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## Novel proposal for a low emittance muon beam using positron beam on target

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### Abstract

Muon beams are customarily obtained via  $K=\pi$  decays produced in proton interaction on target. In this paper we investigate the possibility to produce low emittance muon beams from electron-positron collisions at centre-of-mass energy just above the  $\mu^+\mu^-$  production threshold with maximal beam energy asymmetry, corresponding to a positron beam of about 45 GeV interacting on electrons on target. We present the main features of this scheme with an outline of the possible applications.

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## Abstract

Muon beams are customarily obtained via  $K/\pi$  decays produced in proton interaction on target. In this paper we investigate the possibility to produce low emittance muon beams from electron-positron collisions at centre-of-mass energy just above the  $\mu^+\mu^-$  production threshold with maximal beam energy asymmetry, corresponding to a positron beam of about 45 GeV interacting on electrons on target. We present the main features of this scheme with an outline of the possible applications.

*Keywords:* muon production, muon collider

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## 1. Introduction

Muon beams are customarily obtained via  $K/\pi$  decays produced in proton interaction on target. Their use in high energy physics experiments has a continuous increasing interest for rare decays searches, precision measurement experiments, neutrino physics and for muon colliders feasibility studies. Several dedicated experiments are ongoing to produce high intensity muon beams with low emittance. In this paper we will investigate the possibility to produce low emittance muon beams from a novel approach, using the electron-positron collisions at centre-of-mass energy just above the  $\mu^+\mu^-$  production threshold with maximal beam energy asymmetry, that corresponds to about 45 GeV positron beam interacting on an electron target. Previous studies on this subject are reported in ref. [1, 2]. Our proposal is simpler with respect to present conventional projects where muons are produced by a proton source. One important aspect is that in our proposal muon cooling would not be necessary. The most important key properties of the muons produced by the positrons on target are:

- the low and tunable muon momentum in the centre of mass frame

- large boost, being about  $\gamma \sim 200$ .

These characteristic results in the following advantages:

- the final state muons are highly collimated and have small emittance;
- the muons have an average laboratory lifetime of about 500  $\mu\text{s}$ .

In the section 2 we describe the main processes at the energy of interest. In section two we describe the key issue of the options on the target. The value of the  $e^+e^- \rightarrow \mu^+\mu^-$  cross section is of about  $1 \mu\text{b}$  just above threshold, requiring a target with very high electron density to obtain a reasonable muon production efficiency. We estimate the requirement on the electron density on the target above  $10^{20}$  electrons/ $\text{cm}^{-3}$ . Such high-density values can be obtained either in a liquid or solid target or, possibly, in a more exotic solution like in crystals. We discuss the solid target solution and the crystals. Also a plasma excited via a synchronized electron beam could be a solution. In section 3 we discuss our first estimates that show how this option does not seem practicable. However, we think that that further studies are worth to be performed on this option. Studies of the positron source will be reviewed in section 4, followed, in section 5, by the schemes of the muon production. Finally, in section 6, rate and beam properties estimates for muon

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collider that can be obtained with our proposal are given. We conclude with a proposal for a design study, especially for the key innovative aspects that need to be investigated.

## 2. Processes at $\sqrt{s}$ around 0.212 GeV

The dominant processes at  $\sqrt{s}$  around 0.212 GeV are mainly three:

1.  $e^+e^- \rightarrow \mu^+\mu^-$  :  $\mu^+\mu^-$  production
2.  $e^+e^- \rightarrow e^+e^-\gamma$  : Bhabha scattering
3.  $\gamma\gamma$  scattering.

Our goal is to maximize the muons production and muon beam parameters. In the following sub-section 2.1 we analyse their dependencies and features. The second and third process are instead side effect that reduce the efficiency of the first one. We will discuss in the sub-section 2.2 the second process. They have been simulated with the BabaYaga event generator [3] with the exception of the collinear radiative Bhabha scattering, for which we used BBBrem [4]. The last process, the  $\gamma\gamma$  scattering, will not be discussed in detail, having a cross section that is smaller than that for the  $\mu^+\mu^-$  production. We note that, if needed, the method proposed here for the muons production could also allow the production of high energy collimated photons.

### 2.1. The Process $e^+e^- \rightarrow \mu^+\mu^-$

We discuss here the dependencies that can maximize the muons production and at the same time also minimize the muon bunch emittance and energy spread, when required. The main parameters that play a role are: the dependence of the scattering angle distribution of the outgoing muons, the muons energy distribution and the cross section on the centre-of-mass energy. The cross section for continuum muon pair production  $e^+e^- \rightarrow \mu^+\mu^-$  just above threshold is obtained using the Born cross section, enhanced by the Sommerfeld-Schwinger-Sakharov (SSS) threshold Coulomb resummation factor [5]. The value of this cross section is shown in figure 1 as a function of the centre-of-mass energy. In this figure we see that the cross section approaches its maximum value of about  $1\mu\text{b}$  at  $\sqrt{s} \sim 0.230$  GeV.

In our proposal these values of  $\sqrt{s}$  can be obtained from fixed target interactions with a positrons beam energy of

$$E_+ \approx s/(2m_e) \approx 45 \text{ GeV}$$

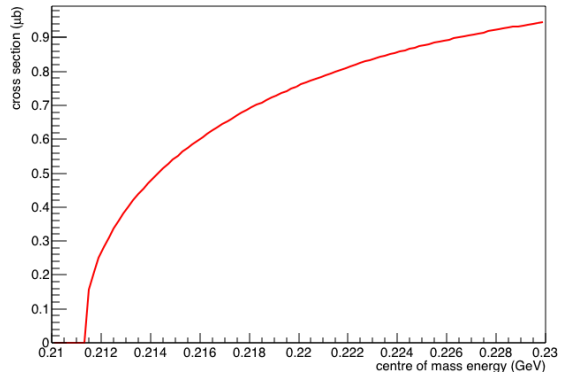


Figure 1: Cross section as a function of  $\sqrt{s}$  of the  $e^+e^-$  collisions.

where  $m_e$  is the electron mass, with a boost of  $\gamma \approx E_+/\sqrt{s} \approx \sqrt{s}/(2m_e) \approx 220$ . The scattering angle of the outgoing muons  $\theta_\mu$  is maximum for the muons emitted orthogonally to positron beam (in the rest frame) and its value depends on  $\sqrt{s}$  (see figure 2).

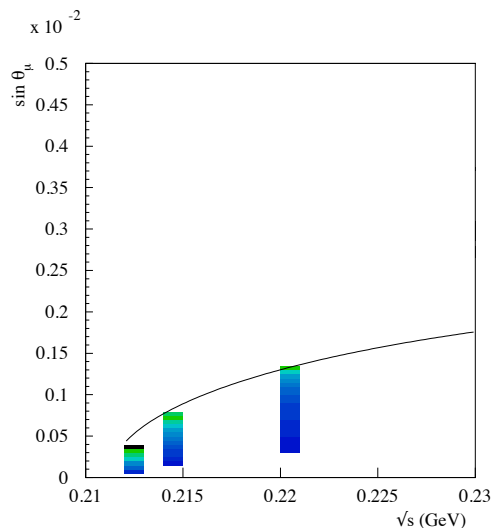


Figure 2: Muon scattering angle distribution as a function of  $\sqrt{s}$  of the  $e^+e^-$  collisions.

In the approximation of  $\beta_\mu = 1$ , where  $\beta_\mu$  is the muons velocity, one can easily obtain for the maximum scattering angle:

$$\theta_\mu^{max} = \frac{4m_e}{s} \sqrt{\frac{s}{4} - m_\mu^2} \quad (1)$$

The value of the scattering angle  $\theta_\mu$  increases with the  $\sqrt{s}$  with approximately the same shape

as the cross section of the  $\mu^+\mu^-$  production. The difference between the maximum and the minimum energy of the muons produced at the positron target ( $\Delta E_\mu$ ) also depends on  $\sqrt{s}$ , and with the  $\beta_\mu = 1$  approximation we get:

$$\Delta E_\mu = \frac{\sqrt{s}}{2m_e} \sqrt{\frac{s}{4} - m_\mu^2} \quad (2)$$

105 These values have to be folded with the muons angular distribution in the rest frame, that is:  $(1 + \cos^2 \theta_\mu^*)$ , where  $\theta_\mu^*$  is the muon scattering angle in the rest frame. The value of  $\sqrt{s}$  has to be optimized to maximize the  $\mu^+\mu^-$  production and to minimize the beam angular divergence and eventually the energy spread. The  $\theta_\mu$  distribution obtained with the **BabaYaga** generator is shown in figure 2 for different  $\sqrt{s}$  values. Muons produced with very small momentum in the rest frame are well contained in a cone of about  $5 \cdot 10^{-4}$  rad for  $\sqrt{s}=0.212$  GeV, the cone size increases to  $\sim 1.2 \cdot 10^{-4}$  rad at  $\sqrt{s}=0.220$  GeV. Similarly, the energy distribution of the muons, as shown in figure 3, has an *RMS* that increases with  $\sqrt{s}$ , from about 1 GeV at  $\sqrt{s}=0.212$  GeV to 3 GeV at  $\sqrt{s}=0.220$  GeV. The muons beam energy has the typical correlation with the muons emission angle as shown in figure 4 for  $\sqrt{s}=0.214$  GeV.

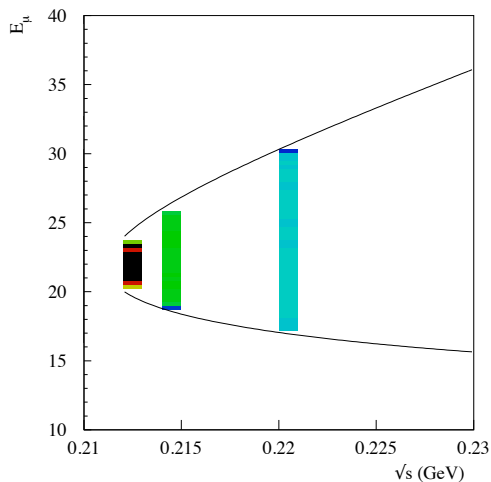


Figure 3: Muons energy distribution as a function of  $\sqrt{s}$ .

Another solution in principle could be to produce the muonium below the  $\mu^+\mu^-$  threshold that can be eventually dissociated in the interaction with the medium. It has been studied in Ref. [5], where it is shown that the  $e^+e^-$  width is proportional to  $1/n$

where  $n$  indicates the muonium energy level. The cross section for the  $S^1$  state in the narrow width approximation is about:

$$10^{-9} \text{ mb } E_+/\sigma_{E_+},$$

where  $\sigma_{E_+}$  is the positrons beam energy spread. This value implies that the use of this process for copious muons production is not realistic.

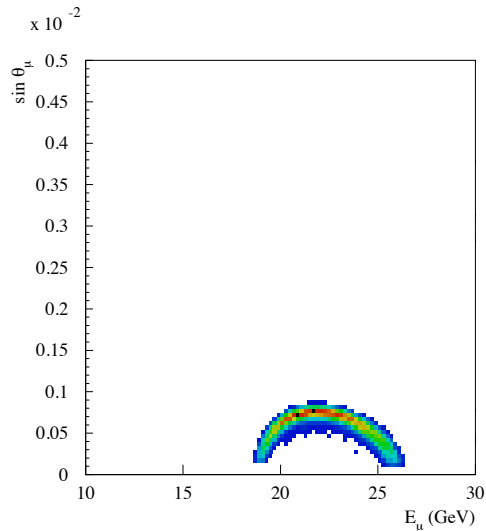


Figure 4: Scattering angle vs muons energy distribution for the  $\sqrt{s}=0.214$  GeV case

## 2.2. The process $e^+e^- \rightarrow e^+e^-\gamma$

The Bhabha scattering represents the largest source of beam loss in this study, setting an upper limit on the muons production from positrons on target. The large angle case has been simulated in the rest frame using **BabaYaga** with radiative photons energy,  $E_\gamma^* < 10$  MeV, and a scattering angle  $\theta_{\text{gamma}} > 10^\circ$ . The total cross section in the region of  $\sqrt{s} = 0.2$  GeV is

$$\sigma_{\text{Bhabha}} \approx 0.6 \text{ mb.}$$

As expected, the process proceeds via t-channel and most of the generated events are produced at a very small positrons scattering angle  $\theta_+$ . Figure 5 shows the distribution of the outgoing positrons scattering angle as a function of the energy of the outgoing positrons, for a beam positron energy of  $E_+ = 46$  GeV impinging the target.

The distribution indicates that the beam loss due to this process can be substantially decreased with

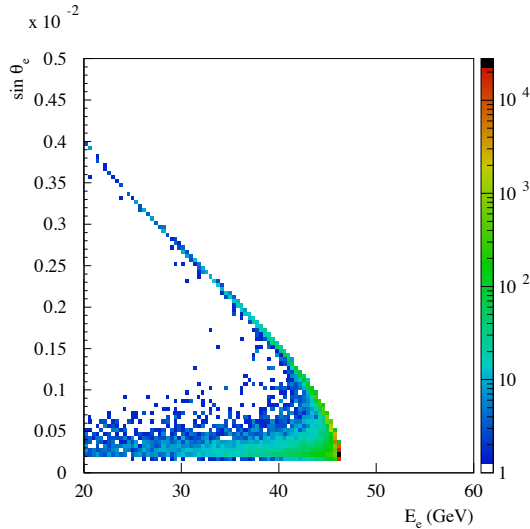


Figure 5: Distribution of the outgoing positrons scattering angle as a function of the energy of the positrons beam after its scattering on the target, for a positron beam energy before the scattering of  $E_+ = 46$  GeV.

reasonable acceptances. The largest part of the  $e^+e^-$  cross section comes from the collinear radiative Bhabha scattering. It has been simulated with BBBrem[4]. The total cross section is about 150 mb for a  $E_\gamma > 0.01E_+$  and it gets to about 60 mb for  $E_\gamma > 0.1E_+$ . This process sets a limit to the  $\mu$  pairs production, as it sets limits to the beam lifetime in high luminosity  $e^+e^-$  colliders.

### 3. Target options

The number of  $\mu^+\mu^-$  pairs produced per positron bunch on target is:

$$n(\mu^+\mu^-) = n^+\rho^-l\sigma(\mu^+\mu^-) \quad (3)$$

where  $n^+$  is the number of positrons in the bunch,  $\rho^-$  is the electron density in the medium,  $l$  is the thickness of the target, and  $\sigma(\mu^+\mu^-)$  is the muon pairs production cross section. As described in the previous section, the dominant process at these energies is the collinear radiative Bhabha scattering with a cross section of about of 150 mb actually setting the value of the positron beam interaction length for a given pure electron target density value. Using as reference value for the positron beam degradation when its current is decreased by  $1/e$ , i.e. one beam lifetime, one can determine the

maximum achievable value for the target density and length:

$$(\rho^-l)_{max} = 1/\sigma(rad.bhabha) \approx 10^{25}cm^{-2} \quad (4)$$

The ratio of the muon pair production cross section to the radiative bhabha cross section determines the maximum value of the *muons conversion efficiency*  $eff(\mu^+\mu^-)$ , that can be obtained with a pure electrons target. In the following we will refer to  $eff(\mu^+\mu^-)$  defined as the ratio of the number of produced  $\mu^+\mu^-$  pair to the number of the incoming positrons. Easily one can see that the upper limit of  $eff(\mu^+\mu^-)$  is of the order of  $10^{-5}$ , so that:

$$n(\mu^+\mu^-)_{max} \approx n^+10^{-5}. \quad (5)$$

#### 3.1. Plasma target option

The option of the plasma target has been considered and studied in some details. It is known that an enhanced electron density can be obtained at the border of the blow-out region in excited plasma. This solution provides with good approximation an ideal electron target and will also benefit from a strong and continuous beam focalization thanks to the Pinch effect [6]. An enhancement of about  $10^2$  in the number of electrons can be obtained in a region of about  $100 \mu m$  just before the blow-out, for a plasma with density of  $n_p = 10^{16} electrons/cm^3$  [7]. The size of the electrons high density region scales as  $1/n_p$ , such that at useful positrons densities values in the order of  $n_p = O(10^{20})$  will be reached in regions in the  $\mu m$  range, making the plasma option hardly practicable.

#### 3.2. Conventional targets option

Electromagnetic interactions with nuclei are dominant in conventional targets. In addition, no intrinsic focusing effects are expected in this case, thus setting limits for the target thickness to not increase the muons beam emittance  $\epsilon_\mu$ . Assuming a uniform distribution in the transverse  $x-x'$  plane<sup>1</sup> the emittance contribution due to the target thickness is :

$$\epsilon_\mu = \frac{xx'^{max}}{12} = \frac{l(\theta_\mu^{max})^2}{12} \quad (6)$$

<sup>1</sup>actually the distribution has an exponential fall with  $e^{-z/\lambda}$  being  $z$  the longitudinal target coordinate and  $\lambda$  the smaller interaction length among all involved processes. The approximation is valid for  $\lambda < l$ .

The number of  $\mu^+\mu^-$  pairs produced per crossing has the form given by the relation 3, with

$$\rho = N_A/A\rho Z$$

being  $Z$  the atomic number,  $A$  the mass number,  $N_A$  the Avogadro constant and  $\rho$  the material density. In addition, the multiple scattering contributes to the emittance increase according to:

$$x'_{RMS} \sim \frac{0.0136}{P(\text{GeV})} \sqrt{0.5l}$$

with  $l$  expressed in radiation length unit and

$$x_{RMS} \sim x'_{RMS} 0.5l\sqrt{3}$$

The bremsstrahlung process governs the positrons beam degradation in this case and it scales with the radiation length.

On one side to minimize the emittance there is the need of a small length  $l$ , on the other side compact materials have typically small radiation length causing an increase of the emittance due multiple scattering and fast positron beam degradation due to bremsstrahlung. The production efficiency is instead proportional to the electrons density. Positrons survival probability is also an issue to be considered not only for long targets (as long as one radiation length:  $l \sim X_0$ ) but also if the positron beam is recirculated to increase the positron rate impinging the target. Relevant properties of the materials considered in our study are given in Table 1; together with the atomic and mass numbers  $Z$  and  $A$  are reported also the radiation length  $X_0$ , the interaction length  $\lambda(\mu^+\mu^-)$  and the ratio  $\lambda(\mu^+\mu^-)/X_0$ , being inversely proportional to the maximum value that can be obtained for  $eff(\mu^+\mu^-)$ .

The criteria we considered for the target choice are:

- the maximization of the number of  $\mu$  pairs produced;
- the minimization of the muon emittance;
- the largest positrons survival, if needed for the positrons recirculation.

Positrons interactions on four different targets have been studied with GEANT4[8]: Beryllium, Carbon, Diamond and Copper. For these four cases, we optimized the target thickness and the positron beam energy to maximize our key parameters. As

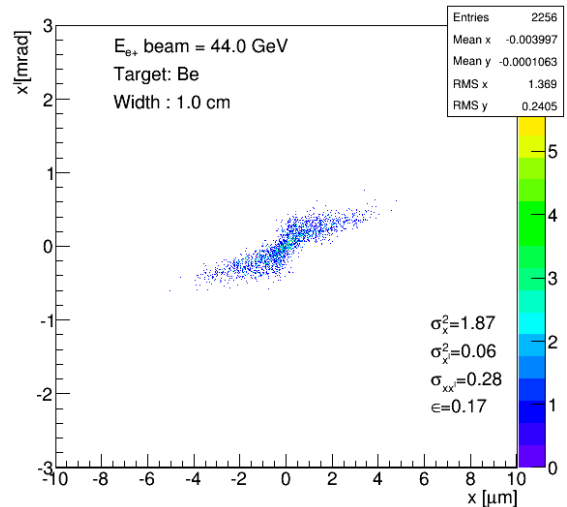


Figure 6: Horizontal phase space distribution  $x - x'$  of the muons exiting the target, for a positron beam energy of 44 GeV and a Beryllium target of 1 cm.

expected, it has been found that light materials: Beryllium, Carbon, and Diamond, have a better performance with respect to heavier materials (i.e. Copper), having a larger muon production efficiency  $eff(\mu^+\mu^-)$ . In addition in these cases the muon beam is produced with a smaller emittance. Finally, the positron survival probability is larger for light materials. These characteristics can be understood looking at the values of the ratio  $\lambda(\mu^+\mu^-)/X_0$  in Table 1. Table 2 shows the results of simulations performed for the positron energy of 44 GeV for the four targets, where the thickness is chosen in order to have the same muon production rate.

The actual value of the muon production efficiency for the Copper target is lower than that expected because of the positron loss due to bremsstrahlung. This effect is also seen in figures 6 and 7, where it is shown the  $x - x'$  distribution of the muons at the target exit.  $x$  and  $x'$  are the transverse displacement and the angle with respect to the positron beam direction. The  $x - x'$  distribution for Beryllium target has the characteristic shape expected for cases in which the emittance is dominated by target length effect and multiple scattering contributions cannot be appreciated. The situation is reversed for Copper target where the shape is fully dominated by multiple scattering.

Table 1: Relevant properties for some materials considered suitable for the target,  $\lambda(\mu^+\mu^-)$  has been calculated with a cross section of 1 mb.

Z	A	$X_0(\text{cm})$	$\lambda(\mu^+\mu^-)(\text{cm})$	$\lambda(\mu^+\mu^-)/X_0$
1	1.00794	900.6	$2.4 \cdot 10^7$	$2.7 \cdot 10^4$
2	4.0026	786	$2.8 \cdot 10^7$	$3.5 \cdot 10^4$
3	6.941	156.2	$7.2 \cdot 10^6$	$4.6 \cdot 10^4$
4	9.01218	35.2	$2.0 \cdot 10^6$	$5.7 \cdot 10^4$
5	10.811	22.2	$1.5 \cdot 10^6$	$6.8 \cdot 10^4$
6	12.0107	21.4	$1.7 \cdot 10^6$	$7.8 \cdot 10^4$
6	12.0107	19.3	$1.5 \cdot 10^6$	$7.8 \cdot 10^4$
6	12.0107	18.8	$1.5 \cdot 10^6$	$7.8 \cdot 10^4$
6	12.0107	12.1	$9.4 \cdot 10^5$	$7.8 \cdot 10^4$
29	63.546	1.4	$4.1 \cdot 10^5$	$2.8 \cdot 10^5$
74	183.84	0.4	$2.1 \cdot 10^5$	$6.1 \cdot 10^5$
82	207.2	0.6	$3.7 \cdot 10^5$	$6.6 \cdot 10^5$

Table 2: Summary of simulation results for 44 GeV positrons.

	Cu	C	Diamond	Be
$L(\text{cm})$	0.4	0.9	0.5	1.0
$L(\lambda(\mu))(10^{-7})$	2.7	1.6	1.6	1.6
$L(X_0)$	0.29	0.04	0.04	0.03
$\epsilon(\mu\text{m-mrad})$	0.19	0.16	0.09	
$\mu$ efficiency ( $10^{-7}$ )	1.6	1.6	1.6	1.6
$e^+$ efficiency ( $\delta E/E < 10\%$ )	0.46	0.90	0.90	0.93

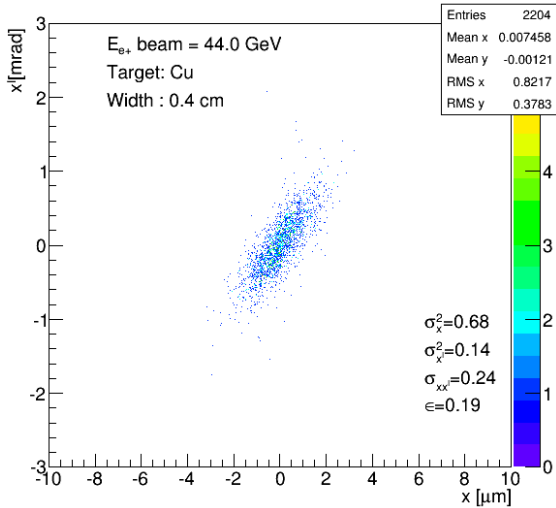


Figure 7: Horizontal phase space distribution  $x - x'$  of the muons exiting the target, for a positron beam energy of 44 GeV and a Copper target of 1 cm.

### 3.3. Crystal target option

It is known that channeling phenomena are present in crystals with particles incident angles with respect to the crystal structure smaller than  $\sqrt{2U/E}$  where  $E$  is the particle energy and  $U$  is the typical crystal energy level ( $O(100 \text{ eV})$  for Diamond). For complete channeling there is no contribution to emittance increase due to the target thickness and very low emittances can be obtained with target thickness of the order of the radiation length. For a 22 GeV muon the critical angle is about 0.1 mrad. The value of  $\theta_\mu^{max}$  is around 0.1 mrad for  $E_+=43.72 \text{ GeV}$ . At this energy the dimuon production cross section is slightly above  $0.1 \mu\text{b}$  and the muon energy spread at 22 GeV is below 1.5%. We think this could be a good option in the case of an Higgs factory at center of mass energy of 125 GeV where a beam energy spread of about  $5 \cdot 10^{-5}$  is needed.

## 4. Positron source

A superior positron source is required to compensate the extremely low muon production efficiency

Table 3: Positron sources parameters for future projects from ref. [9].

	SLC	CLIC	ILC	LHeC	LHeC ERL
E [GeV]	1.19	2.86	4	140	60
$\gamma\epsilon_x$ [ $\mu\text{m}$ ]	30	0.66	10	100	50
$\gamma\epsilon_y$ [ $\mu\text{m}$ ]	2	0.02	0.04	100	50
$e^+$ [ $10^{14}\text{s}^{-1}$ ]	0.06	1.1	3.9	18	440

$eff(\mu^+\mu^-) < 10^{-5}$ . The present record positrons production rate has been reached at the SLAC linac SLC. A summary of the parameters of the positron sources for the future facilities is reported in Table 3. ILC positron source has been designed to provide  $3.9 \cdot 10^{14}e^+$ /s. Two order of magnitudes more intense sources are foreseen for LHeC.

## 5. Muon production

In this section we present the study performed both on the single pass and the multipass of the positron bunch on target. In both cases the muon production rate has been maximized. The low value of the muon conversion efficiency requires a muon accumulator ring to reach  $O(10^8)$  muons per bunch. The muons could be recombined in two rings intercepting the positron beamline in the interaction point of positrons on target in order to preserve the emittance. The muon laboratory lifetime  $\tau_\mu^{lab}$  is about  $460 \mu\text{s}$  so that the recombination of the muon bunches need to be fast. The number of bunches  $n_b$  effectively accumulated in the bunch circulating in the combiner ring at the turn  $N_T$  is:

$$n_b = \sum_{i=1}^{N_T} e^{-\Delta t(N_T-i)/\tau_\mu^{lab}}$$

where  $\Delta t$  is the positron bunch spacing equal to the muon ring revolution frequency. Figure 8 shows the number of bunches  $n_b$  as a function of the turn number  $N_T$  for  $\Delta t=1 \mu\text{s}$ ; from the figure it is clear that there is a saturation at  $\sim 2\tau_\mu^{lab}$  and that a good working point is around  $\tau_\mu^{lab}$  (500 turns).

Muons accumulating in the storage ring pass the target many times and they receive an emittance increase due to multiple scattering:

$$\theta_{MS} \sim \frac{1}{N_T} \sum_{i=1}^{N_T} \frac{0.0136}{P(\text{GeV})} \sqrt{0.5L(X_0)(N_T-i)} e^{-\Delta t(N_T-i)/\tau_\mu^{lab}} \quad (7)$$

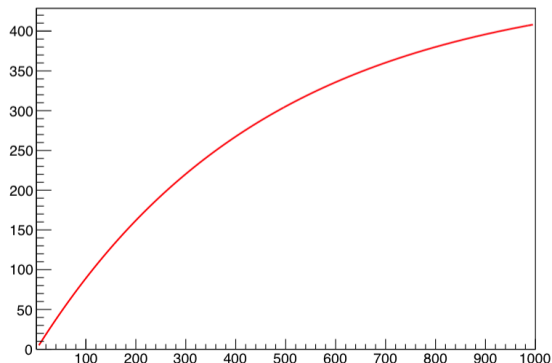


Figure 8: Muon accumulation function: number of bunches vs number of turns in the accumulator ring.

$$X_{MS} \sim \frac{1}{N_T} \sum_{i=1}^{N_T} \frac{0.0136}{P(\text{GeV})} \sqrt{0.5L(X_0)(N_T-i)} \frac{0.5L}{\sqrt{3}} e^{-\Delta t(N_T-i)/\tau_\mu^{lab}} \quad (8)$$

We considered as a first set of parameters the number of positrons per bunch equal to:  $N_b(e^+) = 4 \cdot 10^{11}$ , a bunch train of 2500 bunches with a bunch spacing of 200 ns. This would give a number of positrons per bunch train of  $N_{tot}(e^+) = 10^{15}$ . According to the LHeC positron rate design, up to four bunch trains per second are feasible. We propose that positrons at the exit of the target are collected and conveniently reused. This scheme foresees a bunch structure that can be obtained in an Energy Recovery Linac (ERL) with a “single pass scheme”, or in a positron storage ring with a “multipass scheme”. In the following sub-sections we briefly discuss requirements and advantages for these two schemes.

### 5.1. Single pass scheme

Single pass option requires a target with a length of about  $\sim X_0$ ; with such a thickness in light materials the emittance increases sizably. For example, a simulation of positron beam of 44.5 GeV impinging on a Diamond target of 4 cm (corresponding



to  $1/3 X_0$ ) gives a muon efficiency  $eff(\mu^+\mu^-)$  of the order of  $10^{-6}$  with an emittance of  $2.5 \mu\text{m}$ -mrad. Emittances smaller by two order of magnitudes can be obtained with crystal targets with structures aligned to the beamline. In this case a positron beam energy of about of 43.7 GeV has to be used and a factor of  $0.3 \cdot 10^{-6}$  for the  $\mu$  efficiency  $eff(\mu^+\mu^-)$  is expected. Using for the positron parameters  $\sigma_x = 0.5 \mu\text{m}$ ,  $\sigma_{x'} = 0.05$  mrad, an emittance of  $\epsilon_x = 0.028 \mu\text{m}$ -mrad can be obtained. The total power on target for energy loss is about 24 kW for positron beam parameters reported in the previous section.

### 5.2. Multipass scheme

A multipass scheme allows to increase the  $\mu$  conversion efficiency; it can be implemented with a large momentum acceptance storage ring. A 6 km positron ring with a bending radius  $\rho$  of 0.6 km has been considered. A total positron beam current of  $I_{tot}(e^+) = 240$  mA, corresponding to  $N_b(e^+) = 3 \cdot 10^{11}$  positrons per bunch,  $n_b = 100$  bunches provide a rate of  $1.5 \cdot 10^{18}$  positrons on target per second. Muons could be recombined in two rings with a circumference of 60 m intercepting the positron ring in the interaction point on target.

Table 4: Parameters related to synchrotron emission for the positron ring.

B [T]	0.245
$E_{critical}$ [keV]	315
$e^+$ rate [Hz]	$1.5 \cdot 10^{18}$
$\langle N_\gamma \rangle$	5763
$U_0$ [GeV]	0.578
$P_{tot}$ [MW]	139

Energy loss due to synchrotron radiation in the positron ring has been evaluated. A summary is reported in Table 4. The energy loss per turn is about 600 MeV corresponding to a total power required of about 139 MW, for a positron rate of  $1.5 \cdot 10^{18}$  Hz. The positron loss rate has to match the positron source capability. Using LHeC positron source rate the positron loss on target has to be below 1%. A Beryllium target 3 mm thick provides a positron survival probability of 2% and 0.5% for an energy acceptance of 5% and 25%, respectively.

The multiple scattering contribution to the emittance amounts to 0.5 mrad and  $0.5 \mu\text{m}$  for a 3 mm Be target. Such a thin target allows for a higher positron energy with a small emittance increase.

Table 5: Energy loss in target due to bremsstrahlung.

length Be target [cm]	0.3
$e^+$ rate [Hz]	$1.5 \cdot 10^{18}$
$\Delta E/E$ (5%) [GeV]	0.040
$\Delta E/E$ (25%) [GeV]	0.180
$P_{tot}$ (5%) [MW]	11
$P_{tot}$ (25%) [MW]	48

At 45 GeV we obtain an emittance of  $0.19 \mu\text{m}$  mrad with a  $\mu$  efficiency  $eff(\mu^+\mu^-)$  of  $10^{-7}$ . The muons produced with this technique have a large energy spread, being about  $\Delta E/E \approx 9\%$ , thus resulting interesting for high energy muon collider and neutrino factory applications. The value of the ratio of the number of produced muon pairs to the number of produced positrons is strongly related to the ring energy acceptance, it is about  $50 \cdot 10^{-7}$  for  $\Delta E/E < 5\%$  and about  $200 \cdot 10^{-7}$  for  $\Delta E/E < 25\%$ . Energy loss due to the radiation emitted within target has also to be considered. For a 0.3 cm Beryllium target the power dissipated has an increase from 3 MW to 13 MW depending on the ring energy acceptance. The power on the target due to ionization energy is about 300 kW requiring.

## 6. Beam properties estimate for muon collider

The performances of low emittance muon beams have been studied for two cases: multi-TeV and the Higgs factory case at  $\sqrt{s}=125$  GeV. This work has been performed to assess the potentiality of the method; a more reliable estimate needs a design study to prove the feasibility of the critical issues and to optimize the beams parameters.

### 6.1. Multi-TeV case

We consider that  $\mu^+$  and  $\mu^-$  beams are produced, as described in section 4.2, from a 45 GeV positron beam impinging on a 3 mm Beryllium target. We considered  $3 \cdot 10^{11}$  positrons per bunch with 100 bunches that circulate in a 6 km positron ring with an energy acceptance as large as  $\pm 5\%$ . The muon bunches that are produced by the positron beam are accumulated in two separate combiner rings, one for  $\mu^+$  and one for  $\mu^-$ , with a circumference of 60 m and circulating for 2500 turns.

The muon collider ring would have bunches of  $\mu^+$  and  $\mu^-$  with energy of 22 GeV with  $4.5 \cdot 10^7$  muon

Table 6: Comparison of muon beam properties for high energy applications obtained with our proposal from a positron source and with the conventional proton source.

	positron source	proton source
$\mu$ rate[Hz]	$9 \cdot 10^{10}$	$2 \cdot 10^{13}$
$\mu$ /bunch	$4.5 \cdot 10^7$	$2 \cdot 10^{12}$
normalized $\epsilon$ [ $\mu\text{m-mrad}$ ]	40	25000

Table 7: Comparison of muon beam properties for low beam energy spread positron source proton source.

	positron source	proton source
$\mu$ rate[Hz]	$2 \cdot 10^9$	$2 \cdot 10^{13}$
$\mu$ /bunch	$10^9$	$2 \cdot 10^{12}$
normalized $\epsilon$ [ $\mu\text{m-mrad}$ ]	0.059	25000

particles, emittance 0.19  $\mu\text{m-mrad}$ , and beam energy spread of 9%, produced with a spacing of 500  $\mu\text{s}$  (2 KHz rate). Bunches can be accelerated to the nominal energy as studied by the Muon Acceleration Program (MAP) working group [10].

The relevant parameters needed to determine the luminosity in our proposal of muon collider are reported in table 6. These performances can be compared with those reported in Ref. [10], also shown in table 6. From this table it is clear that the quality of the muons produced from a positron source as we propose in this paper is much better than the one obtainable with a proton source; however, the muons rate is a key parameter. We think that further studies are needed to set a maximum limit in our scheme.

Promising values of luminosities can be obtained with these parameters, being in the range of  $L \approx 10^{32} \text{ cm}^{-2}\text{s}^{-1}$ .

### 6.2. Muon collider at the Higgs boson energy: $E_{cm} = 125 \text{ GeV}$

The optimal scheme for a muon Higgs factory collider is with a single pass scheme with an interaction of positron beam on target just above the dimuon threshold. We propose a positron beam structure similar to the ILC one, with 500  $\mu\text{s}$  long bunch trains. Each bunch train contains 2500 bunches spaced by 200 ns providing a total rate of positrons of  $6 \cdot 10^{15}$  Hz. The natural muon beam energy spread is about 0.04% at 62.5 GeV for a positron beam of 43.8 GeV. It might be reduced to the required values with an increase of the bunch length of 50 times. At these positrons energies a crystal target can be used to obtain a very low emittance.

In a 4 cm Diamond crystal a muon conversion efficiency of  $3 \cdot 10^{-7}$  with an emittance of 0.028  $\mu\text{m-mrad}$  can be obtained with muon rates of  $2 \cdot 10^9$  Hz. Comparison with proton source results from MAP is given in table 7.

## 7. Conclusion

We have presented a novel scheme for the production of muons starting from a positron beam on target, discussing the critical aspects and key parameters of this idea and giving a consistent set of possible parameters that show its feasibility. This scheme has several advantages, the most important one is that it solves the problem of muon cooling. In fact, muon beams are generated already with very low emittances i.e. comparable to that obtained with electron beams. In addition, it might be able to provide luminosity with low muon fluxes avoiding the problems of irradiation typical with the conventional proposal. A critical point is the production of the necessary muon rate that requires detailed studies to assess the maximum possible value. First results presented in this paper shows that first class positron sources proposed for ILC and LHeC need are marginally sufficient to this purpose. An improvement in the positron rate is required for a muon collider purpose. Target survival needs also deep studies. A first set of parameters for a muon collider at high energy and 125 GeV has been shown to assess the potentiality of this proposal. We think that the promising results discussed in this paper encourage a serious design study of the proposal.

[1] M. Antonelli, P. Raimondi, Snowmass report: Ideas for muon production from positron beam interaction on a plasma target (2013).

URL <http://www.slac.stanford.edu/econf/C1307292/>

- 435 [2] M. Antonelli, P. Raimondi, Snowmass report: Ideas for muon production from positron beam interaction on a plasma target, INFN-13-22/LNF.
- [3] C. M. Carloni Calame, G. Montagna, O. Nicosini, F. Piccinini, The BABAYAGA event generator, Nucl. Phys. Proc. Suppl. 131 (2004) 48–55, [48(2003)].  
440 [arXiv:hep-ph/0312014](https://arxiv.org/abs/hep-ph/0312014), [doi:10.1016/j.nuclphysbps.2004.02.008](https://doi.org/10.1016/j.nuclphysbps.2004.02.008).
- [4] R. Kleiss, H. Burkhardt, BBBREM: Monte Carlo simulation of radiative Bhabha scattering in  $t^*$  very forward direction, Comput. Phys. Commun. 81 (1994) 372–380. [arXiv:hep-ph/9401333](https://arxiv.org/abs/hep-ph/9401333), [doi:10.1016/0010-4655\(94\)90085-X](https://doi.org/10.1016/0010-4655(94)90085-X).  
445
- [5] S. J. Brodsky, R. F. Lebed, Production of the Smallest QED Atom: True Muonium ( $\mu^+\mu^-$ ), Phys. Rev. Lett. 102 (2009) 213401. [arXiv:0904.2225](https://arxiv.org/abs/0904.2225), [doi:10.1103/PhysRevLett.102.213401](https://doi.org/10.1103/PhysRevLett.102.213401).  
450
- [6] P. Chen, Grand Disruption: A Possible Final Focusing Mechanism for Linear Colliders, Part. Accel. 20 (1987) 171–182.
- 455 [7] C. Gatti, P. Londrillo, Private communication.
- [8] S. Agostinelli, et al., GEANT4: A Simulation toolkit, Nucl. Instrum. Meth. A506 (2003) 250–303. [doi:10.1016/S0168-9002\(03\)01368-8](https://doi.org/10.1016/S0168-9002(03)01368-8).
- [9] F. Zimmerman, et al., Positron Options for the Linac-ring LHeC, Conf. Proc. C1205201 (2012) 3108–3110.  
460
- [10] The muon accelerator program.  
URL <http://map.fnal.gov>