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Noise lower limit calculation for SuperB DCH Cluster Counting front-end electronics

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Abstract

Cluster counting technique [1] can improve particles identification in tracking devices, like drift chambers, by removing Landau tails. The technique is based on the counting of primary ionization clusters and, to be implemented, it requires slow drift velocity gas mixtures, fast amplifiers, fast sampling devices (> 1GS/s) and, finally, correct termination of the sense wire (to avoid signal reflection). Because the termination resistor low value, its noise contribution is not negligible then setting a lower limit on the readout chain noise.

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1 Introduction

SuperB drift chamber will be operated with a 90% Helium and 10% Isobutane gas mixture. Such mixture has a large radiation length and it is characterized by a low drift velocity (of the order of 1 μs/cm) then envisaging the use of cluster counting technique for dE/dx measurements.
The measurement consist in the detection (and counting) of all ionization clusters. The overall S/N ratio must be enough high to allows single electron cluster detection and, therefore, all noise contributions must be minimized.
Considering the chamber sense wires as a lossless transmission line and discarding external noise pick-up, the major noise contributions of the front-end chain are due to the termination resistor and the preamplifier intrinsic noise.
At the present time the front-end preamplifier has not yet been defined, nevertheless termination resistor noise contribution can be evaluated by means of the Equivalent Noise Charge (ENC) definition.

2 Equivalent Noise Charge (ENC) of the termination resistor

The circuit used to evaluate the contribution of the termination resistor to the ENC (i.e. the input signal amplitude that generates a preamplifier output signal equal to the rms noise) is shown in fig.1.
To simplify calculation we can discard the HV coupling capacitors , because their effects are limited to low frequencies, and we can use the preamplifier-shaper network shown in fig.2, assuming a negligible contribution of the preamplifier itself to the total noise.
Two methods will be used to carry out noise evaluation. The first one is based on direct

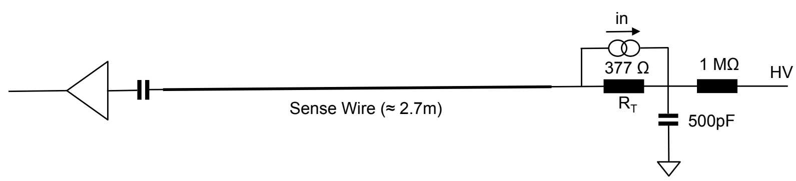

Figure 1: Circuit for the calculation of termination resistor noise contribution.

calculation of the system transfer function (frequency domain), while the second one is based on $'formfactors'$ evaluation (time domain).

2.1 Frequency domain calculation (transfer function)

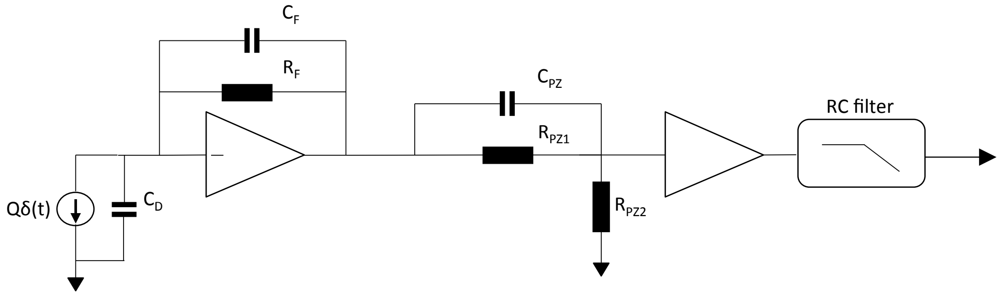

Figure 2: Front end chain for noise calculation

The front-end chain used in our calculation is made of a preamplifier and a shaper with pole-zero cancellation (fig. 2). Its transfer function is (Appendix A):

$$H(s) = \frac{V_{out}}{I_{in}} = \frac{R_{PZ2}}{R_{PZ1} + R_{PZ2}} \frac{R_F}{1 + sR_F C_F} \frac{1 + sR_{PZ1}C_{PZ}}{1 + sR_{PZ}C_{PZ}} \frac{1}{1 + sR_C C_C} \tag{1}$$

where

$$R_{PZ} = \frac{R_{PZ1}R_{PZ2}}{R_{PZ1} + R_{PZ2}}. \tag{2}$$

Assuming $R_F C_F = R_{PZ1}C_{PZ}$ equation (1) can be simplified as shown in (3)

$$H(s) = \frac{R_{PZ2}R_F}{R_{PZ1} + R_{PZ2}} \frac{1}{1 + sR_{PZ}C_{PZ}} \frac{1}{1 + sR_C C_C} \tag{3}$$

Choosing the filter time constant in such a way $R_{PZ}C_{PZ} = R_C C_C = \tau$ equation (3) turn as shown in (4)

$$H(s) = \alpha\tau \cdot \frac{1}{(1 + s\tau)^2} \tag{4}$$

where

$$\alpha\tau = \frac{R_{PZ2}R_F}{R_{PZ1} + R_{PZ2}} = \frac{R_{PZ2}R_{PZ1}}{R_{PZ1} + R_{PZ2}} \frac{1}{R_{PZ1}} R_F \frac{C_{PZ}}{C_{PZ}} = \frac{R_F}{R_{PZ1}C_{PZ}} \tau \tag{5}$$

Finally, by a change of variable $(s = j\omega)$ we obtain the front-end chain transfer function in the frequency domain

$$H(j\omega) = \alpha\tau \cdot \frac{1}{(1 + j\omega\tau)^2}. \tag{6}$$

The rms output voltage noise due to the termination resistor (Appendix B) can be calculated evaluating the integral

$$v_n^2(rms) = i_n^2 \alpha^2 \tau^2 \int_0^\infty |H(j\omega)|^2 df \tag{7}$$

where

$$i_n = \sqrt{\frac{4KT}{R_T}} \tag{8}$$

By changing the integration variable ($\omega\tau = z = 2\pi f\tau$) and extending the integration range to $-\infty$ (the integrand is an even function of z) we obtain

$$v_n^2(rms) = i_n^2 \alpha^2 \tau^2 \frac{1}{4\pi\tau} \int_{-\infty}^{\infty} \frac{dz}{(1+z^2)^2} \tag{9}$$

Evaluating the integral (from tabular computation)

$$\int_{-\infty}^{\infty} \frac{dz}{(1+z^2)^2} = \frac{\pi}{2} \tag{10}$$

we obtain the rms output voltage noise

$$v_n^2(rms) = \frac{4KT}{R_T} \frac{1}{4\pi\tau} \tau^2 \alpha^2 \frac{\pi}{2} = \frac{4KT}{R_T} \frac{1}{8} \tau \alpha^2 \tag{11}$$

To evaluate the system ENC, the square root of the rms output voltage noise must be normalized to the output peak voltage generated by a unit test pulse $\delta(t)$. The system output response to the unit pulse $\delta(t)$ can be obtained by applying the Laplace anti-transform to equation (4)

$$\mathcal{L}^{-1}(\alpha\tau \frac{1}{\tau^2} \frac{1}{(1/\tau + s)^2}) = \alpha \frac{t}{\tau} e^{-t/\tau} \tag{12}$$

Equation (12) has a maximum for $t = \tau$ (peak voltage) and its value is

$$V_{peak} = \alpha \cdot \frac{1}{e} \tag{13}$$

Finally, assuming a peaking time τ of $\simeq$ 3 ns, the ENC due to the 377 Ω termination resistor can be evaluated:

$$ENC = e \cdot \sqrt{\frac{kT}{R_T} \frac{1}{2} \cdot \tau} \simeq \ 0.35 \ fC \simeq \ 2200 \ erms \tag{14}$$

2.2 Time domain calculation (weighting function)

In the previous paragraph noise resistor contribution has been estimated in the frequency domain. Nevertheless it can be also estimated in the time domain, as shown by Radeka, by means of the Parseval's theorem.

The system output noise variance as a function of input white noise spectral density and system time response (see Radeka [2]) obtained in such a way is:

$$\sigma^2 = \frac{1}{2} W_0 \int_{-\infty}^{\infty} h^2(t) dt \tag{15}$$

where W_0 is the input white noise power spectral density. In our case W_0 corresponds to the thermal noise due to the resistor connected to the sense wire and *h(t)* is the system impulse response (the inverse Laplace transform shown in equation (12)).
Evaluating equation (15) for $\forall\ t \geq 0$ (Appendix D) we find

$$\sigma^2 = \frac{1}{2} W_0 \int_0^{\infty} \alpha^2 \left(\frac{t}{\tau}\right)^2 e^{-2\frac{t}{\tau}} dt = \frac{1}{2} \frac{4kT}{R_T} \frac{\alpha^2 \tau}{4} \tag{16}$$

Normalizing with respect to the output peak voltage (equation 13) we get the same result of equation (14)

$$ENC^2 = \frac{1}{2} \frac{kT}{R_T} e^2 \tau \tag{17}$$

More generally, starting from the two basic noise mechanism (input noise current i_n and input noise voltage v_n) the Equivalent Noise charge (ENC) can be expressed as

$$ENC^2 = i_n^2 \tau_S F_i + C^2 v_n^2 \frac{F_v}{\tau_S} \tag{18}$$

where F_i, F_v are the "Form Factors" and depend on the shape of the pulse, τ_S is the shaping (or peaking) time, i_n and v_n are the input current and voltage noise and C is the total input capacitance (detector capacitance + preamplifier input capacitance + stray capacitance).
As we are interested in the evaluation of termination resistor noise contribution and the resistor is in parallel with the preamplifier input we can model it as a current source. Thus, using the ENC general equation (18) and discarding the voltage noise contribution we have:

$$ENC_P^2 = i_n^2 \tau_S F_i \tag{19}$$

The shaping factor F_i is a dimensionless parameter function of the $CR{-}nRC$ filter order. It can be calculated by means of

$$F_i = \frac{1}{2T_S} \int_{-\infty}^{\infty} |W(t)|^2 dt \tag{20}$$

where, for time-invariant filters pulse shaping, $W(t)$ corresponds to the normalized system impulse response and T_S correspond to the system shaping factor. A general solution for $CR - nRC$ unipolar shaping filter can be found in [3] and it is shown in (21).

$$F_i = \frac{1}{2} \left(\frac{e}{2n}\right)^{2n} (2n-1)!\ ;\ F_v = \frac{1}{2} \left(\frac{e}{2n}\right)^{2n} n^2 (2n-2)! \tag{21}$$

The shaping factor F_i affects both the shaper output noise and peaking time as shown in fig.3 where filter output ENC and peaking time ($t_r = n\tau$) have been plotted as a function

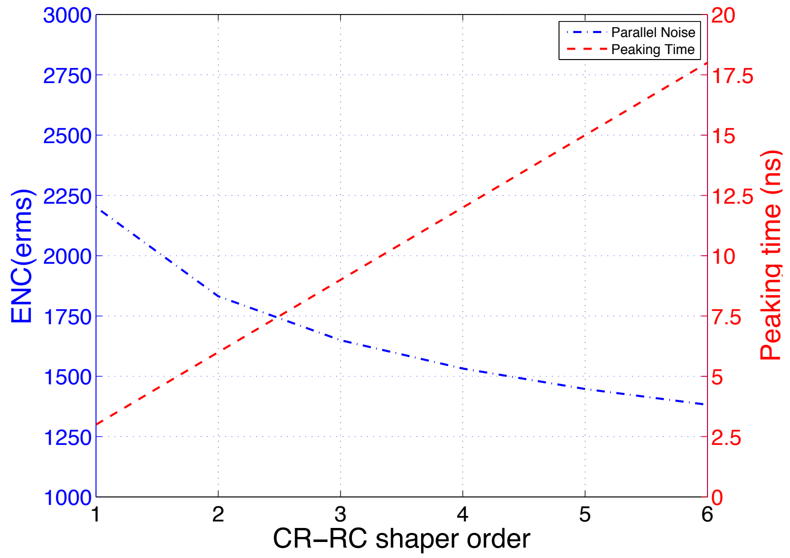

Figure 3: $ENC(erms)$ and peaking time as a function of CR-RC shaping filter order

of the filter order (n).

Finally, including the effect of parasitic capacitances on total noise, we can use the equation (18) to optimize the shaping time in our system. Again, to simplify calculation, let we consider only the contributions of the protection circuit against gas discharges and the detector capacitance.

The simplest protection circuit is, generally, made of one resistor (value around $10-20\ \Omega$) and two diodes. It contributes to the overall noise by means of two factors: the diodes parasitic capacitance and the resistor voltage noise ($\sqrt{4KTR}$). As an example fast switching diodes, like $BAV99$, have a parasitic capacitance of about $1\ pF$, then, considering 2 diodes we get about $2\ pF$ that must be added to the $21\ pF$ of the detector capacitance (2.7 mt sense wire length) for a total parasitic capacitance of $\sim\ 23\ pF$.

Figure 4 shows the plot of the parallel and series contribution together with the total ENC noise assuming a 20 Ω series resitor. A minimum for ENC noise is obtained for a shaping time around 2 ns.

3 Conclusions

The correct implementation of Cluster Counting technique requires sense wire termination to avoid signal reflection. This introduces, considering a front-end chain made of preamplifier and $CR-RC$ shaping circuit and a $3\ ns$ shaping time, an unavoidable noise

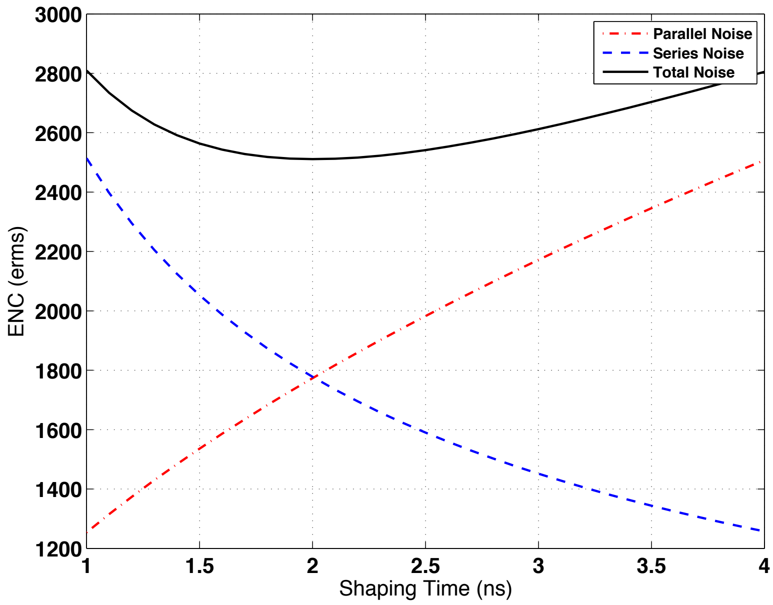

Figure 4: Total $ENC(erms)$ as a function of shaping time

baseline of about 2200 $erms$.

References

[1] A.H. Walenta. IEEE Trans. Nucl. Sci. NS- 26 (1979) 73.

[2] Ann. Rev. Nud. ParI. Sci. 1988. 38: 217-77.

[3] Particle detection with drift chamber. Springer

4 Appendix A - Pole-Zero transfer function calculation

with reference to fig.2 for the pole-zero network we have

$$H_{PZ}(s) = \frac{R_{PZ2}}{R_{PZ2} + \frac{R_{PZ1}}{1+sR_{PZ1}C_{PZ}}} = \frac{R_{PZ2}(1 + sR_{PZ1}C_{PZ})}{R_{PZ1} + R_{PZ2}(1 + sR_{PZ1}C_{PZ})} \tag{22}$$

expanding

$$H_{PZ}(s) = \frac{R_{PZ2} + sR_{PZ1}R_{PZ2}C_{PZ}}{R_{PZ1} + R_{PZ2} + SR_{PZ1}R_{PZ2}C_{PZ}} \tag{23}$$

highlighting $(R_{PZ1} + R_{PZ2})$ equation (23) can be written as

$$H_{PZ}(s) = \frac{\frac{R_{PZ2}}{R_{PZ1}+R_{PZ2}} + s\frac{R_{PZ1}R_{PZ2}}{R_{PZ1}+R_{PZ2}}C_{PZ}}{1 + s\frac{R_{PZ1}R_{PZ2}}{R_{PZ1}+R_{PZ2}}C_{PZ}} \tag{24}$$

and, finally

$$H_{PZ}(s) = \frac{R_{PZ2}}{R_{PZ1} + R_{PZ2}} \frac{1 + sR_{PZ1}C_{PZ}}{1 + sR_{PZ}C_{PZ}} \tag{25}$$

where

$$R_{PZ} = \frac{R_{PZ1}R_{PZ2}}{R_{PZ1} + R_{PZ2}} \tag{26}$$

5 Appendix B - Resistor thermal noise

Thermal noise in resistor is generated by electron density fluctuations. Its spectral noisy density can be expressed as

$$\frac{dv_n^2}{df} = e_n^2 = 4kTR \; or \; \frac{di_n^2}{df} = i_n^2 = \frac{4kT}{R} \tag{27}$$

Because spectral noise components are not correlated the total noise power (in term of root mean square) at the output of a filter with transfer function $H(j\omega)$ can be be obtained by integrating the product of the input noise spectral densityt and the transfer function squared module, i.e.

$$v_{no}^2 = \int_0^\infty v_n^2|H(\omega)|^2 d\omega \; or \; i_{no}^2 = \int_0^\infty \imath_n^2|H(\omega)|^2 d\omega \tag{28}$$

6 Appendix C - ENC calculation in time domain

$$\alpha^2 \int_{-\infty}^{\infty} \left(\frac{t}{\tau}\right)^2 e^{-2\frac{t}{\tau}} dt = \alpha^2\tau \int_{-\infty}^{\infty} \left(\frac{t}{\tau}\right)^2 e^{-2\frac{t}{\tau}} d\frac{t}{\tau} = \alpha^2\tau \int_{-\infty}^{\infty} x^2 e^{-2x} dx \tag{29}$$

where $x = t/\tau$.
Because

$$\int_0^\infty x^m e^{-ax} dx = \frac{\Gamma(m+1)}{a^{(m+1)}} \tag{30}$$

thus for $t \geq 0$

$$\alpha^2 \tau \int_{-\infty}^{\infty} x^2 e^{-2x} dx = \frac{\alpha^2 \tau}{4} \tag{31}$$