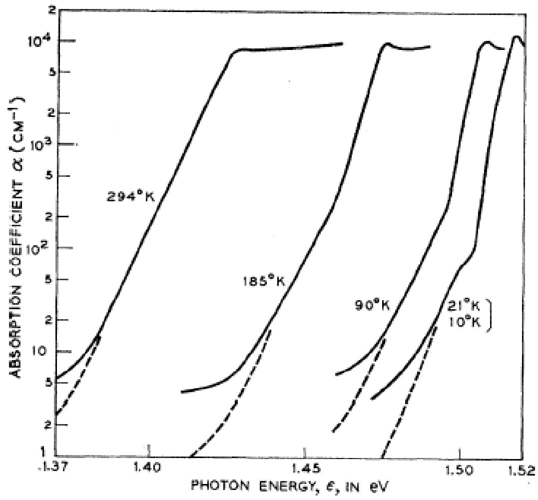

Figure 4: The intrinsic absorption edge in semi-insulating GaAs at five temperatures. The upper parts of these curves are shown (with a linear ordinate scale) in Fig. 3. [28]

and thus $\alpha = \beta = \sqrt{\pi f \mu \sigma}$, with $f = w/2\pi$.

Recalling the solution of the electric wave equation (17), we define δ as the distance which corresponds to a reduction of the amplitude of the wave of a factor e.

The electromagnetic wave gets into this distance, called *skin depth*:

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} = \frac{1}{\alpha} = \frac{1}{\beta} \tag{18}$$

For a typical microwave frequency ($f \simeq 10^{10}$ Hz), the approximation $\frac{\sigma}{w\epsilon} \gg 1$ is valid for conductivities greater than 10 $[\Omega\mathrm{m}]^{-1}$. [2] We observe that δ, even if it is defined with an approximation, is valid in general to estimate the penetration of the wave inside a medium, but the exponential attenuation is not valid any more.

In Tab.3 are shown some values of skin depth for Si and GaAs[3] in normal conditions compared with copper.

The reflection coefficients for real photons at normal incidence do not depend on polarization and are

$$\Gamma = \left| \frac{\sqrt{\epsilon} - 1}{\sqrt{\epsilon} + 1} \right| \tag{19}$$

[2]This approximation is called *low frequency approximation* and it applies when the electric field of the incident wave does not have rapid spatial variations within the distance of the mean free path *l* of electrons. For metals the relaxation time τ is typically 10^{-14} to 10^{-15} s and the average electronic speed v_0 is of the order 10^8 cm/s at room temperature, and hence a mean free path $l = v_0\tau$ of 1 to 10 nm.

[3]values for AXT and Wafer Technology: $\varrho \geq 5{\times}10^7$ Ω cm and $\mu \geq$ 5000 cm^2/V·s.

Table 3: Skin depth in Si and copper for 10 GHz microwaves.

	n (cm^{-3})	σ $[\Omega\mathrm{m}]^{-1}$	δ
Si pure	4.6×10^{11}	2×10^{-2}	20 cm
Si n-type	4.6×10^{14}	20	6.3 mm
Cu	8.5×10^{22}	5.76×10^{7}	7 μm

where $\epsilon = \epsilon_0 + i4\pi\frac{\sigma(\omega)}{\omega}$, ϵ_0 is the dielectric permettivity at zero frequency, and $\sigma(\omega)$ describes the frequency-dependent conductivity.
If the conductivity is rather large, that is $|\epsilon| \gg 1$, the reflection coefficient can be written as

$$\Gamma = \left|\frac{1-\frac{1}{\sqrt{\epsilon}}}{1+\frac{1}{\sqrt{\epsilon}}}\right| = \left|\frac{1-\eta}{1+\eta}\right| \approx 1 - 2\mathrm{Re}\eta \tag{20}$$

η is the *surface impedance*. This is valid for frequencies $\omega \ll \omega_p = \sqrt{\frac{4\pi ne^2}{m^*}}$, where ω_p is the *plasma frequency*, n the density of conducting particles, m^* is their effective mass.

If $\omega \ll \frac{\omega_p^2}{4\pi\sigma_0}$, where σ_0 is the conductivity defined in (11),

$$\Gamma \approx 1 - \sqrt{\frac{\omega}{2\pi\sigma_0}} = 1 - \frac{c_1}{\sqrt{n}} \tag{21}$$

c_1 is a constant.

If the inequality $\omega > \gamma$ holds, there are two different cases:

1. $\omega < \sqrt{\frac{2\hbar\omega_p}{c^2m^*}}\omega_p$ in this case $\Gamma = 1 - \frac{c_2}{\sqrt{n}}$,
   c_2 constant.
2. $\omega > \sqrt{\frac{2\hbar\omega_p}{c^2m^*}}\omega_p$ here $\Gamma = 1 - \frac{c_3^{(1)}}{\sqrt{n}} - c_3^{(2)} - \frac{c_3^{(3)}}{n}$,
   $c_3^{(1)}$, $c_3^{(2)}$, $c_3^{(3)}$ are connected, respectively, with the processes of scattering of free carriers at the defects and impurities, with surface effects and interparticle collisions.

In both cases, the expressions obtained show how with increasing concentrations of free electrons the reflection coefficient Γ approximates unity.

1.1.3 Microwave Measurements

In Fig. 5 is shown a possible experimental setup to verify if the plasma produced by the laser inside the semiconductor is a good mirror for microwaves.

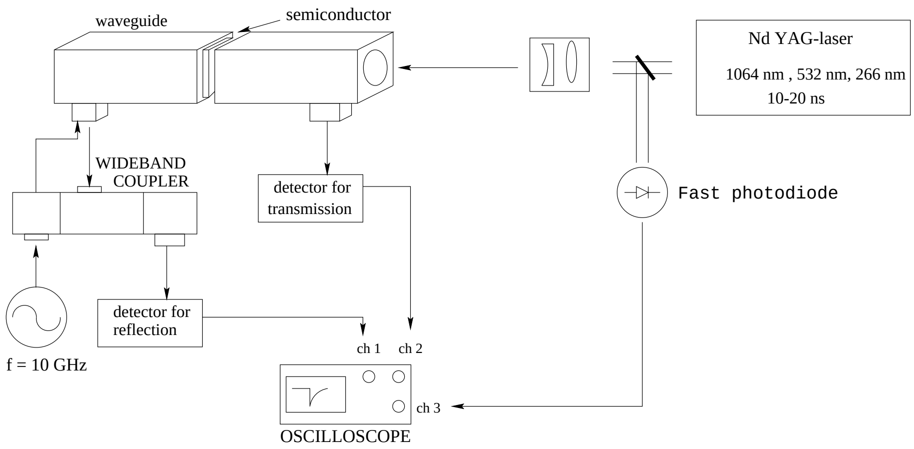

Figure 5: Experimental setup to measure the times involved and reflection coefficients.

The silicon wafer is inserted inside a waveguide and the reflected and transmitted power under laser illumination are measured by means of two antennas coupled to diode detectors.

When the wafer is illuminated a transient in the transmitted and reflected power is observed as shown in Fig.6 and 7. The figures display single-shot measurements ($\lambda = 1064$ nm) with some values of laser power: the value that is displayed for each curve in volts is proportional to the number of free carriers created (see Tab.4).

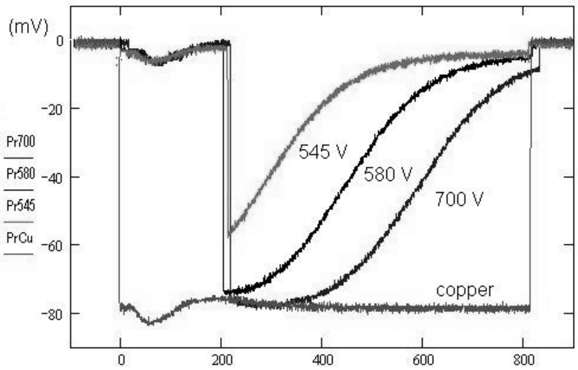

Figure 6: Measured reflection power in mV versus time in μs. When the laser is turned on, 200 μs after the RF power source (t = 0), the reflected power goes from zero to the measured value reflected by copper (square wave).

As soon as the laser is turned on the reflected power goes to the maximum value (the same reflected by the copper slab) and the transmission is zero. Subsequently the curves in Fig.6 and 7 show the transient during recombination of carriers for the reflected and transmitted power: the reflected power goes back to zero while the transmitted increases

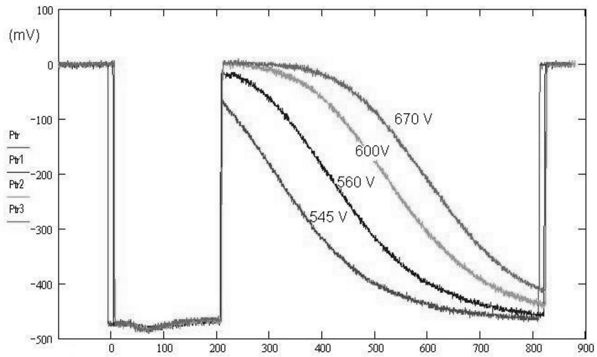

Figure 7: Measured transmission power in mV versus time in μs.

to the value without illumination.

Table 4: Values of the magnitudes describing the mirror created with an incident energy of 100 μJ/cm^2.

λ (nm)	p (μm)	n (cm^{-3})	σ (Ωm)$^{-1}$	δ (m)
1064	400	1.33×10^{16}	5.74×10^{2}	1.17×10^{-3}
532	1	2.67×10^{18}	1.1×10^{5}	8.5×10^{-5}

Note: The second column shows the calculated carrier concentration $n = \frac{E}{E_{ph}\cdot V} \times 0.5$, where E_{ph} is the value of the optical transition and the laser intensity *E* in electronvolts, V volume of a 1 cm^2 mirror of thickness p. We have assumed that the conversion photons-electrons efficiency is 0.5. The conductivity σ has been estimated inserting these concentrations in the formula (11) and the depletion depth δ is obtained with definition (18).

2 The cavity

In the introduction to this report we underlined the fact that the dynamical Casimir effect can be better measured inside a cavity, instead of using only one mirror, because the effect is amplified if observed inside such a structure. If the cavity is rectangular, the permitted modes can be found through Maxwell equations. Each solution of the field equations inside the cavity is called a *cavity mode*. We choose the simplest TE (or *transverse electric*) mode, called TE_{101} and shown in Fig. 8; in this way the expected photons from vacuum are present in the form of an electromagnetic wave, and the lines of the associated electric field are just the ones in Fig. 8.

For each one of the experimental solutions we discussed in section 1, the properties of the cavity will be different. To characterize the cavity, we will use the quality factor Q. The measurement requires a value for Q as much high as possible, so we will give an estimate of it when one of the walls is not a perfect conductor.

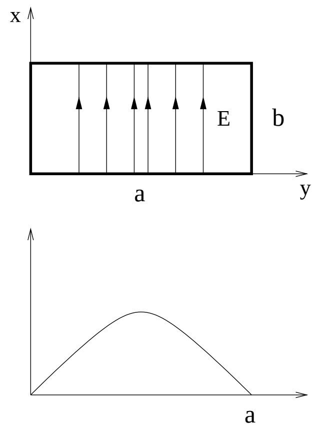

Figure 8: The electric field in the cavity at some value *z*.

2.1 The Q-factor of a cavity

We restrict our study to cavities made of copper at room temperature. We saw in section 1.1.2 that the electromagnetic field penetrates into the cavity wall by the skin depth δ. At 10 GHz this skin depth for copper, and in general for good conductors, is about 7 μm. The losses per unit surface area P_S produced in this thin layer can be expressed as [32]:

$$\mathrm{P}_S = \frac{1}{2}\mathrm{R}_S\mathrm{H}_S^2$$

where H_S is the magnetic surface field and R_S is the surface resistance, which in a normal conducting cavity is given by:

$$\mathrm{R}_S = \left(\frac{\mu\omega}{2\sigma}\right)^{1/2} = \frac{1}{\sigma\delta}$$

This gives at 10 GHz a surface resistance of 2.5 mΩ. It is immediately evident that a semiconductor wall becomes critical as it adds a series resistance considerably higher. For example with $n-$type silicon we get 20 Ω. (see Tab.3).

In general to obtain the power dissipated in the walls it is necessary to get the currents that flow on the surfaces, through the tangent components of the magnetic field. So, the losses for conduction are:

$$P_c = \iint \frac{1}{2\sigma\delta}|\mathbf{n}\times\mathbf{H}|^2 dA \tag{22}$$

The quality factor of a cavity is directly related to its surface resistance. Q is defined as the ratio between the average stored energy U and the energy lost in a period T$= 2\,\pi/\omega_0$:

$$Q = \frac{\omega_0 U}{W_L} \tag{23}$$

where ω_0 is the natural *frequency of the cavity*, and W_L the *mean power loss*.

For a resonant cavity the total energy is converted from the electric field to the magnetic field and viceversa, so it is possible to obtain the total energy stored using the maximum amplitude of the electric field. If we take the rectangular cavity and the field as in Fig.9, we obtain:

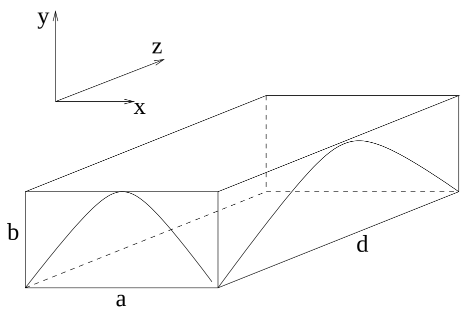

Figure 9: A rectangular cavity in which there is a standing wave such that the length d corresponds exactly to λ/2.

$$(U_E)_{\max} = \frac{\epsilon}{2}\int_0^d\int_0^b\int_0^a |E_y|^2\,dx\,dy\,dz = \frac{\epsilon}{2}\int_0^d\int_0^b\int_0^a E_0^2\sin^2\frac{\pi x}{a}\sin^2\frac{\pi z}{d}\,dx\,dy\,dz$$

$$U = \frac{\epsilon abd}{8}E_0^2 \tag{24}$$

where we used the field shown in Fig.9, $E = E_y = E_0\sin\frac{\pi x}{a}\sin\frac{\pi z}{d}$ (vertically polarized).

The general expression for the energy in the volume V,

$$W = \iiint_V \frac{\mathbf{E}\cdot\mathbf{D}+\mathbf{H}\cdot\mathbf{B}}{2}d^3V = \frac{1}{2}\epsilon_0\epsilon_r\iiint|\mathbf{E}|^2d^3V + \frac{1}{2}\mu_0\mu_r\iiint|\mathbf{H}|^2d^3V \tag{25}$$

where

E = *electric field intensity*

D = *electric flux density*

H = *magnetic field intensity*

B = *magnetic flux density*

with the constitutive relations:

$$\mathbf{D} = \epsilon\mathbf{E}$$

$$\mathbf{B} = \mu\mathbf{H}$$

where $\epsilon = \epsilon_r\epsilon_0$ is the dielectric permettivity of the medium and $\mu = \mu_r\mu_0$ the magnetic susceptibility.

If instead of vacuum inside the cavity there is a medium of charactersitics

$$\epsilon = \epsilon^{'} + i\epsilon^{''}$$

$$\mu = \mu^{'} + i\mu^{''}$$

the power dissipated by the medium results to be:

$$P_t = \iiint \left(\frac{\epsilon_0 \epsilon^{''} |\mathbf{E}|^2}{2} + \frac{\mu_0 \mu^{''} |\mathbf{H}|^2}{2} \right) dV \tag{26}$$

For a rectangular cavity, of sizes $3 \times 4 \times 5$ cm^3, computer simulations give a Q$= 1.35 \times 10^4$ for the fundamental frequency $f_c = 3.903$ GHz. If one of the walls is made of silicon with $\sigma_{Si} = 2.52 \times 10^{-4} \Omega^{-1} \mathrm{m}^{-1}$, the frequency shifts to 3.313 GHz and the Q is reduced to 3500, thereupon the necessity of cooling down. In fact, if the semiconductor is cooled to a value of temperature such that there are no more free carriers (in Si the freeze-out is obtained at 77 K), inside the cavity it does not dissipate any more because it is an insulator. Besides, the semiconductor slab does not alter the Q value if positioned where the electric field is nearly zero (eq. (26) and Fig.9), as designed in this feasibility study.

3 Detecting motion-induced radiation and Noise sources

In the introduction we saw that due to excitation of quantum vacuum it is possible to obtain several thousand of photons inside the cavity; it will be necessary to discriminate this motion-induced radiation from photons coming from stray effects. In this section are analyzed possible sources of noise.

3.1 Black body radiation

One of the possible sources of noise in this measure is not connected to the measure instruments. Consider a cavity with metallic walls at uniform temperature T; the walls emit electromagnetic radiation in the *thermal* range of frequencies. This radiation inside the cavity exists in the form of standing waves of frequency ν_i with nodes at the metallic surfaces. The number N of such waves in a given frequency interval times the average energy of the waves $E(\nu)$, divided by the volume of the cavity *V*, gives the average energy content per unit volume in the frequency interval $[\nu, \nu + d\nu]$. This is the energy density $\varrho_T(\nu)$. The formula that Planck obtained for the energy density in the blackbody spectrum is:

$$\varrho_T(\nu) d\nu = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1} d\nu \tag{27}$$

To estimate the number of thermal photons it is better to introduce the *exitance*, defined as the amount of power per unit area that leaves a surface of a source of radiation. In the case of a cavity source, the expression for the exitance is obtained with the following thought experiment [36].

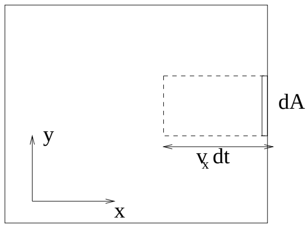

Figure 10: Source area from an infinite conducting box full of photons.

Consider the box in Fig.10 full of photons, moving at the speed of light, *c* in all directions. If there is a small hole of area *dA* in the wall, some photons will escape; the number of photons Q_q that will escape in a time *dt* is equal to half of the photons in the volume *dA* v_x*dt* (only the photons with positive velocities have a chance to escape):

$$Q_q = \frac{1}{2} N dA v_x dt$$

with $v_x = c/2$ and $N = \int \frac{8\pi}{c^3} \frac{\nu^2}{e^{h\nu/kT}-1} d\nu$ the number of photons per unity cavity volume. Combining these results, we find for the photon exitance M_q

$$M_q = \frac{Q_q}{dA\, dt} = \frac{\nu^2 d\nu}{c^2(e^{h\nu/kT} - 1)} \left[\frac{\text{photons}}{\text{cm}^2\text{s}}\right] \tag{28}$$

To get the total exitance from a source over a finite spectral region, an integration is required:

$$M_q = \int_{\nu_1}^{\nu_2} M_{q,\nu}(\nu, T) d\lambda = \frac{1}{c^2} \int_{\nu_1}^{\nu_2} \frac{\nu^2 d\nu}{e^{h\nu/kT} - 1} \left[\frac{\text{photons}}{\text{cm}^2\text{s}}\right] \tag{29}$$

If we consider a cavity with frequency $f = 10^9$ Hz and $Q = \frac{f}{\Delta f} = 10^4$, a bandwidth of 1 MHz is set and the integration is carried out between ν_1 =(1 GHz - 0.5 MHz) and ν_2 =(1 GHz + 0.5 MHz). With numerical integration we obtain, at 4 K, $M_q \simeq 92 \times 10^3$ photons/cm^2 s.

If we reduce the bandwidth to 1 kHz, as it is possible if the quality factor of the cavity is enhanced to 10^6, the integration in equation (29) gives out 92 photons/cm^2 s. During $\tau = 2Q/f = 2$ ms the photons emitted are 18.4$\times 10^{-2}$ photons/cm^2.

The number of thermal photons during the time of observation Δt is 92 photons/cm^2 s $\times\Delta$t(s) multiplied by the cavity emisssion surface of 94 cm^2 (if we will take a cavity with sizes $3 \times 4 \times 5$ cm^3).

3.2 Generation/Recombination noise

The plasma that is formed on the surface of the semiconductor when illuminated can be considered, to all practical purposes, a radiatior inside the cavity: the collisions between the free electrons and the ions of the lattice give rise to bremsstrahlung that can add noise to our measure.

The radiated power for one electron accelerated in the field of the charge Q is in good approximation is given by [30,31]

$$\mathrm{P_{brems}} = \frac{n_e e^4 Q^2 v}{48 \varepsilon_0^3 m_e c^3 h} \qquad [\mathrm{watt/ion}] \tag{30}$$

with e, ε_0, m_e and h physical constants and Q charge of the ion, v and n_e respectively velocity and concentration of electrons.

Substituting the values for e, ε_0, m_e and h in eq.(30) we obtain the form

$$\mathrm{P_{brems}} = 1.85 \times 10^{-38} n_e n_i \sqrt{E} \qquad [\mathrm{watt/m^3}] \tag{31}$$

with $E = E_{ph} - E_{bg}$ *excess* kinetic energy of electrons in eV. The ionic charge Q in eq. (30) is here the charge of each silicon atom which has lost one electron during laser irradiation, so the concentration of ions is $n_i = n_e$.

For example, if the concentration of electrons due to the laser is $n_e = 1.33 \times 10^{22}$ e$^-$/m^3 (as in Tab. 4) and $E \simeq 50$ meV, it turns out that the plasma formed on a 1 cm^2 silicon illuminated surface ($\lambda = 1.064$ μm, p = 400 μm) will radiate away approximately 10 mW.

The spectrum of the radiated power is flat, that is, the intensity of the radiation at each frequency up to $\nu_{\max}$ is roughly constant and then drops to zero for frequencies $\nu > \nu_{\max}$.

The maximum value $\nu_{\max}$ can be obtained estimating the *collision time* $\frac{1}{\nu} = \tau \approx$ b/v, where v is the speed of the electron and b the distance over which the accelerating force is large. The distance b is found with the equation $\frac{e^2}{4\pi\varepsilon_0}\frac{1}{\mathrm{b}} = E$, and comes out 2.88$\times 10^{-8}$ m [4]; v is 1.32 $\times 10^5$ m/s, so $\nu = \frac{1}{\tau} \simeq 4.6$ THz.

With the relation $E = h\nu$ we obtain $\nu = 12$ THz.

[4] b + r$_{\mathrm{at}}$ $\cong$ b, in fact b$\gg r_{\mathrm{at}}$ (the atomic radius for silicon is r$_{\mathrm{at}} = 100$ pm)

The bremsstrahlung intensity in the photo-generated plasma is proportional to n_e^2 (see eq. (31)), so it can be considerably reduced decreasing the laser intensity to the minimum value of carriers that is necessary to reflect microwaves. Besides, this model overestimates the power radiated by the plasma formed on the semiconductor: in fact, it relies on the hypothesis that the *excess* energy is entirely expended in radiative processes.

3.3 Antenna and Minimum detectable signal

Recalling the definition (23) of the quality factor, we write $W_L = \frac{\omega_0}{Q}U$. In a closed cavity, the power loss W_L will be equal to the rate of change of the stored energy U,

$$W_L = -\frac{dU}{dt} \tag{32}$$

so that the decay of the stored energy will be

$$W_L = W_0 e^{-t/\tau_W}$$

where $\tau_W = \frac{Q}{\omega\,0}$ is the *decay time of the stored energy*.

Since $U \propto E^2$, the field will decay as

$$E(t) = E_0 e^{-\frac{t}{2\tau_W}} e^{i\omega t}$$

and the decay time of the field is two times the decay time of the stored energy.

The limit of detectability of radiofrequency photons is set by the noise associated to the antenna itself and the measurement system.

To optimize detection inside the cavity the antenna should be positioned where the electric field is maximum, and parallel to the field lines. The incident wave induces an electromagnetic field that gives rise to a distribution of currents on the antenna. These induced currents feed a load (usually through a transmission line or a waveguide): in this way power is conveyed to the load impedance Z_L and the signal can be detected.

The smallest number of detectable photons is related to Johnson noise in this resistance. This noise is expressed in terms of the voltage fluctuation which appears at the ends of a resistor R. Physically this noise is gaussian, white up to frequencies 10^{13} Hz and its spectral density is

$$V_n^2(\nu) = 4\mathrm{R}k\mathrm{T} \tag{33}$$

where k=1.38$\times 10^{-23}$ J/K is the Boltzmann constant and T is the temperature in Kelvin. In the interval $\Delta\nu$

$$V_{n,\Delta\nu}^2 = \int_0^{\Delta\nu} V_n^2(\nu) d\nu = 4\mathrm{R}k\mathrm{T}\Delta\nu$$

At 4 K, with an amplifier of bandwidth 1 kHz the number of equivalent electron noise is nearly 6.
This thermal noise can be described in a more practical way with electronic magnitudes: if we call B the effective noise bandwidth the mean square noise voltage is $\overline{e^2}$

$$\overline{e^2} = 4k\mathrm{TRB} \tag{34}$$

The quantity *k*TB is defined as the *maximum available noise power for conjugate matching*. In Tab.5 we give some values for *k*TB for increasing bandwidths, expressed in dBm[5].

Table 5: Values for *k*TB at 4 K and 290 K.

B	*k*TB (dBm) at 4 K	*k*TB (dBm) at 290 K
1 Hz	-174	-114
10 Hz	-164	-104
100 Hz	-154	-94
1 kHz	-144	-84
10 kHz	-134	-74
100 kHz	-124	-64
1 MHz	-114	-54

For example, if the noise level of the amplifier (coupled to the cavity) measured with an oscilloscope is -135 dBm, the *noise power density* is $V^2/f = 10^{-17}$ W/Hz and the number of noise photons is:

$$n_{RF} = \frac{V^2/f}{E_{ph} \cdot G} = 2500[\mathrm{ph/Hz}]$$

where E_{ph} is the energy of an RF photon and G is the gain of the amplifier. With a 10 kHz bandwidth cavity ($Q = 10^5$, $w_0 = 2$ GHz) and the noise photons integrated by the cavity are 2.5$\times 10^7$. This is a number that can be reduced using ultra-low-noise microwave receiver, such as the most recent cryogenic amplifiers used by radioastronomers [37].

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[5]dBm is dB referred to 10^{-3}W. We remind that dB= $10\log\frac{P_2}{P_1} = 20\log\frac{V_2}{V_1}$; the following relation is useful to convert dBW (dB referred to watt) to power in watts: dbW= $10\log\frac{P}{1W}$, that implies $P = \log^{-1}\frac{\mathrm{dBW}}{10}$

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