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THE TRANSVERSITY FUNCTION AND DOUBLE SPIN AZIMUTHAL ASYMMETRY IN SEMI-INCLUSIVE PION LEPTOPRODUCTION

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Abstract

We show that the transverse momentum dependent transversity function is proportional to the longitudinal polarization of a quark in a transversely polarized proton. This result suggests an alternative, convenient method for determining transversity, without knowing unusual fragmentation functions. The method consists of measuring the double spin azimuthal asymmetry in semi-inclusive pion leptoproduction by a transversely polarized proton target. The asymmetry, which is twist 3, is estimated to be more than 10% under the most favourable conditions. The experiment we suggest is feasible at facilities like DESY and CERN.

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1 Introduction

For several years high energy spin physicists have been concentrating their efforts on determining the quark transversity distribution[1-5], which appears a particularly difficult task[6-10]. Indeed, different observables have been singled out, which are sensitive to this quantity; among them the Drell-Yan double spin asymmetry[9] and the interference fragmentation functions[7].

For the moment the most promising experiments in this sense are those realized or planned by SMC[11] and HERMES[12,13] collaborations. These experiments are based on the Collins effect[14] and consist of measuring the azimuthal single spin asymmetry[4,15–19] in semi-inclusive pion electroproduction by a longitudinally[12] or transversely[13] polarized proton target. Up to now such experiments have provided a rough evaluation[19] of the transversity function, h_1. The single spin asymmetry is sensitive to the product $h_1(x)c(z)$, where c is the azimuthal asymmetry fragmentation function of a transversely polarized quark into a pion[4,19], and, as usual, x and z are the longitudinal fractional momenta, respectively, of the active quark and of the pion with respect to the fragmenting quark. As claimed by Jaffe[4], this may become the "classic" way of determining the proton transversity distribution functions, provided $c(z)$ is known to some precision and is not too small. But at present we know very little about this function[19]. Analogous considerations could be done about the method suggested by Jaffe and Ji[20] (JJ). They propose to measure the double spin asymmetry in a semi-inclusive deep inelastic scattering (SIDIS) experiment of the type

$$\vec{\ell} p^{\uparrow} \to \ell' \pi X, \tag{1}$$

where $\vec{\ell}$ is a longitudinally polarized charged lepton and $p^{\uparrow}$ a transversely polarized proton target. The asymmetry is defined as

$$A(|\mathbf{k}|; Q, \nu; \Pi_{\parallel}) = \frac{d\sigma_{\uparrow\rightarrow} - d\sigma_{\uparrow\leftarrow}}{d\sigma_{\uparrow\rightarrow} + d\sigma_{\downarrow\leftarrow}}. \tag{2}$$

Here, as usual, ν is the lepton energy transfer and $Q^2 = -q^2$, q being the four-momentum transfer. Furthermore $\mathbf{k}$ is the momentum of the initial lepton and $\Pi_{\parallel}$ the component of the final pion momentum along the momentum transfer. Lastly $d\sigma_{\uparrow\rightarrow}$ and $d\sigma_{\uparrow\leftarrow}$ are the polarized differential cross sections for reaction (1), integrated over the transverse momentum of the final pion with respect to the momentum transfer; arrows indicate the proton and lepton polarization. Asymmetry (2) includes the product $h_1(x)\hat{e}(z)$[20], where $\hat{e}(z)$ is the twist-3 fragmentation function of the pion. The extraction of h_1 depends again critically on an unknown function.

If the cross section is not integrated over the transverse momentum of the final pion, reaction (1)[21,16] and the analogous one with a longitudinally polarized target[21,16,22] exhibit an azimuthal asymmetry. This is particularly suitable for determining the transverse momentum dependent (t.m.d.) distribution functions[15,16,21,23,24], defined as ”new” by Kotzinian and Mulders[21] (KM): see also Mulders and Tangerman[15] (MT). In particular, the double spin azimuthal asymmetry arising from a tranversely polarized target has been found[21] to be sensitive to the ”new” function g_{1T}, proportional to the longitudinal quark polarization in a transversely polarized proton.

The aim of this paper is to re-examine such azimuthal double spin asymmetries. We derive the differential cross section for reaction (1), starting from the definition of t.m.d. transversity function as given by Jaffe and Ji[2] (JJ1), *i. e.*,

$$\delta q_\perp(x, \mathbf{p}_\perp) = \sum_{T=\pm 1/2} 2T q_T(x, \mathbf{p}_\perp), \tag{3}$$

where $q_T(x, \mathbf{p}_\perp)$ is the probability density to find, in a transversely polarized proton, a quark whose spin is parallel (T = 1/2) or opposite (T = -1/2) to the proton spin. This amounts to taking, instead of the usual helicity representation, a canonical one, such that the quantization axis is along the proton transverse polarization. The asymmetry we calculate turns out to coincide with the one by KM, provided we identify g_{1T} with $\delta q_\perp$. We shall prove this identity for massless quarks, which amounts to saying that, owing to transverse momentum, a quark in a transversely polarized proton has a longitudinal polarization, related to transversity. Therefore $\delta q_\perp$ - denoted as h_{1T} by MT - plays a major role in the azimuthal asymmetry of reaction (1). Moreover, as we shall see, this distribution is somewhat relevant also in the case of a longitudinally polarized target. All this suggests an alternative, convenient method for determining the transversity. Indeed, as a consequence of our result, $\delta q_\perp$ contributes not only to the to the chiral-odd component of the t.m.d. correlation matrix[15,23,25], but also to its chiral-even part. Therefore this distribution function may be coupled - and this is the case of the azimuthal double spin asymmetry - to a chiral-even fragmentation function, which is generally easier to determine than a chiral-odd one. This result is not completely surprising, since we have shown in a previous paper[26] that the inclusive muon pair production from singly polarized proton-hadron collisions, at a fixed transverse momentum of the pair with respect to the initial beams, causes a muon polarization sensitive to $\delta q_\perp$. But it is well-known[14] that SIDIS is kinematically isomorphic to Drell-Yan. Therefore we expect an analogous effect in reaction (1), provided we fix the pion direction.

In this paper we calculate the improved parton model contribution and evaluate the Q^{-1} power corrections, but we do not take into account the QCD evolution effects. Sect. 2

is dedicated to the derivation of the formulae for the cross section and for the asymmetry we are interested in. In sect. 3 we make some remarks, concerning power corrections and comparison with other authors; in particular, we illustrate some consequences of the identity $g_{1T} = \delta q_\perp$ in the chiral limit, which we prove in the appendix. In sect. 4 we outline some methods for inferring the transversity from the azimuthal asymmetry. Lastly sect. 5 is devoted to numerical estimates and to a conclusion.

2 Cross section and azimuthal asymmetry

2.1 Cross section

We calculate the SIDIS differential cross section in the framework of a QCD-improved parton model[30]. In the laboratory frame the differential cross section reads, in one-photon exchange approximation,

$$d\sigma = \frac{1}{4|\mathbf{k}|M}\frac{e^4}{Q^4}L_{\mu\nu}H^{\mu\nu}\, d\Gamma, \tag{4}$$

where M is the proton rest mass and $L_{\mu\nu}$ ($H_{\mu\nu}$) the leptonic (hadronic) tensor. $d\Gamma$, the phase space element, reads

$$d\Gamma = \frac{1}{(2\pi)^6}d^4k\;\delta(k^2)\;\theta(k_0)\;\; d^4P\;\delta(P^2 - m_\pi^2)\;\theta(P_0). \tag{5}$$

Here k and P are, respectively, the four-momenta of the initial lepton and of the pion, whose rest mass is m_π. The leptonic tensor is, in the massless approximation,

$$L_{\mu\nu} = \frac{1}{4}Tr[k(1+\lambda_\ell\gamma_5)\gamma_\mu k'\gamma_\nu], \tag{6}$$

λ_ℓ being the helicity of the initial lepton and $k' = k - q$ the four-momentum of the final lepton. Trace calculation yields

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}k\cdot k' + i\lambda_\ell\varepsilon_{\alpha\mu\beta\nu}k^\alpha k'^\beta. \tag{7}$$

As regards the hadronic tensor, the generalized factorization theorem[31,23,32] in the covariant formalism[33] yields, at zero order in the QCD coupling constant,

$$H_{\mu\nu} = \frac{1}{3}\sum_{f=1}^{6}e_f^2\int d\Gamma_q\varphi^f(p';P)h^f_{\mu\nu}(p,p';S), \tag{8}$$

$$h^f_{\mu\nu} = \sum_L q^f_L(p)Tr(\rho^L\gamma_\mu\rho'\gamma_\nu). \tag{9}$$

Here the factor 1/3 comes from color averaging in the elementary scattering process and f runs over the three light flavors (u, d, s) and antiflavors ($\bar{u}, \bar{d}, \bar{s}$), $e_1 = -e_4 = 2/3$, $e_2 = e_3 =$

$-e_5 = -e_6$ = -1/3. p and p' are respectively the four-momenta of the active parton before and after being struck by the virtual photon. S is the Pauli-Lubanski (PL) four-vector of the proton. q_L^f is the probability density function of finding a quark (or an antiquark) in a pure spin state, whose third component along the proton polarization is L. Analogously φ^f is the fragmentation function of a quark of four-momentum p' into a pion of four-momentum P. Moreover

$$d\Gamma_q = \frac{1}{(2\pi)^2} d^4p\ \delta(p^2)\theta(p_0) d^4p'\ \delta(p^{'2})\theta(p'_0)\ \ \delta^4(p'-p-q), \tag{10}$$

the active parton being taken on shell and massless. Lastly the ρ's are the spin density matrices of the initial and final active parton, *i. e.*[26],

$$\rho^L = \frac{1}{2}\not{p}[1 + 2L\gamma_5(\lambda + \not{\eta})] \qquad \text{and} \qquad \rho' = \frac{1}{2}\not{p}'. \tag{11}$$

Here $2L\eta$ is the transverse PL four-vector of the active parton, while λ is the longitudinal component of the quark spin vector. Formulae (11) are consistent with the Politzer theorem[34] in the parton model approximation. These imply, together with eq. (9), that η does not contribute to $h^f_{\mu\nu}$. For later convenience we re-write this last tensor as

$$h^f_{\mu\nu} = \frac{1}{4}\left[q^f(p)s_{\mu\nu} + \lambda\delta q^f(p)a_{\mu\nu}\right]. \tag{12}$$

Here

$$s_{\mu\nu} = Tr(\not{p}\gamma_\mu\not{p}'\gamma_\nu), \qquad\qquad a_{\mu\nu} = Tr(\gamma_5\not{p}\gamma_\mu\not{p}'\gamma_\nu). \tag{13}$$

Moreover $q^f(p) = \sum_{L=\pm 1/2} q^f_L(p)$ is the unpolarized quark distribution function and $\delta q^f(p) = \sum_{L=\pm 1/2} 2Lq^f_L(p)$. λ is a Lorentz scalar, such that $|\lambda| \leq 1$. If we neglect the parton transverse momentum, the only way of constructing such a quantity with the available vectors is

$$\lambda = \lambda_\parallel = \mathbf{S}\cdot\frac{\mathbf{q}}{|\mathbf{q}|} = \frac{-S\cdot q}{\sqrt{\nu^2+Q^2}}. \tag{14}$$

Here we have exploited the fact that ν is a Lorentz scalar and that in the laboratory frame $q \equiv (\nu, \mathbf{q})$ and $S \equiv (0, \mathbf{S})$, where $\mathbf{S}$ is the proton spin vector, $\mathbf{S}^2 = 1$. $\lambda_\parallel$ can be viewed as the helicity of the proton in a frame moving along $\mathbf{q}$. Now, in order to take into account the transverse momentum, we have to adopt a frame where the proton momentum is large in comparison to M[35]. But, in this more refined approximation, we are still faced with the problem of defining λ in a Lorentz invariant way. As we are going to show, the only way to do this is to consider the Breit frame, that is, where the virtual photon has four-momentum $q = (0, \mathbf{q}_B)$, with $|\mathbf{q}_B| = Q$. In this frame the proton momentum is $-\frac{1}{2x}\mathbf{q}_B$, therefore the active parton carries a momentum $\mathbf{p}_B = -\frac{1}{2}\mathbf{q}_B + \mathbf{p}_\perp$, where, as usual,

$x = Q^2/2M\nu$ is the longitudinal fractional momentum and $\mathbf{p}_\perp$ the transverse momentum with respect to $\mathbf{q}_B$. We decompose the proton spin vector $\mathbf{S}$ into a longitudinal and a transverse component, *i. e.*,

$$\mathbf{S} = \lambda_\| \frac{\mathbf{q}}{|\mathbf{q}|} + \mathbf{S}_\perp, \qquad \mathbf{S}_\perp \cdot \mathbf{q} = 0. \tag{15}$$

But the average helicity of the quark is independent of the quantization axis, therefore the decomposition (15) implies, together with eq. (14),

$$\lambda \delta q^f(x, \mathbf{p}_\perp^2) = \lambda_\| \delta q_\|^f(x, \mathbf{p}_\perp^2) + \lambda_\perp \delta q_\perp^f(x, \mathbf{p}_\perp). \tag{16}$$

Here

$$\lambda_\perp = \frac{\mathbf{S}_\perp \cdot \mathbf{p}_B}{|\mathbf{p}_B|} \simeq \frac{2\mathbf{S} \cdot \mathbf{p}_\perp}{Q} \tag{17}$$

and $\delta q_\|^f(x, \mathbf{p}_\perp^2)$ (denoted as $g_{1L}(x, \mathbf{p}_\perp^2)$ by MT) is the t.m.d. helicity distribution function. Now we carry on the integration (8) over the time and longitudinal components of p, taking the z-axis opposite to $\mathbf{q}$. We get, in the light cone formalism,

$$H_{\mu\nu} = \frac{1}{4\pi^2 Q^2} \sum_{f=1}^{6} e_f^2 \int d^2 p_\perp \varphi^f(z, \mathbf{P}_\perp^2) h_{\mu\nu}^f(x, \mathbf{p}_\perp; \mathbf{S}), \tag{18}$$

where the tensor $h_{\mu\nu}^f$ (see eq. (12)) reads[2], after insertion of eq. (16),

$$h_{\mu\nu}^f = \frac{1}{4} \left\{ q^f(x, \mathbf{p}_\perp^2) s_{\mu\nu} + \left[\lambda_\| \delta q_\|^f(x, \mathbf{p}_\perp^2) + \lambda_\perp \delta q_\perp^f(x, \mathbf{p}_\perp) \right] a_{\mu\nu} \right\}. \tag{19}$$

Here $z = (P_\| + P_0)/(2|\mathbf{p}'|)$ is the longitudinal fractional momentum of the pion resulting from fragmentation of the struck parton, whose momentum is $\mathbf{p}'$. We have defined $P_0 = \sqrt{m_\pi^2 + \mathbf{P}^2}$, $P_\| = \mathbf{P} \cdot \mathbf{p}'/|\mathbf{p}'|$ and $\mathbf{P}_\perp = \mathbf{P} - P_\| \mathbf{p}'/|\mathbf{p}'|$, $\mathbf{P}$ being the pion momentum in the laboratory frame. Denoting by $\mathbf{\Pi}_\perp$ the transverse momentum of the pion with respect to the photon momentum, we get

$$\mathbf{P}_\perp = \mathbf{\Pi}_\perp - z\mathbf{p}_\perp. \tag{20}$$

Therefore, if we keep $\mathbf{\Pi}_\perp$ fixed, $\mathbf{P}_\perp$ depends on $\mathbf{p}_\perp$. Since we want to pick up a pion resulting from fragmentation of the active quark, we pick up events such that $|\mathbf{\Pi}_\perp| << |\mathbf{P}|$.

We notice that, although we have chosen a particular frame - coincident with the one adopted by Feynman[36] -, the tensor (19) is covariant. Indeed, the spatial direction of the virtual photon could also be defined covariantly by means of the four-momenta of the photon and of the proton[18].

2.2 Azimuthal asymmetry

In order to calculate the asymmetry $A(|\mathbf{k}|; Q, \nu; \mathbf{P})$ - defined analogously to (2), but keeping the pion momentum $\mathbf{P}$ fixed - we have to substitute the leptonic tensor (7) and the hadronic tensor (18) into the cross section (4), taking into account relations (19) and (17). The result is

$$A(|\mathbf{k}|; Q, \nu; \mathbf{P}) = \mathcal{F} \; \frac{\sum_{f=1}^{6} e_f^2 \delta Q^f}{\sum_{f=1}^{6} e_f^2 Q^f}, \qquad \mathcal{F} = \frac{k_+ k'_- - k_- k'_+}{k_+ k'_- + k_- k'_+}. \tag{21}$$

$\mathcal{F}$ is the depolarization of the virtual photon with respect to the parent lepton[21]. Moreover we have introduced the quantities

$$Q^f = Q^f(x, z, \mathbf{\Pi}_\perp^2) = \int d^2p_\perp q^f(x, \mathbf{p}_\perp^2) \varphi^f(z, \mathbf{P}_\perp^2), \tag{22}$$

$$\delta Q^f = \delta Q_\parallel^f(x, z, \mathbf{\Pi}_\perp^2) + \mathbf{\Pi}_\perp \cdot \mathbf{S} \delta Q_\perp^f(x, z, \mathbf{\Pi}_\perp), \tag{23}$$

$$\delta Q_\parallel^f(x, z, \mathbf{\Pi}_\perp^2) = \lambda_\parallel \int d^2p_\perp \delta q_\parallel^f(x, \mathbf{p}_\perp^2) \varphi^f(z, \mathbf{P}_\perp^2), \tag{24}$$

$$\delta Q_\perp^f(x, z, \mathbf{\Pi}_\perp) \mathbf{\Pi}_\perp \cdot \mathbf{S} = \int d^2p_\perp \lambda_\perp \delta q_\perp^f(x, \mathbf{p}_\perp) \varphi^f(z, \mathbf{P}_\perp^2). \tag{25}$$

The pseudoscalar character of δQ^f (eq. (23)) follows from assuming a massless lepton: indeed, the expression we have deduced for the asymmetry (see the first eq. (21)) holds in any frame where the lepton mass is negligible and the lepton helicity has the same value as in the laboratory frame. Below we shall show that δQ^f is twist 3. From formula (23) we deduce that, in order to maximize the contribution of $\delta q_\perp^f$ to our asymmetry, one has to take the vector $\mathbf{\Pi}_\perp$ parallel to or opposite to $\mathbf{S}$, that is, to select pions whose momenta lie in the ($\mathbf{q}$, $\mathbf{S}$) plane. Furthermore the second term of eq. (23) is especially sensitive to $\delta q_\perp^f(x, \mathbf{p}_\perp)$ if $\mathbf{q} \cdot \mathbf{S}$ = 0. In this situation the first term of eq. (23) - and more generally the JJ asymmetry - vanishes. Therefore events such that the lepton scattering plane is perpendicular to the proton polarization are particularly relevant to our aims.

Since the products $k_+ k'_-$ and $k_- k'_+$ are invariant under boosts along the z-axis, we calculate them in the laboratory frame, where

$$k_\pm = \frac{|\mathbf{k}|}{\sqrt{2}}(1 \pm cos\beta), \qquad k'_\pm = \frac{|\mathbf{k}'|}{\sqrt{2}}[1 \pm cos(\theta + \beta)]. \tag{26}$$

Here $\mathbf{k}' = \mathbf{k} - \mathbf{q}$ is the final lepton momentum. Moreover β and θ are, respectively, the angle between $\mathbf{k}$ and $\mathbf{q}$ and between $\mathbf{k}$ and $\mathbf{k}'$:

$$|\mathbf{q}| cos\beta = |\mathbf{k}| - |\mathbf{k}'| cos\theta. \tag{27}$$

Now we consider the scaling limit, *i. e.*, $Q^2 \to \infty$, $\nu \to \infty$, $Q^2/2M\nu \to x$. Since $Q^2 \simeq 2|\mathbf{k}||\mathbf{k}'|(1 - cos\theta)$, θ tends to zero in that limit, as well as β:

$$\theta \simeq \frac{M}{Q} \frac{y}{x(1-y)^{1/2}}, \qquad \beta \simeq \theta \frac{1-y}{y}, \qquad y = \frac{\nu}{|\mathbf{k}|}. \tag{28}$$

Then the second eq. (21) and eq. (14) yield, respectively,

$$\mathcal{F} = \frac{y(2-y)}{1+(1-y)^2}, \qquad \lambda_{\parallel} = \frac{1-y}{y} sin\theta cos\phi, \tag{29}$$

where ϕ is the azimuthal angle between the $(\mathbf{k}, \mathbf{k}')$ plane and the $(\mathbf{k}, \mathbf{S})$ plane. Therefore $\delta Q^f_{\parallel}$, eq. (24), is twist 3, as follows from the second eq. (29) and from the first eq. (28). But also the second term of eq. (23) is twist 3, as is immediate to check. Therefore our asymmetry is twist 3.

3 Remarks

At this point some remarks are in order.

(i) We write, in quite a general way,

$$\delta q^f_{\perp} = \delta q^f_{e\perp} + \delta q^f_{o\perp}, \tag{30}$$

where the indices *e* and *o* mean, respectively, *even* and *odd* terms under time reversal[29]. The T-odd term $\delta q^f_{o\perp}$ corresponds to the so-called Sivers effect[30, 29]. Invariance under parity, together with a rotation by π around the proton momentum, implies

$$\delta q^f_{e\perp}(x, \mathbf{p}_{\perp}) = \delta q^f_{e\perp}(x, -\mathbf{p}_{\perp}), \qquad \delta q^f_{o\perp}(x, \mathbf{p}_{\perp}) = -\delta q^f_{o\perp}(x, -\mathbf{p}_{\perp}). \tag{31}$$

From these relations it follows that, if we integrate the asymmetry (21) over $\mathbf{\Pi}_{\perp}$, the second term of eq. (23) derives its contribution solely from the T-odd term. On the other hand, upon integration, the first term of eq. (23) goes over into $\lambda_{\parallel}\Delta q^f(x)D^f(z)$, corresponding to the "kinematic" twist-3 term of the numerator in the JJ asymmetry[20]. Here $\Delta q^f(x)$ is the helicity distribution function and $D^f(z)$ the usual fragmentation function of the pion. The above mentioned numerator includes also a "dynamic" twist-3 term of the type $\lambda_{\parallel}[\frac{1}{x}h^f_1(x)\frac{1}{z}\hat{e}^f(z) + g^f_T(x)D^f(z)]$[20], where $g^f_T(x)$ is the transverse spin distribution function.

(ii) The asymmetry derives contributions also from the one-gluon exchange[31] terms, demanded by gauge invariance. More precisely, a gluon emitted by spectator partons may interact with the active quark *a)* before or *b)* after being struck by the photon.

These amplitudes interfere with those in which the active parton interacts with the sole photon, giving rise to ”dynamic” twist-3 contributions[25,26] (see also refs. [27,28]). In our approach these contributions are conveniently calculated in the light cone gauge. The terms of type *a)*, which may be also deduced from the equations of motion[25], result in the distribution $\delta g_T^f(x, \mathbf{p}_\perp)$[25,15], whose integral over the transverse momentum is $g_T^f(x)$. Calculations[26] within the model proposed by Qiu and Sterman[31] assure that such contributions are about 10% of the parton model term.

(iii) Formulae (21) to (25) and the first eq. (29) hold true independent of the orientation of the proton spin. However, if $\mathbf{S}$ is not oriented perpendicularly to the lepton beam, $\lambda_\parallel$ does not decrease with Q. Therefore if, *e. g.*, the proton is polarized longitudinally, the asymmetry has still a contribution sensitive to the t.m.d. tranversity function, which, however, risks to be masked by the JJ term[22]. We shall see in the next section a method for extracting $\delta q_\perp^f$ under such unfavourable conditions, as, *e. g.*, in the HERMES experiment[12].

(iv) The hadronic tensor (18), which we have derived starting from the definition of transversity by JJ1, turns out to coincide with the tensor found by KM, provided we assume $g_{1T}^f \propto \delta q_\perp^f$, with an undetermined proportionality constant. In the appendix we prove this relation, showing that the constant may be chosen so as to identify g_{1T}^f with the t.m.d. transversity function for massless quarks. This result may be read as follows. Owing to the intrinsic transverse momentum, a quark in a transversely polarized proton has a nonvanishing longitudinal polarization, related to $\delta q_\perp^f$ according to the definition by JJ1, but also described by g_{1T}^f according to the parametrizations by MT and by Ralston and Soper[1]. As a consequence of this result, all KM’s considerations and deductions on g_{1T}^f - like, *e. g.*, its relation with g_2^f - can be applied to $\delta q_\perp^f$. The above identification implies that, although the transversity is a typical chiral-odd distribution, the t.m.d. transversity function has a chiral-even component, coincident with g_{1T}^f, owing to the non-collinearity of the quark with respect to the proton momentum. It is just this chiral-even component that appears in formula (25). Therefore, according to chirality conservation, our asymmetry formula (21) - unlike those previously considered for determining the transversity function[4,7,19,20] - does not contain any chiral-odd (and therefore unusual) distribution or fragmentation functions. Indeed, the t.m.d. functions q^f and φ^f, involved in the asymmetry, can be parametrized in a well defined way. Incidentally, in the appendix we establish other useful connections among the transverse momentum dependent distribution functions defined by MT (see also ref.[37]).

4 Extracting transversity from data

In this section we discuss how to extract $\delta q_\perp^f$ from data. To this end we may recur either to a best fit with a suitable parametrization or to the method of the weighted asymmetries[21, 18]. Here we examine both options.

4.1 Gaussian parametrization

A frequently used parametrization of the t.m.d. unpolarized distributions and fragmentation functions consists of[15,23,32]

$$q^f(x, \mathbf{p}_\perp^2) = (a/\pi) q^f(x) exp(-a\mathbf{p}_\perp^2), \tag{32}$$

$$\varphi^f(z, \mathbf{P}_\perp^2) = (a_\pi/\pi) D^f(z) exp(-a_\pi \mathbf{P}_\perp^2). \tag{33}$$

Here $q^f(x)$ is the usual unpolarized distribution function and $a \sim 0.53\ (GeV/c)^{-2}$, as results from Drell-Yan[38]. a_π may be determined from observation of two-jet events in $e^+e^- \to \pi X$. As regards the t.m.d. transversity, the transverse spin induces an anisotropy around the direction of the proton momentum, so that it looks appropriate to set

$$\delta q_\perp^f(x, \mathbf{p}_\perp) = (\sqrt{ab}/\pi) h_1^f(x) exp(-ap_1^2 - bp_2^2). \tag{34}$$

Here

$$p_1 = \mathbf{p}_\perp \cdot \mathbf{s} \times \mathbf{t}, \qquad p_2 = \mathbf{p}_\perp \cdot \mathbf{s}, \tag{35}$$

where $\mathbf{s} = \mathbf{S}_\perp/|\mathbf{S}_\perp|$ and $\mathbf{t} = \mathbf{q}/|\mathbf{q}|$. h_1^f may be parametrized according to the suggestion of ref.[39], *i. e.*,

$$h_1^f(x) = \frac{1}{2} N \frac{x^\alpha (1-x)^\beta}{\alpha^\alpha \beta^\beta} (\alpha+\beta)^{\alpha+\beta} \left[q(x) + \Delta q(x)\right], \tag{36}$$

where N, α, β and b are free parameters, with $|N| \leq 1$.

4.2 Weighted asymmetries

A weighted asymmetry is defined as[21,18]

$$\langle A_w \rangle = \frac{\int W(\mathbf{\Pi}_\perp) d\sigma}{\int d\sigma}. \tag{37}$$

Here W is a given weight function of the transverse momentum $\mathbf{\Pi}_\perp$, over which we make the integrations indicated in eq. (37). We present two kinds of weight functions, suitable for extracting the t.m.d. transversity function from the asymmetry we have proposed.

4.2.1 The KM weight function

For the reaction we are studying, KM have proposed $W(\mathbf{\Pi}_\perp) = 2\mathbf{\Pi}_\perp \cdot \mathbf{S}/M$, where M is the proton rest mass. Then, according to the formulas of sect. 2, we get

$$\langle A_w \rangle = \mathcal{F}\frac{\sum_{f=1}^{6} e_f^2 h_1^{f(1)}(x) D^f(z)}{\sum_{f=1}^{6} e_f^2 q^f(x) D^f(z)}, \tag{38}$$

where $h_1^{f(1)}(x) = 2/QM \int d^2p_\perp(\mathbf{p}_\perp \cdot \mathbf{S})^2 \delta q_\perp^f$. This quantity is quite analogous to the one defined by KM, *i. e.*,

$$g_{1T}^{f(1)}(x) = \frac{1}{2M^2}\int d^2p_\perp \mathbf{p}_\perp^2 g_{1T}^f. \tag{39}$$

KM determine $\sum_{f=1}^{6} e_f^2 g_{1T}^{f(1)}(x)$ (see fig. 2 of that paper). But in appendix we show that, according to the normalization adopted by KM, $g_{1T}^f = 2M/Q\ \delta q_\perp^f$. We substitute this equality into eq. (39) and assume the parametrization (34) for $\delta q_\perp^f$, with $b = a$. Taking into account the KM deduction, we get an approximate evaluation of h_1 for a single quark. The behavior is similar to the bag model prediction (see, *e. g.*, JJ1), especially at small x, and the accord between the two calculations can be made quantitative for Q of order 12 to 15 GeV.

4.2.2 Harmonic oscillator weight functions

More refined information on the t.m.d. transversity function could be extracted from data by using a different set of weight functions, inspired to the Gaussian parametrization. We expand φ^f and $\delta q_\perp^f$ as a series of eigenfunctions of the harmonic oscillator, *i. e.*,

$$\varphi^f(z, \mathbf{P}_\perp^2) = \sum_K c_K^f(z)\Phi_K(P), \qquad \delta q_\perp^f(x, \mathbf{p}_\perp) = \sum_K \tilde{c}_K^f(x)\tilde{\Phi}_K(\tilde{p}). \tag{40}$$

Here $P = (\mathbf{P}_\perp^2)^{1/2}$ and $\tilde{p} = \sqrt{b/ap_1^2 + p_2^2}$, where p_1 and p_2 are given by eqs. (35). The $\Phi_K(P)$ are the eigenfunctions of the equation

$$\left[a_\pi^2 P^2 + \frac{1}{4}\xi^2 - a_\pi(K + \frac{1}{2})\right]\Phi_K(P) = 0, \tag{41}$$

where ξ is the variable conjugate to P. The $\tilde{\Phi}_K$ fulfil an analogous equation, with a instead of a_π. These functions have been chosen in such a way that the terms with $K = 0$ reproduce the parametrizations of subsection 4.1. $c_K^f(z)$ and $\tilde{c}_K^f(x)$ are real coefficients. The set of weight functions we propose is

$$W_K = 2\frac{\mathbf{\Pi}_\perp \cdot \mathbf{S}}{M}\Phi_K(P). \tag{42}$$

Denoting by $\langle A_w^K \rangle$ the quantity obtained by substituting eq. (42) into (37), eq. (38) implies

$$\langle A_w^K \rangle \sum_{f=1}^{6} e_f^2 q^f(x) D^f(z) = \mathcal{F} \sum_{f=1}^{6} \sum_L e_f^2 M_{KL}^f(x,z) \tilde{c}_L^f(x), \tag{43}$$

where

$$M_{KL}^f(x,z) = \frac{2}{M} \int d^2 P_\perp \Phi_K(P) \varphi(z, P^2) \int d^2 p_\perp p_1^2 \tilde{\Phi}_L(\tilde{p}). \tag{44}$$

The linear system (43) can be solved with respect to $\tilde{c}_L^f(x)$, provided we assume dominance of some fragmentation mechanism[20], which reduces the sum over f to a single term. Alternatively, if data relative to asymmetry for $\pi^\pm$ and to K-mesons are simultaneously available, the system (43) can be written as

$$\langle A_w^{KF} \rangle \sum_{f=1}^{6} e_f^2 q^f(x) D_F^f(z) = \mathcal{F} \sum_{f=1}^{6} \sum_L e_f^2 M_{KL}^{fF}(x,z) \tilde{c}_L^f(x), \tag{45}$$

where F runs over the final hadrons π^+, π^- and K. This new system can be solved if we make some assumptions, so as to reduce the sum over f to three or less terms. For example, we may consider separately small and large x. In the latter region we may neglect sea contribution and the system is overdetermined, since f runs over two flavors. For small x, where the sea prevails, we may solve the system by assuming a relation between quark and antiquark distributions, in such a way that f runs over three flavors. The parameter b may be determined so as to minimize, *e. g.*, $\sum_{L>0} |\tilde{c}_L^f(x)|$.

The method of weighted cross sections washes out the unwanted JJ contribution, therefore it is particularly suitable in an experiment, like HERMES[12], where the target is longitudinally polarized.

5 Results and conclusion

Now we calculate the order of magnitude of the asymmetry (21) under optimal conditions. To this end, first of all, according to the considerations of subsection 2.2, we take into account events such that the azimuthal angle ϕ (see the second eq. (29)) is about $\pi/2$, and such that $\mathbf{\Pi}_\perp$ is parallel (or antiparallel) to $\mathbf{S}$. Moreover, eqs. (23) to (25) and the first eq. (29) suggest that y and z should be chosen as close as possible to 1. As regards the functions involved in our asymmetry, we assume the parametrizations (32) to (34), with $a_\pi = b = a$ for the sake of simplicity. Lastly we set $|\mathbf{\Pi}_\perp| \simeq 1\ GeV$ and $Q = 2.5\ GeV$. With such inputs, the asymmetry (21) results in $A \sim 0.4R$, where $R = h_1^f(x)/q^f(x)$ has been determined by HERMES[12], $|R| = (50 \pm 30)\%$.

To conclude, we have shown (see appendix) that the t.m.d. distribution function g_{1T} by RS and MT turns out to coincide with the t.m.d. transversity function, $\delta q_\perp$, in the chiral limit. Therefore $\delta q_\perp$ may be coupled to a chiral-even fragmentation function, in particular to the twist-two, unpolarized t.m.d. fragmentation function. This result suggests an alternative, convenient method for extracting h_1, circumventing the usual drawbacks that plague determination of transversity. Specifically, we propose to measure the the double spin azimuthal asymmetry in pion semi-inclusive leptoproduction. For reasonable values of Q^2 (4 to 10 GeV^2), and under the most favourable kinematic conditions, the order of magnitude of the asymmetry is estimated to be at least $\sim 10\%$. The suggested experiment could be performed at facilities like CERN (COMPASS coll.) and DESY (HERMES coll.), where similar asymmetry measurements are being realized or planned. As a last comment, our results confirm the crucial role, on the one hand, of the intrinsic transverse momentum[15,16,35,29] and, on the other hand, of polarized SIDIS experiments[42,43], in extracting quark distribution functions.

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Appendix

Here we establish some relationships among transverse momentum dependent (t.m.d.) distribution functions, for which we adopt the notations by Mulders and Tangerman[15] (MT) (see also Ralston and Soper[1] (RS) and other authors[16,21]). In particular we show that, in the chiral limit, g_{1T} equals h_{1T} ($\delta q_\perp$ in the present paper). First of all, we write the quark correlation matrix in the QCD parton model. Secondly we compare it with the general expression of the correlation matrix. As a result we find some relationships among t.m.d. distribution functions, which turn out to hold true even taking into account quark-gluon interactions and renormalization effects. Lastly, we give an alternative proof of the relationship between g_{1T} and h_{1T} in the chiral limit.

A.1 - Correlation matrix in QCD parton model

A.1.1 - The correlation matrix

We define the correlation matrix as

$$\Phi(x, p_\perp; P, S) = \int dp^- \int \frac{d^4y}{(2\pi)^4} e^{ipy} \langle P, S|\psi(y)\bar{\psi}(0)|P, S\rangle, \tag{A.1}$$

assuming the light cone gauge $A^+ = 0$. Here P and S are, respectively, the four-momentum and the Pauli-Lubanski (PL) four-vector of the proton, with $S^2 = -1$; ψ is the quark field. p is the quark four-momentum, whose transverse component with respect to the proton momentum is $p_\perp$, *i. e.*,

$$p = p^+ n_+ + p_\perp + p^- n_-, \qquad P = P^+ n_+ + P^- n_-, \tag{A.2}$$

$$n_+^2 = n_-^2 = 0, \qquad n_+ \cdot n_- = 1. \tag{A.3}$$

Moreover $x = p^+/P^+$ is the light cone fraction of the quark momentum.

We take a frame such that the proton has a large momentum, $\mathcal{P}$, much greater than its rest mass M, and it is transversely polarized. Moreover we choose the z-axis along the proton momentum and the y-axis along the proton polarization, so that

$$P \equiv (\sqrt{M^2 + \mathcal{P}^2}, 0, 0, \mathcal{P}), \qquad S \equiv (0, 0, 1, 0), \tag{A.4}$$

$$n_\pm \equiv \frac{1}{\sqrt{2}}(1, 0, 0, \pm 1), \qquad p_\perp \equiv (0, \mathbf{p}_\perp), \tag{A.5}$$

$$\mathbf{p}_\perp \equiv (p_1, p_2, 0), \qquad |\mathbf{p}_\perp| = O(M). \tag{A.6}$$

From now on this frame will be called $\mathcal{P}$-frame. For the sake of simplicity, we exclude T-odd terms in the correlation matrix (A.1).

A.1.2 - QCD-improved parton model

The correlation matrix for free, on-shell[23] quarks in a transversely polarized proton reads

$$\Phi_\perp^{free} = \sum_{T=\pm 1/2} q_T(x, \mathbf{p}_\perp) \frac{1}{2} (\not{p} + m)(1 + 2T\gamma_5 \not{S}_q). \tag{A.7}$$

Here m and p are, respectively, the rest mass and the four-momentum of the quark, such that $p^2 = m^2$. $2TS_q$ is the *quark* PL vector, with $S_q^2 = -1$. $q_T(x, \mathbf{p}_\perp)$ is the probability density of finding a quark with its spin aligned with (T = 1/2) or opposite to (T = -1/2) the proton spin.

In the *quark* rest frame - named q-frame from now on - we have $S_q = S_q^{(0)} = S \equiv (0, 0, 1, 0)$[35], taking the axes of the q-frame parallel to those of the $\mathcal{P}$-frame. But the quark has a nonzero momentum with respect to the proton: choosing a frame at rest with respect to the proton, whose axes are parallel to those of the $\mathcal{P}$-frame, we get

$$\mathbf{p}_r \equiv (p_1, p_2, p_3), \qquad p_3 = O(M). \tag{A.8}$$

In the q-frame, we decompose $S_q = S_q^{(0)} = S$ into a transverse and a longitudinal component with respect to the quark momentum, *i. e.*,

$$S_q^{(0)} = S = \Sigma_\perp cos\theta' + \nu sin\theta'. \tag{A.9}$$

Here

$$sin\theta' = sin\theta sin\phi, \qquad sin\theta = \frac{|\mathbf{p}_\perp|}{|\mathbf{p}|}, \qquad sin\phi = \frac{-p_\perp \cdot S}{|\mathbf{p}_\perp|}, \tag{A.10}$$

$$\begin{aligned}
\mathbf{p} &\equiv (\mathbf{p}_\perp, x\mathcal{P}), \qquad \nu \equiv (0, \mathbf{t}), \qquad \Sigma_\perp \equiv (0, \mathbf{n}), && \text{(A.11)}\\
\mathbf{t} &\equiv (sin\theta cos\phi, sin\theta sin\phi, cos\theta), && \text{(A.12)}\\
\mathbf{n} &\equiv (cos\theta cos\phi, cos\theta sin\phi, -sin\theta). && \text{(A.13)}
\end{aligned}$$

In order to calculate S_q in the $\mathcal{P}$-frame, we perform a boost along the quark momentum. This boost leaves $\Sigma_\perp$ invariant and transforms ν into $\tilde{p}/m$, where

$$\tilde{p} \equiv (|\mathbf{p}|, E_q\mathbf{t}), \qquad E_q = \sqrt{m^2 + \mathbf{p}^2}. \tag{A.14}$$

As a result we get

$$S_q = S + \left[\frac{p}{m} - (\delta + \nu)\right] sin\theta'. \tag{A.15}$$

Here we have defined

$$\delta = \frac{m}{\sqrt{2}x\mathcal{P}} n'_- \left[1 + O(\mathcal{P}^{-2})\right], \qquad n'_\pm \equiv \frac{1}{\sqrt{2}}(1, \pm\mathbf{t}). \tag{A.16}$$

We stress that it is essential that the relative momentum of the quark with respect to the proton - see eq. (A.8) - is nonzero; otherwise x would be fixed and equal to m/M[35]. Substituting eq. (A.15) into eq. (A.7), and taking into account the definitions (A.11) and (A.16) of ν and δ, we get

$$\begin{aligned}
\Phi_\perp^{free} &= \frac{1}{2} q(x, \mathbf{p}_\perp^2)(\not{p} + m)\\
&+ \frac{1}{2}\delta q_\perp(x, \mathbf{p}_\perp)\gamma_5 \left\{\frac{1}{2}[\not{S}, \not{p}] + \not{p} sin\theta' - A + mB\right\} + O(\mathcal{P}^{-1}).
\end{aligned} \tag{A.17}$$

Here we have set

$$q(x, \mathbf{p}_\perp) = \sum_{T=\pm 1/2} q_T(x, \mathbf{p}_\perp^2) \qquad \delta q_\perp(x, \mathbf{p}_\perp) = \sum_{T=\pm 1/2} 2T q_T(x, \mathbf{p}_\perp), \tag{A.18}$$

$$A = E_q \frac{1}{2}[\not{n}'_+, \not{n}'_-] sin\theta', \tag{A.19}$$

$$B = \not{S} + \frac{1}{\sqrt{2}}\left\{\not{n}'_-\left(1 - \frac{m}{|\mathbf{p}|}\right) - \not{n}'_+ + \frac{1}{\sqrt{2}}[\not{n}'_+, \not{n}'_-]\right\} sin\theta'. \tag{A.20}$$

Moreover we have exploited the relations $-\not{p}\not{S} = 1/2[\not{S},\not{p}] - p\cdot S$, $p\cdot S = p_\perp \cdot S$ and $\nu = \frac{1}{\sqrt{2}}(n'_+ - n'_-)$.

A.2 - Parametrization of the correlation matrix

A.2.1 - The general formula

Now we parametrize the correlation matrix of a transversely polarized proton in the $\mathcal{P}$-frame. We normalize this matrix in such a way that, at the leading twist, in the case of an unpolarized proton, integrating over the quark tranverse momentum, it coincides with the usual quark density matrix times the unpolarized quark density. Taking into account the operators of the Dirac algebra[23,15,24], we get

$$\Phi_\perp = \Phi_{0a} + \Phi_{0b} + \Phi_1 + \Phi_2. \tag{A.21}$$

Here

$$\begin{aligned} \Phi_{0a} &= \frac{1}{\sqrt{2}} x\mathcal{P}\left(f_1 \not{n}_+ + \lambda_\perp g_{1T}\gamma_5 \not{n}_+ + \frac{1}{2} h_{1T}\gamma_5[\not{S},\not{n}_+]\right) \\ &+ \frac{1}{4\sqrt{2}}\lambda_\perp h_{1T}^\perp \gamma_5[\not{p}_\perp, \not{n}_+], \end{aligned} \tag{A.22}$$

$$\begin{aligned} \Phi_{0b} &= \frac{1}{2}\left(f_1^\perp + \lambda_\perp g_T^\perp \gamma_5\right)\not{p}_\perp \\ &+ \frac{1}{4}\lambda_\perp\left(h_T^\perp \gamma_5[\not{S},\not{p}_\perp] + h_T \mu\gamma_5[\not{n}_-,\not{n}_+]\right), \end{aligned} \tag{A.23}$$

$$\Phi_1 = \frac{1}{2} M\left(e + g_T\gamma_5\not{S}\right), \tag{A.24}$$

while Φ_2 contains terms of $O(\mathcal{P}^{-1})$. We have set

$$\lambda_\perp = -S\cdot p_\perp/\mu \tag{A.25}$$

and μ is an arbitrary constant, which we shall fix below. Moreover the distributions involved, for which we have used the notations of MT, are functions of the Bjorken variable x and of the intrinsic transverse momentum $\mathbf{p}_\perp$. The term

$$\Phi_0 = \Phi_{0a} + \Phi_{0b} \tag{A.26}$$

is interaction independent. This is evident for Φ_{0a}, which consists of twist-2 operators. We shall show that also Φ_{0b} shares this feature, although the operators involved are classified as ”twist-3”.

A.2.2 - Comparison with the QCD parton model

We equate the coefficients of the independent Dirac operators in eqs. (A.17) and (A.21), taking into account the first eq. (A.2), which reads, in the $\mathcal{P}$-frame,

$$p = \sqrt{2}x\mathcal{P}n_+ + p_\perp + O\left(\mathcal{P}^{-1}\right). \tag{A.27}$$

We get

$$\begin{aligned}
f_1 &= f_1^\perp = q, && \text{(A.28)}\\
\lambda_\perp h_T^\perp &= sin\theta'\delta q_\perp, && \text{(A.29)}\\
\lambda_\perp h_{1T}^\perp &= (1-\epsilon_1)sin\theta'\delta q_\perp, && \text{(A.30)}\\
\mu\lambda_\perp h_T &= (1-\epsilon_1)sin\theta' E_q\delta q_\perp. && \text{(A.31)}\\
\lambda_\perp g_{1T} &= (1-\epsilon_2)sin\theta'\delta q_\perp, && \text{(A.32)}\\
\lambda_\perp g_T^\perp &= (1-\epsilon_3)sin\theta'\delta q_\perp. && \text{(A.33)}
\end{aligned}$$

Here $\epsilon_1 = m/E_q$, $\epsilon_2 = m/2x\mathcal{P}$ and $\epsilon_3 = m/2|\mathbf{p}|$ are the correction terms to the chiral limit, which are generally small for light quarks. The terms of order $O\left[(m^2 + \mathbf{p}_\perp^2)/\mathcal{P}^2\right]$ have been neglected. As regards μ, RS and MT have set it equal to M. We require the various functions to be normalized, in the chiral limit, as $\delta q_\perp$, which is a difference of two probability densities. Therefore we assume

$$\lambda_\perp = sin\theta', \tag{A.34}$$

which implies $\mu = |\mathbf{p}|$. In particular, this choice leads to the relationship

$$g_{1T} = (1-\epsilon_2)h_{1T}. \tag{A.35}$$

In the chiral limit, $sin\theta' h_{1T}$ = $sin\theta' g_{1T}$ is the average helicity of a quark in a transversely polarized proton.

A.2.3 - Equations of motion

Owing to the Politzer theorem[34], the Fourier transform of $\Phi_\perp$ must fulfil the equation of motion (e.o.m.) for a Dirac particle interacting with the gluon field. We set $\Phi_\perp$ = $\Phi_\perp^{free} + \Phi_\perp^{int}$. Since the term $\Phi_\perp^{free}$ (eq. (A.7)) fulfils the Dirac equation for a plane wave, the e.o.m. implies that $\Phi_\perp^{int}$ depends on the quark-gluon interaction and is of order $g\mathcal{P}^{-1}$[25,44], g being the strong interaction coupling constant. But $\Phi_\perp^{free}$ includes the term Φ_0 (eq. (A.26)), with the constraints (A.28)-(A.33). On the other hand, $\Phi_\perp - \Phi_0$ includes the interaction dependent term $\Phi_\perp^{int}$ and is orthogonal to Φ_0: $Tr\left[\Phi_0(\Phi_\perp - \Phi_0)\right]$

= 0. Therefore Φ_0 is interaction independent and the relationships (A.28) to (A.33), although deduced from the naive parton model[23], hold true even after inserting interactions. In particular the term Φ_{0b}, although made up with twist-3 operators, is interaction independent and should be classified as a "kinematic" higher twist term.

The Politzer theorem survives renormalization and off-shell effects[34]. Therefore relationships (A.28) to (A.33) have a quite general validity; in particular they hold also when QCD evolution is taken into account. In the next subsection we shall give an alternative proof of this fact as regards relationship (A.35) in the chiral limit.

A.3 - Relationship between h_{1T} and g_{1T}

A.3.1 - Proof

We consider the projection

$$x\mathcal{P}\lambda_\perp g_{1T} = -\frac{1}{2}Tr\left[\Phi\gamma_5 \not{n}_-\right] = -\frac{1}{2}\mathcal{H}\left[\langle P,S|\bar{\psi}(0)\gamma_5\not{n}_-\psi(y)|P,S\rangle\right]. \tag{A.36}$$

Here we have defined the functional

$$\mathcal{H}\left[\rho(y)\right] = \int dp^- \int \frac{d^4y}{(2\pi)^4}e^{ipy}\rho(y). \tag{A.37}$$

On the other hand,

$$x\mathcal{P}h_{1T} = \frac{1}{4}Tr\left\{\Phi_0\gamma_5[\not{S},\not{n}_-]\right\} = \frac{1}{4}\mathcal{H}\left[\langle P,S|\bar{\psi}(0)\gamma_5[\not{S},\not{n}_-]\psi(y)|P,S\rangle\right]. \tag{A.38}$$

Now we decompose the quark field into the eigenstates of the canonical representation where the quantization axis is taken along the proton polarization:

$$\psi(y) = \psi'_\uparrow(y) + \psi'_\downarrow(y) + \psi''(y). \tag{A.39}$$

Here ψ' and ψ'' denote respectively the "good" and "bad" component of the quark field, *i. e.*,

$$\psi'_{\uparrow(\downarrow)}(y) = \frac{1}{4}(1\pm\gamma_5\not{S})\not{n}_+\not{n}_-\psi(y), \qquad \psi''(y) = \frac{1}{2}\not{n}_-\not{n}_+\psi(y). \tag{A.40}$$

The field may also be decomposed into chirality eigenstates, *i. e.*,

$$\psi(y) = \psi'_R(y) + \psi'_L(y) + \psi''(y), \qquad \psi'_{R(L)}(y) = \frac{1}{4}(1\pm\gamma_5)\not{n}_+\not{n}_-\psi(y). \tag{A.41}$$

Then

$$\bar{\psi}(0)\gamma_5\not{n}_-\psi(y) = -\frac{1}{\sqrt{2}}\left[\psi'^\dagger_R(0)\psi'_R(y) - \psi'^\dagger_L(0)\psi'_L(y)\right], \tag{A.42}$$

$$\frac{1}{2}\bar{\psi}(0)\gamma_5[\not{S},\not{n}_-]\psi(y) = +\frac{1}{\sqrt{2}}\left[\psi'^\dagger_\uparrow(0)\psi'_\uparrow(y) - \psi'^\dagger_\downarrow(0)\psi'_\downarrow(y)\right]. \tag{A.43}$$

Substituting eqs. (A.42) and (A.43) respectively into (A.36) and (A.38), we get

$$\lambda_\perp g_{1T}(x, p_\perp) = q_R^{\Uparrow}(x, p_\perp) - q_L^{\Uparrow}(x, p_\perp), \tag{A.44}$$
$$h_{1T}(x, p_\perp) = q_\uparrow^{\Uparrow}(x, p_\perp) - q_\downarrow^{\Uparrow}(x, p_\perp). \tag{A.45}$$

Here we have set

$$q_R^{\Uparrow} = \frac{1}{2\sqrt{2}x\mathcal{P}}\mathcal{H}\left[\langle P, S|\psi_R'^{\dagger}(0)\psi_R'(y)|P, S\rangle\right], \tag{A.46}$$
$$q_\uparrow^{\Uparrow} = \frac{1}{2\sqrt{2}x\mathcal{P}}\mathcal{H}\left[\langle P, S|\psi_\uparrow'^{\dagger}(0)\psi_\uparrow'(y)|P, S\rangle\right] \tag{A.47}$$

and we have defined analogously $q_L^{\Uparrow}$ and $q_\downarrow^{\Uparrow}$. From eqs. (A.46) and (A.47) it results that $q_{R(L)}^{\Uparrow}$ is the probability of finding the quark in a chirality state, whereas $q_{\uparrow(\downarrow)}^{\Uparrow}$ is the probability for a quark to be in a state of the canonical represention defined above. In this representation the spin density matrix of a quark in a transversely polarized proton reads

$$\rho = q_\uparrow^{\Uparrow}|\uparrow\rangle\langle\uparrow| + q_\downarrow^{\Uparrow}|\downarrow\rangle\langle\downarrow|. \tag{A.48}$$

On the other hand, the helicity operator can be written as

$$\Lambda = |+\rangle\langle+| - |-\rangle\langle-|, \tag{A.49}$$

where $|\pm\rangle$ denote the helicity states. Therefore the average helicity of a quark in a transversely polarized proton results in

$$\begin{aligned}\langle\lambda_{\Uparrow}\rangle &= Tr(\rho\Lambda) = q_\uparrow^{\Uparrow}\left[|\langle+|\uparrow\rangle|^2 - |\langle-|\uparrow\rangle|^2\right] + q_\downarrow^{\Uparrow}\left[|\langle+|\downarrow\rangle|^2 - |\langle-|\downarrow\rangle|^2\right] \\ &= \left(q_\uparrow^{\Uparrow} - q_\downarrow^{\Uparrow}\right) sin\theta' = h_{1T} sin\theta'.\end{aligned} \tag{A.50}$$

But $\langle\lambda_{\Uparrow}\rangle$ equals $q_R^{\Uparrow} - q_L^{\Uparrow}$ in the chiral limit. Therefore eqs. (A.50), (A.46) and (A.47) imply

$$\mathcal{H}\left[\langle P, S|\bar{\psi}(0)O\psi(y)|P, S\rangle\right] = 0 \qquad \text{for} \qquad m = 0, \tag{A.51}$$

having set

$$O = \gamma_5\left\{\not{n}_- - \frac{1}{2}sin\theta'[\not{S}, \not{n}_-]\right\}. \tag{A.52}$$

Therefore, in the chiral limit, relationship (A.35) holds true independent of renormalization and off-shell effects.

A.3.2 - Discussion

The result we have just found appears in contrast with the behavior of the ordinary distribution functions, that is, those integrated over the transverse momentum. Indeed,

$g_1(x)$, a chiral even function, is independent of $h_1(x)$, which is chiral odd. Therefore a measure for determining $g_1(x)$ automatically excludes the possibility of inferring $h_1(x)$ and *vice-versa*; moreover $g_1(x)$ is coupled to a polarized gluon distribution, whereas $h_1(x)$ is not.

But transverse momentum attenuates these differences. A quark polarized perpendicularly to the proton momentum has a nonzero helicity if its transverse momentum is different from zero. And also the converse is true: a longitudinally polarized quark contributes to the proton transversity if it has a nonvanishing $\mathbf{p}_\perp$. Quantitatively, in a transversely polarized proton, a massless quark, whose average spin component along the proton spin is $h_{1T}(x, \mathbf{p}_\perp)$, has an average helicity $sin\theta' h_{1T}(x, \mathbf{p}_\perp)$, which implies $h_{1T}(x, \mathbf{p}_\perp) = g_{1T}(x, \mathbf{p}_\perp)$ in the chiral limit. The fact that h_{1T} is associated to a chiral odd operator and g_{1T} to a chiral even one simply means that the same function can be deduced from different types of experiment: either from single polarization, exploiting the Collins effect[14], or from double polarization, as suggested in the present paper and by Kotzinian and Mulders[21]. The effects of transverse momentum imply also that, in the QCD evolution equations, h_{1T} is coupled to the t.m.d. gluon transverse polarization, $\delta g_\perp$. In fact, in a transversely polarized proton, gluons with nonzero transverse momentum have a nonvanishing transverse and longitudinal polarization, as well as quarks; therefore they give rise to longitudinally polarized quark-antiquark pairs, which, in turn, owing to transverse momentum, contribute to the t.m.d. tranversity function h_{1T}. The coupling between $\delta g_\perp$ and h_{1T} is washed out by integration over transverse momentum.

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