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ABOUT THE EXTRACTION OF BEAMS WITH A VERY SMALL MOMENTUM DISPERSION

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Abstracts

The properties of the m/4 nonlinear resonance are proposed as a tool for attaining beams with very small momentum dispersion and horizontal emittance.

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1 Introduction

From a beam circulating in a storage ring slices with very small momentum spread can be extracted by means of the $\frac{m}{4}$ nonlinear resonance, improving the results obtained with the $\frac{m}{3}$ nonlinear resonance. At the level of approximation we intend to operate, the equation of this resonance is

$$\frac{d^2x}{d\theta^2} + \left(\frac{m}{3} + \Delta Q\right)^2 x = \varepsilon_2 x^2 \sin(p\theta) \tag{1}$$

where x is the radial coordinate and the azimuth θ is proportional to the time. Moreover, either a radial displacement or a momentum variation (or both together) imply a shrinkage (see upper part of Fig. 1) of the stable area, which is followed by a coherent growth of the amplitude of the radial betatron oscillations. This made possible the slow extrac-

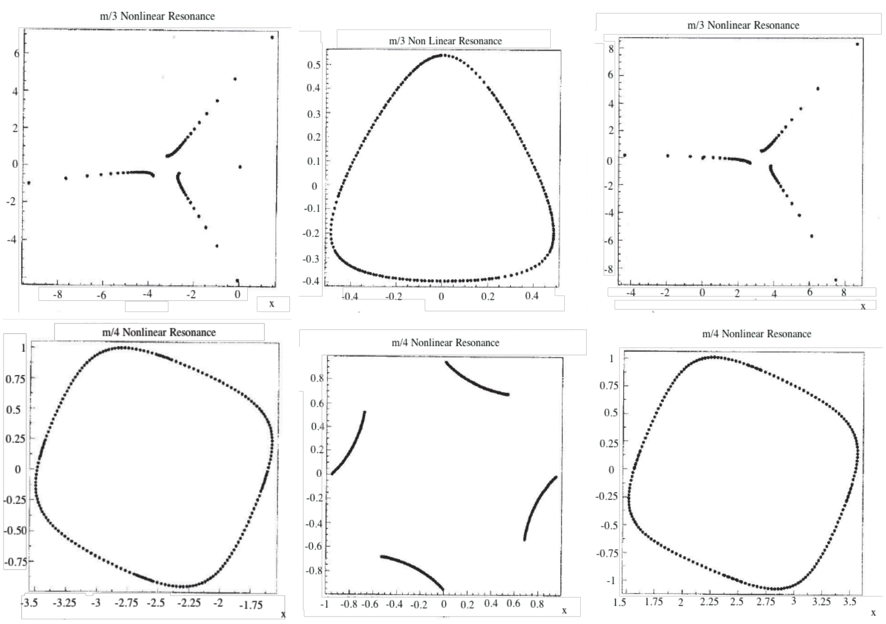

Figure 1: Stroboscopic plots in the $(x, \beta_{\rm H} x')$-plane generated by a sextupolar perturbation (upper row) or an octupolar perturbation (lower row) with, from left to right, negative, null and positive momentum spread.

tion from the Frascati [1] electron synchrotron, where the radio frequency was slowly switched off, letting the electrons lose energy and spiral inwards. A simple simulation program [2] demonstrates that the offset of the beam, with respect to the center of the perturbation, squeezes the stable area. The $\frac{m}{3}$ resonance has been used for realizing the stochastic extraction from LEAR [3],[4],[5] and, more recently, from COSY [6] and from a few synchrotrons [7] dedicated to oncological hadron therapy. In this method a noise, superimposed to the radio frequency, generates random energy kicks to the circulating particles, thus producing a very slow extraction.

In the m-fourth example [8], characterized by the equation

$$\frac{d^2x}{d\theta^2} + \left(\frac{m}{4} + \Delta Q\right)^2 x = \varepsilon_3 x^3 \sin(q\theta) \tag{2}$$

the effect of the momentum spread is just the opposite than in the $\frac{m}{3}$ case, i.e. the stable area increases (see lower part of Fig. 1), meaning that particles with a momentum spread with modulus bigger than a certain value are stable. All the plots deal with an idealized perfectly circular ring. The sextupoles are located every 90^0 with alternate polarities; the octupoles are located every 60^0 with alternate polarities.

2 Role of the momentum distribution

Let us consider all particles characterized by the same design momentum p_0 under the action of the $\frac{m}{4}$ nonlinear resonance. According to the values of the horizontal tune-shift ΔQ we can have two situations, described in the upper part of Fig. 2, with either $\Delta Q \neq 0$ or $\Delta Q = 0$. In the latter case the stable ares shrinks to a point. While the extraction of these particles is taking place, all the other off-momentum particles are stable (dark circle) as shown in the bottom picture. Notice that each separatrix consists of a couple of hyperbolas (see Refs. [8] and [9]) which cross each other in four fixed points located on a circle of radius

$$r = 2\left(\frac{m}{\varepsilon_3}\right)^{1/3} \Delta Q^{1/3}$$

as found from Eqs. (1.4) and (4.3c) of Ref. [9]. In the longitudinal phase-plane, the

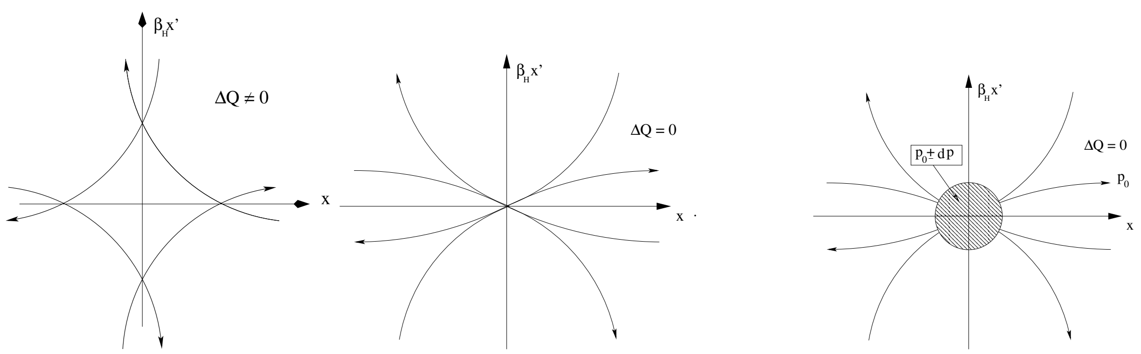

Figure 2: Plots of the $\frac{m}{4}$ resonance for different values of ΔQ.

bunches representing the actual sets of particles are delimited by a separatrix curve which sharply divides the phase plane into stable and unstable regions. Let us assume a parabolic distribution (see Fig. 3) of N_b particles inside the bunch; namely

$$dN = f(\sigma)d\sigma, \quad \text{with } f(\sigma) = \frac{3}{4}\frac{N_b}{\Sigma}\left[1 - \left(\frac{\sigma}{\Sigma}\right)^2\right] \tag{3}$$

and

$$\sigma = \frac{p - p_0}{p_0}, \quad \Sigma = \frac{\Delta p}{p_0} \tag{4}$$

where p_0 is the design momentum, σ is the relative momentum deviation of a particle, Σ is the maximum momentum spread of the beam and $\frac{\delta p}{p_0}$ is the desired very small momentum dispersion.

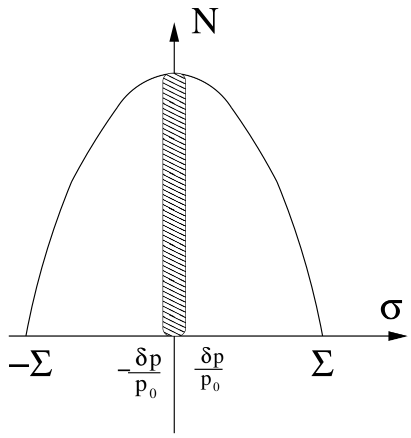

Figure 3: Parabolic distribution of momentum dispersion in the bunch

Following what stated before, only particles with momentum-spread contained between $-\frac{\delta p}{p_0} < \sigma_0 < \frac{\delta p}{p_0}$ (see the dark stripe of Fig. 3) are unstable. Hence synchrotron oscillations will continuously change the particles' energy until their momentum spreads will enter into that dark stripe and the following number of particles will be extracted.

$$\delta N = \int_{-\sigma_0}^{\sigma_0} f(\sigma)d\sigma = \frac{3}{2}N_b\frac{\sigma_0}{\Sigma}\left[1 - \frac{1}{3}\left(\frac{\sigma_0}{\Sigma}\right)^2\right] \simeq \frac{3}{2}N_b\sigma_0\Sigma \tag{5}$$

Therefore, we could obtain a beam with very small momentum dispersion and a horizontal emittance almost point-like, since the particles contained in the dark circle of Fig. 2 have momenta included between $-p_0 - dp$ and $p_0 + dp$, with dp that could be made extremely small. The vertical emittance remains unaltered. These properties of a beam extracted by means of the octupolar resonance appears promising for high precision measurements or, above all, for hadron therapy.

In Fig. 4 we demonstrate that even with a single octupolar lens we obtain results very similar to what shown in Fig. 1.

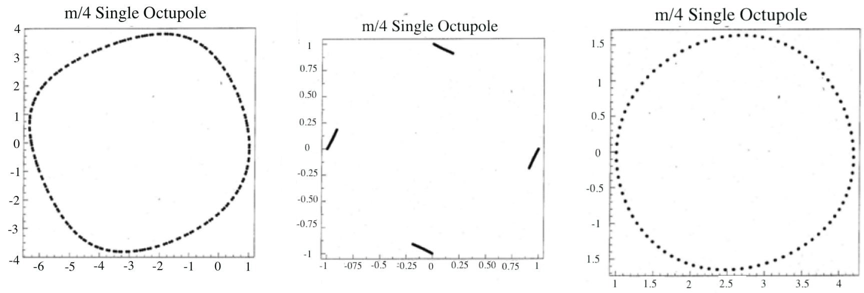

Figure 4: Stroboscopic plots for smaller, null and bigger momentum spread with a single octupole in the ring.

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