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MUFASA: MUon FAst Simulation Algorithm

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Abstract

This note describes the simulation code named MUFASA (MUon FAst Simulation Algorithm), developed to study the muon pair production due to the interaction of a positron beam with a target using a custom Monte Carlo, while also providing an interface with the particle tracking code MADX_PTC. MUFASA is a powerful tool developed as a *start-to-end* simulation code for the study of the muon production and accumulation process for the Low EMittance Muon Accelerator (LEMMA) [1], in order to test different targets, optics and configurations. In this paper the structure of the code is described and validation tests performed with GEANT4 [2] together with some example of results are shown.

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1 Description of the code

The main purpose of MUFASA is to perform a fast simulation of processes involving a stored beam interacting several times with one or more targets placed along the optics, studying the evolution of the beam dynamics through the targets and the lattice. While similar programs already exist (e.g. MDIsim [3]), in order to have a fast response the interaction of the particles with the target is performed with a custom Monte Carlo algorithm in ROOT [4] including only the most relevant cross-sections. The transport of the beam through the lattice is performed using the particle tracking code MADX_PTC [5][6]. After the accumulation process, all the informations about both the alive and lost particles are then saved in a `.root` file for the offline analysis.

In LEMMA muons are produced via annihilation of a positron beam on atomic electrons of a target. Due to the low production cross-section $\sigma_{e^+e^-\to\mu^+\mu^-}$, two rings are necessary in order to accumulate the muons over several iterations of positron bunches impinging on the target. Muons are recirculated and arrive back to the target together with a new positron bunch, so that the new muons get produced in the same phase space of the accumulated bunch. Moreover, the target can be split in several fractions each separated by drift spaces or transport lines common to the three beams (e^+, μ^+ and μ^-). A schematic of the muon production and accumulation setup is shown in figure 1, and a more accurate description of the LEMMA production scheme can be found in [7].

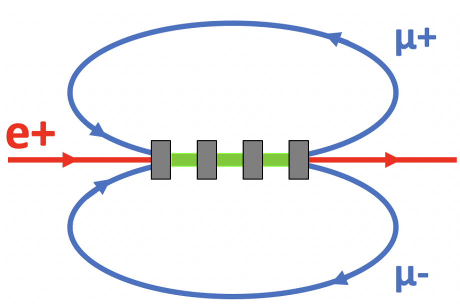

Figure 1: Concept of the muon production and accumulation process in LEMMA. In red the incoming positron beam, in blue the muon accumulator rings, in grey the targets and in green the transport lines (in common for the three beams e^+, μ^+ and μ^-).

From the conceptual scheme shown in figure 1 it can be inferred that the key parameters for the simulation are: target, impinging positron beam and the optics.

The target The target is characterised by 3 main properties: material, length and width. The material can be chosen among the ones currently embedded (liquid H_2, liquid He, liquid Li, Be, C), but a complete library can be easily implemented in the future. Custom materials can also be easily added. For each material the radiation length X_0 $[g\ cm^{-2}]$, density ρ $[g\ cm^{-3}]$, atomic weight and number A and Z, and the muon average energy loss dE/dx $[GeV/m]$ have been taken from the PDG Atomic and Nuclear Properties table [9].

As an example, Beryllium is described in the code with the following string:

```
// material[] = {X0, rho, A, Z, dE/dx};
double beryllium[] = {65.19,1.848,9.01218,4,0.2947};
```

About the geometry of the target, the length (expressed in meters) is the longitudinal dimension, while the width (also expressed in meters) is the transverse section of the target and it is referred to the x-axis, but it can also be changed according to the design of the machine. A sketch of the target is shown in figure 2.

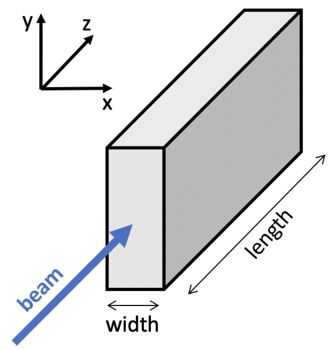

Figure 2: Sketch of the simulated target.

Description of the beams In order to reduce the CPU time, a small number of macro-particles `N_PARTICLES` is simulated, and the results are then weighted to the total number of particles in the bunch `N_BUNCH`.

The input positron beam is produced by a gaussian generator whose inputs are:

- `N_PARTICLES`
- nominal energy `E` $[GeV]$
- energy spread
- bunch length
- beam size and divergence (or emittance and Twiss parameters at the IP)

Each positron macro-particle is characterised by a sequential ID number `partID` and the 6D coordinates. Muons in addition have the weight `w` of the production reaction (explained in section 1.1), the longitudinal position in the target where it was produced `z` and the number of turns it has performed in the accumulator `n_cycle`, both used in the evaluation of the muon decay probability, described in section 1.1.2. Other additional variables can also be stored.

Optics for beam transport Particle tracking in the accumulator ring or in any transport line is performed using MADX_PTC. The modular structure of MUFASA allows an easy way to study how different optics behave with the insertion of a target; indeed it is sufficient to have the sequence file generated by MADX in the dedicated folder. The starting point of the sequence must be the Interaction Point (IP), which corresponds to the center of the target. The only change that must to be done on the sequence before launching the simulation is to "cut" a piece of the drift in the IP region of the same length of the target, in order not to track that length twice at each turn. This process will be rendered automatic in a future upgrade of the code.

In order to interface MADX_PTC with the rest of the code, several conversions between the inputs/outputs are needed and are performed using simple `C++` scripts. In MUFASA the string containing informations of a particle (for instance a muon) is written using the following template:

`partID n_cycle x px y py ct dE_E E w z`

while the MADX_PTC input string for tracking a particle - for example - with all 6D coordinates equal to zero is:

`ptc_start, x=0, px=0, y=0, py=0, t=0, pt=0;`

where `t` and `pt` correspond to the variables `ct` and `dE_E` in MUFASA. Due to the fact that informations other than the 6D coordinates of the particles cannot be stored in the MADX_PTC input file, it is necessary to write them to a temporary file and then merge them back with the 6D coordinates of the particle after the tracking. This is possible because MADX_PTC also internally assigns a sequential ID number to each particle; therefore after the tracking the output (containing only the particles that survived) and the temporary file are checked for particles with the same ID and the informations are merged back in the MUFASA template showed before. Lost particles have their information saved in a different output containing the 6D coordinate and where the particle was lost along the optics; for these particles the informations saved in the temporary file are not merged back as they are not relevant, but such feature could be added. A flowchart of this process is shown in figure 3. Particles which survived the tracking will be fed back into the code.

1.1 Interaction with the target

When a particle goes through a piece of material a wide range of different interactions can happen. MUFASA uses a Monte Carlo algorithm that generates events with a distribution that matches the actual cross-section. For both positrons and muons the tracking is performed particle by particle, and the target divided in 100 steps of equal length. At each step the particle undergoes Multiple Coulomb Scattering and energy loss, evaluated as described later. On the other hand, muon production and decay occour only once.

In this section the interactions included in MUFASA are described.

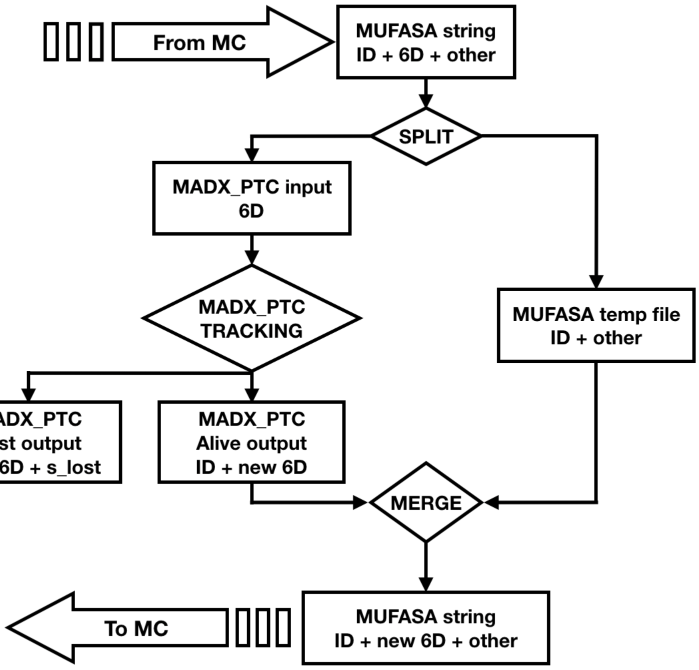

Figure 3: Flowchart of the data handling process for tracking.

1.1.1 Positrons

When a positron arrives on a target, the first operation to perform is to select at which step to attempt the muon production. This is achieved by extracting the longitudinal coordinate of the production vertex z from the exponential distribution

$$f(z) = e^{-z/X_0} \quad ; \quad 0 < z < l_{tgt} \tag{1}$$

where X_0 is the radiation length of the material. This value is then compared to the target length in order to find which step corresponds to it; let's call this number s_{prod}. After this process the positron is tracked step by step through the target using the following criteria:

- for each step before s_{prod} the positron is tracked applying multiple scattering and energy loss due to bremsstrahlung
- at s_{prod} the positron energy is checked against the threshold energy for muon production $E_{e^+} > E_{th} = 43.7\ GeV$; if this condition is satisfied a muon is produced
- whether or not a muon was produced at the previous step, the positron is tracked from s_{prod} to the end of the target, again applying multiple scattering and energy loss at each step

The positron informations are then stored, and the process is repeated for the next particle. The following paragraphs describe the implementation in the code of the processes introduced above; a comparison between the outputs of MUFASA and GEANT4 are shown in section 2.

Multiple Coulomb Scattering The effect of the multiple scattering on the trajectory of a charged particle traversing a medium is a net deflection angle described by a gaussian distribution with an RMS width given by

$$\sigma_\theta[rad] = \frac{0.0136}{E[GeV]}\sqrt{\frac{L[m]}{X_0[m]}} \tag{2}$$

where L is the length of the medium and X_0 is its radiation length. In MUFASA every time a particle is tracked through a step, first σ_θ is evaluated using the energy of the particle and the length of the step. After that a random number is extracted from a normal distribution (i.e. a gaussian with mean zero and variance one); then the particle is tracked for half a step using the drift transport matrix, then the kick is applied and finally the particle is tracked another half a step again using the drift transport matrix. The algorithm for the horizontal plane is shown below, while for the vertical plane the same process is repeated.

```
rnd = gRandom->Gaus(0,1);
x = x + L_step*px/2;
px = px + sigma_theta*rnd;
x = x + L_step*px/2;
```

Energy loss via Bremsstrahlung The Bremsstrahlung differential cross-section for a positron (or electron) can be approximated with the following equation

$$\frac{d\sigma}{dk} = \frac{A}{X_0 N_A k}\left(\frac{4}{3} - \frac{4}{3}y + y^2\right) \tag{3}$$

where X_0 is the radiation length expressed in g/cm^2, k is the energy loss and $y = k/E$ is the ratio between the energy loss and the particle energy. In MUFASA this effect is applied to the particle at each step as follows:

- the energy of the positron is checked: if it is lower than 0.001% of the nominal energy the particle is declared dead, and the next positron is tracked through the target, otherwise the tracking continues;
- the probability of a Bremsstrahlung event is evaluated by

$$P = \frac{N_A \rho}{A} L_{step}\sigma_{tot} \sim \frac{L_{step}[m]}{X_0[m]}\frac{4}{3}log\left(\frac{k_{max}}{k_{min}}\right) \tag{4}$$

  where the ratio $\frac{k_{max}}{k_{min}}$ between the maximum and the minimum energy loss where the cross section is integrated is set to 10^{-4}.
- A random number between 0 and 1 is extracted from a flat distribution: if it is lower than the probability P a Bremsstrahlung event occurs, and the energy lost by the

positron k is extracted from the distribution in Eq. (3) and subtracted to the particle energy.

Muon production Once the positron arrives to the designed step for production, the center-of-mass energy of the system composed by the incoming positron and a rest atomic electron is evalued by

$$S = 2m_e E \tag{5}$$

In order to have muon pair production, this value needs to be at least equal to the rest mass of two muons $\sqrt{S} \geq 2m_\mu$. If this condition is satisfied a muon is generated in the code. Only one muon (instead of two) is generated in order to have a beam composed by all particles of the same charge for tracking, but the other beam would have the same characteristics by symmetry.

If the requirement on the center-of-mass energy is satisfied, the muon production begins. The production cross-section $\sigma_{e^+e^-\to\mu^+\mu^-}$ is evaluated from the distribution in figure 4 using the incoming positron energy. Then a weight is associated to each muon, in order to estimate how many "true" muons correspond to the macroparticle; this value is calculated by:

$$w = \frac{N_{e+}^{true}\ \rho N_A Z/A\ l_{tgt}\ \sigma_{e^+e^-\to\mu^+\mu^-}}{N_{e+}^{macro}} \tag{6}$$

where $\rho N_A Z/A\ l_{tgt}$ are the target parameters while N_{e+}^{macro} and N_{e+}^{true} are respectively the number of positron macroparticles and true particles in the bunch. At the end of the simulation, the sum of the weights of all the muon macroparticles still alive will correspond to the number of true muons that would be accumulated in a bunch.

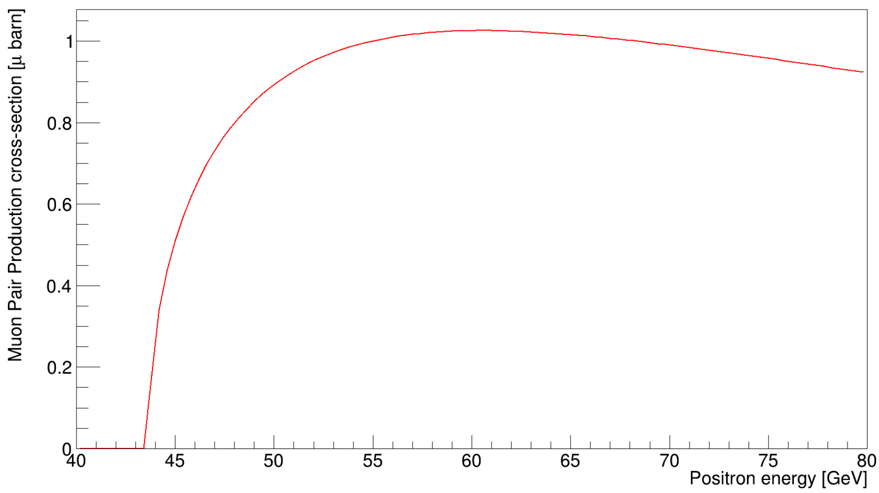

Figure 4: Muon pair production cross-section as a function of the positron energy.

The 6D coordinates of the produced muon are determined in the center-of-mass (noted with the apex *) frame of reference and are later boosted to the laboratory frame (no apex). Starting from the differential cross-section of point-like fermions in e^+e^- annihilation shown in Ref. [9], the radial angle θ^* is extracted from the following distribution

$$f(\theta^*) = \left[1 + \frac{E_{th}}{E} + \left(1 - \frac{E_{th}}{E}\right) cos^2\theta^*\right] sin\theta^* \tag{7}$$

where E_{th} is the muon production energy threshold and E is the positron energy in the laboratory frame, as also described in [10]. The azimuthal angle ϕ^* is instead extracted from a flat distribution in the range $0 \leq \phi^* \leq 2\pi$.

The muon four-momentum in the center-of-mass is then defined as:

$$p^* = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \sqrt{S}/2 \\ p_\mu\ sin\theta^* sin\phi^* \\ p_\mu\ sin\theta^* cos\phi^* \\ p_\mu\ cos\theta^* \end{pmatrix} \tag{8}$$

where $p_\mu = \sqrt{S/4 - m_\mu^2}$. This four-momentum is then boosted to the laboratory frame ($p_{boost} = \left(E - m_e, 0, 0, \sqrt{E^2 - m_e^2}\right)$), so to obtain the angles θ and ϕ in the laboratory frame. These values are then used to calculate the transverse momentum of the muon in the laboratory frame according to:

$$\begin{aligned} p_x^\mu/p^\mu &= p_x^{e+}/p^{e+} + sin\theta\ sin\phi \\ p_y^\mu/p^\mu &= p_y^{e+}/p^{e+} + sin\theta\ cos\phi \end{aligned} \tag{9}$$

The energy of the particle is also obtained from the boosted four-momentum, and it is normalised to the nominal muon beam energy, equal to half the nominal positron beam energy. The transverse and longitudinal positions are instead considered to be the same of the positron. The freshly produced muon can now be transported to the end of the target with the same *step-by-step* approach previously described.

1.1.2 Muons

Because in LEMMA the target is embedded in the muon accumulation rings as shown in figure 5, during the accumulation process the muon beams will pass several times through it. Therefore this interaction is simulated at every turn for the accumulated muon beams, considering Multiple Coulomb Scattering and energy loss via ionization.

Multiple Coulomb Scattering This process is applied in the same way described above for the positrons, and therefore will not be repeated here.

Energy loss via ionization One of the stored parameters for the materials is the average energy loss for muons dE/dx $[GeV/m]$. At each step in the target, the particle energy is reduced by an amount equal to $dE/dx \times l_{step}$. At the moment the same value is used for all the muons, but a dedicated Monte Carlo could be developed also for the evaluation of the muon energy loss in the future.

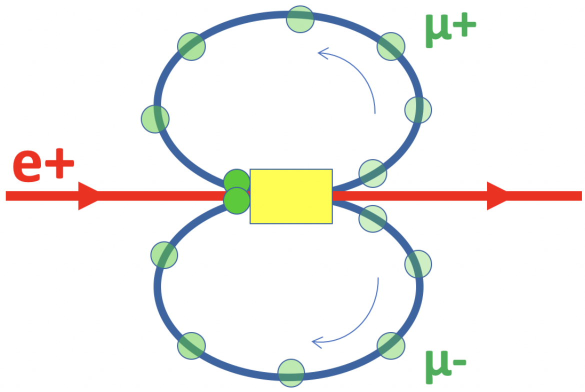

Figure 5: Concept of the muon production and accumulation process in LEMMA.

Muon decay The decay of the muons is evaluated at the end of the simulation. For each particle the survival probability is calculated by:

$$P = e^{-L/\tau\beta c} \tag{10}$$

where $\tau = \gamma \cdot 2.2\mu s$ is the muon mean lifetime at the particle's energy, while $L =$ `l_tgt - z + turn*(l_acc + l_tgt)` is the total length the particle has traveled in the ring. A random number is then extracted from a flat distribution between 0 and 1; if this number is lower than the survival probability the particle is considered alive, and it gets transcribed to the final output file, otherwise the particle is considered decayed.

2 Validation with GEANT4

A first validation of MUFASA has been performed against previously published results. In [1] and in [8] the horizontal phase space of the muons produced by a pointlike positron beam impinging on a Beryllium target and in both cases data are obtained using a GEANT4 simulation. The beam energies and target thicknesses used for the two simulations are respectively 44GeV / 10mm and 45GeV / 3mm. The comparison with MUFASA simulations is shown in figures 6 and 7, proving an excellent agreement between the results.

A more accurate dedicated benchmarking with GEANT4 has been performed [11]. Here are shown the results for a 45GeV point-like positron beam impinging on a 0.106m (0.3 X_0) Beryllium target.

The validation of the positron energy loss via Bremsstrahlung is made by comparing the energy spectrum of the positrons at the end of the target. The plot in figure 8 shows a great agreement between the MUFASA and the GEANT4 outputs.

In order to reduce the CPU time for GEANT4 the muon production cross-section has been enhanced of a factor 10^4. This value for the enhancement has been chosen in order to have a correct distribution of the production vertex along the length of the target. As shown on the left plot in figure 9, higher values (in this case 10^6) would increase the cross-section so much that most of the events happen in the very first fractions of the target; such

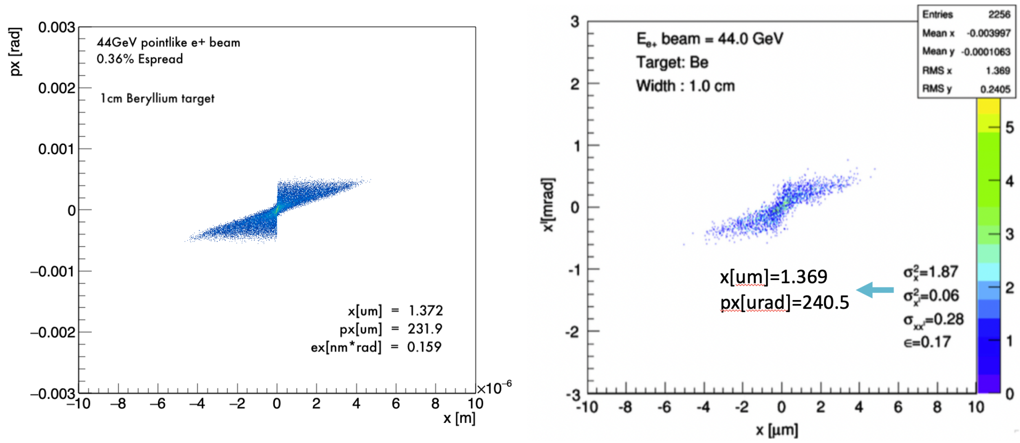

Figure 6: Horizontal phase space for muons produced by a MUFASA simulation (left) and results published in [1] (right) for a 44GeV positron beam impinging on a 1cm Beryllium target.

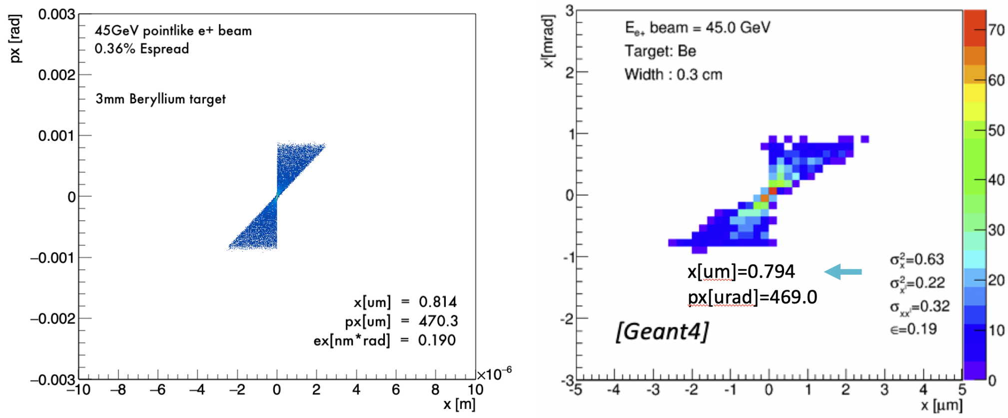

Figure 7: Horizontal phase space for muons produced by a MUFASA simulation (left) and results published in [8] (right) for a 45GeV positron beam impinging on a 3mm Beryllium target.

high values could be used for very thin targets ($\sim 0.01X_0$) but for thicker targets a lower enhancement factor is more suited. On the right plot in figure 9 instead is shown that using an enhancement factor of 10^3 or 10^4 produces the same results. Therefore for the purpose of this simulation an enhancement factor of 10^4 has been used, as it is the higher value that produces the correct vertex distribution.

The produced muon transverse phase space is shown in figure 10. There is a good consistency between the results of MUFASA (left) and GEANT4 (right), and the slight differences can be addressed to the different sample size.

Finally the energy distribution of the produced muons is shown in figure 11. The agreement between the results of MUFASA and GEANT4 is quite good, even if toward the edge values MUFASA distribution is a little smoother.

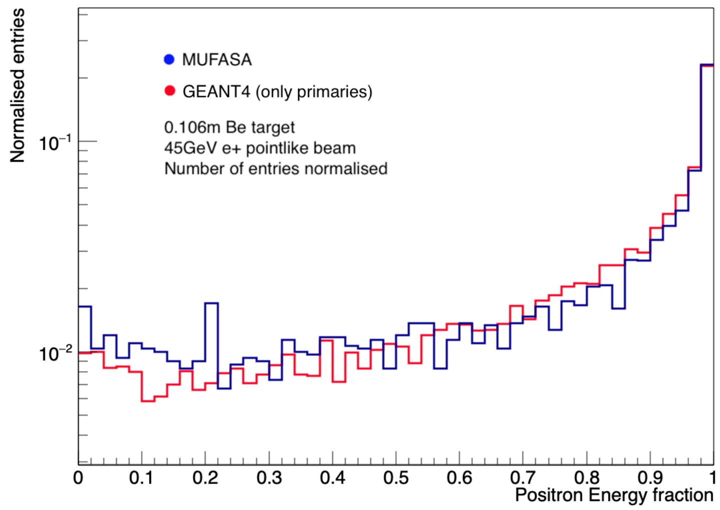

Figure 8: Energy distribution of a 45GeV positron beam after a 0.106m Beryllium target.

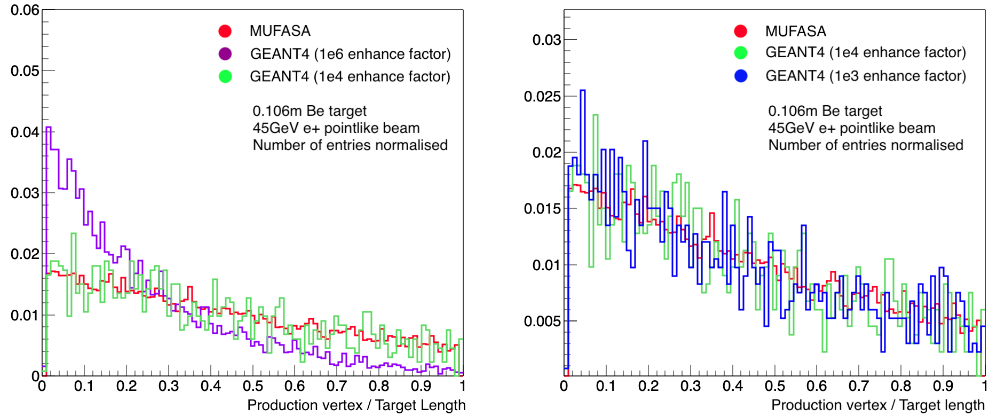

Figure 9: Distribution of muon production vertex along the target for a 45GeV pointlike positron beam on a 0.106m Beryllium target.

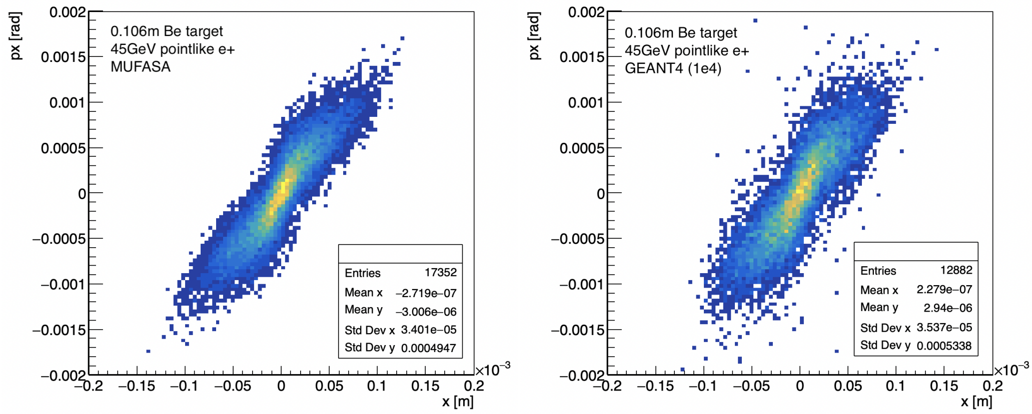

Figure 10: Horizontal muon phase space for a 45GeV pointlike positron beam on a 0.106m Beryllium target.

3 Application to a Lithium film jet target

The results produced by MUFASA simulation of the muon accumulation process for 1500 45GeV positron bunches impinging on a $0.3X_0$ long, $50\mu m$ wide liquid Lithium film jet

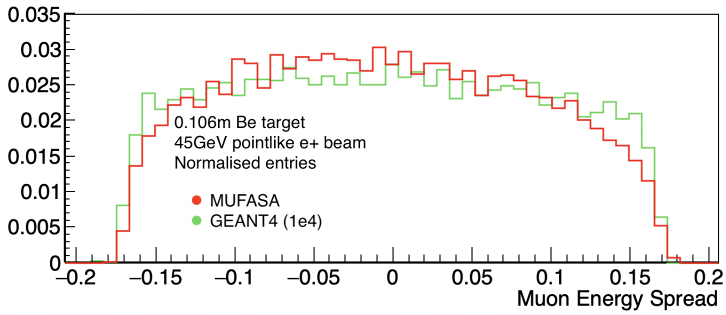

Figure 11: Energy distribution muons produced by a 45GeV positron beam on a 0.106m Beryllium target.

target, as described in section 1 and in [7]. The positron beam emittance is $\epsilon = 700pm$, and the optics used is MUACC_82n, also described in [7].

The muon bunch population before (light blue) and after (dark blue) applying the decay is shown in figure 12. The sharp loss of particles visible in the first tens of turns is due to muons produced outside the optics acceptance and therefore being lost. In the light blue curve it can be seen that after the first one hundred turns, the number of muons is mostly constant, with only a slight loss due to particles going outside the ring acceptance due to multiple scattering, as they pass several times through the target. Applying the decay (dark blue curve) the muon population in the bunch follows an exponential trend.

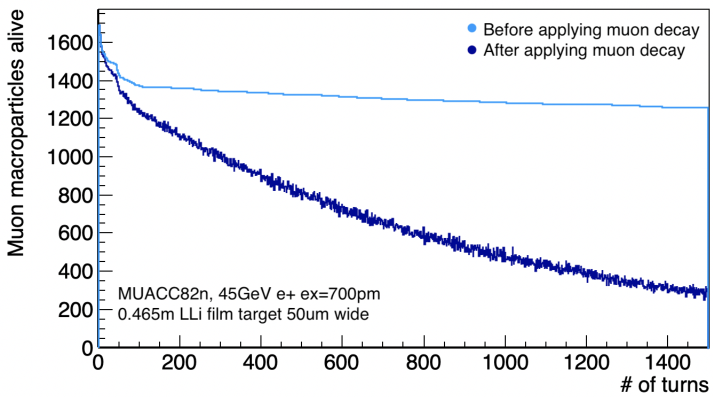

Figure 12: Muon population in a bunch (without accumulation) as a function of the number of turns in the accumulator ring. The population of muons before applying decay is shown in light blue, in dark blue the same distribution after muon decay is applied.

In figure 13 the number of muons per bunch during accumulation considering positron bunches of 5×10^{11} particles is shown. In this plot the result of the $50\mu m$ wide target is compared also with targets of different widths ($200\mu m$, $400\mu m$, extended), a simulation without the multiple scattering contribution, and a theoretical curve of the accumulated number of muons taking into account only the time decay (i.e. supposing ideal perfect optics). This comparison (shown in [7]) shows clearly that using a film jet target helps to mitigate the effect of multiple scattering during the accumulation process.

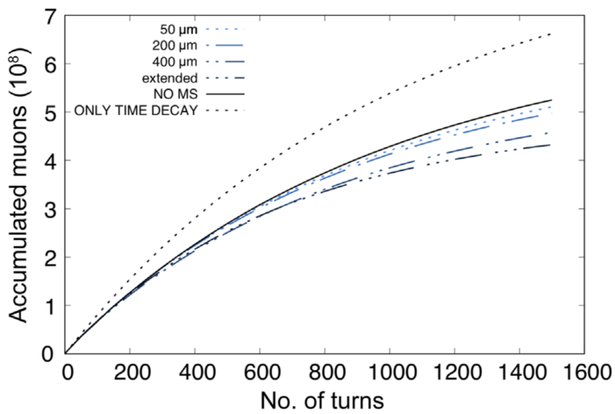

Figure 13: Muon population per bunch during accumulation as a function of the number of turns in the accumulator ring.

The evolution of the muon emittance during the accumulation is shown in figure 14.

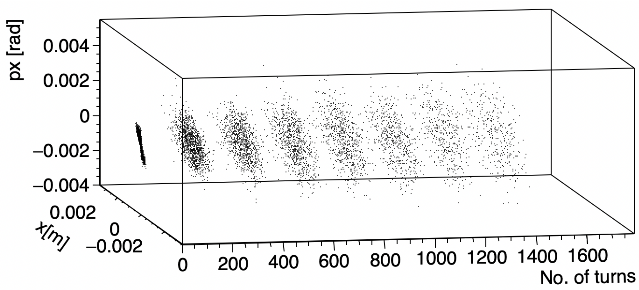

Figure 14: Muon emittance as a function of the number of turns.

4 Conclusions

MUFASA is a powerful tool developed for the study of the muon production and accumulation system for the LEMMA project, being capable of handling both particle transport and interaction with the target. The code has been fully benchmarked against GEANT4 showing good agreement in several different scenarios. Still under constant improvement, MUFASA has been already very useful for the development of the accumulator rings optics and optimisation of the target, as in [7].

5 Acknowledgements

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