INFN

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

FRASCATI PHYSICS SERIES

HANDBOOK ON NEUTRAL KAON INTERFEROMETRY AT A Φ-FACTORY

Editor
A. Di Domenico

HANDBOOK ON
NEUTRAL KAON INTERFEROMETRY AT A Φ-FACTORY

FRASCATI PHYSICS SERIES

Series Editor
Stefano Bianco

Technical Editor
Luigina Invidia

Cover:
Photograph by A. Di Domenico

Volume XLIII

Istituto Nazionale di Fisica Nucleare – Laboratori Nazionali di Frascati
Divisione Ricerca – SIS – Ufficio Pubblicazioni
P.O. Box 13, I–00044 Frascati (Roma) Italy
email: sis.publications@lnf.infn.it

FRASCATI PHYSICS SERIES

Volume XLIII

HANDBOOK ON NEUTRAL KAON INTERFEROMETRY AT A Φ-FACTORY

Istituto Nazionale di Fisica Nucleare

Laboratori Nazionali di Frascati

Editor
A. Di Domenico

Frascati, 2007

FRASCATI PHYSICS SERIES

HANDBOOK ON
NEUTRAL KAON INTERFEROMETRY AT A Φ-FACTORY

ISBN 978—88–86409–50–8

Printed in Italy
by Poligrafica Laziale
P.le della Stazione 4/6, 00044 Frascati

PREFACE

At the beginning of 2006 the KLOE experiment concluded its data-taking phase at the DAΦNE e^+e^- collider, the ϕ-factory of the Frascati National Laboratories of INFN (LNF), collecting about 2.5 fb^{-1} of total integrated luminosity. At the same time there was an intense activity to outline the future LNF research programs[1], and more generally the INFN roadmap for the following years. A couple of working groups, set-up in the framework of the INFN roadmap studies, investigated in detail the prospects for e^+e^- physics at LNF in the hypothesis of a new DAΦNE machine upgraded in luminosity and energy[2]. In the meanwhile a proposal for the continuation of the KLOE physics program was submitted by the KLOE-2 collaboration[3] to the LNF directorate, where the hypothesis of a ϕ-factory able to deliver an integrated luminosity of about 50 fb^{-1} in few years of data taking was considered.

A unique feature of a ϕ-factory is the production of neutral kaon pairs in a pure quantum state with the consequent possibility to study quantum interference effects, and to have pure monochromatic tagged K_S and K_L beams. Besides the possibility to measure to high accuracy most, if not all, of the properties of the kaon system, the correlation between the two kaons could open up new horizons in the study of discrete symmetries and of the basic principles of quantum mechanics. For instance possible CPT violations could manifest in conjunction with tiny modifications of the initial correlation, decoherence effects, or Lorentz symmetry violations, which, in turn, might be justified in a quantum theory of gravity. At KLOE the sensitivity to some observable effects reached the level of the interesting Planck's scale region, i.e. $O(m_K^2/M_{planck})\sim 2 \times 10^{-20}$ GeV, which is a very remarkable level of accuracy (presently unreachable in other similar systems, e.g. the B meson

---

[1] http://www.lnf.infn.it/lnfadmin/direzione/roadmap/roadmap.html
[2] F. Ambrosino et al., Eur. Phys. J C 50 (2006) 729, see also http://www.roma1.infn.it/people/bini/roadmap.html, http://www.infn.it/csn1/Roadmap/Gruppok/index.html
[3] http://www.lnf.infn.it/lnfadmin/direzione/roadmap/LoIKLOE.pdf

system), and significant improvements are expected with an integrated luminosity of 50 fb^{-1}.

Moreover recent theoretical studies demonstrated that entangled neutral kaons at a ϕ-factory are suitable to test the foundations of quantum mechanics, such as Bohr's complementarity principle, the quantum erasure and marking concepts, and the coherence of states over macroscopic distances, while for the more "classical" test with Bell's inequalities, new ideas have been put forward.

During the working group activity, it immediately appeared evident the necessity of a comprehensive and updated review on neutral kaon interferometry, and of an extended assessment of its physics potentials. In fact, the few excellent papers in the DAΦNE Physics Handbook[4], after more than ten years since its publication, needed at least an update to take into account the vast subsequent literature on this subject.

Therefore as a first step toward this aim, a mini-workshop entitled "Neutral kaon interferometry at a ϕ-factory: from quantum mechanics to quantum gravity" was held on March 24th 2006 in Frascati[5]. Review talks were given by G. Amelino-Camelia, J. Bernabeu, R. Bertlmann, A. Bramon, R. Floreanini, A. Go, B. Hiesmayr, G. Isidori, R. Lehnert, N. Mavromatos, and myself. I thank all the speakers for having accepted the invitation, for their interesting presentations, and for their contribution to the success of the workshop.

As a second more ambitious project, the idea was put forward to write a comprehensive report gathering all relevant and updated information on the subject, which was scattered in the literature. The report would have been in the form of a handbook (as a sort of addendum to the DAΦNE Physics Handbook), with extensive and comprehensible contributions, useful to experimental physicists and to people willing to have a comprehensive overview on neutral kaon interferometry at a ϕ-factory. In this spirit, written contributions were given by all speakers of the above mentioned workshop, joined by M. Arzano, F. Benatti, J. Ellis, G. Garbarino, A. Marcianò, D. Nanopoulos, J. Papavassiliou, and S. Sarkar. I warmly acknowledge

---

[4] The second DAΦNE Physics Handbook, edited by L. Maiani, G. Pancheri, N. Paver, INFN-LNF, Frascati, 1995

[5] The slides are available at http://www.roma1.infn.it/people/didomenico/roadmap/kaoninterferometry.html.

them, without whom contribution the present Handbook would have not become a reality.

A special thank is due to the Director of the Frascati Laboratories M. Calvetti, for his interest and constant support for the workshop and the handbook project, to the President of the INFN CSN-1 F. Ferroni, who supported this activity inside the INFN roadmap working group, and to the Director of the INFN Sezione di Roma S. Falciano, for the partial financial support of the workshop.

I express my gratitude to the Spokesperson of the KLOE experiment P. Franzini for useful discussions on the subject and for being for me a constant point of reference, to R. Baldini for stimulating discussions and suggestions, to J. Lee-Franzini, G. Capon, M. Curatolo and all people attending with interest, and contributing to the lively atmosphere and the success of the workshop.

I am grateful to M. Fidecaro for her suggestions and many useful discussions on the subject.

I wish to thank L. Sabatini for her invaluable help in the logistic preparation of the workshop, and all the SIS staff for their constant support. Finally, a special thank to L. Invidia for her crucial help and patience in the completion of this volume.

Antonio Di Domenico

NEUTRAL KAON INTERFEROMETRY AT A Φ–FACTORY:
from Quantum Mechanics to Quantum Gravity

Laboratori Nazionali di Frascati dell'INFN
March 24th, 2006 (9:30 – 18:30)
Aula Touschek

A one-day workshop devoted to discuss the potentialities of neutral kaon interferometry at a ϕ-factory:

Decoherence and novel CPT violation in quantum gravity
Open quantum dynamics and complete positivity
CPT and Lorentz symmetry breaking
Entanglement, Bell's inequalities and other tests of Quantum Mechanics
CPT violation and Bell-Steinberger relation
Review of experimental results and perspectives

Speakers:
G. Amelino-Camelia (Roma), J. Bernabeu (Valencia), R. Bertlmann (Vienna), A. Bramon (Barcelona), R. Floreanini (Trieste), A. Go (CERN), B. Hiesmayr (Vienna), G. Isidori (LNF), R. Lehnert (MIT), N. Mavromatos (London)

Organization: A. Di Domenico (Roma)
E-mail: antonio.didomenico@roma1.infn.it

Secretariat: L. Sabatini (lia.sabatini@lnf.infn.it)
web page: http://www.lnf.infn.it/conference/

INFN Sezione di Roma
c/o Dipartimento di Fisica
Università degli Studi di Roma "La sapienza"

INFN Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati

CONTENT

Preface V

A. Di Domenico 1

Neutral kaon interferometry at a ϕ-factory 1

1 Introduction 1

2 The neutral kaon system 3

3 Correlated kaons 6

4 Kaon interferometry at a ϕ-factory 7

4.1 Decays into two charged and two neutral pions 9

4.2 Double semileptonic decays 11

4.3 Semileptonic and two pion decays 12

4.4 Decays into identical final states 13

5 Entanglement and neutral kaons 14

6 Decoherence and CPT violation 16

6.1 Furry's hypothesis and a simple decoherence model 16

6.2 A general approach to decoherence 17

6.3 Decoherence and CPT violation due to quantum gravity effects 17

7 CPT violation and Lorentz symmetry breaking 20

7.1 Measurement of Δa_0 at a ϕ-factory 22

7.2 Measurement of Δa_Z at a ϕ-factory 23

7.3 Measurement of Δa_X , Δa_Y at a ϕ-factory 24

8 The KLOE experiment at DAΦNE 24

8.1 Decoherence and CPT symmetry tests 25

8.2 CPT symmetry tests with $K_S \rightarrow \pi e \nu$ decays 28

8.3 CPT test from unitarity 29

8.4 Lorentz and CPT symmetries tests 29

9 The KLOE-2 program 30

10 Conclusions 33

References 35

J. Bernabéu, J. Ellis, N. Mavromatos, D. Nanopoulos, J. Papavassiliou 39
CPT and quantum mechanics tests with kaons 39
1 CPT Symmetry and Quantum Gravity: Motivations for its Possible Violation 40
2 Standard Model Extension, Lorentz Violation and Neutral Kaons 45
2.1 Formalism and Order-of-Magnitude Estimates 45
2.2 Tests of Lorentz Violation in Neutral Kaons 47
3 QG Decoherence and CPTV in Neutral Kaons 49
3.1 Stochastically Fluctuating Geometries, Light Cone Fluctuations and Decoherence: General Ideas 49
3.2 Formalism for the Phenomenology of QG-induced Decoherence 51
3.3 Experiments involving Single-Kaon States 54
4 CPTV and Modified EPR Correlations of Entangled Neutral Kaon States 61
4.1 EPR Correlations in Particle Physics 61
4.2 CPTV and Modified EPR-Correlations in ϕ Factories: the ω-Effect 62
4.3 ω-Effect Observables 64
4.4 Microscopic Models for the ω-Effect and Order-of-Magnitude Estimates 66
4.5 Disentangling the ω-Effect from the C(even) Background 71
4.6 Disentangling the ω-Effect from Decoherent Evolution Effects 73
5 Precision T, CP and CPT Tests with Charged Kaons 73
6 Instead of Conclusions 78
Acknowledgements 79
References 80

S. Sarkar 85
Methods and models for the study of decoherence 85
1 The role of decoherence 85
2 General Features of Decoherence 86
3 Particles propagating in a medium and master equations 89
4 Master Equations from (Non-critical) String Theory 94
5 CPT and Recoil 100

Acknowledgements 102
References 102

F. Benatti, R. Floreanini 107
Open quantum dynamics: complete positivity and correlated neutral kaons 107
1 Introduction 107
2 Positivity and complete positivity 110
3 Test of complete positivity at a ϕ-factory 115
4 Tests of dissipative effects in kaon dynamics 122
References 126

R. Lehnert 131
CPT- and Lorentz-symmetry breaking: a review 131
1 Introduction 131
2 Spacetime symmetries and their interplay 133
3 Sample mechanisms for spacetime-symmetry breaking 134
4 The Standard-Model Extension 138
5 CPT and Lorentz tests with mesons 140
6 Experiments 146
7 Summary 150
Acknowledgements 151
References 152

G. Amelino-Camelia, M. Arzano, A. Marcianò 155
On the quantum gravity phenomenology of multiparticle states 155
1 Introduction 155
2 A perspective on multiparticle states in the quantum-gravity realm 156
3 A simple toy model 159
4 On other areas of quantum-gravity phenomenology 163
4.1 Quantum-Gravity Phenomenology exists 163

4.2 Falsifying theories ... 165
4.3 Concerning quantum-gravity effects and the status of Quantum-Gravity theories ... 166
4.3.1 Planck-scale departures from classical spacetime symmetries ... 167
4.3.2 Planck-scale departures from CPT symmetry ... 167
4.3.3 Distance fuzziness and spacetime foam ... 168
4.3.4 Decoherence ... 169
4.3.5 Planck-scale departures from the Equivalence Principle ... 169
4.3.6 Critical-dimension SuperString Theory ... 169
4.3.7 Loop Quantum Gravity ... 170
4.3.8 Approaches based on noncommutative geometry ... 171
4.4 On the status of different areas of this phenomenology ... 172
4.4.1 Planck-scale modifications of Poincaré symmetries ... 172
4.4.2 Planck-scale modifications of CPT symmetry and Decoherence ... 172
4.4.3 Distance fuzziness and spacetime foam ... 173
4.4.4 Decoherence ... 174
4.4.5 Planck-scale departures from the Equivalence Principle ... 174
4.5 Aside on Doubly-Special Relativity ... 174
4.6 More on the phenomenology of departures from Poincaré symmetry ... 177
4.6.1 On the test theories with modified dispersion relation ... 177
4.6.2 Photon stability ... 178
4.6.3 Threshold anomalies ... 180
4.6.4 Time-of-travel analyses ... 180
4.6.5 Synchrotron radiation ... 181
4.6.6 Neutrino observations ... 181
References ... 182

G. Isidori ... 187
Testing CPT in the neutral kaon system by means of the Bell-Steinberger relation ... 187
1 Introduction ... 187

2 Theoretical framework 188
2.1 Two-pion modes 189
2.2 Three-pion modes 190
2.3 Semileptonic channels 191
2.4 Determination of Re(ε) and Im(δ) 191
3 Present experimental status and future prospects 192
Acknowledgements 195
References 195

R. Bertlmann, B. Hiesmayr 197
Strangeness measurements of kaon pairs, CP violation and Bell inequalities 197
1 Introduction 197
2 Kaons as qubits 199
2.1 Quantum states of kaons 199
2.2 Strangeness oscillation 200
2.3 Quasi–spin of kaons and analogy to photons 201
2.4 Measurement procedures 202
3 Entangled kaons, Bell inequalities, CP violation 204
3.1 Entanglement 204
3.2 Bell inequality for quasi–spin variation 206
3.3 Semileptonic decays 206
3.4 Bell inequality for time variation 208
4 Conclusions 211
Acknowledgements 212
References 212

A. Bramon, R. Escribano, G. Garbarino 217
A review of Bell inequality tests with neutral kaons 217
1 Introduction 217
2 Requirements for a genuine Bell inequality 221

3 Bases in 'quasi–spin' space 223
3.1 Strangeness basis: {K^0, $\underline{K}^0$} 224
3.2 Free–space basis: {K_S, K_L} 224
3.3 Inside–matter basis: {K_S', K_L'} 226
4 Dichotomic measurements on neutral kaon states 228
4.1 Strangeness measurements 228
4.2 Lifetime measurements 229
4.3 Active vs passive measurements 229
5 Entangled states of neutral kaon pairs 230
5.1 Maximally entangled states 230
5.2 Non–maximally entangled states 232
6 Bell inequalities with neutral kaon pairs 233
6.1 Clauser–Horne inequalities 233
6.2 Eberhard inequalities 235
6.3 Clauser–Horne–Shimony–Holt inequalities 236
6.4 Wigner inequalities 238
7 A review of the proposals 239
7.1 Assuming fair sampling and perfect anticorrelation 239
7.1.1 A first proposal 239
7.1.2 Improved proposal with strangeness detection and thin regenerators 240
7.2 Assuming fair sampling 241
7.2.1 Proposal with strangeness detection 241
7.2.2 Proposals with lifetime detection and regenerators 242
7.2.3 Proposal with both lifetime and strangeness detection 243
7.3 An attempt of genuine test 245
8 Conclusions 249
Acknowledgements 249
References 249

A. Bramon, G. Garbarino, B. Hiesmayr 255
Kaonic quantum erasers at a ϕ–factory: "Erasing the Present, changing the Past" ... 255
1 Introduction 255
2 Kaons as double slits 256
3 Kaonic quantum eraser 259
(a) Active eraser with active measurements 262
(b) Partially passive quantum eraser with active measurements 265
(c) Passive eraser with passive measurements on the meter 266
(d) Passive eraser with passive measurements 267
4 Conclusions 268
Acknowledgements 270
References 270

A. Go 273
Kaon interferometry at CPLEAR 273
1 Introduction 273
2 CPLEAR experiment 273
3 Loss of quantum coherence 274
4 Non-separability of the $K^0\underline{K}^0$ wave function 275
5 Testing Bell Inequality at KLOE2 277
6 Conclusion 280
References 280

NEUTRAL KAON INTERFEROMETRY AT A Φ-FACTORY

Antonio Di Domenico
Dipartimento di fisica, Universitá di Roma "La Sapienza"
& INFN Sezione di Roma, Rome, Italy
E-mail: antonio.didomenico@roma1.infn.it

Abstract

Interferometric methods applied to neutral kaon pairs at a ϕ-factory offer unique possibilities to perform fundamental tests of discrete symmetries, as well as of the basic principles of quantum mechanics. In this paper a general review on neutral kaon interferometry at a ϕ-factory is given. The most recent results obtained by the KLOE experiment at the DAΦNE e^+e^- collider, the Frascati ϕ-factory, are reviewed. A recent proposal for continuing the KLOE physics program (KLOE-2) at an upgraded DAΦNE machine is discussed in this context.

1 Introduction

The neutral kaon doublet is one of the most intriguing systems in nature. During its time evolution a neutral kaon oscillates between its particle and antiparticle states with a beat frequency $\Delta m \approx 5.3 \times 10^9$ s^{-1} , where Δm is the small mass difference between the exponentially decaying states K_L and K_S. The fortunate coincidence that Δm is about half the decay width of K_S makes possible to observe a variety of intricate interference phenomena in the production and decay of neutral kaons. In turn, such observations enable us to test the linear superposition principle of quantum mechanics, the interplay of different conservation laws and the validity of various symmetry principles.

A unique feature of a ϕ-factory is the production of neutral kaon pairs in a pure quantum state with the consequent possibility to study quantum interference effects, and to have pure monochromatic tagged K_S and K_L beams. Besides the possibility to measure to high accuracy most, if not all, of the properties of the kaon system, the correlation between the two kaons could open up new horizons in the study of discrete symmetries and of the basic principles of quantum mechanics. For instance a possible violation of the CPT

symmetry[1] (where C is charge conjugation, P is parity, and T is time reversal) could manifest in conjunction with tiny modifications of the initial correlation, decoherence effects, or Lorentz symmetry violations, which, in turn, might be justified in a quantum theory of gravity. At a ϕ-factory the sensitivity to some observable effects can reach the level of the interesting Planck's scale region, i.e. $\mathcal{O}(m_K^2/M_{Planck}) \sim 2\times 10^{-20}$ GeV, which is a very remarkable level of accuracy, presently unreachable in other similar systems (e.g. the B meson system). Moreover recent theoretical studies demonstrated that entangled neutral kaons at a ϕ-factory are suitable to test the foundations of quantum mechanics, such as Bohr's complementarity principle, the quantum erasure and marking concepts, and the coherence of states over macroscopic distances, while for the more *classical* test using Bell's inequalities, new ideas have been put forward. Therefore neutral kaon interferometry constitutes a powerful tool and a very attractive opportunity to be fully exploited at a ϕ-factory.

This paper is organized as follows: a brief introduction on the neutral kaon system is given in Sects. 2 and 3; the basic concepts of neutral kaon interferometry and the description of the most important "standard" tests on discrete symmetries that can be performed at a ϕ-factory are reviewed in Sect. 4; a brief introduction is given on possible tests of quantum mechanics (Sect. 5), decoherence and CPT violation effects that could be induced in a quantum gravity framework (Sect. 6), and CPT and Lorentz symmetry violation effects (Sect. 7). Detailed reviews on these subjects can be found in the other contributions of this handbook. The most recent results of the KLOE experiment at DAΦNE, the Frascati ϕ-factory, are reviewed in Sect. 8; finally, the improved sensitivities and prospects for the proposed KLOE-2 experiment are discussed in Sect. 9.

[1]The CPT theorem [1, 2, 3, 4] ensures that exact CPT invariance holds for any quantum field theory assuming (1) Lorentz invariance, (2) Locality, and (3) Unitarity. Testing the validity of CPT invariance therefore probes the most fundamental assumptions of our present understanding of particles and their interactions.

2 The neutral kaon system

The time evolution of a neutral kaon that is initially a generic superposition of K^0 and $\bar{K^0}$,

$$|K(0)\rangle = a(0)|K^0\rangle + b(0)|\bar{K^0}\rangle \ , \tag{1}$$

can be described by the state vector

$$|K(t)\rangle = a(t)|K^0\rangle + b(t)|\bar{K^0}\rangle + \sum_j c_j(t)|f_j\rangle \ , \tag{2}$$

where t is the time in the kaon rest frame, f_j's with $\{j - 1, 2, ...\}$ represent all possible decay final states, and $a(t)$, $b(t)$, and $c_j(t)$ are time dependent functions. In the Wigner-Weisskopf approximation [5], which is valid for times larger than the typical strong interaction formation time, the functions $a(t)$ and $b(t)$, describing the time evolution of the state in the $\{K^0, \bar{K^0}\}$ sub-space, obey the Schrödinger-like equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \mathbf{H}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \ , \tag{3}$$

where the effective Hamiltonian $\mathbf{H}$ is a 2×2 complex, not Hermitian, and time independent matrix. It can be decomposed in terms of its hermitian and anti-hermitian parts

$$\begin{aligned} \mathbf{H} &= \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \\ &= \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix} \ , \end{aligned} \tag{4}$$

where $\mathbf{M}$ and $\mathbf{\Gamma}$ are two hermitian matrices with positive eigenvalues, usually called *mass* and *decay* matrices, and indices 1 and 2 stand for K^0 and $\bar{K^0}$, respectively.

The true Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{wk}$, where $\mathcal{H}_0$ governs the strong and electromagnetic interactions and conserve strangeness ($\mathcal{H}_0|K^0\rangle = M_0|K^0\rangle$, $\mathcal{H}|\bar{K^0}\rangle = M_0|\bar{K^0}\rangle$, $S|K^0\rangle = |K^0\rangle$, $S|\bar{K^0}\rangle = -|\bar{K^0}\rangle$), while $\mathcal{H}_{wk}$ is a small perturbation governing weak interactions and not conserving strangeness, is related to the effective Hamiltonian $\mathbf{H}$ as follows:

$$M_{ij} = M_0\delta_{ij} + \langle i|\mathcal{H}_{wk}|j\rangle + \mathcal{P}\sum_f \left(\frac{\langle i|\mathcal{H}_{wk}|f\rangle\langle f|\mathcal{H}_{wk}|j\rangle}{M_0 - E_f}\right) \tag{5}$$

$$\Gamma_{ij} \;=\; 2\pi \sum_f \langle i|\mathcal{H}_{wk}|f\rangle\langle f|\mathcal{H}_{wk}|j\rangle\delta(M_0 - E_f) \tag{6}$$

where $i, j = 1, 2$, $\mathcal{P}$ stands for the principal part, and the intermediate states f correspond to virtual ($\mathbf{M}$) or real ($\mathbf{\Gamma}$) decay channels.

The matrix $\mathbf{H}$ is characterized by eight independent real parameters; seven of them are observables, while one phase is arbitrary and unphysical. In fact the flavor symmetry of the strong interaction leaves the freedom to redefine the relative phase of $|K^0\rangle$ and $|\bar{K^0}\rangle$ states:

$$\begin{aligned} |K^0\rangle &\rightarrow e^{i\vartheta}|K^0\rangle \\ |\bar{K^0}\rangle &\rightarrow e^{-i\vartheta}|\bar{K^0}\rangle \ , \end{aligned} \tag{7}$$

implying that the off-diagonal elements of H depend on the arbitrary phase ϑ

$$\begin{aligned} H_{12} &\rightarrow e^{-2i\vartheta}H_{12} \\ H_{21} &\rightarrow e^{2i\vartheta}H_{21} \ . \end{aligned} \tag{8}$$

Thus expressions which depend on ϑ are not suited to represent experimental results, unless ϑ is fixed to a definite value by convention. However the diagonal elements of $\mathbf{H}$, the product of the off-diagonal elements, their absolute values, the trace of $\mathbf{H}$, its determinant and eigenvalues are all phase convention independent quantities.

The conservation of discrete symmetries constrains the matrix elements of $\mathbf{H}$, and the following phase-invariant conditions hold[2]:

$$H_{11} = H_{22} \qquad \text{for } CPT \text{ conservation,} \tag{9}$$

$$|H_{12}| = |H_{21}| \qquad \text{for } T \text{ conservation,} \tag{10}$$

$$H_{11} = H_{22} \ \text{ and } \ |H_{12}| = |H_{21}| \qquad \text{for } CP \text{ conservation.} \tag{11}$$

The eigenvalues of $\mathbf{H}$ are

$$\begin{aligned} \lambda_S &= m_S - i\Gamma_S/2 \\ \lambda_L &= m_L - i\Gamma_L/2 \ , \end{aligned} \tag{12}$$

[2]For a general review on discrete symmetries in the neutral kaon system see Refs. 6, 7, 8, 9, 10, 11).

where $m_{S,L}$ and $\Gamma_{S,L}$ are the masses and widths of the physical states, respectively. It is also useful to define the differences

$$\begin{aligned} \Delta m &= m_L - m_S > 0 \\ \Delta\Gamma &= \Gamma_S - \Gamma_L > 0 \end{aligned} \tag{13}$$

and the so called *superweak* phase

$$\tan\phi_{SW} = \frac{2\Delta m}{\Delta\Gamma} \ . \tag{14}$$

The physical states that diagonalize $\mathbf{H}$ are the short- and long-lived states; they evolve in time as pure exponentials

$$\begin{aligned} |K_S(t)\rangle &= e^{-i\lambda_S t}|K_S\rangle \\ |K_L(t)\rangle &= e^{-i\lambda_L t}|K_L\rangle \ , \end{aligned} \tag{15}$$

and are usually written as:

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{2(1+|\epsilon_S|^2)}}\{(1+\epsilon_S)|K^0\rangle + (1-\epsilon_S)|\bar{K}^0\rangle\} \\ |K_L\rangle &= \frac{1}{\sqrt{2(1+|\epsilon_L|^2)}}\{(1+\epsilon_L)|K^0\rangle - (1-\epsilon_L)|\bar{K}^0\rangle\} \ , \end{aligned} \tag{16}$$

where $\epsilon_{S,L}$ are two small complex parameters describing the CP impurity in the physical states; one can equivalently define the parameters

$$\bar{\epsilon} \equiv (\epsilon_S + \epsilon_L)/2 \ , \qquad \delta \equiv (\epsilon_S - \epsilon_L)/2 \ . \tag{17}$$

Ignoring negligible quadratic terms, they can be expressed in terms of the elements of $\mathbf{H}$ as:

$$\bar{\epsilon} = \frac{H_{12} - H_{21}}{2(\lambda_S - \lambda_L)} = \frac{-i\Im M_{12} - \frac{1}{2}\Im\Gamma_{12}}{\Delta m + i\,(\Delta\Gamma)\,/2} \tag{18}$$

$$\delta = \frac{H_{11} - H_{22}}{2(\lambda_S - \lambda_L)} = \frac{\frac{1}{2}\left(M_{22} - M_{11} - \frac{i}{2}\,(\Gamma_{22} - \Gamma_{11})\right)}{\Delta m + i\,(\Delta\Gamma)\,/2} \ . \tag{19}$$

It is worth noting that the parameter $\bar{\epsilon}$ is phase convention dependent. The arbitrariness in the choice of the phase ϑ can be conveniently used to have either $\arg(\Gamma_{12}) = 0$ (in this case $\bar{\epsilon} = |\bar{\epsilon}|e^{i\phi_{SW}}$), or the phase of some decay amplitude such that $\arg(\Gamma_{12}) \ll 1$ (as in the Wu-Yang phase convention [12]).

In this case it can be shown [9, 11, 13, 14] that the real part of $\bar{\epsilon}$ does not depend on $\arg(\Gamma_{12})$, and the following relation holds[3]:

$$\frac{|H_{12}|^2 - |H_{21}|^2}{|H_{12}|^2 + |H_{21}|^2} \simeq 4\Re\bar{\epsilon} \ . \tag{20}$$

Then it is easy to show that

- $\delta \neq 0$ implies CPT violation;
- $\Re\bar{\epsilon} \neq 0$ implies T violation;
- $\Re\bar{\epsilon} \neq 0$ or $\delta \neq 0$ implies CP violation.

The effective Hamiltonian **H** can thus be expressed in terms of the following 7 observable quantities: 4 being in the complex eigenvalues $\lambda_{S,L}$, 2 in the complex parameter δ, and 1 in the real part of $\bar{\epsilon}$.

3 Correlated kaons

The correlations between the decay modes of a system consisting of a $K\bar{K}$ pair produced in nucleon-antinucleon annihilation were first considered in 1958 by Goldhaber, Lee and Yang [15]. Neutral kaon pairs can also be produced in the strong decay of some scalar, vector, or tensor unflavored neutral mesons, e.g. f_0, ϕ, or f_2', with definite $J^{PC} = 0^{++}, 1^{--}, 2^{++}$ quantum numbers. In such a case only the two following zero strangeness states need to be considered:

$$\begin{array}{c} |K^0(+\vec{p})\rangle|\bar{K}^0(-\vec{p})\rangle \\ |\bar{K}^0(+\vec{p})\rangle|K^0(-\vec{p})\rangle \end{array} \tag{21}$$

where the kaon momentum $+\vec{p}$ (or $-\vec{p}$) is specified in the decaying meson rest frame. Neutral kaons are spinless bosons and the physical $K^0\bar{K}^0$ state is required to be symmetric under the combined operation of charge conjugation C and permutation of space coordinates $\mathcal{P}$, i.e. $C\mathcal{P} = +1$. For an arbitrary and well defined orbital angular momentum L, the system is an eigenstate of C with eigenvalue $(-1)^L$. Hence, for the decay of scalar or tensor mesons into

[3] Always neglecting $|\bar{\epsilon}|^2 \ll 1$ and $|\delta|^2 \ll 1$.

$K^0\bar{K}^0$, one has $L = 0, 2$ ($C = \mathcal{P} = +1$) and necessarily the following symmetric combination of states (21):

$$\begin{aligned}
|i\rangle &= \frac{1}{\sqrt{2}}\{|K^0(+\vec{p})\rangle|\bar{K}^0(-\vec{p})\rangle + |\bar{K}^0(+\vec{p})\rangle|K^0(-\vec{p})\rangle\} \\
&= \frac{1}{\sqrt{2}}\{[|K_S(+\vec{p})\rangle|K_S(-\vec{p})\rangle - |K_L(+\vec{p})\rangle|K_L(-\vec{p})\rangle] \\
&\quad -2\delta[|K_S(+\vec{p})\rangle|K_L(-\vec{p})\rangle + |K_L(+\vec{p})\rangle|K_S(-\vec{p})\rangle]\} \qquad (22)
\end{aligned}$$

while, for the decay of vector mesons, one has $L = 1$ ($C = \mathcal{P} = -1$) and the antisymmetric combination:

$$\begin{aligned}
|i\rangle &= \tfrac{1}{\sqrt{2}}\{|K^0(+\vec{p})\rangle|\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(+\vec{p})\rangle|K^0(-\vec{p})\rangle\} \\
&= \tfrac{N}{\sqrt{2}}\{|K_S(+\vec{p})\rangle|K_L(-\vec{p})\rangle - |K_L(+\vec{p})\rangle|K_S(-\vec{p})\rangle\} \qquad (23)
\end{aligned}$$

where

$$N = \frac{\sqrt{(1+|\epsilon_S|^2)(1+|\epsilon_L|^2)}}{(1-\epsilon_S\epsilon_L)} \simeq 1 \qquad (24)$$

is a normalization factor.

It is worth noting that:

- for identical spinless bosons, Bose statistics forbids states with odd angular momentum; hence in the case $L = 1$ terms like K_SK_S or K_LK_L (or K^0K^0, etc.) cannot appear; this is also true for simultaneous kaon states at any time in the evolution of the system after production; the state results totally antisymmetric, and eq.(23) is exact regardless of any CP or CPT violation in the neutral kaon system (apart the case of a possible CPT violation in which Bose statistics does not apply, as described in Ref. [16, 17]);

- in the case $L = 0, 2$ terms of the order $\bar{\epsilon}^2$ and δ^2 have been neglected in eq.(22) but the effect of possible CPT violation has been included, leading to the appearance of K_SK_L and K_LK_S terms.

4 Kaon interferometry at a ϕ-factory

Neutral kaon pairs in the antisymmetric state (23) are ideally and copiously produced at a ϕ-factory ($J^{PC} = 1^{--}$ for the ϕ meson) in the reaction $e^+e^- \to$

$\phi \to K^0\bar{K^0}$. According to quantum mechanics, one can evaluate the decay amplitude for state (23) into final states f_1 and f_2 produced in the $+\vec{p}$ and $-\vec{p}$ directions at kaon proper times t_1 and t_2, respectively:

$$\begin{aligned} A(f_1, t_1; f_2, t_2) &= \frac{N}{\sqrt{2}}\{\langle f_1|T|K_S(t_1)\rangle\langle f_2|T|K_L(t_2)\rangle \\ &\quad -\langle f_1|T|K_L(t_1)\rangle\langle f_2|T|K_S(t_2)\rangle\} \\ &= \frac{N}{\sqrt{2}}\{\langle f_1|T|K_S\rangle\langle f_2|T|K_L\rangle e^{-i\lambda_S t_1}e^{-i\lambda_L t_2} \\ &\quad -\langle f_1|T|K_L\rangle\langle f_2|T|K_S\rangle e^{-i\lambda_L t_1}e^{-i\lambda_S t_2}\} \ . \end{aligned} \tag{25}$$

The double differential decay rate into final states f_1 and f_2 at proper times t_1 and t_2 can be readily computed from eq.(25):

$$\begin{aligned} I(f_1, t_1; f_2, t_2) = C_{12}\{&|\eta_1|^2 e^{-\Gamma_L t_1 - \Gamma_S t_2} + |\eta_2|^2 e^{-\Gamma_S t_1 - \Gamma_L t_2} \\ &-2|\eta_1||\eta_2| e^{-\frac{(\Gamma_S+\Gamma_L)}{2}(t_1+t_2)} \cos[\Delta m(t_1 - t_2) + \phi_2 - \phi_1]\} \end{aligned} \tag{26}$$

where

$$\eta_i \equiv |\eta_i|e^{i\phi_i} = \frac{\langle f_i|T|K_L\rangle}{\langle f_i|T|K_S\rangle} \ , \tag{27}$$

$$C_{12} = \frac{|N|^2}{2}|\langle f_1|T|K_S\rangle\langle f_2|T|K_S\rangle|^2 \ ,$$

and a proper account of phase-space integrals is implicitly assumed. After integration in $(t_1 + t_2)$, at fixed difference of time $\Delta t = t_1 - t_2$, the following distribution is obtained, sometimes simpler to manipulate and compare to data:

$$\begin{aligned} I(f_1, f_2; \Delta t \geq 0) = \frac{C_{12}}{\Gamma_S + \Gamma_L}\{&|\eta_1|^2 e^{-\Gamma_L \Delta t} + |\eta_2|^2 e^{-\Gamma_S \Delta t} \\ &-2|\eta_1||\eta_2| e^{-\frac{(\Gamma_S+\Gamma_L)}{2}\Delta t} \cos[\Delta m \Delta t + \phi_2 - \phi_1]\} \end{aligned} \tag{28}$$

valid for $\Delta t \geq 0$, while for $\Delta t < 0$ the substitutions $\Delta t \to |\Delta t|$ and $1 \leftrightarrow 2$ have to be applied.

Both eqs.(26) and (28) show a time interference term (in the second line of their expressions) giving rise to a characteristic correlation between the two kaon decays. It can be exploited to study the neutral kaon system and discrete symmetries. In fact the decay amplitude ratios η_i defined in eq.(27), as well as the kinematical properties of neutral kaons, i.e. Γ_S, Γ_L and Δm, can be evaluated by measuring the distribution (28) with different choices of final states f_1 and f_2. From these measurements several parameters describing the neutral kaon system can be extracted.

In general two kind of asymmetries can be constructed from eq.(28); the first one can be obtained by considering eq.(28) for positive and negative Δt's:

$$A(|\Delta t|) = \frac{I(f_1, f_2; \Delta t > 0) - I(f_1, f_2; \Delta t < 0)}{I(f_1, f_2; \Delta t > 0) + I(f_1, f_2; \Delta t < 0)} \; ; \tag{29}$$

for $|\Delta t| \gg \tau_S$ (where $\tau_{S,L} = 1/\Gamma_{S,L}$ is the $K_{S,L}$ lifetime) it becomes:

$$A(|\Delta t| \gg \tau_S) \simeq \frac{|\eta_1|^2 - |\eta_2|^2}{|\eta_1|^2 + |\eta_2|^2} \; , \tag{30}$$

while for $|\Delta t| < 5\tau_S$ it depends on the complex ratio η_2/η_1, and therefore from the phase difference $\phi_2 - \phi_1$.

The second asymmetry can be defined by considering three different final states f_1, f_2, and f_3:

$$A_{f_1,f_2}(\Delta t) = \frac{I(f_1, f_3; \Delta t) - I(f_2, f_3; \Delta t)}{I(f_1, f_3; \Delta t) + I(f_2, f_3; \Delta t)} \; . \tag{31}$$

For large positive Δt one obtains:

$$A_{f_1,f_2}(\Delta t \gg \tau_S) \simeq \frac{\Gamma(K_L \to f_1) - \Gamma(K_L \to f_2)}{\Gamma(K_L \to f_1) + \Gamma(K_L \to f_2)} \; , \tag{32}$$

while for large negative Δt one has:

$$A_{f_1,f_2}(\Delta t \ll -\tau_S) \simeq \frac{\Gamma(K_S \to f_1) - \Gamma(K_S \to f_2)}{\Gamma(K_S \to f_1) + \Gamma(K_S \to f_2)} \; . \tag{33}$$

For $|\Delta t| < 5\tau_S$ the asymmetry (31) depends on the ratios η_1/η_3 and η_2/η_3.

4.1 Decays into two charged and two neutral pions

The parameter ϵ'/ϵ signaling direct CP violation [18, 6] in $K \to \pi\pi$ decays can be measured with the choice $f_1 = \pi^+\pi^-$ and $f_2 = 2\pi^0$; in this case the corresponding η_i parameters are defined as follows:

$$\begin{aligned} \eta_{+-} &\equiv |\eta_{+-}| e^{i\phi_{+-}} = \epsilon + \epsilon' \\ \eta_{00} &\equiv |\eta_{00}| e^{i\phi_{00}} = \epsilon - 2\epsilon' \end{aligned} \tag{34}$$

where[4]:

$$\epsilon = \bar{\epsilon} - \delta + i\frac{\Im A_0}{\Re A_0} + \frac{\Re B_0}{\Re A_0} \tag{35}$$

$$\epsilon' = \frac{1}{\sqrt{2}} e^{i(\delta_2-\delta_0)} \frac{\Re A_2}{\Re A_0}\left[i\left(\frac{\Im A_2}{\Re A_2} - \frac{\Im A_0}{\Re A_0}\right) + \left(\frac{\Re B_2}{\Re A_2} - \frac{\Re B_0}{\Re A_0}\right)\right] , \tag{36}$$

and the decay amplitudes of K^0 and $\bar{K^0}$ into a $\pi\pi$ final state of definite isospin $I = 0, 2$ are written as

$$\begin{aligned} \langle \pi\pi; I|T|K^0\rangle &= (A_I + B_I)e^{i\delta_I} \\ \langle \pi\pi; I|T|\bar{K^0}\rangle &= (A_I^* - B_I^*)e^{i\delta_I} , \end{aligned} \tag{37}$$

with δ_I the $\pi\pi$ strong interaction phase shift for channel of total isospin I. Here A_I (B_I) describe the CPT-conserving (CPT-violating) part of $\pi\pi$ decay amplitudes (see Refs. [6, 19, 18]) for a detailed discussion).

The distribution (28) in the case of $f_1 = \pi^+\pi^-$ and $f_2 = 2\pi^0$ is shown in Fig.1 (where the effect of $\epsilon'/\epsilon \neq 0$ is emphasized). One can construct an asymmetry of the kind of eq.(29):

$$\begin{aligned} A_{\epsilon'/\epsilon}(|\Delta t|) &= \frac{I\left(\pi^+\pi^-, \pi^0\pi^0; \Delta t > 0\right) - I\left(\pi^+\pi^-, \pi^0\pi^0; \Delta t < 0\right)}{I\left(\pi^+\pi^-, \pi^0\pi^0; \Delta t > 0\right) + I\left(\pi^+\pi^-, \pi^0\pi^0; \Delta t < 0\right)} \\ &= A_R(|\Delta t|)\Re\left(\frac{\epsilon'}{\epsilon}\right) - A_I(|\Delta t|)\Im\left(\frac{\epsilon'}{\epsilon}\right) \end{aligned} \tag{38}$$

where terms proportional to $\left(\frac{\epsilon'}{\epsilon}\right)^2$ have been neglected in the last equality, and

$$\begin{aligned} A_R(|\Delta t|) &= 3\frac{e^{-\Gamma_L|\Delta t|} - e^{-\Gamma_S|\Delta t|}}{e^{-\Gamma_L|\Delta t|} + e^{-\Gamma_S|\Delta t|} - 2e^{-\frac{(\Gamma_S+\Gamma_L)}{2}|\Delta t|}\cos(\Delta m|\Delta t|)} \\ A_I(|\Delta t|) &= 3\frac{2e^{-\frac{(\Gamma_S+\Gamma_L)}{2}|\Delta t|}\sin(\Delta m|\Delta t|)}{e^{-\Gamma_L|\Delta t|} + e^{-\Gamma_S|\Delta t|} - 2e^{-\frac{(\Gamma_S+\Gamma_L)}{2}|\Delta t|}\cos(\Delta m|\Delta t|)} \end{aligned} \tag{39}$$

The asymmetry is sensitive to $\Im(\epsilon'/\epsilon)$ for $|\Delta t| \leq 5\tau_S$, while for $|\Delta t| \gg \tau_S$ tends to $3\Re(\epsilon'/\epsilon)$.

[4]It is worth remarking that ϵ and ϵ' are measurable quantities independent of any phase convention.

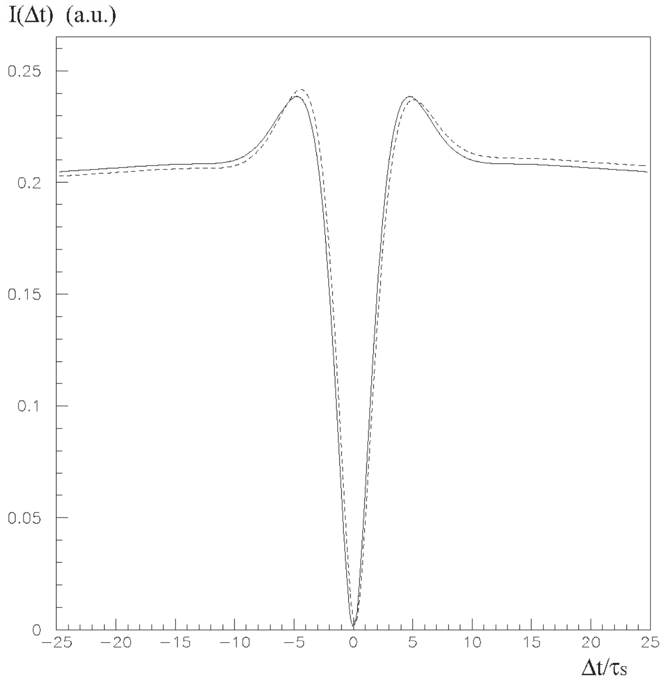


Figure 1: *The* $I\left(\pi^+\pi^-, \pi^0\pi^0; \Delta t\right)$ *distribution in the case of* $\epsilon'/\epsilon = 0$ *(solid line), and in the case of* $\Re(\epsilon'/\epsilon) = 0.005$, $\Im(\epsilon'/\epsilon) = 0.05$ *(dashed line).*

4.2 Double semileptonic decays

The semileptonic decay amplitudes can be parametrized as follows [6]:

$$\begin{aligned} \langle \pi^- l^+ \nu | T | K^0 \rangle &= a + b\ , & \langle \pi^+ l^- \bar{\nu} | T | \bar{K}^0 \rangle = a^* - b^* \\ \langle \pi^+ l^- \bar{\nu} | T | K^0 \rangle &= c + d\ , & \langle \pi^- l^+ \nu | T | \bar{K}^0 \rangle = c^* - d^* \end{aligned} \tag{40}$$

where a, b, c, d are complex quantities; CPT invariance implies $b = d = 0$, $\Delta S = \Delta Q$ rule implies $c = d = 0$, T invariance implies $\Im a = \Im b = \Im c = \Im d = 0$, while CP invariance implies $\Im a = \Re b = \Im c = \Re d = 0$. Then three measurable parameters can be defined:

$$y = -b/a\ , \qquad x_+ = c^*/a\ , \qquad x_- = -d^*/a\ ; \tag{41}$$

x_+ (x_-) describes the violation of the $\Delta S = \Delta Q$ rule in CPT conserving (violating) decay amplitudes, while y parametrizes CPT violation for $\Delta S =$

ΔQ transitions. Then the semileptonic charge asymmetries for K_S and K_L states can be expressed as

$$\begin{aligned} A_S &= \frac{\Gamma(K_S \to \pi^- l^+ \nu) - \Gamma(K_S \to \pi^+ l^- \bar{\nu})}{\Gamma(K_S \to \pi^- l^+ \nu) + \Gamma(K_S \to \pi^+ l^- \bar{\nu})} \\ &= 2\Re\bar{\epsilon} + 2\Re\delta - 2\Re y + 2\Re x_- \ , \end{aligned} \tag{42}$$

and

$$\begin{aligned} A_L &= \frac{\Gamma(K_L \to \pi^- l^+ \nu) - \Gamma(K_L \to \pi^+ l^- \bar{\nu})}{\Gamma(K_L \to \pi^- l^+ \nu) + \Gamma(K_L \to \pi^+ l^- \bar{\nu})} \\ &= 2\Re\bar{\epsilon} - 2\Re\delta - 2\Re y - 2\Re x_- \ . \end{aligned} \tag{43}$$

With the choice $f_1 = \pi^- l^+ \nu$ and $f_2 = \pi^+ l^- \bar{\nu}$, the corresponding η_i parameters are

$$\begin{aligned} \eta_{l^+} &\simeq 1 - 2\delta - 2x_+ - 2x_- \\ \eta_{l^-} &\simeq -1 - 2\delta + 2x_+^* - 2x_-^* \ . \end{aligned} \tag{44}$$

The decay intensity (28) in this case is shown in Fig.2 (where a possible effect of $\delta \neq 0$ is emphasized). The following asymmetry can be constructed:

$$\begin{aligned} A_{CPT}(|\Delta t|) &= \frac{I\left(\pi^- l^+ \nu, \pi l^- \bar{\nu}; \Delta t > 0\right) - I\left(\pi^- l^+ \nu, \pi^+ l^- \bar{\nu}; \Delta t < 0\right)}{I\left(\pi^- l^+ \nu, \pi^+ l^- \bar{\nu}; \Delta t > 0\right) + I\left(\pi^- l^+ \nu, \pi^+ l^- \bar{\nu}; \Delta t < 0\right)} \\ &= -\frac{4}{3}\left\{A_R(|\Delta t|)\delta_R + A_I(|\Delta t|)\delta_I\right\} \\ &\quad \times \frac{\cosh\left(\Delta\Gamma|\Delta t|/2\right) - \cos(\Delta m|\Delta t|)}{\cosh\left(\Delta\Gamma|\Delta t|/2\right) + \cos(\Delta m|\Delta t|)} \end{aligned} \tag{45}$$

that is sensitive to CPT and/or $\Delta S = \Delta Q$ rule violations. In fact for $|\Delta t| \gg \tau_S$ it tends to $-4\delta_R$, where $\delta_R = \Re\delta + \Re x_-$, while for $|\Delta t| \leq 5\tau_S$ it is sensitive to $\delta_I = \Im\delta + \Im x_+$.

4.3 Semileptonic and two pion decays

The decay intensity (28) in the cases of $f_1 = \pi^- l^+ \nu$, $f_2 = \pi^+ l^- \bar{\nu}$, and $f_3 = \pi\pi$ is shown in Fig.3; an asymmetry of the kind of eq.(31) can be constructed

$$A_{l^+ l^-}(\Delta t) = \frac{I\left(\pi^- l^+ \nu, \pi\pi; \Delta t\right) - I\left(\pi^+ l^- \bar{\nu}, \pi\pi; \Delta t\right)}{I\left(\pi^- l^+ \nu, \pi\pi; \Delta t\right) + I\left(\pi^+ l^- \bar{\nu}, \pi\pi; \Delta t\right)} \ , \tag{46}$$

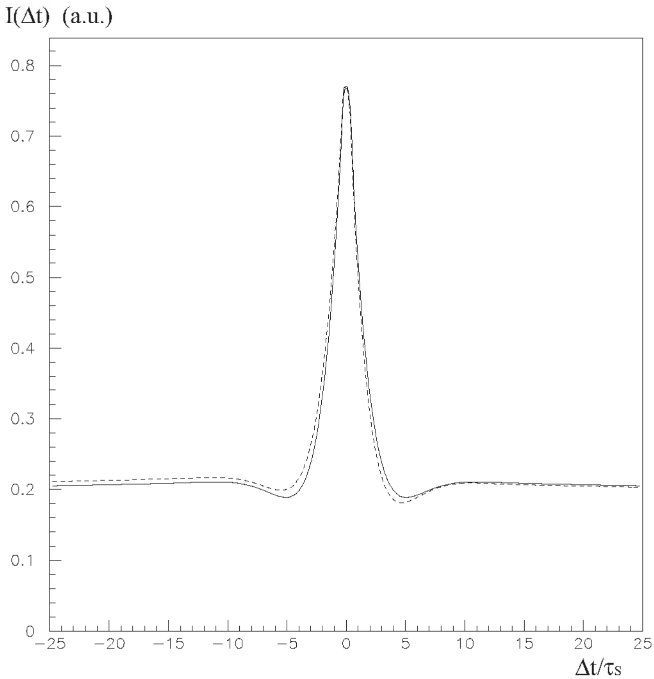


Figure 2: *The* $I(\pi^- l^+ \nu, \pi l^- \bar{\nu}; \Delta t)$ *distribution in the case of* $\delta = x_+ = x_- = 0$ *(solid line), and in the case* $\Re\delta = 5 \cdot 10^{-4}$, $\Im\delta = 0.05$, $x_+ = x_- = 0$ *(dashed line).*

that at large positive times $\Delta t \gg \tau_S$ coincides with the K_L semileptonic asymmetry A_L given in eq.(43), while for short times it is sensitive to $|\eta_{\pi\pi}|$ and $\phi_{\pi\pi}$.

4.4 Decays into identical final states

In the case of $f_1 = f_2 = \pi^+\pi^-$ the dependence on the η_{+-} parameter factorizes out, and the shape of distribution (28) is sensitive only to the kinematical quantities Γ_S, Γ_L and Δm, as shown in Fig.4. The same holds for any choice of identical final states, i.e. with $f_1 = f_2$.

More detailed reviews on this subject can be found in Refs. [18, 6, 19, 20].

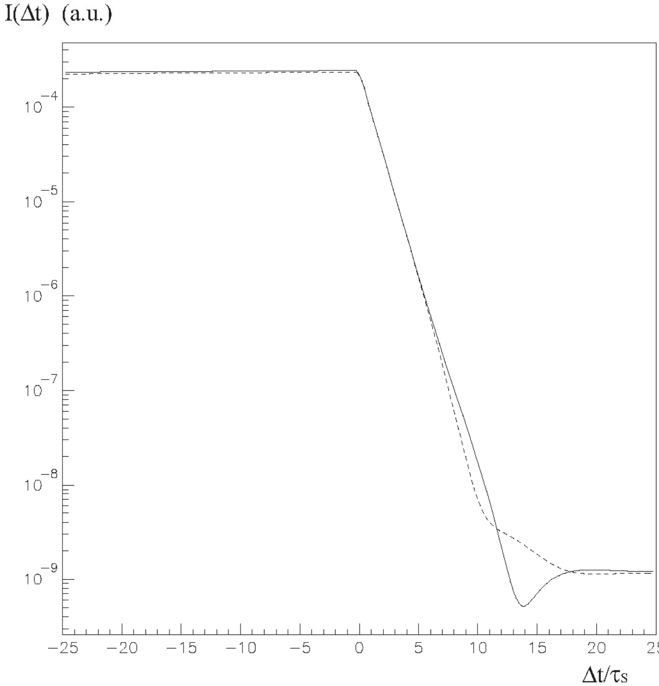


Figure 3: *The* $I\left(\pi^{-}l^{+}\nu,\pi\pi;\Delta t\right)$ *(solid line), and* $I\left(\pi l^{-}\bar{\nu},\pi\pi;\Delta t\right)$ *(dashed line) distributions.*

5 Entanglement and neutral kaons

As mentioned above the interference term in eqs.(26) and (28) gives rise to a characteristic correlation between the two kaon decays. For instance, a complete destructive interference prevents the two kaons from decaying into the same final state f at the same time t, i.e.:

$$I(f,t;f,t)=0 \qquad (47)$$

for any f and t (as it can be also noticed in Fig.(4) for $|\Delta t|=0$). This is a consequence of the antisymmetry of state (23). From an intuitive point of view, once produced, the two kaons can be viewed as two freely propagating independent particles. However even though the two decays can be regarded as separated space-like events (the kaons are produced with opposite momentum in the ϕ meson rest frame), it is like the kaon flying in the $+\vec{p}$ direction

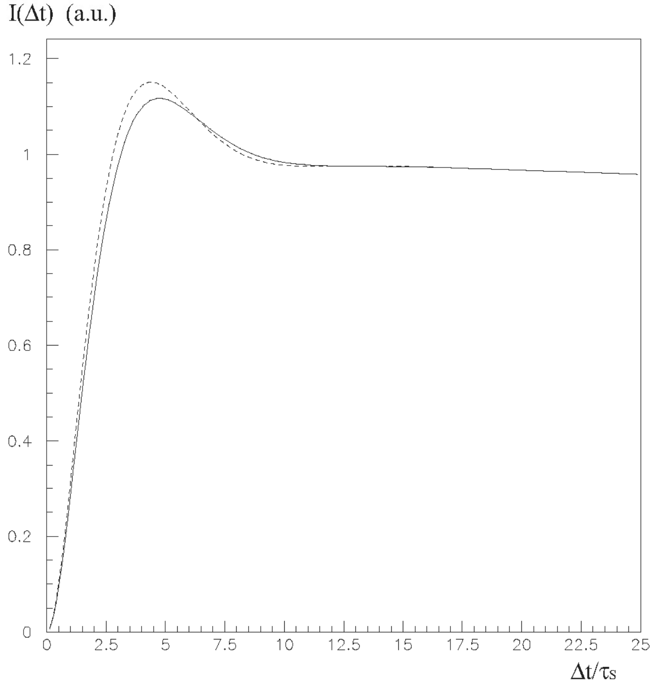


Figure 4: *The* $I(\pi^+\pi^-, \pi^+\pi^-; |\Delta t|)$ *distribution as a function of* $|\Delta t|$ *(solid line), and the same distribution in the case of a fractional variation of* Δm *of* $+10\%$ *(dashed line).*

cannot "freely" decay into a certain final state f at a certain proper time t, but its behaviour depends on what the other kaon flying in the opposite $-\vec{p}$ direction does. This kind of correlation (*entanglement*) for neutral kaon pairs was emphasized already in 1960 by Lee and Yang [21], and later on by several authors [22, 23, 24]. It cannot be simply explained in terms of conservation laws, and is of the type first pointed out by Einstein, Podolsky and Rosen (EPR) in their famous paper [25].

This feature of the initial state (23) has long reaching consequences in terms of potentialities of the neutral kaon system in testing fundamental aspects of quantum mechanics. This can be easily understood by recognizing that the quantum number strangeness ± 1 for a neutral kaon can play the same role of spin *up* or *down* along a chosen direction. Then, the correlations implied by the state (23) for a kaon pair lead to a quite straightforward formal analogy with the system of spin 1/2 particles in the singlet state. Therefore, kaon pairs produced at a ϕ-factory might be suitable for a significant test of Bell's inequality, as it is discussed in detail in the contributions of Bertlmann and Hiesmayr [26], and Bramon, Escribano and Garbarino [27] (see also the contribution of Go [28]), or for the study of Bohr's complementarity principle with an interesting implementation of the quantum erasure concepts, as described in the contribution of Bramon, Garbarino and Hiesmayr [29].

6 Decoherence and CPT violation

6.1 Furry's hypothesis and a simple decoherence model

Most of the key features of the entangled state (23) resides in its non-separability. It has been suggested that the state soon after the ϕ-meson decay, spontaneously factorizes to an equally weighted statistical mixture of states $|K_S\rangle|K_L\rangle$ and $|K_L\rangle|K_S\rangle$, (commonly known as Furry's hypothesis [30]). In this case the characteristic quantum interference term would disappear from expressions (26) and (28); to be more specific, this means that the calculation of intensity $I(f_1,t_1;f_2,t_2)$ is no more given by eq.(26), as in *orthodox* quantum mechanics, but is given by the incoherent sum:

$$I(f_1,t_1;f_2,t_2)_{\rm Furry} = \frac{|N|^2}{2}\{|\langle f_1|T|K_S(t_1)\rangle\langle f_2|T|K_L(t_2)\rangle|^2 + |\langle f_1|T|K_L(t_1)\rangle\langle f_2|T|K_S(t_2)\rangle|^2\} \ . \qquad (48)$$

One of the most direct way to search for such deviations from quantum mechanics [31] is to introduce a decoherence parameter ζ_{SL}, and a factor $(1-\zeta)$ multiplying the interference term in eq.(26):

$$I(f_1,t_1;f_2,t_2;\zeta) = C_{12}\{|\eta_1|^2 e^{-\Gamma_L t_1-\Gamma_S t_2} + |\eta_2|^2 e^{-\Gamma_S t_1-\Gamma_L t_2} -2(1-\zeta)|\eta_1||\eta_2| e^{-\frac{(\Gamma_S+\Gamma_L)}{2}(t_1+t_2)}\cos[\Delta m(t_1-t_2)+\phi_2-\phi_1]\} \ . \qquad (49)$$

The case $\zeta=0$ corresponds to the usual *orthodox* quantum theory, while for $\zeta=1$ the case of spontaneous factorization of state, as in eq.(48), is obtained, i.e. total decoherence. Different ζ values correspond to intermediate situations between these two. However, the state could also spontaneously factorize into another mixture of states, e.g. $|K^0\rangle|\bar{K^0}\rangle$ and $|\bar{K^0}\rangle|K^0\rangle$, giving rise to a different decay intensity expression. As pointed out in Ref. [32], in general the definition of ζ depends on the basis in which is written the initial state (23) because the interference term changes with the basis (obviously in the *orthodox* quantum theory the final result does not depend on the basis choice). For a generic basis $|K_\alpha\rangle, |K_\beta\rangle$, distribution (26) is modified as follows:

$$I(f_1,t_1;f_2,t_2;\zeta_{\alpha\beta}) = \frac{|N'|^2}{2}\{|\langle f_1|T|K_\alpha(t_1)\rangle\langle f_2|T|K_\beta(t_2)\rangle|^2 + |\langle f_1|T|K_\beta(t_1)\rangle\langle f_2|T|K_\alpha(t_2)\rangle|^2 - 2(1-\zeta_{\alpha\beta})\Re[\langle f_1|T|K_\beta(t_1)\rangle\langle f_2|T|K_\alpha(t_2)\rangle\langle f_1|T|K_\alpha(t_1)\rangle^*\langle f_2|T|K_\beta(t_2)\rangle^*]\} , \tag{50}$$

defining the basis dependent decoherence parameter $\zeta_{\alpha\beta}$.

6.2 A general approach to decoherence

In general decoherence is the time evolution of a pure state into an incoherent mixture of states. The density matrix formalism correctly treats pure and mixed states in a unique consistent framework. According to quantum mechanics, the time evolution of the density matrix ρ of a system is given by the Liouville - von Neumann equation:

$$\frac{d\rho}{dt} = -i[\mathcal{H}, \rho] \tag{51}$$

Decoherence can be introduced at a more fundamental level than inserting *by hand* the parameter ζ, by suitably modifying eq.(51). Very general modifications have been proposed in Ref. [33, 34, 35, 36, 37, 38] for single kaon and correlated pair of kaon systems. In the broad framework of open quantum systems, neutral kaons can be modeled as being small subsystems in *weak* interaction with large environments. The reduced dynamics for the subsystem is obtained by tracing over the environment degrees of freedom, and the time evolution is assumed to be described by a completely positive dynamical map. A detailed review on this subject can be found in the contribution of Benatti and Floreanini [39].

6.3 Decoherence and CPT violation due to quantum gravity effects

The decoherence mechanism can be made more specific in the case it is induced by quantum gravity effects. In fact one of the main open problem in quantum gravity is related to what is commonly known as the black hole *information-loss paradox.* In 1976 Hawking showed [40] that the formation and evaporation of black holes, as described in the semiclassical approximation, appear to transform pure states near the event horizon of black holes into mixed states. This corresponds to a loss of information about the initial state, in striking conflict with quantum mechanics and its unitarity description.

At a microscopic level, in a quantum gravity picture, space-time might be subjected to inherent non-trivial quantum metric and topology fluctuations at the Planck scale ($\sim 10^{-33}$ cm), called generically *space-time foam*, with associated microscopic event horizons. As further suggested by Hawking himself [41], this space-time structure, might induce a pure state to evolve into a mixed one, i.e. decoherence of apparently isolated matter systems. This decoherence, in turn, necessarily implies, by means of a theorem [42], CPT violation, in the sense that the quantum mechanical operator generating CPT transformations cannot be consistently defined.

The information-loss paradox generated a lively debate during the last decades with no generally accepted solution. Even the recent proposed solutions in favor of no-loss and preservation of information do not completely solve the problem, some aspects of which still remaining a puzzle (see for instance Refs. [43, 44, 45]). It seems therefore extremely interesting to put experimental limits at the level of the Planck's scale region on possible decoherence effects.

The above mentioned decoherence mechanism lead Ellis and coworkers [46] to formulate a model in which a single kaon is described by a density matrix ρ that obeys a modified Liouville-von Neunmann equation:

$$\frac{d\rho}{dt} = -i\mathbf{H}\rho + i\rho\mathbf{H}^\dagger + i\delta\not{\mathbf{H}}\rho \tag{52}$$

where now $\mathbf{H} = \mathbf{M} - i\mathbf{\Gamma}/2$ is the usual neutral kaon effective Hamiltonian, and the extra term $\delta\not{\mathbf{H}}$ would induce decoherence in the system. Taking as orthonormal basis for ρ the states $|K_1\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\bar{K^0}\rangle\right]$ and $|K_2\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\bar{K^0}\rangle\right]$, and expanding ρ in terms of Pauli spin matrices σ_i and the identity σ_0, i.e. $\rho = \rho_\mu\sigma_\mu$, the extra term can be represented by a 4×4 matrix

$\delta \not{H}_{\mu\nu}$ $(\mu, \nu = 0, 1, 2, 3)$ acting on a column vector with ρ_μ as components:

$$\delta \not{H}_{\mu\nu} = -2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & \beta & \gamma \end{pmatrix} \tag{53}$$

where α, β and γ are three new real parameters, which violate CPT symmetry and quantum mechanics, and satisfy the inequalities α, $\gamma > 0$ and $\alpha\gamma > \beta^2$ (see Refs. [46, 47]). They have mass dimension and are guessed to be at most of $\mathcal{O}(m_K^2/M_{Planck}) \sim 2\times 10^{-20}$ GeV, where $M_{Planck} = 1\sqrt{G_N} = 1.22\times 10^{19}$ GeV is the Planck mass.
The formalism described above is for single kaons. Its extension to the correlated kaon pair (23) has been described in Refs. [48, 17].

It is worth noting that the assumption of complete positivity [34, 35] introduces additional constraints on these three parameters, i.e. $\alpha = \gamma$ and $\beta = 0$, reducing them to only one independent parameter.

As discussed above, in a quantum gravity framework inducing decoherence, the CPT operator is *ill-defined*. This consideration lead Bernabeu, Mavromatos and Papavassiliou [16, 17] to investigate intriguing consequences in correlated neutral kaon states. In fact the resulting loss of particle-antiparticle identity could induce a breakdown of the correlation of state (23) imposed by Bose statistics. As a result the initial entangled state (23) can be parametrized in general as:

$$\begin{aligned} |i\rangle &= \frac{1}{\sqrt{2}} \{ [|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle] + \omega\, [|K^0\rangle|\bar{K}^0\rangle + |\bar{K}^0\rangle|K^0\rangle] \} \\ &\propto \{ [|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle] + \omega\, [|K_S\rangle|K_S\rangle - |K_L\rangle|K_L\rangle] \} \end{aligned} \tag{54}$$

where ω is a complex parameter describing a completely novel CPT violation phenomenon, not included in previous analyses. Its order of magnitude might be at most $|\omega| \sim \left[\frac{(m_K^2/M_{Planck})}{\Delta\Gamma}\right]^{1/2} \sim 10^{-3}$, with $\Delta\Gamma = \Gamma_S - \Gamma_L$.
From eq.(54) it is evident that the best decay channel to look for such CPT violation effects is the one with $f_1 = f_2 = \pi^+\pi^-$: in fact in this case the leading $K_S K_L$ terms are CP suppressed while the new CPT violating $K_S K_S$ term is not.

A general review on the theoretical motivations for possible CPT violation induced by quantum gravity in the neutral kaon system can be found in the contribution of Bernabeu, Ellis, Mavromatos, Nanopoulos and Papavassiliou [49]; a review on decoherence models in this framework can be found in the contribution of Sarkar [50], while general considerations on quantum gravity phenomenology with a special focus on correlated states can be found in the contribution of Amelino-Camelia, Arzano and Marcianò [51].

7 *CPT* violation and Lorentz symmetry breaking

CPT invariance holds for any realistic Lorentz-invariant quantum field theory. However a very general theoretical possibility for CPT violation is based on spontaneous breaking of Lorentz symmetry, as developed by Kostelecky [52, 53, 5 which appears to be compatible with the basic tenets of quantum field theory and retains the property of gauge invariance and renormalizability (Standard Model Extension - SME). A detailed review on this subject can be found in the contribution of Lehnert [55]. Here, after a brief introduction, some measurement methods at a ϕ-factory are discussed.

In SME for neutral kaons, CPT manifests to lowest order only in the CPT violation parameter δ (e.g. B_I, y and x_- vanish at first order), and exhibits a dependence on the 4-momentum of the kaon:

$$\delta \approx i \sin\phi_{SW} e^{i\phi_{SW}} \gamma_K (\Delta a_0 - \vec{\beta_K} \cdot \Delta\vec{a})/\Delta m \tag{55}$$

where γ_K and $\vec{\beta_K}$ are the kaon boost factor and velocity in the observer frame, and Δa_μ are four CPT- and Lorentz-violating coefficients for the two valence quarks in the kaon.

The implications of the momentum dependence in the CPT violation parameter can be substantial, as it is evident in eq.(55). The analysis of experimental data requires a particular care in considering meson boost, momentum orientation, and possible diurnal effects arising from the rotation of the Earth relative to the constant vector $\Delta\vec{a}$, in order to avoid cancellations of the CPT violation effects.

Following Ref. [53], the time dependence arising from the rotation of the Earth can be explicitly displayed in eq.(55) by choosing a three-dimensional basis $(\hat{X}, \hat{Y}, \hat{Z})$ in a non-rotating frame, with the $\hat{Z}$ axis along the Earth's rotation axis, and a basis $(\hat{x}, \hat{y}, \hat{z})$ for the rotating (laboratory) frame (see Fig.5). The CPT violating parameter δ may then be expressed as:

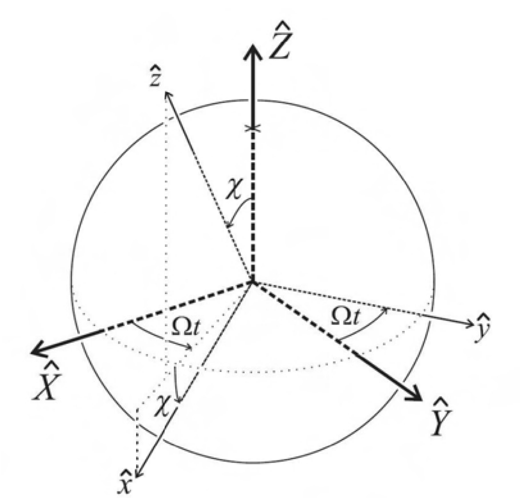


Figure 5: *Basis* $(\hat{x}, \hat{y}, \hat{z})$ *for the rotating frame, and basis* $(\hat{X}, \hat{Y}, \hat{Z})$ *for the fixed non-rotating frame. The laboratory frame precesses around the Earth's rotation axis* $\hat{Z}$ *at the sidereal frequency* Ω*;* χ *is the angle between the axes* $\hat{Z}$ *and* $\hat{z}$*.*

$$\begin{aligned}\delta(\vec{p},t) \;=\;& \frac{i\sin\phi_{SW}e^{i\phi_{SW}}}{\Delta m}\gamma_K\{\Delta a_0 + \beta_K\Delta a_Z(\cos\theta\cos\chi - \sin\theta\cos\phi\sin\chi)\\ &-\beta_K\Delta a_X\sin\theta\sin\phi\sin\Omega t\\ &+\beta_K\Delta a_X(\cos\theta\sin\chi + \sin\theta\cos\phi\cos\chi)\cos\Omega t\\ &+\beta_K\Delta a_Y(\cos\theta\sin\chi + \sin\theta\cos\phi\cos\chi)\sin\Omega t\\ &+\beta_K\Delta a_Y\sin\theta\sin\phi\cos\Omega t\} \qquad (56)\end{aligned}$$

where Ω is the Earth's sidereal frequency, $\cos\chi = \hat{z}\cdot\hat{Z}$, and θ and ϕ are the conventional polar and azimuthal angles defined in the laboratory frame about the $\hat{z}$ axis.

The sensitivity to the four Δa_μ parameters can be very different for fixed target and collider experiments, showing complementary features [53]. At a fixed target experiment usually the kaon momentum direction is fixed, while $|\vec{p}|$ might vary within a certain interval. On the contrary, at a ϕ-factory kaons are emitted with the characteristic *p-wave* angular distribution $dN/d\Omega \propto \sin^2\theta$, while $|\vec{p}|$ is fixed[5]. Assuming a symmetric decay distribution[6] in the azimuthal angle ϕ, and an integration on this variable, the following expression is obtained for δ:

$$\begin{aligned}
\delta &= \frac{1}{2\pi}\int_0^{2\pi} \delta(\vec{p},t)d\phi \\
&= \frac{i\sin\phi_{SW}e^{i\phi_{SW}}}{\Delta m}\gamma_K\{\Delta a_0 + \beta_K\Delta a_Z\cos\theta\cos\chi \\
&\quad +\beta_K(\Delta a_Y\sin\chi\cos\theta\sin\Omega t + \Delta a_X\sin\chi\cos\theta\cos\Omega t)\}\ , \qquad (57)
\end{aligned}$$

showing different angular and time dependences of the various terms proportional to Δa_μ.

7.1 Measurement of Δa_0 at a ϕ-factory

The Δa_0 parameter can be measured through the difference of the semileptonic charge asymmetries for K_S and K_L, given in eqs.(42) and (43), by performing the measurement of each asymmetry with a symmetric integration over the polar angle θ, thus averaging to zero any possible contribution from the terms proportional to $\cos\theta$ in eq.(57). Then one obtains that the difference $(A_S - A_L)$ is proportional to Δa_0, i.e.:

$$A_S - A_L \simeq \left[\frac{4\Re\left(i\sin\phi_{SW}e^{i\phi_{SW}}\right)\gamma_K}{\Delta m}\right]\Delta a_0\ . \qquad (58)$$

An alternative method to measure Δa_0 consists in exploiting the correlation between the two kaons in double semileptonic decays $\phi \to K_SK_L \to \pi^+\ell^-\bar{\nu}, \pi^-\ell^+\nu$ with opposite lepton charges. The two kaons are practically emitted back-to-back, and terms proportional to $\cos\theta$ have opposite sign for the two kaons; Δa_0 can be evaluated through the asymmetry (45), which for large Δt becomes:

$$A_{CPT}(|\Delta t| \gg \tau_S) \simeq -\left[\frac{4\Re\left(i\sin\phi_{SW}e^{i\phi_{SW}}\right)\gamma_K}{\Delta m}\right]\Delta a_0\ . \qquad (59)$$

[5] Apart small variations due to the small ϕ meson momentum in the laboratory frame.

[6] This simplifying assumption will be maintained throughout the following; however small non-symmetric ϕ angle effects could be easily included in the formulas without significantly modifying the main conclusions below.

The above two methods are largely independent and could be useful for systematics cross-checks.

7.2 Measurement of Δa_Z at a ϕ-factory

The Δa_Z parameter can be measured through the A_L asymmetry measured separately for K_L's emitted in the forward ($\cos\theta > 0$) and backward ($\cos\theta < 0$) direction; assuming data have been uniformly taken as a function of sidereal time, thus averaging to zero any possible contribution from the terms proportional to $\cos\Omega t$ and $\sin\Omega t$ in eq.(57) (otherwise a proper t-dependent analysis has to be performed), one has:

$$\begin{aligned} \Delta A_L &\equiv A_L(\cos\theta > 0) - A_L(\cos\theta < 0) \\ &\simeq -\left[\frac{4\Re\left(i\sin\phi_{SW}e^{i\phi_{SW}}\right)\beta_K\gamma_K\cos\chi\,\langle\cos\theta\rangle}{\Delta m}\right]\Delta a_Z \end{aligned} \quad (60)$$

where $\langle\cos\theta\rangle$ is a proper average of $\cos\theta$ over the forward (backward) hemisphere.

Also for the measurement of Δa_Z an alternative and independent method exists, based on neutral kaon interferometry with $\phi \to K_S K_L \to \pi^+\pi^-, \pi^+\pi^-$ decays. In this case the intensity $I\left(\pi^+\pi^-(+), \pi^+\pi^-(-); \Delta t\right)$ can be measured, where the two identical final states are distinguished by their forward or backward emission (the symbols $+$ and $-$ represent $\cos\theta > 0$ and $\cos\theta < 0$, respectively), and the following asymmetry evaluated:

$$A(|\Delta t|) = \frac{I\left(\pi^+\pi^-(+), \pi^+\pi^-(-); \Delta t > 0\right) - I\left(\pi^+\pi^-(+), \pi^+\pi^-(-); \Delta t < 0\right)}{I\left(\pi^+\pi^-(+), \pi^+\pi^-(-); \Delta t > 0\right) + I\left(\pi^+\pi^-(+), \pi^+\pi^-(-); \Delta t < 0\right)} \quad (61$$

To first order in small quantities, the above asymmetry for $\Delta t \gg \tau_S$ tends to zero, because ϵ and δ are 90° out of phase (see Ref. [18]):

$$A(|\Delta t| \gg \tau_S) \simeq -2\Re\left(\frac{\delta}{\epsilon}\right) \sim 0 \quad (62)$$

while for $|\Delta t| \leq 5\tau_S$ it is sensitive to $\Im\left(\delta/\epsilon\right)$, and therefore to Δa_Z:

$$A(0 < |\Delta t| < 5\tau_S) \propto \Im\left(\frac{\delta}{\epsilon}\right) \simeq \left[\frac{\sin\phi_{SW}\beta_K\gamma_K\cos\chi\,\langle\cos\theta\rangle}{\Delta m|\epsilon|}\right]\Delta a_Z\ . \quad (63)$$

Also in this case the two methods could be used for cross-checks.

7.3 Measurement of Δa_X, Δa_Y at a ϕ-factory

The Δa_X, Δa_Y and Δa_Z parameters can be all simultaneously measured by performing a proper sidereal time dependent analysis of asymmetries in eqs.(60) and (61).

8 The KLOE experiment at DAΦNE

DAΦNE, the Frascati ϕ-factory [56], is an e^+e^- collider working at a center of mass energy of $\sqrt{s} \sim 1020$ MeV, corresponding to the peak of the ϕ resonance. The ϕ production cross section is $\sim 3\mu$b; the main ϕ decays and branching ratios are listed in tab. 1. The beams collide at the interaction point (IP)

Table 1: *Main decay channels and branching fractions of the ϕ meson*

Decay channel	Branching fraction (% units)
$\phi \to K^+K^-$	49.1
$\phi \to K^0\bar{K}^0$	34.0
$\phi \to \rho\pi, \pi^+\pi^-\pi^0$	15.4
$\phi \to \eta\gamma$	1.3

with a crossing angle $\theta_x \simeq 25$ mrad, therefore ϕ's are produced with a small momentum of ~ 12.5 MeV in the horizontal plane. The beams collide with a frequency up to 370 MHz corresponding to a bunch crossing period of $T_{bunch} = 2.7$ ns and a maximum number of circulating bunches of 120. The KLOE interaction region is equipped with three low-β quadrupoles, which reduce the beam-size in the vertical (y) direction. The typical sizes of the beam are $\sigma_x = 0.2\,\text{cm}; \sigma_y = 20\,\mu\text{m}; \sigma_z = 3\,\text{cm}$. The maximum peak luminosity reached during KLOE data taking is $L \simeq 1.4 \times 10^{32}\text{cm}^{-2}\,\text{s}^{-1}$

The KLOE detector consists mainly of a large volume drift chamber surrounded by an electromagnetic calorimeter. A superconducting coil around the calorimeter provides a 0.52 T solenoidal magnetic field.

The fine sampling lead-scintillating fiber calorimeter [57] consists of a barrel and two end-caps, and has solid angle coverage of 98%. Photon energies and arrival times are measured with resolutions $\sigma_E/E = 5.7\%/\sqrt{E(\text{GeV})}$ and $\sigma_t = 54\text{ps}/\sqrt{E(\text{GeV})} \oplus 50\text{ps}$, respectively. Photon entry points are determined with an accuracy $\sigma_z \sim 1\ \text{cm}/\sqrt{E(\text{GeV})}$ along the fibers and $\sigma_\perp \sim 1$ cm in the transverse direction.

The tracking detector is a 4 m diameter and 3.3 m long cylindrical drift chamber [58] with a total of $\sim$ 52000 wires, of which $\sim$ 12000 are sense wires. In order to minimize multiple scattering and K_L regeneration and to maximize detection efficiency of low energy photons, the chamber works with a helium based gas mixture and its walls are made of light materials (mostly carbon fiber composites). The momentum resolution for tracks produced at large polar angle is $\sigma_p/p \leq 0.4\%$. Vertices are reconstructed with a resolution of $\sim$ 3 mm.

Kaon regeneration in the beam pipe is a non negligible disturbance. The beam pipe is spherical around the interaction point, with a radius of 10 cm. The walls of the beam pipe, 500 μm thick, are made of a 62%-beryllium/38%-aluminum alloy. A beryllium cylindrical tube of 4.4 cm radius and 50μm thick, coaxial with the beam, provides electrical continuity.

KLOE completed the data taking in March 2006 with a total integrated luminosity of ~ 2.5 fb^{-1}, corresponding to $\sim 7.5 \times 10^9$ ϕ-mesons produced.

8.1 Decoherence and CPT symmetry tests

The quantum interference between the two kaon decays in the CP violating channel $\phi \to K_S K_L \to \pi^+\pi^-\pi^+\pi^-$ has been observed for the first time by KLOE [59]. A data sample corresponding to ~ 380 pb^{-1} has been analysed; the selection of the signal requires two vertices, each with two opposite curvature tracks inside the drift chamber, with an invariant mass and total momentum compatible with the two neutral kaon decays. The experimental resolution on the time difference $|\Delta t|$ in the case of $\pi^+\pi^-$ decays can be improved exploiting the good momentum resolution of the KLOE detector [60] and the closed kinematics of the event. After a kinematic fit, a resolution $\sigma_{|\Delta t|} \sim 0.9\tau_S$ is obtained. The measured $I(\pi^+\pi^-, \pi^+\pi^-; |\Delta t|)$ distribution as a function of $|\Delta t|$ can be fitted with the expression given in eq.(28). After having included resolution and detection efficiency effects, having taken into account the background due to coherent and incoherent K_S-regeneration on the beam pipe wall, the small contamination of non-resonant $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ events, and keeping fixed in the fit Γ_S and Γ_L to the PDG [61] values, Δm can be evaluated. The fit result is $\Delta m = (5.61 \pm 0.33) \times 10^9 \mathrm{s}^{-1}$, which is compatible with the more precise value given by the PDG: $\Delta m = (5.290 \pm 0.015) \times 10^9 \mathrm{s}^{-1}$.

A similar analysis can be done on the same data sample, by fixing Δm to the PDG value, using the modified expression given in eq.(50) after integration

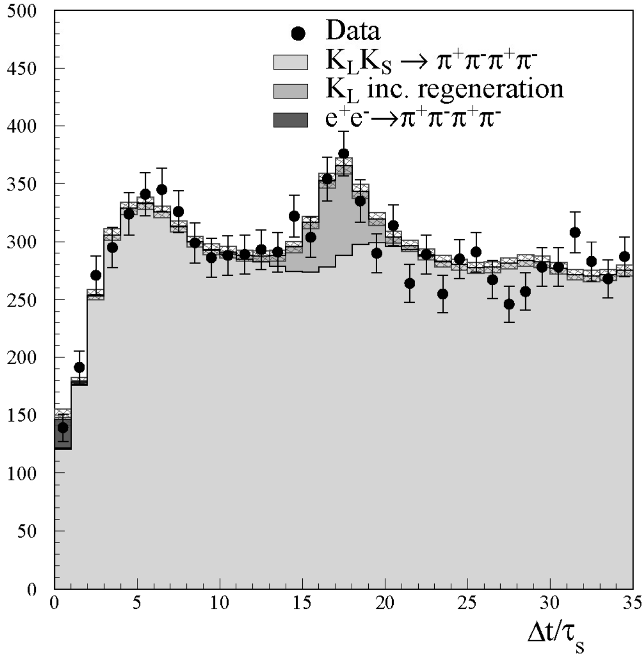


Figure 6: *Fit of the $I(\pi^+\pi^-, \pi^+\pi^-; |\Delta t|)$ distribution. The black points are the experimental data, while the histogram is the fit result in the case of ζ_{SL} determination. The uncertainty arising from the detection efficiency evaluation is shown as the hatched area. The peak at $|\Delta t| \sim 17\tau_S$ is due to coherent and incoherent K_S-regeneration on the spherical beam pipe.*

in $(t_1 + t_2)$, and leaving the decoherence parameter ζ as a free parameter in the fit. The results in the two main basis, $\{K_S, K_L\}$ and $\{K^0, \bar{K^0}\}$, are

$$\begin{aligned} \zeta_{SL} &= 0.018 \pm 0.040_{\rm stat} \pm 0.007_{\rm syst} \\ \zeta_{0\bar{0}} &= (1.0 \pm 2.1_{\rm stat} \pm 0.4_{\rm syst}) \times 10^{-6} \ , \end{aligned}$$

compatible with the quantum mechanics prediction, i.e. $\zeta_{SL} = \zeta_{0\bar{0}} = 0$, and no decoherence effects. As an example, the fit of the $|\Delta t|$ distribution used to determine ζ_{SL} is shown in Fig.6.

The result on $\zeta_{0\bar{0}}$ has a high accuracy, of $\mathcal{O}(10^{-6})$, due to the CP suppression present in the specific $f_1 = f_2 = \pi\pi$ decay channel which makes the function (50) very sensitive to $\zeta_{0\bar{0}}$ deviations from zero. This result improves by five orders of magnitude the previous limit obtained by Bertlmann and co-workers [32] in a re-analysis of CPLEAR data [62] (a review of the CPLEAR results can be found in the contribution of Go [28]). It can also be compared to a similar result recently obtained in the B meson system [63], where an accuracy of $\mathcal{O}(10^{-2})$ can be reached.

Another analysis based on the same data constrains the parameters α, β and γ related to possible decoherence effects induced by quantum gravity, as discussed above. The theoretical expression of the $I(\pi^+\pi^-,\pi^+\pi^-;|\Delta t|)$ distribution including these effects can be found in Refs. [48, 17]. The KLOE preliminary results are [64]:

$$\begin{aligned} \alpha &= \left(-10^{+41}_{-31}{}_{\rm stat} \pm 9_{\rm syst}\right) \times 10^{-17}\,{\rm GeV} \\ \beta &= \left(3.7^{+6.9}_{-9.2}{}_{\rm stat} \pm 1.8_{\rm syst}\right) \times 10^{-19}\,{\rm GeV} \\ \gamma &= \left(-0.5^{+5.8}_{-5.1}{}_{\rm stat} \pm 1.2_{\rm syst}\right) \times 10^{-21}\,{\rm GeV} \end{aligned} \tag{64}$$

In the simplifying hypothesis of complete positivity, i.e. $\alpha = \gamma$ and $\beta = 0$, the KLOE result is [59]:

$$\gamma = \left(1.3^{+2.8}_{-2.4} \pm 0.4\right) \times 10^{-21}\,{\rm GeV}\ , \tag{65}$$

These results can be compared to the ones obtained by the CPLEAR collaboration, studying single neutral kaon decays to $\pi^+\pi^-$ and $\pi e\nu$ final states [65]:

$$\begin{aligned} \alpha &= (-0.5 \pm 2.8) \times 10^{-17}\,{\rm GeV} \\ \beta &= (2.5 \pm 2.3) \times 10^{-19}\,{\rm GeV} \\ \gamma &= (1.1 \pm 2.5) \times 10^{-21}\,{\rm GeV}\ . \end{aligned} \tag{66}$$

All results are compatible with no *CPT* violation, while the sensitivity approaches the interesting level of $\mathcal{O}(10^{-20}\,{\rm GeV})$.

The uncertainties on the KLOE measurements of the ζ, α, β, γ, and ω parameters should improve by more than a factor two with the analysis of the full KLOE data sample of 2.5 fb^{-1}.

As discussed above *CPT* violation effects might also induce a breakdown of the correlation of state (23), as given in eq.(54). A similar analysis performed on the same KLOE data as before, including in the fit the modified initial state (54), yields the first measurement of the complex parameter ω [59]:

$$\Re(\omega) = \left(1.1^{+8.7}_{-5.3} \pm 0.9\right) \times 10^{-4} \qquad \Im(\omega) = \left(3.4^{+4.8}_{-5.0} \pm 0.6\right) \times 10^{-4}\ ;$$

compatible with no CPT violation, and with an accuracy that already reaches the interesting Planck's scale region.

8.2 CPT symmetry tests with $K_S \to \pi e \nu$ decays

For $t_1 \gg t_2, \tau_S$ (or $t_2 \gg t_1, \tau_S$), the amplitude (25) factorizes, and everything behaves like the initial state were an incoherent mixture of states $|K_S\rangle|K_L\rangle$ and $|K_L\rangle|K_S\rangle$. Hence the detection of a kaon at large times *tags* a $|K_S\rangle$ state in the opposite direction. This is a unique feature at a ϕ-factory, not possible at fixed target experiments, that can be exploited to select a pure K_S beam.

At KLOE a K_S is tagged by identifying the interaction of the K_L in the calorimeter (K_L-crash). In fact about 50% of the produced K_L's in $\phi \to K_S K_L$ events reach the calorimeter before decaying; their associated interactions are identified by a high energy, neutral and delayed deposit in the calorimeter, i.e. not associated to any charged track in the event, and delayed of $\sim$ 30 ns (as $\beta_K \sim 0.22$) with respect to a photon coming from the interaction region. Pure K_S samples have been selected exploiting this tagging technique. In particular $K_S \to \pi e \nu$ decays are selected requiring a K_L-crash and two tracks forming a vertex close to the IP, and associated with two energy deposits in the calorimeter. Pions and electrons are recognized using a time-of-flight technique. The number of signal events is normalized to the number of $K_S \to \pi^+\pi^-$ in the same data set. Then the first measurement of the K_S semileptonic charge asymmetry has been performed [66]:

$$A_S = (1.5 \pm 9.6_{\mathrm{stat}} \pm 2.9_{\mathrm{syst}}) \times 10^{-3} \ .$$

The uncertainty on A_S can be reduced at the level of $\approx 3 \times 10^{-3}$ with the analysis of the full data sample of 2.5 fb^{-1}.

From the sum and the difference of the K_S and K_L semileptonic charge asymmetries one can test CPT conservation. Using the values of A_L, $\Re\delta$, and $\Re\bar{\epsilon}$ from other experiments [61], the real part of the CPT violating and $\Delta S = \Delta Q$ violating (conserving) parameter x_- (y) in semileptonic decay amplitudes (see eqs.(42) and (43)), can be evaluated [66]:

$$\Re x_- = \frac{A_S - A_L}{4} - \Re\delta = (-0.8 \pm 2.5) \times 10^{-3}$$

$$\Re y = \Re\bar{\epsilon} - \frac{A_S + A_L}{4} = (0.4 \pm 2.5) \times 10^{-3} \ . \quad (67)$$

8.3 CPT symmetry test from unitarity

The unitarity relation, originally derived by Bell and Steinberger [67],

$$\left(\frac{\Gamma_S+\Gamma_L}{\Gamma_S-\Gamma_L}+i\tan\phi_{SW}\right)\left[\frac{\Re\bar{\epsilon}}{1+|\bar{\epsilon}|^2}-i\Im\delta\right]=$$
$$=\frac{1}{\Gamma_S-\Gamma_L}\sum_f A^*(K_S\to f)A(K_L\to f)\equiv\sum_f\alpha_f \qquad (68)$$

can be used to bound $\Im\delta$, after having provided all the α_i parameters, Γ_S, Γ_L, and ϕ_{SW} as inputs. A detailed review on this subject is given in the contribution of Isidori [68].

Using KLOE measurements, PDG [61] values, and a combined fit of KLOE and CPLEAR data, the following result is obtained [69]:

$$\Re\bar{\epsilon}=(159.6\pm1.3)\times10^{-5}\ , \qquad \Im\delta=(0.4\pm2.1)\times10^{-5}\ , \qquad (69)$$

the main limiting factor of this result being the uncertainty on the phase ϕ_{+-} of the η_{+-} parameter entering in $\alpha_{\pi^+\pi^-}$.

The limits on $\Im(\delta)$ and $\Re(\delta)$ [70] can be used (see eq.(19)) to constrain the mass and width difference between K^0 and $\bar{K}^0$. In the limit $\Gamma_{11}=\Gamma_{22}$, i.e. neglecting CPT-violating effects in the decay amplitudes, the best bound on the neutral kaon mass difference is obtained:

$$-5.3\times10^{-19}\ \text{GeV}<M_{11}-M_{22}<6.3\times10^{-19}\ \text{GeV} \quad \text{at 95 \% CL}\ . \qquad (70)$$

8.4 Lorentz and CPT symmetries tests

From the measured value [66] of A_S and a preliminary evaluation of A_L by KLOE, the difference $A_S-A_L=(-2\pm10)\times10^{-3}$, and a first preliminary evaluation of the Δa_0 parameter can be obtained [71]:

$$\Delta a_0=(0.4\pm1.8)\times10^{-17}\ \text{GeV}\ . \qquad (71)$$

With the analysis of the full KLOE data sample ($L=2.5$ fb^{-1}) a statistical sensitivity $\delta(\Delta a_0)\sim7\times10^{-18}$ GeV could be reached. In the case of the method based on double semileptonic decays (see eq.(45)), the analysis of the full data sample could yield a sensitivity $\delta(\Delta a_0)\sim1\times10^{-17}$ GeV.

An analysis has been performed on the same sample ($L=380$ pb^{-1}) of $\phi\to K_SK_L\to\pi^+\pi-,\pi^+\pi^-$ events used for the measurement of decoherence parameters, exploiting the method based on eq.(61). It yields a first preliminary evaluation of Δa_Z [71]:

$$\Delta a_Z = (-1 \pm 4) \times 10^{-17} \text{ GeV} . \tag{72}$$

With the analysis of 2.5 fb^{-1} a statistical sensitivity $\delta(\Delta a_Z) \sim 2 \times 10^{-17}$ GeV could be reached. In the case of the method based on eq.(60), the analysis of the full data sample could yield a sensitivity $\delta(\Delta a_Z) \sim 3 \times 10^{-17}$ GeV.

The same level of accuracy could also be reached on the Δa_X and Δa_Y parameters by means of a proper sidereal time dependent analysis. However in this case the sensitivity would not be competitive with a preliminary measurement performed by the KTeV collaboration [72] based on the search of sidereal time variation of the phase ϕ_{+-}, that constrains Δa_X and Δa_Y to less than 9.2×10^{-22} GeV at 90% C.L. .

The Δa_μ parameters have also been recently constrained in the B-meson system [73] with an accuracy of $\mathcal{O}(10^{-12}$ GeV).

9 The KLOE-2 program

A proposal [74] has been recently submitted for a physics program to be carried out with an upgraded KLOE detector, KLOE-2, at a new Frascati e^+e^- collider, which is expected to deliver an integrated luminosity of the order of 50 fb^{-1} at the $\phi(1020)$ peak. The high luminosity is necessary to reach significant sensitivities in the tests discussed above by means of neutral kaon interferometry.

As discussed above, the decay mode $\phi \to K_S K_L \to \pi^+\pi^-\pi^+\pi^-$ is very rich in physics. In general all decoherence effects show a deviation from the quantum mechanical prediction (47). Hence the reconstruction of events in the region at $\Delta t \approx 0$, i.e. with vertices near the IP, is crucial for precise determination of the parameters related to CPT violation and to the decoherence. The vertex resolution affects the $I(\pi^+\pi^-, \pi^+\pi^-; |\Delta t|)$ distribution precisely in that region, as shown in Fig. 7, and its impact on the decoherence parameter measurements has to be carefully evaluated. In fact, the resolution has two main effects: (1) to reduce the statistical sensitivity of the fit to the parameters; (2) to introduce a source of systematic uncertainties. In Figs. 8, 9 the statistical uncertainty on several decoherence and CPT-violating parameters is shown as a function of the integrated luminosity for the case $\sigma_{|\Delta t|} \approx \tau_S$ (present KLOE

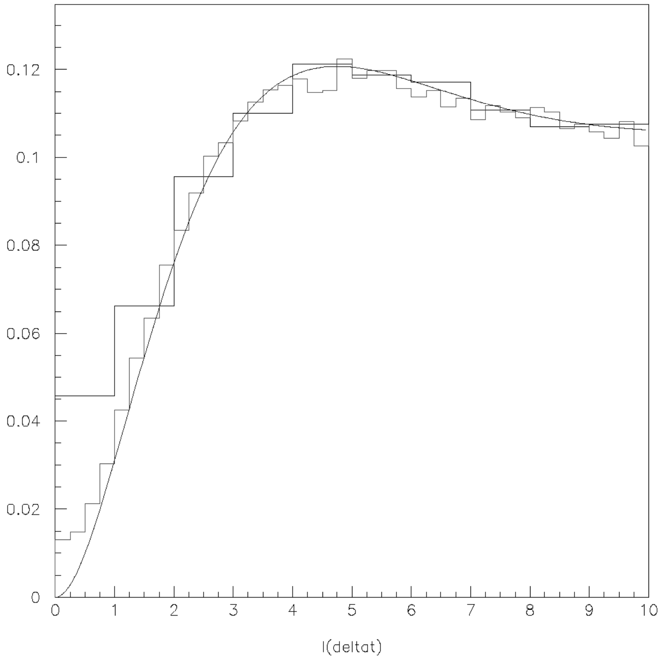


Figure 7: *The $I(\pi^+\pi^-, \pi^+\pi^-; |\Delta t|)$ distribution as a function of $|\Delta t|$ (in τ_S units) with the present KLOE resolution $\sigma_{|\Delta t|} \approx \tau_S$ (histogram with large bins), with an improved resolution $\sigma_{|\Delta t|} \approx 0.25\,\tau_S$ (histogram with small bins), and in the ideal case (solid line).*

resolution), and for $\sigma_{|\Delta t|} \approx 0.25\,\tau_S$. As it can be seen in the last case an improvement of about a factor two could be achieved. Therefore the addition of a vertex detector between the spherical beam pipe and the drift chamber, improving the vertex resolution in that region in order to have $\sigma_{|\Delta t|} \approx 0.25\,\tau_S$, is the major upgrade of the KLOE detector that has been considered in the KLOE-2 proposal. The KLOE-2 physics program concerning interferometry measurements is summarized in table 2, where the KLOE-2 statistical sensitivities to the main parameters that can be extracted from the experimental time distributions $I(f_1, f_2; \Delta t)$ with different choices of final states f_i, are listed in the hypothesis of an integrated luminosity $L = 50$ fb^{-1}, and compared to the best presently published measurements.

Table 2: KLOE-2 statistical sensitivities on several parameters.

f_1	f_2	parameter	best published meas.	KLOE-2 (50 fb^{-1})
$K_S \to \pi e \nu$		A_S	$(1.5 \pm 11) \times 10^{-3}$	$\pm 1 \times 10^{-3}$
$\pi^+\pi^-$	$\pi l \nu$	A_L	$(3322 \pm 58 \pm 47) \times 10^{-6}$	$\pm 25 \times 10^{-6}$
$\pi^+\pi^-$	$\pi^0\pi^0$	$\Re\frac{\epsilon'}{\epsilon}$	$(1.66 \pm 0.26) \times 10^{-3}$	$\pm 0.2 \times 10^{-3}$
$\pi^+\pi^-$	$\pi^0\pi^0$	$\Im\frac{\epsilon'}{\epsilon}$	$(1.2 \pm 2.3) \times 10^{-3}$	$\pm 3 \times 10^{-3}$
$\pi^+ l^- \bar{\nu}$	$\pi^- l^+ \nu$	$(\Re\delta + \Re x_-)$	$\Re\delta = (0.29 \pm 0.27) \times 10^{-3}$ $\Re x_- = (-0.8 \pm 2.5) \times 10^{-3}$	$\pm 0.2 \times 10^{-3}$
$\pi^+ l^- \bar{\nu}$	$\pi^- l^+ \nu$	$(\Im\delta + \Im x_+)$	$\Im\delta = (0.4 \pm 2.1) \times 10^{-5}$ $\Im x_+ = (0.8 \pm 0.7) \times 10^{-2}$	$\pm 3 \times 10^{-3}$
$\pi^+\pi^-$	$\pi^+\pi^-$	Δm	$5.288 \pm 0.043 \times 10^9 s^{-1}$	$\pm 0.03 \times 10^9 s^{-1}$
$\pi^+\pi^-$	$\pi^+\pi^-$	ζ_{SL}	$(1.8 \pm 4.1) \times 10^{-2}$	$\pm 0.2 \times 10^{-2}$
$\pi^+\pi^-$	$\pi^+\pi^-$	$\zeta_{0\bar{0}}$	$(1.0 \pm 2.1) \times 10^{-6}$	$\pm 0.1 \times 10^{-6}$
$\pi^+\pi^-$	$\pi^+\pi^-$	α	$(-0.5 \pm 2.8) \times 10^{-17}$ GeV	$\pm 2 \times 10^{-17}$ GeV
$\pi^+\pi^-$	$\pi^+\pi^-$	β	$(2.5 \pm 2.3) \times 10^{-19}$ GeV	$\pm 0.1 \times 10^{-19}$ GeV
$\pi^+\pi^-$	$\pi^+\pi^-$	γ	$(1.1 \pm 2.5) \times 10^{-21}$ GeV	$\pm 0.2 \times 10^{-21}$ GeV (compl. pos. hyp.) $\pm 0.1 \times 10^{-21}$ GeV
$\pi^+\pi^-$	$\pi^+\pi^-$	$\Re\omega$	$(1.1^{+8.7}_{-5.3} \pm 0.9) \times 10^{-4}$	$\pm 2 \times 10^{-5}$
$\pi^+\pi^-$	$\pi^+\pi^-$	$\Im\omega$	$(3.4^{+4.8}_{-5.0} \pm 0.6) \times 10^{-4}$	$\pm 2 \times 10^{-5}$
$K_{S,L} \to \pi e \nu$ $\pi^+ l^- \bar{\nu}$	 $\pi^- l^+ \nu$	Δa_0	(prelim.: $(0.4 \pm 1.8) \times 10^{-17}$ GeV)	$\pm 2 \times 10^{-18}$ GeV $\pm 2 \times 10^{-18}$ GeV
$\pi^+\pi^-$ $\pi^+\pi^-$	$\pi l \nu$ $\pi^+\pi^-$	Δa_Z	(prelim.: $(-1 \pm 4) \times 10^{-17}$ GeV)	$\pm 5 \times 10^{-18}$ GeV $\pm 3 \times 10^{-18}$ GeV
$\pi^+\pi^-$ $\pi^+\pi^-$	$\pi l \nu$ $\pi^+\pi^-$	$\Delta a_X, \Delta a_Y$	(prelim.: $< 9.2 \times 10^{-22}$ GeV)	$\mathcal{O}(10^{-18})$ GeV $\mathcal{O}(10^{-18})$ GeV

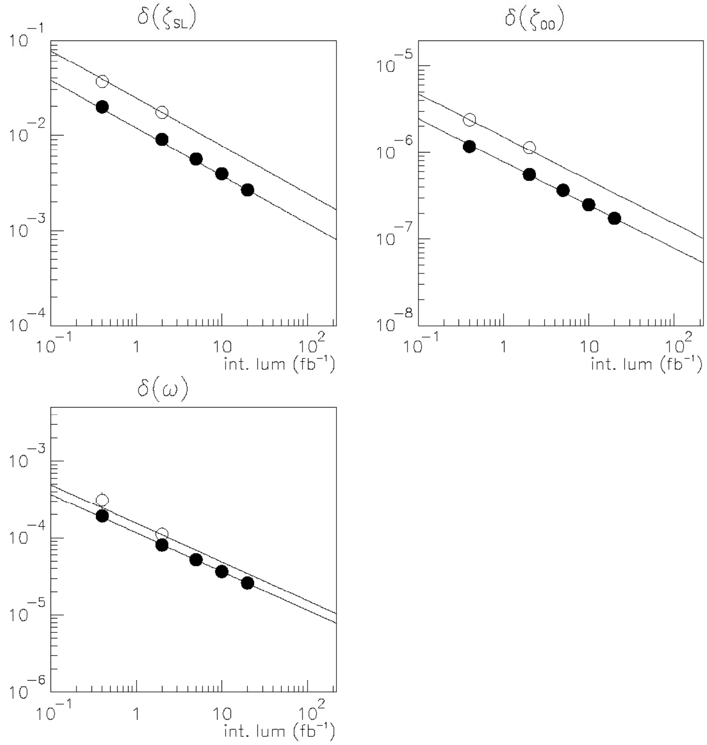


Figure 8: *The statistical sensitivity to the ζ_{SL}, $\zeta_{0\bar{0}}$ and $\Re\omega$ parameters with the present KLOE resolution $\sigma_{|\Delta t|} \approx \tau_S$ (open circles), with an improved resolution $\sigma_{|\Delta t|} \approx 0.25\,\tau_S$ (full circles).*

10 Conclusions

A ϕ-factory represents a unique opportunity to study the neutral kaon system, and the related fundamental discrete symmetries. It is also an ideal place to investigate the entanglement and correlation properties of the produced $K^0\bar{K}^0$ pairs, as well as CPT violation effects that might be induced by quantum gravity effects.

The KLOE experiment concluded the data taking at the beginning of 2006 with a total integrated luminosity of ~ 2.5 fb^{-1}. Several parameters related to possible CPT violations in conjunction with decoherence or Lorentz symmetry violations, have been measured, some of them for the first time, and with a

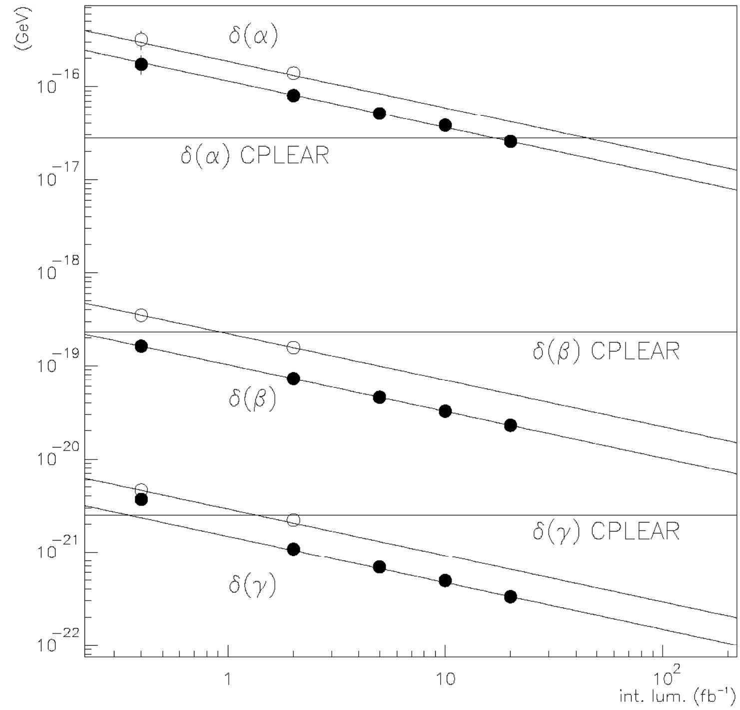


Figure 9: *The statistical sensitivity to the parameters* α*,* β*,* γ *with the present KLOE resolution* $\sigma_{|\Delta t|} \approx \tau_S$ *(open circles), and with an improved resolution* $\sigma_{|\Delta t|} \approx 0.25\,\tau_S$ *(full circles); the horizontal lines represent the CPLEAR results.*

precision reaching the interesting Planck's scale region; with the analysis of the full KLOE data sample further improvements are expected on all results.

The search for such CPT violation effects and precision tests of quantum mechanics by means of neutral kaon interferometry constitute one of the main physics issues of the KLOE-2 proposal. With an integrated luminosity of about 50 fb^{-1}, significant improvements in a variety of observables involving different final states are expected.

References

1. G. Lueders, Ann. Phys. (NY) **2** (1957) 1, reprinted in Ann. Phys. (NY) **281** (2000) 1004.

2. W. Pauli, *Exclusion principle, Lorentz group and reflexion of space-time and charge* in *Niels Bohr and the development of physics*, edited by W. Pauli, Pergamon, London, 1955, p.30.

3. J. S. Bell, Proc. R. Soc. London A **231** (1955) 479

4. R. Jost, Helv. Phys. Acta **30** (1957) 409.

5. V. Weisskopf and E. P. Wigner, Z. Phys. **63** (1930) 54; see also appendix A of: P. K. Kabir, *The CP puzzle*, Academic Press, London, 1968, or appendix I of: O. Nachtmann, *Elementary Particle Physics: Concepts and Phenomena*, Springer-Verlag, Berlin, 1990.

6. L. Maiani, in *The second DaΦne handbook*, ed. L. Maiani, G. Pancheri, N. Paver, Vol.I, INFN-LNF, Frascati, 1995.

7. G. C. Branco, L. Lavoura, J. P. Silva, *CP Violation*, Oxford University Press, Oxford, 1999.

8. I. I. Bigi, A. I. Sanda, *CP Violation*, Cambridge University press, Cambridge, 2000.

9. M. Fidecaro, H. J. Gerber, Rep. Progr. Phys. **69** (2006) 1713.

10. L. Lavoura, Ann. Phys. **207** (1991) 428.

11. J. P. Silva, Phys. Rev. D **62** (2000) 116008.

12. T. T. Wu, C. N. Yang, Phys. Rev. Lett. **13** (1964) 380.

13. P. Bloch, *CPT invariance tests in neutral kaon decay*, in Ref. [61]; see also D.-D. Wu, Mod. Phys. Lett. A **13** (1998) 1413 and references therein.

14. A. Angelopoulos et al., CPLEAR collaboration, Phys. Rep. **374** (2003) 165.

15. M. Goldhaber, T. D. Lee, C. N. Yang, Phys. Rev. **112** (1958) 1796; see also M. Goldhaber, C. N. Yang *The K^0-$\bar{K}^0$ system in p-$\bar{p}$ annihilation at rest* in "Evolution of Particle Physics" Edited by M. Conversi, Academic Press Inc., New York, 1970.

16. J. Bernabeu, N. Mavromatos, J. Papavassiliou, Phys. Rev. Lett. **92** (2004) 131601.

17. J. Bernabeu, N. Mavromatos, J. Papavassiliou, A. Waldron-Lauda, Nucl. Phys. B **744** (2006) 180.

18. C. D. Buchanan et al., Phys. Rev. D **45** (1992) 4088.

19. G. D'Ambrosio, G. Isidori, A. Pugliese, in *The second DaΦne handbook*, ed. L. Maiani, G. Pancheri, N. Paver, Vol.I, INFN-LNF, Frascati, 1995.

20. M. Hayakawa and A. I. Sanda, Phys. Rev. D **48** (1993) 1150.

21. T. D. Lee and C. N. Yang, reported by T. D. Lee at Argonne National Laboratory, May, 1960 (unpublished).

22. D. R. Inglis, Rev. Mod. Phys. **33** (1961) 1.

23. T. B. Day, Phys. Rev.**121** (1961) 1204.

24. H. J. Lipkin, Phys. Rev. **176** (1968) 1715.

25. A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. **47** (1935) 777.

26. R. Bertlmann, B. Hiesmayr, *Strangeness measurements of kaon pairs, CP violation and Bell inequalities*, this handbook.

27. A. Bramon, R. Escribano, G. Garbarino, *A review of Bell inequality tests with neutral kaons*, this handbook.

28. A. Go, *Kaon interferometry at CPLEAR* , this handbook.

29. A. Bramon, G. Garbarino, B. Hiesmayr, *Kaonic quantum erasers at a phi-factory: "erasing the present, changing the past"*, this handbook.

30. W. H. Furry, Phys. Rev. **49** (1936) 393.

31. P. H. Eberhard, in *The second DaΦne handbook*, ed. L. Maiani, G. Pancheri, N. Paver, Vol.I, INFN-LNF, Frascati, 1995.

32. R. A. Bertlmann, W. Grimus, B. C. Hiesmayr, Phys. Rev. D **60** (1999) 114032.

33. F. Benatti, R. Floreanini, Nucl. Phys. B **488** (1997) 335.

34. F. Benatti, R. Floreanini, Nucl. Phys. B **511** (1998) 550.

35. F. Benatti, R. Floreanini, Phys. Lett. B **468** (1999) 287.

36. R. A. Bertlmann, K. Durstberger, B. C. Hiesmayr, Phys. Rev. A **68** (2003) 012111.

37. H. J. Gerber, Phys. Rev. Lett. **80** (1998) 2969

38. H. J. Gerber, Eur. Phys. J. C **32** (2004) 229

39. F. Benatti, R. Floreanini, *Open quantum dynamics: complete positivity and correlated kaons*, this handbook.

40. S. Hawking, Phys. Rev. D **14** (1976) 2460.

41. S. Hawking, Commun. Math. Phys. **87** (1982) 395.

42. R. Wald, Phys. Rev. D **21** (1980) 2742.

43. S. Hawking, Phys. Rev. D **72** (2005) 084013.

44. J. Smolin, J. Oppenheim, Phys. Rev. Lett. **96** (2006) 081302.

45. C. Kiefer, Annalen Phys. **15** (2005) 129

46. J. Ellis, J. S. Hagelin, D. V. Nanopoulos, M. Srednicki , Nucl. Phys. B **241** (1984) 381.

47. J. Ellis, J. L. Lopez, N. .E. Mavromatos, D. V. Nanopoulos, Phys. Rev. D **53** (1996) 3846.

48. P. Huet, M. Peskin, Nucl. Phys. B **434** (1995) 3.

49. J. Bernabeu, J. Ellis, N. Mavromatos, D. Nanopoulos and J. Papavassiliou, *CPT and quantum mechanics tests with kaons* , this handbook.

50. S. Sarkar, *Methods and models for the study of decoherence*, this handbook.

51. G. Amelino-Camelia, M. Arzano, A. Marcianò *On the quantum gravity phenomenology of multiparticle states*, this handbook.

52. V. A. Kostelecky, Phys. Rev. Lett. **80**, 1818 (1998).

53. V. A. Kostelecky, Phys. Rev. D **61**, 016002 (1999).

54. V. A. Kostelecky, Phys. Rev. D **64**, 076001 (2001).

55. R. Lehnert, *CPT and Lorentz symmetry breaking: a review* , this handbook.

56. S. Guiducci, *Status of DAΦNE*, in Eds. P. Lucas, S. Webber, Proc. of the 2001 Particle Accelerator Conference, Chicago, IL, 2001, p.353.

57. M. Adinolfi et al., KLOE collaboration, Nucl. Instr. and Meth. A **482** (2002) 364.

58. M. Adinolfi et al., KLOE collaboration, Nucl. Instr. and Meth. A **488** (2002) 51.

59. F. Ambrosino et al., KLOE collaboration, Phys. Lett. B **642** (2006) 315.

60. F. Ambrosino et al., Nucl. Instr. and Meth. A **534** (2004) 403.

61. W.-M. Yao et al., Particle Data Group, J. Phys. G**33** (2006) 1.

62. A. Apostolakis et al., CPLEAR collaboration, Phys. Lett. B **422** (1998) 339.

63. A. Go et al, Belle collaboration, Phys. Rev. Lett. **99** (2007) 131802.

64. A. Di Domenico, *Correlations in ϕ decays into $K^0\bar{K}^0$*, Proceedings of the International School of Physics "Enrico Fermi" CLXIII course "CP violation: from quarks to leptons", 19-29 July 2005, Varenna, Italy.

65. R. Adler et al., CPLEAR collaboration, Phys. Lett. B **364** (1995) 239.

66. F. Ambrosino et al., KLOE collaboration, Phys. Lett. B **636** (2006) 173.

67. J.S. Bell and J. Steinberger, *Proc. Oxford Int. Conf. on Elementary Particles* (1965).

68. G. Isidori, *Testing CPT in the neutral kaon system by means of the Bell-Steinberger relation* , this handbook.

69. F. Ambrosino et al., KLOE collaboration, JHEP **12** 011 (2006).

70. A. Angelopoulos et al., CPLEAR collaboration, Eur. Phys. J. C **22** (2001) 55.

71. A. Di Domenico, (KLOE collaboration), *CPT and Lorentz Symmetry IV*, V.A. Kostelecký, ed., World Scientific, Singapore, 2007 in publication.

72. H. Nguyen, (KTeV collaboration), in *CPT and Lorentz Symmetry II*, V.A. Kostelecký, ed., World Scientific, Singapore, 2002.

73. B. Aubert et al., BABAR collaboration, arXiv: hep-ex/0607103.

74. R. Beck et al, KLOE-2 collaboration, *Expression of interest for the continuation of the KLOE physics program at DAΦNE upgraded in luminosity and in energy*, March 31, 2006, available at http://www.lnf.infn.it/lnfadmin/direzione/roadmap/LoIKLOE.pdf .

CPT AND QUANTUM MECHANICS TESTS WITH KAONS

Jose Bernabéu

Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46100 Burjassot (Valencia), Spain

John Ellis

CERN, Physics Department, Theory Division, CH-1211 Geneva 23, Switzerland

Nick E. Mavromatos

King's College London, Department of Physics, Strand, London WC2R 2LS, United Kingdom

Dimitri V. Nanopoulos

George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA; Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA; Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece

Joannis Papavassiliou

Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46100 Burjassot (Valencia), Spain

Abstract

In this review we first discuss the theoretical motivations for possible CPT violation and deviations from ordinary quantum-mechanical behavior of field-theoretic systems in the context of an extended class of quantum-gravity models. Then we proceed to a description of precision tests of CPT symmetry using mainly neutral kaons. We emphasize the possibly unique rôle of neutral meson factories in providing specific tests of models where the quantum-mechanical CPT operator is not well-defined, leading to modifications of Einstein-Podolsky-Rosen particle correlators. Finally, we present tests of CPT, T, and CP using charged kaons, and in particular $K^{\pm}_{\ell 4}$ decays, which are interesting due to the high statistics attainable in experiments.

1 CPT Symmetry and Quantum Gravity: Motivations for its Possible Violation

Any complete theory of *quantum gravity* (QG) is bound to address fundamental issues, directly related to the emergence of space-time and its structure at energies beyond the Planck energy scale $M_P \sim 10^{19}$ GeV. From our experience with low-energy local quantum field theories on flat space-times, we are tempted to expect that a theory of QG should respect most of the fundamental symmetries that govern the standard model of electroweak and strong interactions, specifically Lorentz symmetry and CPT invariance, that is invariance under the combined action of Charge Conjugation (C), Parity (P) and Time Reversal Symmetry (T).

CPT invariance is guaranteed in flat space-times by a theorem applicable to any local quantum field theory of the type used to describe the standard phenomenology of particle physics to date. The *CPT theorem* can be stated as follows [1)]: *Any quantum theory formulated on flat space-times is symmetric under the combined action of CPT transformations, provided the theory respects (i) Locality, (ii) Unitarity (i.e. conservation of probability) and (iii) Lorentz invariance.*

The extension of this theorem to QG is far from obvious. In fact, it is still a wide open and challenging issue, linked with our (very limited at present) understanding of QG, as well as the very nature of space-time at (microscopic) Planckian distances 10^{-35} m. The important point to notice is that the CPT theorem *may not* be valid (at least in its strong form) in highly curved (singular) space-times, such as black holes, or more general in some QG models involving *quantum space-time foam* backgrounds [2)]. The latter are characterized by singular quantum fluctuations of space-time geometry, such as black holes, *etc.*, with event horizons of microscopic Planckian size. Such backgrounds result in *apparent* violations of *unitarity* in the following sense: there is some part of the initial information (quantum numbers of incoming matter) which "disappears" inside the microscopic event horizons, so that an observer at asymptotic infinity will have to trace over such "trapped" degrees of freedom. One faces therefore a situation where an initially pure state evolves in time and becomes mixed. The asymptotic states are described by density matrices, defined as

$$\rho_{\rm out} = {\rm Tr}_M |\psi >< \psi| \ , \tag{1}$$

where the trace is over trapped (unobserved) quantum states that disappeared inside the microscopic event horizons in the foam. Such a non-unitary evolution makes it impossible to define a standard quantum-mechanical scattering matrix. In ordinary local quantum field theory, the latter connects asymptotic state vectors in a scattering process

$$|\mathrm{out}>= S\ |\mathrm{in}>,\ S = e^{iH(t_f - t_i)}\ , \tag{2}$$

where $t_f - t_i$ is the duration of the scattering (assumed to be much longer than other time scales in the problem, i.e. $\lim\, t_i \to -\infty,\ t_f \to +\infty$). Instead, in foamy situations, one can only define an operator that connects asymptotic density matrices [3]:

$$\rho_{\mathrm{out}} \equiv \mathrm{Tr}_M|\mathrm{out}><\mathrm{out}| = \$\ \rho_{\mathrm{in}}, \qquad \$ \neq S\ S^\dagger. \tag{3}$$

The lack of factorization is attributed to the apparent loss of unitarity of the effective low-energy theory, defined as the part of the theory accessible to low-energy observers performing scattering experiments. In such situations particle phenomenology has to be reformulated [4, 5] by viewing our low-energy world as an open quantum system and using (3). Correspondingly, the usual Hamiltonian evolution of the wave function is replaced by the Liouville equation for the density matrix [4]

$$\partial_t \rho = i[\rho, H] + \delta \not{H} \rho\ , \tag{4}$$

where $\delta \not{H} \rho$ is a correction of the form normally found in open quantum-mechanica systems [6].

The \$ matrix is *not invertible*, and this reflects the effective unitarity loss. It is this property that leads to a violation of CPT invariance, since one of the requirements of CPT theorem (unitarity) is violated. But in this particular case there is something more than a mere violation of the symmetry. The CPT operator itself is *not well-defined*, at least from an effective field theory point of view. This is a strong form of CPT violation (CPTV). There is a corresponding theorem by Wald [7] describing the situation: *In an open (effective) quantum theory, interacting with an environment, e.g., quantum gravitational, where* $\$ \neq SS^\dagger$, *CPT invariance is violated, at least in its strong form.*

The proof is based on elementary quantum mechanical concepts and the above-mentioned non-invertibility of \$, as well as the relation (3) connecting asymptotic *in* and *out* density matrices. Let one suppose that there is invariance under CPT, then there must exist a unitary, invertible operator Θ acting on density matrices, such that $\Theta\overline{\rho}_{in} = \rho_{out}$, where the barred quantities denote antiparticles. Using (3), after some elementary algebraic manipulations we obtain $\rho_{out} = \$ \ \rho_{in} \rightarrow \Theta\overline{\rho}_{in} = \$ \ \Theta^{-1}\overline{\rho}_{out} \rightarrow \overline{\rho}_{in} = \Theta^{-1}\$ \ \Theta^{-1}\overline{\rho}_{out}$. But, since $\overline{\rho}_{out} = \$\overline{\rho}_{in}$, one arrives at $\overline{\rho}_{in} = \Theta^{-1}\$ \ \Theta^{-1} \ \$ \ \overline{\rho}_{in}$.

The latter relation, if true, would imply that \$ has an inverse $\Theta^{-1} \ \$\Theta^{-1}$; but this can be shown to be impossible when one has a mixed final state, i.e., *decoherence* (which is related to information loss). We omit here the details of this last but important part, due to lack of space. The interested reader is referred to the original literature [7].

From the above considerations one concludes that, under the special circumstances described, the generator of CPT transformations cannot be a well-defined quantum-mechanical operator (and thus CPT is violated at least in its strong form). This form of violation introduces a fundamental arrow of time/microscopic time irreversibility, unrelated in principle to CP properties. The reader's attention is called to the fact that such decoherence-induced CPT violation (CPTV) would occur in effective field theories, i.e., when the low-energy experimenters do not have access to all the degrees of freedom of QG (e.g., back-reaction effects, *etc.*). It is unknown whether full CPT invariance could be restored in the (still elusive) complete theory of QG.

In such a case, however, there may be [7] a *weak form of CPT invariance*, in the sense of the possible existence of *decoherence-free subspaces* in the space of states of a matter system. If this situation is realized, then the strong form of CPTV will not show up in any measurable quantity (that is, scattering amplitudes, probabilities *etc.*).

The weak form of CPT invariance may be stated as follows: *Let $\psi \in \mathcal{H}_{in}$, $\phi \in \mathcal{H}_{out}$ denote pure states in the respective Hilbert spaces $\mathcal{H}$ of in and out states, assumed accessible to experiment. If θ denotes the (anti-unitary) CPT operator acting on pure state vectors, then weak CPT invariance implies the following equality between transition probabilities*

$$\mathcal{P}(\psi \rightarrow \phi) = \mathcal{P}(\theta^{-1}\phi \rightarrow \theta\psi) \ . \quad (5)$$

Experimentally it is possible, at least in principle, to test equations like (5), in the sense that, if decoherence occurs, it induces (among other modifications) damping factors in the time profiles of the corresponding transition probabilities. The diverse experimental techniques for testing decoherence range from terrestrial laboratory experiments (in high-energy, atomic and nuclear physics) to astrophysical observations of light from distant extragalactic sources and high-energy cosmic neutrinos [5].

In the present article, we restrict ourselves to decoherence and CPT invariance tests within the neutral kaon system [4, 8, 9, 10, 11]. As we argue later on, this type of (decoherence-induced) CPTV exhibits some fairly unique effects in ϕ factories [12], associated with a possible modification of the Einstein-Podolsky-Rosen (EPR) correlations of the entangled neutral kaon states produced after the decay of the ϕ-meson (similar effects could be present for B mesons produced in Υ decays).

Another possible mechanism of CPTV in QG is the *spontaneous breaking of Lorentz symmetry (SBL)* [13]; this type of CPTV does not necessarily imply (nor does it invoke) decoherence. In this case the ground state of the field theoretic system is characterized by non-trivial vacuum expectation values of certain tensorial quantities,

$$\langle \mathcal{A}_\mu \rangle \neq 0 \ , \quad \text{or} \quad \langle \mathcal{B}_{\mu_1 \mu_2 \ldots} \rangle \neq 0 \quad etc. \ . \tag{6}$$

This may occur in (non-supersymmetric ground states of) string theory and other models, such as loop QG [14]. Again there is an extensive literature on the subject of experimental detection/bounding of potential Lorentz violation, which we do not discuss here [15, 16]. Instead we restrict ourselves to Lorentz tests using neutral kaons [17]. We stress at this point that quantum-gravitational decoherence and Lorentz violation are in principle independent, in the sense that there exist quantum-coherent Lorentz-violating models as well as Lorentz-invariant decoherence scenarios [18].

The important difference between the CPTV in SBL models and the CPTV due to the space-time foam is that in the former case the CPT operator is well-defined, but *does not commute* with the effective Hamiltonian of the matter system. In such cases one may parametrize the Lorentz and/or CPT breaking terms by local field theory operators in the effective Lagrangian, leading to a construction known as the "standard model extension" (SME) [13], which is a framework for studying precision tests of such effects.

CPTV may also be caused due to deviations from locality, e.g., as advocated in [19], in an attempt to explain observed neutrino 'anomalies', such as the LSND result [20]. Violations of locality could also be tested with high precision, by studying discrete symmetries in meson systems.

If present, CPT-violating effects are expected to be strongly suppressed, and thus difficult to detect experimentally. Naively, QG has a dimensionful constant, $G_N \sim 1/M_P^2$, where $M_P = 10^{19}$ GeV is the Planck scale. Hence, CPT violating and decohering effects may be expected to be suppressed by E^3/M_P^2, where E is a typical energy scale of the low-energy probe. However, there could be cases where loop resummation and other effects in theoretical models result in much larger CPT-violating effects, of order $\frac{E^2}{M_P}$. This happens, for instance, in some loop gravity approaches to QG [14], or some non-equilibrium stringy models of space-time foam involving open string excitations [21]. Such large effects may lie within the sensitivities of current or immediate future experimental facilities (terrestrial and astrophysical), provided that enhancements due to the near-degeneracy take place, as in the neutral-kaon case.

When interpreting experimental results in searches for CPT violation, one should pay particular attention to disentangling ordinary-matter-induced effects, that mimic CPTV, from genuine effects due to QG [5]. The order of magnitude of matter induced effects, especially in neutrino experiments, is often comparable to that expected in some models of QG, and one has to exercise caution, by carefully examining the dependence of the alleged "effect" on the probe energy, or on the oscillation length (in neutrino oscillation experiments). In most models, but *not always*, since the QG-induced CPTV is expressed as a back-reaction effect of matter onto space-time, it increases with the probe energy E (and oscillation length L in the appropriate situations). In contrast, ordinary matter-induced "fake" CPT-violating effects increase with L.

We emphasize that the phenomenology of CPTV is complicated, and there does *not* seem to be a *single* figure of merit for it. Depending on the precise way CPT might be violated in a given model or class of models of QG, there are different ways to test the violation [5]. Below we describe only a selected class of such sensitive probes of CPT symmetry and quantum-mechanical evolution (unitarity, decoherence). We commence the discussion by examining CPT and decoherence tests in neutral kaon decays, and then continue with some tests at meson factories, which are associated uniquely with a breaking of CPT in the sense of its ill-defined nature in "fuzzy" decoherent space-times. We then finish with a brief discussion of high-precision tests in some charged kaon decays, specifically four-body $K^{\pm}_{\ell 4}$ decays, which have recently become very relevant, as a result of the (significantly) increased statistics of recent experiments [22].

The structure of the article is as follows: in Section 2 we discuss kaon tests of Lorentz symmetry within the SME framework [17], and give the latest bounds and prospects, especially from the point of view of meson factories [23]. In Section 3 we describe tests of decoherence-induced CPTV using (single-state) neutral kaon systems. In Section 4 we discuss the novel EPR-like modifications in meson factories; the latter may arise if the CPT operator is not well-defined, as happens in some space-time foam models of QG. We argue in favour of the unique character of such tests in providing information on the stochastic nature of quantum space-time, and we give some order-of-magnitude estimates within some string-inspired models. As we show, such models can be falsified (or severely constrained) in next-generation (upgraded) ϕ-meson factories, such as DAΦNE [24]. The enhancement of the effect provided by the identical decay channels $(\pi^+\pi^-, \pi^+\pi^-)$ is unique. Finally, in Section 5 we discuss precision tests of the discrete symmetries T, CP and CPT using a specific type of charged kaon decays [25, 26], namely $K^{+(-)} \to \pi^+ + \pi^- + \ell(\overline{\ell}) + \nu_\ell(\overline{\nu}_\ell)$. Recently, high statistics has been attained by the NA48 experiment [22], thereby increasing the prospects of using such decays for precision tests of CPT symmetry. This could be accomplished through the study of (appropriately constructed [27]) T-odd observables between the $K^\pm$ modes, involving triple momentum products of the lepton and the di-pion state $\vec{p} \cdot (\vec{p}_1 \times \vec{p}_2)$, which we discuss briefly.

2 Standard Model Extension, Lorentz Violation and Neutral Kaons

2.1 Formalism and Order-of-Magnitude Estimates

As mentioned earlier, there is a case where Lorentz symmetry is (spontaneously) violated, in the sense of certain tensorial quantities acquiring vacuum expectation values (6). Hence CPT is violated, but no quantum decoherence or unitarity loss occurs. The generator of the CPT symmetry is a well-defined operator, which, however, does not commute with the effective (low-energy) Hamiltonian of the matter system.

Most microscopic models where such a violation is realized are based on string theory with exotic (non-supersymmetric) ground states (backgrounds) [13] characterized by tachyonic instabilities. In the corresponding effective low-energy string action tachyon fields couple to tensorial fields (gauge, *etc.*), leading to non-zero v.e.v.s of certain tensorial quantities, thus inducing Lorentz symmetry violation in these exotic string ground states. Models from loop gravity [14] or non-commutative geometries may also display similar types of Lorentz violation, described by analogous terms in a SME effective Hamiltonian.

The upshot of SME is that there is a *Modified Dirac Equation* for spinor fields ψ, representing leptons and quarks with charge q:

$$\left(i\gamma^\mu D^\mu - M - a_\mu\gamma^\mu - b_\mu\gamma_5\gamma^\mu - \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu} + ic_{\mu\nu}\gamma^\mu D^\nu + id_{\mu\nu}\gamma_5\gamma^\mu D^\nu\right)\psi = 0 \ ,$$

where $D_\mu = \partial_\mu - A^a_\mu T^a - qA_\mu$ is an appropriate gauge-covariant derivative. The non-conventional terms proportional to the coefficients $a_\mu,\ b_\mu,\ c_{\mu\nu},\ d_{\mu\nu},\ H_{\mu\nu},\ldots$ stem from corresponding local operators of the effective Lagrangian, which are phenomenological at this stage. The set of terms pertaining to $a_\mu\ ,b_\mu$ entail CPT & Lorentz violation, while the terms proportional to $c_{\mu\nu}\ ,d_{\mu\nu}\ ,H_{\mu\nu}$ exhibit Lorentz violation only.

It should be stressed that, within the SME framework (as also with the decoherence approach to QG), CPTV does *not necessarily* imply mass differences between particle and antiparticles.

Some remarks are now in order, regarding the form and order-of-magnitude estimates of the Lorentz and/or CPT violating effects. In the approach of [13, 15, 17] the SME coefficients have been taken to be constants. Unfortunately there is not yet a detailed microscopic model available, that would allow for concrete predictions of the order of magnitude to be made. Theoretically, the (dimensionful, with dimensions of energy) SME parameters can be bounded by applying renormalization group and naturalness assumptions to the effective local SME Hamiltonian; for example, the bounds on b_μ so obtained are of the order of 10^{-17} GeV. At present all SME parameters should be considered as phenomenological, to be constrained by experiment.

In general, however, the SME coefficients may not be constant. In fact, in certain string-inspired or stochastic models of space-time foam with Lorentz symmetry violation, the coefficients a_μ, b_μ... are probe-energy (E) dependent, as a result of back-reaction effects of matter onto the fluctuating space-time.

Specifically, in stochastic models of space-time foam, one may find that on average there is no CPT and/or Lorentz violation, i.e., the respective statistical v.e.v.s (over stochastic space-time fluctuations) $\langle a_\mu\ , b_\mu\rangle = 0$, but this is not true for higher order correlators of these quantities (fluctuations), i.e., $\langle a_\mu a_\nu \rangle \neq 0$, $\langle b_\mu a_\nu \rangle \neq 0$, $\langle b_\mu b_\nu \rangle \neq 0$, ... In such a case the SME effects will be much more suppressed, since, by dimensional arguments, such fluctuations are expected to be of order E^4/M_P^2, probably with no chance of being observed in upcoming facilities, and certainly not in neutral kaon systems in the foreseeable future.

2.2 Tests of Lorentz Violation in Neutral Kaons

We now turn to a brief description of experimental tests of Lorentz symmetry within the SME framework, using neutral kaons, both single [17] and as entangled states at a ϕ factory [23].

We begin our analysis with the single-kaon case. To determine the relevant observable, we first recall that the wave function of the neutral kaon, Ψ, is represented as a two-component $\Psi^T = \left(K^0, \overline{K}^0\right)$ vector (the superscript T denotes matrix transposition).

Time evolution within the rules of quantum mechanics (but with CPT- and Lorentz-violation) is described by the equation

$$\partial_t \Psi = \mathcal{H}\Psi \, ,$$

where the effective Hamiltonian $\mathcal{H}$ includes CP-violating effects, the latter being parametrized by the conventional CP-(and T-)violating parameter of order $\epsilon_K \sim 10^{-3}$, as well as CPT-(and CP-) violating effects parametrized by the (complex) parameter [11] $\delta_K \sim (\mathcal{H}_{11} - \mathcal{H}_{22})/2\Delta\lambda$, with $\Delta\lambda$ the eigenvalue difference.

In order to isolate the terms in the SME effective Hamiltonian that are pertinent to neutral kaon tests, one should notice [17] that $\mathcal{H}_{11} - \mathcal{H}_{22}$ is flavour-diagonal, and that the parameter δ_K must be C-violating but P,T-preserving, as a consequence of strong-interaction properties in neutral meson evolution.

Hence one should look for terms in the SME formalism that share the above features, namely are flavour-diagonal and violate C, but preserve T, P. These considerations imply that δ_K is sensitive *only* to the $-a_\mu^q \overline{q}\gamma_\mu q$ quark terms in SME, where q denote quark fields, with the meson composition being denoted by $M = q_1\overline{q}_2$. The analysis of [17], then, leads to the following relation of the Lorentz- and CPT-violating parameter a_μ to the CPT-violating parameter δ_K of the neutral kaon system,

$$\delta_K \simeq i\sin\widehat{\phi}\exp(i\widehat{\phi})\gamma\left(\Delta a_0 - \vec{\beta}_K \cdot \Delta\vec{a}\right)/\Delta m,$$

with the usual short-hand notation S=short-lived, L=long-lived, I=interference term, $\Delta m = m_L - m_S$, $\Delta\Gamma = \Gamma_S - \Gamma_L$, $\widehat{\phi} = \arctan(2\Delta m/\Delta\Gamma)$, $\Delta a_\mu \equiv a_\mu^{q_2} - a_\mu^{q_1}$, and $\beta_K^\mu = \gamma(1, \vec{\beta}_K)$ the 4-velocity of the boosted kaon.

The experimental bounds on a_μ from the neutral-kaon experiments are based on searches for sidereal variations of δ_K (day-night effects). The experimental situation is depicted schematically in Fig. 1.

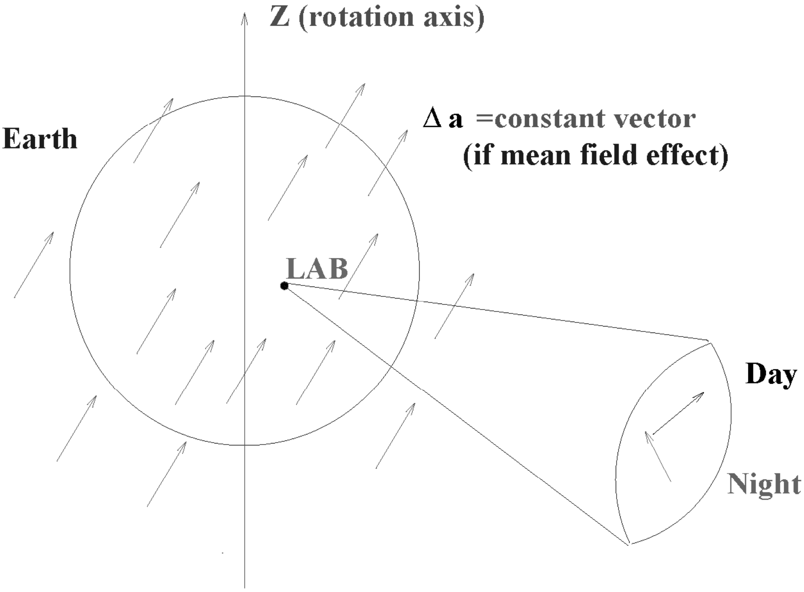


Figure 1: Schematic representation of searches for sidereal variations of the CPT-violating parameter δ_K in the SME framework. The green arrows, crossing the Earth indicate a constant Lorentz-violating vector that characterizes the Lorentz-violating ground state.

From the KTeV experiment [28] the following bounds on the X and Y components of the a_μ parameter have been obtained

$$\Delta a_X, \Delta a_Y < 9.2 \times 10^{-22} \text{ GeV} ,$$

where X, Y, Z denote sidereal coordinates (see Fig. 1).

Complementary probes of the a_Z component can come from ϕ-factories [23]. In the case of ϕ-factories there is additional dependence of the CPT-violating parameter δ_K on the polar (θ) and azimuthal (ϕ) angles

$$\delta_K^\phi(|\vec{p}|,\theta,t) = \frac{1}{\pi}\int_0^{2\pi} d\phi\delta_K(\vec{p},t) \simeq$$
$$i\sin\widehat{\phi}\exp(i\widehat{\phi})(\gamma/\Delta m)\left(\Delta a_0 + \beta_K\Delta a_Z\cos\chi\cos\theta + \right.$$
$$\left.\beta_K\Delta a_X\sin\chi\cos\theta\cos(\Omega t) + \beta_K\Delta a_Y\sin\chi\cos\theta\sin(\Omega t)\right)$$

where Ω denotes the Earth's sidereal frequency, and χ is the angle between the laboratory Z-axis and the Earth's axis.

The experiment KLOE at DAΦNE is sensitive to a_Z: limits on $\delta(\Delta a_Z)$ can be placed from forward-backward asymmetry measurements $A_L = 2\text{Re}\epsilon_K - 2\text{Re}\delta_K$. For more details on the relevant experimental bounds we refer the reader to the literature [23].

We only mention at this stage that in an upgraded DAΦNE facility, namely experiment KLOE-2 at DAΦNE-2, the expected sensitivity is [23] $\Delta a_\mu = \mathcal{O}(10^{-18})$ GeV which, however, is not competitive with the current KTeV limits on $a_{X,Y}$ given above.

We close this subsection by pointing out that additional precision tests can be performed using other meson factories (using B-mesons, *etc.*...), which would also allow one to test the universality of QG Lorentz-violating effects, if observed.

3 QG Decoherence and CPTV in Neutral Kaons

3.1 Stochastically Fluctuating Geometries, Light Cone Fluctuations and Decoherence: General Ideas

If the ground state of QG consists of "fuzzy" space-time, i.e., stochastically-fluctuating metrics, then a plethora of interesting phenomena may occur, including light-cone fluctuations [29, 21] (c.f. Fig. 2). Such effects will lead to stochastic fluctuations in, say, arrival times of photons with common energy, which can be detected with high precision in astrophysical experiments [30, 29]. In addition, they may give rise to decoherence of matter, in the sense of induced time-dependent damping factors in the evolution equations of the (reduced) density matrix of matter fields [21, 31].

Such "fuzzy" space-times are formally represented by metric deviations which are fluctuating randomly about, say, flat Minkowski space-time: $g_{\mu\nu} =$

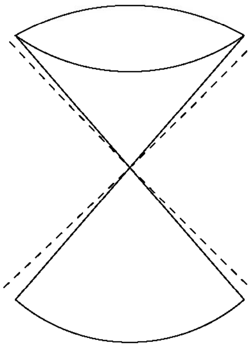


Figure 2: In stochastic space-time models of QG the light cone may fluctuate, leading to decoherence and quantum fluctuations of the speed of light in "vacuo".

$\eta_{\mu\nu} + h_{\mu\nu}$, with $\langle \cdots \rangle$ denoting statistical quantum averaging, and $\langle g_{\mu\nu} \rangle = \eta_{\mu\nu}$ but $\langle h_{\mu\nu}(x) h_{\lambda\sigma}(x') \rangle \neq 0$, i.e., one has only quantum (light cone) fluctuations but not mean-field effects on dispersion relations of matter probes. In such a situation Lorentz symmetry is respected on the average, *but* not in individual measurements.

The path of light follows null geodesics $0 = ds^2 = g_{\mu\nu} dx^\mu dx^\nu$, with non-trivial fluctuations in geodesic deviations, $\frac{D^2 n^\mu}{D\tau^2} = -R^\mu_{\alpha\nu\beta} u^\alpha n^\nu u^\beta$; in a standard general-relativistic notation, $D/D\tau$ denotes the appropriate covariant derivative operation, $R^\mu_{\alpha\nu\beta}$ the (fluctuating) Riemann curvature tensor, and u^μ (n^μ) the tangential (normal) vector along the geodesic.

Such an effect causes primarily fluctuations in the arrival time of photons at the detector ($|\phi\rangle$=state of gravitons, $|0\rangle$= vacuum state)

$$\Delta t^2_{obs} = |\Delta t^2_\phi - \Delta t^2_0| = \frac{|\langle \phi | \sigma_1^2 | \phi \rangle - \langle 0 | \sigma_1^2 | 0 \rangle|}{r^2} \equiv \frac{|\langle \sigma_1^2 \rangle_R|}{r} ,$$

where

$$\langle \sigma_1^2 \rangle_R = \frac{1}{8} (\Delta r)^2 \int_{r_0}^{r_1} dr \int_{r_0}^{r_1} dr' \; n^\mu n^\nu n^\rho n^\sigma$$
$$\langle \phi | h_{\mu\nu}(x) h_{\rho\sigma}(x') + h_{\mu\nu}(x') h_{\rho\sigma}(x) | \phi \rangle$$

and the two-point function of graviton fluctuations can be evaluated using standard field theory techniques [29].

Apart from the stochastic metric fluctuations, however, the aforementioned effects could also induce decoherence of matter propagating in these types of backgrounds [31], a possibility of particular interest for the purposes of the present article. Through the theorem of Wald [7], this implies that the CPT operator is not well-defined, and hence one also has a breaking of CPT symmetry.

We now proceed to describe briefly the general formalism used for parametri ing such QG-induced decoherence, as far as the CPT-violating effects on matter are concerned.

3.2 Formalism for the Phenomenology of QG-induced Decoherence

In this subsection we shall be very brief, giving the reader a flavor of the formalism underlying such decoherent systems. We shall discuss first a model-independent parametrization of decoherence, applicable not only to QG media, but covering a more general situation.

If the effects of the environment are such that the modified evolution equation of the (reduced) density matrix of matter ρ [32] is linear, one can write down a Lindblad evolution equation [6], provided that (i) there is (complete) positivity of ρ, so that negative probabilities do not arise at any stage of the evolution, (ii) the energy of the matter system is conserved on the average, and (iii) the entropy is increasing monotonically.

For N-level systems, the generic decohering Lindblad evolution for ρ reads

$$\frac{\partial \rho_\mu}{\partial t} = \sum_{ij} h_i \rho_j f_{ij\mu} + \sum_\nu L_{\mu\nu}\rho_\nu \ ,$$
$$\mu, \nu = 0, \dots N^2 - 1, \quad i, j = 1, \dots N^2 - 1 \ ,$$

where the h_i are Hamiltonian terms, expanded in an appropriate basis, and the decoherence matrix L has the form:

$$L_{0\mu} = L_{\mu 0} = 0 \ ,$$
$$L_{ij} = \frac{1}{4} \sum_{k,\ell,m} c_{l\ell} \left(-f_{i\ell m} f_{kmj} + f_{kim} f_{\ell mj}\right) \ ,$$

with c_{ij} a positive-definite matrix and f_{ijk} the structure constants of the appropriate $SU(N)$ group. In this generic phenomenological description of decoherence, the elements $L_{\mu\nu}$ are free parameters, to be determined by experiment.

We shall come back to this point in the next subsection, where we discuss neutral kaon decays.

A rather characteristic feature of this equation is the appearance of exponential damping, $e^{-(\ldots)t}$, in interference terms of the pertinent quantities (for instance, matrix elements ρ, or asymmetries in the case of the kaon system, see below). The exponents are proportional to (linear combinations) of the elements of the decoherence matrix [6, 4, 32]. Note, however, that Lindblad type evolution is *not* the most generic evolution for QG models. In cases of space-time foam corresponding to *stochastically (random) fluctuating space-times*, such as the situations causing light-cone fluctuations examined previously, there is a different kind of decoherent evolution, with damping that is quadratic in time, i.e., one has a $e^{-(\ldots)t^2}$ suppression of interference terms in the relevant observables.

A specific model of stochastic space-time foam is based on a particular kind of gravitational foam [21, 33, 31], consisting of "real" (as opposed to "virtual") space-time defects in higher-dimensional space times, in accordance with the modern viewpoint of our world as a brane hyper-surface embedded in the bulk space-time [34]. This model is quite generic in some respects, and we will use it later to estimate the order of magnitude of novel CPT violating effects in entangled states of kaons.

A model of space-time foam [33] can be based on a number (determined by target-space supersymmetry) of parallel brane worlds with three large spatial dimensions. These brane worlds move in a bulk space-time, containing a "gas" of point-like bulk branes, termed "D-particles", which are stringy space-time solitonic defects. One of these branes is the observable Universe. For an observer on the brane the crossing D-particles will appear as twinkling space-time defects, i.e. microscopic space-time fluctuations. This will give the four-dimensional brane world a "D-foamy" structure. Following work on gravitational decoherence [21, 31], the target-space metric state, which is close to being flat, can be represented schematically as a density matrix

$$\rho_{\rm grav} = \int d^5r \; f\left(r_\mu\right) \left|g\left(r_\mu\right)\right\rangle \left\langle g\left(r_\mu\right)\right| . \tag{7}$$

The parameters r_μ $(\mu = 0, 1 \ldots)$ pertain to appropriate space-time metric deformations and are stochastic, with a Gaussian distribution $f\left(r_\mu\right)$ characterized by the averages

$$\langle r_\mu \rangle = 0, \ \langle r_\mu r_\nu \rangle = \Delta_\mu \delta_{\mu\nu} \,.$$

This model will be studied in more detail in section 4.

We will assume that the fluctuations of the metric felt by two entangled neutral mesons are independent, and $\Delta_\mu \sim O\left(\frac{E^2}{M_P^2}\right)$, i.e., very small. As matter moves through the space-time foam in a typical ergodic picture, the effect of time averaging is assumed to be equivalent to an ensemble average. For our present discussion we consider a semi-classical picture for the metric, and therefore $|g(r_\mu)\rangle$ in (7) is a coherent state.

In the specific model of foam discussed in [31], there is a recoil effect of the D-particle, as a result of its scattering with stringy excitations that live on the brane world and represent low-energy ordinary matter. As the space-time defects, propagating in the bulk space-time, cross the brane hyper-surface from the bulk in random directions, they scatter with matter. The associated distortion of space-time caused by this scattering can be considered dominant only along the direction of motion of the matter probe. Random fluctuations are then considered about an average flat Minkowski space-time. The result is an effectively two-dimensional approximate fluctuating metric describing the main effects [31]

$$g^{\mu\nu} = \begin{pmatrix} -(a_1+1)^2 + a_2^2 & -a_3(a_1+1) + a_2(a_4+1) \\ -a_3(a_1+1) + a_2(a_4+1) & -a_3^2 + (a_4+1)^2 \end{pmatrix} . \tag{8}$$

The a_i represent the fluctuations and are assumed to be random variables, satisfying $\langle a_i \rangle = 0$ and $\langle a_i a_j \rangle = \delta_{ij}\sigma_i$.

Such a (microscopic) model of space-time foam is not of Lindblad type, as can be seen [31] by considering the oscillation probability for, say, two-level scalar systems describing oscillating neutral kaons, $K^0 \leftrightarrow \overline{K}^0$. In the approximation of small fluctuations one finds the following form for the oscillation probability of the two-level scalar system:

$$\langle e^{i(\omega_1 - \omega_2)t} \rangle = \frac{4\tilde{d}^2}{(P_1 P_2)^{1/2}} \exp\left(\frac{\chi_1}{\chi_2}\right) \exp(i\tilde{b}t) \,,$$

where $\omega_i,\ i=1,2$ are the appropriate energy levels [31] of the two-level kaon system in the background of the fluctuating space-time (8), and

$$
\begin{aligned}
\chi_1 &= -4(\tilde{d}^2\sigma_1 + \sigma_4 k^4)\tilde{b}^2 t^2 + 2i\tilde{d}^2\tilde{b}^2\tilde{c}k^2\sigma_1\sigma_4 t^3, \\
\chi_2 &= 4\tilde{d}^2 - 2i\tilde{d}^2(k^2\tilde{c}\sigma_4 + 2\tilde{b}\sigma_1)t + \\
&\quad \tilde{b}k^2\left(\tilde{b}k^2 - 2\tilde{d}^2\tilde{c}\right)\sigma_1\sigma_4, \\
P_1 &= 4\tilde{d}^2 + 2i\tilde{d}\tilde{b}\left(k^2 - \tilde{d}\right)\sigma_2 t + \tilde{b}^2 k^4\sigma_2\sigma_3 t^2, \\
P_2 &= 4\tilde{d}^2 - 2i\tilde{d}^2\left(k^2\tilde{c}\sigma_4 + 2\tilde{b}\sigma_1\right)t + \mathcal{O}\left(\sigma^2\right)\ ,
\end{aligned}
$$

with

$$
\begin{aligned}
\tilde{b} &= \sqrt{k^2+m_1^2} - \sqrt{k^2+m_2^2}, \\
\tilde{c} &= m_1^2(k^2+m_1^2)^{-3/2} - m_2^2(k^2+m_2^2)^{-3/2}, \\
\tilde{d} &= \sqrt{k^2+m_1^2}\sqrt{k^2+m_2^2}.
\end{aligned}
$$

From this expression one can see [31] that the stochastic model of space-time foam leads to a modification of oscillation behavior quite distinct from that of the Lindblad formulation. In particular, the transition probability displays a Gaussian time-dependence, decaying as $e^{-(\ldots)t^2}$, a modification of the oscillation period, as well as additional power-law fall-off.

From this characteristic time-dependence, one can obtain bounds for the fluctuation strength of space-time foam in kaon systems. In the context of this presentation, we restrict ourselves to Lindblad decoherence tests using only neutral kaons. However, when discussing the CPTV effects of foam on entangled states we make use of this specific model of stochastically fluctuating D-particle foam [33, 31], in order to demonstrate the effects explicitly and obtain definite order-of-magnitude estimates [35].

3.3 Experiments involving Single-Kaon States

As mentioned in the previous subsection, QG may induce decoherence and oscillations $K^0 \leftrightarrow \overline{K}^0$ [4, 8], thereby implying a two-level quantum mechanical system interacting with a QG "environment". Adopting the general assumptions of average energy conservation and monotonic entropy increase, the simplest model for parametrizing decoherence (in a rather model-independent way) is the (linear) Lindblad approach mentioned earlier. Not all entries of a general decoherence matrix are physical, and in order to isolate the physically relevant entries one must invoke specific assumptions, related to the symmetries of the particle system in question. For the neutral kaon system, such an extra assumptions is that the QG medium respects the $\Delta S = \Delta Q$ rule. In such a case, the modified Lindblad evolution equation (4) for the respective density matrices of neutral kaon matter can be parametrized as follows [4]:

$$\partial_t \rho = i[\rho, H] + \delta\not{H}\rho \ ,$$

where

$$H_{\alpha\beta} = \begin{pmatrix} -\Gamma & -\frac{1}{2}\delta\Gamma & -\mathrm{Im}\Gamma_{12} & -\mathrm{Re}\Gamma_{12} \\ -\frac{1}{2}\delta\Gamma & -\Gamma & -2\mathrm{Re}M_{12} & -2\mathrm{Im}M_{12} \\ -\mathrm{Im}\Gamma_{12} & 2\mathrm{Re}M_{12} & -\Gamma & -\delta M \\ -\mathrm{Re}\Gamma_{12} & -2\mathrm{Im}M_{12} & \delta M & -\Gamma \end{pmatrix}$$

and

$$\delta\not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix} .$$

Positivity of ρ requires: $\alpha, \gamma > 0, \quad \alpha\gamma > \beta^2$. Notice that α, β, γ violate *both* CPT, due to their decohering nature [7], and CP symmetry, as they do not commute with the CP operator $\widehat{CP}$ [8]: $\widehat{CP} = \sigma_3 \cos\theta + \sigma_2 \sin\theta, \quad [\delta\not{H}_{\alpha\beta}, \widehat{CP}] \neq 0$.

An important remark is now in order. As pointed out in [10], although the above parametrization is sufficient for a single-kaon state to have a positive definite density matrix (and hence probabilities) this is *not* true when one considers the evolution of entangled kaon states (ϕ-factories). In this latter case, complete positivity is guaranteed only if the further conditions

$$\alpha = \gamma \text{ and } \beta = 0 \tag{9}$$

are imposed. When incorporating entangled states, one should either consider possible new effects (such as the ω-effect considered below) or apply the constraints (9) also to single kaon states [10]. This is not necessarily the case when other non-entangled particle states, such as neutrinos, are considered, in which case the α, β, γ parametrization of decoherence may be applied. Experimentally the complete positivity hypothesis can be tested explicitly. In what follows, as far as single-kaon states are concerned, we keep the α, β, γ parametrization, and give the available experimental bounds for them, but we always have in mind the constraint (9) when referring to entangled kaon states in a ϕ-factory.

As already mentioned, when testing CPT symmetry with neutral kaons one should be careful to distinguish two types of CPTV: **(i)** CPTV within Quantum Mechanics [11], leading to possible differences between particle-antiparticle masses and widths: $\delta m = m_{K^0} - m_{\overline{K}^0}$, $\delta\Gamma = \Gamma_{K^0} - \Gamma_{\overline{K}^0}$. This type of CPTV could be, for instance, due to (spontaneous) Lorentz violation [13]. In that case the CPT operator is well-defined as a quantum mechanical operator, but does not commute with the Hamiltonian of the system. This, in turn, may lead to mass and width differences between particles and antiparticles, among other effects. **(ii)** CPTV through decoherence [4, 5] via the parameters α, β, γ (entanglement with the QG "environment", leading to modified evolution for ρ and $\$ \neq S\, S^\dagger$). In the latter case the CPT operator may not be well-defined, which implies novel effects when one uses entangled states of kaons, as we shall discuss in the next subsection.

Process	QMV	QM
$A_{2\pi}$	$\neq$	$\neq$
$A_{3\pi}$	$\neq$	$\neq$
$A_{\rm T}$	$\neq$	$=$
$A_{\rm CPT}$	$=$	$\neq$
$A_{\Delta m}$	$\neq$	$=$
ζ	$\neq$	$=$

Table 1: Qualitative comparison of predictions for various observables in CPT-violating theories beyond (QMV) and within (QM) quantum mechanics. Predictions either differ ($\neq$) or agree ($=$) with the results obtained in conventional quantum-mechanical CP violation. Note that these frameworks can be qualitatively distinguished via their predictions for $A_{\rm T}$, $A_{\rm CPT}$, $A_{\Delta m}$, and ζ.

The important point to notice is that the two types of CPTV can be *disentangled experimentally* [8]. The relevant observables are defined as $\langle O_i \rangle = {\rm Tr}\,[O_i \rho]$. For neutral kaons, one looks at decay asymmetries for $K^0, \overline{K}^0$, defined

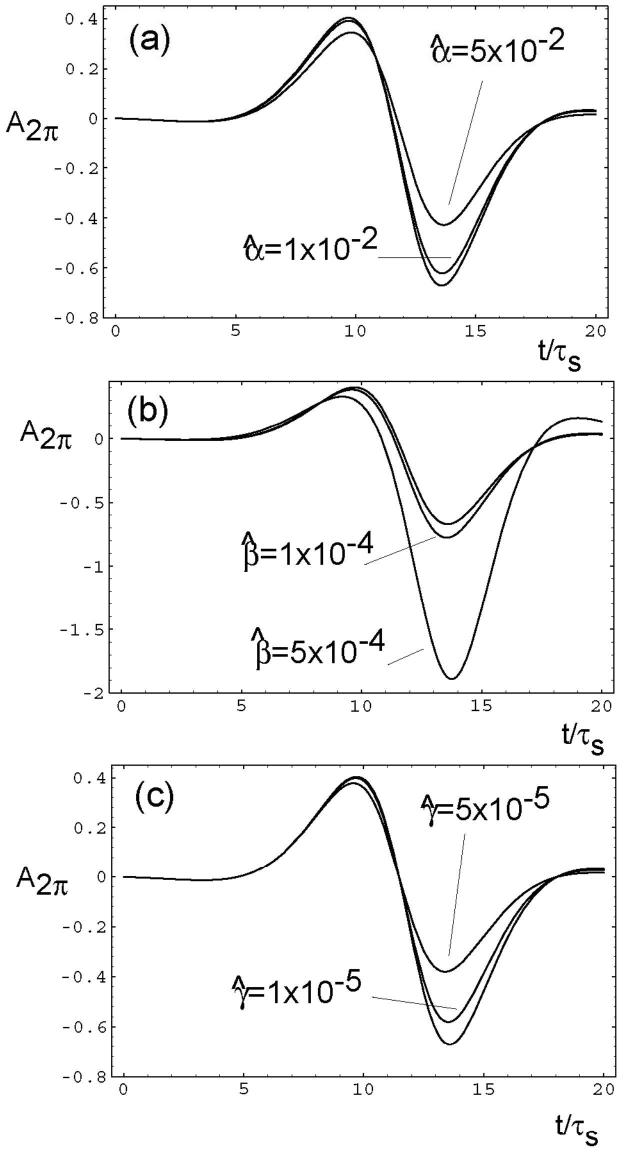


Figure 3: Neutral kaon decay asymmetries $A_{2\pi}$ [8] indicating the effects of QG-induced decoherence.

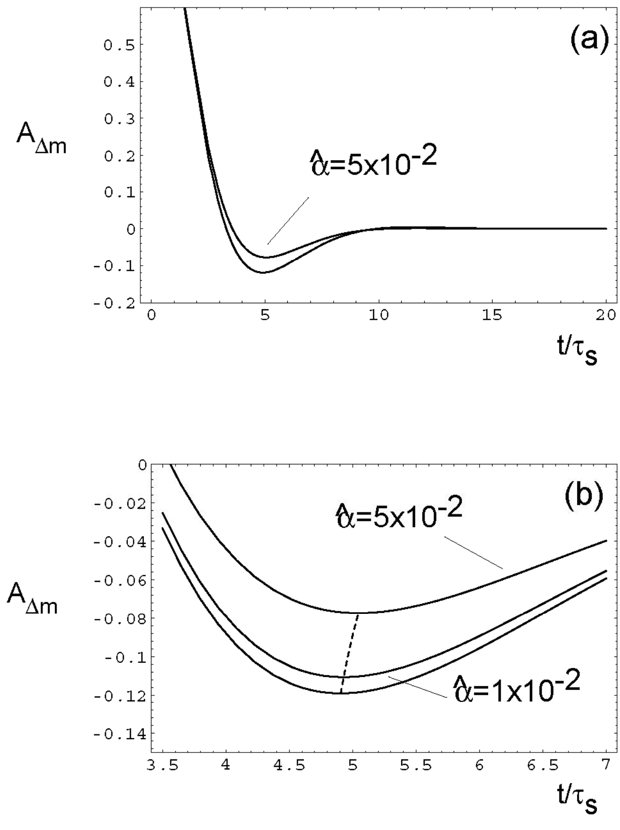
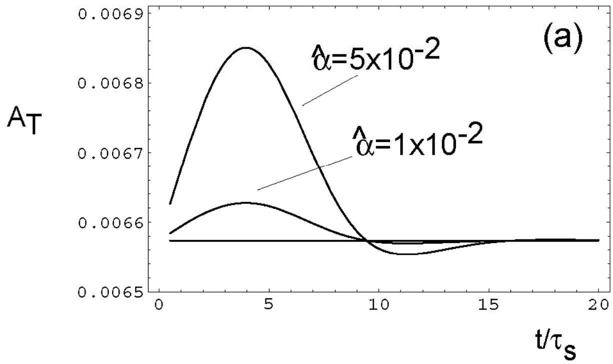


Figure 4: Typical neutral kaon decay asymmetries $A_{\Delta m}$ [8] indicating the effects of QG-induced decoherence.



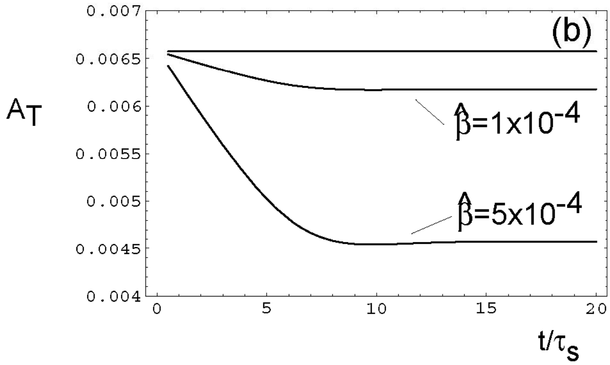


Figure 5: Typical neutral kaon decay asymmetries A_T [8] indicating the effects of QG-induced decoherence.

as:

$$A(t) = \frac{R(\bar{K}^0_{t=0} \to \bar{f}) - R(K^0_{t=0} \to f)}{R(\bar{K}^0_{t=0} \to \bar{f}) + R(K^0_{t=0} \to f)} ,$$

where $R(K^0 \to f) \equiv \mathrm{Tr}\,[O_f \rho(t)] =$ denotes the decay rate into the final state f (starting from a pure K^0 state at $t = 0$).

In the case of neutral kaons, one may consider the following set of asymmetries: (i) *identical final states*: $f = \bar{f} = 2\pi$: $A_{2\pi}$, $A_{3\pi}$, (ii) *semileptonic* : A_T (final states $f = \pi^+ l^- \bar{\nu} \neq \bar{f} = \pi^- l^+ \nu$), A_{CPT} ($\overline{f} = \pi^+ l^- \bar{\nu}$, $f = \pi^- l^+ \nu$), $A_{\Delta m}$. Typically, for instance when final states are 2π, one has a time evolution of the decay rate $R_{2\pi}$: $R_{2\pi}(t) = c_S\, e^{-\Gamma_S t} + c_L\, e^{-\Gamma_L t} + 2 c_I\, e^{-\Gamma t} \cos(\Delta m t - \phi)$, where S=short-lived, L=long-lived, I=interference term, $\Delta m = m_L - m_S$, $\Delta\Gamma = \Gamma_S - \Gamma_L$, $\Gamma = \frac{1}{2}(\Gamma_S + \Gamma_L)$. One may define the *decoherence parameter* $\zeta = 1 - \frac{c_I}{\sqrt{c_S c_L}}$, as a (phenomenological) measure of quantum decoherence induced in the system [11]. For larger sensitivities one can look at this parameter in the presence of a regenerator [8]. In our decoherence scenario, ζ corresponds to a particular combination of the decoherence parameters [8]:

$$\zeta \to \frac{\widehat{\gamma}}{2|\epsilon^2|} - 2\frac{\widehat{\beta}}{|\epsilon|}\sin\phi ,$$

with the notation $\widehat{\gamma} = \gamma/\Delta\Gamma$, *etc.* Hence, ignoring the constraint (9), the best bounds on β, or -turning the logic around- the most sensitive tests of complete positivity in kaons, can be placed by implementing a regenerator [8].

The experimental tests (decay asymmetries) that can be performed in order to disentangle decoherence from quantum-mechanical CPT violating effects are summarized in Table 1. In Figures 3, 4, 5 we give typical profiles of several decay asymmetries [8], from where bounds on QG decohering parameters can be extracted. At present there are experimenatl bounds available from CPLEAR measurements [36] $\alpha < 4.0 \times 10^{-17}$ GeV , $|\beta| < 2.3. \times 10^{-19}$ GeV , $\gamma < 3.7 \times 10^{-21}$ GeV, which are not much different from theoretically expected values in some optimistic scenarios [8] $\alpha\, , \beta\, , \gamma = O(\xi \frac{E^2}{M_P})$.

Recently, the experiment KLOE at DAΦNE updated these limits by measuring for the first time the γ decoherence parameter for entangled kaon states [23], as well as the (naive) decoherence parameter ζ (to be specific, the KLOE Collaboration has presented measurements for two ζ parameters, one, ζ_{LS}, pertaining to an expansion in terms of K_L, K_S states, and the other, $\zeta_{0\bar{0}}$, for an expansion in terms of $K^0, \overline{K}^0$ states). We remind the reader once more that, under the assumption of complete positivity for entangled meson states [10)], theoretically there is only one parameter to parametrize Lindblad decoherence, since $\alpha = \gamma$, $\beta = 0$. In fact, the KLOE experiment has the greatest sensitivity to this parameter γ. The latest KLOE measurement [23)] for γ yields $\gamma_{\rm KLOE} = (1.3^{+2.8}_{-2.4} \pm 0.4) \times 10^{-21}$ GeV, i.e. $\gamma < 6.4 \times 10^{-21}$ GeV, competitive with the corresponding CPLEAR bound [36)] discussed above. It is expected that this bound could be improved by an order of magnitude in upgraded facilities, such as KLOE-2 at DAΦNE-2 [23)], where one expects $\gamma_{\rm upgrade} \to \pm 0.2 \times 10^{-21}$ GeV.

The reader should also bear in mind that the Lindblad linear decoherence is not the only possibility for a parametrization of QG effects, see for instance the stochastically fluctuating space-time metric approach discussed in Section 3.1 above. Thus, direct tests of the complete positivity hypothesis in entangled states, and hence the theoretical framework *per se*, should be performed by independent measurements of all the three decoherence parameters α, β, γ; as far as we understand [1], such data are currently available in kaon factories, but not yet analyzed in detail [23)].

4 CPTV and Modified EPR Correlations of Entangled Neutral Kaon States

4.1 EPR Correlations in Particle Physics

We now come to a description of an entirely novel effect [12)] of CPTV due to the ill-defined nature of the CPT operator, which is *exclusive* to neutral-meson factories, for reasons explained below. The effects are qualitatively similar for kaon and B-meson factories [37)], with the important observation that in kaon factories there is a particularly good channel, that of both correlated kaons decaying to $\pi^+\pi^-$. In that channel the sensitivity of the effect increases because the complex parameter ω, parametrizing the relevant EPR modifications [12)], appears in the particular combination $|\omega|/|\eta_{+-}|$, with $|\eta_{+-}| \sim 10^{-3}$. In the case of B-meson factories one should focus instead on the "same-sign" di-lepton channel [37)], where high statistics is expected.

In this article we restrict ourselves to the case of ϕ-factories, referring the interested reader to the literature [37)] for the B-meson applications. We

[1]We thank A. Di Domenico for informative discussions on this point.

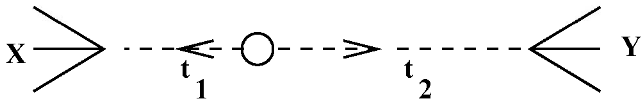


Figure 6: Schematic representation of the decay of a ϕ-meson at rest (for definiteness) into pairs of entangled neutral kaons, which eventually decay on the two sides of the detector.

commence our discussion by briefly reminding the reader of EPR particle correlations.

The EPR effect was originally proposed as a *paradox*, testing the foundations of Quantum Theory. There was the question whether quantum correlations between spatially separated events implied instant transport of information that would contradict special relativity. It was eventually realized that no super-luminal propagation was actually involved in the EPR phenomenon, and thus there was no conflict with relativity.

The EPR effect has been confirmed experimentally, e.g., in meson factories: (i) a pair of particles can be created in a definite quantum state, (ii) move apart and, (iii) eventually decay when they are widely (spatially) separated (see Fig. 6 for a schematic representation of an EPR effect in a meson factory). Upon making a measurement on one side of the detector and identifying the decay products, we *infer* the type of products appearing on the other side; this is essentially the EPR correlation phenomenon. It does *not* involve any *simultaneous measurement* on both sides, and hence there is no contradiction with special relativity. As emphasized by Lipkin [38)], the EPR correlations between different decay modes should be taken into account when interpreting any experiment.

4.2 CPTV and Modified EPR-Correlations in ϕ Factories: the ω-Effect

In the case of ϕ factories it was *claimed* [39)] that due to EPR correlations, *irrespective* of CP, and CPT violation, the *final state* in ϕ decays: $e^+e^- \Rightarrow \phi \Rightarrow K_S K_L$ always contains $K_L K_S$ products. This is a direct consequence of imposing the requirement of *Bose statistics* on the state $K^0\overline{K}^0$ (to which the ϕ decays); this, in turn, implies that the physical neutral meson-antimeson state must be *symmetric* under $C\mathcal{P}$, with C the charge conjugation and $\mathcal{P}$ the operator that permutes the spatial coordinates. Assuming *conservation* of angular momentum, and a proper existence of the *antiparticle state* (denoted by a bar), one observes that: for $K^0\overline{K}^0$ states which are C-conjugates with C$= (-1)^\ell$ (with ℓ the angular momentum quantum number), the system has to be an eigenstate of the permutation operator $\mathcal{P}$ with eigenvalue $(-1)^\ell$. Thus, for $\ell = 1$: C$= - \to \mathcal{P} = -$. Bose statistics ensures that for $\ell = 1$ the state of two *identical* bosons is *forbidden*. Hence, the initial entangled state:

$$|i> = \frac{1}{\sqrt{2}}\left(|K^0(\vec{k}), \overline{K}^0(-\vec{k}) > -|\overline{K}^0(\vec{k}), K^0(-\vec{k}) >\right)$$
$$= \mathcal{N}\left(|K_S(\vec{k}), K_L(-\vec{k}) > -|K_L(\vec{k}), K_S(-\vec{k}) >\right)$$

with the normalization factor $\mathcal{N} = \frac{\sqrt{(1+|\epsilon_1|^2)(1+|\epsilon_2|^2)}}{\sqrt{2}(1-\epsilon_1\epsilon_2)} \simeq \frac{1+|\epsilon^2|}{\sqrt{2}(1-\epsilon^2)}$, and $K_S = \frac{1}{\sqrt{1+|\epsilon_1^2|}}(|K_+ > +\epsilon_1|K_- >)$, $K_L = \frac{1}{\sqrt{1+|\epsilon_2^2|}}(|K_- > +\epsilon_2|K_+ >)$, where ϵ_1, ϵ_2 are complex parameters, such that $\epsilon \equiv \epsilon_1 + \epsilon_2$ denotes the CP- & T-violating parameter, whilst $\delta \equiv \epsilon_1 - \epsilon_2$ parametrizes the CPT & CP violation within quantum mechanics [11], as discussed previously. The $K^0 \leftrightarrow \overline{K}^0$ or $K_S \leftrightarrow K_L$ correlations are apparent after evolution, at any time $t > 0$ (with $t = 0$ taken as the moment of the ϕ decay).

In the above considerations there is an implicit assumption, which was noted in [12]. The above arguments are valid independently of CPTV, provided such violation occurs within quantum mechanics, e.g., due to spontaneous Lorentz violation, where the CPT operator is well defined.

If, however, CPT is *intrinsically* violated, due, for instance, to decoherence scenarios in space-time foam, then the factorizability property of the super-scattering matrix \$ breaks down, $\$ \neq SS^\dagger$, and the generator of CPT is not well defined [7]. Thus, the concept of an "antiparticle" may be *modified* perturbatively! The perturbative modification of the properties of the antiparticle is important, since the antiparticle state is a physical state which exists, despite the ill-definition of the CPT operator. However, the antiparticle Hilbert space will have components that are *independent* of the particle Hilbert space.

In such a case, the neutral mesons K^0 and $\overline{K}^0$ should *no longer* be treated as *indistinguishable particles*. As a consequence [12], the initial entangled state in ϕ factories $|i>$, after the ϕ-meson decay, will acquire a component with opposite permutation ($\mathcal{P}$) symmetry:

$$
\begin{aligned}
|i> &= \frac{1}{\sqrt{2}}\left(|K_0(\vec{k}),\overline{K}_0(-\vec{k})> - |\overline{K}_0(\vec{k}),K_0(-\vec{k})>\right) \\
&+ \frac{\omega}{2}\left(|K_0(\vec{k}),\overline{K}_0(-\vec{k})> + |\overline{K}_0(\vec{k}),K_0(-\vec{k})>\right)\Big] \\
&= \Big[\mathcal{N}\left(|K_S(\vec{k}),K_L(-\vec{k})> - |K_L(\vec{k}),K_S(-\vec{k})>\right) \\
&+ \omega\left(|K_S(\vec{k}),K_S(-\vec{k})> - |K_L(\vec{k}),K_L(-\vec{k})>\right)\Big] ,
\end{aligned}
$$

where $\mathcal{N}$ is an appropriate normalization factor, and $\omega = |\omega| e^{i\Omega}$ is a complex parameter, parametrizing the intrinsic CPTV modifications of the EPR correlations. Notice that, as a result of the ω-terms, there exist, in the two-kaon state, $K_S K_S$ or $K_L K_L$ combinations, which entail important effects to the various decay channels. Due to this effect, termed the ω-effect by the authors of [12], there is *contamination* of $\mathcal{P}$(odd) state with $\mathcal{P}$(even) terms. The ω-parameter controls the amount of contamination of the final $\mathcal{P}$(odd) state by the "wrong" ($\mathcal{P}$(even)) symmetry state.

Later in this section we will present a microscopic model where such a situation is realized explicitly. Specifically, an ω-like effect appears due to the evolution in the space-time foam, and the corresponding parameter turns out to be purely imaginary and time-dependent [35].

4.3 ω-Effect Observables

To construct the appropriate observable for the possible detection of the ω-effect, we consider the ϕ-decay amplitude depicted in Fig. 6, where one of the kaon products decays to the final state X at t_1 and the other to the final state Y at time t_2. We take $t = 0$ as the moment of the ϕ-meson decay.

The relevant amplitudes read:

$$
A(X,Y) = \langle X|K_S\rangle\langle Y|K_S\rangle\,\mathcal{N}\,(A_1 + A_2) \ ,
$$

with

$$
\begin{aligned}
A_1 &= e^{-i(\lambda_L+\lambda_S)t/2}[\eta_X e^{-i\Delta\lambda\Delta t/2} - \eta_Y e^{i\Delta\lambda\Delta t/2}] \\
A_2 &= \omega[e^{-i\lambda_S t} - \eta_X \eta_Y e^{-i\lambda_L t}]
\end{aligned}
$$

denoting the CPT-allowed and CPT-violating parameters respectively, and $\eta_X = \langle X|K_L\rangle/\langle X|K_S\rangle$ and $\eta_Y = \langle Y|K_L\rangle/\langle Y|K_S\rangle$. In the above formulae, t is the sum of the decay times t_1, t_2 and Δt is their difference (assumed positive).

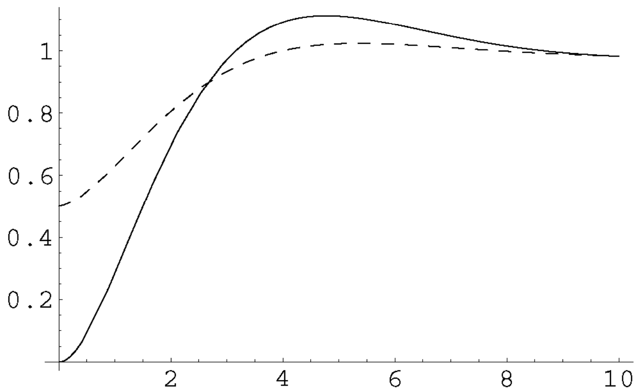


Figure 7: A characteristic case of the intensity $I(\Delta t)$, with $|\omega| = 0$ (solid line) vs $I(\Delta t)$ (dashed line) with $|\omega| = |\eta_{+-}|$, $\Omega = \phi_{+-} - 0.16\pi$, for definiteness [12].

The "intensity" $I(\Delta t)$ is the desired *observable* for a detection of the ω-effect,

$$I(\Delta t) \equiv \frac{1}{2}\int_{\Delta t}^{\infty} dt\, |A(X,Y)|^2 \ .$$

depending only on Δt.

Its time profile reads [12]:

$$I(\Delta t) \equiv \frac{1}{2}\int_{|\Delta t|}^{\infty} dt\, |A(\pi^+\pi^-, \pi^+\pi^-)|^2 =$$
$$|\langle \pi^+\pi^-|K_S\rangle|^4 |\mathcal{N}|^2 |\eta_{+-}|^2 \Big[I_1 + I_2 + I_{12}\Big] \ ,$$

where

$$I_1(\Delta t) = \frac{e^{-\Gamma_S \Delta t} + e^{-\Gamma_L \Delta t} - 2e^{-(\Gamma_S+\Gamma_L)\Delta t/2}\cos(\Delta m \Delta t)}{\Gamma_L + \Gamma_S}$$

$$
\begin{aligned}
I_2(\Delta t) &= \frac{|\omega|^2}{|\eta_{+-}|^2}\frac{e^{-\Gamma_S\Delta t}}{2\Gamma_S} \\
I_{12}(\Delta t) &= -\frac{4}{4(\Delta m)^2 + (3\Gamma_S+\Gamma_L)^2}\frac{|\omega|}{|\eta_{+-}|} \times \\
&\Big[2\Delta m\Big(e^{-\Gamma_S\Delta t}\sin(\phi_{+-}-\Omega) - \\
&e^{-(\Gamma_S+\Gamma_L)\Delta t/2}\sin(\phi_{+-}-\Omega+\Delta m\Delta t)\Big) \\
&-(3\Gamma_S+\Gamma_L)\Big(e^{-\Gamma_S\Delta t}\cos(\phi_{+-}-\Omega) - \\
&e^{-(\Gamma_S+\Gamma_L)\Delta t/2}\cos(\phi_{+-}-\Omega+\Delta m\Delta t)\Big)\Big] ,
\end{aligned}
$$

with $\Delta m = m_S - m_L$ and $\eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}}$ in the usual notation [11].

A typical case for the relevant intensities, indicating clearly the novel CPTV ω-effects, is depicted in Fig. 7.

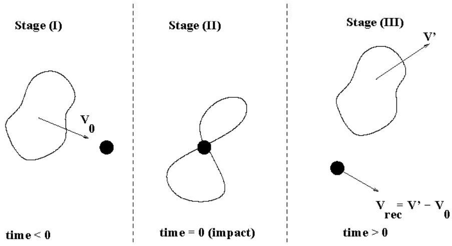
As announced, the novel ω-effect appears in the combination $\frac{|\omega|}{|\eta_{+-}|}$, thereby implying that the decay channel to $\pi^+\pi^-$ is particularly sensitive to the ω effect [12], due to the enhancement by $1/|\eta_{+-}| \sim 10^3$, implying sensitivities up to $|\omega| \sim 10^{-6}$ in ϕ factories. The physical reason for this enhancement is that ω enters through $K_S K_S$ as opposed to $K_L K_S$ terms, and the $K_L \to \pi^+\pi^-$ decay is CP-violating.

4.4 Microscopic Models for the ω-Effect and Order-of-Magnitude Estimates

For future experimental searches for the ω-effect it is important to estimate its expected order of magnitude, at least in some models of foam.

A specific model is that of the D-particle foam [33, 31, 35], discussed already in connection with the stochastic metric-fluctuation approach to decoherence. An important feature for the appearance of an ω-like effect is that, during each scattering with a D-particle defect, there is (momentary) capture of the string state (representing matter) by the defect, and a possible change in phase and/or flavour for the particle state emerging from such a capture (see Fig. 8).

The induced metric distortions, including such flavour changes for the emergent post-recoil matter state, are:



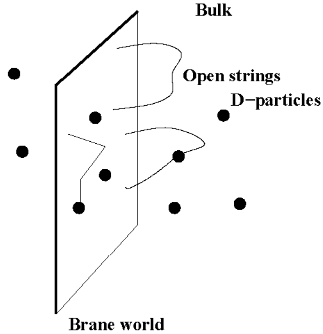


Figure 8: Upper: Recoil of closed string states with D-particles (space-time defects). Lower: A supersymmetric brane world model of D-particle foam. In both cases the recoil of (massive) D-particle defect causes distortion of space-time, stochastic metric fluctuations are possible and the emergent post-recoil string state may differ by flavour and CP phases.

$$g^{00} = (-1 + r_4)\,1 \ ,$$
$$g^{01} = g^{10} = r_0 1 + r_1\sigma_1 + r_2\sigma_2 + r_3\sigma_3,$$
$$g^{11} = (1 + r_5)\,1$$

where the σ_i are Pauli matrices.

The target-space metric state is the density matrix $\rho_{\rm grav}$ defined at (7) [35], with the same assumptions for the parameters r_μ stated there. The order of magnitude of the metric elements $g_{0i} \simeq \overline{v}_{i,rec} \propto g_s \frac{\Delta p_i}{M_s}$, where $\Delta p_i \sim \tilde{\xi} p_i$ is the momentum transfer during the scattering of the particle probe (kaon) with the D-particle defect, $g_s < 1$ is the string coupling, assumed weak, and M_s is the string scale, which in the modern approach to string/brane theory is not necessarily identified with the four-dimensional Planck scale, and is left as a phenomenological parameter to be constrained by experiment.

To estimate the order of magnitude of the ω-effect we construct the gravitationally-dressed initial entangled state using stationary perturbation theory for degenerate states [12], the degeneracy being provided by the CP-violating effects. As Hamiltonian function we use

$$\widehat{H} = g^{01}\left(g^{00}\right)^{-1}\widehat{k} - \left(g^{00}\right)^{-1}\sqrt{\left(g^{01}\right)^{2}k^{2} - g^{00}\left(g^{11}k^{2} + m^{2}\right)}$$

describing propagation in the above-described stochastically-fluctuating space-time. To leading order in the variables r the interaction Hamiltonian reads:

$$\widehat{H_I} = -\left(r_1\sigma_1 + r_2\sigma_2\right)\widehat{k} \tag{10}$$

with the notation $|K_L\rangle = |\uparrow\rangle \;,\quad |K_S\rangle = |\downarrow\rangle$. The gravitationally-dressed initial states then can be constructed using stationary perturbation theory:

$$\left|k^{(i)},\downarrow\right\rangle_{QG}^{(i)} = \left|k^{(i)},\downarrow\right\rangle^{(i)} + \left|k^{(i)},\uparrow\right\rangle^{(i)}\alpha^{(i)} \;,$$

where $\alpha^{(i)} = \frac{{}^{(i)}\langle\uparrow,k^{(i)}|\widehat{H_I}|k^{(i)},\downarrow\rangle^{(i)}}{E_2-E_1}$. For $\left|k^{(i)},\uparrow\right\rangle^{(i)}$ the dressed state is obtained by $|\downarrow\rangle \leftrightarrow |\uparrow\rangle$ and $\alpha \to \beta$ where $\beta^{(i)} = \frac{{}^{(i)}\langle\downarrow,k^{(i)}|\widehat{H_I}|k^{(i)},\uparrow\rangle^{(i)}}{E_1-E_2}$.

The totally antisymmetric "gravitationally-dressed" state of two mesons (kaons) is then:

$$\begin{aligned}
&|k,\uparrow\rangle_{QG}^{(1)}|-k,\downarrow\rangle_{QG}^{(2)} - |k,\downarrow\rangle_{QG}^{(1)}|-k,\uparrow\rangle_{QG}^{(2)} = \\
&|k,\uparrow\rangle^{(1)}|-k,\downarrow\rangle^{(2)} - |k,\downarrow\rangle^{(1)}|-k,\uparrow\rangle^{(2)} \\
&+|k,\downarrow\rangle^{(1)}|-k,\downarrow\rangle^{(2)}\left(\beta^{(1)} - \beta^{(2)}\right) + \\
&|k,\uparrow\rangle^{(1)}|-k,\uparrow\rangle^{(2)}\left(\alpha^{(2)} - \alpha^{(1)}\right) \\
&+\beta^{(1)}\alpha^{(2)}|k,\downarrow\rangle^{(1)}|-k,\uparrow\rangle^{(2)} - \alpha^{(1)}\beta^{(2)}|k,\uparrow\rangle^{(1)}|-k,\downarrow\rangle^{(2)} \;.
\end{aligned}$$

Notice here that, for our order-of-magnitude estimates, it suffices to assume that the initial entangled state of kaons is a pure state. In practice, due to the omnipresence of foam, this may not be entirely true, but this should not affect our order-of-magnitude estimates based on such an assumption.

With these remarks in mind we then write for the initial state of two kaons after the ϕ decay:

$$|\psi\rangle = |k,\uparrow\rangle^{(1)} |-k,\downarrow\rangle^{(2)} - |k,\downarrow\rangle^{(1)} |-k,\uparrow\rangle^{(2)} + \\ \xi |k,\uparrow\rangle^{(1)} |-k,\uparrow\rangle^{(2)} + \xi' |k,\downarrow\rangle^{(1)} |-k,\downarrow\rangle^{(2)} \ ,$$

where for $r_i \propto \delta_{i1}$ we have $\xi = \xi'$, that is strangeness violation, whilst for $r_i \propto \delta_{i2} \longrightarrow \xi = -\xi'$) (since $\alpha^{(i)} = \beta^{(i)}$) we obtain a strangeness conserving ω-effect.

Upon averaging the density matrix over r_i, only the $|\omega|^2$ terms survive:

$$|\omega|^2 = \mathcal{O}\left(\frac{1}{(E_1 - E_2)^2}(\langle\downarrow, k|H_I|k,\uparrow\rangle)^2\right) \sim \\ \frac{\Delta_2 k^2}{(m_1 - m_2)^2}$$

for momenta of order of the rest energies, as is the case of a ϕ factory.

Recalling that in the recoil D-particle model under consideration we have [21] $\Delta_2 = \tilde{\xi}^2 k^2/M_P^2$, we obtain the following order of magnitude estimate of the ω effect:

$$|\omega|^2 \sim \frac{\tilde{\xi}^2 k^4}{M_P^2 (m_1 - m_2)^2} \ . \tag{11}$$

For neutral kaons with momenta of the order of the rest energies $|\omega| \sim 10^{-4}|\tilde{\xi}|$. For $1 > \tilde{\xi} \geq 10^{-2}$ this not far below the sensitivity of current facilities, such as KLOE at DAΦNE. In fact, the KLOE experiment has just released the first measurement of the ω parameter [23]):

$$\mathrm{Re}(\omega) = \left(1.1^{+8.7}_{-5.3} \pm 0.9\right) \times 10^{-4} \ ,$$
$$\mathrm{Im}(\omega) = \left(3.4^{+4.8}_{-5.0} \pm 0.6\right) \times 10^{-4} \ .$$

One can constrain the ω parameter (or, in the context of the above specific model, the momentum-transfer parameter $\tilde{\xi}$) significantly in upgraded facilities.

For instance, there are the following perspectives for KLOE-2 at (the upgraded) DAΦNE-2 [23]: $\mathrm{Re}(\omega)$, $\mathrm{Im}(\omega) \longrightarrow 2 \times 10^{-5}$.

Let us now mention that ω-like effects can also be generated by the Hamiltonian evolution of the system as a result of gravitational medium interactions. To this end, let us consider the Hamiltonian evolution in our stochastically-fluctuating D-particle-recoil distorted space-times,

$$|\psi(t)\rangle = \exp\left[-i\left(\widehat{H}^{(1)} + \widehat{H}^{(2)}\right)\frac{t}{\hbar}\right]|\psi\rangle \ .$$

Assuming for simplicity $\xi = \xi' = 0$, it is easy to see [35] that the time-evolved state of two kaons contains strangeness-conserving ω-terms:

$$|\psi(t)\rangle \sim e^{-i\left(\lambda_0^{(1)}+\lambda_0^{(2)}\right)t}\varpi(t) \times \\ \left\{|k,\uparrow\rangle^{(1)}|-k,\uparrow\rangle^{(2)} - |k,\downarrow\rangle^{(1)}|-k,\downarrow\rangle^{(2)}\right\} \ .$$

The quantity $\varpi(t)$ obtained within this specific model is purely imaginary,

$$\mathcal{O}(\varpi) \simeq i\frac{2\Delta_1^{\frac{1}{2}}k}{(k^2+m_1^2)^{\frac{1}{2}} - (k^2+m_2^2)^{\frac{1}{2}}} \times \\ \cos\left(\left|\lambda^{(1)}\right|t\right)\sin\left(\left|\lambda^{(1)}\right|t\right) = \varpi_0\sin\left(2\left|\lambda^{(1)}\right|t\right),$$

with $\Delta_1^{1/2} \sim \left|\tilde{\xi}\right|\frac{|k|}{M_P}$, $\varpi_0 \equiv \frac{\Delta_1^{\frac{1}{2}}k}{\left(k^2+m_1^2\right)^{\frac{1}{2}}-\left(k^2+m_2^2\right)^{\frac{1}{2}}}$, $|\lambda^{(1)}| \sim \left(1+\Delta_4^{\frac{1}{2}}\right)\sqrt{\chi_1^2+\chi_3^2}$,

$$\chi_3 \sim \left(k^2+m_1^2\right)^{\frac{1}{2}} - \left(k^2+m_2^2\right)^{\frac{1}{2}} \ .$$

It is important to notice the time dependence of the medium-generated effect. It is also interesting to observe that, if in the initial state we have a strangeness-conserving (-violating) combination, $\xi = -\xi'$ ($\xi = \xi'$), then the time evolution generates time-dependent strangeness-violating (-conserving ω-) imaginary effects.

The above description of medium effects using Hamiltonian evolution is approximate, but suffices for the purposes of obtaining order-of-magnitude estimates for the relevant parameters. In the complete description of the above model there is of course decoherence [35, 21], which affects the evolution and induces mixed states for kaons. A complete analysis of both effects, ω-like and decoherence in entangled neutral kaons of a ϕ-factory, has already been carried out [12], with the upshot that the various effects can be disentangled experimentally, at least in principle (see Section 4.6 below).

Finally, as the analysis of [35] demonstrates, no ω-like effects are generated by thermal bath-like (rotationally-invariant, isotropic) space-time foam situations, argued to simulate the QG environment in some models [40]. In this way, the potential observation of an ω-like effect in EPR-correlated meson states would in principle distinguish various types of space-time foam.

4.5 Disentangling the ω-Effect from the C(even) Background

When interpretating experimental results on delicate violations of CPT symmetry, it is important to disentangle (possible) genuine effects from those due to ordinary physics. Such a situation arises in connection with the ω-effect, that must be disentangled from the C(even) background characterizing the decay products in a ϕ-factory [39].

The C(even) background $e^+e^- \Rightarrow 2\gamma \Rightarrow K^0\overline{K}^0$ leads to states of the form

$$|b>=|K^0\overline{K}^0(C(\text{even}))>= \frac{1}{\sqrt{2}}\left(K^0(\vec{k})\overline{K}^0(-\vec{k})+\overline{K}^0(\vec{k})K^0(-\vec{k})\right) ,$$

which at first sight mimic the ω-effect, as such states would also produce contamination by terms $K_SK_S,\ K_LK_L$.

Closer inspection reveals, however, that the two types of effects can be clearly disentangled experimentally. The reason is two-fold.

(i) First of all, the order of magnitude of the C(even) background is *much smaller* than the C(odd) resonant contribution, as we have seen in the previous discussion, at least in the context of a class of models [35]. Indeed, unitarity bounds [39, 41] imply for the C(even) background:

$$\frac{\sigma(e^+e^- \to K^0\overline{K}^0, J^P=0^+)}{\sigma(e^+e^- \to \phi \to K_SK_L)} \geq 3.6\times 10^{-10} ,$$

and actually one expects the inequality to be saturated. In contrast, the order

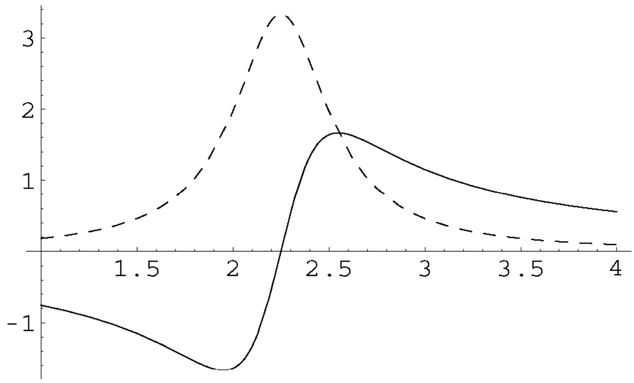


Figure 9: Disentangling the ω-effect from C(even) background: different behaviour at the resonance. The $C = -$ contribution (solid line) vanishes at the top of the resonance, while the C=+ genuine effect (dashed line) still exhibits a resonance peak.

of magnitude of the ω-effect might be much larger, at least in some models (11).

(ii) A more important feature, which clearly distinguishes the ω-effect from the "fake" background effects, is its *different interference* with the C(odd) background [12]. Terms of the type $K_S K_S$ (which dominate over $K_L K_L$) coming from the ϕ-resonance as a result of ω-CPTV can be distinguished from those coming from the $C = +$ (even) background because they interfere differently with the regular $C = -$ (odd) resonant contribution with $\omega = 0$.

Indeed, in the CPTV case, the $K_L K_S$ and $\omega K_S K_S$ terms have the same dependence on the center-of-mass energy s of the colliding particles producing the resonance, because both terms originate from the ϕ-particle. Their interference, therefore, being proportional to the real part of the product of the corresponding amplitudes, still displays a peak at the resonance.

On the other hand, the amplitude of the $K_S K_S$ coming from the $C = +$ background has no appreciable dependence on s and has practically vanishing imaginary part. Therefore, given that the real part of a Breit-Wigner amplitude vanishes at the top of the resonance, this implies that the interference of the $C = +$ background with the regular $C = -$ resonant contribution vanishes at the top of the resonance, with opposite signs on both sides of the latter (see

Fig 9). This clearly distinguishes experimentally the two cases.

4.6 Disentangling the ω-Effect from Decoherent Evolution Effects

As a final point in this section we discuss briefly the experimental disentanglement of the ω-effect from decoherent evolution effects [12].

In models of space-time foam, the initial entangled state of two kaons, after the ϕ-meson decay, is actually itself a density matrix $\tilde{\rho}_\phi = \mathrm{Tr}|\phi >< \phi|$. For $\omega = 0$, the density matrix assumes the form (we remind the reader that the requirement of complete positivity in the entangled-kaon case implies [10] that the decoherent coefficients are $\alpha = \gamma, \beta = 0$) [9]:

$$\begin{aligned}\tilde{\rho}_\phi &= \rho_S \otimes \rho_L + \rho_L \otimes \rho_S - \rho_I \otimes \rho_{\overline{I}} - \rho_{\overline{I}} \otimes \rho_I \\ &-\frac{i\alpha}{\Delta m}(\rho_I \otimes \rho_I - \rho_{\overline{I}} \otimes \rho_{\overline{I}}) - \frac{2\gamma}{\Delta\Gamma}(\rho_S \otimes \rho_S - \rho_L \otimes \rho_L)\ ,\end{aligned}$$

where $\rho_S = |S >< S|$, $\rho_L = |L >< L|$, $\rho_I = |S >< L|$, $\rho_{\overline{I}} = |L >< S|$, and an overall multiplicative factor of $\frac{1}{2}\frac{(1+2|\epsilon|^2)}{1-2|\epsilon|^2\cos(2\phi_\epsilon)}$ has been suppressed.

Now, for $\omega \neq 0$ but $\gamma = 0$ the initial entangled state becomes [12]:

$$\begin{aligned}\rho_\phi &= \rho_S \otimes \rho_L + \rho_L \otimes \rho_S - \rho_I \otimes \rho_{\overline{I}} - \rho_{\overline{I}} \otimes \rho_I \\ &- \omega(\rho_I \otimes \rho_S - \rho_S \otimes \rho_I) - \omega^*(\rho_{\overline{I}} \otimes \rho_S - \rho_S \otimes \rho_{\overline{I}}) \\ &- \omega(\rho_{\overline{I}} \otimes \rho_L - \rho_L \otimes \rho_{\overline{I}}) - \omega^*(\rho_I \otimes \rho_L - \rho_L \otimes \rho_I) \\ &- |\omega|^2(\rho_I \otimes \rho_I + \rho_{\overline{I}} \otimes \rho_{\overline{I}}) + |\omega|^2(\rho_S \otimes \rho_S + \rho_L \otimes \rho_L)\end{aligned}$$

with the same suppressed multiplicative factor as in the previous equation.

The experimental disentanglement of ω from the decoherence parameter γ is possible as a result of different symmetry properties and different structures generated by the time evolution of the pertinent terms. A detailed phenomenological analysis in various channels for ϕ factories has been performed in [12], where we refer the interested reader for details.

5 Precision T, CP and CPT Tests with Charged Kaons

It turns out that precision tests of discrete symmetries can also be performed with charged kaons. This realization has generated great interest [42], mainly due to the (recently acquired) high statistics of the NA48 experiment [22] in certain decay channels. In fact, as we will argue in this section, while the primary objective of this experiment is to probe in detail certain aspects of chiral perturbation theory, it could also furnish strong constraints for various new physics scenarios.

For the purpose of testing CPT symmetry we shall restrict ourselves to one particular charged kaon decay, $K^{\pm} \to \pi^+ + \pi^- + \ell^{\pm} + \nu_\ell(\overline{\nu}_\ell)$, abbreviated as $K^{\pm}_{\ell 4}$. The CPT symmetry can be tested with this reaction [25, 26] by comparing the decay rates of K^+ with the corresponding decays of the K^- mode.

If CPT is well-defined but does not commute with the Hamiltonian, we have the relations: $|K^+\rangle = \widehat{CPT}|K^-\rangle$, $|\pi^+\rangle = \widehat{CPT}|\pi^-\rangle$, $|\pi^0\rangle = \widehat{CPT}|\pi^0\rangle$. If CPT does not commute with the Hamiltonian, then differences between particle antiparticle masses may occur, but this is not the end of the story. In fact, as emphasized earlier, this is not true in certain models of Lorentz- and/or unitarity-violating QG [4, 8, 9, 13].

If, on the other hand, CPT is ill-defined, as is the case of QG-induced decoherence, then there are (perturbative) ambiguities in the antiparticle state, which is still well-defined but with modified properties (see previous section). However, in contrast to the neutral kaon case, the two charged pions in this decay are already distinguishable by means of their electromagnetic interactions (charge), which are, of course, much stronger than their (quantum) gravitational counterparts. Hence, in this respect the ill-defined nature of the CPT operator is not relevant.

A breaking of CPT through unitarity violations (e.g., non-hermitean effective Hamiltonians) could lead in principle to different decay widths for the two decay modes $K^{\pm}$. This would constitute a straightforward precision test of CPT symmetry, if sufficiently high statistics for charged kaons were available [25, 26]. Unlike the entangled neutral kaon case, however, such tests could not distinguish between the various types of CPT breaking.

We next proceed to review briefly the precision tests of T, CP and CPT symmetry using $K^{\pm}_{\ell 4}$ decays. With the exception of tests of T-odd triple correlations that we present at the end of this section, the discussion parallels that of [25, 26], where we refer the reader for further details.

It is important to stress once more that in QG, especially in stochastic space-time foam models, the CPTV is essentially a microscopic Time Reversal (T) Violation, independent of CP properties. It is therefore important to discuss precision tests of T symmetry independent of CP, CPT.

By using $K^{\pm}_{\ell 4}$ for precision tests of T, CP, CPT, one can check in parallel the validity of the $\Delta S = \Delta Q$ rule, exploiting the high statistics a of the NA48 experiment [22], e.g., one can look for the $\Delta S = -\Delta Q$ reaction: $K^+ \rightarrow \pi^+ + \pi^+ + e + \nu$, whose non-observation would establish stringent bounds on the violation of the rule.

In our analysis below, following [25, 26] we assume the $|\Delta I| = 1/2$ isospin rule, which, by the way, can be checked experimentally, as we shall see.

We use the following notation for the corresponding amplitudes:

$$
\begin{aligned}
& e^{i\xi} A = \\
& \langle \pi^+\pi^- | \ell = 0, m = 0 \rangle \langle \ell = 0, m = 0 | S_z + ivS_4 | K^+ \rangle \times \\
& m_K^{5/2} (\omega_+ \omega_-)^{1/2} \ , \\
& e^{i\xi + i\eta_0} B_0 \cos\theta = \\
& \langle \pi^+\pi^- | \ell = 1, m = 0 \rangle \langle \ell = 1, m = 0 | S_z + ivS_4 | K^+ \rangle \times \\
& m_K^{5/2} (\omega_+ \omega_-)^{1/2} \ , \\
& e^{i\xi + i\eta_\pm} B_\pm \sin\theta e^{\pm i\phi} = \\
& \langle \pi^+\pi^- | \ell = 1, m = \pm 1 \rangle \langle \ell = 1, m = \pm 1 | \frac{1}{\sqrt{2}} (S_x + iS_y) | K^+ \rangle \times \\
& m_K^{5/2} (\omega_+ \omega_-)^{1/2} \ ,
\end{aligned}
$$

with ℓ the orbital angular momentum quantum number, m its z-axis component. The phase conventions are chosen such that $A, B_0, B_\pm$ are real and positive; the polar angles θ, ϕ pertain to the di-pion center-of-mass system $\Sigma_{2\pi}$ and x, y, z are Cartesian coordinates in the laboratory system $\Sigma_{\rm Lab}$. An angle α in the di-lepton center-of-mass system $\Sigma_{\ell\nu_\ell}$ will also be used. The $\omega_\pm$ denote the laboratory energies of the $\pi^\pm$, $v = -[m_K - (\vec{P}^2 + M^2)^{1/2}]^{-1} |\vec{P}|$ is the velocity of Lorentz transformation connecting $\Sigma_{\ell\nu_\ell}$ to $\Sigma_{\rm Lab}$ frames, with $\vec{P}$ the total momentum of the two pions in $\Sigma_{\rm Lab}$.

The action of CPT is obtained by replacing the corresponding amplitudes by barred quantities: $\overline{(...)}$, and implementing the following substitutions: $K^+ \rightarrow K^-$, $\pi^+(\vec{k}_1)\pi^-(\vec{k}_2) \rightarrow \pi^-(\vec{k}_1)\pi^+(\vec{k}_2)$, plus appropriate complex conjugates.

We outline below various possible precision tests of discrete symmetries based on the reaction $K^{\pm}_{\ell 4}$:

- CPT invariance implies:

  $A = \overline{A}, \quad B_0 = \overline{B}_0, \quad B_\pm = \overline{B}_\mp, \qquad \eta_+ + \overline{\eta}_- = \eta_- + \overline{\eta}_+ = \eta_0 + \overline{\eta}_0 = 2\,(\delta_p - \delta_s)$, independently of T invariance, with $\delta_p(\delta_s)$ the strong-interaction $\pi - \pi$ scattering phase shifts for the states $I = 1, \ell = 1 (I = 0, \ell = 0)$.

  Also, CPT invariance, independently of the $|\Delta I| = 1/2$ rule, implies: rate$(K^+ \to \pi^+\pi^- e^+ \nu_e)$+rate$(K^+ \to \pi^0\pi^0 e^+ \nu_e)$ = rate$(K^- \to \pi^+\pi^- e^- \overline{\nu}_e)$ rate$(K^- \to \pi^0\pi^0 e^- \overline{\nu}_e)$.

  Under the assumption of the $|\Delta I| = 1/2$ rule, on the other hand, CPT invariance implies for the differential rates d^5N of $K^{\pm}_{\ell 4}$:

  $\int d\phi d\cos\theta \quad d^5N(K^+ \to \pi^+\pi^- e^+ \nu_e) = \int d\phi d\cos\theta \quad d^5N(K^- \to \pi^+\pi^- e^- \overline{\nu}_e$

- T invariance (independent of CP, CPT) implies:

  $\eta_\sigma = \delta_p - \delta_s \text{ (modulo } \pi) , \qquad \sigma = 0, \pm .$

- CP Invariance (independent of T, CPT), which in terms of the angles means: $\theta, \phi, \alpha \to \theta, -\phi, \alpha$, implies:

  $A = \overline{A}, \quad B_0 = \overline{B}_0, \quad B_\pm = \overline{B}_\mp, \quad \eta_0 = \overline{\eta}_0, \quad \eta_\pm = \overline{\eta}_\mp$, and for the differential rates $[d^5N(K^+)]_{\alpha,\theta,\phi} = [d^5N(K^-)]_{\alpha,\theta,-\phi}$, leading also to

$$\int d\phi d\cos\theta \quad d^5N(K^+ \to \pi^+\pi^- e^+ \nu_e) = \int d\phi d\cos\theta \quad d^5N(K^- \to \pi^+\pi^- e^- \overline{\nu}_e) \tag{12}$$

  but *without* the assumption about $|\Delta I| = 1/2$.

Let us now deviate sligtly from the main scope of this article, and comment on the possibility of testing physics beyond the Standard Model (SM) by looking for T-odd triple correlators [27] in the NA48 data for the reaction modes $K^{\pm}_{\ell 4}$. Such tests are not directly related to CPT but rather to different aspects of new physics, such as supersymmetry; the latter, in turn, could be essential for formulating consistent theories of QG.

Within the SM, direct CP violation or CP violation of pure $\Delta S = 1$ origin, which, due to CPT symmetry, would imply T-odd correlators [2], is very strongly suppressed in non-leptonic decays $K^{\pm} \to (3\pi)^{\pm}$: $\mathcal{O}\left(10^{-5} - 10^{-6}\right)$ and absolutely negligible in $K_{\ell 4}$ [43]. Evidence for such violations in $K_{\ell 4}$ charged-kaon decays would, therefore, constitute evidence for physics beyond the SM.

As was pointed out in [27], one can construct appropriate CP observables for charged kaon decays $K_{\ell 4}$ that do not involve the lepton polarization, a quantity difficult to measure in the NA48 experiment [22]. This is achieved by considering appropriate combinations of matrix elements pertaining to *both* decay modes $K^{\pm}_{\ell 4}$. The construction makes use of the fact that, under the assumption of only left-handed neutrinos, the most general local effective Hamiltonian, relevant to charged-kaon $K_{\ell 4}$ (and $K_{\ell 3}$) decays, can be expanded in terms of appropriate local dimension six field operators $\mathcal{O}_i$ [27]:

$$\mathcal{H}_{\rm eff} = 2\sqrt{2} G_F V^*_{us} \sum_i C_i \mathcal{O}_i + h.c. \ ,$$

where the operators $\mathcal{O}_i$ are four-fermion operators involving left-handed neutrinos (e.g. $\mathcal{O}^V_L = \overline{s}_L \gamma^\mu u_L \overline{\nu}_L \gamma_\mu \ell_L$, $\mathcal{O}^S_L = \overline{s}_R u_L \overline{\nu}_L \ell_R$, $\mathcal{O}^T_L = \overline{s}_R \sigma^{\mu\nu} u_L \overline{\nu}_L \sigma_{\mu\nu} \ell_R$, etc ($\mathcal{O}^i_R : s_R \to s_L,\ u_L \to u_R$)). In the SM only $C^V_L = 1$, while the others are negligible.

Within the SM, the relevant matrix elements for the $K_{\ell 4}$ decay are

$$\langle \pi^+ \pi^- | \overline{s} \gamma^\mu u | K^+ \rangle, \langle \pi^+ \pi^- | \overline{s} \gamma^\mu \gamma_5 u | K^+ \rangle \ .$$

Beyond the SM, one has more structures; for instance [27]

$$\langle \pi^+ \pi^- | \overline{s} \gamma_5 u | K^+ \rangle, \langle \pi^+ \pi^- | \overline{s} \sigma^{\mu\nu} \gamma_5 u | K^+ \rangle \ .$$

Using such structures, one can construct [27] appropriate combinations of T-odd correlators in $K_{\ell 4}$ decays, proportional to momentum triple products, $\vec{p}_\ell \cdot (\vec{p}_{\pi_1} \times \vec{p}_{\pi_2})$, by using *both* K^+ and K^- modes. This leads to new CP-violating observables, free from strong final-state interactions. These observables can be used for precision CP tests without the need of measuring lepton polarization.

[2] It should be stressed at this point that, on account of the anti-unitarity of the time-reversal operator, T-odd correlators are not necessarily T-violating.

The results are complementary to those obtained through normal-to-the-decay-plane muon polarization in $K_{\mu 3}$ decays, and of comparable accuracy. For details and related references we refer the interested reader to the literature [27].

We close this section by pointing out that the NA48 data could also provide new stringent constraints on exotic (beyond the SM) scenarios, such as R-parity breaking in supersymmetric theories, complementary to those obtained through $K_{\ell 3}$ or other decays. In fact, as has been known for some time [44], the existence of complex coupling constants allows to test supersymmetry in weak decays (in particular rare kaon decays involving leptons). Specifically, T-invariance may be studied with appropriate T-odd observables, such as triple correlators of polarizations and momenta (for instance, in $K^+_{\mu 3}$ decays the appropriate observable is the normal-to-the-decay-plane muon polarization $\langle \vec{\sigma}_\mu \cdot (\vec{p}_\mu \times \vec{p}_\pi)/|\vec{p}_\mu \times \vec{p}_\pi|\rangle$). This type of analyses can be complemented by the above-mentioned study of lepton-polarization-independent T-odd observables in $K^{\pm}_{\ell 4}$ decays, and also serve as precision tests of CPT symmetry. To the best of our knowledge this has not been done yet.

6 Instead of Conclusions

In this review we have outlined several aspects of CPTV and the corresponding experiments. We have attempted to convey a general feel for the interesting and challenging precision tests that can be performed using kaon systems. Such experiments could shed light on many aspects of an extended class of QG models, featuring decoherence of low-energy matter due to its propagation in foamy backgrounds.

We hope to have presented sufficient theoretical motivation and estimates of the associated effects to support the case that testing QG experimentally at present fascilities may turn out to be a worthwhile endeavour. In fact, as we have argued, CPTV may be a real feature of QG, that can be tested and observed, if true, in the foreseeable future.

We have outlined various, schemes for CPT breaking, that are in principle independent. We have stressed that decoherence and Lorentz violation (LV) are independent effects: in QG one may have Lorentz-invariant (LI) decoherence [18]. The frame dependence of LV effects (e.g. day-night differences) could serve to disentangle LV from LI CPTV. The example discussed in this article is a comparison between results in meson factories. If there is LV, then there should in principle be frame-dependent differences between ϕ-factories, where the initial meson is produced at rest, and B-meson factories, where the initial Υ-state is boosted.

As mentioned above, precision tests of fundamental symmetry in meson factories could provide sensitive probes of QG-induced decoherence and CPTV. In particular, one might observe novel effects (ω-effects) exclusive to entangled neutral meson states, modified EPR correlations, and, as a consequence, theoretical (intrinsic) limitations on flavour tagging for B-factories [37]. As we have seen, some theoretical (string-inspired) models of space-time foam predict ω-like effects of an order of magnitude that is already well within the reach of the next upgrade of ϕ-factories, such as DAΦNE-2.

Precision experiments to test discrete space-time symmetries can also be performed with charged kaons: the pertinent experiments [22] can carry out high-precision tests of T, CPT and CP invariance, including aspects of physics beyond the Standard Model, such as supersymmetry, using $K_{\ell 4}$ decays.

The current experimental situation for QG signals appears exciting, and several experiments are reaching interesting regimes, where many theoretical models can be falsified. More precision experiments are becoming available, and many others are being designed for the immediate future. Searching for tiny effects of this elusive theory may at the end be very rewarding. Surprises may be around the corner, so it is worth investing time and effort. Nevertheless, much more work, both theoretical and experimental, needs to be done before (even tentative) conclusions regarding QG effects are reached.

Acknowledgements

We thank A. Di Domenico for the invitation to write this review and for many illuminating discussions. We also thank E. Alvarez, G. Amelino-Camelia, F. Botella, M. Nebot, Sarben Sarkar, A. Waldron-Lauda, and M. Westmuckett for discussions and collaboration on some of the topics reviewed here; J.B. and N.E.M. thank B. Bloch-Devaux, L. Di Lella, G. Isidori and B. Peyaud for informative discussions on charged-kaon decays. The work of J.B. and J.P. is supported by Spanish MEC and European FEDER under grant FPA 2005-01678. The work of D.V.N. is supported by D.O.E. grant DE-FG03-95-ER-40917.

References

1. G. Lüders, Ann. Phys. (NY) 2 (1957) 1.
2. See: J. A. Wheeler and K. Ford, *Geons, Black Holes, And Quantum Foam: A Life In Physics,* (New York, USA: Norton (1998)).
3. S. W. Hawking, Comm. Math. Phys. 87 (1982) 395.
4. J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos and M. Srednicki, Nucl. Phys. B 241 (1984) 381.
5. For a review see: N. E. Mavromatos, Lect. Notes Phys. **669**(2005) 245 [arXiv:gr-qc/0407005] and references therein.
6. G. Lindblad, Commun. Math. Phys. **48**, 119 (1976); V. Gorini, A. Kossakowski and E. C. G. Sudarshan, J. Math. Phys. **17**, 821 (1976); R. Alicki and K. Lendi, Lect. Notes Phys. **286** (Springer Verlag, Berlin (1987)) and references therein.
7. R. Wald, Phys. Rev. D21 (1980) 2742.
8. J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B **293** (1992) 142. [arXiv:hep-ph/9207268]; J. R. Ellis, J. L. Lopez, N. E. Mavromatos and D. V. Nanopoulos, Phys. Rev. D **53** (1996) 3846 [arXiv:hep-ph/9505340].
9. P. Huet and M. E. Peskin, Nucl. Phys. B **434** (1995) 3 [arXiv:hep-ph/9403257].
10. F. Benatti and R. Floreanini, Nucl. Phys. B **511** (1998) 550 [arXiv:hep-ph/9711240]; Phys. Lett. B **468** (1999) 287 [arXiv:hep-ph/9910508].
11. For a recent review see: M. Fidecaro and H. J. Gerber, Rept. Prog. Phys. **69** (2006) 1713 [arXiv:hep-ph/0603075] and references therein.
12. J. Bernabeu, N. E. Mavromatos and J. Papavassiliou, Phys. Rev. Lett. **92** (2004) 131601 [arXiv:hep-ph/0310180]; J. Bernabeu, N. E. Mavromatos, J. Papavassiliou and A. Waldron-Lauda, Nucl. Phys. B **744** (2006) 180 [arXiv:hep-ph/0506025].

13. For a recent review see: V. A. Kostelecky, *Prepared for 3rd Meeting on CPT and Lorentz Symmetry (CPT 04), Bloomington, Indiana, 4-7 Aug 2004* (World Scientific, Singapore 2005) and references therein.

14. L. Smolin, Lect. Notes Phys. **669** (2005) 363, and references therein.

15. R. Bluhm, V. A. Kostelecky and N. Russell, Phys. Rev. Lett. 82 (1999) 2254; R. Bluhm, arXiv:hep-ph/0111323; V. A. Kostelecky, arXiv:hep-ph/0403088; V. A. Kostelecky and M. Mewes, Phys. Rev. D **70** (2004) 076002 [arXiv:hep-ph/0406255].

16. T. Jacobson, S. Liberati and D. Mattingly, Annals Phys. **321** (2006) 150 [arXiv:astro-ph/0505267] and references therein.

17. V. A. Kostelecky, Phys. Rev. Lett. **80** (1998) 1818 [arXiv:hep-ph/9809572]; arXiv:hep-ph/9809584.

18. G. J. Milburn, arXiv:gr-qc/0308021.

19. G. Barenboim and J. Lykken, Phys. Lett. B 554 (2003) 73.

20. A. Aguilar *et al.* [LSND Collaboration], Phys. Rev. D **64** (2001) 112007 [arXiv:hep-ex/0104049]; G. Drexlin, Nucl. Phys. Proc. Suppl. **118** (2003) 146.

21. J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B **293** (1992) 37 [arXiv:hep-th/9207103]; *A microscopic Liouville arrow of time*, Invited review for the special Issue of *J. Chaos Solitons Fractals*, Vol. 10, p. 345-363 (eds. C. Castro amd M.S. El Naschie, Elsevier Science, Pergamon 1999) [arXiv:hep-th/9805120]; J. R. Ellis, P. Kanti, N. E. Mavromatos, D. V. Nanopoulos and E. Winstanley, Mod. Phys. Lett. A **13** (1998) 303 [arXiv:hep-th/9711163]; N.E. Mavromatos and Sarben Sarkar, Phys. Rev. D **72** (2005) 065016 [arXiv:hep-th/0506242].

22. C. Lazzeroni [NA48 Collaboration], PoS **HEP2005** (2006) 096 and references therein; see also G. Anzivino, Proc. *BEACH2006 Conference* (Lancaster, U.K. July 2-7 2006) and references therein (http://www.hep.lancs.ac.uk/Beach2006/).

23. F. Ambrosino [KLOE Collaboration], Phys. Lett. B **642** (2006) 315;
see also: A. Di Domenico, contribution to this handbbok and references therein.

24. P. Franzini and M. Moulson, Ann. Rev. Nucl. Part. Sci. **56** (2006) 207 [arXiv:hep-ex/0606033] and references therein.

25. T. D. Lee and C. S. Wu. Ann. Review Nuclear Science, **16** (1966) 471.

26. A. Pais and S.B. Treiman, Phys. Rev. **168** (1968) 1858.

27. A. Retico, Phys. Rev. D **65** (2002) 117901 [arXiv:hep-ph/0203044].

28. See M. J. Wilking, arXiv:hep-ex/0606044 and references therein.

29. L. H. Ford, Int. J. Theor. Phys. **38** (1999) 2941; H. w. Yu and L. H. Ford, arXiv:gr-qc/0004063.

30. J. R. Ellis, K. Farakos, N. E. Mavromatos, V. A. Mitsou and D. V. Nanopoulos, Astrophys. J. **535** (2000) 139 [arXiv:astro-ph/9907340].

31. N. E. Mavromatos and Sarben Sarkar, arXiv:hep-ph/0606048.

32. D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O Stamatescu and H.D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer 1996).

33. J. R. Ellis, N. E. Mavromatos and M. Westmuckett, Phys. Rev. D **70**, 044036 (2004) [arXiv:gr-qc/0405066]; Phys. Rev. D **71**, 106006 (2005) arXiv:gr-qc/0501060

34. J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond* (Cambridge University Press, Cambrige (UK), 1998).

35. J. Bernabeu, N. E. Mavromatos and S. Sarkar, arXiv:hep-th/0606137.

36. R. Adler *et al.* [CPLEAR collaboration], Phys. Lett. B 364 (1995) 239.

37. E. Alvarez, J. Bernabeu, N. E. Mavromatos, M. Nebot and J. Papavassiliou, Phys. Lett. B **607** (2005) 197 [arXiv:hep-ph/0410409]; E. Alvarez, J. Bernabeu and M. Nebot, arXiv:hep-ph/0605211; E. Alvarez, J. Bernabeu and M. Nebot, PoS **HEP2005** (2006) 252 [arXiv:hep-ph/0512073]; E. Alvarez, arXiv:hep-ph/0603102, Ph.D. Thesis.

38. See for instance: H. J. Lipkin, Phys. Rev. **176** (1968) 1715; preprint WIS-89/72/PH *Proc. of the Nathan Rosen 80th Birthday Symp., Developments in General Relativity, Astrophysics and Quantum Mechanics, Nov 1989* and references therein.

39. I. Dunietz, J. Hauser and J. L. Rosner, Phys. Rev. D **35** (1987) 2166; J. Bernabeu, F. J. Botella and J. Roldan, Phys. Lett. B **211** (1988) 226.

40. L. J. Garay, Int. J. Mod. Phys. A **14** (1999) 4079 (1999) [arXiv:gr-qc/9911002] and references therein; Phys. Rev. D**58** (1998) 124015.

41. L. Maiani, G. Pancheri and N. Paver, eds., *The Second Daphne Physics Handbook* Vol. 1, 2," (LN Frascati, Italy INFN 1995)

42. See: M. Ali, Proc. *BEACH2006 Conference* (Lancaster, U.K. July 2-7 2006) and references therein (http://www.hep.lancs.ac.uk/Beach2006/).

43. G. D'Ambrosio and G. Isidori, Int. J. Mod. Phys. A **13** (1998) 1 [arXiv:hep-ph/9611284] and references therein.

44. E. Christova and M. Fabbrichesi, Phys. Lett. B **315** (1993) 113 [arXiv:hep-ph/9302303]; T. G. Rizzo, *Rare K decays and new physics beyond the standard model*, arXiv:hep-ph/9809526 and references therein.

METHODS AND MODELS FOR THE STUDY OF DECOHERENCE

Sarben Sarkar
King's College London, University of London, Department of Physics, Strand WC2R 2LS, London, U.K.

Abstract

We shall review methods used in the description of decoherence on particle probes in experiments due to surrounding media. This will include conventional media as well as a model for space-time foam arising from non-critical string theory.

1 The role of decoherence

Until recently in elementary particle physics the environment was not considered. Scatterings were calculated in a vacuum background and S-matrix elements were calculated within the paradigm of the standard gauge theory model. The latter is a successful theory overall. However, recently systems which oscillate coherently have been investigated with increasing precision, e.g. neutrino and neutral meson flavour oscillations. Clearly neutrinos produced in the sun, on going through it, encounter an obvious scattering environment. In laboratory experiments however there does not seem to be the need for such considerations; of course there are uncertainties in determining time and position which lead to features akin to decoherence [1]. However, triggered again by increased precision, the effect of fluctuations in the space-time metric due to space-time defects such as microscopic black holes, and D branes in string theory are being estimated. Given the smallness of the gravitational coupling compared to the other interactions in the past the search for such effects was regarded as optimisitc. Progress in experimental techniques is making such effects more testable [2].

In this Handbook it was considered to be desirable to split the discussion of decoherence between two chapters.This one will render a brief account of the methods of decoherence that are used in the analysis of experiments given in the companion chapter [3]. We shall demonstrate why there is a large universality class in the space of theories describing decoherence with most analyses using models from this class. However we should stress that the universality is for descriptions where the system-environment interaction is in some sense conventional. Indeed when we introduce descriptions emanating from string theory we can and do produce descriptions which can give qualitatively different effects [4]. Such non-conventional descriptions are to be expected since it is natural for quantum space-time to be somewhat different from the paradigm of Brownian phenomena in condensed matter. Moreover the manifestation of gravitational decoherence in a theory, which is diffeomorphic covariant at the classical level, is not just restricted to fluctuation and dissipation. It is pivotal in the breakdown of discrete symmetries such as CPT and more obviously T. This is an exciting role for decoherence because it gives rise to qualitatively new phenomena [5] which is being tested now and in the next generation of laboratory experiments.

This paper will be divided into three sections:

- decoherence in a general setting with a discussion of how coherence is lost and the implication for discrete symmetries
- generic treatment of system-reservoir interactions and the Lindblad formalism from Markovian approximations
- non-critical string theory and D-particle foam and the phenomenolgy of stochastic metrics

2 General Features of Decoherence

The fact that an environment $\mathcal{E}$ interacts with a system $\mathcal{S}$ and is affected by it is obvious whether they interact classically or quantum mechanically. However classically the measurement of $\mathcal{E}$ can only locally affect $\mathcal{S}$. This is in sharp contrast to the quantum mechanical situation where non-local effects can take place. The associated distinguishing property is that of entanglement. For the compound system $\mathcal{ES}$ Schmidt bases allow us to write the state $|\Psi\rangle$ as

$$|\Psi\rangle = \sum_n \sqrt{p_n}\,|\phi_n\rangle\,|\Phi_n\rangle$$

where the Hilbert space $H_{\mathcal{S}}$ of states $|\phi_n\rangle$ are associated with $\mathcal{S}$ and the Hilbert space $H_{\mathcal{E}}$ of states $|\Phi_n\rangle$are associated with $\mathcal{E}$. In the Schmidt basis the states for different n in the different spaces have to be mutually orthogonal i.e.

$$\langle\phi_n|\,\phi_m\rangle = \langle\Phi_n|\,\Phi_m\rangle = \delta_{nm}$$

and the non-negative coefficients p_n satisfy $\sum_n p_n^2 = 1$.

The corresponding density matrix is

$$\rho = \rho_{class.} + \sum_{n\neq m} \sqrt{p_n p_m}\,|\phi_n\rangle\langle\phi_m| \otimes |\Phi_n\rangle\langle\Phi_m|$$

where $\rho_{class.} = \sum_n p_n |\phi_n\rangle\langle\phi_n| \otimes |\Phi_n\rangle\langle\Phi_n|$. The term $\rho - \rho_{class.}$ is known as the entanglement. Clearly entanglement is a measure of the departure of the compound system from a product state of states of $\mathcal{S}$ and $\mathcal{E}$. A classic example of a pure entangled state is the EPR state (Einstein-Podolsky-Rosen) written conventionally in terms of spin $\frac{1}{2}$ systems

$$\frac{|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}}$$

which is clearly not factorisable. Now let us see how the interaction between $\mathcal{S}$ and $\mathcal{E}$ leads to decoherence by considering a simple interaction

$$\lambda H_{\mathcal{E}\mathcal{S}} = \sum_n |\phi_n\rangle\langle\phi_n| \otimes \widehat{A}_n$$

where $\widehat{A}_n$ are operators on the $H_{\mathcal{E}}$. For an initial pure unentangled state i.e. a product state

$$|\Psi\rangle = \sum_n c_n |\phi_n\rangle|\Theta_0\rangle$$

(where $|\Theta_0\rangle$ can be expressed in terms of the $|\Phi_n\rangle$s) under time evolution

$$|\phi_n\rangle|\Theta_0\rangle \xrightarrow{t} |\phi_n\rangle \exp\left(-i\widehat{A}_n t\right)|\Theta_0\rangle \equiv |\phi_n\rangle|\Theta_n(t)\rangle$$

The density matrix traced over the environment $\rho_{\mathcal{S}}(t)$ gives

$$\rho_{\mathcal{S}}(t) = \sum_{n,m} c_m^* c_n \langle\Theta_m(t)\,|\Theta_n(t)\rangle\,|\phi_m\rangle\langle\phi_n|$$

If the circumstances are such that $\langle \Theta_m(t) | \Theta_n(t) \rangle \longrightarrow \delta_{mn}$ as $t \longrightarrow \infty$, then asymptotically

$$\rho_S(t) \longrightarrow \sum_n |c_n|^2 |\phi_n\rangle \langle \phi_n| .$$

All coherences embodied by off-diagonal matrix elements have vanished, i.e. there is complete decoherence [6].

We will now consider an *associated* aspect of the interaction of the system with the environment, the lack of an invertible scattering matrix. Consider schematically three spaces $\mathfrak{H}_1, \mathfrak{H}_2$ and $\mathfrak{H}_3$ where $\mathfrak{H}_1$ is the space of states of the initial states, $\mathfrak{H}_2$ is the state space for inaccessible environmental degrees of freedom (e.g. states inside a black hole horizon) and $\mathfrak{H}_3$ is the space of final states. Within a scattering matrix formalism consider an in-state $\sum_A x_A |X_A\rangle_1 |0\rangle_2 |0\rangle_3$ (where the subscripts 1, 2 and 3 are related to the spaces $\mathfrak{H}_1, \mathfrak{H}_2$ and $\mathfrak{H}_3$) this is scattered to $\sum_A \mathbb{S}_A^{bc} x_A |0\rangle_1 \left|\overline{Y}^b\right\rangle_2 \left|\overline{Z}^c\right\rangle_3$ where the bar above the state labels indicates the CPT transform [7]. (On introducing the operator $\theta = CPT$ we have explicitly $\left|\overline{Y}^b\right\rangle = \theta |Y_b\rangle$ etc.) Now on tracing over the inaccessible degrees of freedom (in $\mathfrak{H}_2$) we obtain

$$|X_A\rangle \langle X_A| \longrightarrow \sum_{c,c'} \not{S}_{A\,A}^{\,c\,c'} \left|\overline{Z}^c\right\rangle \left\langle \overline{Z}^{c'}\right|$$

with the effective scattering matrix $\not{S}$ given by

$$\not{S}_{A\,A}^{\,c\,c'} = \sum_{b,b'} \mathbb{S}_A^{bc} \mathbb{S}_A^{*\,b'c'} .$$

This does not factorise, which it would have to, for $/S$ to be of the form $UU^\dagger$. Consequently evolution is non-unitary. This is generic to environmental decoherence. Of course with space-time defects the inaccessible degrees of freedom can be behind causal horizons.

For local relativistic interacting quantum field theories there is the CPT theorem. Such theories show unitary evolution. A violation of CPT for Wightman functions (i.e. unordered correlation functions for fields) implies violation of Lorentz invariance [8]. However CPT invariance of course is not sufficient for Lorentz invariance. For physical systems, which in the absence of gravity show CPT invariance, the incorporation of a gravitational environment can lead to non-unitary evolution as we have argued. In fact we shall sketch arguments from non-critical string theory which produce such non-unitary evolution. There is then a powerful argument due to Wald which argues that an operator θ incorporating strong CPT invariance does not exist. The argument proceeds via reduction ad absurdum. For strong CPT invariance to hold we should have in states and out states connected by $\not{S}$ and θ and their operations commute in the following sense. For an in state ρ_{in} there is an out state ρ_{out} such that

$$\rho_{out} = \not{S}\rho_{in}.$$

Also there is another out state $\rho'_{out} = \theta\rho_{in}$ associated with ρ_{in}. If the CPT transforms of states have the same $\not{S}$ evolution as the untransformed states then there is strong CPT invariance. In such situations

$$\theta\ \not{S}\theta\ \not{S}\rho_{in} = \rho_{in}$$

and so

$$\theta\ \not{S}\theta\ \not{S} = I,$$

i.e. $\not{S}$ has an inverse. In most circumstances interaction with an environment produces dissipation and so the inverse of $\not{S}$ would not exist. Hence the assumption of strong CPT is incompatible with non-unitary evolution [9].

3 Particles propagating in a medium and master equations

Particles reaching us from outside a laboratory always travel through some physical medium which can often be described by a conventional medium. For the moment we will be general and call the medium $\mathcal{E}$ and the particle $\mathcal{S}$. We are ignoring particle-particle interactions and so the approximation of a single body point of view is appropriate. This bipartite separation can be subtle since different degrees of freedom of the same particle can be distributed between $\mathcal{E}$ and $\mathcal{S}$. Initially (at $t = t_0$) the state ρ of the compound system is assumed to have a factorised form

$$\rho(t_0) = \rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}} \qquad (1)$$

with $\rho_{\mathcal{S}}$ being a normalised density operator on the Hilbert space $\mathfrak{H}_{\mathcal{S}}$ of states of $\mathcal{S}$ and analogously for $\rho_{\mathcal{E}}$. This condition may be not hold in the very early universe and for an ever present meidum such as space-time foam; $\mathcal{E}$ and $\mathcal{S}$ would then always be entangled. Certainly for laboratory experiments the condition 1 is acceptable [10] and the analysis is simplified. Write the total hamiltonian H as

$$H = H_{\mathcal{S}} + H_{\mathcal{E}} + \widehat{H}_{\mathcal{S}\mathcal{E}}$$

where $H_{\mathcal{S}\mathcal{E}}$ represents the interaction coupling the system and environment.The Heisenberg equation is

$$\frac{\partial \rho}{\partial t} = -i\left[H_{\mathcal{S}} + H_{\mathcal{E}} + H_{\mathcal{S}\mathcal{E}}, \rho\right] \equiv L\rho \tag{2}$$

and we will also find it useful to let $-i\left[H_{\mathcal{S}}, \rho\right] \equiv L_{\mathcal{S}}\rho$, $-i\left[H_{\mathcal{E}}, \rho\right] \equiv L_{\mathcal{E}}\rho$ and $-i\left[H_{\mathcal{S}\mathcal{E}}, \rho\right] \equiv L_{\mathcal{S}\mathcal{E}}\rho$. ρ evolves unitarily. For measuring with operators acting on $\mathfrak{H}_{\mathcal{S}}$ it is sufficient to consider

$$\rho_{\mathcal{S}} = Tr_{B}\rho \tag{3}$$

but given a $\rho_{\mathcal{S}}$ there is in general no unique ρ associated with it. Hence the evoultion of $\rho_{\mathcal{S}}$ is not well defined. However by choosing a reference environment state $\rho_{\mathcal{E}}$ satisfying

$$L_{\mathcal{E}}\rho_{\mathcal{E}} = 0 \tag{4}$$

we can associate with a $\rho_{\mathcal{S}}$ a unique state $\rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}$ of $S\mathcal{E}$. In this way a well defined evolution can be envisaged.

We will obtain a master equation for $\rho_{\mathcal{S}}$ by using the method of projectors [11]. Let us define

$$P\rho = \left(Tr_{\mathcal{E}}\rho\right) \otimes \rho_{\mathcal{E}}.$$

Clearly

$$P^{2}\rho = \left[Tr_{\mathcal{E}}\rho_{\mathcal{E}}\right]\left(Tr_{\mathcal{E}}\rho\right) \otimes \rho_{\mathcal{E}} = \left(Tr_{\mathcal{E}}\rho\right) \otimes \rho_{\mathcal{E}} = P\rho$$

and so P is a projector. Also we define $Q = 1 - P$. Acting on 2 with P gives

$$P\frac{\partial \rho}{\partial t} = PL\rho = PLP\rho + PLQ\rho. \tag{5}$$

Similarly

$$Q\frac{\partial \rho}{\partial t} = QL\rho = QLP\rho + QLQ\rho. \tag{6}$$

These give two coupled equations for $P\rho$ and $Q\rho$. 6 can be solved for $Q\rho$ on noting that

$$\left(\frac{\partial}{\partial t} - QL\right) Q\rho = QLP\rho$$

and then on formally integrating

$$\int_0^t \frac{\partial}{\partial t'} \left(e^{-QLt'} Q\rho(t')\right) dt' = \int_0^t e^{-QLt'} QLP\rho(t')\, dt'$$

i.e.

$$e^{-QLt} Q\rho(t) = Q\rho(0) + \int_0^t e^{-QLt'} QLP\rho(t')\, dt'. \tag{7}$$

This expression for $Q\rho$ is substituted in 5 to give

$$Tr_{\mathcal{E}}\left[P\frac{\partial \rho}{\partial t}\right] = \frac{\partial \rho_{\mathcal{S}}}{\partial t} = Tr_{\mathcal{E}}\left[PLP\rho\right] + Tr_{\mathcal{E}}\left[PL\left(e^{-QLt}Q\rho(0) + \int_0^t e^{-QL(t-t')}QL\right.\right.$$

and can be simplified further on noting that

$$PL_{\mathcal{E}}Q\rho = PL_{\mathcal{E}}(\rho - P\rho) = PL_{\mathcal{E}}\rho = 0 \Longrightarrow PL_{\mathcal{E}} = 0 \tag{8}$$

owing to the cyclic properties of traces. Also

$$PL_{\mathcal{S}}Q\rho = PL_{\mathcal{S}}(\rho - \rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}) = L_{\mathcal{S}}P\rho - (L_{\mathcal{S}}\rho_{\mathcal{S}}) \otimes \rho_{\mathcal{E}} = 0 \Longrightarrow PL_{\mathcal{S}} = PL_{\mathcal{S}}P. \tag{9}$$

Hence

$$\begin{aligned} Tr_{\mathcal{E}}(PL_{\mathcal{S}}P\rho) &= Tr_{\mathcal{E}}(PL_{\mathcal{S}}\rho) \\ &= Tr_{\mathcal{E}}(Tr_{\mathcal{E}}(L_{\mathcal{S}}\rho) \otimes \rho_{\mathcal{E}}) = Tr_{\mathcal{E}}(L_{\mathcal{S}}\rho) = L_{\mathcal{S}}\rho_{\mathcal{S}}. \end{aligned}$$

Also we assume that $H_{\mathcal{SE}} = V_{\mathcal{S}} \otimes V_{\mathcal{E}}$ (which is standard for local quantum field theory) and so

$$\begin{aligned} Tr_{\mathcal{E}}(PL_{\mathcal{SE}}P\rho) &= Tr_{\mathcal{E}}(PL_{\mathcal{SE}}\rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}) \\ &= Tr_{\mathcal{E}}\left[P(V_{\mathcal{S}}\rho_{\mathcal{S}}) \otimes (V_{\mathcal{E}}\rho_{\mathcal{E}}) - P(\rho_{\mathcal{S}}V_{\mathcal{S}}) \otimes (\rho_{\mathcal{E}}V_{\mathcal{E}})\right] \\ &= Tr_{\mathcal{E}}\left[V_{\mathcal{S}}\rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}Tr_{\mathcal{E}}(V_{\mathcal{E}}\rho_{\mathcal{E}}) - \rho_{\mathcal{S}}V_{\mathcal{S}} \otimes \rho_{\mathcal{E}}Tr_{\mathcal{E}}(\rho_{\mathcal{E}}V_{\mathcal{E}})\right] \\ &= [V_{\mathcal{S}}, \rho_{\mathcal{S}}] \otimes \rho_{\mathcal{E}}(Tr_{\mathcal{E}}(V_{\mathcal{E}}\rho_{\mathcal{E}})) \\ &= Tr_{\mathcal{E}}(L_{\mathcal{SE}}\rho_{\mathcal{E}})\rho_{\mathcal{S}}. \end{aligned}$$

The analysis would go through also when $H_{\mathcal{S}\mathcal{E}}$ is a sum of factorised terms. Similarly on using 8 and 9

$$Tr_{\mathcal{E}}\left(PLe^{QLt}Q\rho\left(0\right)\right) = Tr_{\mathcal{E}}\left(L_{\mathcal{S}\mathcal{E}}e^{QLt}Q\rho\left(0\right)\right)$$

and

$$Tr_{\mathcal{E}}\left(PLe^{QLt'}QLP\rho\left(t-t'\right)\right) = Tr_{\mathcal{E}}\left(L_{\mathcal{S}\mathcal{E}}e^{QLt'}QL\rho_{S}\left(t-t'\right)\otimes\rho_{\mathcal{E}}\right).$$

In summary the master equation reduces to

$$\frac{\partial}{\partial t}\rho_{S}\left(t\right) = L_{\mathcal{S}}^{eff}\left[\rho_{S}\left(t\right)\right] + \int_{0}^{t}\mathcal{K}\left(t'\right)\left[\rho_{S}\left(t-t'\right)\right] + \mathcal{J}\left(t\right) \tag{10}$$

with

$$\begin{aligned} L_{\mathcal{S}}^{eff} &\equiv L_{S} + Tr_{\mathcal{E}}\left(L_{\mathcal{S}\mathcal{E}}\rho_{\mathcal{E}}\right), \\ \mathcal{K}\left(t\right)\left[\rho_{S}\right] &= Tr_{\mathcal{E}}\left(L_{\mathcal{S}\mathcal{E}}e^{QLt}QL\left[\rho_{S}\otimes\rho_{\mathcal{E}}\right]\right), \\ \mathcal{J}\left(t\right) &= Tr_{\mathcal{E}}\left(L_{\mathcal{S}\mathcal{E}}e^{QLt}Q\rho\left(0\right)\right). \end{aligned}$$

In general it is an integro-differential equation with a memory kernel $\mathcal{K}$. Since the evolution of ρ is unitary, the positivity of ρ is maintained. The partial trace $\rho_{S}\left(t\right)$ of the positive operator ρ preserves the positivity. (10) is exact and so guarantees a positive $\rho_{S}\left(t\right)$. It is only when approximations (truncations) are made that positivity may be lost. The Markov approximation occurs if there is a timescale $\tau_{\mathcal{E}}$ associated with $\mathcal{K}\left(t\right)$ which is much shorter than τ_{S} the natural time scale of the system $\mathcal{S}$ i.e. $\frac{\tau_{\mathcal{S}}}{\tau_{\mathcal{E}}} \longrightarrow \infty$. This Markov approximation has to be done carefully for otherwise positivity can be lost [12]. Mathematically there is another singular solution of this limit, $\tau_{S} \longrightarrow \infty$ with $\tau_{\mathcal{E}}$ finite [13] which leads to the phenomenonology of dynamical semi-groups and the Lindblad formalism [14].

Definition 1 *Time evolutions Λ_t with $t \geq 0$ form a dynamical semi-group if a) $\Lambda_{t_1}\circ\Lambda_{t_2} = \Lambda_{t_1+t_2}$, b) $Tr\left[\Lambda_t\rho\right] = Tr\left[\rho\right]$ for all t and ρ and c) are positive i.e map positive operators into positive operators.*

There are other technical conditions such as strong continuity which we will not dwell on. As far as applications are concerned the most important characterisation of dynamical semi-groups is that they arise from the singular limit mentioned above and are governed by the following theorem due to Lindblad:

Theorem 2 *If $P(\mathfrak{H})$ denotes the states on a Hilbert space $\mathfrak{H}$, and L is a bounded linear operator which is the generator of a dynamical semi-group (i.e. $\Lambda_t = e^{Lt}$), then*

$$L[\rho] = -i[H,\rho] + \frac{1}{2}\sum_j \left(\left[V_j\rho, V_j^\dagger \right] + \left[V_j, \rho V_j^\dagger \right] \right)$$

where $H\left(= H^\dagger\right)$, V_j and $\sum_j V_j^\dagger V_j$ are bounded linear operators on $\mathfrak{H}$.

This is the Lindblad form which has been used extensively in high energy physics phenomenology. $L[\rho]$, in the absence of the terms involving the Vs, is the Liouville operator. H is the hamiltonian which generally could be in the presence of a background stochastic classical metric [15] (as we will discuss later). Such effects may generally arise from back-reaction of matter within a quantum theory of gravity [16] which decoheres the gravitational state to give a stochastic ensemble description. In phenomenological analyses a theorem due to Gorini, Kossakaowski and Sudarshan [17] on the structure of L, the generator of a quantum dynamical semi-group [14, 17] is of importance. This states that for a non-negative matrix c_{kl} (i.e. a matrix with non-negative eigenvalues) such a generator is given by

$$\frac{d\rho}{dt} = \mathcal{L}[\rho] = -i[H,\rho] + \frac{1}{2}\sum_{k,l} c_{kl}\left([F_k\rho, F_l^\dagger] + [F_k, \rho F_l^\dagger] \right),$$

where $H = H^\dagger$ is a hermitian Hamiltonian, $\{F_k, k = 0, ..., n^2-1\}$ is a basis in $M_n(\mathbf{C})$ such that $F_0 = \frac{1}{\sqrt{n}} I_n$, $\mathrm{Tr}(F_k) = 0\ \forall k \neq 0$ and $\mathrm{Tr}(F_i^\dagger F_j) = \delta_{ij}$ [17]. In applications we can take $F_i = \frac{\Lambda_i}{2}$ (where, for example, Λ_i are the Gell-Mann matrices) and satisfy the Lie algebra $[F_i, F_j] = i\sum_k f_{ijk}F_k, (i = 1, ...8)$, f_{ijk} being the standard structure constants, antisymmetric in all indices. It can always be arranged that the sum over k and l run over $1, \ldots, 8$. Without a microscopic model, in the three generation case, the precise physical significance of the matrix c_{kl} cannot be understood. Moreover a general parametrisation of c_{kl} is too complicated to have any predictive power.

It is precise in formulation but gives no inkling of its $\mathcal{SE}$ compound system progenitor. Therein lies its weakness [18] but nonetheless it has been useful in providing 'test' theories and estimating orders of magnitudes for the strength of effects. If the strength of effects are in accord with a theoretical picture then it has been customary to conclude that the source of the decoherence is compatible with the theoretical picture. Recently it has been argued that this may be too simplistic and it is necessary to delve into the background $\mathcal{SE}$ to be able to argue in favour of a picture.

4 Master Equations from (Non-critical) String Theory

When neutrinos from the Sun are produced (e.g. from the nuclear $p-p$ cycle) and pass through it, the nature of $\mathcal{E}$ and $L_{\mathcal{SE}}$ can be understood from the gauge theories of the weak interactions [19]. Consequently the programme outlined in the previous paragraph with a perturbative evaluation of $\mathcal{K}(t)$ is feasible in principle. However in recent years there has been a debate on whether microscopic black holes can induce quantum decoherence at a microscopic level. The presence of quantum-fluctuating microscopic horizons, of radius of the order of Planck length (10^{-35} m), may give space-time a "foamy" structure, causing decoherence of matter propagating in it. In particular, it has been suggested [20] that such Planck-scale black holes and other topological fluctuations in the space-time background cause a breakdown of the conventional S-matrix description of asymptotic scattering in local quantum field theory.Hence when we consider space-time foam we are on less firm ground for applying the Lindblad formalism. Clearly gleaning an understanding of the nature of space-time itself raises a huge number of foundational issues. String theory is one attempt to address such questions but is still far from the goal of clarifying strong gravity. There are some who even believe that gravity is an emergent feature and consequently that an attempt to understand the quantum aspects of gravity may be fundamentally futile. It is not appropriate to enter this debate here. As far as experiments are concerned, both now and in the near future, it is reasonable to ask what the current theories have to say concerning quantum effects where a nearly flat metric gravity is clearly reasonable.

The issue of quantum-gravity-induced decoherence is controversial and worthy of further phenomenological exploitation. We shall restrict ourselves to a specific framework for analyzing decoherent propagation of low-energy matter in foamy space-time backgrounds in the context of string theory [21, 22], the so-called Liouville-string [23] decoherence [24]. One motivation for using string theory is that it appears to be the best controlled theory of quantum gravity available to date. At this juncture we should also mention that there are other interesting approaches to quantum space-time foam, which also lead to experimental predictions, e.g. the "thermal bath" approach advocated in [25], according to which the foamy gravitational environment may behave as a thermal bath; this induces decoherence and diffusion in the propagating matter, as well as quantum damping in the evolution of low-energy observables, features which are, at least in principle, testable experimentally. As we shall see presently, similar behaviour is exhibited by the specific models of foam that we study here; the D-particle foam model of [26, 27] may characterize modern versions of string theory [22], and are based on point-like membrane defects in space-time (D-particles). Such considerations have more recently again come to the fore because of current neutrino data including LSND data [28]. There is experimental evidence, that the neutrino has mass which leads to neutrino oscillations. However LSND results appear consistent with the dominance of anti-neutrino oscillations $\overline{\nu}_e \rightleftarrows \overline{\nu}_\mu$ over neutrino oscillations. In particular, provided LSND results turn out to be correct, which at present is quite unclear, there is evidence for CPT violation. It has been suggested recently [5] that Planck scale quantum decoherence may be a relevant contribution to the CPT violation seen in the experiments of LSND. Other examples of flavour oscillating systems with quite different mass scales are furnished by $B\overline{B}$ and $K\overline{K}$ systems [29]. The former because of the large masses involved provides a particularly sensitive system for investigating the Planck scale fluctuations embodied by space-time foam. In all these cases, experiments, such as CPLEAR [30], provide very low bounds on CPT violation which are not inconsistent with estimates from dimensional analysis for the magnitudes of effects from space-time foam. These systems have been analyzed within a dynamical semigroup approach to quantum Markov processes. Once the framework has been accepted then a master equation for finite-dimensional systems ensued which was characterized by a small set of parameters. This approach is somewhat phenomenological and is primarily used to fit data [31, 32, 33]. Consequently it is important to obtain a better understanding of the nature of decoherence from a more fundamental viewpoint.

Given the very limited understanding of gravity at the quantum level, the analysis of modifications of the quantum Liouville equation implied by non-critical strings can only be approximate and should be regarded as circumstantial evidence in favour of the dissipative master equation. In the context of two-dimensional toy black holes [34] and in the presence of singular space-time fluctuations there are believed to be inherently unobservable delocalised modes which fail to decouple from light (the observed) states. The effective theory of the light states which are measured by local scattering experiments can be described by a non-critical Liouville string. This results in an irreversible temporal evolution in target space with decoherence and associated entropy production.

The following master equation for the evolution of stringy low-energy matter in a non-conformal σ-model can be derived [24]

$$\partial_t \rho = i\,[\rho, H] + :\beta^i \mathcal{G}_{ij}\left[g^j, \rho\right]: \tag{11}$$

where t denotes time (Liouville zero mode), the H is the effective low-energy matter Hamiltonian, g^i are the quantum background target space fields, β^i are the corresponding renormalization group β functions for scaling under Liouville dressings and $\mathcal{G}_{ij}$ is the Zamolodchikov metric [35, 36] in the moduli space of the string. The double colon symbol in (11) represents the operator ordering $:AB:=[A,B]$. The index i labels the different background fields as well as space-time. Hence the summation over i,j in (11) corresponds to a discrete summation as well as a covariant integration $\int d^{D+1}y\,\sqrt{-g}$ where y denotes a set of $(D+1)$-dimensional target space-time co-ordinates and D is the space-time dimensionality of the original non-critical string.

The discovery of new solitonic structures in superstring theory [22] has dramatically changed the understanding of target space structure. These new non-perturbative objects are known as D-branes and their inclusion leads to a scattering picture of space-time fluctuations. Heuristically, when low energy matter given by a closed (or open) string propagating in a $(D+1)$-dimensional space-time collides with a very massive D-particle embedded in this space-time, the D-particle recoils as a result. Since there are no rigid bodies in general relativity the recoil fluctuations of the brane and their effectively stochastic back-reaction on space-time cannot be neglected. On the brane there are closed and open strings propagating. Each time these strings cross with a D-particle, there is a possibility of being attached to it, as indicated in Fig. 1. The entangled state causes a back reaction onto the space-time, which can be calculated perturbatively using logarithmic conformal field theory formalism [37].

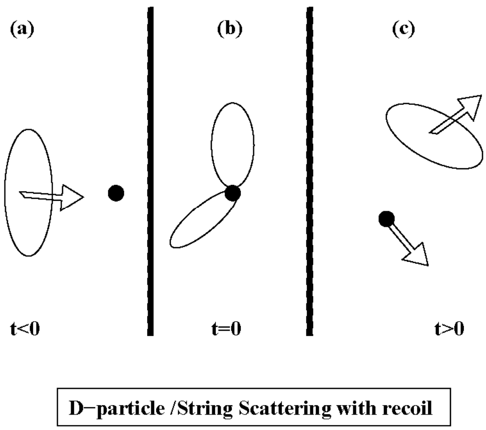


Figure 1: *Schematic picture of the scattering of a string matter state on a D-particle, including recoil of the latter. The sudden impulse at* $t = 0$*, implies a back reaction onto the space time, which is described by a logarithmic conformal field theory. The method allows for the perturbative calculation of the induced space-time distortion due to the entangled state in (b).*

Now for large Minkowski time t, the non-trivial changes from the flat metric produced from D-particle collisions are

$$g_{0i} \simeq \overline{u}_i \equiv \frac{u_i}{\varepsilon} \propto \frac{\Delta p_i}{M_P} \tag{12}$$

where u_i is the velocity and Δp_i is the momentum transfer during a collision, ε^{-2} is identified with t and M_P is the Planck mass (actually, to be more precise $M_P = M_s/g_s$, where $g_s < 1$ is the (weak) string coupling, and M_s is a string mass scale); so g_{0i} is constant in space-time but depends on the energy content of the low energy particle and the Ricci tensor $R_{MN} = 0$ where M and N are target space-time indices. Since we are interested in fluctuations of the metric the indices i will correspond to the pair M, N. However, recent astrophysical observations from different experiments all seem to indicate that 73% of the energy of the Universe is in the form of dark energy. Best fit models give the positive cosmological constant Einstein-Friedman Universe as a good candidate to explain these observations. For such de Sitter backgrounds $R_{MN} \propto \Omega g_{MN}$ with $\Omega > 0$ a cosmological constant. Also in a perturbative derivative expansion (in powers of α' where $\alpha' = l_s^2$ is the Regge slope of the string and l_s is the fundamental string length) in leading order

$$\beta_{\mu\nu} = \alpha' R_{\mu\nu} = \alpha' \Omega g_{\mu\nu} \tag{13}$$

and

$$\mathcal{G}_{ij} = \delta_{ij}. \tag{14}$$

This leads to

$$\partial_t \rho = i\,[\rho, H] + \ \alpha'\Omega : g_{MN}\left[g^{MN}, \rho\right] : \tag{15}$$

For a weak-graviton expansion about flat space-time, $g_{MN} = \eta_{MN} + h_{MN}$, and

$$h_{0i} \propto \frac{\Delta p_i}{M_P}. \tag{16}$$

If an antisymmetric ordering prescription is used, then the master equation for low energy string matter assumes the form

$$\partial_t \dot{\rho}_{Matter} = i\,[\rho_{Matter}, H] - \ \Omega\left[h_{0j}, \left[h^{0j}, \rho_{Matter}\right]\right] \tag{17}$$

(when α' is absorbed into Ω). In view of the previous discussion this can be rewritten as

$$\partial_t \dot{\rho}_{Matter} = i\,[\rho_{Matter}, H] - \ \Omega\left[\overline{u}_j, \left[\overline{u}^j, \rho_{Matter}\right]\right] \ . \tag{18}$$

thereby giving the *master equation for Liouville decoherence* in the model of a D-particle foam with a cosmological constant.

The above D-particle inspired approach deals with possible non-perturbativ quantum effects of gravitational degrees of freedom. The analysis is distinct from the phenomenology of dynamical semigroups which does not embody specific properties of gravity. Indeed the phenomenology is sufficiently generic that other mechanisms of decoherence such as the MSW effect can be incorporated within the same framework. Consequently an analysis which is less generic and is related to the specific decoherence implied by non-critical strings is necessary.It is sufficient to study a massive non-relativistic particle propagating in one dimension to establish qualitative features of D-particle decoherence. The environment will be taken to consist of both gravitational and non-gravitational degrees of freedom; hence we will consider a generalisation of quantum Brownian motion for a particle which has additional interactions with D-particles. This will allow us to compare qualitatively the decoherence due to different environments.The non-gravitational degrees of freedom in the environment (in a thermal state) are conventionally modelled by a collection of harmonic oscillators with masses m_n, frequency ω_n and co-ordinate operator $\widehat{q}_n$ coupled to the particle co-ordinate $\widehat{x}$ by an interaction of the form $\sum_n g_n \widehat{x}\widehat{q}_n$. The master equation which is derived can have time dependent coefficients due to the competing timescales, e.g. relaxation rate due to coupling to the thermal bath, the ratio of the time scale of the harmonic oscillator to the thermal time scale etc. However an ab initio calculation of the time-dependence is difficult to do in a rigorous manner. It is customary to characterise the non-gravitational environment by means of its spectral density $I(\omega)\left(=\sum_n \delta(\omega-\omega_n)\frac{g_n^2}{2m_n\omega_n}\right)$. The existence of the different time scales leads in general to non-trivial time dependences in the coefficients in the master equation which are difficult to calculate in a rigorous manner [38]. The dissipative term in (18) involves the momentum transfer operator due to recoil of the particle from collisions with D-particles (12). This transfer process will be modelled by a classical Gaussian random variable r which multiplies the momentum operator $\widehat{p}$ for the particle:

$$\overline{u_i} \qquad \rightarrow \qquad \frac{r}{M_P}\widehat{p} \tag{19}$$

Moreover the mean and variance of r are given by

$$\langle r\rangle = 0 \ , \qquad \text{and} \qquad \left\langle r^2\right\rangle = \sigma^2 \ . \tag{20}$$

On amalgamating the effects of the thermal and D-particle environments, we have for the reduced master equation [39] for the matter (particle) density matrix ρ (on dropping the Matter index)

$$i\frac{\partial}{\partial t}\rho = \frac{1}{2m}\left[\widehat{p}^2,\rho\right] - i\Lambda\left[\widehat{x},\left[\widehat{x},\rho\right]\right] + \frac{\gamma}{2}\left[\widehat{x},\left\{\widehat{p},\rho\right\}\right] - i\Omega r^2\left[\widehat{p},\left[\widehat{p},\rho\right]\right] \tag{21}$$

where Λ,γ and Ω are real time-dependent coefficients. As discussed in [39] a possible model for $\Omega\left(t\right)$ is

$$\Omega\left(t\right) = \Omega_0 + \frac{\widetilde{\gamma}}{a+t} + \frac{\widetilde{\Gamma}}{1+bt^2} \tag{22}$$

where ω_0, $\widetilde{\gamma}$, a, $\widetilde{\Gamma}$ and b are positive constants. The quantity $\widetilde{\gamma} < 1$ contains information on the density of D-particle defects on a four-dimensional world.The time dependence of γ and Λ can be calculated in the weak coupling limit for general n (i.e. ohmic, $n = 1$ and non-ohmic $n \neq 1$ environments) where

$$I\left(\omega\right) = \frac{2}{\pi}m\gamma_0\omega\left[\frac{\omega}{\varpi}\right]^{n-1} e^{-\omega^2/\varpi^2} \tag{23}$$

and ϖ is a cut-off frequency. The precise time dependence is governed by $\Lambda\left(t\right) - \int_0^t ds\,\nu\left(s\right)$ and $\gamma\left(t\right) - \int_0^t ds\,\nu\left(s\right)s$ where $\nu\left(s\right) - \int_0^\infty d\omega\, I\left(\omega\right)\coth\left(\beta\hbar\omega/2\right.$ For the ohmic case, in the limit $\hbar\varpi \ll k_B T$ followed by $\varpi \to \infty$, Λ and γ are given by $m\gamma_0 k_B T$ and γ_0 respectively after a rapid initial transient. For high temperatures Λ and γ have a powerlaw increase with t for the subohmic case whereas there is a rapid decrease in the supraohmic case.

5 CPT and Recoil

The above model of space-time foam refers to a specific string-inspired construction. However the form of the induced back reaction (12) onto the space-time has some generic features, and can be understood more generally in the context of effective theories of such models, which allows one to go beyond a specific non-critical (Liouville) model. Indeed, the D-particle defect can be viewed as an idealisation of some (virtual, quantum) black hole defect of the ground state of quantum gravity, viewed as a membrane wrapped around some small extra dimensions of the (stringy) space time, and thus appearing to a four-dimensional observer as an "effectively" point like defect. The back reaction on space-time due to the interaction of a pair of neutral mesons, such as those produced in a meson factory, with such defects can be studied generically as follows: consider the non-relativistic recoil motion of the heavy defect, whose coordinates in space-time,in the laboratory frame, are $y^i = y^i_0 + u^i t$, with u^i the (small) recoil velocity.One can then perform a (infinitesimal) general coordinate transformation $y^\mu \to x^\mu + \xi^\nu$ so as to go to the rest (or co-moving) frame of the defect after the scattering. From a passive point of view, for one of the mesons, this corresponds to an induced change in metric of space-time of the form (in the usual notation, where the parenthesis in indices denote symmetrisation) $\delta g_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}$, which in the specific case of non-relativistic defect motion yields the off-diagonal metric elements (12). Such transformations cannot be performed simultaneously for both mesons, and moreover in a full theory of quantum gravity the recoil velocities fluctuate randomly, as we shall discuss later on. This means that the effects of the recoil of the space-time defect are observable. The mesons will feel such effects in the form of induced fluctuating metrics (12). It is crucial to note that the interaction of the matter particle (meson) with the foam defect may also result in a "flavour" change of the particle (e.g. the change of a neutral meson to its antiparticle). This feature can be understood in a D-particle Liouville model by noting that the scattering of the matter probe off the defect involves first a splitting of a closed string representing matter into two open ones, but with their ends attached to the D-particle, and then a joining of the string ends in order to re-emit a closed string matter state. The re-emitted (scattered) state may in general be characterised by phase, flavour and other quantum charges which may not be required to be conserved during black hole evaporation and disparate space-time-foam processes. In our application we shall restrict ourselves only to effects that lead to flavour changes. The modified form of the metric fluctuations (12) of each component of the metric tensor $g^{\alpha\beta}$ will not be simply given by the simple recoil distortion (12), but instead can be taken to have a 2×2 ("flavour") structure [4]:

$$
\begin{aligned}
g^{00} &= (-1 + r_4)\,\mathsf{1} \\
g^{01} &= g^{10} = r_0 \mathsf{1} + r_1\sigma_1 + r_2\sigma_2 + r_3\sigma_3 \\
g^{11} &= (1 + r_5)\,\mathsf{1}
\end{aligned}
\qquad (24)
$$

where $\mathsf{1}$, is the identity and σ_i are the Pauli matrices. The above parametrisation has been taken for simplicity and we can also consider motion to be in the x- direction which is natural since the meson pairs move collinearly. A metric with this type of structure is compatible with the view that the D-particle defect is a “point-like” approximation for a compactified higher-dimensional brany black hole, whose no hair theorems permit non-conservation of flavour.(In the case of neutral mesons the concept of “flavour” refers to either particle/antiparticle species or the two mass eigenstates). The detailed application of this model to the ω effect for neutral mesons can be found in [4].

Acknowledgements

I would like to thank A. Di Domenico for the invitation to contribute to the handbook on neutral kaon interferometry at a ϕ-factory. Discussions with N E Mavromatos, J Bernabeu and A Waldron-Lauda are gratefully acknowledged.

References

1. T. Ohlsson, Phys. Lett. B **502**, 159 (2001) [arXiv:hep-ph/0012272]; M. Blennow, T. Ohlsson and W. Winter, arXiv:hep-ph/0502147
2. http://antares.in2p3.fr; http://icecube.wisc.edu
3. J. Bernabeu, J. Ellis, N. E. Mavromatos, D. V. Nanapoulos and J. Papavassiliou, CPT and Quantum Mechanics Tests with Kaons, this handbook [arXiv:hep-ph/0607322].
4. J. Bernabeu, N E Mavromatos and S Sarkar, Phys. Rev. D**74**, 045014 (2006)
5. G. Barenboim and N. E. Mavromatos, JHEP **0501**, 034 (2005) [arXiv:hep-ph/0404014].
6. D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu and H. D. Zeh, Decoherence and the Appearance of a Classical World in Quantum Theory, Springer-Verlag 1996
7. D. Page, Phys. Rev. Lett. **44** 301 (1980).
8. O. W. Greenberg, Why is CPT fundamental?, hep-ph 0309309.

9. R. M. Wald, Phys. Rev. D **21**, 2742 (1980).

10. E. R. Pike and S. Sarkar, The Quantum Theory of Radiation, Oxford Univ Press 1995

11. S. Nakajima, Prog. Theor. Phys. **20** 948 (1958); R. Zwanzig, Lect. in Theor. Phys. **3** 106 (1960) (Interscience , New York).

12. E. B. Davies, Quantum Theory of Open Systems, Academic Press 1976

13. V. Gorini and A. Kossakowski, J. Math. Phys. **17** 1298 (1976)

14. G. Lindblad, Commun. Math. Phys. **48**, 119 (1976); R. Alicki and K. Lendi, Lect. Notes Phys. **286** (Springer Verlag, Berlin (1987)).

15. N. E. Mavromatos and S. Sarkar, Phys.Rev. D **74** 036007 (2006).

16. B. L. Hu and E. Verdaguer, Class. Quant. Grav. **20** R1 (2003)

17. V. Gorini, A. Kossakowski and E.C.G. Sudarshan, J. Math. Phys. **17** 821 (1976)

18. G. Barenboim, N.E. Mavromatos, S. Sarkar and A. Waldron-Lauda, Quantum Decoherence and Neutrino Data, Nucl. Phys. B to be published (hep-ph/0603028).

19. C. P. Burgess and D. Michaud, Ann. Phys. **256** 1 (1997)

20. S. W. Hawking, Commun. Math. Phys. **87**, 395 (1982).

21. M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vols. 1, 2" (Cambridge Univ. Press 1987).

22. J. Polchinski, "String theory. Vol. 2: Superstring theory and beyond,"

23. F. David, Mod. Phys. Lett. A **3**, 1651 (1988); J. Distler and H. Kawai, Nucl. Phys. B **321**, 509 (1989); see also: N. E. Mavromatos and J. L. Miramontes, Mod. Phys. Lett. A **4**, 1847 (1989); E. D'Hoker and P. S. Kurzepa, Mod. Phys. Lett. A **5**, 1411 (1990); for earlier works see: E. D'Hoker and R. Jackiw, Phys. Rev. D **26**, 3517 (1982), and references therein.

24. J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B **293**, 37 (1992) [arXiv:hep-th/9207103]; arXiv:hep-th/9805120.

25. L. J. Garay, Int. J. Mod. Phys. A **14**, 4079 (1999) [arXiv:gr-qc/9911002]; Phys. Rev. D **58**, 124015 (1998) [arXiv:gr-qc/9806047]; Phys. Rev. Lett. **80**, 2508 (1998) [arXiv:gr-qc/9801024].

26. J. R. Ellis, N. E. Mavromatos and M. Westmuckett, Phys. Rev. D **70**, 044036 (2004) [arXiv:gr-qc/0405066];Phys. Rev. D **71**, 106006 (2005) [arXiv:gr-qc/0501060] ;

27. J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Rev. D **62**, 084019 (2000) [gr-qc/0006004], and references therein.

28. for a recent review see: P. Langacker, arXiv:hep-ph/0411116, and references therein.

29. J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B **293**, 142 (1992) [arXiv:hep-ph/9207268]; J. R. Ellis, J. L. Lopez, N. E. Mavromatos and D. V. Nanopoulos, Phys. Rev. D **53**, 3846 (1996) [arXiv:hep-ph/9505340]; P. Huet and M. E. Peskin, Nucl. Phys. B **434**, 3 (1995) [arXiv:hep-ph/9403257]; F. Benatti and R. Floreanini, Phys. Lett. B **468**, 287 (1999) [arXiv:hep-ph/9910508]; Nucl. Phys. B **511**, 550 (1998) [arXiv:hep-ph/9711240]. J. Bernabeu, N. E. Mavromatos and J. Papavassiliou, Phys. Rev. Lett. **92**, 131601 (2004) [arXiv:hep-ph/0310180]; J. Bernabeu, N. E. Mavromatos, J. Papavassiliou and A. Waldron-Lauda, Nucl. Phys. B **744**, 180 (2006).

30. R. Adler *et al.* [CPLEAR collaboration], Phys. Lett. B **364**, 239 (1995) [arXiv:hep-ex/9511001].

31. J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos and M. Srednicki, Nucl. Phys. B **241**, 381 (1984).

32. E. Lisi, A. Marrone and D. Montanino, Phys. Rev. Lett. **85**, 1166 (2000) [arXiv:hep-ph/0002053].

33. F. Benatti and R. Floreanini, Phys. Rev. D **64**, 085015 (2001) [arXiv:hep-ph/0105303].

34. E. Witten, arXiv:hep-th/9206069; Phys. Rev. **D44**, 44 (1991);

35. A. B. Zamolodchikov, JETP Lett. **43** (1986) 730 [Pisma Zh. Eksp. Teor. Fiz. **43** (1986) 565].

36. D. Kutasov, Mod. Phys. Lett. A **7**, 2943 (1992) [arXiv:hep-th/9207064].

37. I. I. Kogan, N. E. Mavromatos and J. F. Wheater, Phys. Lett. B **387**, 483 (1996) [arXiv:hep-th/9606102]; For a review focusing on D-brane recoil, including supermembranes, see: N. E. Mavromatos, arXiv:hep-th/0407026, in *Shifman, M. (ed.) et al.: From fields to strings, I. Kogan memorial Volume 2*, 1257-1364. (World Sci. 2005), and references therein.

38. B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. D **45** 2843 (1992)

39. N. E. Mavromatos and S. Sarkar, Phys.Rev. D **72** 065016 (2005).

OPEN QUANTUM DYNAMICS: COMPLETE POSITIVITY AND CORRELATED NEUTRAL KAONS

F. Benatti
Dipartimento di Fisica Teorica, Università di Trieste,
and
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Trieste, Italy

R. Floreanini
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Trieste, Italy

Abstract

Neutral kaons can be treated as open systems, *i.e.* as subsystems immersed in an external environment, generated either by the fundamental dynamics of extended objects (strings and branes), or by matter fluctuations in a medium. New, non-standard phenomena are induced at low energies, producing irreversibility and dissipation, whose physical description requires however some care. Meson factories are suitable interferometric set-ups where these new effects can be experimentally studied with great accuracy.

1 Introduction

Standard quantum mechanics usually deals with closed physical systems, *i.e.* with systems that can be considered isolated from any external environment. The time-evolution of such systems is described by one-parameter groups of unitary operators, $U(t) = e^{-iHt}$, generated by the system hamiltonian H; they embody the reversible character of the dynamics.

This description is however just an approximation, valid when the action of the external world on the system under study can be considered vanishingly small. On the contrary, when a system $\mathcal{S}$ interacts with an environment $\mathcal{E}$ in a non-negligible way, it must be treated as an open quantum system, namely as a subsystem embedded within $\mathcal{E}$, exchanging with it energy and entropy, and whose time-evolution is irreversible.[1] Being closed, the total system $\mathcal{S} + \mathcal{E}$

[1]The literature on the theory of open quantum systems and their phenomenological applications is large. General reviews and monographs on the topic can be found in Refs.[1-17].

evolves in time with the unitary dynamics generated by the total hamiltonian $H_{\rm tot}$, that can always be written as:

$$H_{\rm tot} = H_{\mathcal{S}} + H_{\mathcal{E}} + H' \ . \tag{1}$$

In this decomposition, $H_{\mathcal{S}}$ is the hamiltonian driving the dynamics of $\mathcal{S}$ in absence of the environment, $H_{\mathcal{E}}$ describes the internal evolution of $\mathcal{E}$, while H' takes into account the interaction between subsystem and environment.

In many instances, one is interested in the dynamics of the subsystem $\mathcal{S}$ alone and not in the details of the $\mathcal{E}$ motion; one can then eliminate (*i.e.* sum over) the environment degrees of freedom. The resulting time-evolution for $\mathcal{S}$ turns out to be rather involved: it can not anymore be described in terms of a unitary evolution. Indeed, representing the states of $\mathcal{S}$ in terms of density matrices, the map transforming the initial state $\rho(0)$ into the final one $\rho(t)$ is given by:

$$\rho(0) \equiv {\rm Tr}_{\mathcal{E}}\left[\rho_{\mathcal{S}+\mathcal{E}}\right] \mapsto \rho(t) \equiv {\rm Tr}_{\mathcal{E}}\left[e^{-iH_{\rm tot}\,t}\,\rho_{\mathcal{S}+\mathcal{E}}\,e^{iH_{\rm tot}\,t}\right] \ , \tag{2}$$

where $\rho_{\mathcal{S}+\mathcal{E}}$ is the density matrix representing the initial state of the total system, while ${\rm Tr}_{\mathcal{E}}$ constitutes the trace operation over the environment degrees of freedom.[2] Due to the exchange of energy as well as entropy between $\mathcal{S}$ and $\mathcal{E}$, the evolution map $\rho(0) \rightarrow \rho(t)$ gives rise in general to nonlinearities and memory effects, and can not be described in closed form.

Nevertheless, the situation greatly simplifies when the interaction between subsystem and environment can be considered to be weak. In this case, physically plausible approximations lead to reduced dynamics $\rho(0) \rightarrow \rho(t) \equiv \gamma_t[\rho(0)]$ that involve only the $\mathcal{S}$ degrees of freedom: they are represented by linear maps γ_t, that are generated by *master equations*. Such reduced dynamics provides an effective description of how $\mathcal{E}$ affects the time-evolution of $\mathcal{S}$, and typically gives rise to dissipative and noisy effects.[1) – 9)]

However, not all time-dependent linear maps γ_t can represent suitable reduced dynamics: very basic physical requirements need to be satisfied. Although the dynamics is no longer reversible, forward in time composition should

[2] In absence of initial correlations between $\mathcal{S}$ and $\mathcal{E}$, a situation commonly encountered in many physical applications, it can be written in factrorized form: $\rho_{\mathcal{S}+\mathcal{E}} = \rho(0) \otimes \rho_{\mathcal{E}}$, where $\rho_{\mathcal{E}}$ is the density matrix representing the state of the environment.

be guaranteed: $\gamma_s \circ \gamma_t = \gamma_{t+s}$, for all positive times s, t; the one-parameter ($\equiv$ time) family of maps $\{\gamma_t\}$ is then said to be a semigroup. Further, γ_t should preserve probability and positivity.[1) – 7)] Indeed, in order to represent a physical state of the subsystem $\mathcal{S}$, a density matrix ρ must be a positive operator, since its eigenvalues have the meaning of probabilities; this is at the root of the statistical interpretation of quantum mechanics. The time evolution $\rho(0) \mapsto \rho(t) = \gamma_t[\rho(0)]$ must then preserve this fundamental property, and therefore map a positive initial $\rho(0)$ into a positive final $\rho(t)$. Such a property of the linear transformation γ_t is called *positivity.*

Apparently, positivity seems sufficient to assure the physical consistency of the reduced dynamics. In reality, the structure of quantum mechanics requires a more stringent requirement to be satisfied, that of *complete positivity.* [1) – 8)] This property guarantees the positivity not only of γ_t but also of the dynamics of a larger system built with two equal, mutually non interacting systems $\mathcal{S}$, immersed in the same external environment; as we shall see, such a situation is precisely that of correlated neutral kaons coming from the decay of a ϕ meson. The dynamics of this enlarged system is then described by $\Gamma_t = \gamma_t \otimes \gamma_t$. Positivity of Γ_t means complete positivity of the map γ_t. [18)] It is important to note that this property is intimately related to entanglement, *i.e.* to the possibility that the initial state of the compound subsystem $\mathcal{S} + \mathcal{S}$ exihibits quantum correlations.[6), 19)]

A family of one-parameter linear maps γ_t that satisfy all the above mentioned properties, including complete positivity, forms a so-called *quantum dynamical semigroup.*[1) – 8)] In the regime of weak coupling between subsystem and environment, they represent the most general realization of a dissipative reduced dynamics compatible with the probabilistic interpretation of quantum mechanics.[20) – 26), 6)]

This description of the open quantum systems turns out to be very general. Although originally developed in the framework of quantum optics, [10) – 16)] it has been successfully applied to model very different situations, in atomic and molecular physics, quantum chemistry, solid state physics. [1) – 9), 17)] Further, it has been recently applied to the study of irreversibility and dissipation in the evolution of various particle systems, involving atoms, [27)] neutrons, [28), 29)] photons, [30) – 33)] neutrinos [34) – 36)] and in particular neutral mesons. [37) – 52)]

In standard treatments, these systems are usually considered as closed; once more, this is justified only in an idealized situation. Indeed, quantum gravity effects at Planck's scale or more in general, the dynamics of fundamental, extended objects (strings and branes) are expected to act as an effective environment, inducing non-standard, dissipative effects at low energies.[53) – 56), 44)] Similar effects are also produced when neutral kaons or neutrinos travel inside a fluctuating matter medium; in this case, due to the interactions between these elementary particles and the scattering matter centers, the medium plays the role of an environment producing noise and decoherence.[36), 52)]

These new phenomena are nevertheless expected to be very small; they are suppressed by at least one inverse power of the Planck mass, in the case of gravitational or stringy effects, while in presence of matter fluctuations they appear to be second order with respect to ordinary regeneration or oscillation effects. In spite of this, interesting bounds on some of the constants parametrizing the dissipative effects have been already obtained using available experimental data, and improvements are expected in the future.

In this respect, dedicated neutral meson experiments at meson factories, appear to be particularly promising. Indeed, as we shall see in the following, suitable neutral meson observables turn out to be particularly sensitive to the new, dissipative phenomena, so that their presence can be experimentally probed quite independently from other, non-standard effects.

2 Positivity and complete positivity

The evolution and decay of the neutral kaon system can be effectively modeled by means of a two-dimensional Hilbert space.[57) – 60)] A kaon state is then described by means of a 2×2 density matrix ρ, *i.e.* a positive hermitian operator (with real, nonnegative eigenvalues) and constant trace.

The evolution in time of the kaon system can then be formulated in terms of a linear *master equation* for ρ; it takes the general form:[1) – 7)]

$$\frac{\partial \rho(t)}{\partial t} = -iH_{\rm eff}\ \rho(t) + i\rho(t)\, H_{\rm eff}^{\dagger} + L[\rho(t)]\ . \tag{3}$$

The first two terms on the r.h.s. of this equation are the standard quantum mechanical ones: they contain the effective hamiltonian $H_{\rm eff} = M - i\Gamma/2$, which includes a non-hermitian part, characterizing the natural width of the kaon states.

The entries of this matrix can be expressed in terms of the complex parameters ϵ_S, ϵ_L, appearing in the eigenstates of $H_{\rm eff}$,

$$|K_S\rangle = \frac{1}{(1+|\epsilon_S|^2)^{1/2}}\big(|K_1\rangle + \epsilon_S|K_2\rangle\big)\ , \quad |K_L\rangle = \frac{1}{(1+|\epsilon_L|^2)^{1/2}}\big(\epsilon_L|K_1\rangle + |K_2\rangle\big)\ , \tag{4}$$

with $|K_{1,2}\rangle = (|K^0\rangle \pm |\overline{K^0}\rangle)/\sqrt{2}$, and the four real parameters m_S, γ_S and m_L, γ_L, the masses and widths of the states in (4), characterizing the eigenvalues of $H_{\rm eff}$: $\lambda_S = m_S - \frac{i}{2}\gamma_S$, $\lambda_L = m_L - \frac{i}{2}\gamma_L$. For later use, we introduce the following positive combinations: $\Delta\Gamma = \gamma_S - \gamma_L$, $\Delta m = m_L - m_S$, as well as the complex quantities $\Gamma_\pm = \Gamma \pm i\Delta m$ and $\Delta\Gamma_\pm = \Delta\Gamma \pm 2i\Delta m$, with $\Gamma = (\gamma_S + \gamma_L)/2$. One easily checks that CPT-invariance is broken when $\epsilon_S \neq \epsilon_L$, while a nonvanishing $\epsilon_S = \epsilon_L$ implies violation of CP symmetry.

On the other hand, the additional piece $L[\rho]$ in the evolution equation (3) encodes effects leading to dissipation and irreversibility: these are non-standard phenomena that in general give rise to further violations of CP and CPT symmetries.[61), 51)]

It should be stressed that in absence of the piece $L[\rho]$, pure states (*i.e.* states of the form $|\psi\rangle\langle\psi|$) are transformed by the evolution equation (3) back into pure states, even though probability is not conserved, a direct consequence of the presence of a non-hermitian part in the effective hamiltonian $H_{\rm eff}$. Only when the extra piece $L[\rho]$ is also present, $\rho(t)$ becomes less ordered in time due to a mixing-enhancing mechanism, producing possible loss of quantum coherence.

The explicit form of the linear map $L[\rho]$ can be uniquely fixed by taking into account the basic physical requirements that the complete time evolution, $\gamma_t : \rho(0) \mapsto \rho(t)$, generated by (3) needs to satisfy. As mentioned in the introductory remarks, in order to represent a physically consistent dynamical evolution, the one-parameter family of maps γ_t should obey the semigroup composition law, $\gamma_t[\rho(s)] = \rho(t+s)$, for t, $s \geq 0$, while transforming density matrices into density matrices, in particular preserving their positivity.

One can show that the semigroup request fixes $L[\rho]$ to be of Kossakowski-Lindblad form: [20)]

$$L[\rho] = \sum_{i,j=1}^{3} C_{ij}\Big[\sigma_j\rho\,\sigma_i - \frac{1}{2}\Big\{\sigma_i\sigma_j\ ,\ \rho\Big\}\Big]\ , \tag{5}$$

where σ_i, $i = 1,2,3$ are the Pauli matrices, while $[C_{ij}]$ is a 3×3 matrix, called the *Kossakowski matrix.* We shall consider dissipative evolutions for which the von Neumann entropy, $S = -\mathrm{Tr}[\rho \ln \rho]$, is increasing; this is a condition that is very well satisfied in usual phenomenological treatments of open systems consisting of elementary particles. In this case, the matrix $[C_{ij}]$ turns out to be real symmetric: its six entries parametrize the noise effects induced by the presence of the environment.[1), 6)]

The condition that the map γ_t generated by (3) preserve the positivity of the single kaon state ρ for all times gives further constraints on these real parameters. [29)] In order to explicitly show this, it is convenient to decompose the density matrix ρ along the Pauli matrices σ_j, $j = 1, 2, 3$, and the identity matrix σ_0, and represent ρ as a 4-dimensional ket-vector $|\rho\rangle$,

$$\rho = \begin{pmatrix} \rho_1 & \rho_3 \\ \rho_4 & \rho_2 \end{pmatrix} = \sum_{\mu=0}^{3} \rho^\mu \sigma_\mu \longmapsto |\rho\rangle = \begin{pmatrix} \rho^0 \\ \rho^1 \\ \rho^2 \\ \rho^3 \end{pmatrix} \tag{6}$$

$$\rho^0 = \frac{\rho_1 + \rho_2}{2}\ , \ \rho^1 = \frac{\rho_3 + \rho_4}{2}\ , \ \rho^2 = \frac{\rho_4 - \rho_3}{2i}\ , \ \rho^3 = \frac{\rho_1 - \rho_2}{2}\ . \tag{7}$$

Then, the action of the linear operator $L[\cdot]$ in (5) on the state ρ can be equivalently expressed as the action of a real symmetric 4×4 matrix $\big[L_{\mu\nu}\big]$ on the column vector $|\rho\rangle$. This matrix can be parametrized by six real constants a, b, c, α, β, and γ as follows:

$$\big[L_{\mu\nu}\big] = -2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & \alpha & \beta \\ 0 & c & \beta & \gamma \end{pmatrix}\ . \tag{8}$$

We know that any hamiltonian evolution preserves the positivity of the density matrices; then, one can limit the discussion to the contribution of the dissipative piece L only.[3] Since $\mathrm{Tr}(L[\rho]) = 0$, as easily checked from the expression in (5), we have $\rho^0(t) = \rho^0(0)$. Further, the positivity of the spectrum of ρ is preserved

[3] Let us indicate by ω_t the time-evolution generated by (3) in absence of the dissipative term L, and λ_t the one generated just by L in absence of the hamiltonian term. The complete evolution γ_t can then be expressed via the Lie-Trotter formula as: $\gamma_t = \lim_{n\to\infty}(\omega_{t/n} \circ \lambda_{t/n})^n$. [7)] Being the evolution ω_t positive, the positivity properties of γ_t are directly connected to those of λ_t.

at all times if and only if $\mathrm{Det}(\rho) = (\rho^0)^2 - \sum_{j=1}^3 (\rho^j)^2 \geq 0$. We now set $\rho \equiv \rho(0)$ and use

$$\frac{\mathrm{dDet}(\rho(t))}{\mathrm{d}t}\Big|_{t=0} = -2\sum_{ij=1}^{3} [L\,]_{ij}\, \rho^i \rho^j \ . \tag{9}$$

If $|\rho\rangle = (\rho^0, \rho^1, \rho^2, \rho^3)$ is a pure state, $\mathrm{Det}(\rho) = 0$ and the right hand side of (9) cannot be negative, otherwise a negative eigenvalue would appear for $t > 0$. By varying ρ^j while keeping $\sum_j (\rho^j)^2 = (\rho^0)^2$, from $\sum_{ij=1}^3 [L\,]_{ij}\, \rho^i \rho^j \geq 0$ one gets that the real symmetric submatrix $[L\,]_{ij}$ must necessarily be positive, therefore that the following inequalities must be fulfilled,

$$\begin{cases} a \geq 0 \\ \alpha \geq 0 \\ \gamma \geq 0 \end{cases} , \quad \begin{cases} a\alpha \geq b^2 \\ a\gamma \geq c^2 \\ \alpha\gamma \geq \beta^2 \end{cases} , \quad \mathrm{Det}([L_{ij}]) \geq 0 \ . \tag{10}$$

These conditions are also sufficient for preservation of positivity. In fact, since $-[L_{ij}] \geq 0$, we can write $-[L] = \mathcal{B}^2$, with $\mathcal{B}$ a symmetric 3×3 matrix. Then, the term in the right hand side of the equality in (9) is given by $\|\mathcal{B}|\rho\rangle\|^2$. Let us suppose $\mathrm{Det}[\rho(t')] < 0$, at time $t' > 0$; it follows that $\mathrm{Det}[\rho(t^*)] = 0$ at some time t^* such that $0 \leq t^* < t'$. Thus, $\mathcal{B}|\rho(t^*)\rangle = 0$, otherwise $\mathrm{Det}[\rho(t)] > 0$ for $t \geq t^*$; but this implies $|\dot\rho(t^*)\rangle = L\,|\rho(t^*)\rangle = -\mathcal{B}^2|\rho(t^*)\rangle = 0$. Therefore, for all $t > t^*$, $|\rho(t)\rangle = |\rho(t^*)\rangle$, and the dissipative dynamics generated by L is positivity-preserving.

Although the conditions (10) guarantee that the evolution $\rho(0) \mapsto \rho(t) \equiv \gamma_t[\rho(0)]$ of a single neutral kaon is physically consistent, more stringent constraints are needed in order to get a positive dissipative evolution when correlated kaons produced in a ϕ-meson decay are considered.[39),41),45)]

Since the ϕ-meson has spin 1, the two neutral spinless kaons produced in a ϕ-decay, and flying apart with opposite momenta in the meson ϕ rest-frame, are produced in an antisymmetric state:

$$|\psi_A\rangle = \frac{1}{\sqrt{2}}\Big(|K_1, -p\rangle \otimes |K_2, p\rangle - |K_2, -p\rangle \otimes |K_1, p\rangle\Big) \ . \tag{11}$$

Their corresponding density matrix $\rho_A \equiv |\psi_A\rangle\langle\psi_A|$ is antisymmetric in the spatial labels. By means of the projectors onto the CP eigenstates,

$$P_1 = |K_1\rangle\langle K_1| \ , \qquad P_2 = |K_2\rangle\langle K_2| \ , \tag{12}$$

and of the off-diagonal operators,

$$P_3 = |K_1\rangle\langle K_2| \ , \qquad P_4 = |K_2\rangle\langle K_1| \ , \tag{13}$$

we can write

$$\rho_A = \frac{1}{2}(P_1 \otimes P_2 + P_2 \otimes P_1 - P_3 \otimes P_4 - P_4 \otimes P_3) \ . \tag{14}$$

The time evolution of a system of two correlated neutral K-mesons, initially described by ρ_A, can be analyzed using the sigle K-meson dynamics so-far discussed. Indeed, as in standard quantum mechanics, it is natural to assume that, once produced in a ϕ decay, the kaons evolve in time each according to the map γ_t generated by (3), (5).[4]

Within this framework, the density matrix that describes a situation in which the first K-meson has evolved up to proper time t_1 and the second up to proper time t_2 is given by:

$$\begin{aligned}\rho_A(t_1,t_2) \equiv \big(\gamma_{t_1} \otimes \gamma_{t_2}\big)\big[\rho_A\big] = \frac{1}{2}\Big[&P_1(t_1) \otimes P_2(t_2) \\ &+ P_2(t_1) \otimes P_1(t_2) \ - P_3(t_1) \otimes P_4(t_2) - P_4(t_1) \otimes P_3(t_2)\Big]\end{aligned} \tag{15}$$

where $P_i(t_1)$ and $P_i(t_2)$, $i = 1, 2, 3, 4$, represent the evolution according to (3) of the initial operators (12), (13), up to the time t_1 and t_2, respectively. In the following, we shall set $t_1 = t_2 = t$, and simply call $\rho_A(t) \equiv \rho_A(t,t)$ the evolution of (14) up to proper time t.

Consider then the state $|\psi_+\rangle$ which is as in (11) but with a plus sign between the two terms in parenthesis; it is an entangled state which is orthogonal to $|\psi_A\rangle$. The following quantity

$$\Delta(t) = \langle\psi_+|\rho_A(t)|\psi_+\rangle \ , \tag{16}$$

being a mean value, must be positive for all times. In particular, since $\Delta(0) = 0$, its time evolution must start at $t = 0$ with a positive derivative, otherwise

[4] We stress that this choice is the only natural possibility if one requires that after tracing over the degrees of freedom of one particle, the resulting dynamics for the remaining one be positive, of semigroup type and independent from the initial state of the other particle.

it would become negative as soon as $t > 0$. In other terms, presevation of positivity of the density matrix $\rho_A(t)$ implies the condition

$$\frac{d}{dt}\Delta(0) \equiv a + \alpha - \gamma \geq 0 \ . \tag{17}$$

By substituting for $|\psi_+\rangle$ the most general entangled state orthogonal to $|\psi_A\rangle$, one can show that the preservation of the positivity of the matrix $\rho_A(t)$ describing correlated kaons is equivalent to the following inequalities:

$$2R \equiv \alpha + \gamma - a \geq 0 \ , \qquad RS \geq b^2 \ , \tag{18}$$

$$2S \equiv a + \gamma - \alpha \geq 0 \ , \qquad RT \geq c^2 \ , \tag{19}$$

$$2T \equiv a + \alpha - \gamma \geq 0 \ , \qquad ST \geq \beta^2 \ , \tag{20}$$

$$RST \geq 2\,bc\beta + R\beta^2 + Sc^2 + Tb^2 \ , \tag{21}$$

that in turn are equivelent to the positivity of the Kossakowski matrix C_{ij} appearing in (5).

These constraints on the dissipative parameters a, b, c, α, β, γ are more stringent than those in (10); indeed, with the above conditions the master equation (3) generates not just a positive, but a *completely positive* evolution.[1) – 6)] This conclusion can be formalized in a Theorem: [18)] the dynamics $\gamma_t \otimes \gamma_t$, describing the dissipative evolution of correlated neutral kaons, is positive if and only if the single kaon dynamics γ_t is completely positive.

It should be stressed that it is the intimate structure of quantum mechanics, *i.e.* the existence of entangled states, that require any physically consistent dissipative dynamics to be completely positive. [6)] Any attempt to model noisy effects induced by a weakly coupled external environment via a positive, but not completely positive time evolution will unavoidably lead to unphysical results. As we shall see in the next section, such a conclusion can be experimentally exposed at a ϕ-factory.

3 Test of complete positivity at a ϕ-factory

As discussed in the previous section, a consistent statistical description of the initial single kaon density matrix ρ as a state requires the positivity of its eigenvalues that are interpreted as probabilities: for this description to hold for all times, the evolution map γ_t must be positive, *i.e.* it must preserve the positivity of the eigenvalues of $\rho(t)$, for any t.

On the other hand, complete positivity is a more stringent condition; it guarantees the positivity of the eigenvalues of density matrices describing states of correlated kaons, as those produced in ϕ-meson decays. We have seen that states of entangled, but not dynamically interacting kaons, evolve according to the factorized product $\gamma_t \otimes \gamma_t$ of the single-kaon dynamical maps. If γ_t is not completely positive, there are instances of correlated states that develop negative eigenvalues; in such cases, their statistical and physical interpretation is lost.

Therefore, the issue of complete positivity is not only theoretical, but can be given experimental relevance. Indeed, in the following we shall give explicit examples of experimentally accessible kaon observables, defined to be positive, that would return, in absence of complete positivity, negative mean values. [45)]

Let us consider again the dissipative evolution of two initially correlated neutral kaons coming from the decay of a ϕ-meson, as given in (15), with $t_1 = t_2 = t$. Recalling the definitions (12), (13), the statistical description of $\rho_A(t) \equiv \rho_A(t,t)$ allows us to give a meaningful interpretation of the quantities

$$\mathcal{P}_{ij}(t) = \mathrm{Tr}[\rho_A(t) P_i \otimes P_j] \ , \quad i,j = 1,2 \ , \tag{22}$$

as the probabilities to have one kaon in the state $|K_i\rangle$ at proper time t, while the other is in the state $|K_j\rangle$ at the same proper time. When $i,j = 3,4$, the quantities $\mathcal{P}_{ij}(t)$ are complex and do not represent directly joint probabilities. However, as we shall see, they can still be obtained from data accessible to experiments.

On the basis of rough dimensional estimates,[54), 55), 44)] the parameters a, b, c, α, β and γ appearing in (8) are expected to be very small, since they are suppressed by at least one power of a very large energy scale, the one that characterizes the dynamics of fundamental objects (strings or branes). Assimilating this scale with the Planck mass M_P, one finds that the above dissipative parameters, being of dimension of energy, can be estimated to be at most of order $m_K^2/M_P \sim 10^{-19}$ GeV, with m_K the kaon mass. This value is roughly of the same order of magnitude of $\epsilon_S \Delta\Gamma$ and $\epsilon_L \Delta\Gamma$; therefore, in finding explicit solutions of the evolution equation (3) for the kaon density matrix $\rho(t)$ one can use an expansion in all these small parameters, and approximate expressions for the entries of $\rho(t)$ can be explicitly worked out.[38), 41)]

Up to first order in all small parameters, one then finds:

$$\begin{aligned}
\mathcal{P}_{11}(t) &= \frac{\gamma}{\Delta\Gamma}\, e^{-2\Gamma t}\Big(1-e^{-\Delta\Gamma t}\Big)\ , && (23)\\
\mathcal{P}_{12}(t) &= \frac{e^{-2\Gamma t}}{2}\ , && (24)\\
\mathcal{P}_{13}(t) &= 2\, e^{-2\Gamma t}\,\frac{c+i\beta}{\Delta\Gamma_+}\Big(1-e^{-t\Delta\Gamma_+/2}\Big)\ , && (25)\\
\mathcal{P}_{22}(t) &= \frac{\gamma}{\Delta\Gamma}\, e^{-2\Gamma t}\Big(e^{\Delta\Gamma t}-1\Big)\ , && (26)\\
\mathcal{P}_{23}(t) &= 2\, e^{-2\Gamma t}\,\frac{c+i\beta}{\Delta\Gamma_-}\Big(1-e^{t\Delta\Gamma_-/2}\Big)\ , && (27)\\
\mathcal{P}_{33}(t) &= e^{-2\Gamma t}\,\frac{2b+i(\alpha-a)}{2\Delta m}\Big(1-e^{-2it\Delta m}\Big)\ , && (28)\\
\mathcal{P}_{34}(t) &= -\frac{e^{-2\Gamma t}}{2}\Big(1-2(\alpha+a-\gamma)t\Big)\ . && (29)
\end{aligned}$$

The remaining quantities $\mathcal{P}_{ij}(t)$ can be derived from the previous expressions by using the following properties:

$$\begin{aligned}
&\mathcal{P}_{ij}(t)=\mathcal{P}_{ji}(t)\ , \qquad i,j=1,2,3,4\ , && (30)\\
&\mathcal{P}_{i3}(t)=\mathcal{P}^*_{i4}(t)\ , \qquad i=1,2\ , && (31)\\
&\mathcal{P}_{44}(t)=\mathcal{P}^*_{33}(t)\ . && (32)
\end{aligned}$$

Putting $a=b=c=\alpha=\beta=\gamma=0$, one obtains the standard quantum mechanical effective description that evidentiates the singlet-like anti-correlation in $\rho_A(t)$: $\mathcal{P}_{ii}(t)\equiv 0$.

We emphasize that none of the above expressions contain the standard CP, CPT-violating parameters ϵ_S, ϵ_L. This fact makes possible, at least in principle, a direct determination of the non-standard parameters irrespectively of the values of ϵ_S, ϵ_L; one needs to fit the previous expressions of the quantities $\mathcal{P}_{ij}(t)$ with actual data from experiments at ϕ-factories.

To be more specific, we shall now explicitly show how the quantities $\mathcal{P}_{ij}$ can be directly related to frequency countings of decay events. First, notice that, given any single-kaon time-evolution $\rho \mapsto \rho(t)$, the matrix elements of the state $\rho(t)$ at time t can be measured by identifying appropriate orthogonal bases in the two-dimensional single kaon Hilbert space. The choice of the CP-eigenstates $|K_1\rangle$, $|K_2\rangle$ is rather suited to experimental tests. Indeed, since a two-pion state has the same CP eigenvalue of $|K_1\rangle$, the probability

$P_t(K_1) = \langle K_1|\rho(t)|K_1\rangle$ of having a kaon state K_1 at time t is directly related to the frequency of two-pion decays at time t. Possible direct CP violating effects, the only ones allowing $K_2 \to 2\pi$, can be safely neglected; they are proportional to the phenomenological parameter ε', that has been found to be very small. [62]

On the other hand, while the decay state $\pi^0\pi^0\pi^0$ has $CP = -1$, the state $\pi^+\pi^-\pi^0$ may have $CP = \pm 1$. Thus, the probability $P_t(K_2) = \langle K_2|\rho(t)|K_2\rangle$ to have a kaon state K_2 at time t is not as conveniently measured by counting the frequency of the three-pion decays. To avoid the difficulty, the following strangeness eigenstates can be used:

$$|K^0\rangle = \frac{|K_1\rangle + |K_2\rangle}{\sqrt{2}}\ , \qquad |\overline{K^0}\rangle = \frac{|K_1\rangle - |K_2\rangle}{\sqrt{2}}\ . \tag{33}$$

Then, the probabilities $P_t(K^0) = \langle K^0|\rho(t)|K^0\rangle$ and $P_t(\overline{K^0}) = \langle\overline{K^0}|\rho(t)|\overline{K^0}\rangle$, that the kaon state at time t be a K^0, respectively a $\overline{K^0}$, may be experimentally determined by counting the semileptonic decays $K^0 \mapsto \pi^-\ell^+\nu$, respectively $\overline{K^0} \mapsto \pi^+\ell^-\overline{\nu}$, the exchanged decays being forbidden by the $\Delta S = \Delta Q$ rule. (In the Standard Model, this selection rule is expected to be valid up to order 10^{-14}. [63]) Further, the probability $P_t(K_2) = \langle K_2|\rho(t)|K_2\rangle$ of having a kaon state K_2 at proper time t can be expressed as $P_t(K_2) = P_t(K^0) + P_t(\overline{K^0}) - P_t(K_1)$, by writing

$$|K_2\rangle\langle K_2| = |K^0\rangle\langle K^0| + |\overline{K^0}\rangle\langle\overline{K^0}| - |K_1\rangle\langle K_1|\ . \tag{34}$$

Hence, $P_t(K_2)$ can be measured by counting the frequencies of semileptonic decays and of decays into two pions.

In order to measure the off-diagonal elements $\langle K_1|\rho|K_2\rangle$, $\langle K_2|\rho|K_1\rangle$, we use the identity

$$|K^0\rangle\langle K^0| - |\overline{K^0}\rangle\langle\overline{K^0}| = |K_1\rangle\langle K_2| + |K_2\rangle\langle K_1|\ , \tag{35}$$

and extract $|K_1\rangle\langle K_2|$ from it. To do this, we need a third orthonormal basis of vectors whose projectors are measurable observables in actual experiments. An interesting possibility is based on the phenomenon of kaon-regeneration (see Refs.[64, 65] and references therein). The idea is to insert a slab of material across the neutral kaons path; the interactions of the K^0, $\overline{K^0}$ mesons with the nuclei of the material "rotate" in a known way the initial kaon states entering the regenerator into new ones. As initial states, consider the orthogonal vectors

$$|\widetilde{K}_S\rangle = \frac{|K_1\rangle - \eta^*|K_2\rangle}{\sqrt{1+|\eta|^2}}\ , \qquad |\widetilde{K}_L\rangle = \frac{\eta|K_1\rangle + |K_2\rangle}{\sqrt{1+|\eta|^2}}\ , \tag{36}$$

where η is a complex parameter which depends on the regenerating material. By carefully choosing the material and the tickness of the slab, one can tune the modulus and phase of η in such a way to completely suppress the $\widetilde{K}_L$ component and to regenerate the $\widetilde{K}_S$ state into a K_1, just outside the material slab. Thus, the probability $P_t(\widetilde{K}_S) = \langle\widetilde{K}_S|\rho(t)|\widetilde{K}_S\rangle$ that a kaon, impinging on a slab of regenerating material in a state $\rho(t)$ at time t, be a $\widetilde{K}_S$, can be measured by counting the decays into 2π just beyond the slab. Now, the projector onto the state $|\widetilde{K}_S\rangle$ reads

$$\begin{aligned} |\widetilde{K}_S\rangle\langle\widetilde{K}_S| &= \frac{1}{1+|\eta|^2}|K_1\rangle\langle K_1| + \frac{|\eta|^2}{1+|\eta|^2}|K_2\rangle\langle K_2| \\ &- \frac{\eta}{1+|\eta|^2}|K_1\rangle\langle K_2| - \frac{\eta^*}{1+|\eta|^2}|K_2\rangle\langle K_1|\ . \end{aligned} \tag{37}$$

Then, from (33)–(35) and (37) it follows that

$$\begin{aligned} |K_1\rangle\langle K_2| &= \zeta_1\,|K_1\rangle\langle K_1| + \zeta_2\,|\widetilde{K}_S\rangle\langle\widetilde{K}_S| \\ &+ \zeta_3\,|K^0\rangle\langle K^0| + \zeta_4\,|\overline{K^0}\rangle\langle\overline{K^0}|\ , \end{aligned} \tag{38}$$

where

$$\zeta_1 = \frac{1-|\eta|^2}{2i\mathcal{I}m(\eta)}\ , \qquad \zeta_2 = -\frac{1+|\eta|^2}{2i\mathcal{I}m(\eta)}\ , \tag{39}$$

$$\zeta_3 = \frac{|\eta|^2-\eta^*}{2i\mathcal{I}m(\eta)}\ , \qquad \zeta_4 = \frac{|\eta|^2+\eta^*}{2i\mathcal{I}m(\eta)}\ . \tag{40}$$

In this way, the determination of the off-diagonal elements of $\rho(t)$ amounts to counting the frequencies of decays into two pions with or without regeneration and the frequencies of semileptonic decays:

$$\begin{aligned} \langle K_1|\rho(t)|K_2\rangle &- \zeta_1\,P_t(K_1) + \zeta_2\,P_t(\widetilde{K}_S) \\ &+ \zeta_3\,P_t(K^0) + \zeta_4\,P_t(\overline{K^0})\ . \end{aligned} \tag{41}$$

The application of these results to the case of correlated kaons is now straightforward. For sake of compactness, we identify the various kaon states with the projections Q_μ, $\mu = 1, 2, 3, 4$, where:

$$Q_1 = |K_1\rangle\langle K_1| \ , \qquad Q_3 = |K^0\rangle\langle K^0| \ , \tag{42}$$

$$Q_2 = |\widetilde{K}_S\rangle\langle\widetilde{K}_S| \ , \qquad Q_4 = |\overline{K^0}\rangle\langle\overline{K^0}| \ . \tag{43}$$

As discussed, these operators can be measured by identifying 2π final states, in absence and presence of a regenerator (Q_1 and Q_2), and semileptonic decays (Q_3 and Q_4); the same holds for the projectors in (12) and (13), since:

$$P_1 \ = \ |K_1\rangle\langle K_1| \equiv Q_1 \ , \tag{44}$$

$$P_2 \ = \ |K_2\rangle\langle K_2| = Q_3 + Q_4 - Q_1 \ , \tag{45}$$

$$P_3 \ = \ |K_1\rangle\langle K_2| \equiv P_4^\dagger = \sum_{\mu=1}^{4} \zeta_\mu\, Q_\mu \ . \tag{46}$$

Further, we denote by $P_t(Q_\mu, Q_\nu)$ the probability that, at proper time t after a ϕ-decay, the two kaons be in the states identified by Q_μ and Q_ν, respectively. Then, the determination of the quantities $\mathcal{P}_{ij}(t)$ reduces to measuring joint probabilities, *i.e.* to counting frequencies of events of certain specificied types. Indeed, one explicitly finds:

$$\mathcal{P}_{11}(t) = P_t(Q_1, Q_1) \ , \tag{47}$$

$$\mathcal{P}_{12}(t) = P_t(Q_1, Q_3) + P_t(Q_1, Q_4) - P_t(Q_1, Q_1) \ , \tag{48}$$

$$\mathcal{P}_{13}(t) = \sum_{\mu=1}^{4} \zeta_\mu\, P_t(Q_1, Q_\mu) \ , \tag{49}$$

$$\mathcal{P}_{22}(t) = P_t(Q_1, Q_1) + P_t(Q_3, Q_3) + P_t(Q_4, Q_4) + 2\Big[P_t(Q_3, Q_4) - P_t(Q_1, Q_4) - P_t(Q_1, Q_4)\Big] \ , \tag{50}$$

$$\mathcal{P}_{23}(t) = \sum_{\mu=1}^{4} \zeta_\mu \Big[P_t(Q_3, Q_\mu) + P_t(Q_4, Q_\mu) - P_t(Q_1, Q_\mu)\Big] \ , \tag{51}$$

$$\mathcal{P}_{33}(t) = \sum_{\mu=1}^{4}\sum_{\nu=1}^{4} \zeta_\mu\, \zeta_\nu\, P_t(Q_\mu, Q_\nu) \ . \tag{52}$$

As a result of the previous analysis, the inconsistencies of models without complete positivity, besides being theoretically unsustainable, turn out to be experimentally exposable. Let P_{φ_+} and P_{ψ_+} project onto the correlated states

$$|\varphi_+\rangle \ = \ \frac{1}{\sqrt{2}}\Big(|K_1\rangle \otimes |K_1\rangle + |K_2\rangle \otimes |K_2\rangle\Big) \ , \tag{53}$$

$$|\psi_+\rangle \;=\; \frac{1}{\sqrt{2}}\Big(|K_1\rangle\otimes|K_2\rangle+|K_2\rangle\otimes|K_1\rangle\Big)\ , \tag{54}$$

that are orthogonal to the state $|\psi_A\rangle$ in (11) produced in a ϕ decay. The averages of these two positive observables with respect to the state $\rho_A(t)$ read

$$\begin{aligned}\Phi(t) &= \mathrm{Tr}[\rho_A(t)\,P_{\varphi_+}] \equiv \langle\varphi_+|\rho_A(t)|\varphi_+\rangle\\ &= \frac{1}{2}\big(\mathcal{P}_{11}(t)+\mathcal{P}_{22}(t)+\mathcal{P}_{33}(t)+\mathcal{P}_{44}(t)\big) \qquad (55)\\ \Psi(t) &= \mathrm{Tr}[\rho_A(t)\,P_{\psi_+}] \equiv \langle\psi_+|\rho_A(t)|\psi_+\rangle\\ &= \frac{1}{2}\big(\mathcal{P}_{12}(t)+\mathcal{P}_{21}(t)+\mathcal{P}_{34}(t)+\mathcal{P}_{43}(t)\big)\ , \qquad (56)\end{aligned}$$

and, as explained before, can be directly obtained by measuring joint probabilities in experiments at ϕ-factories. On the other hand, (23)–(29) give, up to first order in the small parameters,

$$\Phi(t)=e^{-2\Gamma t}\left[\frac{\gamma}{\Delta\Gamma}\sinh(t\Delta\Gamma)+\frac{b}{\Delta m}\Big(1-\cos(2t\Delta m)\Big)\right.\\ \left.+\frac{a-\alpha}{2\Delta m}\sin(2t\Delta m)\right]\ , \tag{57}$$

$$\Psi(t)=e^{-2\Gamma t}\big(a+\alpha-\gamma\big)\,t\ . \tag{58}$$

Thus, $\Phi(0)=\Psi(0)=0$, whereas

$$\frac{\mathrm{d}\Phi(0)}{\mathrm{d}t}=a+\gamma-\alpha\ ,\qquad \frac{\mathrm{d}\Psi(0)}{\mathrm{d}t}=a+\alpha-\gamma\ , \tag{59}$$

are both positive because of conditions (19) and (20). More in general, the mean values (55) and (56) are surely positive, for the complete positivity of the single-kaon dynamical maps γ_t implies $\rho(t)=\sum_j V_j(t)\,\rho\,V_j^\dagger(t)$,[1) – 6)] where the $V_j(t)$ are 2×2 matrices such that $\sum_j V_j^\dagger(t)V_j(t)$ is a bounded 2×2 matrix.[5] Then, the complete evolution $\rho_A\to\rho_A(t)=\sum_{i,j}[V_i(t)\otimes V_j(t)]\,\rho_A\,[V_i^\dagger(t)\otimes V_j^\dagger(t)]$ will never develop negative eigenvalues.

[5]Notice that, in absence of the extra contribution L in (3), the time evolution $\rho(t)$ is realized with a single matrix V, *i.e.* $j=1$, and $V_1(t)=e^{-iH_{\rm eff}\,t}$; in other words, in ordinary quantum mechanics the condition of complete positivity is trivially satisfied.

On the other hand, if the single-kaon dynamical map ω_t is not completely positive, inconsistencies may emerge. As an example, take the phenomenological models studied in Refs.[55, 56], where the non-standard parameters a, b, c are set to zero and $\alpha \neq \gamma$, $\alpha\gamma \geq \beta^2$. The corresponding dynamics is not completely positive: the inequalities (18)–(21) are in fact violated. In this case, one still has $\Phi(0) = \Psi(0) = 0$, but

$$\frac{\mathrm{d}\Phi(0)}{\mathrm{d}t} = \gamma - \alpha = -\frac{\mathrm{d}\Psi(0)}{\mathrm{d}t} \ . \tag{60}$$

Therefore, one of the mean values (55), (56) starts assuming negative values as soon as $t > 0$. The inconsistence is avoided only if $\alpha = \gamma$, which is a necessary condition for getting back the property of complete positivity. As explictly discussed above, planned set-ups at ϕ-factories can measure, at least in principle, the two mean values in (55) and (56) and therefore directly check the positivity of the two combinations in (59), thus clarifying also from the experimental point of view the need of complete positivity for the description of the dissipative dynamics of neutral kaons.

4 Tests of dissipative effects in kaon dynamics

From the discussion of the previous section, it is apparent that a physically consistent description of the dissipative dynamics of neutral kaons weakly coupled to an external environment can be realized only through the use of completely positive dynamical semigroups; these are generated by master equations of the form (3) and (5), with a positive Kossakowski matrix C. Indeed, only evolutions of this type satisfy the physical requirements that are at the basis of the statistical interpretation of quantum mechanics, so that the eigenvalues of the kaon density matrix can be correctly interpreted as probabilities. Modelling dissipative kaon evolutions with linear maps that are not completely positive will unavoidably lead to the appearence of negative values for some of those probabilities when correlated kaons are involved, thus spoiling the physical consistency of the whole treatment. Indeed, only completely positive time evolutions are compatible with the presence of entanglement. [6)]

In view of these considerations, it is apparent that the form (3), (5) of the kaon time-evolution is very general and quite independent from the detailed mechanism leading to the phenomena of noise and dissipation, that can be of gravitational, stringy or fluctuating medium origin. Indeed, the evolution of any quantum open system, immersed in a weakly coupled environment can be effectively modeled using quantum dynamical semigroups. In this respect, the equations (3), (5) should be regarded as phenomenological in nature, and therefore quite suitable to experimental test: any signal of non-vanishing value for some of the parameters in (8) would certainly attest in a model independent way the presence of non-standard, dissipative effects in kaon physics.

Physical observables of the neutral kaon system are associated with the decays of the kaons into suitable final state f, typically pion states, and semileptonic states. In the language of density matrices, these final decay states are described by suitable hermitian operators $\mathcal{O}_f$; taking the trace of these operators with $\rho(t)$, solution of the master equation (3), (5), allows computing the explicit time dependence of various experimentally relevant observables (*e.g.* see Refs.[38, 40, 41, 51, 52]).[6]

For instance, the operators $\mathcal{O}_{+-}$ and $\mathcal{O}_{00}$ that describe the $\pi^+\pi^-$ and $2\pi^0$ final states have the form:

$$\mathcal{O}_{+-} \sim \begin{bmatrix} 1 & r_{+-} \\ r_{+-}^* & |r_{+-}|^2 \end{bmatrix}, \qquad \mathcal{O}_{00} \sim \begin{bmatrix} 1 & r_{00} \\ r_{00}^* & |r_{00}|^2 \end{bmatrix}. \tag{61}$$

To lowest order in all small parameters, the complex constants r_{+-} and r_{00}, can be written as:

$$r_{+-} = \varepsilon - \epsilon_L + \varepsilon' , \qquad r_{00} = \varepsilon - \epsilon_L - 2\varepsilon' , \tag{62}$$

where ε and ε' are the familiar phenomenological parameters signaling direct CP and CPT violating effects.[57) – 60)] Similar results hold for the matrices $\mathcal{O}_{\pi^+\pi^-\pi^0}$, $\mathcal{O}_{3\pi^0}$, $\mathcal{O}_{\ell^-}$ and $\mathcal{O}_{\ell^+}$ that describe the decays into $\pi^+\pi^-\pi^0$, $3\pi^0$, $\pi^+\ell^-\bar{\nu}$ and $\pi^-\ell^+\nu$; for explicit expressions, see Refs.[39, 40, 50, 51].

With the help of these matrices, one can compute the time-dependence of various useful observables of the neutral kaon system, like decay rates and asymmetries. These quantities are accessible to the experiment, so that they can be used to obtain bounds on the dissipative effects, encoded in the parameters in (8); using the most recent available results on single kaon experi-

[6]Similar appraoches are employed also in Refs.[55, 56, 66, 67], where however, as discussed in the previous section, a physically inconsistent time evolution is adopted.

ments, [62), 68), 69)] one can indeed obtain estimates on some of them:

$$\begin{aligned}
&a \leq 5.0 \times 10^{-17}\ \text{GeV}\ ,\\
&c \leq 2.0 \times 10^{-17}\ \text{GeV}\ ,\\
&\alpha \leq 6.0 \times 10^{-17}\ \text{GeV}\ ,\\
&\beta \leq 1.0 \times 10^{-17}\ \text{GeV}\ ,\\
&\gamma \leq 22.0 \times 10^{-20}\ \text{GeV}\ .
\end{aligned} \tag{63}$$

Unfortunately, the precision of the present experimental results on single neutral kaons is not high enough, so that only rough upper bounds can be obtained. Although more complete and precise data will surely be available in the future, the most promising venues for studying the consequences of the dissipative dynamics in (3), (5) are certainly the experiments on correlated neutral kaons at ϕ-factories.

The typical observables that can be studied in such physical situations are double decay rates, *i.e.* the probabilities that a kaon decays into a final state f_1 at proper time t_1, while the other kaon decays into the final state f_2 at proper time t_2:

$$\mathcal{G}(f_1, t_1; f_2, t_2) \equiv \text{Tr}\Big[\Big(\mathcal{O}_{f_1} \otimes \mathcal{O}_{f_2}\Big)\rho_A(t_1, t_2)\Big]\ ; \tag{64}$$

as before, the operators $\mathcal{O}_{f_1}$ and $\mathcal{O}_{f_2}$ are the 2×2 hermitian matrices describing the final decay states. By studying these observables in a high-luminosity ϕ-factory it will be possible to determine the values of the non-standard parameters a, b, c, α, β, γ.[7] For instance, the long time behaviour ($t \gg 1/\gamma_S$) of the three pion probability gives direct informations on the parameter γ:

$$\mathcal{G}(\pi^+\pi^-\pi^0, t\,; \pi^+\pi^-\pi^0, t) \sim \frac{\gamma}{\Delta\Gamma}\ e^{-2\gamma_L t}\ . \tag{65}$$

Similarly, the small time behaviour of the ratio of semileptonic probabilities,

$$\frac{\mathcal{G}(\ell^\pm, t; \ell^\pm, t)}{\mathcal{G}(\ell^\pm, t; \ell^\mp, t)} \sim 2\ a\ t\ , \tag{66}$$

[7] As observed before, notice that the time-behaviour of these decay rates is completely different from the one required by ordinary quantum mechanics, which for instance predicts $\mathcal{G}(f, t; f, t) \equiv 0$ for all final states f, due to the antisymmetry of the initial state ρ_A.

is sensitive to the parameter a. [41)]

However, much of the analysis at ϕ-factories is carried out using integrated distributions at fixed time interval $t = t_1 - t_2$. [70)] One then deals with single-time distributions, defined by:

$$\Gamma(f_1, f_2; t) = \int_0^\infty d\tau\ \mathcal{G}(f_1, \tau + t; f_2, \tau)\ , \qquad t > 0\ . \tag{67}$$

Starting with these integrated probabilities, one can form asymmetries that are sensitive to various parameters in the theory. A particularly interesting example is given by the following observable, involving two-pion final states,

$$\mathcal{A}_{\varepsilon'}(t) = \frac{\Gamma(\pi^+\pi^-, 2\pi^0; t) - \Gamma(2\pi^0, \pi^+\pi^-; t)}{\Gamma(\pi^+\pi^-, 2\pi^0; t) + \Gamma(2\pi^0, \pi^+\pi^-; t)}\ ; \tag{68}$$

it is used for the determination of the ratio ε'/ε. The clear advantage of using the asymmetry $\mathcal{A}_{\varepsilon'}$ to determine the value of ε'/ε in comparison to the familiar "double ratio" method, [62)] is that, at least in principle, both real and imaginary part can be extracted from the time behaviour of (68). [70)] Due to the presence of the dissipative parameters however, this appears to be much more problematic than in the standard case; a meaningful determination of ε'/ε is possible provided independent estimates on c, β and γ are obtained from the measure of other independent asymmetries. This is particularly evident if one looks at the large-time limit ($t \gg 1/\gamma_S$) of (68):

$$\mathcal{A}_{\varepsilon'}(t) \sim 3\,\mathcal{R}e\left(\frac{\varepsilon'}{\varepsilon}\right)\ \frac{|\varepsilon|^2 + 2\,\mathcal{R}e\big(\varepsilon\, C/\Delta\Gamma_+\big)}{|\varepsilon|^2 + \mathcal{D}} - 6\,\mathcal{I}m\left(\frac{\varepsilon'}{\varepsilon}\right)\ \frac{\mathcal{I}m\big(\varepsilon\, C/\Delta\Gamma_+\big)}{|\varepsilon|^2 + \mathcal{D}}\ , \tag{69}$$

where

$$\mathcal{D} = \frac{\gamma}{\Delta\Gamma} - 4\left|\frac{C}{\Delta\Gamma_+}\right|^2 + 4\,\mathcal{R}e\left(\frac{\varepsilon\, C}{\Delta\Gamma_+}\right) - 4\,\mathcal{R}e\left(\frac{\epsilon_L C}{\Delta\Gamma}\right)\ , \tag{70}$$

with $C = c + i\beta$; only when $c = \beta = \gamma = 0$, the expression in (69) reduces to the standard result: $\mathcal{A}_{\varepsilon'} \sim 3\,\mathcal{R}e(\varepsilon'/\varepsilon)$. Therefore, if the non-standard, dissipative parameters in (8) are found to be non-zero, even neglecting the contribution from the imaginary part, the actual value of $\mathcal{R}e(\varepsilon'/\varepsilon)$ could be significantly different from the measured value of the quantity $\mathcal{A}_{\varepsilon'}/3$. [46)]

In conclusion, dissipative effects in the dynamics of both single and correlated neutral kaon systems could affect the precise determination of various relevant quantities in kaon physics; dedicated experiments, in particular those involving correlated kaons, will certainly provide stringent bounds on these dissipative effects, and possibly allow a definite clarification of the role played by complete positivity in open quantum system dynamics.

References

1. R. Alicki, K. Lendi, *Quantum Dynamical Semi-Groups and Applications*, Lect. Notes Phys. **286**, (Springer-Verlag, Berlin, 1987)
2. V. Gorini, A. Frigerio, M. Verri, A. Kossakowski and E.G.C. Sudarshan, Rep. Math. Phys. **13** (1976) 149
3. H. Spohn, Rev. Mod. Phys. **52**, 569 (1980)
4. E.B. Davies, *Quantum Theory of Open Systems*, (Academic Press, New York, 1976)
5. R. Alicki, Invitation to quantum dynamical semigroups, in: Lect. Notes Phys. **597**, P. Garbaczewski and R. Olkiewicz, Eds., (Springer-Verlag, Berlin, 2002), p.239
6. F. Benatti and R. Floreanini, Int. J. Mod. Phys. B **19** (2005) 3063
7. R. Alicki and M. Fannes, *Quantum Dynamical Systems*, (Oxford University Press, Oxford, 2001)
8. *Dissipative Quantum Dynamics*, F. Benatti and R. Floreanini, Eds., Lect. Notes Phys. **622**, (Springer-Verlag, Berlin, 2003)
9. H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002)
10. W.H. Louisell, *Quantum Statistical Properties of Radiation*, (Wiley, New York, 1973)
11. F. Haake, Statistical treatment of open systems by generalized master equations, in Springer Tracts in Mod. Phys. **95**, (Springer-Verlag, Berlin, 1973)
12. C. Cohen-Tannoudji, J. Dupont-Roc and G. Grynberg, *Atom-Photon Interactions*, (Wiley, New York, 1988)

13. D.F. Walls and G.J. Milburn, *Quantum Optics*, (Springer-Verlag, Berlin, 1994)
14. M.O. Scully and M.S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997)
15. C.W. Gardiner and P. Zoller, *Quantum Noise*, 2nd. ed., (Springer-Verlag, Berlin, 2000)
16. R.R. Puri, *Mathematical Methods of Quantum Optics*, (Springer-Verlag, Berlin, 2001)
17. U. Weiss, *Quantum Dissipative Systems*, 2nd. ed., (World Scientific, Singapore, 1999)
18. F. Benatti, R. Floreanini and R. Romano, J. Phys. A **35** (2002) L551
19. F. Benatti, R. Floreanini and M. Piani, Phys. Rev. A **67** (2003) 042110
20. A. Kossakowski, Bull. Acad. Pol. Sc. **12** (1972) 1021
21. V. Gorini, A. Kossakowski and E.C.G. Sudarshan, J. Math. Phys. **17** (1976) 821
22. G. Lindblad, Comm. Math. Phys. **48** (1976) 119
23. R. Dümcke and H. Spohn, Z. Phys. **B34** (1979) 419
24. E.B. Davies, Comm. Math. Phys. **39** (1974) 91
25. E.B. Davies, Math. Ann. **219** (1976) 147
26. A. Kossakowski, Open Sys. Inf. Dyn. **10** (2003) 1
27. F. Benatti and R. Floreanini, Phys. Rev. A **66** (2002) 043617
28. F. Benatti and R. Floreanini, Phys. Lett. **B451** (1999) 422
29. F. Benatti, R. Floreanini and R. Romano, J. Phys. A **35** (2002) 4955
30. F. Benatti and R. Floreanini, Phys. Rev. D **62** (2000) 125009
31. F. Benatti, R. Floreanini and A. Lapel, Cybernetics and Sys., **32** (2001) 343

32. F. Benatti and R. Floreanini, J. Optics B, **4** (2002) S238
33. F. Benatti and R. Floreanini, Tests of complete positivity in fiber optics, *Open Sys. Inf. Dyn.*, 2006, to appear
34. F. Benatti and R. Floreanini, JHEP **02** (2002) 032
35. F. Benatti and R. Floreanini, Phys. Rev. D **64** (2001) 085015
36. F. Benatti and R. Floreanini, Phys. Rev. D **71** (2005) 013003
37. F. Benatti and R. Floreanini, Phys. Lett. **B389** (1996) 100
38. F. Benatti and R. Floreanini, Nucl. Phys. **B488** (1997) 149
39. F. Benatti and R. Floreanini, Mod. Phys. Lett. **A12** (1997) 1465
40. F. Benatti and R. Floreanini, Phys. Lett. **B401** (1997) 337
41. F. Benatti and R. Floreanini, Nucl. Phys. **B511** (1998) 550
42. F. Benatti and R. Floreanini, Phys. Lett. **B428** (1998) 149
43. F. Benatti and R. Floreanini, Banach Center Publications, **43** (1998) 71
44. F. Benatti and R. Floreanini, Ann. of Phys. **273** (1999) 58
45. F. Benatti and R. Floreanini, Phys. Lett. **B468** (1999) 287
46. F. Benatti and R. Floreanini, Mod. Phys. Lett. **A14** (1999) 1519
47. F. Benatti and R. Floreanini, Phys. Rev. D **57** (1998) R1332
48. F. Benatti and R. Floreanini, Eur. Phys. J. **C13** (2000) 267
49. F. Benatti and R. Floreanini, Chaos, Solitons and Fractals **12** (2001) 2631
50. F. Benatti, R. Floreanini and R. Romano, Phys. Lett. **B465** (1999) 260
51. F. Benatti, R. Floreanini and R. Romano, Nucl. Phys. **B602** (2001) 541
52. F. Benatti, R. Floreanini and R. Romano, Phys. Rev. D **68** (2003) 094007
53. 53 M.S. Marinov, JETP Lett. **15** (1972) 479; Sov. J. Nucl. Phys. **19** (1974) 173; Nucl. Phys. **B253** (1985) 609

54. S. Hawking, Comm. Math. Phys. **87** (1983) 395; Phys. Rev. D **37** (1988) 904; Phys. Rev. D **53** (1996) 3099; S. Hawking and C. Hunter, Phys. Rev. D **59** (1999) 044025

55. J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. **B241** (1984) 381;

56. P. Huet and M.E. Peskin, Nucl. Phys. **B434** (1995) 3

57. L. Maiani in *The Second DaΦne Physics Handbook*, L. Maiani, G. Pancheri and N. Paver, eds., (INFN, Frascati, 1995)

58. G.C. Branco, L. Lavoura and J.P. Silva, *CP Violation*, (Clarendon Press, Oxford, 1999)

59. R. Belusevic, *Neutral Kaons* (Springer-Verlag, Berlin, 1999)

60. I.I. Bigi and A.I. Silva, *CP Violation*, (Cambridge Univeristy Press, Cambridge, 2000)

61. F. Benatti and R. Floreanini in, *Physics and Detectors for DaΦne*, S. Bianco, F. Bossi, G. Capon, F.L. Fabbri,, P. Gianotti, G. Isidori and F. Murtas, eds., (INFN, Frascati, 1999), p. 307

62. Particle Data Group, Phys. Lett. **B592** (2004) 1

63. N.W. Tanner and R.H. Dalitz, Ann. of Phys. **171** (1986) 463

64. P.H. Eberhard and F. Uchiyama, Nucl. Instr. Met. A **350** (1994) 144

65. A. Di Domenico, Nucl. Phys. **B450** (1995) 293

66. J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. **B293** (1992) 37; Int. J. Mod. Phys. **A11** (1996) 1489

67. J. Ellis, J.L. Lopez, N.E. Mavromatos and D.V. Nanopoulos, Phys. Rev. D **53** (1996) 3846

68. CPLEAR Collaboration, Phys. Rep. **374** (2003) 165

69. M. Fidecaro and H.-J. Gerber, Rep. Prog. Phys. **69** (2006) 1713

70. G. D'Ambrosio, G. Isidori and A. Pugliese, in *The Second DaΦne Physics Handbook*, L. Maiani, G. Pancheri and N. Paver, eds., (INFN, Frascati, 1995)

CPT- AND LORENTZ-SYMMETRY BREAKING: A REVIEW

Ralf Lehnert
Center for Theoretical Physics
Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A

Abstract

The breakdown of spacetime symmetries has recently been identified as a promising candidate signal for underlying physics, possibly arising through quantum-gravitational effects. This paper gives an overview over various aspects of CPT- and Lorentz-violation research. Particular emphasis is given to the interplay between CPT, Lorentz, and translation symmetry, mechanisms for CPT and Lorentz breaking, and the construction of a low-energy quantum-field description of such effect. This quantum field framework, called the SME, is employed to determine possible phenomenological consequences of CPT and Lorentz violation for neutral-meson interferometry.

1 Introduction

Although phenomenologically successful, the Standard Model of particle physics leaves unanswered a variety of theoretical questions. At present, a significant amount of theoretical work is therefore directed toward the search for an underlying theory that includes a quantum description of gravity. However, observational tests of such ideas face a major obstacle of practical nature: most quantum-gravity effects in virtually all leading candidate models are expected to be extremely small due to Planck-scale suppression. For example, low-energy measurements are likely to require sensitivities of at least one part in 10^{17}. This paper gives an overview of a recent approach to this issue that involves spacetime symmetries.

The presumed minute size of candidate quantum-gravity effects requires a careful choice of experiments. A promising idea that one may pursue is testing physical laws that satisfy three primary criteria. First, one should consider fundamental laws that are believed to hold *exactly* in established physics. Any measured deviations would then definitely indicate qualitatively new physics. Second, the likelihood of observing such effects is increased by testing laws that may be *violated* in credible candidate fundamental theories. Third, from a practical point of view, these laws must be amenable to *ultrahigh-precision* tests.

One example of a physics law that satisfies all of these criteria is CPT invariance. [1] As a brief reminder, this law requires that the physics remains unchanged under the combined operations of charge conjugation (C), parity inversion (P), and time reversal (T). Here, the C transformation connects particles and antiparticles, P corresponds to a spatial reflection of physics quantities through the coordinate origin, and T reverses a given physical process in time. The Standard Model of particle physics is CPT-invariant by construction, so that the first criterion is satisfied. With regards to criterion two, we mention that a variety of approaches to fundamental physics can lead to CPT violation. Such approaches include strings, [2] spacetime foam, [3] nontrivial spacetime topology, [4] and cosmologically varying scalars. [5] The third of the above criteria is met as well. Consider, for instance, the conventional figure of merit for CPT conservation in the kaon system: its value lies currently at 10^{-18}, as quoted by the Particle Data Group. [6]

Since the CPT transformation relates a particle to its antiparticle, one would expect that CPT invariance implies a symmetry between matter and antimatter. One can indeed prove that the magnitude of the mass, charge, decay rate, gyromagnetic ratio, and other intrinsic properties of a particle are exactly equal to those of its antiparticle. This proof can be extended to systems of particles and their dynamics. For instance, atoms and anti-atoms must exhibit identical spectra and a particle-reaction process and its CPT-conjugate process must possess the same reaction cross section. It follows that experimental matter–antimatter comparisons can serve as probes for the validity of CPT invariance. In particular, the extraordinary sensitivities offered by meson interferometry yield high-precision tools in this context.

This paper is organized as follows. Section 2 discusses the interplay of various spacetime symmetries. Two mechanisms for CPT and Lorentz breakdown in Lorentz symmetric underlying theories are reviewed in Sec. 3. The basic ideas behind the construction of the SME are contained in Sec. 4. Section 5 extracts CPT observables from the SME. In Sec. 6, we comment on CPT tests involving neutral-meson systems. A brief summary is presented in Sec. 7.

2 Spacetime symmetries and their interplay

Spacetime transformation fall into two classes: continuous and discrete. The continuous transformations include translations, rotations, and boosts. Examples of discrete transformations are C, P, and T discussed in the introduction. Suppose symmetry is lost under one or more of these transformations. It is then a natural question as to whether the remaining transformations can still remain symmetries, or whether the breaking of one set of spacetime symmetry is typically associated with the violation of other spacetime invariances. This sections contains a brief discussion of this issue.

Suppose translational symmetry is broken (one possible mechanism for this effect is discussed in the next section). Then, the generator of translations, which is the energy–momentum tensor $\theta^{\mu\nu}$, is typically no longer conserved. Would this also affect Lorentz symmetry? To answer this question, let us look at the generator for Lorentz transformations, which is given by the the angular-momentum tensor $J^{\mu\nu}$:

$$J^{\mu\nu} = \int d^3x \, \big(\theta^{0\mu}x^\nu - \theta^{0\nu}x^\mu\big). \tag{1}$$

Note that this definition contains the non-conserved energy–momentum tensor $\theta^{\mu\nu}$. It follows that in general $J^{\mu\nu}$ will exhibit a nontrivial dependence on time, so that the usual time-independent Lorentz-transformation generators do not exist. As a result, Lorentz symmetry is no longer assured. We see that (with the exception of special cases) translation-symmetry violation leads to Lorentz breakdown.

We next consider CPT invariance. The celebrated CPT theorem of Bell, Lüders, and Pauli states that CPT symmetry arises under a few mild assumptions through the combination of quantum theory and Lorentz invariance. If CPT symmetry is broken one or more of the assumptions necessary to prove the CPT theorem must be false. This leads to the obvious question which one of the fundamental assumptions in the CPT theorem should be dropped. Since both CPT and Lorentz invariance involve spacetime transformations, it is natural to suspect that CPT violation implies Lorentz-symmetry breakdown. This has recently been confirmed rigorously in Greenberg's "anti-CPT theorem," which roughly states that in any unitary, local, relativistic point-particle field theory CPT breaking implies Lorentz violation. [7, 8] Note, however, that the converse of this statement—namely that Lorentz breaking implies CPT violation—is not true in general. In any case, it follows that CPT tests also probe Lorentz invariance. *As a result, potential CPT violation in the kaon system would typically be direction and energy dependent.* We will confirm this result explicitly in Sec. 5. Other types of CPT violation would require further deviations from conventional physics.[1]

3 Sample mechanisms for spacetime-symmetry breaking

In the previous section, we have found that the violation of a particular spacetime symmetry can lead to the breaking of another spacetime invariance. However, the question of *how* exactly a translation-, Lorentz-, and CPT-invariant candidate theory can lead to the violation of a spacetime symmetry in the first place has thus far been left unaddressed. The purpose of this section is to provide some intuition about such mechanisms for spacetime-symmetry breaking in underlying physics. Of the various possible mechanisms mentioned in Sec. 1, we will focus on spontaneous CPT and Lorentz breakdown as well as CPT and Lorentz violation through varying scalars.

Spontaneous CPT and Lorentz breaking. The mechanism of spontaneous symmetry violation is well established in various subfields of physics, such as the physics of elastic media, condensed-matter physics, and elementary-particle theory. From a theoretical viewpoint, this mechanism is very attractive because the invariance is essentially violated through a non-trivial ground-state solution. The underlying dynamics of the system, which is governed by the hamiltonian, remains completely invariant under the symmetry. To gain intuition about spontaneous Lorentz and CPT violation, we will consider three sample systems, whose features will gradually lead us to a better understanding of the effect. Figure 1 contains an illustration supporting these three examples.

We first look at classical electrodynamics. Any electromagnetic-field configuration is associated with an energy density $V(\vec{E},\vec{B})$, which is given by

$$V(\vec{E},\vec{B}) = \tfrac{1}{2}\left(\vec{E}^2 + \vec{B}^2\right) . \qquad (2)$$

[1]One could consider violations of unitarity, so that the usual quantum-mechanical probability conservation no longer holds. See, for example, J. Bernabeu *et al.*contribution to this handbook.

Here, we have employed natural units, and $\vec{E}$ and $\vec{B}$ denote the electric and magnetic fields, respectively. Equation (2) determines the field energy of any given solution of the Maxwell equations. Note that if the electric field, or the magnetic field, or both are nonzero in some spacetime region, the energy stored in these fields will be strictly positive. The field energy can only vanish when both $\vec{E}$ and $\vec{B}$ are zero everywhere. The ground state (or vacuum) is usually identified with the lowest-energy configuration of a system. We see that in conventional electromagnetism the configuration with the lowest energy is the field-free one, so that the Maxwell vacuum is empty (disregarding Lorentz- and CPT-symmetric quantum fluctuations).

Second, let us consider the Higgs field, which is part of the phenomenologically very successful Standard Model of particle physics. As opposed to the electromagnetic field, the Higgs field is a scalar. In what follows, we may adopt some simplifications without distorting the features important in the present context. The expression for the energy density of our Higgs scalar ϕ in situations with spacetime independence is given by

$$V(\phi) = (\phi^2 - \lambda^2)^2 \,. \tag{3}$$

Here, λ is a constant. As in the electrodynamics case discussed above, the lowest possible field energy is zero. Note, however, that this configuration *requires* ϕ to be non-vanishing: $\phi = \pm\lambda$. It therefore follows that the vacuum for a system containing a Higgs-type field is not empty; it contains, in fact, a constant scalar field $\phi_{vac} \equiv \langle\phi\rangle = \pm\lambda$. In quantum physics, the quantity $\langle\phi\rangle$ is called the vacuum expectation value (VEV) of ϕ. One of the physical effects caused by the VEV of the Standard-Model Higgs is to give masses to many elementary particles. We remark that $\langle\phi\rangle$ is a scalar and does *not* select a preferred direction in spacetime.

We finally take a look at a vector field $\vec{C}$ (the relativistic generalization is straightforward) not contained in the Standard-Model. Clearly, there is no observational evidence for such a field at the present time, but fields like $\vec{C}$ frequently arise in approaches to more fundamental physics. In analogy to the Higgs case, we take its expression for energy density in cases with constant $\vec{C}$ to be

$$V(\vec{C}) = (\vec{C}^2 - \lambda^2)^2 \,. \tag{4}$$

Just as in the previous two examples, the lowest-possible energy is exactly

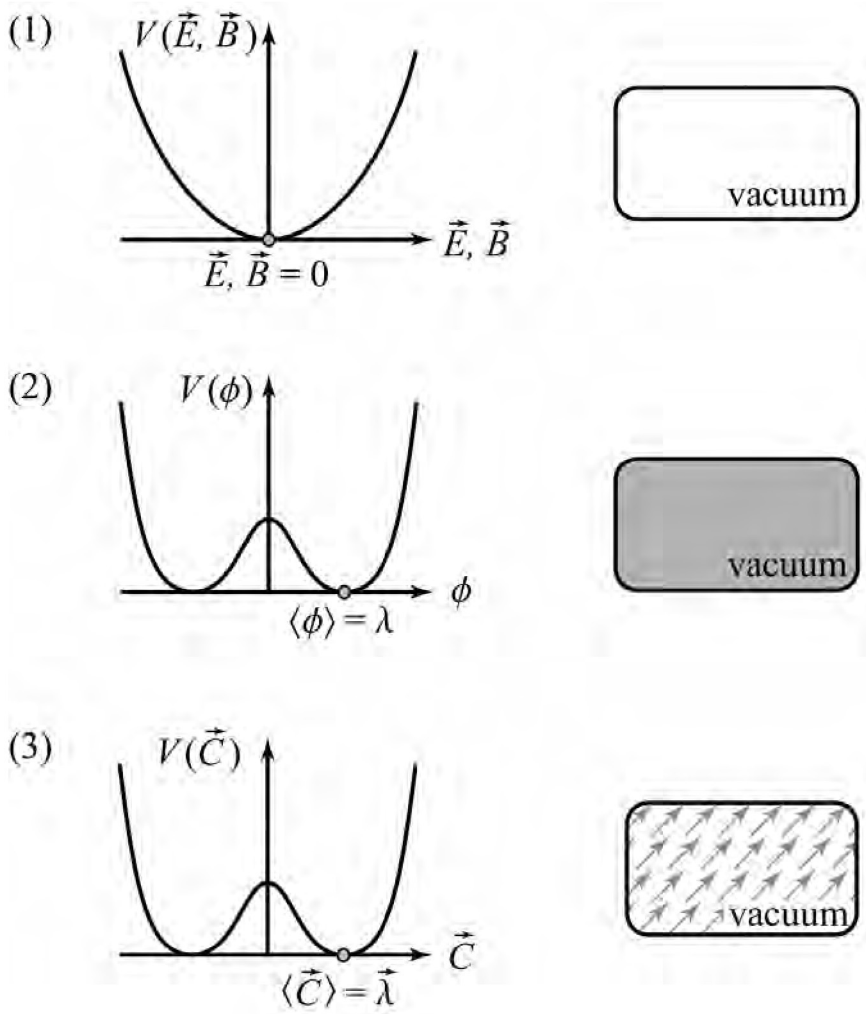


Figure 1: Spontaneous symmetry violation. In conventional electromagnetism (1), the lowest-energy state is attained for zero $\vec{E}$ and $\vec{B}$ fields. The vacuum remains essentially field free. For the Higgs-type field (2), interactions lead to an energy density $V(\phi)$ that forces a non-vanishing value of ϕ in the ground state. The vacuum is filled with a scalar condensate shown in gray. CPT and Lorentz invariance still hold (other, internal symmetries may be violated though). Vector fields occurring, for example, in string theory (3) can exhibit interactions similar to those of the Higgs requiring a nonzero field value in the lowest-energy state. The VEV of a vector field selects a preferred direction in the vacuum, which violates Lorentz and possibly CPT symmetry.

zero. As for the Higgs, this lowest energy configuration requires a nonzero $\vec{C}$. More specifically, we must demand $\vec{C}_{vac} \equiv \langle\vec{C}\rangle = \vec{\lambda}$, where $\vec{\lambda}$ is any constant vector satisfying $\vec{\lambda}^2 = \lambda^2$. Again, the vacuum does not remain empty, but it contains the VEV of our vector field. Because we have only considered constant solutions $\vec{C}$, $\langle\vec{C}\rangle$ is also spacetime independent (x dependence would lead to positive definite derivative terms in Eq. (4) raising the energy density). The true vacuum in the above model therefore contains an intrinsic direction determined by $\langle\vec{C}\rangle$ *violating rotation invariance and thus Lorentz symmetry.* We remark that interactions leading to energy densities like those in Eq. (4) are absent in conventional renormalizable gauge theories, but can be found in the context of strings, for example.

Spacetime-dependent scalars. A varying scalar, regardless of the mechanism driving the variation, typically implies the breaking of spacetime-translation invariance. [5] In Sec. 2 we have argued that translations and Lorentz transformations are closely linked in the Poincaré group, so that translation-symmetry violation typically leads to Lorentz breakdown. In the remainder of this section, we will focus on an explicit example for this effect.

Consider a system with a varying coupling $\xi(x)$ and scalar fields ϕ and Φ, such that the lagrangian $\mathcal{L}$ contains a term $\xi(x)\,\partial^\mu\phi\,\partial_\mu\Phi$. We may integrate the action for this system by parts (e.g., with respect to the first partial derivative in the above term) without affecting the equations of motion. An equivalent lagrangian $\mathcal{L}'$ would then be

$$\mathcal{L}' \supset -K^\mu\phi\,\partial_\mu\Phi\,. \tag{5}$$

Here, $K^\mu \equiv \partial^\mu\xi$ is an external nondynamical 4-vector, which selects a preferred direction in spacetime violating Lorentz symmetry. Note that for variations of ξ on cosmological scales, K^μ is constant locally to an excellent approximation—say on solar-system scales.

Intuitively, the violation of Lorentz symmetry in the presence of a varying scalar can be understood as follows. The 4-gradient of the scalar must be nonzero in some spacetime regions. This 4-gradient then selects a preferred direction in such regions (see Fig. 2). Consider, for instance, a particle that interacts with the scalar. Its propagation properties might be different in the directions parallel and perpendicular to the gradient. But physically inequivalent directions imply the violation of rotation invariance. Since rotations are contained in the Lorentz group, Lorentz symmetry must be broken.

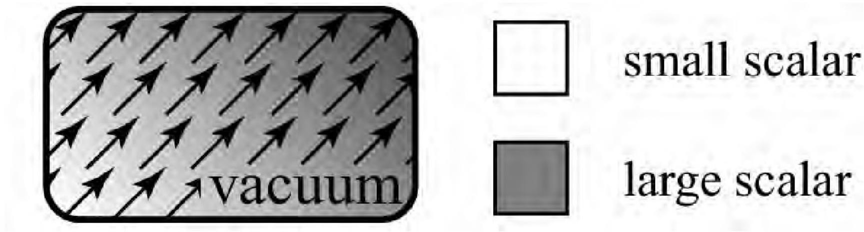


Figure 2: Lorentz violation through varying scalars. The background shade of gray corresponds to the value of the scalar: the lighter regions are associated with smaller values of the scalar. The gradient represented by the black arrows picks a preferred direction in the vacuum. It follows that Lorentz invariance is violated.

4 The Standard-Model Extension

To determine general low-energy manifestations of CPT and Lorentz violation and to identify specific experimental signatures for these effects, a suitable test model is needed. Many Lorentz tests are motivated and analyzed in purely kinematical frameworks allowing for small violations of Lorentz invariance. Examples are Robertson's framework and its Mansouri–Sexl extension, as well as the c^2 model and phenomenologically constructed modified dispersion relations. However, the CPT properties of these test models are unclear, and the lack of dynamical features severely restricts their scope. For this reason, the SME, already mentioned in Sec. 1, has been developed. The present section gives a brief review of the ideas behind the construction of the SME.

We begin by arguing in favor of dynamical rather than kinematical test models. The construction of a dynamical test framework is constrained by the demand that known physics must be recovered in certain limits, despite some residual freedom in introducing dynamical features compatible with a given set of kinematical rules. In addition, it appears difficult and may even be impossible to develop an effective theory containing the Standard Model with dynamics significantly different from that of the SME. We also mention that kinematical analyses are limited to only a subset of potential Lorentz-breakdown signatures from fundamental physics. From this perspective, it seems to be desirable to implement explicitly dynamical features of sufficient generality into test models for CPT and Lorentz symmetry.

The generality of the SME. To appreciate the generality of the SME, we review the main cornerstones of its construction. [9] Starting from the conventional Standard-Model lagrangian $\mathcal{L}_{\rm SM}$, Lorentz-breaking modifications $\delta\mathcal{L}$ are added:

$$\mathcal{L}_{\rm SME} = \mathcal{L}_{\rm SM} + \delta\mathcal{L} \ . \tag{6}$$

Here, the SME lagrangian is denoted by $\mathcal{L}_{\rm SME}$. The correction term $\delta\mathcal{L}$ is constructed by contracting Standard-Model field operators of any dimensionality with Lorentz-violating tensorial coefficients that describe a nontrivial vacuum with background vectors or tensors originating from the presumed effects in the underlying theory. Examples of such effects were discussed in the previous section. To guarantee coordinate independence, these contractions must give coordinate Lorentz scalars. It becomes thus apparent that all possible contributions to $\delta\mathcal{L}$ give the most general effective dynamical description of Lorentz breakdown at the level of observer Lorentz-invariant unitary quantum field theory. For simplicity, we have focused on nongravitational physics in the above line of reasoning. We remark, however, that the complete SME also contains an extended gravity sector.

Potential Planck-scale features, such as non-pointlike elementary particles or a discrete spacetime, are unlikely to invalidate the above effective-field-theory approach at currently attainable energies. On the contrary, the phenomenologically successful Standard Model is widely believed to be an effective-field-theory limit of more fundamental physics. If underlying physics indeed leads to minute Lorentz-violating effects, it would seem contrived to consider low-energy effective models outside the framework of effective quantum field theory. We finally remark that the necessity for a low-energy description beyond effective field theory is also unlikely to arise in the context of candidate fundamental models with novel Lorentz-*symmetric* aspects, such as additional particles, new symmetries, or large extra dimensions. Lorentz-invariant modifications can therefore be implemented into the SME, if needed. [10]

Advantages of the SME. The SME permits the identification and direct comparison of virtually all currently feasible experiments searching for Lorentz and CPT violation. Furthermore, certain limits of the SME correspond to classical kinematics test models of relativity (such as the previously mentioned Robertson's framework, its Mansouri-Sexl extension, or the

c^2 model). [11] Another advantage of the SME is the possibility of implementing further desirable features besides coordinate independence. For instance, one can choose to impose spacetime-translation invariance, SU(3)×SU(2)×U(1) gauge symmetry, power-counting renormalizability, hermiticity, and pointlike interactions. These demands further restrict the parameter space for Lorentz violation. One could also adopt simplifying choices, such as a residual rotational invariance in certain coordinate systems. This latter assumption together with additional simplifications of the SME has been considered in the literature. [12]

Analyses performed within the SME. At present, the flat-spacetime limit of the minimal SME has provided the basis for numerous investigations of CPT and Lorentz violation involving mesons, [13, 14, 15, 16, 17, 18, 19] baryons, [20, 21, 22] electrons, [23, 24, 25] photons, [26, 11] muons, [27] and the Higgs sector. [28] Studies involving the gravity sector have recently also been performed. [29] We remark that neutrino-oscillation experiments offer the potential for discovery. [9, 30, 31] CPT and Lorentz tests with mesons will be discussed further in the next section.

5 CPT and Lorentz tests with mesons

Some of the CPT and Lorentz tests listed in the previous section involve some form of antimatter. As pointed out earlier, certain matter–antimatter comparisons are extremely sensitive to CPT violations because CPT symmetry connects particles and antiparticles. This idea can be adopted for studies with mesons. Neutral-meson oscillations are essentially controlled by the energy difference between the meson and its antimeson. Although the SME contains the same mass parameter for quarks and antiquarks, these particles are affected differently by the CPT- and Lorentz-violating background. This allows the dispersion relations for mesons and antimesons to differ, so that mesons and antimesons can have distinct energies. This effect is potentially observable with interferometric methods. The present section contains a more detailed discussion of this idea.

We begin by recalling that any neutral-meson state is a linear combination of the Schrödinger wave functions for the meson P^0 and its antimeson $\overline{P^0}$. If this state is viewed as a two-component object $\Psi(t)$, its time evolution is controlled by a 2×2 effective hamiltonian Λ according to the Schrödinger-type equation [32] $i\partial_t\Psi = \Lambda\Psi$. Although the effective hamiltonian Λ is different for each neutral-meson system, we use a single symbol here for simplicity. The eigenstates $|P_a\rangle$ and $|P_b\rangle$ of Λ are the physical propagating states of the neutral-meson system. They exhibit the usual time evolution

$$|P_a(t)\rangle = \exp(-i\lambda_a t)|P_a\rangle \ , \quad |P_b(t)\rangle = \exp(-i\lambda_b t)|P_b\rangle \ , \tag{7}$$

where the complex parameters λ_a and λ_b are the eigenvalues of Λ. They can be written in terms of the physical masses m_a, m_b and decay rates γ_a, γ_b of the propagating particles:

$$\lambda_a \equiv m_a - \tfrac{1}{2}i\gamma_a \ , \quad \lambda_b \equiv m_b - \tfrac{1}{2}i\gamma_b \ . \tag{8}$$

For convenience, one usually works with the sum and difference of the eigenvalues instead:

$$\begin{aligned} \lambda &\equiv \lambda_a + \lambda_b = m - \tfrac{1}{2}i\gamma \ , \\ \Delta\lambda &\equiv \lambda_a - \lambda_b = -\Delta m - \tfrac{1}{2}i\Delta\gamma \ . \end{aligned} \tag{9}$$

Here, we have defined $m = m_a + m_b$, $\Delta m = m_b - m_a$, $\gamma = \gamma_a + \gamma_b$, and $\Delta\gamma = \gamma_a - \gamma_b$.

The effective hamiltonian Λ is a 2×2 complex matrix, and as such it contains eight real parameters for the neutral-meson system under consideration. Four of these correspond to the two masses and decay rates. Among the remaining four parameters are three that determine the extent of indirect CP violation in the neutral-meson system and one that is an unobservable phase. Indirect CPT violation in this system occurs if and only if the difference $\Delta\Lambda \equiv \Lambda_{11} - \Lambda_{22}$ of the diagonal elements of Λ is nonzero. It follows that Λ contains two real parameters for CPT breakdown. On the other hand, indirect T violation occurs if and only if the magnitude of the ratio $|\Lambda_{21}/\Lambda_{12}|$ of the off-diagonal components of Λ differs from 1. The effective hamiltonian therefore contains also one real parameter for T violation.

Various explicit parametrizations of Λ are possible. However, for the heavy meson systems D, B_d, B_s, less is known about CPT and T violation than for the K system. It is therefore desirable to employ a general parametrization of the effective hamiltonian Λ that is independent of phase conventions, [33)] valid for arbitrary-size CPT and T breaking, model independent, and expressed in terms of mass and decay rates insofar as possible. Such a convenient parametrization can be achieved by writing two diagonal elements of

Λ as the sum and difference of two complex numbers, and the two off-diagonal elements as the product and ratio of two other complex numbers: [19]

$$\Lambda = \tfrac{1}{2}\Delta\lambda \begin{pmatrix} U+\xi & VW^{-1} \\ VW & U-\xi \end{pmatrix} . \qquad (10)$$

In this definition, $UVW\xi$ are dimensionless complex numbers. The requirement that the trace of Λ is tr $\Lambda = \lambda$ and that its determinant is $\det\Lambda = \lambda_a\lambda_b$ fixes the complex parameters U and V:

$$U \equiv \lambda/\Delta\lambda \, , \quad V \equiv \sqrt{1-\xi^2} \, . \qquad (11)$$

The CPT and T properties of the effective hamiltonian (10) are now determined in the complex numbers $W = w\exp(i\omega)$ and $\xi = \mathrm{Re}\,\xi + i\mathrm{Im}\,\xi$. Of the four real components, the phase angle ω of W is physically irrelevant. The remaining three components are physical, with $\mathrm{Re}\,\xi$ and $\mathrm{Im}\,\xi$ describing CPT violation and the modulus $w \equiv |W|$ of W governing T breaking. Their relation to the components of Λ are

$$\xi = \Delta\Lambda/\Delta\lambda \, , \quad w = \sqrt{|\Lambda_{21}/\Lambda_{12}|} \, . \qquad (12)$$

CPT conservation requires $\mathrm{Re}\,\xi = \mathrm{Im}\,\xi = 0$, while T conservation requires $w = 1$. The eigenstates of Λ, which are the physical states of definite masses and decay rates, can also be obtained in a straightforward way. [19]

We remark in passing that the $w\xi$ formalism above can be related to other formalisms used in the literature provided appropriate assumptions about the phase conventions and the smallness of CP violation are made. [19] For example, in the K system the widely adopted [32] formalism involving ϵ_K and δ_K depends on the phase convention, and it can be applied only if CPT and T violation are small. Under this assumption and in a special phase convention, δ_K is related to ξ_K by $\xi_K \approx 2\delta_K$.

Thus far, we have discussed the phenomenological description of neutral-meson oscillations with particular emphasis on CPT violation. We next review how the phenomenological CPT-breaking parameters above are connected to coefficients in the SME. Since the minimal SME is a relativistic unitary quantum field theory, it satisfies the conditions for Greenberg's "anti-CPT theorem," which states that CPT breaking must come with Lorentz violation. Without any calculations we can therefore conclude already at this point that δ_K, for example, cannot be constant. In particular, it will typically be direction dependent. This fact is further illustrated in Fig. 3.

The leading CPT-breaking contributions to Λ can be calculated perturbatively in the coefficients for CPT and Lorentz violation that appear in the SME. These corrections are expectation values of CPT- and Lorentz-violating interactions in the hamiltonian for the theory, [17] evaluated with the unperturbed wave functions $|P^0\rangle$, $|\overline{P^0}\rangle$ as usual. Note that the hermiticity of the perturbation hamiltonian ensures real contributions.

To determine an expression for the parameter $\xi_K \approx 2\delta_K$, one needs to find the difference $\Delta\Lambda = \Lambda_{11} - \Lambda_{22}$ of the diagonal terms of Λ. A calculation within the SME gives [19]

$$\Delta\Lambda \approx \beta^\mu \Delta a_\mu \ , \tag{13}$$

where $\beta^\mu = \gamma(1, \vec{\beta})$ is the four-velocity of the meson state in the observer frame. In this equation, we have defined $\Delta a_\mu = r_{q_1} a_\mu^{q_1} - r_{q_2} a_\mu^{q_2}$, where $a_\mu^{q_1}$, $a_\mu^{q_2}$ are coefficients for CPT and Lorentz breaking for the two valence quarks in the P^0 meson. These coefficients have mass dimension one, and they arise from lagrangian terms of the form $-a_\mu^q \overline{q}\gamma^\mu q$, where q specifies the quark flavor. The quantities r_{q_1}, r_{q_2} characterize normalization and quark-binding effects. [17]

We see that among the consequences of CPT and Lorentz breakdown are the 4-velocity and hence 4-momentum dependence of observables, as expected from our above considerations involving the "anti-CPT theorem." It follows that the standard assumption of a constant parameter ξ for CPT violation fails under the very general condition of unitary quantum field theory. In particular, the presence of the 4-velocity in Eq. (13) implies that CPT observables will typically vary with the magnitude and orientation of the meson momentum. This can have major consequences for experimental investigations, since the meson momentum spectrum and angular distribution now contribute directly to the determination of the experimental CPT reach.

An important effect of the 4-momentum dependence is the appearance of sidereal variations in some CPT observables: the vector $\Delta\vec{a}$ is constant, while the Earth rotates in a celestial equatorial frame. Because a laboratory frame is employed for the derivation of Eq. (13), and since this frame is rotating, observables can exhibit sidereal variations. This is schematically depicted in Fig. 3. To display explicitly this sidereal-time dependence, one can transform

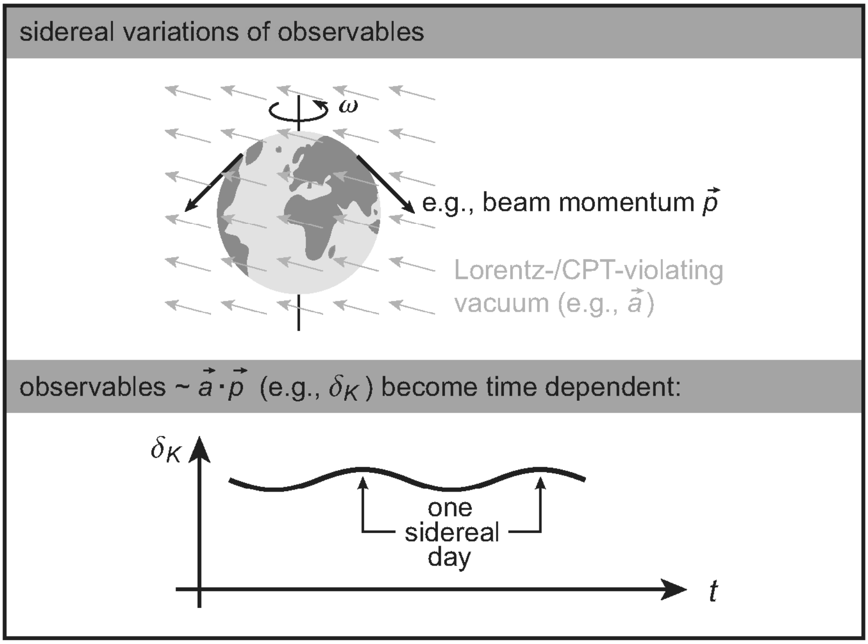


Figure 3: Sidereal variations. Experiments are typically associated with an intrinsic direction. For instance, particle-accelerator experiments have a characteristic beam direction determined by the set-up of the accelerator. As the Earth rotates, this direction will change because the accelerator is attached to the Earth. In the above figure, a beam direction $\vec{p}$ pointing south is shown at two times separated by approximately 12 hours (black arrows). The angle between the Lorentz-violating background (gray $\vec{a}$ arrows) and the orientation of the beam direction is clearly different at these two times. An observable, such as the phase δ_K, may for example acquire a correction $\sim \vec{p}\cdot\vec{a}$ that leads to the shown sidereal modulation.

the expression (13) for $\Delta\Lambda$ from the laboratory frame to a nonrotating frme. To this end, let us denote the spatial basis in the laboratory frame by $(\hat{x}, \hat{y}, \hat{z})$ and that in the nonrotating frame by $(\hat{X}, \hat{Y}, \hat{Z})$. We next choose the $\hat{z}$ axis in the laboratory frame for maximal convenience. For instance, the beam direction is a natural choice for the case of collimated mesons, while the collision axis could be adopted in a collider. We further define the nonrotating-frame basis $(\hat{X}, \hat{Y}, \hat{Z})$ to be consistent with celestial equatorial coordinates, with $\hat{Z}$ aligned along the Earth's rotation axis. For the observation of sidereal variations we must have $\cos\chi = \hat{z} \cdot \hat{Z} \neq 0$. It then follows that $\hat{z}$ precesses about $\hat{Z}$ with the Earth's sidereal frequency Ω. The complete transformation between the two bases can be found in the literature. [20] In particular, any coefficient $\vec{a}$ for Lorentz breakdown with laboratory-frame components (a^1, a^2, a^3) possesses nonrotating-frame components (a^X, a^Y, a^Z). This transformation determines the time dependence of $\Delta\vec{a}$ and hence the sidereal variation of $\Delta\Lambda$. The entire momentum and sidereal-time dependence of the CPT-breaking parameter ξ in any P system can then be extracted.

To give an explicit expression for the final answer for ξ, define θ and ϕ to be standard polar coordinates about the $\hat{z}$ axis in the laboratory frame. In general, the laboratory-frame 3-velocity of a P meson can then be written as $\vec{\beta} = \beta(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. It follows that the magnitude of the momentum obeys $p \equiv |\vec{p}| = \beta m_P \gamma(p)$, where $\gamma(p) = \sqrt{1 + p^2/m_P^2}$ as usual. In terms of these quantities and the sidereal time $\hat{t}$, the result for ξ takes the form [19]

$$\begin{aligned}
\xi &\equiv \xi(\hat{t}, \vec{p}) \equiv \xi(\hat{t}, p, \theta, \phi) \\
&= \frac{\gamma(p)}{\Delta\lambda} \big\{ \Delta a_0 + \beta \Delta a_Z (\cos\theta\cos\chi - \sin\theta\cos\phi\sin\chi) \\
&\qquad + \beta \big[\Delta a_Y (\cos\theta\sin\chi + \sin\theta\cos\phi\cos\chi) \\
&\qquad\qquad - \Delta a_X \sin\theta\sin\phi \big] \sin\Omega\hat{t} \\
&\qquad + \beta \big[\Delta a_X (\cos\theta\sin\chi + \sin\theta\cos\phi\cos\chi) \\
&\qquad\qquad + \Delta a_Y \sin\theta\sin\phi \big] \cos\Omega\hat{t} \big\} \ . \qquad (14)
\end{aligned}$$

The experimental challenge is the measurement the four independent coefficients Δa_μ for CPT breakdown allowed by quantum field theory. The result (14) shows that suitable binning of data in sidereal time, momentum magnitude, and orientation has the potential to extract four independent constraints from any observable with a nontrivial ξ dependence. Note that each one of the neutral-meson systems may have different values of these coefficients. As a result of the distinct masses and decay rates, the physics of each system is distinct. A complete experimental study of CPT breaking requires four independent measurements in each system.

6 Experiments

To date, various CPT tests with neutral mesons have been analyzed within the SME. Other current and future experiments offer the possibility to tighten these existing constraints or extract bounds on other CPT-violation coefficients in the SME. This section contains a brief account of this topic with focus on the KLOE or KLOE-II detectors.

As argued in the previous section, a key issue in the analysis of experimental data is magnitude of the meson momentum and its orientation relative to the CPT- and Lorentz-violating coefficient Δa^μ. The orientation depends on the experimental set-up, so that different experiments are sensitive to different combinations of Δa^μ components. One important parameter is the beam direction, which is usually fixed with respect to the laboratory. Since the Earth, and thus the laboratory, rotates with respect to Δa^μ, the beam direction relative to Δa^μ is determined by the date and the time of the day. This requires time binning for any neutral-meson experiment with sensitivity to $\Delta\vec{a}$.

In a fixed-target measurement at high enough energies, the momenta of the produced mesons are aligned with the beam direction to a good approximation, and no further directional information in addition to the time stamp of the event needs to be recorded. These experiments typically involve uncorrelated mesons, which further simplifies their conceptual analysis. We have $\beta_\mu \Delta a^\mu = (\beta^0 \Delta a^0 - \Delta\vec{a}_\| \cdot \vec{\beta}_\|) - (\Delta\vec{a}_\perp \cdot \vec{\beta}_\perp)$, where $\|$ and $\perp$ are taken with respect to the Earth's rotation axis. We see that in principle all four components of Δa^μ can be determined: the $\perp$ components via their sidereal variations and the sidereally constant components in the first parentheses via their dependence on the momentum magnitude. However, under our initial assumption of high energies the variation of $|\vec{\beta}|$ with the energy is tiny, which makes it difficult to disentangle the individual components Δa^0 and $\Delta\vec{a}_\|$. On the other hand, high energies are associated with large boost factors, which increase the overall CPT reach for the other combinations of Δa^μ components.

These ideas have been applied in experiments with the K and D systems. For the K system, two independent CPT measurements of different combinations of the coefficients Δa_μ have been performed. [13, 19] One measurement constrains a linear combination of Δa_0 and Δa_Z to about 10^{-20} GeV, and the other bounds a combination of Δa_X and Δa_Y to 10^{-21} GeV. These experiments were performed with mesons highly collimated in the laboratory frame. In this case, ξ simplifies because the 3-velocity takes the form $\vec{\beta} = (0, 0, \beta)$. Binning in $\hat{t}$ yields sensitivity to the equatorial components Δa_X, Δa_Y. On the other hand, averaging over $\hat{t}$ eliminates these components altogether.

For the D-meson system, two independent bounds have been obtained by the FOCUS experiment. [14] They constrain a linear combination of Δa_0 and Δa_Z to about 10^{-16} GeV, and they bound Δa_Y also to roughly 10^{-16} GeV. Notice that CPT constraints in the D system are unique in that the valence quarks involved are the u and the c, whereas the other neutral mesons involve the d, s, and b.

CPT measurements are also possible for correlated meson pairs in a symmetric collider. This experimental set-up is relevant for the KLOE and KLOE-II experiments at the Frascati laboratory, and it differs significantly from that in the previous paragraph. In particular, the energy dependence is essentially irrelevant: the kaon pairs are produced in the decay of ϕ quarkonium just above threshold leading to approximately monoenergetic kaons. Moreover, the boost factor does not substantially improve the CPT reach. On the other hand, the wide angular distribution of the kaons in the laboratory frame requires angular binning in addition to date/time binning to reconstruct the direction of β^μ with respect to Δa^μ. Moreover, the correlation of the meson pairs can give additional observational information. We will see that these two features would allow the extraction of independent constraints on four components of Δa^μ.

Consider a ϕ quarkonium state with $J^{PC} = 1^{--}$ decaying at time t in its rest frame into a correlated K-$\overline{K}$ pair.[2] Since the laboratory frame is unboosted relative to the quarkonium rest frame, the time t may be taken as the sidereal time. Subsequently, one of the kaons decays into f_1 at time $t + t_1$, while the other decays into f_2 at time $t + t_2$. Then, standard arguments yield

$R_{12}(\vec{p}, t, \bar{t}, \Delta t) =$

[2]The line of reasoning for B_d, B_s, and D mesons would be similar.

$$|\hat{N}|^2 e^{-\overline{\gamma}\overline{t}/2} \left[|\eta_1|^2 e^{-\Delta\gamma\Delta t/2} + |\eta_2|^2 e^{\Delta\gamma\Delta t/2} - 2|\eta_1\eta_2|\cos(\Delta m \Delta t + \Delta\phi) \right] \quad (15)$$

for the double-decay rate. In this equation, η_α denotes the following ratio of amplitudes $A(K_L \to f_\alpha)/A(K_S \to f_\alpha)$, and $\hat{N}$ is a normalization containing the factor $A(K_L \to f_1)A(K_S \to f_2)$. We have further defined $\overline{t} = t_1 + t_2$, $\Delta t = t_2 - t_1$, $\overline{\gamma} = \gamma_S + \gamma_L$, $\Delta\gamma = \gamma_L - \gamma_S$, and $\Delta\phi = \phi_1 - \phi_2$. The amplitudes $A(K_{L/S} \to f_\alpha)$ may be functions of the momentum $\vec{p}_1 = -\vec{p}_2 \equiv \vec{p}$ and the sidereal time t via a possible dependence on $\Delta\Lambda$. It follows that the effects of potential CPT violations in $R_{12}(\vec{p}, t, \overline{t}, \Delta t)$ are contained in η_α and $\hat{N}$.

A detailed study of the CPT signals from symmetric-collider experiments with correlated kaons requires analyses with expressions of the type (15) for various final states f_1, f_2. With sufficient experimental resolution, the dependence of certain decays on the two meson momenta $\vec{p}_1$, $\vec{p}_2$ and on the sidereal time t could be measured by appropriate data binning and analysis. We note that different asymmetries can be sensitive to distinct components of $\Delta\Lambda$, so that some care is required in such investigations.

Let us consider the sample case of double-semileptonic decays of correlated kaon pairs in a symmetric collider. Assuming the $\Delta S = \Delta Q$ rule, one can show that the double-decay rate $R_{l^+l^-}$ can be regarded as proportional to an expression depending on the ratio [19]

$$\left| \frac{\eta_{l^+}}{\eta_{l^-}} \right| \approx 1 - \frac{4\mathrm{Re}\,(i \sin\hat{\phi}\ e^{i\hat{\phi}})}{\Delta m} \gamma(\vec{p}) \Delta a_0 \quad . \quad (16)$$

In this expression, $\hat{\phi} \equiv \tan^{-1}(2\Delta m/\Delta\gamma)$ is sometimes called the superweak angle. Note the absence of all angular and time dependence in Eq. (16). This fact arises because for a symmetric collider we have $\vec{\beta}_1 \cdot \Delta\vec{a} = -\vec{\beta}_2 \cdot \Delta\vec{a}$, which leads to a cancellation between the contributions from each kaon.

In this form for the double-decay rate $R_{l^+l^-}$, any angular and momentum dependence can therefore only enter through the overall factor of $|\hat{N}\eta_{l^-}|^2$. The measurement of such a normalizing factor is experimentally challenging. For example, the normalization factor would cancel in a conventional analysis to extract the physics using the usual asymmetry. Another obstacle is the line spectrum mentioned above, so that the dependence on $|\vec{p}|$ is unobservable. We conclude that the double-semileptonic decay channel is well suited to place a clean bound on the timelike parameter Δa_0 for CPT breakdown, and the experimental data may be collected for analysis without regard to their angular locations in the detector or their sidereal time stamps.

Apart from the double-semileptonic channel, there are also other decay possibilities for the two kaons. Among these are mixed double decays, in which only one of the two kaons has a ξ_K-sensitive mode. For such asymmetric decay products, there is no longer a cancellation of the spatial contributions of Δa^μ, and independent bounds on three of its components may become possible. One example for such a double-decay mode is a channel with one semileptonic prong and one double-pion prong. Note that in a conventional CPT analysis, a given double-decay mode of this type is inextricably connected with other parameters for CP violation. [34, 35, 36] However, in the present context the possibility of angular and time binning implies that clean tests of CPT breaking are feasible even for these mixed modes.

As a sample set-up, consider a detector with acceptance independent of the azimuthal angle ϕ. The distribution of mesons from the quarkonium decay is symmetric in ϕ, so the ξ_K dependence of a ϕ-averaged dataset is determined by

$$\begin{aligned} \delta_K^{\rm av}(|\vec{p}|,\theta,t) &\equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi\ \xi_K(\vec{p},t)/2 \\ &= \frac{i\sin\hat{\phi}\ e^{i\hat{\phi}}}{\Delta m}\gamma\,[\Delta a_0 + \beta\Delta a_Z\cos\chi\cos\theta \\ &\qquad +\beta\Delta a_Y\sin\chi\cos\theta\sin\Omega t \\ &\qquad +\beta\Delta a_X\sin\chi\cos\theta\cos\Omega t]\ . \end{aligned} \tag{17}$$

Inspection of this equation establishes that by measuring the θ and t dependences an experiment with asymmetric double-decay modes can in principle extract separate constraints on each of the three components of the parameter $\Delta\vec{a}$ for CPT breakdown. We remark that this result holds independent of other CP parameters that may appear because the latter neither possess angular nor time dependence. It follows that a combination of data from asymmetric double-decay modes and from double-semileptonic modes permits in principle the extraction of independent constraints on each of the four components of Δa_μ.

Similar arguments can be made for other experimental observables. Consider, for instance, the standard rate asymmetry for K_L semileptonic decays [6)]

$$\delta_l \equiv \frac{\Gamma(K_L \to l^+\pi^-\nu) - \Gamma(K_L \to l^-\pi^+\overline{\nu})}{\Gamma(K_L \to l^+\pi^-\nu) + \Gamma(K_L \to l^-\pi^+\overline{\nu})}$$
$$\approx 2\mathrm{Re}\,\epsilon_K - \mathrm{Re}\,\xi_K(\vec{p},t) \ . \qquad (18)$$

Here, the symbol Γ denotes a partial decay rate, and violations of the $\Delta S = \Delta Q$ rule have been neglected. In principle, this asymmetry could also be investigated for angular and time dependencies, which would lead to bounds on Δa_μ. From the forward–backward asymmetry of this expression, a preliminary bound at the level of 10^{-17} GeV on the Δa_Z coefficient for the kaon can been obtained by KLOE. [37)] If confirmed, this would be the first clean constraint on this coefficient.

We finally mention another experimental set-up. Suppose the quarkonium is not produced at rest, but with a sufficient net momentum, such as in an asymmetric collider. Then, $\xi_1 + \xi_2$ does not cancel and could be sensitive to all four coefficients Δa_μ for the neutral-meson system under investigation. It follows that appropriate data binning would also allow up to four independent CPT measurements. The existing asymmetric B_d factories BaBar and BELLE would be able to undertake measurements of these types. [15)] Preliminary results from the BaBar experiment constrain various component combinations of Δa^μ for the B_d meson to about 10^{-13} GeV. [16)] We also mention that the same study does find a 2.2σ signal for sidereal variations. [16)] While this level of significance is still consistent with no effect, it clearly motivates further experimental CPT- and Lorentz-violation searches in neutral-meson systems.

7 Summary

Although both CPT and Lorentz invariance are deeply ingrained in the currently accepted laws of physics, there are a variety of candidate underlying theories that could generate the breakdown of these symmetries. The sensitivity attainable in matter–antimatter comparisons offers the possibility for CPT-breakdown searches with Planck precision. Lorentz-symmetry tests open an additional avenue for CPT measurements because CPT violation implies Lorentz violation.

A potential source of CPT and Lorentz breaking is spontaneous symmetry violation in string field theory. Because this mechanism is theoretically very attractive, and because strings show great potential as a candidate fundamental theory, this Lorentz-violation origin is particularly promising. CPT and Lorentz breaking can also originate from spacetime-dependent scalars: the gradient of such scalars selects a preferred direction in the effective vacuum. This mechanism for Lorentz violation might be of interest in light of recent claims of a time-dependent fine-structure parameter and the presence of time-dependent scalar fields in various cosmological models.

The leading-order CPT- and Lorentz-violating effects that would emerge from Lorentz-symmetry breaking in approaches to fundamental physics are described by the SME. At the level of effective quantum field theory, the SME is the most general dynamical framework for Lorentz and CPT violation that is compatible with the fundamental principle of unitarity. Experimental studies are therefore best performed within the SME.

Neutral-meson interferometry is an excellent high-sensitivity tool in experimental searches for Planck-scale physics. In the context of unitary quantum field theory, potential CPT violations come with Lorentz breaking, which then typically leads to direction- and energy-dependent CPT-violation observables. For Earth-based tests, this effect leads to sidereal variations, which typically requires momentum and time binning in experiments. Within the minimal SME, there are four independent coefficients for CPT breaking in each meson system. Observational constraints in the order of 10^{-13} down to 10^{-21} GeV have been obtained for a subset of these coefficients. In general, tests with neutral mesons bound parameter combinations of the SME inaccessible by other experiments. The KLOE and the planned KLOE-II experiments with their symmetric set-up offer unique opportunities for CPT tests along these lines. Such measurements would give further insight into the enigmatic kaon system, and they have the potential to probe Planck-scale physics.

Acknowledgments

The author would like to thank Antonio Di Domenico for organizing this stimulating meeting, for the invitation to attend, and for financial support. This work was supported in part by the European Commission under Grant No. MOIF-CT-2005-008687.

References

1. See, e.g., R.G. Sachs, *The Physics of Time Reversal*, University of Chicago Press, Chicago, 1987.
2. V.A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); Phys. Rev. Lett. **63**, 224 (1989); **66**, 1811 (1991); V.A. Kostelecký and R. Potting, Nucl. Phys. B **359**, 545 (1991); Phys. Lett. B **381**, 89 (1996); Phys. Rev. D **63**, 046007 (2001); V.A. Kostelecký *et al.*, Phys. Rev. Lett. **84**, 4541 (2000).
3. G. Amelino-Camelia *et al.*, Nature (London) **393**, 763 (1998); D. Sudarsky *et al.*, Phys. Rev. D **68**, 024010 (2003).
4. F.R. Klinkhamer, Nucl. Phys. B **578**, 277 (2000).
5. V.A. Kostelecký *et al.*, Phys. Rev. D **68**, 123511 (2003); O. Bertolami *et al.*, Phys. Rev. D **69**, 083513 (2004).
6. W.-M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).
7. O.W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002).
8. For a somewhat more pedestrian exposition, see O.W. Greenberg, hep-ph/0309309.
9. D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); **58**, 116002 (1998); V.A. Kostelecký and R. Lehnert, Phys. Rev. D **63**, 065008 (2001); V.A. Kostelecký, Phys. Rev. D **69**, 105009 (2004); R. Bluhm and V.A. Kostelecký, Phys. Rev. D **71**, 065008 (2005).
10. M.S. Berger and V.A. Kostelecký, Phys. Rev. D **65**, 091701(R) (2002); H. Belich *et al.*, Phys. Rev. D **68**, 065030 (2003); M.S. Berger, Phys. Rev. D **68**, 115005 (2003).
11. V.A. Kostelecký and M. Mewes, Phys. Rev. D **66**, 056005 (2002).
12. S. Coleman and S.L. Glashow, Phys. Rev. D **59**, 116008 (1999).
13. KTeV Collaboration, H. Nguyen, hep-ex/0112046; Y.B. Hsiung *et al.*, Nucl. Phys. Proc. Suppl. **86**, 312 (2000).

14. FOCUS Collaboration, J.M. Link *et al.*, Phys. Lett. B **556**, 7 (2003).

15. OPAL Collaboration, R. Ackerstaff *et al.*, Z. Phys. C **76**, 401 (1997); DELPHI Collaboration, M. Feindt *et al.*, preprint DELPHI 97-98 CONF 80 (1997); BELLE Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **86**, 3228 (2001); BaBar Collaboration, B. Aubert *et al.*, hep-ex/0303043.

16. BaBar Collaboration, B. Aubert *et al.*, hep-ex/0607103.

17. V.A. Kostelecký and R. Potting, Phys. Rev. D **51**, 3923 (1995).

18. D. Colladay and V.A. Kostelecký, Phys. Lett. B **344**, 259 (1995); Phys. Rev. D **52**, 6224 (1995); V.A. Kostelecký and R. Van Kooten, Phys. Rev. D **54**, 5585 (1996); O. Bertolami *et al.*, Phys. Lett. B **395**, 178 (1997); N. Isgur *et al.*, Phys. Lett. B **515**, 333 (2001).

19. V.A. Kostelecký, Phys. Rev. Lett. **80**, 1818 (1998); Phys. Rev. D **61**, 016002 (2000); Phys. Rev. D **64**, 076001 (2001).

20. D. Bear *et al.*, Phys. Rev. Lett. **85**, 5038 (2000); D.F. Phillips *et al.*, Phys. Rev. A **62**, 063405 (2000); Phys. Rev. D **63**, 111101 (2001); M.A. Humphrey *et al.*, Phys. Rev. A **68**, 063807 (2003); V.A. Kostelecký and C.D. Lane, Phys. Rev. D **60**, 116010 (1999); J. Math. Phys. **40**, 6245 (1999).

21. R. Bluhm *et al.*, Phys. Rev. Lett. **88**, 090801 (2002).

22. F. Canè *et al.*, Phys. Rev. Lett. **93**, 230801 (2004); P. Wolf *et al.*, Phys. Rev. Lett. **96**, 060801 (2006).

23. H. Dehmelt *et al.*, Phys. Rev. Lett. **83**, 4694 (1999); R. Mittleman *et al.*, Phys. Rev. Lett. **83**, 2116 (1999); G. Gabrielse *et al.*, Phys. Rev. Lett. **82**, 3198 (1999); R. Bluhm *et al.*, Phys. Rev. Lett. **82**, 2254 (1999); Phys. Rev. Lett. **79**, 1432 (1997); Phys. Rev. D **57**, 3932 (1998); C.D. Lane, Phys. Rev. D **72**, 016005 (2005).

24. L.-S. Hou *et al.*, Phys. Rev. Lett. **90**, 201101 (2003); R. Bluhm and V.A. Kostelecký, Phys. Rev. Lett. **84**, 1381 (2000); B.R. Heckel *et al.*, Phys. Rev. Lett. **97**, 021603 (2006).

25. H. Müller *et al.*, Phys. Rev. D **68**, 116006 (2003); R. Lehnert, J. Math. Phys. **45**, 3399 (2004); B. Altschul, Phys. Rev. Lett. **96**, 201101 (2006).

26. S.M. Carroll *et al.*, Phys. Rev. D **41**, 1231 (1990); V.A. Kostelecký and M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001); J. Lipa *et al.*, Phys. Rev. Lett. **90**, 060403 (2003); Q. Bailey and V.A. Kostelecký, Phys. Rev. D **70**, 076006 (2004); R. Lehnert and R. Potting, Phys. Rev. Lett. **93**, 110402 (2004); Phys. Rev. D **70**, 125010 (2004); B. Feng *et al.*, Phys. Rev. Lett. **96**, 221302 (2006); V.A. Kostelecký and M. Mewes, Phys. Rev. Lett., in press (hep-ph/0607084); B. Altschul, hep-th/0609030.

27. V.W. Hughes *et al.*, Phys. Rev. Lett. **87**, 111804 (2001); R. Bluhm *et al.*, Phys. Rev. Lett. **84**, 1098 (2000); E.O. Iltan, JHEP **0306**, 016 (2003).

28. D.L. Anderson *et al.*, Phys. Rev. D **70**, 016001 (2004).

29. Q.G. Bailey and V.A. Kostelecký, Phys. Rev. D **74**, 045001 (2006).

30. V. Barger *et al.*, Phys. Rev. Lett. **85**, 5055 (2000); J.N. Bahcall *et al.*, Phys. Lett. B **534**, 114 (2002); V.A. Kostelecký and M. Mewes, Phys. Rev. D **70**, 031902 (2004); Phys. Rev. D **70**, 076002 (2004); T. Katori, V.A. Kostelecký, and R. Tayloe, hep-ph/0606154.

31. V.A. Kostelecký and M. Mewes, Phys. Rev. D **69**, 016005 (2004).

32. See, for example, T.D. Lee and C.S. Wu, Annu. Rev. Nucl. Sci. **16**, 511 (1966).

33. L. Lavoura, Ann. Phys. **207**, 428 (1991).

34. C.D. Buchanan *et al.*, Phys. Rev. D **45**, 4088 (1992).

35. I. Dunietz, J. Hauser, and J.L. Rosner, Phys. Rev. D **35**, 2166 (1987).

36. M. Hayakawa and A.I. Sanda, Phys. Rev. D **48**, 1150 (1993).

37. A. Di Domenico, this handbook.

ON THE QUANTUM-GRAVITY PHENOMENOLOGY OF MULTIPARTICLE STATES

Giovanni Amelino-Camelia
Dipartimento di Fisica Università di Roma La Sapienza and Sez. Roma1 INFN, P. le A. Moro 2, 00185 Roma, Italy
Michele Arzano
Perimeter Institute for Theoretical Physics, 31 Caroline St. N, N2L 2Y5, Waterloo, Ontario, Canada
Antonino Marcianò
Dipartimento di Fisica Università di Roma La Sapienza and Sez. Roma1 INFN, P. le A. Moro 2, 00185 Roma, Italy

Abstract

We discuss some general expectations concerning the structure of multiparticle states in the quantum-gravity realm, and we introduce the first elements of a toy model which could be used as guidance in the estimate of some associated effects. We also provide a brief review of "quantum gravity phenomenology" and comment on how the study of multiparticle states could contribute to the overall development of this field.

1 Introduction

The "quantum-gravity problem" has been discussed for more than 70 years [1] assuming that no guidance could be obtained from experiments. Indeed, it is not unlikely that experiments might never give us any clear lead toward quantum gravity, especially if our intuition concerning the role of the tiny Planck length ($\sim 10^{-35}m$) in setting the magnitude of the characteristic effects of the new theory turns out to be correct. But over the past decade or so a growing number of research groups is working hard [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] at trying to find ways to uncover experimentally some manifestations of quantum gravity, even if the new effects were really so small.

Our estimate that the quantum-gravity corrections should be very small in low-energy experiments is based on our experience with other similar situations; in fact, we expect that the Planck scale, since it is the energy scale where the current theories appear to break down, should also govern the magnitude of quantum-gravity corrections to the analysis of processes involving particles with energies smaller than the Planck scale. For example in processes involving two particles both with energy E the magnitude of the new effects should be set by some power of the ratio between E and the Planck scale E_p ($\sim 10^{28} eV$). Since in all cases accessible to us experimentally E/E_p is extremely small, this is a key challenge for quantum-gravity phenomenology. A challenge which however can be dealt with also relying on experience with other analogous situations in physics: for example, as emphasized in Ref. [2], ongoing studies of proton stability from the grandunification perspective and early 1900s studies of Brownian motion could be described as facing a very similar challenge.

In the second part of these notes we shall review some key results obtained in quantum-gravity phenomenology. We intend to convey the point that this phenomenology has already established some (however humble but) valuable constraints for quantum-gravity model building, but these constraints are essentially confined to the behaviour of isolated particles, or systems of particles interacting for a very short time. In the next section we argue that some key hints for the search of quantum gravity might be uncovered in the study of the evolution over time of certain types of multiparticle states. And in Section 3 we introduce the first elements of a toy model which could be used as guidance in the estimate of some peculiar Planck-scale effects for multiparticle states.

Our model is at present too crude to make definite predictions, but our intuition is that, when fully developed, it could be sensitively probed through the study of certain multi-kaon systems, such as the states of two neutral kaons produced by decay of the ϕ resonance. Indeed, as we shall discuss, the primary source of inspiration for the toy model discussed in Section 3 is the framework based on the κ-Minkowski noncommutative spacetime, which already inspired a picture for CPT-violation mechanism [22] which could be tested in studies of neutral-kaon systems.

2 A perspective on multiparticle states in the quantum-gravity realm

It is probably fair to say that we are still rather far from a comprehensive solution of the quantum-gravity problem. We do have some proposals, such as String Theory and Loop Quantum Gravity, that provide tentative solutions for some (but not all) aspects of the problem, but these theories still have absolutely no support in experimental and from a robust conservative scientific perspective must therefore be viewed as mere theoretical speculations.

In more than 70 years of work on the quantum gravity problem the community has developed some intuition for features to expect in the quantum-gravity realm, such as the mentioned expected role of the Planck scale in setting the magnitude of effects, and, although of course this intuition must be treated cautiously (with no less caution than the one that should be adopted in relying on String Theory or Loop Quantum Gravity), it is natural to use this intuition as guidance for at least some of our efforts searching for experimentally-established facts about the quantum-gravity realm. In this section we intend to discuss briefly (our perspective on) the part of this intuition that concerns the relationship between the structure of one-particle states and the structure of multiparticle states.

In our current (pre-quantum-gravity) theories one obtains multiparticle states from single-particle states by a standard use of the trivial tensor product of Hilbert spaces, but there is (conceptual/theoretical) evidence that this recipe might not be applicable in the quantum-gravity realm. This expectation emerges not really from a single robust argument but rather from the fact that various lines of reasoning on multiparticle states all appear to suggest that novel features must be introduced.

A first observation which we should report here relies on our present understanding of gravity in 2+1 spacetime dimensions. 2+1D gravity is a topological field theory, rather similar to the Chern-Simons gauge theories that can be considered in a 2+1D spacetime. Especially for the case of a Chern-Simons theory with a $U(1)$ gauge field the literature is very large and it is well established that multiparticle states are not obtained by standard tensor product of single-particle states. The particle excitations of the Chern-Simons gauge field are the so-called "anyons", and it has emerged that for any given Hamiltonian governing the evolution of the anyon system it actually makes sense to treat as completely separate problems each of the n-anyon sectors: there is no simple recipe for obtaining two-particle states from single-particle states, or for obtaining three-particle states from the acquired knowledge of two-particle states. In this anyon example the complexity of multiparticle states is such that one cannot meaningfully introduce some creation-annihilation operators capable of producing from a vacuum state the different n-anyon sectors.

Some of our intuition for multiparticle states in the quantum-gravity originates from familiarity with this multianyon problem [33]. This intuition is directly applicable to 2+1D gravity, and might play an (however indirect) role also in 3+1D gravity, at least when viewed as a "broken topological field theory": guided by the known facts about 2+1D gravity one could set up 3+1D as a theory which is itself "topological up to correction terms".

As an example of argument suggesting complexity for the construction of multiparticle states without relying on the peculiarities of 2+1D spacetimes, we find useful here to mention one aspect of the quantum-gravity problem, which is often set aside but universally acknowledged. The differences between the gravity field and, say, the electromagnetic field are such that for quantum gravity it appears to be necessary to contemplate an in-principle obstruction [1, 34, 35] for a full decoupling of "apparatus" from "system". It is well-established that electromagnetism admits a limiting procedure such that (in the limit) the apparatus actually establishes facts about the system without interfering/affecting the evolution of the system, but the Equivalence Principle (by identifying the inertial mass and the gravitational charge) appears to provide an obstruction for this limiting procedure. And this opens at least an opportunity for complexity in the construction of multiparticle states: it appears to be rather plausible that the relationship between the way in which the apparatus "interferes" with a single-particle system and the way in which the apparatus "interferes" with a two-particle system might be more complex than what is codified in a standard tensor-product rule.

While not often discussed in papers and seminars, these issues for multiparticle states in quantum gravity are rather widely acknowledged. For example, some careful readers from the community of researchers involved in neutral-kaon studies (which is one of the communities toward which we are hoping to direct these notes, because of the possible use of neutral kaons in the investigation of the features discussed in the next section) might have noticed that the debate on the choice of parametrization for the phenomenology of Planck-scale-induced CPT violation [36, 37, 38, 39, 40] reflects in part some differences in the intuition for the structure of multiparticle states.

3 A simple toy model

To give some substance to the arguments presented in the previous section we now intend to introduce the first elements of a possible toy model for the description of a class of effects which could characterize multiparticle states at the Planck scale. This toy model is loosely inspired by the results of our investigations [41, 42, 43, 44, 45, 46] of field theories in κ-Minkowski spacetime, an example of "noncommutative spacetime" (spacetime with noncommuting coordinates) characterized by the following commutators of spacetime coordinates [47, 48, 49]

$$[x_j, x_0] = i\lambda x_j \,, \qquad [x_k, x_j] = 0 \,, \tag{1}$$

where λ is an observer-independent length scale usually expected to be of the order of the minute Planck length ($\sim 10^{-35}m$). Preliminary evidence suggests (but are still inconclusive [50] on the fact) that the observer independence of the noncommutativity parameter may result in the necessity to describe symmetry transformations somehow in terms of the κ-Poincaré Hopf algebra [51, 47, 48] (rather than the Poincaré, or other, Lie algebra), but this will not be used explicitly in our reasoning. It is however important for us that the role played by the κ-Poincaré Hopf algebra in the structure of theories in κ-Minkowski spacetime, has led to a peculiar proposal for the law of composition of momenta, and it is this deformed law of composition of momenta that provides the key ingredient of our rudimentary toy model.

The phenomenological scheme for quantum fields that we intend to describe in this section is only loosely based on our work on κ-Minkowski [41, 42, 43, 44, 45, 46] partly because of the present limitations of our understanding of quantum field theories in κ-Minkowski and partly because of the hope that, by not borrowing too much from detailed aspects of the κ-Minkowski, we might have a chance to gain an intuition for the properties of multiparticle states which is of wider relevance for Planck-scale theories. It appears indeed plausible (but it is difficult to test this conjecture presently because of the huge mathematical complexity of some of these frameworks) that structures at least somewhat similar to the ones we contemplate here might arise not only in κ-Minkowski but also in other approaches to the Planck scale problem, perhaps most notably the Loop Quantum Gravity approach [52, 53, 54, 55].

Even for the understanding of classical field theories in κ-Minkowski, while some noteworthy results have been obtained [41, 43, 44, 45], several issues still remain to be clarified. And in the analysis of Ref. [46], which does provide a proposal for quantum fields in κ-Minkowski and is the main source of intuition for the scheme here considered, one encounters structures that are somewhat more complex that the simplified scheme we are here using as illustrative example. Readers who would consider contributing to further development of this scheme should therefore consider Ref. [46] as a natural entry point into the literature devoted to the issues that must deal with in attempting to discuss more rigorously the relevant framework.

The first ingredient of our construction is the assumption that the noncommutativity properties of single-particle states of given fourmomentum k_μ would be in agreement with the ones of the much studied [41, 43, 46, 48] time-ordered plane waves on κ-Minkowski space-time

$$|\Psi_{\vec{k}}> \;\leftrightarrow\; e^{i\vec{k}\cdot\vec{x}}e^{-i\omega^+(\vec{k})x_0} \tag{2}$$

in which $\omega^+(\vec{k})$ represents the (real) positive root of the equation

$$0 = -m^2 + (2/\lambda)^2 \sinh^2(\lambda\omega/2) - \vec{k}^2 \exp(\lambda\,\omega) \tag{3}$$

This "on-shell condition" (3) comes from the form of the deformed Klein-Gordon equation one adopts in κ-Minkowski, which in turn is dictated by the form of the mass-Casimir of the relevant Hopf algebra of symmetries [41, 43, 46, 47, 48]:

$$(2/\lambda)^2 \sinh^2(\lambda P_0/2) - \vec{P}^2 \exp(\lambda\,P_0) \tag{4}$$

with P_μ the energy-momentum[1] operator, i.e. $\{P_0, \vec{P}\}|\Psi_{\vec{k}}>= \{\omega^+(\vec{k}), \vec{k}\}|\Psi_{\vec{k}}>$.

The other structure for which we take inspiration from the κ-Minkowski literature is a candidate for the total momentum of a two-particle state. From the observation that the commutators (1) imply

$$e^{i\vec{k}\cdot\vec{x}}\,e^{-i\omega^+(\vec{k})x_0}\quad e^{i\vec{q}\cdot\vec{x}}\,e^{-i\omega^+(\vec{q})x_0} = e^{i(\vec{k}+\vec{q}\,e^{-\lambda\omega^+(\vec{k})})\cdot\vec{x}}\,e^{-i(\omega^+(\vec{k})+\omega^+(\vec{q}))\,x_0} \tag{5}$$

[1]The identification of P_μ with the energy-momentum observable is a key point in which we are to be considered only loosely inspired by the κ-Minkowski literature. There is rather robust evidence that these operators P_μ should appear in the relevant formulas for energy-momentum, but they might be have to be combined with other structures (see, *e.g.*, Ref. [46]).

one is led to suggesting the possibility that

$$\{\vec{K}^{tot}, K_0^{tot}\} = \{\vec{k} + \vec{q}\, e^{-\lambda\omega^+(\vec{k})},\, \omega^+(\vec{k}) + \omega^+(\vec{q})\} \equiv \{\vec{k}\dot{+}\vec{q},\, \omega^+(\vec{k}) + \omega^+(\vec{q})\} \quad (6)$$

where $\dot{+}$, such that $\vec{k}\dot{+}\vec{q} \equiv \vec{k} + \vec{q}\, e^{-\lambda\omega^+(\vec{k})}$ is a nonabelian addition rule based on (5).

Within this setup it is obvious that the description of multiparticle states must require new structures with respect to the usual construction. Let us consider for example a state with two indistinguishable particles in a 1+1-dimensional κ-Minkowski spacetime. If we measure the energy-momentum of each of the two particles the indistinguishability would require a description of the state of the following form

$$|\Psi^{(2)}_{\{k,\, q\}} > = \frac{1}{\sqrt{2}} \left(|\psi_k > \otimes |\psi_q > + |\psi_q > \otimes |\psi_k > \right) \quad (7)$$

However, this already exposes a peculiarity: according to (6) the state (7) obtained by “indistinguishability symmetrization” based on the information gained by measurement of the energy-momentum of the two particles is not an eigenstate of total energy-momentum. In fact the action of P_0 on both the states $|\psi_q > \otimes |\psi_k >$ and $|\psi_k > \otimes |\psi_q >$ gives the eigenvalue $\omega^+(\vec{q}) + \omega^+(\vec{k})$, while the action of P on $|\psi_q > \otimes |\psi_k >$, which gives $P\, |\psi_q > \otimes |\psi_k > = (q + k\, e^{-\lambda\omega^+(q)})\, |\psi_q > \otimes |\psi_k >$, differs from the action of P on $|\psi_k > \otimes |\psi_q >$, which gives $P\, |\psi_k > \otimes |\psi_q > = (k + q\, e^{-\lambda\omega^+(k)})\, |\psi_k > \otimes |\psi_q >$.

This invites us to ask how a framework with these peculiarities should describe the case in which for a system of two identical particles one measures the total energy-momentum of the system. For on-shell particles the measurement of the total momentum $\{K^{tot}, K_0^{tot}\}$ translates into constraints of the type

$$\begin{aligned} K^{tot} &= p' \dot{+} p'' \\ K_0^{tot} &= \omega^+(p') + \omega^+(p'') \end{aligned} \quad (8)$$

These admit as solutions two possible pairs of on-shell momenta, and it is easy to establish a relationship between these two solutions: denoting by $\{k, \omega^+(k)\}, \{q, \omega^+(q)\}$ a first solution the second solution is related to the first by

$$\begin{aligned} \{\tilde{q}_0; \tilde{q}\} &= \{\omega^+(q\, e^{-\lambda\omega^+(k)}),\, q\, e^{-\lambda\omega^+(k)}\} \\ \{\tilde{k}_0; \tilde{k}\} &= (\omega^+(k\, e^{\lambda\tilde{q}_0}),\, k\, e^{\lambda\tilde{q}_0}) \end{aligned} \quad (9)$$

From this we infer that this state selected by a total-energy-momentum measurement should have the form

$$|\Psi^{(2)}_{\{K^{tot}\}} >= \frac{1}{\sqrt{2}} \left(|\psi_k > \otimes |\psi_q > + |\psi_{\tilde{q}} > \otimes |\psi_{\tilde{k}} > \right) \tag{10}$$

Evidently just like the state of two particles with definite energy-momenta turned out not to be an eigenstate of total momentum, this state of two particles with definite total energy-momentum does not provide a sharp prediction for the energy-momentum of the individual particles.

We have therefore produced a scheme with very peculiar relationship between one-particle states and multiparticle states, which in particular introduces a sort of new uncertainty principle: there is an incompatibility between measurements of total energy-momentum and measurements of the individual energy-momenta of particles. Sharp measurements of total energy-momentum introduce an irreducible uncertainty in the individual energy-momenta, and sharp measurements of individual energy-momenta introduce an irreducible uncertainty in the total energy-momentum.

We exposed this peculiarities thinking of identical particles, but it seems clear that they are structural to the tensor product of Hilbert spaces, so related (though possibly different) peculiarities should be expected also for distinguishable particles.

While it might be difficult to test directly the new energy-momentum-measurement uncertainty principle, it might be possible to test, *e.g.*, Eq. (10) by looking in data analysis for a sort of contamination by an unexpected state. The analysis could be inspired by the following perspective on Eq. (10)

$$\begin{aligned} |\Psi^{(2)}_{\{K^{tot}\}} > &= \frac{1}{\sqrt{2}} \left(|\psi_k > \otimes \ |\psi_q > + |\psi_{\tilde{q}} > \otimes \ |\psi_{\tilde{k}} > \right) = \\ &= \frac{1}{\sqrt{2}} \left(|\psi_k > \otimes \ |\psi_q > + |\psi_q > \otimes \ |\psi_k > + |\Delta > \right) \end{aligned} \tag{11}$$

with $|\Delta > \equiv |\psi_{\tilde{q}} > \otimes |\psi_{\tilde{k}} > - |\psi_q > \otimes |\psi_k >$. Clearly the limitations of the theoretical basis of our toy model do not allow us at present to characterize $|\Delta >$ sharply enough to be of real help for phenomenology. But the situation should improve gradually as we develop a better understanding of the κ-Minkowski (and possibly other) framework. In particular, it will be interesting to establish if this theory framework ends up having at least a partial overlap with the phenomenological proposal for multi particle states put forward in Ref. [56].

4 On other areas of quantum-gravity phenomenology

The phenomenology for multiparticle states discussed in the previous sections would be in many ways complementary to the topics so far studied in an ongoing effort searching for some first experimental manifestation of effects with quantum-gravity origin. Indeed these previous "quantum-gravity phenomenology" [2] studies mainly focused on the behaviour of isolated particles, or systems of particles interacting for a very short time. In this section we intend to review briefly some of these topics previously considered from the quantum-gravity-phenomenology perspective, also hoping to provide some intuition for the potential relevance of the studies of multiparticle states discussed in the previous sections.

Of course, the first concern for quantum-gravity phenomenology was to show that it was really possible to test experimentally some effects introduced genuinely at the Planck scale. This is by now well established, and we discuss one explicit example in Subsection 4.1.

In Subsection 4.2 we comment on the possibility for quantum-gravity phenomenology to actually falsify theories (something worth our efforts even when the results of experiments are negative, rather than merely keep trying to catch lucky through a positive/discovery result).

Subsection 4.3 is devoted to a (incomplete but representative) list of effects that should be considered as candidate quantum-gravity effects, and a brief descriptions of the experiments and/or observations which are being analyzed as opportunities to provide related insight.

The rest of this section focuses on the most studied area of quantum-gravity phenomenology, the one that concerns the possibility of Planck-scale departures from Poincaré (Lorentz) symmetry.

4.1 Quantum-Gravity Phenomenology exists

Task number one for any phenomenology (usually an easy task but a challenging one here) is to show that effects of the type that could be expected from the relevant class of theories could be seen. The key source of pride for quantum-gravity phenomenologists comes from the fact that over these past few years, and over a time that indeed spanned over only a handful of years, we managed to change the perception of quantum-gravity research from the traditional "no help from experiments possible" to the present intuition, shared by most workers in the field, that these effects could be seen. We might need some luck to actually see them, but clearly it is not possible. There is therefore a legitimate phenomenology to be developed for quantum gravity.

Once task one is accomplished it is important to show that the type of observations that are doable not only provide opportunities to luckily stumble upon a manifestation of the new theory, but actually the data could be used to falsify candidate theories. This task two clearly requires much more of task one, both at the level of our understanding of the theories and for what concerns the quality of the data and their phenomenological analysis.

Concerning task one it is of course significant that over these past few years several authors have shown in different ways and for different candidate Planck-scale effects that, in spite of the horrifying smallness of these effects, some classes of doable experiments and observations could see the effects. Just to make absolutely clear the fact that effects genuinely introduced at the Planck scale could be seen, let us just exhibit here one very clear illustrative example.

The Planck-scale effect we consider is codified by the following energy-momentum (dispersion) relation

$$m^2 \simeq E^2 - \vec{p}^2 + \eta \vec{p}^2 \left(\frac{E^2}{E_p^2} \right) , \qquad (12)$$

where E_p denotes again the Planck scale ($E_p = 1/L_p \sim 10^{28} eV$) and η is a phenomenological parameter. This is a good choice because convincing the reader that we are dealing with an effect introduced genuinely at the Planck scale is in this case effortless. It is in fact well known (see, *e.g.*, Ref. [57]) that this type of E_p^{-2} corrections to the dispersion relation can result from discretization of spacetime on a lattice with E_p^{-1} lattice spacing[2].

If such a modified dispersion relation is part of a framework where the laws

[2]The idea of a rigid lattice description of spacetime is not really one of the most advanced for quantum-gravity research, but this consideration is irrelevant for task one: in order to get this phenomenology started we first must establish that the sensitivities we have are sufficient for effects as small as typically obtained from introducing structure at the Planck scale. The smallness of the effect in (13) is clearly representative of the type of magnitude that quantum-gravity effects are expected to have, and the fact that it can also be obtained from a lattice with E_p^{-1} spacing confirms this point.

of energy-momentum conservation are unchanged one easily finds [3, 4, 5, 6] significant implications for the cosmic-ray spectrum. In fact, the "GZK cutoff", a key expected feature of the cosmic-ray spectrum, is essentially given by the threshold energy for cosmic-ray protons to produce pions in collisions with CMBR photons. In the evaluation of the threshold energy for $p + \gamma_{CMBR} \rightarrow p + \pi$ the correction term $\eta \vec{p}^2 E^2 / E_p^2$ of (13) can be very significant. Whereas the classical-spacetime prediction for the GZK cutoff is around $5 \cdot 10^{19} eV$, at those energies the Planck-scale correction to the threshold turns out [3, 4, 5, 6] to be of the order of $\eta E^4/(\epsilon E_p^2)$, where ϵ is the typical CMBR-photon energy. For positive values of η, even somewhat smaller[3] than 1, this amounts to an observably large shift of the threshold energy, which should easily be seen (or excluded) once the relevant portion of the cosmic-ray spectrum becomes better known, with observatories such as the Pierre Auger Observatory.

Of course, the same effect is present and is even more significant if instead of a E_p^{-2} correction one introduces in the dispersion relation a correction of E_p^{-1} type.

4.2 Falsifying theories

Arguments such as the one offered in the previous subsection clearly show that this phenomenology has a right to existence. Task one is settled. We do have at least a chance (perhaps slim, but this is not the point here) to see Planck-scale effects, and if we ever do see one such effect it will be wonderful. But a phenomenology should also be valuable when it does not find the effects it looks for, by setting limits on (and in some cases ruling out) corresponding theories. Have we proven that quantum-gravity phenomenology can rule out Planck-scale theories?

Of course the phenomenology will be based on some "test theories" and the parameters of the test theories will be increasingly constrained as data become available. But beyond the level of test theories there is the truly sought level of "theories", models which are not merely introduced (as is the case of test theories) as a language used in mapping the progress of experimental limits on some effects, but rather models which are originally motivated by some ideas for the solution of the quantum-gravity problem. And in order to falsify one such theory we need to prove experimentally the absence of an effect which has

[3]Of course the quantum-gravity intuition for η is $\eta \sim 1$.

been rigorously established to be a necessary consequence of the theory. These are the ingredients of the task two described above. But the theories used in quantum-gravity research are so complex that we can rarely really establish that a given effect is necessarily present in the theory. What usually happens is that we find some "theoretical evidence" for the effect in a given quantum-gravity theory and then we do the phenomenology of that effect using some test theories. The link from theory to effect is too weak to be used in reverse: we are usually not able to say that the absence of the effect really amounts to ruling out the theory.

Think for example of Loop Quantum Gravity. Because of the so-called "classical-limit problem" at present one is never really able to use that theory to provide a definite prediction for an effect to be looked for by experimentalists. And for String Theory the situation might be worse, at least in the sense that one might not even be able to hope better things for the future: it is in fact at present not clear whether string theory is in principle able to make any definite predictions, since the formalism is so flexible, so capable to say anything, that it is feared to amount basically to saying nothing.

So concerning task two the situation does not look very healthy, but the problem resides on the theory side, not the phenomenology side.

If indeed, at least for now, we cannot falsify Loop Quantum Gravity and String Theory, can we at least falsify some other theory used in quantum-gravity research? It is of course extremely important for quantum-gravity phenomenology to find one such example. If we do find a first example then we can legitimately hope that the falsifiability of more and more theories will gradually be achieved. Theories in the κ-Minkowski noncommutative spacetime considered in Section 3 could turn out to be falsifiable, and the needed mathematics is probably within our reach.

4.3 Concerning quantum-gravity effects and the status of Quantum-Gravity theories

At the present stage of investigation of the quantum-gravity problem it is actually not so obvious how to identify candidate quantum-gravity effects. Analogous situations in other areas of physics are usually such that there are a few new theories which have started to earn our trust by successfully describing some otherwise unexplained data, and then often we let those theories guide us toward new effects that should be looked for. The theories we have for quantum gravity, in spite of all their truly remarkable mathematical beauty, and their extraordinary contribution to the investigation of the conceptual sides of the quantum-gravity problem, cannot (yet) claim any success in the experimental realm. Moreover, even if we wanted to used them as guidance for experiments the complexity of these theories proves to be a formidable obstruction. What we can do with these theories (and we must be content with it since we do not have many alternatives) is to look at their general structure and use this as a source of intuition for the proposal of a few candidate effects.

A similar type of path toward the identification of some candidate quantum-gravity effects is the one based on the analysis of the general structure of the quantum-gravity problem itself. It happens to be the case that by looking at the type of presently-unanswered questions for which quantum-gravity is being sought, one is automatically led to considering a few candidate effects.

Of course these ideas suggested from our perception of the structure of the quantum-gravity problem and from our analysis of the general structure of some proposed quantum-gravity theories could well turn out to be completely off the mark, but it still makes sense to investigate these ideas.

4.3.1 Planck-scale departures from classical spacetime symmetries

From the general structure of the quantum-gravity problem, which clearly provides at least some encouragement to considering discretized (or otherwise "quantized") spacetimes, one finds encouragement to consider departures from classical spacetime symmetries. Consider for example the Minkowski limit, the one described by the classical Minkowski spacetime in current theories. There is a duality one-to-one relation between the classical Minkowski spacetime and the classical (Lie-) algebra of Poincaré symmetrie. Poincaré transformations are smooth arbitrary-magnitude classical transformations and it is rather obvious that they should be put under scrutiny [58] if the classical description of spacetime is replaced by a quantized/discretized one.

4.3.2 Planck-scale departures from CPT symmetry

Perhaps the most intelligible evidence of a Planck-scale effect would be a violation of CPT symmetry. CPT symmetry is in fact protected by a theorem in our current (Minkowski-limit) theories, mainly as a result of locality and

Poincaré symmetry. The fact that the structure of the quantum-gravity problem invites us to consider spacetimes with some element of nonlocality and/or departures from Poincaré symmetry clearly opens a window of opportunity for Planck-scale violations of CPT symmetry.

4.3.3 Distance fuzziness and spacetime foam

The fact that the structure of the quantum-gravity problem suggests that the classical description of spacetime should give way to a nonclassical one at scales of order the Planck scale has been used extensively as a source of inspiration concerning the proper choice of formalism for the solution of the quantum-gravity problem, but for a long time (decades) it had not inspired ideas relevant for phenomenology. The description that came closer to a physical intuition for the effects induced by spacetime nonclassicality is Wheeler's "spacetime foam", which however does not amount to a definition (at least not a scientific/operative definition). A few years ago one of us proposed [9] a physical/operative definition of (at least one aspect of) spacetime fuzziness/foam, which makes direct reference to interferometry. According to this definition the fuzziness/foaminess of a spacetime is established on the basis of an analysis of strain noise in interferometers set up in that spacetime. In achieving their remarkable accuracy modern interferometers must deal with several classical-physics strain noise sources (*e.g.*, thermal and seismic effects induce fluctuations in the relative positions of the test masses). And importantly strain noise sources associated with effects due to ordinary quantum mechanics are also significant for modern interferometers (the combined minimization of *photon shot noise* and *radiation pressure noise* leads to a noise source which originates from ordinary quantum mechanics). The operative definition of fuzzy/foamy spacetime advocated in Ref. [9] characterizes the corresponding quantum-gravity effects as an additional source of strain noise. A theory in which the concept of distance is fundamentally fuzzy in this operative sense would be such that the read-out of an interferometer would still be noisy (because of quantum-gravity effects) even in the idealized limit in which all classical-physics and ordinary-quantum-mechanics noise sources are completely eliminated.

4.3.4 Decoherence

For approaches to the quantum-gravity problem which assume that, in merging with General Relativity, Quantum Mechanics should be revised one of the most popular effects is decoherence. This may be also motivated using heuristic arguments, based mainly on quantum field theory in curved spacetimes, which suggest that black holes radiate thermally, with an associated "information-loss problem".

4.3.5 Planck-scale departures from the Equivalence Principle

Various perspectives on the quantum-gravity problem appear to suggest departures from one or another (stronger or weaker) form of the Equivalence Principle. For brevity let me just summarize here my preferred argument, which is based on the observation that locality is a key ingredient of the present formulation of the Equivalence Principle. In fact, the Equivalence Principle ensures that (for same initial conditions) two point particles would go on the same geodesic independently of their mass. But it is well established that this is not applicable to extended bodies, and presumably also not applicable to "delocalized point particles" (point particles whose position is affected by uncontrolled uncertainties). If spacetime structure is such to induce an irreducible limit on the localization of particles it would seem then natural to expect some departures from the Equivalence Principle.

4.3.6 Critical-dimension SuperString Theory

The most popular realization of String Theory, with the adoption of supersymmetry and the choice of working in a "critical" number of spacetime dimensions, has given a very significant contribution to the conceptual aspects of the debate on quantum gravity, perhaps most notably the fact that, indeed thanks to research on string theory, we now know that quantum gravity might well be a perturbatively renormalizable theory (whereas this was once thought to be impossible). But for what concerns the prediction of physical effects string theory has not proven (yet?) to be rich. In spite of all the noteworthy mathematical structure that are needed for the analysis of string theory, from a wider perspective this is the approach that by construction assumes that the solution to the quantum-gravity problem should bring about a rather limited amount of novelty. In particular, string theory is still introduced in a classical Minkowski spacetime and it is still a genuinely quantum-mechanical theory. None of the effects possibly due to spacetime quantization are therefore necessarily expected and all the departures-from-quantum-mechanics effects, like decoherence effects, are also not expected.

But on the other hand, as mentioned, string theory is turning out to be a remarkably flexible formalism, and therefore, while one can structure things in such a way that nothing interestingly new happens, one can also mould the formalism in such a way to have some striking new effects[4], and effects that fit within some intuitions concerning the quantum-gravity problem. In particular there is a known scheme for having violations of the equivalence principle [18], and by providing a vacuum expectation value for a relevant antisymmetric tensor one can give rise [59] to departures from Poincaré symmetry (together with the emergence of an effective spacetime noncommutativity).

4.3.7 Loop Quantum Gravity

The only other approach with contributions to the conceptual debate on the quantum-gravity problem of significance comparable to the ones of the string-theory approach is Loop Quantum Gravity. In particular, it is thanks to work on Loop Quantum Gravity that we now know that quantum gravity might fully preserve the diffeomorphism invariance of General Relativity (whereas this was once thought to be impossible). But also Loop Quantum Gravity, while excelling in the conceptual arena, has its difficulties providing predictions to phenomenologists. While String Theory may be perceived as frustratingly flexible, one might perhaps say that at the present stage of development Loop Quantum Gravity appears not to have even the needed room to maneuver it down to the mundane arena of corrections to General Relativity and corrections to the Standard Model of particle physics. As a result of the much debated "classical-limit problem", in a certain sense Loop Quantum Gravity provides a candidate description of everything but does not provide an explicit description of anything. One may attempt however (and several groups have

[4]One of the most noteworthy possibilities is the one of "large extra dimensions". This gives rise to a peculiar brand of quantum-gravity phenomenology, which is not governed by the Planck scale. In these notes we intend to focus on Planck-scale effects.

indeed attempted to do this) to infer from the general structure of the theory some ideas for candidate Loop-Quantum-Gravity effects. In particular, several studies [8, 19] have argued that the type of discretization of spacetime observables usually attributed to Loop Quantum Gravity could be responsible for Planck-scale departures from Lorentz symmetry. This hypothesis also finds encouragement [20] in light of the role apparently played by noncommutative geometry in the description of certain aspects of the theory.

Of course, as long as the "classical-limit problem" is not solved, the evidence of departures from Lorentz symmetry in (the Minkowski limit [21] of) Loop Quantum Gravity must be considered weak, and any attempt to give a concrete formulation of these effects will have to rely at one point or another on heuristics. This remains a very valuable exercise for quantum-gravity phenomenology, since it gives us ideas on effects that are worth looking for, but clearly at present phenomenologists are not given any chance of falsifying Loop Quantum Gravity.

From the phenomenology perspective there is more than the Lorentz-symmetry issue at stake in the "classical-limit problem": it is not unlikely that structures relevant for CPT symmetry and the Equivalence Principle are also present, and Loop Quantum Gravity could be a natural context where to develop a physical intuition for spacetime foam.

4.3.8 Approaches based on noncommutative geometry

Noncommutative spacetimes so far have been considered has opportunities to look at specific aspects of the quantum-gravity problem (whereas string theory and loop quantum gravity attempt to provide a full solution). It is perhaps fair to say that the most significant findings emerged in attempts to describe the Minkowski limit [21] of quantum-gravity. One might say that these studies look at one half of the quantum-gravity problem, the quantum-spacetime aspects. Because of the double role of the gravitational field, which in some ways is just like another field given in spacetime but it is also governs the structure of spacetime, in quantum-gravity research one ends up considering two types of quantization: some sort of quantization of gravitational interactions and some sort of quantization of spacetime structure. At present one might say that only within the Loop Quantum Gravity approach we are truly exploring both aspects of the problem. String Theory, as long as it is formulated in a classical (background) spacetime, focuses in a sense on the quantization of the gravitational interaction, and sets aside the possible "quantization" of spacetime[5]. And the reverse is true of mainstream research on spacetime noncommutativity, which provides a way to quantize spacetime, but, at least for this early stages of development, does not provide a description of gravitational interactions.

The analysis of noncommutative deformations of Minkowski spacetime has provided some intuition for what could be the fate of (Minkowski-limit/Poinca symmetries at the Planck scale. And also valuable for the development of quantum-gravity phenomenology is the fact that in some cases, such as the κ-Minkowski noncommutative spacetime, it is reasonable to hope that these studies will soon provide truly falsifiable predictions.

Unfortunately spacetime fuzziness, which is the primary motivation for most researchers to consider noncommutativity, frustratingly remains only vaguel characterized in current research on noncommutative spacetimes.

4.4 On the status of different areas of this phenomenology

4.4.1 Planck-scale modifications of Poincaré symmetries

The most developed quantum-gravity-phenomenology research area is the one that considers the possibility of Planck-scale departures from Poincaré symmetry. We shall discuss in some detail these studies later in this section.

4.4.2 Planck-scale modifications of CPT symmetry and Decoherence

The most studied opportunity to test CPT symmetry is provided by the neutral-kaon and the neutral-B systems [36, 37]. One finds that in these neutral-meson systems a Planck-scale departure from CPT symmetry could in principle be amplified. In particular, the neutral-kaon system hosts the peculiarly small mass difference between long-lived and the short-lived kaons $|M_L - M_S|/M_{L,S} \sim 7{\cdot}10^{-15}$, and there are scenarios of Planck-scale CPT violation in the literature [36] in which the inverse of this small number amplifies a small (Planck-

[5] Just like in noncommutative geometry one hopes one day to obtain also the quantization of the interaction, by introducing a suitable noncommutative-geometrodynamics, in approaches like string theory one may hope that the quantization of the interaction field may at advanced levels of analysis amount to spacetime quantization. Some string-theory results do encourage this hope [60] but the situation remains puzzling [61]

scale induced) CPT-violation effect. This in particular occurs in the most studied scenario for Planck-scale violations of CPT symmetry in the neutral-kaon system, in which the Planck-scale effects induce a difference between the terms on the diagonal of the K^0,$\bar{K}^0$ mass matrix. An analogous effect would be present in the neutral-B system but if the Planck-scale effect for the terms on the diagonal is momentum independent the best sensitivity is expected from studies of the neutral-kaon system. It is however not implausible [22] that the Planck-scale effects would introduce a correction to the diagonal terms of the neutral-meson mass matrix that depends on the momentum of the particle, and in this case, among the experiments currently done or planned, the best sensitivity would be obtained with the neutral-B system.

4.4.3 Distance fuzziness and spacetime foam

The phenomenology of distance fussiness is being developed mainly in two directions: interferometry and observations of extragalactic sources.

In interferometry the debate [9, 10] involves a variety of phenomenological models and different perspectives on what is the correct intuition that one should implement. It is perhaps best here to just give the simplest observation that can provide encouragement for these studies. As we stressed above in interferometry it is natural to look for Planck-scale contributions to the strain noise. And it is noteworthy that strain noise is naturally described in terms [9] of a function of frequency $\rho(\nu)$ (a tool for spectral analysis) that carries dimensions of Hz^{-1}. If one was to make a naive dimensional estimate of Planck scale effects one could simply pose $\rho \sim L_p/c$, which at first might seem not too encouraging since it leads to a very small estimate of ρ: $\rho \sim 10^{-44} Hz^{-1}$. However, modern interferometers are achieving truly remarkable sensitivities, driven by their main objective of seeing classical gravity waves, and levels of ρ as small as $10^{-44} Hz^{-1}$ are within their reach.

Another much discussed opportunity for constraining models of spacetime fuzziness is provided by the observation of extragalactic sources, such as distant quasars. Essentially it is argued [23, 24] that, given a wave description of the light observed from the source, spacetime fuzziness should introduce an uncertainty in the waves phase that cumulates as the wave travels, and for sufficiently long propagation times this effect should scramble the wave front enough to prevent the observation of interferometric fringes. Also in this case plausible estimates suggest that, in spite of the smallness of the Planck-scale effects, thanks to the amplification provided by the long propagation times the needed sensitivity might soon be within our reach.

4.4.4 Decoherence

The development of test theories for decoherence is of course a challenging area of quantum-gravity phenomenology, since the test theories must go beyond quantum mechanics. Let us just here mention Refs. [25] as a good entry point in the relevant literature, and stress that the neutral-kaon system, with its delicate balance of scales, is also considered [36, 25] to be our best opportunity for laboratory studies of Planck-scale-induced decoherence.

4.4.5 Planck-scale departures from the Equivalence Principle

As mentioned the quantum-gravity problem also provides motivation to contemplate departures from the Equivalence Principle, and in some approaches (in particular in String Theory) some structures suitable for describing departures from the Equivalence Principle are found. The phenomenology is very rich and in many ways goes well beyond the specific interests of quantum-gravity research: the Equivalence Principle continues to be placed under careful scrutiny especially because of its central role in General Relativity. Interested readers could consider as points of entrance in the relevant literature the overall review in Ref. [26] and, more specifically for departures from the Equivalence Principle within the string-theory approach, Ref. [18].

4.5 Aside on Doubly-Special Relativity

In preparation for the next subsection, which focuses on the phenomenology of Planck-scale departures from Poincaré symmetry, it might useful to provide here a short introduction to "doubly-special relativity" (DSR) [17], which recently has been often analyzed as an alternative to the standard scenario of Planck-scale effects that break Lorentz(/Poincaré) symmetry. The doubly-special-relativity scenario was introduced [17] as a sort of alternative perspective on the results on Planck-scale departures from Lorentz symmetry which had been reported in numerous articles [3, 4, 5, 6, 7, 8, 19] between 1997 and 2000. These studies were advocating a Planck-scale modification of the energy-momentum dispersion relation, usually of the form $E^2 =$

$p^2 + m^2 + \eta L_p^n p^2 E^n + O(L_p^{n+1} E^{n+3})$, on the basis of preliminary findings in the analysis of several formalisms in use for Planck-scale physics. The complexity of the formalisms is such that very little else was known about their physical consequences, but the evidence of a modification of the dispersion relation was becoming robust. In all of the relevant papers it was assumed that such modifications of the dispersion relation would amount to a breakup of Lorentz symmetry, with associated emergence of a preferred class of inertial observers (usually identified with the natural observer of the cosmic microwave background radiation).

The DSR idea was proposed [17] on the basis of a striking analogy between these developments and the developments which led to the emergence of Special Relativity, now more than a century ago. In Galilei Relativity there is no observer-independent scale, and in fact the energy-momentum relation is written as $E = p^2/(2m)$. As experimental evidence in favour of Maxwell equations started to grow, the fact that those equations involve a fundamental velocity scale appeared to require the introduction of a preferred class of inertial observers. But in the end we figured out that the situation was not demanding the introduction of a preferred frame, but rather a modification of the laws of transormation between inertial observers. Einstein's Special Relativity introduced the first observer-independent relativistic scale (the velocity scale c), its dispersion relation takes the form $E^2 = c^2 p^2 + c^4 m^2$ (in which c plays a crucial role for what concerns dimensional analysis), and the presence of c in Maxwell's equations is now understood as a manifestation of the necessity to deform the Galilei transformations.

Refs. [17] argued that it is plausible that we might be presently confronted with an analogous scenario. Research in quantum gravity is increasingly providing reasons of interest in Planck-scale modifications of the dispersion relation, of the type mentioned above, and, while it was customary to assume that this would amount to the introduction of a preferred class of inertial frames (a "quantum-gravity ether"), the proper description of these new structures might require yet again a modification of the laws of transformation between inertial observers. The new transformation laws would have to be characterized by two scales (c and L_p) rather than the single one (c) of ordinary Special Relativity.

The "historical motivation" described above leads to a scenario for Planck-scale physics which is not intrinsically equipped with a mathematical formalism for its implementation, but still is rather well defined. With Doubly-Special Relativity one looks for a transition in the Relativity postulates, which should be largely analogous to the Galilei $\rightarrow$ Einstein transition. Just like it turned out to be necessary, in order to describe high-velocity particles, to set aside Galilei Relativity (with its lack of any characteristic invariant scale) and replace it with Special Relativity (characterized by the invariant velocity scale c), it is at least plausible that, in order to describe ultra-high-energy particles, we might have to set aside Special Relativity and replace it with a new relativity theory, a DSR, with two characteristic invariant scales, a new small-length/large-momentum scale in addition to the familiar velocity scale.

A theory will be compatible with the DSR principles if there is complete equivalence of inertial observers (Relativity Principle) and the laws of transformation between inertial observers are characterized by two scales, a high-velocity scale and a high-energy/short-length scale. Since in DSR one is proposing to modify the high-energy sector, it is safe to assume that the present operative characterization of the velocity scale c would be preserved: c is and should remain the speed of massless low-energy particles[6]. Only experimental data could guide us toward the operative description of the second invariant scale λ, which may or may not be based on a deformed dispersion relation, but λ is naturally guessed to be somewhere in the neighborhood of the Planck length L_p.

As a result of the "historical context" that led to the DSR idea most authors have explored the possibility that the second relativistic invariant be introduced through a modifications of the dispersion relation. This is a reasonable choice but it would be incorrect at present to identify (as often done in the literature) the DSR proposal with the proposal of observer-independent modifications of the dispersion relation. For example the dispersion relation might not be modified but there might instead be an observer-independent bound on the accuracy achievable in the measurement of distances.

In the search of a first example of formalism compatible with the DSR

[6]Note however the change of perspective imposed by the DSR idea: within Special Relativity c is the speed of all massless particles, but Special Relativity must be perceived as a low-energy theory (as viewed from the DSR perspective) and in taking Special Relativity as starting point for a high-energy deformation one is only bound to preserving c as the speed of massless low-energy particles.

principles much work has been devoted to the study of the κ-Minkowski spacetime, which inspired our toy model (Section 3) for multiparticle-state phenomenology.

4.6 More on the phenomenology of departures from Poincaré symmetry

In this subsection we comment on some aspects of recent phenomenology work on departures from Poincaré symmetry, mostly as codified in modifications of the energy-momentum dispersion relation. We start by stressing that the same modified dispersion relation can be introduced in very different test theories, leading to completely different physical predictions.

4.6.1 On the test theories with modified dispersion relation

The majority (see, *e.g.*, Refs. [3, 4, 5, 6, 7, 8, 19]) of studies concerning Planck-scale modifications of the dispersion relation adopt the phenomenological formula

$$m^2 \simeq E^2 - \vec{p}^2 + \eta \vec{p}^2 \left(\frac{E^n}{E_p^n} \right) + O(\frac{E^{n+3}}{E_{QG}^{n+1}}) \ , \tag{13}$$

with real η (assumed to be of order $|\eta| \sim 1$) and integer n.

There is at this point a very large literature on the associated phenomenology, but actually some of the different phenomenological studies that compose this literature introduce this type of dispersion relation within different test theories. The limits obtained within different test theories are of course not to be compared. The same parametrization of the dispersion relation, if introduced within different test theories, actually gives rise to independent sets of parameters.

The potential richness of this phenomenology, for what concerns the development of test theories, mainly originates from the need to specify, in addition to the form of the dispersion relation, several other structural properties of the test theory.

It is necessary to state whether the theory is still "Hamiltonian", at least in the sense that the velocity is obtained from the commutator with an Hamiltonian (for example, along the x axis, $v \sim [x, H]$) and whether the Heisenberg commutator preserves its standard form ($[x, p] \sim \hbar$). Especially this second concern is rather significant since some of the heuristic arguments which are used to motivate the presence of modified dispersion relations at the Planck scale also suggest that the Heisenberg commutator should be correspondingly modified.

Then the test theory should formulate a law of energy-momentum conservation. We have discussed the example of The *kappa*-Minkowski which we considered is an example of spacetime that contributed to interest in modified dispersion relations and appears to be such to require also an accompanying modification of the law of energy-momentum conservation. And in particular a link between modification of the dispersion relation and associated modification of the law of energy-momentum conservation is required by the DSR principles (see below).

And one should keep clearly separate the test theories that intend to describe only kynematics and the ones that also adopt a scheme for Planck-scale dynamics. For example, in Loop Quantum Gravity and some noncommutative spacetimes which provided motivation for considering modifications of the dispersion relation, while we might be close to have a correct picture of kinematics it appears that we are still far from understanding Planck-scale corrections to dynamics[7]

An attempt to introduce a few examples of meaningful test theories has been reported in Ref. [27]. Here we shall be content with showing how in different phenomenological studies based on modified dispersion relations one ends up making assumptions about the points listed above.

4.6.2 Photon stability

It has been recently realized (see, *e.g.*, Refs. [28, 29, 30]) that when Lorentz symmetry is broken at the Planck scale there can be significant implications for certain decay processes. At the qualitative level the most significant novelty would be the possibility for massless particles to decay. Let us consider for example a photon decay into an electron-positron pair: $\gamma \rightarrow e^+e^-$. And let us analyze this process using the dispersion relation (13), for $n = 1$, with unmodified law of energy-momentum conservation. One easily finds a relation between the energy E_γ of the incoming photon, the opening angle θ between the

[7]On the Loop Quantum Gravity side this is linked once again with the "classical limit problem", while for the relevant noncommutative spacetime the concern originates from the difficulties encountered in producing consistent theories of quantum matter fields in those spacetimes.

outgoing electron-positron pair, and the energy E_+ of the outgoing positron, which, for the region of phase space with $m_e \ll E_\gamma \ll E_p$, takes the form $\cos(\theta) = (A+B)/A$, with $A = E_+(E_\gamma - E_+)$ and $B = m_e^2 - \eta E_\gamma E_+(E_\gamma - E_+)/E_p$ (m_e denotes of course the electron mass). The fact that for $\eta = 0$ this would require $cos(\theta) > 1$ reflects the fact that if Lorentz symmetry is preserved the process $\gamma \to e^+e^-$ is kinematically forbidden. For $\eta < 0$ the process is still always forbidden, but for positive η and $E_\gamma \gg (m_e^2 E_p/|\eta|)^{1/3}$ one finds that $\cos(\theta) < 1$ in certain corresponding region of phase space.

The energy scale $(m_e^2 E_p)^{1/3} \sim 10^{13} eV$ is not too high for astrophysics. The fact that certain observations in astrophysics allow us to establish that photons of energies up to $\sim 10^{14} eV$ are not unstable (at least not noticeably unstable) could be used [28, 30] to set valuable limits on η.

This is quite a striking result, which however should be reported with caution: this is not a strategy to set direct limits on the parameters of the dispersion relation, since the analysis very explicitly requires us to specify also the form of the energy-momentum conservation law. By changing the form of the law of energy-momentum conservation, for fixed form of the dispersion relation, one can indeed obtain very different results. This is best illustrated contemplating the possibility that such a dispersion relation be introduced within a DSR framework. First of all let us notice that any theory compatible with the DSR principle will have stable massless particles, so that by looking for massless-particle decay one could falsify the DSR idea. A threshold-energy requirement for massless-particle decay (such as the $E_\gamma \gg (m_e^2 E_p/|\eta|)^{1/3}$ mentioned above) cannot of course be introduced as an observer-independent law, and is therefore incompatible with the DSR principles.

An analysis of the stability of massless particles that is compatible with the DSR principles can be obtained by combining the modification of the dispersion relation with an associated modification of the laws of energy-momentum conservation. The form of the new law of energy-momentum conservation can be derived from the requirement of being compatible both with the DSR principles and with the modification of the dispersion relation [17], and in particular in the case of $a \to b + c$ decays one arrives at $E_\gamma \simeq E_+ + E_- - \lambda \vec{p}_+ \cdot \vec{p}_-$, $\vec{p}_\gamma \simeq \vec{p}_+ + \vec{p}_- - \lambda E_+ \vec{p}_- - \lambda E_- \vec{p}_+$. Using these in place of ordinary conservation of energy-momentum one ends up with a result for $\cos(\theta)$ which is still of the form $(A + B)/A$ but now with $A = 2E_+(E_\gamma - E_+) + \lambda E_\gamma E_+(E_\gamma - E_+)$ and

$B = 2m_e^2$. Evidently this formula always gives $\cos(\theta) > 1$, consistently with the fact that $\gamma \to e^+e^-$ is forbidden in DSR.

4.6.3 Threshold anomalies

Another opportunity to investigate Planck-scale departures from Lorentz symmetry is provided by certain types of energy thresholds for particle-production processes that are relevant in astrophysics. This is a very powerful tool for quantum-gravity phenomenology, and in fact we already discussed the evaluation of the threshold energy for $p + \gamma_{CMBR} \to p + \pi$ as a key example in support of the fact that quantum-gravity phenomenology is worth doing.

Numerous quantum-gravity-phenomenology papers (see, *e.g.*, Refs.[3, 4, 5, 6]) have been devoted to the study of Planck-scale-modified thresholds, so the interested readers can find an abundance of related materials.

4.6.4 Time-of-travel analyses

A wavelength dependence of the speed of photons is obtained from a modified dispersion relation, if one assumes the velocity to be still described by $v = dE/dp$. For the dispersion relation here considered one finds that at "intermediate energies" ($m < E \ll E_p$) the velocity law will take the form

$$v \simeq 1 - \frac{m^2}{2E^2} + \eta \frac{n+1}{2} \frac{E^n}{E_p^n} \ . \tag{14}$$

On the basis of this formula one would find that two simultaneously-emitted photons should reach the detector at different times if they carry different energy. And this time-of-arrival-difference effect can be significant[7] in the analysis of short-duration gamma-ray bursts that reach us from cosmological distances. For a gamma-ray burst it is not uncommon that the time travelled before reaching our Earth detectors be of order $T \sim 10^{17}s$. Microbursts within a burst can have very short duration, as short as $10^{-3}s$, and this means that the photons that compose such a microburst are all emitted at the same time, up to an uncertainty of $10^{-3}s$. Some of the photons in these bursts have energies that extend at least up to the GeV range, and for two photons with energy difference of order $\Delta E \sim 1GeV$ a $\Delta E/E_p$ speed difference over a time of travel of $10^{17}s$ would lead to a difference in times of arrival of order $\Delta t \sim T\Delta\frac{E}{E_p} \sim 10^{-2}s$

which is significant (the time-of-arrival differences would be larger than the time-of-emission differences within a microburst).

It is well established that the sensitivities achievable [31] with the next generation of gamma-ray telescopes, such as GLAST [31], could allow to test very significantly (14) in the case $n = 1$, by possibly pushing the limit on $|\eta|$ far below 1. And, as we shall stress later, for the case $n = 2$ neutrino astronomy may lead to valuable insight [14, 15].

4.6.5 Synchrotron radiation

As observed recently in Ref. [32], in the mechanism that leads to the production of synchrotron radiation a key role is played by the special-relativistic velocity law $v = dE/dp \simeq 1 - m^2/(2E^2)$. And an interesting observation is obtained by considering the velocity law (14) for the case $n = 1$. Assuming that all other aspects of the analysis of synchrotron radiation remain unmodified at the Planck scale, one is led [32] to the conclusion that, if $\eta < 0$, the energy/wavelength dependence of the Planck-scale term in (14) can affect the value of the cutoff energy for synchrotron radiation. This originates from the fact that according to (14), for $n = 1$ and $\eta < 0$, an electron cannot have a speed that exceeds the value $v_e^{max} \simeq 1 - (3/2)(|\eta| m_e/E_p)^{2/3}$, whereas in special relativity v_e can take values arbitrarily close to 1. This may be used to argue that for negative η the cutoff energy for synchrotron radiation should be lower than it appears to be suggested by certain observations of the Crab nebula [32].

In making use of this striking observation it is however important to notice that synchrotron radiation is due to the acceleration of the relevant electrons and therefore dynamics plays an implicit role in the derivation of the result [27]. From a field-theory perspective the process of synchrotron-radiation emission is described in terms of Compton scattering of the electrons with the virtual photons of the magnetic field, confirming the need to include a description of some aspects of dynamics and of energy-momentum conservation (for the vertices in the Compton-scattering analysis).

4.6.6 Neutrino observations

In closing we find appropriate to spend a few words on a novel opportunity for quantum-gravity phenomenology: planned neutrino observatories, such as ICE-CUBE, are likely to be very valuable. This had already been timidly suggested in a few earlier papers [11, 12, 13] and should now gain some momentum in light of the analysis reported in Ref. [14] (also see Refs. [15, 16]), which proposes a definite and apparently doable programme of studies.

A key reason of interest in these neutrino studies is the possibility to use them in combination with gamma-ray studies to seek evidence of a spin dependence of the way in which conjectured quantum properties of spacetime affect particle propagation. And, even assuming that there is no such spin dependence (so that gamma rays and neutrinos could serve exactly the same purposes), neutrinos might well be then our best weapon for the study of certain candidate effects. This is due to the fact that it is actually easier to detect high-energy neutrinos (at or above $10^{14}eV$), rather than low energy ones, whereas it is expected that high-energy gamma rays (starting at energies of a few TeV) be absorbed by soft photons in the cosmic background. So neutrinos will effectively extend the energy range accessible to certain classes of studies, and energy is obviously a key factor for the sensitivity of quantum-gravity-phenomenology analyses.

References

1. J. Stachel, "Early History of Quantum Gravity", in *Black Holes, Gravitational Radiation and the Universe*, B.R. Iyer, B. Bhawal eds. (Kluwer Academic Publisher, Netherlands, 1999).

2. G. Amelino-Camelia, gr-qc/9910089, Lect. Notes Phys. **541** (2000) 1; gr-qc/0204051, Mod. Phys. Lett. **A17** (2002) 899.

3. T. Kifune, Astrophys. J. Lett. **518** (1999) L21.

4. R. Aloisio, P. Blasi, P.L. Ghia and A.F. Grillo, Phys. Rev. **D62** (2000) 053010.

5. R.J. Protheroe and H. Meyer: Phys. Lett. **B493** (2000) 1.

6. G. Amelino-Camelia and T. Piran, astro-ph/0008107, Phys. Rev. **D64** (2001) 036005.

7. G. Amelino-Camelia, J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, hep-th/9605211, Int. J. Mod. Phys. **A12** (1997) 607; G. Amelino-Camelia, J.

Ellis, N.E. Mavromatos, D.V. Nanopoulos and S. Sarkar, astro-ph/9712103, Nature **393** (1998) 763.

8. R. Gambini and J. Pullin: Phys. Rev. **D59** (1999) 124021.

9. G. Amelino-Camelia: gr-qc/9808029, Nature **398** (1999) 216; gr-qc/0104086, Nature **410** (2001) 1065.

10. Y.J. Ng and H. van Dam: gr-qc/9906003, Found. Phys. **30**, 795 (2000).

11. J. Ellis, N.E. Mavromatos, D.V. Nanopoulos and G. Volkov, Gen. Rel. Grav. **32** (2000) 1777.

12. S. Choubey and S.F. King, Phys. Rev. **D67** (2003) 073005.

13. G. Amelino-Camelia, Int. J. Mod. Phys. **D12** (2003) 1633-1640.

14. U. Jacob and T. Piran, Nature Physics **3** (2007) 87.

15. G. Amelino-Camelia, Nature Physics **3** (2007) 81.

16. M.C. Gonzalez-Garcia and F. Halzen, hep-ph/0611359.

17. G. Amelino-Camelia: gr-qc/0012051, Int. J. Mod. Phys. **D11** (2002) 35; hep-th/0012238, Phys. Lett. **B510** (2001) 255.

18. T. Damour, F. Piazza and G. Veneziano hep-th/0205111, Phys. Rev. **D66** (2002) 046007.

19. J. Alfaro, H.A. Morales-Tecotl and L.F. Urrutia, Phys. Rev. Lett. **84** (2000) 2318.

20. G. Amelino-Camelia, L. Smolin and A. Starodubtsev, hep-th/0306134, Classical and Quantum Gravity **21** (2004) 3095.

21. G. Amelino-Camelia, gr-qc/0506117, Int. J. Mod. Phys. **D14** (2005) 2167.

22. G. Amelino-Camelia and F. Buccella, hep-ph/0001305, Mod. Phys. Lett. **A15** (2000) 2119.

23. R. Lieu and L.W. Hillman, Astrophys. J. **585** (2003) L77.

24. W.A. Christiansen, Y. Jack Ng and H. van Dam, gr-qc/0508121, Phys. Rev. Lett. **96** (2006) 051301.

25. N.E. Mavromatos and S. Sarkar, hep-ph/0612193.

26. M.P. Haugan and C. Lammerzahl, gr-qc/0103067, Lect. Notes Phys. **562** (2001) 195.

27. G. Amelino-Camelia, gr-qc/0212002, New J. Phys. **6** (2004) 188.

28. T. Jacobson, S. Liberati and D. Mattingly, Phys. Rev. **D66** (2002) 081302.

29. G. Amelino-Camelia: gr-qc/0107086, Phys. Lett. B **528**, 181 (2002).

30. T.J. Konopka and S.A. Major: New J. Phys. **4**, 57 (2002).

31. J.P. Norris, J.T. Bonnell, G.F. Marani and J.D. Scargle: astro-ph/9912136.

32. T. Jacobson, S. Liberati and D. Mattingly: Nature **424** (2003) 1019.

33. G. Amelino-Camelia, hep-th/9402020, Phys. Lett. **B326** (1994) 282; G. Amelino-Camelia and D. Bak, hep-th/9406213, Phys. Lett. **B343** (1995) 231; G. Amelino-Camelia, hep-th/9406212, Phys. Rev. **D51** (1995) 2000.

34. M.P. Bronstein, Physikalische Zeitschrift der Sowietunion **9** (1936) 140.

35. G. Amelino-Camelia, gr-qc/9603014, Mod. Phys. Lett. **A9** (1994) 3415; gr-qc/9603013; Mod. Phys. Lett. **A11** (1996) 1411.

36. J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. **B241** (1984) 381.

37. P. Huet and M.E. Peskin: Nucl. Phys. **B434** (1995) 3.

38. J. Ellis, J. Lopez, N.E. Mavromatos and D.V. Nanopoulos, Phys. Rev. D **53**, 3846 (1996).

39. F. Benatti and R. Floreanini: Nucl. Phys. B **488**, 335 (1997).

40. J. Ellis, N. Mavromatos and D.V. Nanopoulos: Phys. Lett. **B293** (1992) 37-48. Phys. Lett. B **293**, 37 (1992); E. Gravanis and N.E. Mavromatos, hep-th/0108008.

41. G. Amelino-Camelia and S. Majid, hep-th/9907110, Int. J. Mod. Phys. **A15** (2000) 4301.

42. G. Amelino-Camelia and M. Arzano, hep-th/0105120, Phys. Rev. **D65** (2002) 084044.

43. A. Agostini, G. Amelino-Camelia and F. D'Andrea, hep-th/0306013, Int. J. Mod. Phys. **A19** (2004) 5187.

44. A. Agostini, G. Amelino-Camelia, M. Arzano, A. Marcianó and R. A. Tacchi, hep-th/0607221.

45. M. Arzano and A. Marcianó, hep-th/0701268, Phys. Rev. **D75** (2007) 081701.

46. M. Arzano and A. Marcianó, arXiv:0707.1329 [hep-th].

47. S. Majid and H. Ruegg, Phys. Lett. **B334** (1994) 348.

48. J. Lukierski, H. Ruegg and W.J. Zakrzewski Ann. Phys. **243** (1995) 90.

49. J. Kowalski-Glikman and S. Nowak, hep-th/0204245, Int. J. Mod. Phys. **D12** (2003) 299;

50. G. Amelino-Camelia, gr-qc/0402092.

51. J. Lukierski, A. Nowicki, H. Ruegg, and V.N. Tolstoy, Phys. Lett. **B264** (1991) 331.

52. C. Rovelli, gr-qc/9710008, Living Rev. Rel. **1** (1998) 1.

53. L. Smolin, hep-th/0303185; L. Smolin, "Three Roads to Quantum Gravity" (Weidenfeld and Nicolson, London, 2000).

54. A. Ashtekar, gr-qc/0112038.

55. T. Thiemann: gr-qc/0210094, Lect. Notes Phys. **631** (2003) 41.

56. J. Bernabeu, N.E. Mavromatos and J. Papavassiliou, hep-ph/0310180, Phys. Rev. Lett. **92** (2004) 131601.

57. G. 't Hooft: Class. Quant. Grav. **13** (1996) 1023.

58. G. Amelino-Camelia, gr-qc/0205125; gr-qc/0309054.

59. N.R. Douglas and N.A. Nekrasov: Rev. Mod. Phys. **73** (2001) 977.

60. G. Veneziano, Europhys. Lett. **2** (1986) 199; D.J. Gross and P.F. Mende, Nucl. Phys. **B303** (1988) 407; D. Amati, M. Ciafaloni, G. Veneziano, Phys. Lett. **B216** (1989) 41; K. Konishi, G. Paffuti, P. Provero, Phys. Lett. **B234** (1990) 276; T. Yoneya, Mod. Phys. Lett. **A4** (1989) 1587.

61. E. Witten, Phys. Today **49** (1996) 24.

TESTING CPT IN THE NEUTRAL KAON SYSTEM BY MEANS OF THE BELL-STEINBERGER RELATION

Gino Isidori
INFN, Laboratori Nazionali di Frascati, I-00044 Frascati,Italy

Abstract

The possibility to test the basic assumptions of quantum field theories, and in particular the CPT theorem, by means of unitarity relations in the neutral kaon system (Bell-Steinberger relation) is reviewed. The present status of these tests and their future prospects are also briefly outlined.

1 Introduction

The three discrete symmetries of charge conjugation (C), parity (P) and time reversal (T) are known to be violated in nature, both separately and in any bilinear combination. Only CPT, namely the product of the three (in any order), seems to be an exact symmetry in nature. This fact is not surprising: exact CPT invariance is expected in any quantum field theory respecting the general hypotheses of Lorentz invariance, locality and unitarity [1]. For this reason, testing the validity of CPT invariance is equivalent to probe some of the most fundamental assumptions on which the present description of particle physics is based. Interestingly enough, these hypotheses are likely to be violated at very high energy scales, where the quantum effects of gravitational interactions cannot be ignored [2]. On the other hand, since we still miss a consistent theory of quantum gravity, it is hard to predict at which level CPT-violating effects may show up in experimentally accessible systems.

The neutral kaon system offers a unique possibility for phenomenological studies of CPT invariance. One of the most significant tests is the one obtained by means of the Bell-Steinberger (BS) relation [3]. This relation makes use of unitarity (or the conservation of probability) to connect a possible violation of CPT invariance in the time-evolution of the K^0–$\overline{K}^0$ system ($m_{K^0} \neq m_{\overline{K}^0}$ and/or $\Gamma_{K^0} \neq \Gamma_{\overline{K}^0}$) to the observable CP-violating interference of K_L and K_S decays into the same final state f. Because of the involvement of the unitarity hypothesis, the BS relation cannot be considered as model-independent test of CPT invariance. However, this does not diminish the role of this relation in testing the basic assumptions of quantum field theories (we recall that unitarity is also one of the main hypothesis of the CPT theorem).

2 Theoretical framework

Within the Wigner-Weisskopf approximation, the time evolution of the neutral kaon system is described by [4]

$$i\frac{\partial}{\partial t}\Psi(t) = H\Psi(t) = (M - \frac{i}{2}\Gamma)\Psi(t) \ , \tag{1}$$

where M and Γ are 2×2 time-independent Hermitian matrices and $\Psi(t)$ is a two-component state vector in the K^0–$\overline{K}^0$ space. Denoting by m_{ij} and Γ_{ij} the elements of M and Γ in the K^0–$\overline{K}^0$ basis, CPT invariance implies

$$m_{11} = m_{22} \quad (\text{or } m_{K^0} = m_{\overline{K}^0}) \qquad \text{and} \qquad \Gamma_{11} = \Gamma_{22} \quad (\text{or } \Gamma_{K^0} = \Gamma_{\overline{K}^0}) \ . \tag{2}$$

The eigenstates of eq. (1) can be written as

$$K_{S,L} = \frac{1}{\sqrt{2\left(1+|\epsilon_{S,L}|^2\right)}}\left[\left(1+\epsilon_{S,L}\right)K^0 \pm \left(1-\epsilon_{S,L}\right)\bar{K}^0\right] \ , \tag{3}$$

$$\begin{aligned} \epsilon_{S,L} &= \frac{-i\mathrm{Im}\left(m_{12}\right) - \frac{1}{2}\mathrm{Im}\left(\Gamma_{12}\right) \pm \frac{1}{2}\left[m_{\overline{K}^0} - m_{K^0} - \frac{i}{2}\left(\Gamma_{\overline{K}^0} - \Gamma_{K^0}\right)\right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \\ &= \epsilon \pm \delta \ , \end{aligned} \tag{4}$$

such that $\delta = 0$ in the limit of exact CPT invariance.

Unitarity allows us to express the four entries of Γ in terms of appropriate combination of kaon decay amplitudes:

$$\Gamma_{ij} = \sum_f \mathcal{A}_i(f)\mathcal{A}_j(f)^*, \tag{5}$$

where the sum runs over all the accessible final states. Using this decomposition in eq. (4) leads to the BS relation: a link between Re(ϵ), Im(δ) and the physical kaon decay amplitudes. In particular, without any expansion in the CPT-conserving parameters and neglecting only $\mathcal{O}(\epsilon)$ corrections to the coefficient of the CPT-violating parameter δ, we find

$$\left[\frac{\Gamma_S+\Gamma_L}{\Gamma_S-\Gamma_L}+i\tan\phi_{\rm SW}\right]\left[\frac{{\rm Re}(\epsilon)}{1+|\epsilon|^2}-i{\rm Im}(\delta)\right]=\frac{1}{\Gamma_S-\Gamma_L}\sum_f \mathcal{A}_L(f)\mathcal{A}_S^*(f)\;, \quad (6)$$

where $\phi_{\rm SW} = \arctan[2(m_L - m_S)/(\Gamma_S - \Gamma_L)]$. We stress that, contrary to similar expressions which can be found in the literature, eq. (6) is exact and phase-convention independent in the exact CPT limit: an evidence for a non-vanishing Im(δ) resulting from this relation can only be attributed to violations of: i) CPT invariance; ii) unitarity; iii) the time independence of M and Γ in eq. (1).

The advantage of the neutral kaon system is that only few decay modes give significant contributions to the r.h.s. in eq. (6): in practice, only the $\pi\pi(\gamma)$, $\pi\pi\pi$ and $\pi\ell\nu$ modes turn out to be relevant up to the 10^{-7} level. The product of the corresponding decay amplitudes are conveniently expressed in terms of the α_i parameters defined below.

2.1 Two-pion modes

Starting from two pion states, we define

$$\alpha_i = \frac{1}{\Gamma_S}\langle \mathcal{A}_L(i)\mathcal{A}_S^*(i)\rangle = \eta_i\ {\rm BR}({\rm K_S}\to {\rm i})\ , \qquad {\rm i}=\pi^0\pi^0\ ,\ \pi^+\pi^-(\gamma) \quad (7)$$

where $\pi^+\pi^-(\gamma)$ denotes the inclusive sum over bremsstrahlung photons, and $\langle\rangle$ indicates the appropriate phase-space integrals. By construction, the η_i appearing in eq. (7) can also be expressed in terms non-integrated amplitude ratios: $\eta_i = \mathcal{A}_L(i)/\mathcal{A}_S(i)$.

The contributions from $\pi^+\pi^-\gamma$ direct-emission (DE) amplitudes not included in the $\alpha_{\pi^+\pi^-(\gamma)}$ parameter are encoded in

$$\alpha_{\pi\pi\gamma_{\rm DE}} = \alpha_{\pi\pi\gamma_{\rm E1-S}} + \alpha_{\pi\pi\gamma_{\rm E1-L}} + \alpha_{\pi\pi\gamma_{\rm DE\times DE}}\ , \quad (8)$$

where

$$
\begin{aligned}
\alpha_{\pi\pi\gamma_{\rm E1-S}} \quad + \quad \alpha_{\pi\pi\gamma_{\rm E1-L}} &= \qquad (9) \\
&= \frac{1}{\Gamma_S}\left[\langle \mathcal{A}_L(\pi\pi\gamma)\mathcal{A}^*_S(\pi\pi\gamma_{\rm E1})\rangle + \langle \mathcal{A}_L(\pi\pi\gamma_{\rm E1})\mathcal{A}^*_S(\pi\pi\gamma)\rangle\right] \\
&= \Delta B(K_S \to \pi\pi\gamma_{\rm DE})\ \eta_{+-} + (\eta_{+-\gamma} - \eta_{+-})\ {\rm BR}({\rm K_S} \to \pi\pi\gamma)
\end{aligned}
$$

Here $\mathcal{A}_{L,S}(\pi\pi\gamma)$ and $\mathcal{A}_{L,S}(\pi\pi\gamma_{\rm E1})$ denote the leading bremsstrahlung and the electric-dipole DE amplitudes, respectively. Their interference cannot be trivially neglected. ${\rm BR}({\rm K_S} \to \pi\pi\gamma)$ indicates the branching fraction for a real photon emission, with minimum photon-energy cut equivalent to the one used in the corresponding $\eta_{+-\gamma}$ measurement. $\Delta B(K_S \to \pi\pi\gamma_{\rm DE}) = {\rm BR}({\rm K_S} \to \pi\pi\gamma)^{\rm exp} - {\rm BR}({\rm K_S} \to \pi\pi\gamma)^{\rm th-IB}$ is the deviation of the observed $K_S \to \pi\pi\gamma$ decay distribution from the one inferred from a pure bremsstrahlung spectrum.

We have generically denoted by $\alpha_{\pi\pi\gamma_{\rm DE\times DE}}$ the contribution arising from the interference of two DE amplitudes (either electric or magnetic ones). Given the strong experimental suppression of DE amplitudes, this term turns out to be safely negligible up to the 10^{-8} level [5].

2.2 Three-pion modes

For the three pion states we define

$$
\alpha_i = \frac{1}{\Gamma_S}\langle \mathcal{A}_L(i)\mathcal{A}^*_S(i)\rangle = \frac{\tau_{K_S}}{\tau_{K_L}}\eta^*_i\ {\rm BR}({\rm K_L} \to {\rm i}) \qquad {\rm i} = 3\pi^0\ ,\ \pi^0\pi^+\pi^-(\gamma)\ . \quad (10)
$$

Note that in this case the amplitudes are not necessarily constant over the phase space. As a result, the η_i appearing in eq. (10) should be interpreted as appropriate Dalitz-plot averages. In practice, given the poor direct experimental information on η_{000}, in the neutral case it turns out to be more convenient to set a bound on $|\alpha_{\pi^0\pi^0\pi^0}|$ by means of the relation

$$
|\alpha_{\pi^0\pi^0\pi^0}|^2 = \frac{\tau_{K_S}}{\tau_{K_L}}{\rm BR}({\rm K_L} \to 3\pi^0){\rm BR}({\rm K_S} \to 3\pi^0)\ . \quad (11)
$$

2.3 Semileptonic channels

In the case of semileptonic channels, introducing the standard decomposition [6]

$$\begin{aligned} \mathcal{A}(K^0 \to l^+\nu\pi^-) &= A_0(1-y) \, , \\ \mathcal{A}(K^0 \to l^-\nu\pi^+) &= A_0^*(1+y^*)(x_+ - x_-)^* \, , \\ \mathcal{A}(\bar{K}^0 \to l^-\nu\pi^+) &= A_0^*(1+y^*) \, , \\ \mathcal{A}(\bar{K}^0 \to l^+\nu\pi^-) &= A_0(1-y)(x_+ + x_-) \, , \end{aligned} \tag{12}$$

assuming lepton universality, and expanding to first non-trivial order in the small CP- and CPT-violating parameters, leads to

$$\begin{aligned} \textstyle\sum_{\pi l\nu} \langle \mathcal{A}_L(\pi l\nu)\mathcal{A}_S^*(\pi l\nu)\rangle &= 2\Gamma(K_L \to \pi l\nu)\left\{\mathrm{Re}(\epsilon) - \mathrm{Re}(y) - i\left[\mathrm{Im}(x_+) + \mathrm{Im}(\delta)\right.\right. \\ &= 2\Gamma(K_L \to \pi l\nu)\left\{(A_S + A_L)/4 - i\left[\mathrm{Im}(x_+) + \mathrm{Im}(\delta)\right]\right\} \, . \end{aligned}$$

The dependence of $\mathrm{Re}(y)$ has been eliminated taking advantage of the relation $\mathrm{Re}(\epsilon) - \mathrm{Re}(y) = (A_S + A_L)/4$ [6], where $A_{L,S}$ are the observable semileptonic charge asymmetries. The parameter $\mathrm{Im}(x_+)$ can be measured by appropriate time-dependent distributions [7], while $\mathrm{Im}(\delta)$ is one of the two output of the BS relation. In order to get rid of the explicit $\mathrm{Im}(\delta)$ dependence, it is convenient to define

$$\begin{aligned} \alpha_{\pi l\nu} &= \frac{1}{\Gamma_S}\sum_{\pi l\nu}\langle \mathcal{A}_L(\pi l\nu)\mathcal{A}_S^*(\pi l\nu)\rangle + 2i\frac{\tau_{K_S}}{\tau_{K_L}}\mathrm{BR(K_L \to \pi l\nu)Im}(\delta) \\ &= 2\frac{\tau_{K_S}}{\tau_{K_L}}\mathrm{BR(K_L \to \pi l\nu)}\left[\mathrm{(A_S + A_L)/4 - iIm(x_+)}\right] \, . \end{aligned} \tag{14}$$

2.4 Determination of Re(ϵ) and Im(δ)

The α_i defined in eqs. (7), (10), (8), and (14) can be determined (or bounded) in terms of measurable quantities. Taking into account these definitions (in particular the non-standard expression of $\alpha_{\pi l\nu}$), the solution to the unitarity relation in eq. (6) is:

$$\left[\begin{array}{c} \dfrac{\mathrm{Re}(\epsilon)}{1+|\epsilon|^2} \\ \mathrm{Im}(\delta) \end{array}\right] = \frac{1}{N}\left[\begin{array}{cc} 1+\kappa(1-2b) & (1-\kappa)\tan\phi_{\mathrm{SW}} \\ (1-\kappa)\tan\phi_{\mathrm{SW}} & -(1+\kappa) \end{array}\right]\left[\begin{array}{c} \Sigma_i \mathrm{Re}(\alpha_i) \\ \Sigma_i \mathrm{Im}(\alpha_i) \end{array}\right] \, , \tag{15}$$

where $\kappa = \tau_{K_S}/\tau_{K_L}$, $b = \mathrm{BR}(\mathrm{K_L} \to \pi\ell\nu)$, and

$$N = (1+\kappa)^2 + (1-\kappa)^2 \tan^2 \phi_{\mathrm{SW}} - 2b\kappa(1+\kappa) \ . \tag{16}$$

As anticipated, a non-vanishing $\mathrm{Im}(\delta)$ resulting from this relation would signal a major breakthrough in our understinding of fundamental interactions: $\mathrm{Im}(\delta) \neq 0$ could be attirbuted eiter to violations of CPT symmetry, or to violations of unitary (including apparent violations due to undetected final states), or to violations of the Wigner-Weisskopf approximation.

3 Present experimental status and future prospects

The experimental determination of the α_i has recently been reviewed and updated in Ref. [8], taking into account a series of new measurements of K_L and K_S branching ratios by KLOE in conjunction with previous results by other experiements. The complete updated list of inputs is summarized in Table 1. As far as $K \to \pi l \nu$ amplitudes are concerned, the KLOE measurement of the K_S charge asymmetry and the PDG average of the K_L asymmetry have been combined with the time-dependent measurement of K^0 and $\overline{K}^0$ semileptonic rates by CPLEAR [7]. This has allowed an improved determination of the various parameters appering in $K \to \pi l \nu$ amplitudes (see Table 2), and in particular of $\mathrm{Im}(x_+)$, which is the main source of uncertainty in $\alpha_{\pi l \nu}$.

A detailed discussion about the results for all the relevant α_i can be found in Ref. [8]. In Fig. 1 we show the two most representative examples, namely $\alpha_{\pi^+\pi^-}$ and $\alpha_{\pi l \nu}$. Putting all the ingredients together, the values of $\mathrm{Re}(\epsilon)$ and $\mathrm{Im}(\delta)$ obtained by means of the unitarity relation are (see Fig. 2):

$$\mathrm{Re}(\epsilon) = (159.6 \pm 1.3) \times 10^{-5} \ , \qquad \mathrm{Im}(\delta) = (0.4 \pm 2.1) \times 10^{-5} \ . \tag{17}$$

Thanks to the new KLOE data, the error on $\mathrm{Im}(\delta)$ is now completely dominated by $\pi\pi$ states, and in particular by the the $K_L \to \pi^+\pi^-$ channel. The semileptonic term contributes only to about $\sim 10\%$ of the error on $\mathrm{Im}(\delta)$.

The limits on $\mathrm{Im}(\delta)$ and $\mathrm{Re}(\delta)$, which are perfectly compatible with exact CPT invariance, can be translated into constrains on the K^0–$\overline{K^0}$ mass and width differences by means of the relation

$$\delta = \frac{i(m_{K^0} - m_{\overline{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})}{\Gamma_S - \Gamma_L} \cos\phi_{\mathrm{SW}} e^{i\phi_{\mathrm{SW}}} [1 + O(\epsilon)] \ . \tag{18}$$

Table 1: Input values to the Bell-Steinberger relation used in Ref. [8]. For the KLOE averages see Ref. [15].

Observable	Value	Source
τ_{K_S}	0.08958 ± 0.00005 ns	PDG [9]
τ_{K_L}	50.84 ± 0.23 ns	KLOE average
$m_L - m_S$	$(5.290 \pm 0.016) \times 10^9$ s^{-1}	PDG [9]
$\mathrm{BR}(K_S \to \pi^+\pi^-)$	0.69186 ± 0.00051	KLOE average
$\mathrm{BR}(K_S \to \pi^0\pi^0)$	0.30687 ± 0.00051	KLOE average
$\mathrm{BR}(K_S \to \pi e \nu)$	$(11.77 \pm 0.15) \times 10^{-4}$	KLOE [10]
$\mathrm{BR}(K_L \to \pi^+\pi^-)$	$(1.933 \pm 0.021) \times 10^{-3}$	KLOE average
$\mathrm{BR}(K_L \to \pi^0\pi^0)$	$(0.848 \pm 0.010) \times 10^{-3}$	KLOE average
ϕ_{+-}	$(43.4 \pm 0.7)^\circ$	PDG [9]
ϕ_{00}	$(43.7 \pm 0.8)^\circ$	PDG [9]
$R_{S,\gamma}$ $(E_\gamma > 20\mathrm{MeV})$	$(0.710 \pm 0.016) \times 10^{-2}$	E731 [11]
$R^{\mathrm{th-IB}}_{S,\gamma}$ $(E_\gamma > 20\mathrm{MeV})$	$(0.700 \pm 0.001) \times 10^{-2}$	KLOE MC [13]
$\lvert\eta_{+-\gamma}\rvert$	$(2.359 \pm 0.074) \times 10^{-3}$	E773 [12]
$\phi_{+-\gamma}$	$(43.8 \pm 4.0)^\circ$	E773 [12]
$\mathrm{BR}(K_L \to \pi^+\pi^-\pi^0)$	0.1262 ± 0.0011	KLOE average
η_{+-0}	$\big((-2 \pm 7) + i(-2 \pm 9)\big) \times 10^{-3}$	CPLEAR [7]
$\mathrm{BR}(K_L \to 3\pi^0)$	0.1996 ± 0.0021	KLOE average
$\mathrm{BR}(K_S \to 3\pi^0)$	$< 1.5 \times 10^{-7}$ at 95% CL	KLOE [14]
ϕ_{000}	uniform from 0 to 2π	
$\mathrm{BR}(K_L \to \pi \ell \nu)$	0.6709 ± 0.0017	KLOE average
$A_L + A_S$	$(0.5 \pm 1.0) \times 10^{-2}$	$K_{\ell 3}$ average
$\mathrm{Im}(x_+)$	$(0.8 \pm 0.7) \times 10^{-2}$	$K_{\ell 3}$ average

The allowed region in the $(m_{K^0} - m_{\overline{K}^0}), (\Gamma_{K^0} - \Gamma_{\overline{K}^0})$ plane is shown in the right panel of Fig. 2. The strong correlation reflects the high precision of $\mathrm{Im}(\delta)$ compared to $\mathrm{Re}(\delta)$.

Since the total decay widths are dominated by long-distance dynamics, in models where CPT invariance is a pure short-distance phenomenon it is useful to consider the limit $\Gamma_{K^0} = \Gamma_{\overline{K}^0}$. In this limit (i.e. neglecting CPT-violating effects in the decay amplitudes), the following bounds on the neutral kaon mass difference are obtained

$$-5.3 \times 10^{-19}\ \mathrm{GeV} < m_{K^0} - m_{\overline{K}^0} < 6.3 \times 10^{-19}\ \mathrm{GeV} \quad \text{at 95 \% CL}. \qquad (19)$$

Table 2: Results of the combined (CPLEAR+KLOE+PDG) fit of $K \to \pi l \nu$ amplitudes [8].

Amplitude	Value	Correlation coefficients				
$\mathrm{Re}(\delta)$	$(3.4 \pm 2.8) \times 10^{-4}$	1				
$\mathrm{Im}(\delta)$	$(-1.0 \pm 0.7) \times 10^{-2}$	-0.27	1			
$\mathrm{Re}(x_-)$	$(-0.07 \pm 0.25) \times 10^{-2}$	-0.23	-0.58	1		
$\mathrm{Im}(x_+)$	$(0.8 \pm 0.7) \times 10^{-2}$	-0.35	-0.12	0.57	1	
$A_S + A_L$	$(0.5 \pm 1.0) \times 10^{-2}$	-0.12	-0.62	0.99	0.54	1

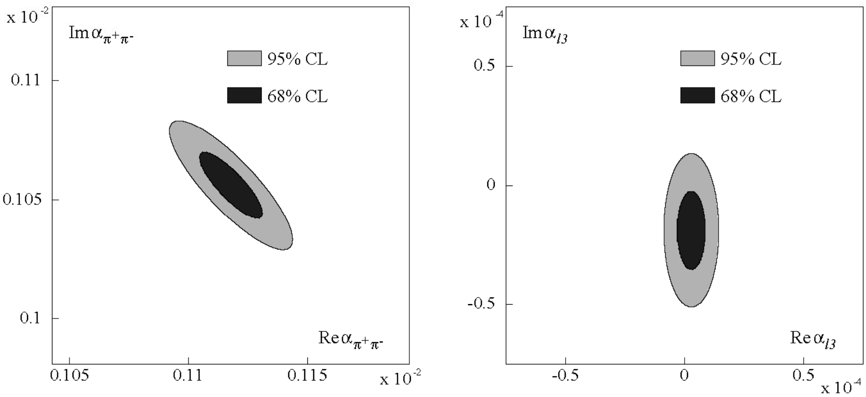


Figure 1: Determination of $\alpha_{\pi^+\pi^-}$ and $\alpha_{\pi l \nu}$ in the complex plane [8]. The two ellipses represent the 68% and the 95% CL contours.

As often emphasized in the literature, this limit provide a significant constraint on models where CPT violating effect scales linearly with the inverse of the Plank mass ($m_K^2/m_{\mathrm{Planck}} \sim 10^{-19}$ GeV). While this fact should not be over emphasized (in several models the power behavior in m^2/m_{Planck} is not linear and the proportionality coefficient is far from unity), there is no doubt that this result is one of the most (if not the most) significant constraint on possible violations of CPT symmetry. It would therefore be very interesting trying to improve it in the future. To this purpose, the analysis of Ref. [8] demonstrates

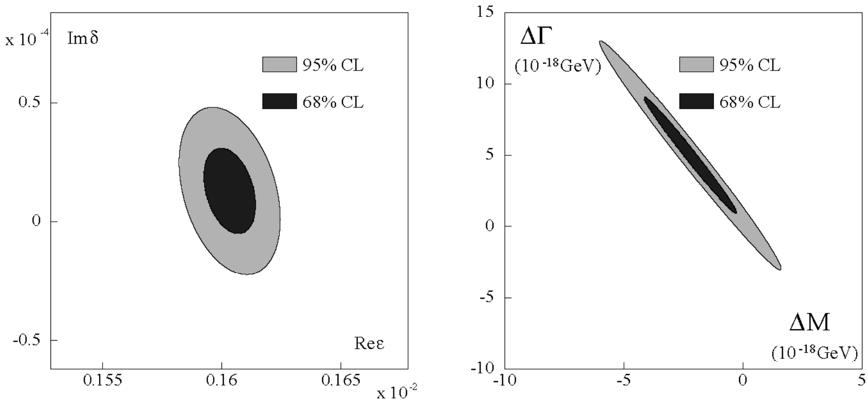


Figure 2: Left: allowed region at 68% and 95% C.L. in the Re(ϵ), Im(δ) plane. Right: allowed region at 68% and 95% C.L. in the $(m_{K^0} - m_{\overline{K}^0})$–$(\Gamma_{K^0} - \Gamma_{\overline{K}^0})$ plane.

that this is possible with new high-precision interference measurements of the CP-violating phases of the $\pi\pi$ final states.

Acknowledgments

It is a pleasure to thank all the authors of Ref. [8], and in particular Mario Antonelli and Matteo Palutan, for interesting discussions and a fruitful collaboration. This work is supported in part by the EU Contract No. MRTN-CT-2006-035482, *FLAVIAnet*.

References

1. G. Lueders, Annals Phys. **2** (1957) 1 [Annals Phys. **281** (2000) 1004].

2. See e.g. V. A. Kostelecky and R. Lehnert, Phys. Rev. D **63** (2001) 065008 [hep-th/0012060]; J. Bernabeu, J. Ellis, N. E. Mavromatos, D. V. Nanopoulos and J. Papavassiliou, hep-ph/0607322; and references therein.

3. J.S.Bell and J.Steinberger, Proceedings Oxford Int. Conf. on Elementary Particles (1965)

4. V. Weisskopf and E. P. Wigner, Z. Phys. **63** (1930) 54; T. D. Lee, R. Oehme and C. N. Yang, Phys. Rev. **106** (1957) 340.

5. G. D'Ambrosio and G. Isidori, Int. J. Mod. Phys. A **13** (1998) 1 [hep-ph/9611284].

6. L. Maiani, G. Pancheri and N. Paver, *The Second DAΦNE Physics Handbook* (Frascati, 1995).

7. CPLEAR Collaboration, A. Angelopoulos et al., Phys. Reports **374** (2003) 165.

8. F. Ambrosino *et al.* [KLOE Collaboration], JHEP **0612** (2006) 011 [arXiv:hep-ex/0610034].

9. Particle Data Group, W.-M. Yao et al., Journal of Physics **G33** (2006) 1.

10. KLOE Collaboration, F. Ambrosino et al., Phys. Lett. **B636** (2006) 173.

11. E. Ramberg et al., Phys. Rev. Lett.**70** (1993) 2525.

12. E. Ramberg et al., Phys. Rev. Lett.**70** (1993) 2529.

13. C. Gatti, Eur. Phys. J. **C45** (2005) 417, and references therein.

14. KLOE Collaboration, F. Ambrosino et al., Phys. Lett. **B619** (2005) 61.

15. KLOE Collaboration, F. Ambrosino et al., Phys. Lett. **B632** (2006) 43; Phys. Lett. **B626** (2005) 15; Phys. Lett. **B638** (2006) 140; hep-ex/0601025.

STRANGENESS MEASUREMENTS OF KAON PAIRS, *CP* VIOLATION AND BELL INEQUALITIES

Reinhold A. Bertlmann and Beatrix C. Hiesmayr
Institute for Theoretical Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria

Abstract

The nonlocal property of quantum mechanics can be nicely tested in high energy physics; in particular, the neutral kaon pairs as produced at DAΦNE, Frascati, are very well suited. The analogies of kaons as compared to polarized photons or spin–$\frac{1}{2}$ particles —the kaonic qubit feature— are reviewed. However, there are also fundamental differences which occur due to the kaon time evolution and due to internal symmetries; in particular, the violation of CP symmetry is related to the violation of Bell inequalities. Two type of Bell inequalities for kaons are presented, one for the variation of the "quasi–spin" and the other for different detection times of the kaon.

1 Introduction

The nonlocality feature of quantum mechanics (QM), as discovered by John Bell in his work "On the Einstein–Podolsky–Rosen Paradox" (EPR) [1], does not conflict with Einstein's relativity, thus it cannot be used for superluminal communication. Nevertheless, Bell's celebrated work [1, 2] initiated new physics, like quantum cryptography [3, 4, 5, 6] and quantum teleportation [7, 8], and it triggered a new technology: quantum information and quantum communication [9, 10]. More about "from Bell to quantum information" can be found in the book [11].

Of course, it is of great interest to investigate the EPR–Bell correlations of measurements also for massive systems in particle physics (for a review see, e.g., Ref. [12]). One of the most exciting systems is the "strange" $K^0\bar{K}^0$ system in a $J^{PC} = 1^{--}$ state [13, 14, 15, 16], where the quantum number *strangeness* $S = +, -$ plays the role of spin $\Uparrow$ and $\Downarrow$ of spin–$\frac{1}{2}$ particles or of polarization V and H of photons. In fact, in comparison to quantum information the kaon can be considered as a "kaonic qubit" [17] but due to its specific internal particle properties (particle–antiparticle oscillation and decay characteristics, symmetry violation) additional fundamental quantum features —not occurring in photon systems— are seen.

Several authors [18, 19, 20, 21, 22, 23, 24, 25, 26] suggested already to investigate the $K^0\bar{K}^0$ pairs which are produced at the Φ resonance, for instance in the e^+e^-–machine DAΦNE at Frascati. There is the great chance to test many different aspects of QM, for instance, Bell inequalities and decoherence models (see, e.g., Ref. [12]), the quantum eraser phenomenon [27, 28, 29] and symmetry violation [30]. In particular, local realistic theories (LRT) have been constructed, which describe the $K^0\bar{K}^0$ pairs, as tests versus quantum mechanics [31, 32, 33, 34]. However, a general test of LRT versus QM is usually performed via Bell inequalities, where —as we shall see— we have more options. We may choose either different "quasi–spins" of the kaon or different kaon detection times (or both); they play the role of the different angles in the photon or spin–$\frac{1}{2}$ case. Due to the kaon decay we have in addition to the *active* measurement procedure the *passive* measurement. Furthermore, an interesting feature of kaons is CP violation in the mixing of particle–antiparticle and indeed it is related to the violation of Bell inequalities.

Besides the kaon system which is an ideal tool to test the amazing features of QM, there is the $B^0\bar{B}^0$ system which is produced to an enormous amount at the asymmetric B–factories at KEK-B [35] and at PEP-II [36]. A Bell inequality (BI) for this system [37] faces, however, with difficulties so that it cannot be considered as a Bell test refuting local realism. The two main drawbacks are: Firstly, "active" measurements —a necessary requirement for the validity of a BI— are missing, therefore one can construct a local realistic model; and secondly, the unitary time evolution of the unstable quantum state —the decay property of the meson, which is part of its nature— has been ignored (for more detailed criticism, see Refs. [38, 39, 40]). Nevertheless, the $B^0\bar{B}^0$ events, the asymmetry of like– and unlike–flavor events for several different times, at KEK-B [41] are ideal to test the validity of the quantum mechanical wavefunction or to confirm the corresponding time dependence of possible decoherence effects, see Refs. [12, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51].

Finally, we want to mention quite different attempts to test QM versus LRT, these are the positron annihilation experiments [52, 53, 54, 55, 56, 57], the proton–proton scattering experiments [58] and the $\Lambda\bar{\Lambda}$ [59, 60] and $\tau^+\tau^-$ pair productions [61, 62]. Unfortunately, all these reactions suffer by loopholes and are not conclusive as Bell tests (for a detailed discussion, see Ref. [32]).

2 Kaons as qubits

Kaons are fantastic quantum systems, we could even say they are selected by Nature to demonstrate fundamental quantum principles such as:

- superposition principle
- oscillation and decay property
- quasi-spin property.

Let us focus on the quantum features which we need in our discussion.

2.1 Quantum states of kaons

Quantum–mechanically we can describe the kaons in the following way. Kaons are characterized by their *strangeness* quantum number $+1, -1$

$$S|K^0\rangle = +|K^0\rangle \ , \qquad S|\bar{K}^0\rangle = -|\bar{K}^0\rangle \ , \tag{1}$$

and the combined operation CP gives

$$CP|K^0\rangle = -|\bar{K}^0\rangle \ , \qquad CP|\bar{K}^0\rangle = -|K^0\rangle \ . \tag{2}$$

It is straightforward to construct the CP eigenstates

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle - |\bar{K}^0\rangle\} \ , \qquad |K_2^0\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle + |\bar{K}^0\rangle\} \ , \tag{3}$$

a quantum number conserved in strong interactions

$$CP|K_1^0\rangle = +|K_1^0\rangle\ , \qquad CP|K_2^0\rangle = -|K_2^0\rangle\ . \tag{4}$$

However, due to weak interactions CP symmetry is *violated* and the kaons decay in physical states, the short– and long–lived states, $|K_S\rangle, |K_L\rangle$, which differ slightly in mass, $\Delta m = m_L - m_S = 3.49 \times 10^{-6}$ eV, but immensely in their lifetimes and decay modes

$$|K_S\rangle = \frac{1}{N}\{p|K^0\rangle - q|\bar{K}^0\rangle\}\ , \qquad |K_L\rangle = \frac{1}{N}\{p|K^0\rangle + q|\bar{K}^0\rangle\}\ . \tag{5}$$

The weights $p = 1+\varepsilon,\ q = 1-\varepsilon$, with $N^2 = |p|^2 + |q|^2$ contain the complex CP *violating parameter* ε with $|\varepsilon| \approx 10^{-3}$. CPT *invariance* is assumed. The short–lived K–meson decays dominantly into $K_S \longrightarrow 2\pi$ with a width or lifetime $\Gamma_S^{-1} \sim \tau_S = 0.89 \times 10^{-10}$ s and the long–lived K–meson decays dominantly into $K_L \longrightarrow 3\pi$ with $\Gamma_L^{-1} \sim \tau_L = 5.17 \times 10^{-8}$ s. However, due to CP violation we observe a small amount $K_L \longrightarrow 2\pi$.

In this description the superpositions (3) and (5) —or quite generally any vector in the 2–dimensional complex Hilbert space of kaons— represent kaonic qubit states in analogy to the qubit states in quantum information.

2.2 Strangeness oscillation

K_S, K_L are eigenstates of a non–Hermitian "effective mass" Hamiltonian

$$H \;=\; M - \frac{i}{2}\,\Gamma \tag{6}$$

satisfying

$$H\,|K_{S,L}\rangle \;=\; \lambda_{S,L}\,|K_{S,L}\rangle \qquad \text{with} \qquad \lambda_{S,L} \;=\; m_{S,L} - \frac{i}{2}\,\Gamma_{S,L}\ . \tag{7}$$

Both mesons K^0 and $\bar{K}^0$ have transitions to common states (due to weak interactions) therefore they mix, that means they *oscillate* between K^0 and $\bar{K}^0$ before decaying. Since the decaying states evolve —according to the Wigner–Weisskopf approximation— exponentially in time

$$|K_{S,L}(t)\rangle \;=\; e^{-i\lambda_{S,L}\,t}|K_{S,L}\rangle\ , \tag{8}$$

the subsequent time evolution for K^0 and $\bar{K}^0$ is given by

$$|K^0(t)\rangle = g_+(t)|K^0\rangle + \frac{q}{p}g_-(t)|\bar{K}^0\rangle\ , \quad |\bar{K}^0(t)\rangle = \frac{p}{q}g_-(t)|K^0\rangle + g_+(t)|\bar{K}^0\rangle \tag{9}$$

with

$$g_{\pm}(t) \;=\; \frac{1}{2}\left[\pm e^{-i\lambda_S t} + e^{-i\lambda_L t}\right] \;. \tag{10}$$

Supposing that a K^0 beam is produced at $t = 0$, e.g. by strong interactions, then the probability for finding a K^0 or $\bar{K}^0$ in the beam is calculated to be

$$\begin{aligned} \left|\langle K^0|K^0(t)\rangle\right|^2 &= \frac{1}{4}\left\{e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2\,e^{-\Gamma t}\cos(\Delta m t)\right\} \;, \\ \left|\langle \bar{K}^0|K^0(t)\rangle\right|^2 &= \frac{1}{4}\frac{|q|^2}{|p|^2}\left\{e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-\Gamma t}\cos(\Delta m t)\right\} \,, \end{aligned} \tag{11}$$

with $\Delta m = m_L - m_S$ and $\Gamma = \frac{1}{2}(\Gamma_L + \Gamma_S)\,.$

The K^0 beam oscillates with frequency $\Delta m/2\pi$, where $\Delta m\,\tau_S = 0.47$. The oscillation is clearly visible at times of the order of a few τ_S, before all K_S's have died out leaving only the K_L's in the beam. So in a beam which contains only K^0 mesons at the beginning $t = 0$ there will occur $\bar{K}^0$ far from the production source through its presence in the K_L meson.

2.3 Quasi–spin of kaons and analogy to photons

In comparison with spin–$\frac{1}{2}$ particles, or with photons having the polarization directions V (vertical) and H (horizontal), it is very instructive to characterize the kaons by a *quasi–spin* (for details see Ref. [63]). We can regard the two states $|K^0\rangle$ and $|\bar{K}^0\rangle$ as the quasi–spin states up $|\!\uparrow\rangle$ and down $|\!\downarrow\rangle$ and can express the operators acting in this quasi–spin space by Pauli matrices. So we identify the strangeness operator S with the Pauli matrix σ_3, the CP operator with $(-\sigma_1)$ and for describing CP violation we also need σ_2. In fact, the Hamiltonian (6) then has the form

$$H \;=\; a\cdot\mathbf{1} + \vec{b}\cdot\vec{\sigma}\;, \tag{12}$$

with

$$\begin{aligned} b_1 = b\cos\alpha, \quad b_2 = b\sin\alpha, \quad b_3 = 0\;, \\ a = \frac{1}{2}(\lambda_L + \lambda_S), \quad b = \frac{1}{2}(\lambda_L - \lambda_S)\;, \end{aligned} \tag{13}$$

and the angle α is related to the CP violating parameter ε by

$$e^{i\alpha} \;=\; \frac{1-\varepsilon}{1+\varepsilon}\;. \tag{14}$$

Summarizing, we have the following kaonic–photonic analogy:

neutral kaon	**quasi–spin**	**photon**
$\vert K^0\rangle$	$\vert\uparrow\rangle_z$	$\vert V\rangle$
$\vert\bar{K}^0\rangle$	$\vert\downarrow\rangle_z$	$\vert H\rangle$
$\vert K_1^0\rangle$	$\vert\nwarrow\rangle$	$\vert -45^0\rangle = \frac{1}{\sqrt{2}}(\vert V\rangle - \vert H\rangle)$
$\vert K_2^0\rangle$	$\vert\nearrow\rangle$	$\vert +45^0\rangle = \frac{1}{\sqrt{2}}(\vert V\rangle + \vert H\rangle)$
$\vert K_S\rangle$	$\vert\rightarrow\rangle_y$	$\vert L\rangle = \frac{1}{\sqrt{2}}(\vert V\rangle - i\vert H\rangle)$
$\vert K_L\rangle$	$\vert\leftarrow\rangle_y$	$\vert R\rangle = \frac{1}{\sqrt{2}}(\vert V\rangle + i\vert H\rangle)$

A good *optical analogy* to the phenomenon of strangeness oscillation can be achieved by using the physical effect of birefringence in optical fibers which leads to the rotation of polarization directions. Thus H (horizontal) polarized light is rotated after some distance into V (vertical) polarized light, and so on. On the other hand, the decay of kaons can be simulated by polarization dependent losses in optical fibres, where one state has lower losses than its orthogonal state [64].

The description of kaons as qubits reveals close analogies to photons but also deep physical differences. Kaons oscillate, they are massive, they decay and can be characterized by symmetries like CP. Even though some kaon features, like oscillation and decay, can be mimicked by photon experiments (see Ref. [64]), they are inherently different since they are intrinsic properties of the kaon given by Nature.

2.4 Measurement procedures

For neutral kaons there exist two physical alternative bases, accordingly we have two observables for the kaons, namely the projectors to the two bases. The first basis is the strangeness eigenstate basis $\{|K^0\rangle, |\bar{K}^0\rangle\}$, it can be measured by inserting along the kaon trajectory a piece of ordinary matter, which corresponds to an *active* measurement of strangeness. Due to strangeness conservation of the strong interactions the incoming state is projected either onto

K^0 by $K^0 p \to K^+ n$ or onto $\bar{K}^0$ by $\bar{K}^0 p \to \Lambda \pi^+$, $\bar{K}^0 n \to \Lambda \pi^0$ or $\bar{K}^0 n \to K^- p$. Here nucleonic matter plays the same role as a two channel analyzer for polarized photon beams.

Alternatively, the strangeness content of neutral kaons can be determined by observing their semileptonic decay modes, eq.(23).

Obviously, the experimenter has no control of the kaon decay, neither of the mode nor of the time. The experimenter can only sort at the end of the day all observed events in proper decay modes and time intervals. We call this procedure opposite to the *active* measurement described above a *passive* measurement procedure of strangeness.

The second basis $\{K_S, K_L\}$ consists of the short– and long–lived states having well defined masses $m_{S(L)}$ and decay widths $\Gamma_{S(L)}$. We have seen that it is the appropriate basis to discuss the kaon propagation in free space, because these states preserve their own identity in time, eq.(8). Due to the huge difference in the decay widths the K_S's decay much faster than the K_L's. Thus in order to observe if a propagating kaon is a K_S or K_L at an instant time t, one has to detect at which time it subsequently decays. Kaons which are observed to decay before $\simeq t + 4.8\,\tau_S$ have to be identified as K_S's, while those surviving after this time are assumed to be K_L's. Misidentifications reduce only to a few parts in 10^{-3} (see Refs. [27, 28]). Note that the experimenter doesn't care about the specific decay mode, she/he records only a decay event at a certain time. We call this procedure an *active* measurement of lifetime.

Since the neutral kaon system violates the CP symmetry (recall Section 2.1) the mass eigenstates are not strictly orthogonal, $\langle K_S | K_L \rangle \neq 0$. However, neglecting CP violation —remember it is of the order of 10^{-3}— the K_S's are identified by a 2π final state and K_L's by a 3π final state. We call this procedure a *passive* measurement of lifetime, since the kaon decay times and decay channels used in the measurement are entirely determined by the quantum nature of kaons and cannot be in any way influenced by the experimenter. It is assumed that *active* and *passive* measurements have the same amount of misidentifications.

The important message for testing Bell inequalities which we are going to discuss in the next section is:

- The *active* measurement procedures are a necessary requirement for the validity of a BI.

3 Entangled kaons, Bell inequalities, CP violation

Having discussed kaons as qubit states and their analogy to photons we consider next two qubit states. A two qubit system of kaons is a general superposition of the 4 states $\{|K^0\rangle \otimes |K^0\rangle, |K^0\rangle \otimes |\bar{K}^0\rangle, |\bar{K}^0\rangle \otimes |K^0\rangle, |\bar{K}^0\rangle \otimes |\bar{K}^0\rangle\}$.

3.1 Entanglement

Interestingly, also for strange mesons entangled states can be obtained, in analogy to the entangled spin up and down pairs, or H and V polarized photon pairs. Such states are produced by e^+e^-–colliders through the reaction $e^+e^- \to \Phi \to K^0\bar{K}^0$, in particular at DAΦNE in Frascati, or they are produced in $p\bar{p}$–collisions, like, e.g., at LEAR at CERN [65]. There, a $K^0\bar{K}^0$ pair is created in a $J^{PC} = 1^{--}$ quantum state and thus antisymmetric under C and P, and is described at the time $t = 0$ by the entangled state

$$\begin{aligned} |\psi(t=0)\rangle &= \frac{1}{\sqrt{2}}\left\{|K^0\rangle_l \otimes |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l \otimes |K^0\rangle_r\right\} , \\ &= \frac{N_{SL}}{\sqrt{2}}\left\{|K_S\rangle_l \otimes |K_L\rangle_r - |K_L\rangle_l \otimes |K_S\rangle_r\right\} , \end{aligned} \tag{15}$$

with $N_{SL} = \frac{N^2}{2pq}$, in complete analogy to the entangled photon case

$$\begin{aligned} |\psi\rangle &= \frac{1}{\sqrt{2}}\left\{|V\rangle_l \otimes |H\rangle_r - |H\rangle_l \otimes |V\rangle_r\right\} , \\ &= \frac{1}{\sqrt{2}}\left\{|L\rangle_l \otimes |R\rangle_r - |R\rangle_l \otimes |L\rangle_r\right\} . \end{aligned} \tag{16}$$

The neutral kaons fly apart and are detected on the left (l) and right (r) hand side of the source. Of course, during their propagation the $K^0\bar{K}^0$ pairs oscillate and the K_S, K_L states decay. This is an important difference to the case of photons which are stable.

Let us measure *actively* at time t_l a K^0 meson on the left hand side and at time t_r a K^0 or a $\bar{K}^0$ on the right hand side then we find an EPR–Bell correlation analogously to the entangled photon case with polarization V–V or V–H. Assuming for simplicity stable kaons ($\Gamma_S = \Gamma_L = 0$) then we get the following result for the quantum probabilities

$$
\begin{aligned}
P(K^0, t_l; K^0, t_r) &= P(\bar{K}^0, t_l; \bar{K}^0, t_r) = \frac{1}{4}\big\{1 - \cos(\Delta m(t_l - t_r))\big\} \,, \\
P(K^0, t_l; \bar{K}^0, t_r) &= P(\bar{K}^0, t_l; K^0, t_r) = \frac{1}{4}\big\{1 + \cos(\Delta m(t_l - t_r))\big\} \,, \qquad (17)
\end{aligned}
$$

which is the analogy to the probabilities of finding simultaneously two entangled photons along two chosen directions $\vec{\alpha}$ and $\vec{\beta}$

$$
\begin{aligned}
P(\vec{\alpha}, V; \vec{\beta}, V) &= P(\vec{\alpha}, H; \vec{\beta}, H) = \frac{1}{4}\big\{1 - \cos 2(\alpha - \beta)\big\} \,, \\
P(\vec{\alpha}, V; \vec{\beta}, H) &= P(\vec{\alpha}, H; \vec{\beta}, V) = \frac{1}{4}\big\{1 + \cos 2(\alpha - \beta)\big\} \,. \qquad (18)
\end{aligned}
$$

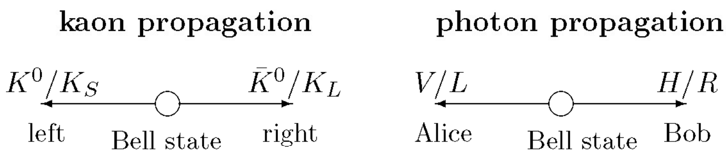
Thus we observe a *perfect analogy* between times $\Delta m(t_l - t_r)$ and angles $2(\alpha - \beta)$.

Alternatively, we also can fix the time and vary the quasi–spin of the kaon, which corresponds to a rotation in quasi–spin space analogously to the rotation of polarization of the photon

$$
\mid k\rangle = a \mid K^0\rangle + b \mid \bar{K}^0\rangle \quad \longleftrightarrow \quad \mid \alpha, \phi; V\rangle = \cos\alpha \mid V\rangle + \sin\alpha\, e^{i\phi} \mid H\rangle \,. \qquad (19)
$$

Note that the weights a, b are not independent and not all kaonic superpositions are realized in Nature in contrast to photons.

Depicting the kaonic–photonic analogy we have:



- $K^0\bar{K}^0$ oscillation
- K_S, K_L decay

- stable

3.2 Bell inequality for quasi–spin variation

Consequently, for establishing a BI for kaons we have the option:

- varying the quasi–spin — fixing time
- fixing the quasi–spin — varying time.

Let us begin with a BI for certain quasi–spins (first option) and demonstrate that its violation is related to a symmetry violation in particle physics. In Ref. [66, 67] it was shown that symmetries quite generally may constrain local realistic theories.

For a BI we need 3 different "quasi–spins" – the "Bell angles" – and we may choose the H, S and CP eigenstates: $|K_S\rangle\,, |\bar{K}^0\rangle$ and $|K_1^0\rangle$.

Denoting the probability of measuring the short–lived state K_S on the left hand side and the anti–kaon $\bar{K}^0$ on the right hand side, both at the time $t = 0$, by $P(K_S, \bar{K}^0)$, and analogously the probabilities $P(K_S, K_1^0)$ and $P(K_1^0, \bar{K}^0)$ we can easily derive under the usual hypothesis of Bell's locality the following *Wigner–like Bell inequality* [68, 69]

$$P(K_S, \bar{K}^0) \;\leq\; P(K_S, K_1^0) + P(K_1^0, \bar{K}^0)\;. \tag{20}$$

BI (20) is rather formal because it involves the unphysical CP–even state $|K_1^0\rangle$, but – and this is now important – it implies an inequality on a *physical* quantity, the CP violation parameter. Inserting the quantum amplitudes

$$\langle\bar{K}^0 \mid K_S\rangle = -\frac{q}{N}\,, \qquad \langle\bar{K}^0 \mid K_1^0\rangle = -\frac{1}{\sqrt{2}}\,, \qquad \langle K_S \mid K_1^0\rangle = \frac{1}{\sqrt{2}N}(p^* + q^*)\,, \tag{21}$$

and optimizing the inequality we can convert (20) into an inequality for the complex kaon transition coefficients p, q

$$|\,p\,| \quad\leq\quad |\,q\,|\;. \tag{22}$$

It's amazing, inequality (22) is *experimentally testable*! How does it work?

3.3 Semileptonic decays

Let us consider the semileptonic decays of the kaons. The strange quark s decays weakly as constituent of $\bar{K}^0$ (see Fig.1):

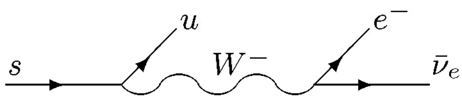


Figure 1: *Strange quark decays weakly.*

Due to the quark content $K^0(\bar{s}d)$ and $\bar{K}^0(s\bar{d})$ have the following decays:

$$\begin{array}{llllll} K^0(d\bar{s}) & \longrightarrow & \pi^-(d\bar{u})\ l^+\ \nu_l & \text{where} & \bar{s} \longrightarrow \bar{u}\ l^+\ \nu_l & \\ \bar{K}^0(\bar{d}s) & \longrightarrow & \pi^+(\bar{d}u)\ l^-\ \bar{\nu}_l & \text{where} & s \longrightarrow u\ l^-\ \bar{\nu}_l\ , & (23) \end{array}$$

with $l = \mu, e$. When studying the leptonic charge asymmetry

$$\delta \;=\; \frac{\Gamma(K_L \to \pi^- l^+ \nu_l) - \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)}{\Gamma(K_L \to \pi^- l^+ \nu_l) + \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)}\ , \tag{24}$$

we notice that l^+ and l^- tag K^0 and $\bar{K}^0$, respectively, in the K_L state, and the leptonic asymmetry (24) is expressed by the probabilities $|p|^2$ and $|q|^2$ of finding a K^0 and a $\bar{K}^0$, respectively, in the K_L state

$$\delta \;=\; \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}\ . \tag{25}$$

Returning to inequality (22) we find consequently the bound

$$\delta \;\leq\; 0 \tag{26}$$

for the leptonic charge asymmetry which measures CP violation.

Experimentally, however, the asymmetry is nonvanishing [70)]

$$\delta = (3.27 \pm 0.12) \cdot 10^{-3}\ . \tag{27}$$

What we find is that bound (26), dictated by BI (20), is in contradiction to the experimental value (27) which is definitely positive.

On the other hand, we can replace $\bar{K}^0$ by K^0 in the BI (20) and obtain the reversed inequality $\delta \geq 0$ so that respecting all possible BI's leads to strict equality $\delta = 0$, CP conservation, in contradiction to experiment.

Conclusion: The premises of LRT are *only* compatible with strict CP conservation in $K^0\bar{K}^0$ mixing. Conversely, CP violation in $K^0\bar{K}^0$ mixing, no matter which sign the experimental asymmetry (24) actually has, always leads to a *violation* of a BI and in consequence rules out a local realistic theory for the description of a $K^0\bar{K}^0$ system!

Remark: We believe that this connection between symmetry violation and BI violation is not just accidental for the CP symmetry case but is more fundamental and should be observed in case of other symmetries as well.

3.4 Bell inequality for time variation

Bell inequalities by fixing the quasi–spin and varying the time we have studied already in detail in Refs. [63, 38, 71, 72]. As we emphasized in a *unitary* time evolution also the decay states are involved, in fact, in the following way.

The complete time evolution of the kaon states is given by a *unitary* operator $U(t,0)$ whose effect can be written as [73, 74]

$$U(t,0)\,|K_{S,L}\rangle \;=\; e^{-i\lambda_{S,L}t}\,|K_{S,L}\rangle + |\Omega_{S,L}(t)\rangle\,, \tag{28}$$

where $|\Omega_{S,L}(t)\rangle$ denotes the state of all decay products. The norm decrease of the state $|K_{S,L}(t)\rangle$ must be compensated by the increase of the norm of the final states, i.e., $\langle\Omega_{S,L}(t)|\Omega_{S,L}(t)\rangle = 1 - e^{-\Gamma_{S,L}\,t}$ and $\langle\Omega_L(t)|\Omega_S(t)\rangle = \langle K_L|K_S\rangle(1 - e^{i\Delta m\,t}e^{-\Gamma t})$, $\langle K_{S,L}|\Omega_S(t)\rangle = \langle K_{S,L}|\Omega_L(t)\rangle = 0$.

Let us start at time $t=0$ with an entangled state of kaon pairs given in the K_SK_L basis choice (recall eq.15)

$$|\psi(t=0)\rangle \;=\; \frac{N_{SL}}{\sqrt{2}}\,\{|K_S\rangle_l\otimes|K_L\rangle_r - |K_L\rangle_l\otimes|K_S\rangle_r\}\,. \tag{29}$$

Then we get the state at time t from (29) by applying the unitary operator

$$U(t,0) \;=\; U_l(t,0)\cdot U_r(t,0)\,, \tag{30}$$

where the operators $U_l(t,0)$ and $U_r(t,0)$ act on the space of the left and of the right mesons according to the time evolution (28).

For the quantum mechanical probabilities for detecting, or not detecting, a specific quasi–spin state on the left side, say $|\bar{K}^0\rangle_l$, and on the right side

$|\bar{K}^0\rangle_r$ of the source we need the projection operators

$$P_{l,r}(\bar{K}^0) = |\bar{K}^0\rangle\langle\bar{K}^0|_{l,r} \qquad \text{and} \qquad Q_{l,r}(\bar{K}^0) = \mathbf{1} - P_{l,r}(\bar{K}^0)\,. \tag{31}$$

Starting from the initial state (29) the unitary time evolution (30) provides the state at a time t_r

$$|\psi(t_r)\rangle \;=\; U(t_r,0)|\psi(t=0)\rangle \;=\; U_l(t_r,0)U_r(t_r,0)|\psi(t=0)\rangle\,. \tag{32}$$

Measuring now $\bar{K}^0$ at t_r on the right side means that we project onto the state

$$|\tilde{\psi}(t_r)\rangle \;-\; P_r(\bar{K}^0)|\psi(t_r)\rangle\,, \tag{33}$$

and state (33) evolves until t_l when we measure next a $\bar{K}^0$ on the left side

$$|\tilde{\psi}(t_l,t_r)\rangle \;=\; P_l(\bar{K}^0)U_l(t_l,t_r)P_r(\bar{K}^0)|\psi(t_r)\rangle\,. \tag{34}$$

The probability of the joint measurement is given by the squared norm of the state (34) and coincides with the norm of the state

$$|\psi(t_l,t_r)\rangle \;=\; P_l(\bar{K}^0)P_r(\bar{K}^0)U_l(t_l,0)U_r(t_r,0)|\psi(t=0)\rangle\,, \tag{35}$$

which corresponds to a factorization of the eigentimes t_l and t_r.

We calculate the quantum mechanical probability $P_{\bar{K}^0,\bar{K}^0}(Y,t_l;Y,t_r)$ for finding a $\bar{K}^0$ at t_l on the left side *and* a $\bar{K}^0$ at t_r on the right side, and the probability $P_{\bar{K}^0,\bar{K}^0}(N,t_l;N,t_l)$ for finding *no* such kaons by the following norms (and similarly the probability $P_{\bar{K}^0,\bar{K}^0}(Y,t_l;N,t_r)$)

$$P_{\bar{K}^0,\bar{K}^0}(Y,t_l;Y,t_r) \;=\; ||P_l(\bar{K}^0)P_r(\bar{K}^0)U_l(t_l,0)U_r(t_r,0)|\psi(t=0)\rangle||^2 \tag{36}$$

$$P_{\bar{K}^0,\bar{K}^0}(N,t_l;N,t_r) \;=\; ||Q_l(\bar{K}^0)Q_r(\bar{K}^0)U_l(t_l,0)U_r(t_r,0)|\psi(t=0)\rangle||^2 \tag{37}$$

$$P_{\bar{K}^0,\bar{K}^0}(Y,t_l;N,t_r) \;=\; ||P_l(\bar{K}^0)Q_r(\bar{K}^0)U_l(t_l,0)U_r(t_r,0)|\psi(t=0)\rangle||^2 . \tag{38}$$

Then the expectation value for measuring the antikaons is expressed by

$$E_{\bar{K}^0,\bar{K}^0}(t_l,t_r) = -1 + 2\left\{P_{\bar{K}^0,\bar{K}^0}(Y,t_l;Y,t_r) + P_{\bar{K}^0,\bar{K}^0}(N,t_l;N,t_r)\right\}, \tag{39}$$

and with expression (39) Bell inequalities are constructed.

For our purpose we use a BI in the familiar expression of Clauser, Horne, Shimony, Holt (CHSH) [75] which in terms of time variation can be formulated in the following way [63, 74]. Defining the function

$$S(t_1,t_2,t_3,t_4) = |E_{\bar{K}^0,\bar{K}^0}(t_1,t_2) - E_{\bar{K}^0,\bar{K}^0}(t_1,t_3)| + |E_{\bar{K}^0,\bar{K}^0}(t_4,t_2) + E_{\bar{K}^0,\bar{K}^0}(t_4,t_3)|\,, \tag{40}$$

the CHSH–Bell inequality is given by

$$S(t_1,t_2,t_3,t_4) \;\leq\; 2\,, \tag{41}$$

where the value 2 is the maximum satisfied by any LRT.

The question is now whether inequality (41) can be violated in the kaon case. As we know [63, 71, 74] the four Bell states ($\psi^{\mp} \sim K_S K_L \pm K_L K_S$, $\phi^{\mp} \sim K_S K_S \mp K_L K_L$) which are maximal entangled do not violate inequality (41). The reason is that the internal physical parameters, the ratio oscillation to decay, $\Delta m/\Gamma$, is experimentally about 1 whereas for a violation a value of 2 is necessary for the $\psi^{\mp}$ states and a smaller value of about 1.7 for the $\phi^{\mp}$ states.

A recent investigation [72] of a quite general initial state

$$|\psi(0)\rangle = r_1 e^{i\phi_1}|K_S\rangle_l \otimes |K_S\rangle_r + r_2 e^{i\phi_2}|K_S\rangle_l \otimes |K_L\rangle_r + r_3 e^{i\phi_3}|K_L\rangle_l \otimes |K_S\rangle_r + r_4 e^{i\phi_4}|K_L\rangle_l \otimes |K_L\rangle_r\,, \tag{42}$$

(with $r_1^2 + r_2^2 + r_3^2 + r_4^2 = 1$) providing the general expectation value

$$\begin{aligned}
E_{\bar{K}^0,\bar{K}^0}(t_l,t_r) = {} & 1 + r_1^2\, e^{-\Gamma_S(t_l+t_r)} + r_2^2\, e^{-\Gamma_S t_l - \Gamma_L t_r} + r_3^2\, e^{-\Gamma_L t_l - \Gamma_S t_r} \\
& + r_4^2\, e^{-\Gamma_L(t_l+t_r)} - r_1^2\,(e^{-\Gamma_S t_l} + e^{-\Gamma_S t_r}) - r_2^2\,(e^{-\Gamma_S t_l} + e^{-\Gamma_L t_r}) \\
& - r_3^2\,(e^{-\Gamma_L t_l} + e^{-\Gamma_S t_r}) - r_4^2\,(e^{-\Gamma_L t_l} + e^{-\Gamma_L t_r}) \\
& + 2\, r_1 r_2\,(1 - e^{-\Gamma_S t_l}) \cos(\Delta m t_r + \phi_1 - \phi_2)\, e^{-\Gamma t_r} \\
& + 2\, r_1 r_3\, \cos(\Delta m t_l + \phi_1 - \phi_3)\, e^{-\Gamma t_l}\,(1 - e^{-\Gamma_S t_r}) \\
& + 2\, r_2 r_4\, \cos(\Delta m t_l + \phi_2 - \phi_4)\, e^{-\Gamma t_l}\,(1 - e^{-\Gamma_L t_r}) \\
& + 2\, r_3 r_4\,(1 - e^{-\Gamma_L t_l})\, \cos(\Delta m t_r + \phi_3 - \phi_4)\, e^{-\Gamma t_r} \\
& + 2\, r_1 r_4 \cos(\Delta m(t_l + t_r) + \phi_1 - \phi_4)\, e^{-\Gamma(t_l+t_r)} \\
& + 2\, r_2 r_3\, \cos(\Delta m(t_l - t_r) + \phi_2 - \phi_3)\, e^{-\Gamma(t_l+t_r)}\,,
\end{aligned} \tag{43}$$

shows that for a certain parameter choice the CHSH–Bell inequality (41) is indeed *violated*!

The S–function value turns out to be $S = 2.12$ for the parameter choice: all phases $\phi_i = 0$ and $r_1 = -0.834$, $r_2 = r_3 = 0.245$ and times $t_1 = t_2 = 0$, $t_3 = t_4 = 5.6\tau_S$; and $S = 2.16$ for the choice: $\phi_1 = -0.275$, $\phi_2 = \phi_3 = -0.678$ and $r_1 = -0.782$, $r_2 = r_3 = -0.146$ and times $t_1 = t_2 = 1.6\tau_S$, $t_3 = t_4 = 0$. (The numerical optimization procedure does not guarantee a global maximum).

Conclusion: There exist initial states for kaons that —by respecting the unitary time evolution, the decay property— violate a Bell inequality and are therefore nonlocal, although not maximal entangled, which agrees with the qutrit results of Refs. [76, 77]. It shows that nonlocality and entanglement are *not* the same features of QM. The question remains, however, how to produce the initial state (42) with the parameter values given above, e.g., at DAΦNE.

4 Conclusions

Kaons are ideal objects to test the fundamental principles of quantum mechanics, in particular the entanglement or nonlocality properties of kaon pairs, which are of great interest in connection with the physics of quantum communication and quantum information. In fact, in analogy to polarized photons the kaons can be considered as qubits as well but —due to their internal symmetries and time evolution— they exhibit further exciting features as compared to photons.

One is that the violation of CP symmetry in the mixing of $K^0\bar{K}^0$ leads to a violation of a Bell inequality for quasi–spin variation refuting in consequence any local realistic theory.

Another feature is that Bell inequalities for time variations are —due to the unitary time evolution which includes the decay states— much more sophisticated than in the photon case. A CHSH–Bell inequality can be violated for a certain initial state thus ruling out local realistic theories. This nonlocal state is not maximally entangled and shows therefore the difference of the conceptions nonlocality and entanglement. The interesting question is how such a nonlocal state (where the K_SK_S and K_LK_L parts dominate) can be produced at DAΦNE.

Furthermore, using the regeneration feature of the kaons other type of Bell inequalities can be established. The analysis of all possible Bell inequalities together with the choice of suitable initial states and experimental set–ups will be of great importance for testing quantum mechanics at DAΦNE. Work in this direction is in progress [78].

Acknowledgements

We would like to thank A. Di Domenico for the invitation to the Workshop on *Neutral Kaon Interferometry at a Φ–Factory* at Frascati, March 2006. We also acknowledge financial support from EURIDICE HPRN-CT-2002-00311.

References

1. J.S. Bell, Physics **1**, 195 (1964).
2. J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press 1987).
3. A.K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
4. D. Deutsch, A.K. Ekert, Physics World **11**, No.3, p.47 (1998).
5. R.J. Hughes, Contemp. Phys. **36**, 149 (1995).
6. W. Tittel, G. Ribordy, N. Gisin, Physics World **11**, No.3, p.41 (1998); W. Tittel, J. Brendel, H. Zbinden, N. Gisin, Phys. Rev. Lett. **81**, 3563 (1998); H. Zbinden, J. Brendel, W. Tittel, N. Gisin, Phys. Rev. A **63**, 022111 (2001).
7. C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W.K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
8. J.-W. Pan, D. Bouwmeester, H. Weinfurter, A. Zeilinger, Nature **390**, 575 (1997).
9. A. Zeilinger, Physics World **11**, No.3, 35 (1998).
10. D. Bouwmeester, A. Ekert, A. Zeilinger, *The physics of quantum information: quantum cryptography, quantum teleportation, quantum computations* (Springer, Berlin Heidelberg New York 2000).
11. R. A. Bertlmann, A. Zeilinger (eds.), *Quantum* [Un]*speakables, from Bell to Quantum Information* (Springer, Berlin Heidelberg New York 2002).

12. R.A. Bertlmann, *Entanglement, Bell inequalities and decoherence in particle physics*, Lecture Notes in Physics **689**, W. Pötz, J. Fabian, U. Hohenester (eds.), p.1, (Springer-Verlag, Berlin 2006), quant-ph/0410028.

13. T.D. Lee, C.N. Yang, reported by T.D. Lee at Argonne National Laboratory, May 28, 1960 (unpublished).

14. D.R. Inglis, Rev. Mod. Phys. **33**, 1 (1961).

15. T.B. Day, Phys. Rev. **121**, 1204 (1961).

16. H.J. Lipkin, Phys. Rev. **176**, 1715 (1968).

17. R. A. Bertlmann, B. C. Hiesmayr, Quantum Information Processing, DOI: 10.1007/s11128-006-0026-1 (2006), quant-ph/0512171.

18. P.H. Eberhard, in *The Second DaΦne Physics Handbook*, edited by L. Maiani, G. Pancheri, N. Paver (SIS–Pubblicazioni dei Laboratori di Frascati, Italy, 1995) Vol I, p. 99.

19. P.H. Eberhard, Nucl. Phys. B **398**, 155 (1992).

20. I.I. Bigi, Nucl. Phys. B (Proc. Suppl.) **24A**, 24 (1991).

21. A. Di Domenico, Nucl. Phys. B **450**, 293 (1995).

22. A. Bramon, M. Nowakowski, Phys. Rev. Lett. **83**, 1 (1999).

23. B. Ancochea, A. Bramon, M. Nowakowski, Phys. Rev. D **60**, 094008 (1999).

24. A. Bramon, G. Garbarino, Phys. Rev. Lett. **88**, 040403 (2002).

25. M. Genovese, C. Novero, E. Predazzi, Phys. Lett. B **513**, 401 (2001).

26. J. Bernabéu, N.E. Mavromatos, J. Papavassiliou, Phys. Rev. Lett. **92**, 131601 (2004).

27. A. Bramon, G. Garbarino, B. C. Hiesmayr, Phys. Rev. Lett. **92**, 020405 (2004).

28. A. Bramon, G. Garbarino, B. C. Hiesmayr, Phys. Rev. A **68**, 062111 (2004).

29. A. Bramon, G. Garbarino, B. C. Hiesmayr, *Kaonic quantum erasers at KLOE 2; Erasing the present, changing the past*, contribution to the Frascati Physics Series.

30. J. Bernabéu, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, J. Papavassiliou, *CPT and quantum mechanics tests with kaons*, contribution to this handbook [quant-ph/0607322].

31. F. Selleri, Lett. Nuovo Cim. **36**, 521 (1983); P. Privitera, F. Selleri, Phys. Lett. B **296**, 261 (1992); F. Selleri, Phys. Rev. A **56**, 3493 (1997); R. Foadi, F. Selleri, Phys. Rev. A **61**, 012106 (1999); R. Foadi, F. Selleri, Phys. Lett. B **461**, 123 (1999).

32. A. Afriat, F. Selleri, *The Einstein, Podolsky, and Rosen Paradox in Atomic, Nuclear and Particle Physics*, (Plenum Press, New York London 1999).

33. J. Six, Phys. Lett. A **150**, 243 (1990).

34. A. Bramon, G. Garbarino, Phys. Rev. Lett. **89**, 160401 (2002).

35. BELLE-homepage, http://belle.kek.jp

36. BABAR-homepage, http://www.slac.stanford.edu/babar

37. A. Go, J. Mod. Optics **51**, 991 (2004).

38. R.A. Bertlmann, A. Bramon, G. Garbarino, B.C. Hiesmayr, Phys. Lett. A **332**, 355 (2004).

39. A. Bramon, R. Escribano, G. Garbarino, J. Mod. Phys. **52**, 1681 (2005).

40. A. Bramon, R. Escribano, G. Garbarino, Found. Phys. **36**, 563 (2006).

41. G. Leder, talk given at the conference SUSY'02, Hamburg June 2002, http://wwwhephy.oeaw.ac.at/p3w/belle/leder_susy02.pdf (unpublished).

42. R.A. Bertlmann, W. Grimus, Phys. Lett. B **392**, 426 (1997).

43. G.V. Dass, K.V.L. Sarma, Eur. Phys. J. C **5**, 283 (1998).

44. R.A. Bertlmann, W. Grimus:, Phys. Rev. D **58**, 034014 (1998).

45. A. Datta, D. Home, Phys. Lett. A **119**, 3 (1986).

46. A. Pompili, F. Selleri, Eur. Phys. J. C **14**, 469 (2000).

47. R.A. Bertlmann, W. Grimus, Phys. Rev. D **64**, 056004 (2001).

48. R. A. Bertlmann, K. Durstberger, B. C. Hiesmayr, Phys. Rev. A **68**, 012111 (2003).

49. P. Caban, J. Rembieliński, K.A. Smoliński, Z. Walczak, M. Włodarczyk, Phys. Lett. A **357**, 6 (2006).

50. KLOE Collaboration, presented by A. Di Domenico, *Kaon interferometry at KLOE: present and future*, hep-ex/0312032.

51. F. Ambrosino [KLOE Collaboration], Phys. Lett. B **642** (2006) 315;

52. L. R. Kasday, J. D. Ullmann, C. S. Wu, Bull. Am. Phys. Soc. **15**, 586 (1970).

53. L. R. Kasday, J. D. Ullmann, C. S. Wu, Nuovo Cimento B **25**, 633 (1975).

54. G. Faraci, D. Gutkowski, S. Notarrigo, A. R. Pennisi, Nuovo Cimento Lett. B **9**, 607 (1974).

55. A. R. Wilson, J. Lowe, D.K. Butt, J. Phys. G **2**, 613 (1976).

56. M. Bruno, M. D'Agostino, C. Maroni, Nuovo Cimento B **40**, 143 (1977).

57. G. Bertolini, E. Diana, A. Scotti, Nuovo Cimento B **63**, 651 (1981).

58. M. Lamehi-Rachti, W. Mittig, Phys. Rev. D **14**, 2543 (1976).

59. N. A. Törnqvist, Phys. Lett. A **117**, 1 (1986).

60. N. A. Törnqvist, in: *Quantum Mechanics versus Local Realism: The Einstein–Podolsky–Rosen Paradox*, F. Selleri (ed.), (Plenum New York 1988), p.115.

61. P. Privitera, Phys. Lett. B **275**, 172 (1992).

62. S. A. Abel, M. Dittmar, H. Dreiner, Phys. Lett. B **280**, 304 (1992).

63. R. A. Bertlmann, B. C. Hiesmayr, Phys. Rev. A **63**, 062112 (2001).

64. N. Gisin and A. Go, Am. J. Phys. **69**, 264 (2001).

65. A. Apstolakis et.al., Phys. Lett. B **422**, 339 (1998).

66. K. Nagata, W. Laskowski, M. Wieśniak, M. Żukowski, Phys. Rev. Lett. **93**, 230403 (2004).

67. C.S. Unnikrishnan, Europhys. Lett. **69**, 489 (2005).

68. F. Uchiyama, Phys. Lett. A **231**, 295 (1997).

69. R.A. Bertlmann, W. Grimus, B.C. Hiesmayr, Phys. Lett. A **289**, 21 (2001).

70. D.E. Groom et al, *Review of Particle Physics*, Eur. Phys. J. C **3**, 1 (1998).

71. B.C. Hiesmayr, PhD-Thesis, University of Vienna, 2002.

72. B.C. Hiesmayr, Eur. Phys. J. **C50** (2007) 73.

73. J.S. Bell, J. Steinberger, *Weak Interactions of Kaons*, Proceedings of the Oxford International Conference, 10-25 Sept., p. 147, (1965).

74. G.C. Ghirardi, R. Grassi, T. Weber, in Proc. of Workshop on *Physics and Detectors for DAΦNE, the Frascati Φ Factory*, April 9-12th, 1991, edited by G. Pancheri.

75. J.E. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Phys. Rev. D **10**, 880 (1969).

76. A. Acin, R. Gill, N. Gisin, Phys. Rev. Lett. **95**, 210402 (2005).

77. A. Acin, T. Durt, N. Gisin, J.I. Latorrre, Phys. Rev. A **65**, 052325 (2002).

78. A. Di Domenico, A. Go, B.C. Hiesmayr, *Testable Bell inequalities at DAΦNE*, in preparation.

A REVIEW ON BELL INEQUALITY TESTS WITH NEUTRAL KAONS

A. Bramon, R. Escribano
Grup de Física Teòrica, Universitat Autònoma de Barcelona, E–08193 Bellaterra, Spain
G. Garbarino
Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, I-10125 Torino, Italy

Dedicated to the memory of R.H.Dalitz
from whom we learnt the 'strangeness' of kaon physics.

Abstract

Recent proposals aiming to confront Local Realistic theories with Quantum Mechanics by performing Bell tests with entangled neutral kaons, such as those produced by ϕ decays at Daphne, are reviewed. Some difficulties appear because of the reduced number of useful, non–commuting kaonic observables and the low efficiency of the strangeness measurements. The possibilities to overcome this and other loopholes are analyzed.

1 Introduction

A classical book by R. H. Dalitz [1] offers an accurate description of the development of the 'strange' particle physics since its origin in the 1950s. The 'strangeness' of their behavior was associated with the fact that these particles were copiously produced in ordinary, non–strange particle reactions always in pairs. Present day examples of such 'associated productions' are the electron–positron and the s–wave proton–antiproton annihilations into the state

$$\frac{1}{\sqrt{2}}\left[|K^0\rangle_l|\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l|K^0\rangle_r\right] \quad (1)$$

consisting of two strange, neutral kaons which, after collimation, form a left– and a right–moving beam as indicated by the subindexes. Independently, another classical book by D. Bohm, 'Quantum Theory', appeared in 1951 [2]. The nowadays famous gedanken experiment by Einstein, Podolsky and Rosen [3] was discussed there in its simplest form, *i. e.*, in terms of the singlet state formed by two spin–1/2 objects which is quite similar to the two–kaon state (1). In the Bohm singlet state, each spin–1/2 points both into any given spatial direction and its opposite one; similarly, each particle in (1) is both a kaon and an antikaon at the very same time. According to quantum mechanics, each separate spin–1/2 particle or kaon in the two–particle states just considered cannot be represented by a wave function or state vector; only the global system, such as that in Eq. (1), has a definite state vector and is thus the single, indivisible quantum. In both considered cases, apparently one has to deal with a rather simple two–particle (bipartite) quantum state, but the *entanglement* or quantum correlations between its two partners adds to the 'strangeness' of kaon physics the weirdness of quantum mechanics.

Indeed, one of the most counterintuitive and subtle aspects of quantum mechanics refers to the correlations shown by the distant parts of composite systems like the above mentioned two. This became evident in 1935, when Einstein, Podolsky and Rosen (EPR) [3], discussing a gedanken experiment with entangled states, arrived at the conclusion that the description of physical reality given by the quantum wave function cannot be complete. Bohr, in his famous response [4], noted that EPR's criterion of physical reality contained an ambiguity if applied to quantum phenomena and gave rise to one of the most important and long standing debates in physics. According to Bohr, EPR's assumption that a quantum system has real and well defined properties also when does not interact with other systems (including measuring apparata) is contradicted by the basic axioms of quantum mechanics.

For about 30 years the debate triggered by EPR and Bohr remained basically a matter of philosophical belief. Then, in 1964, Bell [5] interpreted EPR's argument as the need for the introduction of additional, unobservable variables aiming to restore *completeness*, *relativistic causality* (or *locality*) and *realism* in quantum theory. He established a theorem which proved that any *local hidden–variable* (*i. e.*, *local realistic* [6]) theory is incompatible with some statistical predictions of quantum mechanics. Since then, various forms of Bell inequalities [7]- [11] have been the tool for an experimental discrimination between local realism (LR) and quantum mechanics (QM).

Such a discrimination is possible only if the predictions coming from QM cannot be reproduced with LR models. These models allow the derivation of Bell inequalities which necessarily relate the statistical results one has to expect from a given entangled system when its two members are potentially subjected to alternative joint measurements chosen by the experimenters. If such a choice among experiments exists, we refer to them as *active* measurements. Each one of these experiments projects then each measured kaon into one of the two states of the chosen measurement basis. This is a common feature in Refs. [7)- 11)] but care has to be taken when extrapolating these considerations to unstable systems such as neutral kaons [12)]. Admittedly, this instability allows for different decay modes, which effectively correspond to different quantum measurements. But the inequalities involving these *passive* measurements, with no choice on the experimenter part, are not *Bell* inequalities since they cannot discriminate LR from QM, as we will discuss later on.

Many experiments confronting QM versus LR have been performed, mainly with entangled optical photons [10, 13, 14, 15, 16)] and, more recently, with entangled ions [17)]. All these tests obtained results in good agreement with QM but, according to several authors, they do not represent a conclusive proof against LR. The tests are affected by another type of criticisms, which are certainly less severe than that mentioned in the preceding paragraph but have been discussed for many years [18)]. These tests only seem to show the violation of the so called *non–genuine* Bell inequalities. Indeed, because of non–idealities of the apparata and other technical problems, *supplementary assumptions* not implicit in LR were needed in the interpretation of the experiments. Consequently, no one of these tests has been strictly *loophole free* [10, 18, 19)], *i. e.*, able to test a *genuine* Bell inequality, which has to be a necessary consequence of LR alone.

One of these criticisms, frequently referred to as the *detection* or *efficiency loophole*, is particularly relevant for kaons. It has been proven [9, 10, 20)] that for any bipartite and entangled state one can derive Bell inequalities without the introduction of (plausible but not testable) supplementary assumptions concerning undetected events. In particular, the most appropriate inequality for confronting LR vs QM has been derived long ago by Clauser and Horne [9)]. For maximally entangled (non–maximally entangled) states, if one assumes that all detectors have the same overall detection efficiency η, these genuine Clauser–Horne inequalities are violated by QM only if $\eta > 0.83$ [21)] ($\eta > 0.67$ [22)]). Since such detection thresholds cannot be presently achieved in photon experiments, only non–genuine inequalities have been tested experimentally.

Several of these photonic tests violated non–genuine inequalities by the amount predicted by QM but they could not overcome the detection loophole. Indeed, local realistic models exploiting the detector inefficiencies and reproducing the experimental results can be contrived [9, 23] for these tests. Only the recent test with entangled beryllium ions of Ref. [17], for which $\eta \simeq 0.97$, did close the detection loophole. On the other hand, an experiment with entangled photons [14] closed the other main existing loophole, the *locality loophole.* In this test, the measurements on the two photons were carried out under strict space–like separation conditions, thus avoiding any possible exchange of subluminal signals between the two measurement event regions. But this is not the case for the high efficiency experiment [17] with two beryllium ions separated only by a few microns. In other words, no experiment closing simultaneously both the detection *and* locality loopholes has been performed till now.

Extensions to other kind of entangled systems are thus important. Over the past ten years or so there has been an increased interest on the possibility to test LR vs QM in particle physics,*e. g.*, by using entangled neutral kaons [24)–41)]. This is also a manifestation of the desire to go beyond the usually considered spin–singlet case and to have new entangled systems made of massive particles with peculiar quantum–mechanical properties (apart from the classical book by Dalitz [1], other detailed reviews of neutral kaons are [29, 42, 43]). Entangled $K^0\bar{K}^0$ states (1) are copiously produced in the decay of the $\phi(1020)$ resonance [44] and in proton–antiproton annihilation processes at rest [45, 46]. For kaons, the strong nature of hadronic interactions should contribute to close the detection loophole, since it enhances the efficiencies to detect the products of kaon decays and kaon interactions with ordinary matter (pions, kaons, nucleons, hyperons,...). Moreover, the two kaons produced in ϕ decays or $p\bar{p}$ annihilations at rest fly apart from each other at relativistic velocities and can fulfill the condition of space–like separation. Therefore, contrary to the experiment with ion pairs of Ref. [17], the locality loophole could be closed with kaon pairs by using equipments able to prepare, very rapidly, the alternative kaon measurement settings.

In this contribution, our purpose is to review the Bell inequalities proposed to test LR vs QM using $K^0\bar{K}^0$ entangled pairs. The proposals are discussed in the light of the basic requirements —specified in Section 2— necessary to establish genuine Bell inequalities. Each measurement is associated to a specific basis and the bases relevant for our discussion are studied in Section 3. The alternative measurements one can perform on each neutral kaon at a given time are rather reduced, as we show in Section 4. The preparation of the two–kaon entangled state is discussed in Section 5 and can be performed in many different ways; a given, fixed state, however, has to be used for all the alternative measurements contemplated in a given Bell inequality. The various forms of inequalities are derived and related in Section 6. In Section 7 the different proposals with neutral kaons are discussed.

2 Requirements for a genuine Bell inequality

The requirements for deriving a Bell inequality from LR can be summarized as follows:

(1) A non–factorisable or entangled state must be used. Here, as in most cases, a two–particle (bipartite) state is considered. The simplest example is the state (1);

(2) Alternative and mutually exclusive measurements, corresponding to non–commuting observables, must be chosen at will and performed on both members of that state;

(3) Each one of the different single measurements has to have dichotomic outcomes. However, if the possibility of undetected events is considered, they can count as a third outcome;

(4) The measurement process on each member of the two–particle state must be space–like separated from the measurement on the other member.

At a ϕ–factory, or in proton–antiproton annihilations at rest, the first requirement poses no serious problems. Indeed, entanglement has been confirmed experimentally, over macroscopic distances, for $K^0\bar{K}^0$ pairs at CPLEAR [45] using active strangeness measurements and can be demonstrated at the DaΦne ϕ–factory as well [47]. However, care has to be taken to define the state at a specific (proper) time τ, or specific times τ_l and τ_r if these are different for the left– and right–moving members of the entangled state. Indeed, contrary to what happens in photonic experiments, neutral kaons decay and oscillate in time. Only when these times are fixed we have a well defined state to perform Bell–tests.

Difficulties appear with requirement number (2). Indeed, among the differences between the spin–singlet state of entangled photons and the $K^0\bar{K}^0$ entangled state (1), the most important one is that while for photons one can measure the linear polarization along *any* space direction chosen at will, measurements on neutral kaons essentially reduce to only *two* kinds: one can chose to detect either the strangeness or the lifetime of each kaon. These are then two useful and direct measurement choices which can be somehow enlarged by kaon regeneration effects before the final detection (see Section 3.3). But the problem essentially remains and complicates considerably the possibilities of Bell–tests with neutral kaons.

Another property of neutral kaons, not shared by photons, is that the former are unstable and decay via different modes. Each one of these modes is associated with a specific kaon basis and the observation of a kaon decay into a given mode represents a *passive* measurement [12)]. Indeed, the experimenter has no control on when the kaon decays nor into which of the various channels it decays. In general, the information thus obtained does not refer to the specific state under consideration (because of kaon time evolution), nor to a desired basis *actively* chosen by the experimenter. As a result, the inequalities that some authors have proposed, which make use uniquely of decay–mode observations, cannot discriminate between LR and QM and, in this sense, are not Bell inequalities. The reason is quite obvious: since the experimenter is not allowed to exert his/her free will, a LR model can immediately be constructed which always gives the same predictions as QM *and* violates the proposed inequality. But this is an absurdity since, by definition, a Bell inequality has to contradict some QM prediction. Since there are no active changes of measurements, the LR model is constructed by just adopting the set of decay distributions predicted by QM as the complete set of hidden–variables. This point was first discussed in a related context by Kasday time ago [48)] but has been ignored by many authors when deriving Bell inequalities in the domain of particle physics. In the case of entangled $B^0\bar{B}^0$ pairs, for which only decay mode measurements can be performed, the situation is then more unfortunate than with kaon pairs. Neutral kaons are then unique among pseudo–scalar mesons: the lack of active measurement procedures for B–mesons makes impossible the derivation of relevant Bell inequalities [12, 49]. In this review, centered in discriminating QM from LR, we do not discuss these Bell–tests based on passive measurements, although most of them are of clear interest showing, among other things, the entanglement between pairs of separated particles.

The requirement (4) on locality deserve also some comments. Kaons move at relativistic velocities and can travel macroscopic distances away from the production point before decaying. These distances are certainly much shorter than those involved in photonic experiments (a recent one has shown two–photon entanglement over 144 km [50]) but much larger than those for ions (separated only some microns in Ref. [17]). During these survival distances each kaon has to be submitted to either one or another measurement and this implies changing the experimental setup, typically, by placing or removing material pieces (kaon regenerators). Actively changing from one setup to another in such a way that the two (left and right) distant measurement events are space–like separated could imply serious technical difficulties. For this reason, some authors [51] prefer to consider static measurements setups (fixed pieces acting as different absorbers) which, even if they will not be able to close the locality loophole, look more feasible and still of interest.

Finally, in order to establish the feasibility of the real test, one has to derive the detection efficiencies necessary for a meaningful quantum mechanical violation of the considered Bell inequality. With all this in mind and in the light of the basic requirements (1)–(4), in Section 7 we proceed to analyse various proposals of Bell–tests with entangled kaon–antikaon pairs. Before, we present a general discussion on measurement bases, quantum states and genuine and non–genuine Bell inequalities for neutral kaons.

3 Bases in 'quasi–spin' space

Thanks to the analogy with spin–1/2 particles, neutral kaon states can be conveniently described with the formalism of 'quasi–spin'. The strangeness eigenstates K^0 and $\bar{K}^0$ (specified in subsection 3.1) are considered as members of a quasi–spin doublet, with $|K^0\rangle \equiv \binom{1}{0}$ having 'spin up' and $|\bar{K}^0\rangle \equiv \binom{0}{1}$ having 'spin down'. A particular superposition, with unitary norm, of these strangeness eigenstates together with the corresponding orthogonal state:

$$\begin{aligned} |K_\alpha\rangle &= \alpha|K^0\rangle + \bar{\alpha}|\bar{K}^0\rangle, \\ |K_\alpha^\perp\rangle &= -\bar{\alpha}^*|K^0\rangle + \alpha^*|\bar{K}^0\rangle, \end{aligned} \tag{2}$$

with $\langle K_\alpha|K_\alpha\rangle = \langle K_\alpha^\perp|K_\alpha^\perp\rangle = |\alpha|^2 + |\bar{\alpha}|^2 = 1$ and $\langle K_\alpha|K_\alpha^\perp\rangle = 0$, define the generic basis $\{K_\alpha, K_\alpha^\perp\}$ along the quasi–spin axis α. Any operator acting on the quasi–spin space can be expressed in terms of the Pauli matrices, σ_x, σ_y and σ_z. The formalism is appropriate for all two–level quantum systems or 'qubits' in the novel language of quantum information.

3.1 Strangeness basis: $\{K^0, \bar{K}^0\}$

Neutral kaons are spinless and s–wave quark–antiquark bound meson states, $K^0 \sim d\bar{s}$ and $\bar{K}^0 \sim s\bar{d}$. They define the 'strangeness' or 'strong–interaction' basis which consists of the two eigenstates $|K^0\rangle$ and $|\bar{K}^0\rangle$ with strangeness $S = +1$ and $S = -1$, respectively. This is the suitable basis to analyze S–conserving electromagnetic and strong interaction processes, such as the creation of $K^0\bar{K}^0$ systems from non–strange initial states (*e. g.*., $e^+e^- \to \phi(1020) \to K^0\bar{K}^0$, $p\bar{p} \to K^0\bar{K}^0$), and the detection of neutral kaons via strong kaon–nucleon interactions. This 'strangeness' basis is orthonormal, $\langle K^0|\bar{K}^0\rangle = 0$. In the quasi–spin picture, the strangeness operator evidently corresponds to σ_z:

$$\sigma_z|K^0\rangle = +|K^0\rangle, \quad \sigma_z|\bar{K}^0\rangle = -|\bar{K}^0\rangle.$$

Weak interaction phenomena —such as K^0–$\bar{K}^0$ mixing, K^0–$\bar{K}^0$ oscillations and neutral kaon evolution in time—, as well as kaon propagation in a medium —with the associated regeneration effects— introduce other relevant bases.

3.2 Free–space basis: $\{K_S, K_L\}$

The so called K–short and K–long states, $|K_S\rangle$ and $|K_L\rangle$, are the normalized eigenvectors of the effective weak Hamiltonian H_{free} governing neutral kaon time evolution in free–space:

$$i\frac{d}{d\tau}|K_{S,L}(\tau)\rangle = H_{\text{free}}|K_{S,L}(\tau)\rangle, \quad H_{\text{free}} = \begin{pmatrix} \lambda_+ & \lambda_-/r \\ r\lambda_- & \lambda_+ \end{pmatrix}, \tag{3}$$

where $r \equiv (1-\epsilon)/(1+\epsilon)$, ϵ is the CP–violation parameter [42, 52] and τ is the kaon proper time.

The (complex) eigenvalues of the previous (non–hermitian) Hamiltonian are

$$\begin{aligned} \lambda_S &= \lambda_+ + \lambda_- = m_S - \frac{i}{2}\Gamma_S, \\ \lambda_L &= \lambda_+ - \lambda_- = m_L - \frac{i}{2}\Gamma_L, \end{aligned} \tag{4}$$

where $m_{S,L}$ are the $K_{S,L}$ masses and $\Gamma_{S,L} \equiv 1/\tau_{S,L}$ their decay widths, with lifetimes $\tau_S = (0.8953 \pm 0.0005) \times 10^{-10}$ s and $\tau_L = (5.18 \pm 0.04) \times 10^{-8}$ s [52]. We also introduce $\Delta m \equiv m_L - m_S \simeq 0.475\,\Gamma_S$ and $\Delta\Gamma \equiv \Gamma_L - \Gamma_S \simeq -\Gamma_S$, to be used later on.

The corresponding K–short and K–long eigenstates are:

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle\right], \\ |K_L\rangle &= \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right], \end{aligned} \tag{5}$$

or, ignoring an irrelevant global phase:

$$|K_{S,L}\rangle = \frac{1}{\sqrt{1+|r|^2}}\left[|K^0\rangle \pm r|\bar{K}^0\rangle\right]. \tag{6}$$

The proper time propagation of the short– and long–lived states, having well–defined masses and decay widths, shows no oscillation between these two states and, according to Eqs. (3), is simply given by

$$|K_{S,L}(\tau)\rangle = e^{-im_{S,L}\tau}e^{-\frac{1}{2}\Gamma_{S,L}\tau}|K_{S,L}(\tau=0)\rangle \equiv e^{-i\lambda_{S,L}\tau}|K_{S,L}\rangle. \tag{7}$$

The $\tau = 0$ states $|K_{S,L}\rangle$ define a quasi–orthonormal basis with $\langle K_S|K_S\rangle = \langle K_L|K_L\rangle = 1$ and

$$\langle K_S|K_L\rangle = \langle K_L|K_S\rangle = \frac{1-|r|^2}{1+|r|^2} = \frac{\epsilon+\epsilon^*}{1+|\epsilon|^2} \simeq 0,$$

due to the smallness of the CP–violation parameter ϵ with modulus $|\epsilon| \simeq (2.284 \pm 0.014) \times 10^{-3}$ and phase $\phi \simeq 43.5^\circ$ [52].

In the quasi–spin space, the weak interaction eigenstates are indeed very 'similar' to the CP eigenstates $|K_1\rangle$ ($CP = +1$) and $|K_2\rangle$ ($CP = -1$):

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left[|K_1\rangle + \epsilon|K_2\rangle\right], \qquad (8)$$
$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left[|K_2\rangle + \epsilon|K_1\rangle\right].$$

But, while the $K_{S,L}$ basis is useful to discuss free–space propagation, the CP–basis describes weak kaon decays either into two or three final pions from the K_1 and K_2 states, respectively. These two $CP = \pm$ states are the eigenstates of σ_x, $\sigma_x|K_1\rangle = +|K_1\rangle$ and $\sigma_x|K_2\rangle = -|K_2\rangle$. Thus, the limit of CP–conservation corresponds to the invariance under quasi–spin rotations around the x axis. In this limit one has

$$|K_S\rangle \to |K_1\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\bar{K}^0\rangle\right], \qquad (9)$$
$$|K_L\rangle \to |K_2\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\bar{K}^0\rangle\right],$$

and strict orthogonality between the K_S and K_L states is recovered.

3.3 Inside–matter basis: $\{K'_S, K'_L\}$

The dynamics of neutral kaons propagating inside a homogeneous medium of nucleonic matter, which we can consider as a 'regenerator' and/or an 'absorber', is governed by the Hamiltonian

$$H_{\text{medium}} = H_{\text{free}} - \frac{2\pi\nu}{m_K}\begin{pmatrix} f_0 & 0 \\ 0 & \bar{f}_0 \end{pmatrix}, \qquad (10)$$

showing an additional, strong interaction term where m_K is the mean $K_{S,L}$ mass, f_0 and $\bar{f}_0$ are the forward scattering amplitudes for K^0 and $\bar{K}^0$ on nucleons and ν is the nucleonic density of the homogeneous medium.

The eigenvalues of H_{medium} are

$$\lambda'_S = \lambda_+ - \frac{\pi\nu}{m_K}(f_0 + \bar{f}_0) + \lambda_-\sqrt{1+4\rho^2}, \qquad (11)$$
$$\lambda'_L = \lambda_+ - \frac{\pi\nu}{m_K}(f_0 + \bar{f}_0) - \lambda_-\sqrt{1+4\rho^2},$$

and the corresponding eigenstates

$$|K'_S\rangle = \frac{1}{\sqrt{1+|r\bar{\rho}|^2}}\left[|K^0\rangle + r\bar{\rho}|\bar{K}^0\rangle\right], \quad (12)$$
$$|K'_L\rangle = \frac{1}{\sqrt{1+|r(\bar{\rho})^{-1}|^2}}\left[|K^0\rangle - r(\bar{\rho})^{-1}|\bar{K}^0\rangle\right],$$

where we have introduced the dimensionless regenerator parameter ρ, as well as the auxiliary parameter $\bar{\rho}$ and its inverse $(\bar{\rho})^{-1}$,

$$\rho \equiv \frac{\pi\nu}{m_K}\frac{f_0-\bar{f}_0}{\lambda_S-\lambda_L}, \quad (13)$$
$$\bar{\rho} \equiv \sqrt{1+4\rho^2}+2\rho\ , \quad (\bar{\rho})^{-1} = \sqrt{1+4\rho^2}-2\rho. \quad (14)$$

The proper time propagation of these $K'_{S,L}$ states inside matter is given by

$$|K'_{S,L}(\tau)\rangle = e^{-i\lambda'_{S,L}\tau}|K'_{S,L}\rangle, \quad (15)$$

and shows no K'_S–K'_L oscillations but a decreasing intensity in time given by the imaginary part of $\lambda'_{L,S}$. The latter comes from weak decays, essentially as in free–space propagation, plus absorption via strong kaon interactions with the medium driven by the imaginary part of $f_0+\bar{f}_0$. In this sense, the medium acts as an 'absorber'. The difference between f_0 and $\bar{f}_0$ appearing in ρ is responsible for 'rotations' in the quasi–spin space and transitions between K_S and K_L states. For surviving kaons, the medium acts as a 'regenerator', giving rise to the well known ρ–dependent regeneration effects.

Again, the $\tau=0$ states form a quasi–orthonormal basis,

$$\langle K'_S|K'_L\rangle = \langle K'_L|K'_S\rangle^* = \frac{1-|r|^2(\bar{\rho}^*/\bar{\rho})}{\sqrt{1+|r\bar{\rho}|^2}\sqrt{1+|r/\bar{\rho}|^2}}, \quad (16)$$

due to the smallness of ϵ ($r \simeq 1$), as before, and to the low efficiency of usual regenerators ($\rho \simeq \mathcal{R}e\,\rho \sim 10^{-2}$ and $\bar{\rho} \simeq \bar{\rho}^*$ [26, 42]), in spite of the strong character of the induced kaon–nucleon interactions.

Two limiting cases illustrate the relationships among the three bases we have considered: i) for a very low density medium: $\nu, \rho \to 0$ and $\bar{\rho} \to 1$ imply $|K'_{S,L}\rangle \to |K_{S,L}\rangle$, thus recovering the states in Eq. (6) and ii) for extremely high density media (absorbers): $\nu, |\rho|, |\bar{\rho}| \to \infty$ implies $|K'_S\rangle \to |\bar{K}^0\rangle$ and $|K'_L\rangle \to |K^0\rangle$.

4 Dichotomic measurements on neutral kaon states

4.1 Strangeness measurements

When a kaon–nucleon reaction occurs at a given place of a medium, the distinct strong interactions of the $S = +1$ and $S = -1$ neutral kaons on the bound nucleons inside the medium project the arbitrary state of an incoming kaon into one of the two orthogonal members of the strangeness basis $\{K^0, \bar{K}^0\}$ [1, 29, 42]. The quantum number S of the incoming kaon state is determined by identifying the products (usually pions, kaons, nucleons and hyperons) of the strangeness conserving kaon–nucleon strong interaction. Simple examples of $\bar{K}^0$ identifying reactions at low energies are $\bar{K}^0 p \to \Lambda\pi^+$, $\bar{K}^0 p \to \Sigma\pi$ and $\bar{K}^0 n \to \Lambda\pi^0$, while the lowest threshold reaction $K^0 p \to K^+ n$ identifies incoming states as K^0's. This strangeness measurement is then analogous to the projective von Neumann measurements with two–channel analyzers for polarized photons or Stern–Gerlach setups for spin–1/2 particles.

Unfortunately, the efficiency for such strangeness measurements at moderate kaon energies as in $\phi \to K^0\bar{K}^0$ and $p\bar{p} \to K^0\bar{K}^0$ is certainly less than what people have been naively expecting from the strong nature of these interactions [45]. The reason, rather than being the difficulty in detecting the final state particles (for which one can have rather high efficiencies), stems from the low probability in initiating the strong reaction. Indeed, the efficiency to *induce* either a $\bar{K}^0$–nucleon or a K^0–nucleon interaction at a given time τ turns out to be close to 1 only for infinitely dense absorber materials or for ultrarelativistic kaons, where, by Lorentz contraction, the absorber is seen by the incoming kaon as extremely thin and dense ($\nu \to \infty$). In this case, kaon–nucleon strong interactions occur and the incident kaons are immediately projected into one member of the inside–matter or the strangeness basis which are coincident in the present limit; *i. e.*, the incident kaon is projected into either $|K'_S\rangle \to |\bar{K}^0\rangle$ or $|K'_L\rangle \to |K^0\rangle$. But in ordinary cases, when a thin absorber is placed to measure strangeness, the incident kaon likely fails to interact with a nucleon and, although the kaon can be efficiently detected beyond the absorber, the desired strangeness measurement has not been performed. Ordinary matter is too transparent for kaons. This contrasts with the polarization measurements where the low efficiency corresponds to the final detection of photons having passed through the efficient analyzers. It would be highly desirable to identify very efficient kaon absorbers. Since this does not seem to be viable at present, one has to play with small strangeness detection efficiencies, which originate both conceptual and practical difficulties when discussing Bell–type tests for entangled kaons [37, 38, 39, 41].

4.2 Lifetime measurements

To measure if a kaon is propagating in free–space as a K_S or K_L at a given time τ, one has to allow for further propagation in free–space and then detect at which time it subsequently decays. Kaons which show a decay vertex between times τ and $\tau+\Delta\tau$ have to be identified as K_S's, while those decaying later than $\tau + \Delta\tau$ have to be identified as K_L's. Since there are no K_S–K_L oscillations, such subsequent decays do really identify the state at the desired previous time τ. The probabilities for wrong K_S and K_L identification are then given by $\exp(-\Gamma_S\,\Delta\tau)$ and $1 - \exp(-\Gamma_L\,\Delta\tau)$, respectively. By choosing $\Delta\tau = 4.8\,\tau_S$, both K_S and K_L misidentification probabilities reduce to $\simeq 0.8\%$, which can be further reduced if the decay mode is also identified (see appendix of Ref. [41]).

Recall that the K_S and K_L states are not strictly orthogonal to each other, $\langle K_S|K_L\rangle = 2\,\mathcal{R}e\,\epsilon/(1+|\epsilon|^2) \neq 0$; thus their identification cannot be exact even in principle. However, ϵ is so small [$|\epsilon| \simeq (2.284 \pm 0.014) \times 10^{-3}$ [52]] and the decay probabilities of the two components so different ($\Gamma_S \simeq 579\,\Gamma_L$) that the K_S vs K_L identification can effectively work [37]. Note also that, contrary to strangeness measurements, K_S vs K_L identifications are not affected by the previous low inefficiencies: by using detectors with very large solid angles, one can play with rather high efficiencies for the detection and proper identification of the kaon decay products.

4.3 Active vs passive measurements

The methods described in the last two subsections to discriminate K^0 vs $\bar{K}^0$ and K_S vs K_L are fully appropriate to establish Bell inequalities and tests. On the one hand, the two measurements correspond to complementary and non–commuting observables with dichotomic outcomes in both cases (essentially, σ_z and σ_x in quasi–spin space). On the other hand, they are clearly *active* measurement procedures since they are performed by exerting the free will of the experimenter, another crucial ingredient to establish genuine Bell inequalities. Indeed, at the chosen measurement time τ, either one places a dense slab of matter or allows for free–space propagation. Needless to say, one measurement excludes the other. In the former case, strangeness is measured and no information is obtained on the lifetime of the observed kaon. Conversely, if free propagation is allowed, one identifies K_S vs K_L but nothing is learned on the strangeness quantum number. As previously discussed, the active strangeness measurement is monitored by strangeness conservation, while the active lifetime measurement is possible thanks to the smallness of Γ_L/Γ_S.

Contrary to what happens with other two–level quantum systems such as spin–1/2 particles or photons, *passive* measurements of lifetime and strangeness for neutral kaons are also possible [12], by randomly exploiting the quantum–mechanical dynamics of kaon decays. To this aim, one has to allow for complete free–space propagation and observe the various kaon decay modes. By neglecting the (small) CP–violation effects ($\epsilon \to 0$), non–leptonic kaon decays into two and three pions permit the identification of K_S's and K_L's, respectively. Alternatively, the strangeness of a given neutral kaon state is measured by observing their semileptonic decays. These decays obey the well tested $\Delta Q = \Delta S$ rule, which allows the modes $K^0 \to \pi^- l^+ \nu_l$ and $\bar{K}^0 \to \pi^+ l^- \bar{\nu}_l$, with $l = e, \mu$, but forbids decays into the respective charge–conjugate final states. These procedures for the passive K_S vs K_L and K^0 vs $\bar{K}^0$ discriminations are unambiguous in the approximations given by CP–conservation and the $\Delta Q = \Delta S$ rule, respectively.

However, in passive measurement procedures the experimenter has no control on the time when the lifetime or the strangeness measurement occurs, nor on the basis in which the measurement is performed, in contrast with the previous active, von Neumann projection measurements requiring the intervention of the experimenter, who is free to chose between the two complementary measurements. As discussed in sect.2, for experiments performed with passive measurements only, Kasday construction [48, 49] is therefore possible, thus preventing the derivation of Bell inequalities.

5 Entangled states of neutral kaon pairs

5.1 Maximally entangled states

The simplest and most often discussed two–party or bipartite states are the spin–singlet states consisting of two spin–1/2 particles, as first proposed by

D. Bohm [2]. Let us then first consider the two–kaon entangled state which is the analogous [32, 34, 40, 53] to this standard Bohm state. From both ϕ–resonance decays [44] or s–wave proton–antiproton annihilation [45], one starts at time $\tau = 0$ with an initial state $|\phi(0)\rangle$ with global spin, charge conjugation and parity $J^{PC} = 1^{--}$:

$$\begin{aligned} |\phi(0)\rangle &= \frac{1}{\sqrt{2}} \left[|K^0\rangle_l |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l |K^0\rangle_r \right] \\ &= \frac{1}{\sqrt{2}} \frac{1 + |\epsilon|^2}{|1 - \epsilon^2|} \left[|K_L\rangle_l |K_S\rangle_r - |K_S\rangle_l |K_L\rangle_r \right], \end{aligned} \tag{17}$$

where l and r denote the 'left' and 'right' directions of motion of the two separating kaons. The weak, CP–violating effects enter only in the last equality. Note that this state is antisymmetric and maximally entangled in the two observable bases. The corresponding measurements will always lead to left–right anticorrelated results.

After production, the left and right moving kaons evolve according to Eq. (7) up to (proper) times τ_l and τ_r, respectively. Formally, this leads to the 'two–times' state

$$\begin{aligned} |\phi(\tau_l, \tau_r)\rangle &= \frac{e^{-(\Gamma_L \tau_l + \Gamma_S \tau_r)/2}}{\sqrt{2}} \\ &\times \left\{ |K_L\rangle_l |K_S\rangle_r - e^{i\Delta m(\tau_l - \tau_r)} e^{\Delta\Gamma(\tau_l - \tau_r)/2} |K_S\rangle_l |K_L\rangle_r \right\} \end{aligned} \tag{18}$$

in the lifetime basis, with $\epsilon \to 0$. Equivalently,

$$\begin{aligned} |\phi(\tau_l, \tau_r)\rangle &= \frac{1}{2\sqrt{2}} e^{-(\Gamma_L \tau_l + \Gamma_S \tau_r)/2} \\ &\times \left\{ \left(1 - e^{i\Delta m(\tau_l - \tau_r)} e^{\Delta\Gamma(\tau_l - \tau_r)/2}\right) \left[|K^0\rangle_l |K^0\rangle_r - |\bar{K}^0\rangle_l |\bar{K}^0\rangle_r \right] \right. \\ &+ \left. \left(1 + e^{i\Delta m(\tau_l - \tau_r)} e^{\Delta\Gamma(\tau_l - \tau_r)/2}\right) \left[|K^0\rangle_l |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l |K^0\rangle_r \right] \right\} \end{aligned} \tag{19}$$

in the strangeness basis.

Most usually, one considers two–kaon states at a unique time $\tau_l = \tau_r \equiv \tau$. One then has

$$\begin{aligned} |\phi(\tau, \tau)\rangle &= \frac{1}{\sqrt{2}} e^{-(\Gamma_L + \Gamma_S)\tau/2} \left[|K^0\rangle_l |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l |K^0\rangle_r \right] \\ &= \frac{1}{\sqrt{2}} e^{-(\Gamma_L + \Gamma_S)\tau/2} \left[|K_L\rangle_l |K_S\rangle_r - |K_S\rangle_l |K_L\rangle_r \right], \end{aligned} \tag{20}$$

showing the same maximal entanglement and anticorreations at any time τ.

5.2 Non–maximally entangled states

Apart from the previous maximally entangled state of kaons, other non–maximally entangled states are of interest for testing LR vs QM. To prepare these states we start with the initial state (17). A thin, homogeneous regenerator is fixed on the right beam (say), as close as possible to the point where the two–kaon state originates. If the regenerator is very close to this origin and the proper time $\Delta\tau$ required by the right moving neutral kaon to cross the regenerator is short enough, $\Delta\tau << \tau_S$, weak decays can be ignored and the state leaving the thin regenerator is

$$|\phi(\Delta t)\rangle = \frac{1}{\sqrt{2}}\left[|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle + \eta|K_S\rangle|K_S\rangle - \eta|K_L\rangle|K_L\rangle\right], \qquad (21)$$

where the indexes l and r referring to the kaon propagation directions are omitted from now on. The complex parameter η characterizes the regeneration effects and is defined by [42]:

$$\eta \equiv i\rho(\lambda_S - \lambda_L)\Delta\tau = i\frac{\pi\nu}{m_K}(f_0 - \bar{f}_0)\Delta\tau = i\frac{\pi\nu}{p_K}(f_0 - \bar{f}_0)d, \qquad (22)$$

where m_K is the average neutral kaon mass, p_K the kaon momentum and d the total length of the regenerator.

The states (20) at $\tau = 0$ and (21) only differ in the terms linear in the small parameter η. Indeed, for typical regenerators and at DaΦne energies one has $|\eta| = \mathcal{O}(10^{-3})$ when $d = 1$ mm [26, 42], thus allowing to neglect higher order terms in the state (21). To enhance that difference, we now allow the state (21) to propagate in free space up to a proper time T in the wide range $\tau_S << T << \tau_L \simeq 579\,\tau_S$. One thus obtains the state:

$$\begin{aligned}|\phi(T)\rangle \;=\; & \frac{e^{-(\Gamma_L\,\tau_l+\Gamma_S\,\tau_r)/2}}{\sqrt{2}}\left[|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle\right. \qquad (23)\\ & \left.-\eta e^{-i\Delta m T}e^{\frac{1}{2}(\Gamma_S-\Gamma_L)T}|K_L\rangle|K_L\rangle + \eta e^{i\Delta m T}e^{\frac{1}{2}(\Gamma_L-\Gamma_S)T}|K_S\rangle|K_S\rangle\right],\end{aligned}$$

where the K_LK_L component has survived against weak decays much better than the accompanying terms K_SK_L and K_LK_S and has thus been enhanced. On the contrary, the K_SK_S component has been strongly suppressed and can be neglected if $T/\tau_S >> 1$.

The normalization of state (23) to the surviving pairs leads then to:

$$|\Phi\rangle \;=\; \frac{1}{\sqrt{2+|R_L|^2+|R_S|^2}} \qquad (24)$$

$$[|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle + R_L|K_L\rangle|K_L\rangle + R_S|K_S\rangle|K_S\rangle] ,$$

where:

$$R_L \equiv -re^{\left[-i\Delta m+\frac{1}{2}(\Gamma_S-\Gamma_L)\right]T}, \quad R_S \equiv re^{\left[i\Delta m-\frac{1}{2}(\Gamma_S-\Gamma_L)\right]T}. \qquad (25)$$

Note that the quantity $|R_L| \simeq |r|e^{\frac{1}{2}\Gamma_S T}$ is not necessarily small with an exponential factor compensating the smallness of $|r|$, but we take $R_S \to 0$ from now on. The non–maximally entangled state Φ describes all kaon pairs with both left and right partners surviving up to the common proper time T. Because of the particular normalization of Φ, kaon pairs showing the decay of one (or both) member(s) before time T have to be detected and excluded. Since this occurs prior to any measurement eventually used for a Bell–type test, ours is a 'pre–selection' (as opposed to 'post–selection') procedure which poses no problem when confronting LR with QM.

Once the state is prepared as in Eq. (24), alternative joint measurements on each one of the corresponding kaon pairs have to be considered for a Bell–type test. In Section 7 we will see how one can utilize this state for such tests.

6 Bell inequalities with neutral kaon pairs

In the present Section, our first aim is to show how the Clauser–Horne inequality can be derived from LR and adapted for a generic entangled state of kaon pairs. The obtained CH inequality, equivalent to an inequality which Eberhard proved in a different way, is a *genuine* Bell inequality in the sense that it follows from LR with no need of extra assumptions. When supplementary assumptions, not implicit in LR, are introduced, other, *non–genuine* Bell inequalities can be derived which allow to design more feasible experimental tests of LR vs QM. The potentialities of a Bell inequality derived by Clauser–Horne–Shimony–Holt and of another simple one, due by Wigner, are thus illustrated.

6.1 Clauser–Horne inequalities

In the interpretation with *hidden–variables*, a generic two–kaon entangled state corresponds to a statistical ensemble of kaon pairs specified by different values of these additional, unobservable, deterministic or stochastic variables, which are here globally denoted by the symbol λ. In principle, λ can contain the same information of the quantum mechanical two–kaon state or wave–function but can be further completed, for instance to restore classical determinism in measurement processes. Moreover, in λ one could also include apparatus random hidden–variables, which can influence, locally, the outcomes of measurements. In a general hidden–variable (*i. e.*, realistic) theory, the joint probability to observe particular kaon quasi–spin states $|K_\alpha\rangle = \alpha|K^0\rangle + \bar{\alpha}|\bar{K}^0\rangle$ and $|K_\beta\rangle = \beta|K^0\rangle + \bar{\beta}|\bar{K}^0\rangle$ along the 'left' and 'right' beams, respectively —when measurements along the α and β quasi–spin axes are performed— is given by:

$$P(K_\alpha, K_\beta) = \int d\lambda\, \rho(\lambda)\, p(K_\alpha, K_\beta|\lambda), \tag{26}$$

where $\rho(\lambda)$ is the hidden–variable probability distribution (normalized to unity, $\int d\lambda\, \rho(\lambda) = 1$) and $p(K_\alpha, K_\beta|\lambda)$ the conditional probability that a joint measurement produces the outcome (K_α, K_β) when the kaon pair (and eventually the measuring devices) is in the *state* specified by λ. Note that, since each kaon pair is assumed to be emitted by the source in a way which is independent of the 'adjustable parameters' α and β characterizing the chosen measurement axes, the hidden–variable distribution function $\rho(\lambda)$ is independent of α and β.

By enforcing the *locality* condition, the previous conditional probability $p(K_\alpha, K_\beta|\lambda)$ can be written in the following factorized form:

$$p(K_\alpha, K_\beta|\lambda) = p(K_\alpha, *|\lambda)\, p(*, K_\beta|\lambda), \tag{27}$$

where, for instance:

$$p(K_\alpha, *|\lambda) \equiv p(K_\alpha, K_\gamma|\lambda) + p(K_\alpha, K_\gamma^\perp|\lambda) + p(K_\alpha, U_\gamma|\lambda), \tag{28}$$

taking values between 0 and 1, is independent of the choice of the states $|K_\gamma\rangle$ and $|K_\gamma^\perp\rangle$ forming an orthogonal basis in the quasi–spin space, $\langle K_\gamma|K_\gamma^\perp\rangle = 0$. Note that, in particular $p(*, *|\lambda) = 1$ for any λ. Note also that the last term of the previous equation takes into account eventual undetected events or events for which the proposed measurement failed, denoted by the U_γ argument, due to the various non–perfect efficiencies in measurements along the 'axis' γ in the quasi–spin space. It is important to remark that in the present scheme, the measurement fails or not depending on the values of the hidden–variables. In other words, the possibility of performing or not the desired measurement is correlated with the values of λ. It is also important to emphasize that for fixed

λ, the single–side probabilities $p(K_\alpha, *|\lambda)$ and $p(*, K_\beta|\lambda)$ entering equation (27) are independent of the measurement that one chooses to perform on the other member of the pair: the kaon quasi–spin outcome K_α (K_β) observed along the left (right) beam when measuring along the quasi–spin axis α (β) is independent of the quasi–spin axis β (α) employed to detect the right (left) going kaon.

To derive the Clauser–Horne (CH) inequality, the mathematical lemma of Ref. [9] can be used. It asserts that for any value between 0 and 1 of the real numbers x_1, x_2, x_3 and x_4, the inequality $x_1x_2 - x_1x_4 + x_2x_3 + x_3x_4 \leq x_3 + x_2$ holds. By assigning $x_1 = p(K_\alpha, *|\lambda)$, $x_2 = p(*, K_\beta|\lambda)$, $x_3 = p(K_{\alpha'}, *|\lambda)$ and $x_4 = p(*, K_{\beta'}|\lambda)$, using the factorisable (locality) condition (27) and integrating over the hidden–variable λ as in Eq. (26), one easily obtains the CH inequality:

$$\begin{aligned} -1 \leq S \quad &\equiv \quad P(K_\alpha, K_\beta) - P(K_\alpha, K_{\beta'}) + P(K_{\alpha'}, K_\beta) + P(K_{\alpha'}, K_{\beta'}) \\ &\quad -P(K_{\alpha'}, *) - P(*, K_\beta) \leq 0, \end{aligned} \tag{29}$$

with single–side probabilities given by:

$$\begin{aligned} P(K_{\alpha'}, *) &= P(K_{\alpha'}, K_\gamma) + P(K_{\alpha'}, K_\gamma^\perp) + P(K_{\alpha'}, U_\gamma), \\ P(*, K_\beta) &= P(K_\delta, K_\beta) + P(K_\delta^\perp, K_\beta) + P(U_\delta, K_\beta), \end{aligned} \tag{30}$$

for an arbitrary choice of the quasi–spin axes γ and δ. As noted by Clauser and Horne in Ref. [9], the right–hand side of the CH inequality, $S \leq 0$, has the advantage of being independent of the hidden–variable normalization condition, $\int d\lambda \rho(\lambda) = 1$, thus canceling the influence of the size of the ensemble of detected events.

6.2 Eberhard inequalities

If we chose $\gamma = \beta$ and $\delta = \alpha$ in Eqs. (29), (30), the right–hand side of the CH inequality (29) can be rewritten in the form of the following Bell–like inequality first derived by Eberhard [22]:

$$\begin{aligned} P(K_{\alpha'}, K_{\beta'}) \quad \leq \quad & P(K_{\alpha'}, K_\beta^\perp) + P(K_\alpha^\perp, K_\beta) \\ & +P(K_\alpha, K_{\beta'}) + P(K_{\alpha'}, U_\beta) + P(U_\alpha, K_\beta). \end{aligned} \tag{31}$$

The right–hand side CH and Eberhard inequalities just introduced are both genuine Bell inequalities in the sense discussed in Section 2. Unfortunately, due to their specific form, they hardly provide feasible experimental tests able to discriminate between LR and QM. As explained in Refs. [21, 22], the main shortcoming is originated by the existence of thresholds for the relevant measurement efficiencies which have to be overcome in order to attain violations of the considered inequalities by QM. Since in both performed (photon) and proposed (kaon) experiments such thresholds turn out to be hardly reachable within the current experimental capabilities, additional hypotheses beyond realism and locality must be made in order to obtain testable, but non–genuine, inequalities.

6.3 Clauser–Horne–Shimony–Holt inequalities

We then come to an important example of experimentally testable but non–genuine inequality which has been widely adopted in photon experiments, thus allowing the refutation of a restricted class of LR models. Here we will not repeat its derivation, which is due to Clauser, Horne, Shimony and Holt (CHSH) and can be found in Refs. [8, 10]. Our interest is to illustrate the differences with respect to the previously discussed CH inequality, especially concerning the role played by the supplementary assumptions when testing LR.

First, the CHSH inequality refers to expectation values instead of probabilities. In a local hidden–variable theory, the expectation value for a joint kaon measurement along the quasi–spin axes α and β is defined as:

$$E(\alpha,\beta)=\int d\lambda\rho(\lambda)A(\alpha|\lambda)B(\beta|\lambda), \tag{32}$$

where the locality requirement is evident in the factorized form of the left and right beam outcomes $A(\alpha|\lambda)$ and $B(\beta|\lambda)$, for a given state λ, which do not depend on the other–side measurement axis β and α, respectively.

The result of each single–side measurement can take one of the three possible outcomes, 0 or ± 1. For deterministic hidden–variables, we assume that $A(\alpha|\lambda)$ takes the following values: $+1$ when the measurement outcome is K_α, -1 when the outcome is the state $K_\alpha^\perp$ orthogonal to K_α and 0 when the particle is not projected into the $\{|K_\alpha\rangle, |K_\alpha^\perp\rangle\}$ measurement basis. This third outcome 0 presupposes the ability of the experimenter to know when a particular kaon fails to be measured in the chosen basis. In photon experiments the outcome 0 correspond to undetected photons. Since in this case their number is unknown, in 1971 Bell proposed (see Ref. [10]) to use what he called 'event–ready' detectors in order to enumerate the photon pairs emitted by the source which really reach the regions where they are then subject to Bell–measurements. Only if these undetected or 'unmeasured' pairs are included in the ensemble which defines the hidden–variable distribution $\rho(\lambda)$, one is sure that $\rho(\lambda)$ does not depend on the measurement parameters α and β [10, 20]. In practical cases with photons, any conceivable 'event–ready' detector fatally disturb if not destroy the particles. For kaons, good 'event–ready' detectors seems to be at our disposal: one has to detect kaon decays occurring along the beams prior to the measurement times used in the Bell–test and all the initial two–kaon pairs showing (at least one of) such decays have to be excluded from the sample. This amounts to the previously discussed renormalization of the states. Also, kaons which fail to initiate a kaon–nucleon interaction in a thin absorber when trying to measure their strangeness —this is the low efficiency measurement at our energies— can be further detected to decay as a K_S or K_L state and be properly included in the analysis as 'unmeasured', outcome 0 events.

In the lack of 'event–ready' detectors, as in photon experiments, to be sure that $\rho(\lambda)$ does not depend on α and β, one can follow another approach and introduce an *additional assumption* [8], not inferable in the hypothesis of realism and locality, which is plausible but untestable. For photons, this hypothesis amounts to require that 'if a particle passes through a spin analyser, its probability of detection is independent of the analyser's orientation' [8]. For kaons, with essentially only two possible 'orientations' (strangeness or lifetime), the situation is quite different: it is the low probability of inducing the initial kaon–nucleon interaction in strangeness measurements, rather than the detection of the final state products, what contrasts with highly efficient lifetime measurements. Rather than adapting the previous CHSH additional assumption to the kaon case, one can resort to the so–called 'fair sampling' hypothesis [10]. It amounts to assume that the set of effectively measured events represent a fair or undistorted sample of the whole set of states emitted from the source; in other words, the kaon hidden–variables are not correlated with the efficiencies of the measuring apparata. Under this assumption, the efficiency factors in the Bell inequalities are assumed to be 1 and thus no undetected or unmeasured event appears.

Coming back to Eq. (32), in the most general case of stochastic hidden–variables, for the functions $A(\alpha|\lambda)$ and $B(\beta|\lambda)$ we have the obvious constraints:

$|A(\alpha|\lambda)| \leq 1$ and $|B(\beta|\lambda)| \leq 1$. These last expectation values can be seen as averages of the previous deterministic expectation values. One can thus obtain the CHSH inequality in the form:

$$|E(\alpha,\beta) - E(\alpha,\beta') + E(\alpha',\beta) + E(\alpha',\beta')| \leq 2, \tag{33}$$

where, unlike the case of the right–hand side CH inequality, the hidden–variable normalization condition, $\int d\lambda\rho(\lambda) = 1$, has been employed in the derivation.

In terms of joint probabilities, each expectation values can be expressed as:

$$E(\alpha,\beta) = P(Y_\alpha, Y_\beta) + P(N_\alpha, N_\beta) - P(Y_\alpha, N_\beta) - P(N_\alpha, Y_\beta), \tag{34}$$

both in QM and LR, where $Y_{\alpha(\beta)}$ (Yes) and $N_{\alpha(\beta)}$ (No) answer to the question whether the incoming kaon projects into the state K_α (K_β) or otherwise when measuring along the quasi–spin axis α (β). To establish a complete link with the joint probabilities entering the CH inequality (29), or the Eberhard inequality (31), we have to specify the following relations:

$$\begin{aligned} P(Y_\alpha, Y_\beta) &= P(K_\alpha, K_\beta), \\ P(N_\alpha, N_\beta) &= P(K_\alpha^\perp, K_\beta^\perp) + P(K_\alpha^\perp, U_\beta) + P(U_\alpha, K_\beta^\perp) + P(U_\alpha, U_\beta), \\ P(Y_\alpha, N_\beta) &= P(K_\alpha, K_\beta^\perp) + P(K_\alpha, U_\beta), \\ P(N_\alpha, Y_\beta) &= P(K_\alpha^\perp, K_\beta) + P(U_\alpha, K_\beta), \end{aligned} \tag{35}$$

which hold when undetected or unmeasured events, denoted by $U_{\alpha,\beta}$, can really be identified by efficient 'event–ready' detectors. In such cases, the CHSH inequality (33) is a genuine Bell inequality and is equivalent to the CH inequality (29). In the most common case in which 'event–ready' detectors are not available, and one has to resort to the fair sampling hypothesis, all the detection efficiencies are assumed to be 1, the right–hand sides of relations (35) do not contain probabilities for undetected or unmeasured events and the corresponding CHSH inequality (33) is of the non–genuine type.

6.4 Wigner inequalities

Let us now see how introducing supplementary hypotheses one can obtain testable inequalities from the right–hand side CH inequality (29), or equivalently the Eberhard inequality (31). The first is the fair sampling hypothesis, for which the efficiency factors in the inequalities are assumed to be 1. The corresponding inequality takes the form of what we may call a Wigner–like inequality, but with four instead of three terms and measurement settings:

$$P(K_{\alpha'}, K_{\beta'}) \leq P(K_{\alpha'}, K_{\beta}^{\perp}) + P(K_{\alpha}^{\perp}, K_{\beta}) + P(K_{\alpha}, K_{\beta'}). \tag{36}$$

This Bell inequality is equivalent to the CHSH inequality (33) when the latter is also considered together with the assumption of fair sampling.

If in addition one demands $P(K_{\alpha}^{\perp}, K_{\beta}) = 0$, a standard (*i. e.*, with three terms and three different measurement settings) Wigner inequality [7] is obtained. Note that this requirement restricts the derivability of the standard Wigner inequality to deterministic local realistic theories only. On the contrary, the four–term Wigner inequality (36) is valid for both deterministic and stochastic hidden–variable theories. For maximally entangled and perfectly–anticorrelated states, $P(K_{\alpha}^{\perp}, K_{\beta}) = 0$ is achieved when $K_{\alpha} \equiv K_{\beta}^{\perp}$, which corresponds to require perfect anticorrelation for joint measurements along the same generic quasi–spin axis α, thus obtaining the standard Wigner inequality:

$$P(K_{\alpha'}, K_{\beta'}) \leq P(K_{\alpha'}, K_{\alpha}) + P(K_{\alpha}, K_{\beta'}). \tag{37}$$

7 A review of the proposals

7.1 Assuming fair sampling and perfect anticorrelation

We start by reviewing those proposals of Bell–type tests using maximally entangled two–kaon states and based on the Wigner inequality (37), which needs three different quasi–spin measurement axes. These are non–genuine inequalities since to be derived they require, in addition to the hypotheses of realism and locality, the assumption of fair sampling and the condition of perfect anticorrelation, as explained in Section 6.4.

7.1.1 A first proposal [27, 31]

For the maximally entangled kaon state (17), Uchiyama [27] derived the inequality:

$$P(K_S, K^0) \leq P(K_S, K_1) + P(K_1, K^0), \tag{38}$$

which has been rediscussed in detail by Bertlmann and Hiesmayr [31]. Here, the joint probabilities are assumed to be measured at a proper time $\tau = \tau_l = \tau_r$ very close to the instant of the pair creation, $\tau \to 0$. For this reason, the above inequality would eventually test noncontextuality rather than locality. Inserting the quantum–mechanical probabilities into Eq. (38), one obtains the constraint $\mathcal{R}e\,\epsilon \leq |\epsilon|^2$, which is violated by the presently accepted value of the CP–violation parameter ϵ.

Note that the proposed inequality involves passive measurements along the basis consisting of the two unphysical CP eigenstates. Moreover, the smallness of the parameter ϵ and Eq. (8) preclude any realistic attempt of discriminating between lifetime (K_S vs K_L) and CP (K_1 vs K_2) eigenstates. In this sense, the interest of inequality (38) reduces to that of a clear and well defined gedanken experiment.

7.1.2 Improved proposal with strangeness detection and thin regenerators [32, 3.

The authors of Refs. [32, 33)] based their study on the K^0 vs $\bar{K}^0$ identification and exploited the phenomenon of kaon regeneration to obtain the three different quasi–spin measurement axes needed to establish a Wigner inequality. The weak interaction eigenstates which emerge after crossing a thin, homogeneous regenerator during a time interval $\Delta\tau$ turn out to be [32, 33)]:

$$|K'_S\rangle = |K_S\rangle + \eta|K_L\rangle, \quad |K'_L\rangle = |K_L\rangle + \eta|K_S\rangle, \tag{39}$$

where

$$\eta \equiv i\,(\lambda_S - \lambda_L)\,\rho\Delta\tau = -\left(i\Delta m + \frac{1}{2}\Delta\Gamma\right)\rho\Delta\tau, \tag{40}$$

and ρ, given by Eq. (13), accounts for the (small) regeneration effects. Eqs. (39) are valid at lowest order in the regeneration parameter ρ.

The maximally entangled two–kaon state adopted in Refs. [32, 33)] is the one of Eq. (20), where a single time τ is considered for simultaneous left and right measurements. At this time τ, one kaon enters a regenerator placed on the left hand side and the other kaon enters a right hand side regenerator. The proper time spent by each kaon to cross the corresponding regenerator is $\Delta\tau$. The regenerator parameters are chosen at will by the experimenters: in order to derive a Wigner inequality such as (37), a regenerator is chosen between two available options, both along the left and right beams. The Wigner inequalities thus derived are the following ones:

$$P(K^0, 0; \bar{K}^0, \rho) \leq P(K^0, 0; \bar{K}^0, 2\rho) + P(\bar{K}^0, 2\rho; \bar{K}^0, \rho), \tag{41}$$

$$P(K^0, 0; \bar{K}^0, \rho) \leq P(K^0, 0; K^0, 2\rho) + P(K^0, 2\rho; \bar{K}^0, \rho), \tag{42}$$

where, *e. g.*, $P(K^0, \rho; \bar{K}^0, 2\rho)$ is the probability to detect, at time $\tau + \Delta\tau$, a K^0 on the left after a thin regenerator with parameter ρ and a $\bar{K}^0$ on the right after a double density (2ρ) regenerator. The absence of a regenerator is denoted with a 0. Note that in the present case, each regenerator is considered as part of the corresponding measurement process.

Quantum mechanics predicts a violation of one of the two previous inequalities. In fact, Eq. (41) and (42) imply:

$$\mathcal{R}e\ [i\rho(\lambda_L - \lambda_S)] \geq 0 \iff \mathcal{R}e\,\eta \leq 0, \tag{43}$$

$$\mathcal{R}e\ [i\rho(\lambda_L - \lambda_S)] \leq 0 \iff \mathcal{R}e\,\eta \geq 0. \tag{44}$$

Therefore, inequality (41) [(42)] has to be used for an actual test if the experimental value of $\mathcal{R}e\,\eta$ is positive (negative).

The problem using thin regenerators is that the parameter η is small, typically $|\eta| \simeq 10^{-3} \div 10^{-2}$ [26]. This lowers the level of violation, by quantum mechanics, of the inequalities to some %. Thick regenerators (say larger than a few millimeters) worsen the detector performances and the event reconstruction becomes more difficult because of multiple scatterings. Moreover, one is forced to use thin regenerators since otherwise saturation effects in the regeneration process occurs, due to inevitable K_S–decays (note that the K_S lifetime τ_S corresponds to a distance covered by a K_S coming from the decay $\phi \to K^0\bar{K}^0$ of about 0.6 cm).

7.2 Assuming fair sampling

We now proceed to analyse those proposals based on the fair sampling hypothesis, that is the CHSH inequality (33) or the Wigner inequality (36). Even if these inequalities are non–genuine Bell inequalities, their use opens up the possibility to test the family of both deterministic and stochastic local realistic theories based on the fair sampling.

7.2.1 Proposal with strangeness detection [24]

The analogy between strangeness and linear polarization measurements has been exploited by many authors. In the analysis by Ghirardi et al. [24] one considers the state (19) and performs joint strangeness measurements at two different times on the left beam (τ_1 and τ_2) and at other two different times on the right beam (τ_3 and τ_4). The detection times can be chosen at will and, at least ideally, in accordance with the locality requirement. The proposed inequality, incorporating the fair sampling hypothesis, is in the CHSH form:

$$|E(\tau_1, \tau_3) - E(\tau_1, \tau_4) + E(\tau_2, \tau_3) + E(\tau_2, \tau_4)| \leq 2, \tag{45}$$

where the expectation value $E(\tau_r, \tau_r)$ takes the value $+1$ when either two $\bar{K}^0$'s or no $\bar{K}^0$'s are found in the left (τ_l) and right (τ_r) measurements, and -1 otherwise:

$$E(\tau_l, \tau_r) = P(Y, \tau_l; Y, \tau_r) + P(N, \tau_l; N, \tau_r) - P(Y, \tau_l; N, \tau_r) - P(N, \tau_l; Y, \tau_r). \tag{46}$$

The probabilities entering this correlation function, where Y (Yes) and N (No) answer the question whether a $\bar{K}^0$ is detected at the considered time, can be easily obtained in QM and one gets:

$$E_{\rm QM}(\tau_l, \tau_r) = -\exp\{-(\Gamma_L + \Gamma_S)(\tau_l + \tau_r)/2\} \cos[\Delta m\,(\tau_l - \tau_r)], \tag{47}$$

where, having assumed fair sampling, the inefficiencies in strangeness detection can be ignored.

Because of strangeness oscillations in free–space along both kaon paths, choosing among four different times corresponds to four different choices of measurement directions in the photon case. Unfortunately, the above CHSH inequality is never violated by QM because strangeness oscillations proceed too slowly and cannot compete with the more rapid kaon weak decays.

7.2.2 Proposals with lifetime detection and regenerators [25, 51]

An alternative option is based on K_S vs K_L identification and has been first proposed by Eberhard in Ref. [25]. Here, the two–kaon state of Eq. (20) is considered. To observe if a neutral kaon in a beam is K_S or K_L at a given point (*i. e.*, instant), a kaon detector is located far enough downstream from this point so that the number of undecayed K_S's reaching the detector is negligible. Since $\Gamma_L << \Gamma_S$, almost all K_L's can reach the detector, where they manifest by strong nuclear interactions. In a complementary way, K_S's are identified by their decays (mainly into two–pions) not far from that point of interest. Misidentifications and ambiguous events will certainly appear, but at an acceptably low level, as explained in Ref. [25].

Measurements of K_S vs K_L are thus performed for each one of four experimental setups. In a first setup, the two–kaon state is allowed to propagate in free–space; its normalization is lost because of weak decays, but its perfect antisymmetry is maintained. In the other three setups, regenerators —one thin (4 mm), the other thick (5 cm)— are asymmetrically and alternatively located along one beam, or along the other, or along both. The following interesting inequality relating the number of K_L's and K_S's detected in each experimental setup is then derived from LR:

$$P(K_L, \rho; K_L, \rho') \leq P(K_L, \rho; K_L, 0) + P(K_S, 0; K_S, 0) + P(K_L, 0; K_L, \rho'). \quad (48)$$

Again, ρ and ρ' denote the regenerator parameters and 0 stands for the absence of regenerators. Note that the above Bell inequality is a particular case of the Wigner inequality (36). Due to a constructive interference effect between the two regeneration processes, this Bell inequality turns out to be significantly violated by QM predictions even if the above mentioned misidentifications and ambiguous detection events are taken into account. These successful predictions have some limitations, as already discussed by the author. In particular, they are valid for asymmetric ϕ–factories (where the two neutral kaon beams form a small angle and have a velocity larger than for kaons pairs from symmetric ϕ–factories), whose construction is not foreseen.

Fortunately, it has been recently shown that a measurable QM violation of the previous Wigner inequality can also be achieved when the experiment is performed at a symmetric e^+e^- machine. A proposal for such a test at the Frascati ϕ–factory has been indeed put forward [51].

7.2.3 *Proposal with both lifetime and strangeness detection* [37]

We now discuss a way to use the non–maximally entangled state of Eqs. (24) and (25), which is prepared with the help of a kaon regenerator and corresponds to a proper time T along both kaon beams. Following the approach of Ref. [37], for each kaon on each beam at time T we consider either a strangeness or a lifetime measurement.

With the strategy illustrated in Section 4.2 for lifetime measurements, requiring an extra interval time $\Delta T = 4.8\,\tau_S$ after T, care has to be taken to choose T large enough to guarantee the space–like separation between left and right measurements. Locality then excludes any influence from the experimental setup encountered by one member of the kaon pair at time T on the behaviour of its other–side partner between T and $T+\Delta T$. For kaon pairs from ϕ decays, moving with velocity $\beta \simeq 0.22$, this implies $T > (1/\beta - 1)\Delta T/2 = 1.77\,\Delta T$, with a considerable reduction of the total kaon sample. Indeed, for $\Delta T = 4.8\,\tau_S$ one can choose $T = 2\,\Delta T \simeq 9.6\,\tau_S$, and only 1 in 15000 initial events can be used, having both kaons surviving up to time T. For faster kaons, as in CPLEAR, the situation improves considerably.

In Ref. [37] the following CH inequalities have been derived under the assumption of perfectly efficient experimental apparata:

$$\begin{aligned} -1 \leq\ & P(\bar{K}^0, K_L) - P(\bar{K}^0, \bar{K}^0) + P(K_S, \bar{K}^0) \\ & + P(K_S, K_L) - P(K_S, *) - P(*, K_L) \leq 0, \end{aligned} \tag{49}$$

$$\begin{aligned} -1 \leq\ & P(\bar{K}^0, K_S) - P(\bar{K}^0, \bar{K}^0) + P(K_L, \bar{K}^0) \\ & + P(K_L, K_S) - P(*, K_S) - P(K_L, *) \leq 0, \end{aligned}$$

where, for instance:

$$P(K_S, *) \equiv P(K_S, K^0) + P(K_S, \bar{K}^0). \tag{50}$$

In this and the other one–side probabilities, the joint probabilities for the two possible outcomes on the other side are added to guarantee that both kaons have survived up to time T. This respects the particular normalization of the state (24). Note that each one of the two previous inequalities follows from the other by just inverting left and right measurements on the left–right asymmetric state (24). The right–hand side CH inequalities can be rewritten as:

$$\begin{aligned} \frac{P(\bar{K}^0, K_L) - P(\bar{K}^0, \bar{K}^0) + P(K_S, \bar{K}^0) + P(K_S, K_L)}{P(K_S, *) + P(*, K_L)} &\leq 1, \\ \frac{P(\bar{K}^0, K_S) - P(\bar{K}^0, \bar{K}^0) + P(K_L, \bar{K}^0) + P(K_L, K_S)}{P(*, K_S) - P(K_L, *)} &\leq 1. \end{aligned} \tag{51}$$

The CH–like inequalities (49) and (51), actually incorporating the fair sampling hypothesis, can be easily and equivalently rewritten as four–term Wigner inequalities. By properly writing the single–side probabilities in (51), the result can be put in the following form:

$$\begin{aligned} P(K_S, \bar{K}^0) &\leq P(K_S, K_S) + P(K^0, K_L) + P(\bar{K}^0, \bar{K}^0), \\ P(\bar{K}^0, K_S) &\leq P(\bar{K}^0, \bar{K}^0) + P(K_L, K^0) + P(K_S, K_S). \end{aligned} \tag{52}$$

By choosing $T \geq 9.6\tau_S$, as required by locality, the complex parameter R_S of Eq. (25) turns out to be negligible, while $|R_L| = \mathcal{O}(1)$. By substituting the QM predictions in the left–hand side CH inequalities (51), one easily finds:

$$\frac{2 - \mathcal{R}e\, R_L + \frac{1}{4}|R_L|^2}{2 + |R_L|^2} \leq 1, \quad \frac{2 + \mathcal{R}e\, R_L + \frac{1}{4}|R_L|^2}{2 + |R_L|^2} \leq 1, \tag{53}$$

whose only difference is the sign affecting the linear term in $\mathcal{R}e\, R_L$. According to the sign of $\mathcal{R}e\, R_L$, one of these two inequalities is violated if $|\mathcal{R}e\, R_L| \geq 3|R_L|^2/4$. The greatest violation occurs for a purely real value of R_L, $|R_L| \simeq 0.56$, for which one of the two ratios in Eq. (53) reaches the value 1.14. This 14 % violating effect predicted by QM opens up the possibility for a meaningful Bell–type test with neutral kaons which could refute those LR models based on the fair sampling hypothesis.

Values for the parameter R_L satisfying $\mathcal{I}m\, R_L = 0$ and $|\mathcal{R}e\, R_L| = 0.56$, as required, are not difficult to obtain. Indeed, for kaon pairs from ϕ decays and according to the values of the regeneration parameters [26)], one can use a thin beryllium regenerator 1.55 mm thick to prepare the state (21), which then converts into the state (24) with the desired value of R_L by propagating in free–space up to $T \simeq 11.1\,\tau_S$.

7.3 An attempt of genuine test [38, 41)]

We conclude our review by discussing a proposal which does not assume hypotheses which go beyond the reality and locality requirements. In our opinion, it represent an interesting attempt for a loophole–free test of LR vs QM with neutral kaons.

Hardy's proof without inequalities of Bell's theorem [55)] has been applied in Ref. [38)] to the non–maximally entangled state of Eqs. (24) and (25). This considerably improved the analysis of Ref. [37)]. In such an approach, alternative measurements of strangeness or lifetime are considered, at time T, on each one of the kaon pairs, according to the strategies for active measurement procedures illustrated in Section 4. Hardy's proof is then translated into an Eberhard inequality [41)], which could discriminate between LR and QM conditionally on the detection efficiencies for strangeness and lifetime measurements at disposal in the actual test.

Let us first concentrate on the 'non–locality without inequalities' proof of Ref. [38)]. Neglecting CP–violation and K_L–K_S misidentification effects, from state (24) with $R_S = 0$ and $R_L = -1$ (called Hardy's state) one obtains the following QM predictions:

$$\begin{aligned} P_{\rm QM}(K^0, \bar{K}^0) &= \eta\,\bar{\eta}/12, & (54)\\ P_{\rm QM}(K^0, K_L) &= 0, & (55)\\ P_{\rm QM}(K_L, \bar{K}^0) &= 0, & (56)\\ P_{\rm QM}(K_S, K_S) &= 0, & (57) \end{aligned}$$

where η ($\bar{\eta}$) is the overall efficiency for K^0 ($\bar{K}^0$) detection. We note that the values $R_S = 0$ and $R_L = -1$ can be obtained by using, for instance, a beryllium (carbon) regenerator with thickness $d = 2.83$ mm ($d = 0.78$ mm), a detection time $T = 11.1\,\tau_S$ ($T = 11.3\,\tau_S$) and kaon pairs created at a ϕ–factory (proton–antiproton machine). It is found that the necessity to reproduce, under LR, equalities (54)–(56) requires:

$$P_{\rm LR}(K_S, K_S) \geq P_{\rm LR}(K^0, \bar{K}^0) = \eta\,\bar{\eta}/12 > 0, \qquad (58)$$

which contradicts Eq. (57). In principle, this allows for an 'all–or–nothing', Hardy–like test of LR vs QM. In Ref. [38] it was concluded that, by requiring a perfect discrimination between K_S and K_L states, an experiment measuring the joint probabilities of Eqs. (54)–(57) closes the efficiency loophole even for infinitesimal values of the strangeness detection efficiencies η and $\bar{\eta}$. However, since K_L and K_S misidentifications (due to the finite value of $\Gamma_S/\Gamma_L \simeq 579$) do not permit an ideal lifetime measurement even when the detection efficiency η_τ for the kaon decay products is 100%, the original proposal must be reanalysed paying attention to the inefficiencies involved in the real test [41].

Retaining the effects due to K_S–K_L misidentifications, for Hardy's state one obtains (see the Appendix of Ref. [41] for details):

$$\begin{aligned} P_{\rm QM}(K^0, \bar{K}^0) &= \eta\bar{\eta}/12, & (59)\\ P_{\rm QM}(K^0, K_L) &= 6.77 \times 10^{-4}\,\eta\,\eta_\tau, & (60)\\ P_{\rm QM}(K_L, \bar{K}^0) &= 6.77 \times 10^{-4}\,\bar{\eta}\,\eta_\tau, & (61)\\ P_{\rm QM}(K_S, K_S) &= 1.19 \times 10^{-5}\,\eta_\tau^2, & (62) \end{aligned}$$

which replace the results of Eqs. (54)–(57). In the standard Hardy's proof of non–locality [55], the probabilities corresponding to our (60), (61) and (62) are perfectly vanishing. In our case they are very small but not zero. Nevertheless, this does not prevent us from deriving a contradiction between LR and QM. Indeed, as proved in Ref. [56], the well known criterion of physical reality of Einstein, Podolsky and Rosen [3] can be generalized to include predictions made with *almost* certainty, as it is required in the present case due to the nonvanishing values of probabilities (60)–(62).

According to this generalization, the following Eberhard inequality must be used to demonstrate the incompatibility between LR and QM:

$$H \equiv \frac{P(K^0, \bar{K}^0)}{P(K^0, K_L) + P(K_S, K_S) + P(K_L, \bar{K}^0) + P(K^0, U_{\rm Lif}) + P(U_{\rm Lif}, \bar{K}^0)} \leq 1. \tag{63}$$

Essentially, it is a different writing of the CH inequality:

$$Q \equiv \frac{P(K_S, \bar{K}^0) - P(K_S, K_S) + P(K^0, \bar{K}^0) + P(K^0, K_S)}{P(K^0, *) + P(*, \bar{K}^0)} \leq 1, \tag{64}$$

and the argument $U_{\rm Lif}$ refers to failures in lifetime detection. The QM expression for the probabilities containing lifetime undetection are:

$$P_{\rm QM}(K^0, U_{\rm Lif}) = \frac{1}{6}\eta\,(1-\eta_\tau)\,, \quad P_{\rm QM}(U_{\rm Lif}, \bar{K}^0) = \frac{1}{6}\bar{\eta}\,(1-\eta_\tau)\,. \tag{65}$$

Note that the use of an inequality [57] allows for deviations, existing in real experiments, in the values of R_S and R_L required to prepare Hardy's state and, in addition, takes care of the difficulties associated to 'almost null' measurements, as is the case of probabilities (60)–(62). Both previous inequalities are actually derivable from LR for any value of R_S and R_L. However, Hardy's proof leads to inequality (63) only for Hardy's state ($R_S = 0$ and $R_L = -1$). It is important to stress that the previous Eberhard and CH inequalities have been obtained *without invoking supplementary assumptions* on undetected events. They are both genuine Bell inequalities and provide the same restrictions on the efficiencies η, $\bar{\eta}$ and η_τ required for a detection loophole free experiment.

In order to discuss the feasibility of such an experiment, let us start considering a few ideal cases. Assume first that perfect discrimination between K_S and K_L were always possible ($\eta_\tau = 1$ and $p_L = p_S = 1$; see appendix of Ref. [41]); one could then make a conclusive test of LR for any nonvanishing values of η and $\bar{\eta}$: $H_{\rm QM}^{\eta_\tau = p_L = p_S = 1} \to \infty$, $\forall\, \eta, \bar{\eta} \neq 0$. In a second ideal case with no undetected events, *i. e.* with $\eta = \bar{\eta} = \eta_\tau = 1$, the inequalities are strongly

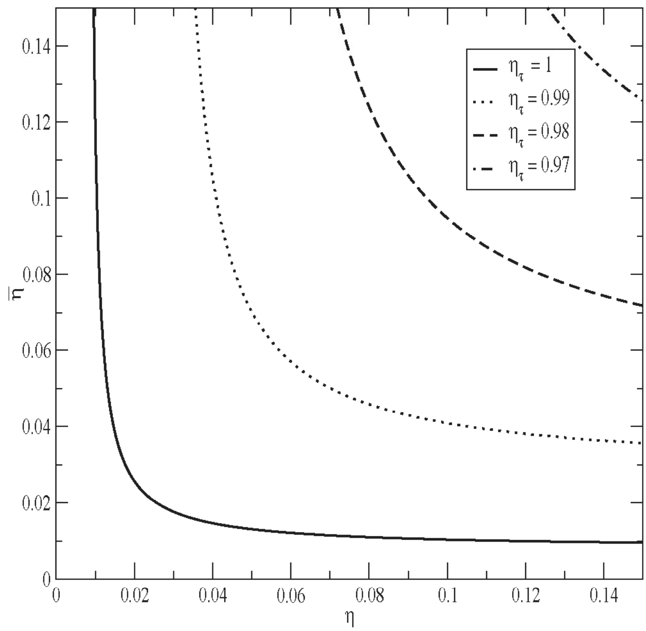


Figure 1: *The four curves (corresponding to $\eta_\mathcal{T} = 1$, $\eta_\mathcal{T} = 0.99$, $\eta_\mathcal{T} = 0.98$ and $\eta_\mathcal{T} = 0.97$) provide the values of η and $\bar{\eta}$ for which $H_{\rm QM} = Q_{\rm QM} = 1$ using Hardy's state. QM violates inequalities (63) and (64) for values of η and $\bar{\eta}$ situated above the corresponding curve.*

violated by QM, $H_{\rm QM}^{\eta=\bar{\eta}=\eta_\mathcal{T}=1} \simeq 60.0$ and $Q_{\rm QM}^{\eta=\bar{\eta}=\eta_\mathcal{T}=1} \simeq 1.25$, even if one allows for unavoidable K_S and K_L misidentifications. Finally, assuming that only the detection efficiency of kaon decay products is ideal ($\eta_\mathcal{T} = 1$), for $\eta = \bar{\eta}$ ($\eta = \bar{\eta}/2$) Eberhard and CH inequalities are contradicted by QM whenever $\eta > 0.023$ ($\eta > 0.017$).

Let us now consider more realistic situations with small and possibly achievable values of η and $\bar{\eta}$. This implies that we have to consider large decay–product detection efficiencies such as $\eta_\mathcal{T} = 0.97$, 0.98, 0.99 and, ideally, 1. For each $\eta_\mathcal{T}$, the values of η and $\bar{\eta}$ that permit a detection loophole free test ($H_{\rm QM}$, $Q_{\rm QM} > 1$) lie above the corresponding curve plotted in Fig. 1. As expected, when $\eta_\mathcal{T}$ decreases, the region of η and $\bar{\eta}$ values which permits a conclusive test diminishes and larger values of η and $\bar{\eta}$ are required.

Note, however, that the strangeness detection efficiencies required for a conclusive test of LR vs QM with neutral kaons are considerably smaller than the limit ($\eta_0 = 0.67$) deduced by Eberhard [22)] for non–maximally entangled photon states.

8 Conclusions

A series of proposals aiming to perform Bell inequality tests with entangled neutral kaon pairs has been reviewed. The relativistic velocities of these kaons and their strong interactions seem to offer the possibility of simultaneously closing the so–called locality and detection loopholes which affect analogous experiments performed with photons and ions. The real situation, however, is not a simple one.

All the proposal we discussed suffer from difficulties coming from the fact that the number of different complementary measurements on neutral kaons one can use for a Bell–test is reduced. Essentially, only strangeness and lifetime measurements are possible. The situation can be improved if the well known effects of kaon regeneration are taken into account. On the one hand, this amounts to an effective increase in the number of non–compatible measurements one can perform. On the other hand, by changing or removing the regenerators, the active presence of the experimenter is guaranteed. A final difficulty could still remain: the rather low efficiency of some of these neutral kaon measurements. A detailed analysis suggests that a Bell–test with neutral kaons free from the detection loophole would require a few % strangeness detection efficiencies and very high efficiencies for the detection of the kaon decay products.

Acknowledgements

This work is partly supported by the Ramón y Cajal program (R. E.), EURIDICE HPRN–CT–2002–00311, MIUR 2001024324_007, INFN and DGICYT BFM-2002-02588.

References

1. R. H. Dalitz, *Strange Particles and Strong Interactions* (Tata Institute of Fundamental Research, Bombay, 1962).

2. D. Bohm, *Quantum Theory* (Prentice Hall, Englewood Cliffs, N. J., 1951).

3. A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47**, 777 (1935).

4. N. Bohr, Phys. Rev. **48**, 696 (1935).

5. J. Bell, Physics **1**, 195 (1964).

6. M. Redhead, *Incompleteness, non–locality and realism* (Oxford University Press, Oxford, 1990).

7. E. P. Wigner, Am. J. Phys. **38**, 1005 (1970).

8. J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).

9. J. F. Clauser and M. A. Horne, Phys. Rev. **D 10**, 526 (1974).

10. J. F. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978).

11. J. S. Bell, *Speakable and unspeakable in quantum mechanics (collected papers on quantum philosophy)* (Cambridge University Press, Cambridge, 1987).

12. A. Bramon, G. Garbarino and B. C. Hiesmayr, Phys. Rev. **A 69**, 062111 (2004).

13. A. Aspect, J. Dalibard and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).

14. G. Weihs, T. Jennewein, C. Simon, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. **81**, 5039 (1998).

15. W. Tittel, J. Brendel, H. Zbinden and N. Gisin, Phys. Rev. Lett. **81**, 3563 (1998); W. Tittel, J. Brendel, N. Gisin and H. Zbinden, Phys. Rev. **A 59**, 4150 (1999).

16. R. A. Bertlmann and A. Zeilinger (Eds.), *Quantum (Un)speakables – From Bell to Quantum Information* (Springer, Berlin, 2002).

17. M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, Nature **409**, 791 (2001).

18. E. Santos, Phys. Lett. **A 327**, 33 (2004); Found. Phys. **34**, 1643 (2004); Studies in History and Philosophy of Modern Physics **36 B**, 544 (2005).

19. L. Vaidman, Phys. Lett. **A 286**, 241 (2001).

20. P. Pearle, Rhys. Rev. **D 2**, 1418 (1970).

21. A. Garg and N. D. Mermin, Phys. Rev. **D 35**, 3831 (1987).

22. P. H. Eberhard, Phys. Rev. **A 47**, R747 (1993).

23. E. Santos, Phys. Rev. **A 46**, 3646 (1992); Phys. Lett. **A 212**, 10 (1996); N. Gisin and B. Gisin, Phys. Lett. **A 260**, 323 (1999).

24. G. C. Ghirardi, R. Grassi and T. Webern, in *Proceedings of the Workshop on Physics and Detectors for DaΦne*, edited by G. Pancheri, p. 261 (INFN, Laboratori Nazionali di Frascati, Frascati, Italy, 1991).

25. P. H. Eberhard, Nucl. Phys. **B 398**, 155 (1993).

26. A. Di Domenico, Nucl. Phys. **B 450**, 293 (1995).

27. F. Uchiyama, Phys. Lett. **A 231**, 295 (1997).

28. F. Selleri, Phys. Rev. **A 56**, 3493 (1997); R. Foadi and F. Selleri, Phys. Lett. **B 461**, 123 (1999); Phys. Rev. **A 61**, 012106 (2000).

29. A. Afriat and F. Selleri, *The Einstein, Podolsky and Rosen paradox in atomic, nuclear and particle physics* (Plenum Press, New York, 1998).

30. F. Benatti and R. Floreanini, Phys. Rev. **D 57**, R1332 (1998); Eur. Phys. J. **C 13**, 267 (2000).

31. R. A. Bertlmann, W. Grimus and B. C. Hiesmayr, Phys. Rev. **D 60**, 114032 (1999); Phys. Lett. **A 289**, 21 (2001); B. C. Hiesmayr, Found. Phys. Lett. **14**, 231 (2001); R. A. Bertlmann and B. C. Hiesmayr, Phys. Rev. **A 63**, 062112 (2001); Phys. Lett. **A 332**, 355 (2004); R. A. Bertlmann, K. Durstberger and B. C. Hiesmayr, Phys. Rev. **A 68**, 012111 (2003).

32. A. Bramon and M. Nowakowski, Phys. Rev. Lett. **83**, 1 (1999).

33. B. Ancochea, A. Bramon and M. Nowakowski, Phys. Rev. **D 60**, 094008 (1999).

34. N. Gisin and A. Go, Am. J. Phys. **69**, 264 (2001).

35. R. Dalitz and G. Garbarino, Nucl. Phys. **B 606**, 483 (2001).

36. M. Genovese, C. Novero and E. Predazzi, Phys. Lett. **B 513**, 401 (2001); Found. Phys. **32**, 589 (2002).

37. A. Bramon and G. Garbarino, Phys. Rev. Lett. **88**, 040403 (2002).

38. A. Bramon and G. Garbarino, Phys. Rev. Lett. **89**, 160401 (2002).

39. M. Genovese, Phys. Rev. **A 69**, 022103 (2004).

40. R. A. Bertlmann, Lect. Notes Phys. **689**, 1 (2006) [quant-ph/0410028].

41. A. Bramon, R. Escribano and G. Garbarino, Found. Phys. **36**, 563 (2006) [quant-ph/0501069].

42. P. K. Kabir, *The CP Puzzle* (Academic Press, London, 1968).

43. M. Fidecaro and H.-J. Gerber, Rept. Prog. Phys. **69**, 1713 (2006).

44. *The Second DaΦne Physics Handbook* edited by L. Maiani, G. Pancheri and N. Paver (INFN, Laboratori Nazionali di Frascati, Frascati, Italy, 1995).

45. A. Apostolakis et al., Phys. Lett. **B 422**, 339 (1998).

46. A. Angelopoulos et al., Phys. Rept. **374**, 165 (2003).

47. A. Di Domenico, **hep-ex/0312032**, published in **eConf C0309101: THWP007, 2003**; this Handbook.

48. L. Kasday, in *Foundations of Quantum Mechanics*, B. d'Espagnat ed., p. 195 (New York, Academic Press, 1971). Proceedings of the International School of Physics 'Enrico Fermi', Course IL.

49. A. Bramon, R. Escribano and G. Garbarino, J. Mod. Opt. **52**, 1681 (2005) [quant-ph/0410122].

50. R. Ursin et al, quant-ph/0607182.

51. A. Go, this Handbook.

52. S. Eidelman et al. (Particle Data Group), Phys. Lett. **B 592**, 1 (2004).

53. A. Bramon, G. Garbarino and B. C. Hiesmayr, Phys. Rev. Lett. **92**, 020405 (2004).

54. R. A. Bertlmann, A. Bramon, G. Garbarino and B. C. Hiesmayr, Phys. Lett. **A 332**, 355 (2004).

55. L. Hardy, *Phys. Rev. Lett.* **68**, 2981 (1992); Phys. Rev. Lett. **71**, 1665 (1993).

56. P. H. Eberhard and P. Rosselet, Universite de Lausanne Report No. IPNL–93–3, 1993; Found. Phys. **25**, 91 (1995).

57. L. Hardy, Phys. Rev. Lett. **73**, 2279 (1994); N. D. Mermin, Am. J. Phys. **62**, 880 (1994); A. Garuccio, Phys. Rev. **A 52**, 2535 (1995).

KAONIC QUANTUM ERASERS AT A Φ-FACTORY: *"ERASING THE PRESENT, CHANGING THE PAST"*

Albert Bramon
Grup de Física Teòrica, Universitat Autònoma de Barcelona, E–08193 Bellaterra, Spain
Gianni Garbarino
Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, I–10125 Torino, Italy
Beatrix C. Hiesmayr
Institut für Theoretische Physik, Faculty of Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria

Abstract

Neutral kaons are unique quantum systems to show some of the most puzzling peculiarities of quantum mechanics. Here we focus on a quantitative version of Bohr's complementary principle and on quantum marking and eraser concepts. In detail we show that neutral kaons (1) are kind of double slit devices encapsulating Bohr's complementarity principle in a simple and transparent way, and (2) offer marking and eraser options which are ***not*** afforded by other quantum systems and which can be performed at the DAΦNE machine.

1 Introduction

During the last fifteen years or so we have witnessed an interesting revival of the research concerning some fundamental issues of quantum mechanics. A very positive aspect of this revival is that it has been driven by a series of impressive results which have been possible thanks to improved experimental techniques and skillful ideas of several experimental groups. As a result, some of the *Gedankenexperimente* proposed and discussed in the earlier days of quantum mechanics, or slight modifications of these proposals, have been finally performed in the laboratory. Most of these experiments belong to the fields of quantum optics and photonics; others make use of (single) atomic or ion states.

Among these kinds of experiments we concentrate on two types. The first type concerns the old complementarity principle of Niels Bohr for which a quantitative version became available in recent years. This 'quantitative complementarity' represents a major improvement over older treatments and can be tested for rather simple quantum systems. The second type of experiments requires more complex states consisting of entangled two–particle systems. With these bipartite states at hand one can test much more subtle quantum phenomena such as the so called 'quantum eraser', which adds puzzling space–time considerations to the previous, Bohr's complementarity issue.

The aim of our contribution is to analyze the role that neutral kaons can play in this two types of experiments. A copious source of entangled neutral meson pairs, such as those produced in the Daphne e^+e^- machine, can be shown to be extremely useful for this purpose.

2 Kaons as double slits

The famous statement about quantum mechanics "*the double slit contains the only mystery*" of Richard Feynman is well known, his statement about neutral kaons is not less to the point: "*If there is any place where we have a chance to test the main principles of quantum mechanics in the purest way —does the superposition of amplitudes work or doesn't it?— this is it*" [1]. In this section we argue that single neutral kaons can be considered as double slits as well.

Bohr's complementarity principle and the closely related concept of duality in interferometric or double–slit like devices are at the heart of quantum mechanics. The well–known qualitative statement that "*the observation of an interference pattern and the acquisition of which–way information are mutually exclusive*" has only recently been rephrased to a quantitative statement [2, 3]:

$$\mathcal{P}^2(y) + \mathcal{V}_0^2(y) \leq 1 \,, \tag{1}$$

where the equality is valid for pure quantum states and the inequality for mixed ones. $\mathcal{V}_0(y)$ is the fringe visibility, which quantifies the sharpness or contrast of the interference pattern (the "wave–like" property), whereas $\mathcal{P}(y)$ denotes the path predictability, i.e., the *a priori* knowledge one can have on the path taken by the interfering system (the "particle–like" property). The path predictability is defined by [2]

$$\mathcal{P}(y) \;=\; |p_I(y) - p_{II}(y)| \,, \tag{2}$$

where $p_I(y)$ and $p_{II}(y)$ are the probabilities for taking each path ($p_I(y) + p_{II}(y) = 1$). Both $\mathcal{V}_0(y)$ and $\mathcal{P}(y)$ depend on the same parameter y related somehow to the geometry of the interferometric setup. It is often too idealized to assume that the predictability and visibility are independent of this external parameter y. For example, consider a usual experiment with a vertical screen having a higher and a lower slit. Then the intensity is generally given by

$$I(y) \;=\; F(y)\left(1+\mathcal{V}_0(y)\cos(\phi(y)\right)\,, \qquad (3)$$

where $F(y)$ is specific for each setup and $\phi(y)$ is the phase–difference between the two paths. The variable y characterizes in this case the position of the detector scanning a vertical plane beyond the double–slit. An accurate description of the interference pattern, whose contrast along a wide scanned region can hardly be constant, thus requires to consider the y–dependence of visibility and predictability.

In Ref. [4] the authors investigated physical situations for which the expressions of $\mathcal{V}_0(y), \mathcal{P}(y)$ and $\phi(y)$ can be calculated analytically. This included interference patterns of various types of double slit experiments (y is linked to position), as well as Mott scattering experiments of identical particles or nuclei (y is linked to a scattering angle). But it also included particle–antiparticle oscillations in time due to particle mixing, as in the neutral kaon system. In this case, y is a time variable indirectly linked to the position of the kaon detector or the kaon decay vertex. Remarkably, all these two–state systems, belonging to quite distinct fields of physics, can then be treated via the generalized complementarity relation (1) in a unified way. Even for specific thermodynamical systems, Bohr's complementarity can manifest itself, see Ref. [5]. Here we investigate the neutral kaon case.

The time evolution of an initial K^0 state is given by

$$|K^0(t)\rangle \;=\; \frac{1}{\sqrt{2}}e^{-im_L t-\frac{\Gamma_L}{2}t}\left\{e^{i\Delta m t+\frac{\Delta\Gamma}{2}t}\,|K_S\rangle+|K_L\rangle\right\}\,, \qquad (4)$$

where (here and in the following) inessential CP violation effects are safely neglected. In our notation $\Delta m \equiv m_L - m_S$ is the (small) mass difference between the long– and short–lived kaon components whose time evolution is simply given by

$$|K_S(t)\rangle = e^{-im_S t}e^{-\frac{1}{2}\Gamma_S t}\,|K_S\rangle\,,\;\; |K_L(t)\rangle = e^{-im_L t}e^{-\frac{1}{2}\Gamma_L t}\,|K_L\rangle\,. \qquad (5)$$

Note that there are no oscillations in time between these two states and that their decay rates are remarkably different, $\Gamma_S \simeq 579\Gamma_L$, so that we can write $\Delta\Gamma \equiv \Gamma_L - \Gamma_S \simeq -\Gamma_S \simeq -2.1\Delta m < 0$.

State (4) can be interpreted as follows. The two mass eigenstates $|K_S\rangle$ and $|K_L\rangle$, i.e. the two terms in the right hand side of eq. (4), represent the two slits. At time $t = 0$ both terms (slits) have the same weight (width) and constructively interfere with a common phase. As time evolves, the K_S component decreases faster than the other one and this can be interpreted as a relative shrinkage of the K_S–slit making more likely the 'passage' through the K_L–slit. In addition, the norm of eq. (4) decreases with time as a consequence of both K_S and K_L decays, an effect which could be mimicked by an hypothetical shrinkage of both slit widths in real double–slit experiments. The analogy gets more obvious if we eliminate this latter effect by restricting to kaons which survive up to a certain time t and are thus described by renormalizing the state (4):

$$|K^0(t)\rangle \;\cong\; \frac{1}{\sqrt{2\cosh(\frac{\Delta\Gamma}{2}t)}}\, e^{-\frac{\Delta\Gamma}{4}t}\left\{e^{i\Delta m t+\frac{\Delta\Gamma}{2}t}\,|K_S\rangle + |K_L\rangle\right\}\,. \tag{6}$$

The probabilities for detecting on this state either a K^0 or a $\bar{K}^0$ are given by

$$\begin{aligned} P(K^0,t) &= \left|\langle K^0|K^0(t)\rangle\right|^2 = \frac{1}{2}\left\{1+\frac{\cos(\Delta m t)}{\cosh(\frac{\Delta\Gamma}{2}t)}\right\} \\ P(\bar{K}^0,t) &= \left|\langle \bar{K}^0|K^0(t)\rangle\right|^2 = \frac{1}{2}\left\{1-\frac{\cos(\Delta m t)}{\cosh(\frac{\Delta\Gamma}{2}t)}\right\}\,, \end{aligned} \tag{7}$$

showing the well–known strangeness oscillations. We observe that the oscillating phase is given by $\phi(t) = \Delta m\, t$ and the time dependent visibility by

$$\mathcal{V}_0(t) = \frac{1}{\cosh(\frac{\Delta\Gamma}{2}t)}\,. \tag{8}$$

The path predictability $\mathcal{P}(t)$, which in our kaonic case corresponds to a "which width" information, can be directly calculated from eq. (6)

$$\mathcal{P}(t) = |P(K_S,t) - P(K_L,t)| = \left|\frac{e^{\frac{\Delta\Gamma}{2}t} - e^{-\frac{\Delta\Gamma}{2}t}}{2\cosh(\frac{\Delta\Gamma}{2}t)}\right| = \left|\tanh\left(\frac{\Delta\Gamma}{2}t\right)\right|\,. \tag{9}$$

The expressions for predictability (9) and visibility (19) satisfy the complementary relation (1) for all times t

$$\mathcal{P}^2(t) + \mathcal{V}_0^2(t) \;=\; \tanh^2\left(\frac{\Delta\Gamma}{2}t\right) + \frac{1}{\cosh^2(\frac{\Delta\Gamma}{2}t)} \;=\; 1\,. \tag{10}$$

For time $t = 0$ there is full interference, the visibility is maximal, $\mathcal{V}_0(t = 0) = 1$, and we have no information about the lifetime, $\mathcal{P}(t = 0) = 0$. This corresponds to the central part (around the zero–order maximum) of the interference pattern in a usual double slit scenario. For very large times, i.e. $t \gg 1/\Gamma_S$, the surviving kaon is most probably in a long lived state K_L and no interference is observed since we have almost perfect information on "which width" is actually propagating. For times between these two extremes and due to the natural instability of the kaons, we obtain partial "which width" information and the expected interference contrast is thus smaller than one. However, the full information on the system is contained in eq. (1) and is always maximal for pure states.

The complementarity principle was proposed by Niels Bohr in an attempt to express the most fundamental difference between classical and quantum physics. According to this principle, and in sharp contrast to classical physics, in quantum physics we cannot capture all aspects of reality simultaneously and the available information on complementary aspects is always limited. Neutral kaons encapsulate indeed this peculiar feature in the very same way as a particle having passed through a double slit. But kaons are double slit devices automatically provided by Nature for free!

3 Kaonic quantum eraser

Two hundred years ago Thomas Young taught us how to observe interference phenomena with light beams. Much more later, interference effects of light have been observed at the single photon level. Nowadays also experiments with very massive particles, like fullerene molecules, have impressively demonstrated that fundamental feature of quantum mechanics [6]. It seems that there is no physical reason why not even heavier particles should interfere except for technical ones. In the previous section we have shown that interference effects disappear if it is possible to know the path through the double slit. The 'quantum eraser', a gedanken experiment proposed by Scully and Drühl in 1982 [7], surprised the physics community: if that knowledge on the path of the particle is erased, interference can be brought back again.

Since that work many different types of quantum erasers have been analyzed and experiments have been performed with atom interferometers [8] and with entangled photon pairs [9, 10, 12, 13, 14, 15]. In most of them, the quantum erasure is performed in the so-called "delayed choice" mode which best captures the essence and the most subtle aspects of the eraser phenomenon. In this case the meter, the quantum system which carries the mark on the path taken, is a system spatially separated from the interfering system which is generally called the object system. The decision *to erase or not* the mark of the meter system —and therefore the possibility *to observe or not* interference— can be taken long after the measurement on the object system has been completed. This was nicely phrased by Aharonov and Zubairy in the title of their review article [16)] as "erasing the past and impacting the future".

Here we want to present four conceptually different types of quantum erasers for neutral kaons, Refs. [17, 18)]. Two of them are analogous to erasure experiments already performed with entangled photons, e.g. Refs. [9, 10)]. For convenience of the reader we added two figures sketching the setups of these experiments, Fig. 1 and Fig. 2. In the first experiment the erasure operation was carried out "actively", i.e., by exerting the free will of the experimenter, whereas in the latter experiment the erasure operation was carried out "partially actively", i.e., the mark of the meter system was erased or not by a well known probabilistic law: photon reflection or transmission in a beam splitter. However, different to photons the kaons can be measured by an *active* or a completely *passive* procedure. This offers new quantum erasure possibilities and proves the underlying working principle of a quantum eraser, namely, sorting events according to the acquired information.

For neutral kaons there exist two relevant alternative physical bases. The first basis is the strangeness eigenstate basis $\{|K^0\rangle, |\bar{K}^0\rangle\}$. The strangeness S of an incoming neutral kaon at a given time t can be measured by inserting at the appropriate point of the kaon trajectory a thin piece of high–density matter. Due to strangeness conservation of the strong interactions between kaons and nucleons, the incoming state is projected either onto K^0, by a reaction like $K^0 p \to K^+ n$, or onto $\bar{K}^0$, by other reactions such as $\bar{K}^0 p \to \Lambda\pi^+$, $\bar{K}^0 n \to \Lambda\pi^0$ or $\bar{K}^0 n \to K^- p$. Here the nucleonic matter plays the same role as a two channel analyzer for polarized photon beams. We refer to this kind of strangeness measurement, which requires the insertion of that piece of matter, as an *active* measurement.

Alternatively, the strangeness content of neutral kaons can be determined by observing their semileptonic decay modes. Indeed, these semileptonic decays obey the well tested $\Delta S = \Delta Q$ rule which allows the modes

$$K^0(\bar{s}d) \to \pi^-(\bar{u}d) + l^+ + \nu_l \ , \ \bar{K}^0(s\bar{d}) \to \pi^+(u\bar{d}) + l^- + \bar{\nu}_l, \qquad (11)$$

where l stands for e or μ, but forbids decays into the respective charge conjugated modes. Obviously, the experimenter has no control on the kaon decay, neither on the mode nor on the time. The experimenter can only sort the set of all observed events in proper decay modes and time intervals. We call this procedure, opposite to the *active* measurement procedure described above, a *passive* strangeness measurement.

The second basis $\{K_S, K_L\}$ consists of the short– and long–lived states having well defined masses $m_{S(L)}$ and decay widths $\Gamma_{(S)L}$. We have seen that it is the appropriate basis to discuss the kaon propagation in free space because these states preserve their own identity in time, eq. (5). Due to the huge difference in the decay widths, the K_S's decay much faster than the K_L's. Thus in order to observe if a propagating kaon is a K_S or K_L at an instant time t, one has to detect at which time it subsequently decays. Kaons which are observed to decay before $\simeq t + 4.8\,\tau_S$ have to be identified as K_S's, while those surviving after this time are assumed to be K_L's. Misidentifications reduce only to a few parts in 10^{-3}, see also Refs. [17, 18]. Note that the experimenter doesn't care about the specific decay mode, he has to allow for free propagation and to record only the time of each decay event. We call this procedure an *active* measurement of lifetime. Indeed, it is by actively placing or removing an appropriate piece of matter that the strangeness (as previously discussed) or the lifetime of a given kaon can be measured. Since one measurement excludes the other, the experimenter has to decide which one is actually performed and the kind of information thus obtained.

On the other hand, neglecting CP violation effects —recall that they are of the order of 10^{-3}, like the just mentioned K_S, K_L misidentifications— the K_S's can be also identified by their specific decay into 2π final states ($CP = +$), while 3π final states ($CP = -$) have to be associated with K_L decays. As before, we call this procedure a *passive* measurement of lifetime, since the kaon decay times and decay channels (two vs three pions) used in the measurement are entirely determined by the quantum nature of kaons and cannot be in any way influenced by the experimenter.

(a) Active eraser with *active* measurements

Let us first discuss the quantum eraser experiments performed with photon pairs in Ref. [9]. In this experiment (see Fig. 1) two interfering two–photon amplitudes are prepared by forcing a pump beam to cross twice the same nonlinear crystal. Idler and signal photons from the first down conversion are marked by subsequently rotating their polarization by 90° and then superposed to the idler (i) and signal (s) photons emerging from the second passage of the beam through the same crystal. If type–II spontaneous parametric down conversion were used, we would had the two–photon state [1]

$$|\psi\rangle \;=\; \frac{1}{\sqrt{2}}\Big\{ \underbrace{|V\rangle_i|H\rangle_s}_{\text{second passage}} - e^{i\Delta\phi} \underbrace{|H\rangle_i|V\rangle_s}_{\text{first passage}} \Big\}\,, \qquad (12)$$

where H and V refer to horizontal and vertical polarizations. The first and second terms in Eq. (12) correspond to pair production at the second and first passage of the pump beam. Their relative phase $\Delta\phi$, which depends on the difference between the paths, is thus under control by the experimenter. The signal photon, the object system, is always measured by means of a two–channel polarization analyzer aligned at $+45^0$. Due to entanglement, the vertical or horizontal idler polarization supplies full *which way* information for the signal system, i.e., whether it was produced at the first or second passage. In this first experimental setup, where nothing is made to erase the polarization marks, no interference can be observed in the signal–idler joint detections. To erase this information, the idler photon has to be detected also in the $+45°/-45°$ basis. This is simply achieved in a second setup by changing the orientation of the half–wave plate in the meter path. Interference fringes or, more precisely, fringes and anti–fringes can then be observed in each one of the two channels when the relative phase $\Delta\phi$ is modified.

In the case of entangled kaons produced by ϕ resonance decays one starts with the state

$$|\phi(0)\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle_l|\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l|K^0\rangle_r\right] \simeq \frac{1}{\sqrt{2}}\left[|K_L\rangle_l|K_S\rangle_r - |K_S\rangle_l|K_L\rangle_r\right], \qquad (13)$$

[1]The authors of Ref. [9] used type–I crystals in their experiment but this doesn't affect the present discussion.

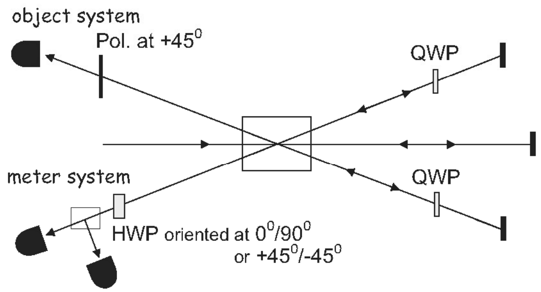


Figure 1: *Sketched setup for an active eraser. A pump beam transverses twice a non–linear crystal producing photon pairs by (type II) parametric down–conversion. The pairs produced in the first passage through the crystal (from left to right) cross two times twice a quarter–wave plate (QWP) which transforms an original horizontal polarized photon into a vertical one and vice versa. The pairs produced in the second passage through the crystal (from right to left) are superposed to the previous ones and directed to the measurement devices. The signal or object photon is always detected after crossing a polarization analyzer aligned at* $+45^\circ$. *The idler or meter photon crosses a half–wave plate (HWP) oriented at* $0^\circ, 90^\circ$ *(first setup) or at* $\pm 45^\circ$ *(second setup) and is then analyzed by a polarization beam splitter. In the first setup —the meter photon is measured in the* H/V *basis— one has full which way information, namely, one knows that the pair was produced at the first or second passage. In the second setup —the meter photon is measured in the* $+45^\circ/-45^\circ$ *basis— the information on the first or second passage is erased. One then observes fringes for one–half of the joint detections and the complementary (anti-)fringes for the other half.*

where the l and r subscripts denote the "left" and "right" directions of motion of the two separating kaons and, as before, CP–violating effects are neglected in the last equality. Kaons evolve in time in such a way that the relevant state turns out to depend on the two measurement times, t_l and t_r, on the left and the right hand side, respectively. More conveniently, this two–kaon state can be made to depend only on $\Delta t = t_l - t_r$ by normalizing to surviving kaon pairs[2]

$$\begin{aligned} |\phi(\Delta t)\rangle &= \frac{1}{\sqrt{1+e^{\Delta\Gamma\Delta t}}}\Big\{|K_L\rangle_l|K_S\rangle_r - e^{i\Delta m\Delta t}e^{\frac{1}{2}\Delta\Gamma\Delta t}|K_S\rangle_l|K_L\rangle_r\Big\} \qquad (14) \\ &= \frac{1}{2\sqrt{1+e^{\Delta\Gamma\Delta t}}}\Big\{\big(1-e^{i\Delta m\Delta t}e^{\frac{1}{2}\Delta\Gamma\Delta t}\big)\{|K^0\rangle_l|K^0\rangle_r - |\bar{K}^0\rangle_l|\bar{K}^0\rangle_r\} \\ &\qquad + \big(1+e^{i\Delta m\Delta t}e^{\frac{1}{2}\Delta\Gamma\Delta t}\big)\{|K^0\rangle_l|\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l|K^0\rangle_r\}\Big\} \end{aligned}$$

We note that the phase $\Delta m\Delta t$ introduces automatically a time dependent relative phase between the two amplitudes. Moreover, there is a complete analogy between the photonic state (12) and the two–kaon state written in the lifetime basis, first eq. (14).

The marking and erasure operations can be performed on entangled kaon pairs (14) as in the optical case discussed above. The object kaon flying to the left hand side is measured always *actively* in the strangeness basis, see Fig. 3(a). This active measurement is performed by placing the strangeness detector at different points of the left trajectory, thus searching for oscillations along a certain t_l range. As in the optical version, the kaon flying to the right hand side, the meter kaon, is always measured *actively* at a fixed time t_r^0. But one chooses to make this measurement either in the strangeness basis by placing a piece of matter in the beam or in the lifetime basis by removing the piece of matter. Both measurements are thus performed *actively*. In the latter case we obtain full information about the lifetime of the meter kaon and, thanks to the entanglement, *which width* the object kaon has. Consequently, no interference in the meter–object joint detections can be observed. This can be immediately seen from eq. (14) once the left and right kaon kets are written

[2]Thanks to this normalization, we work with bipartite two–level quantum systems like polarization entangled photons or entangled spin–1/2 particles. For an accurate description of the time evolution of kaons and its implementation consult Ref. [11)].

in the strangeness and lifetime bases, respectively. Indeed, one obtains

$$P\left[K^0(t_l), K_S(t_r)\right] = P\left[\bar{K}^0(t_l), K_S(t_r)\right] = \frac{1}{2\left(1+e^{\Delta\Gamma\Delta t}\right)}, \qquad (15)$$

$$P\left[K^0(t_l), K_L(t_r)\right] = P\left[\bar{K}^0(t_l), K_L(t_r)\right] = \frac{1}{2\left(1+e^{-\Delta\Gamma\Delta t}\right)}, \qquad (16)$$

showing no oscillations in time. But interferences are recovered by joint strangene measurements on both kaons. From the last eq. (14) one gets the following probabilities to observe like– or unlike–strangeness events on both sides

$$P\left[K^0(t_l), K^0(t_r)\right] = P\left[\bar{K}^0(t_l), \bar{K}^0(t_r)\right] = \frac{1}{4}\left[1 - \mathcal{V}(\Delta t)\cos(\Delta m\, \Delta t)\right], \quad (17)$$

$$P\left[K^0(t_l), \bar{K}^0(t_r)\right] = P\left[\bar{K}^0(t_l), K^0(t_r)\right] = \frac{1}{4}\left[1 + \mathcal{V}(\Delta t)\cos(\Delta m\, \Delta \tau)\right], \quad (18)$$

with a visibility

$$\mathcal{V}(\Delta t) = \frac{1}{\cosh(\Delta\Gamma\Delta t/2)}\,. \qquad (19)$$

(b) Partially passive quantum eraser with *active* measurements

In Fig. 2 a setup is sketched where an entangled photon pair is produced having a common origin in a region of points including, e.g., points A and B. The experiment, realized in Ref. [10] to which we refer for details, comprises a double slit affecting the right moving object photon and a series of static beam splitters and mirrors along the paths possibly followed by the meter photon. A look at Fig. 2 immediately shows that "clicks" on detector $D1$ or $D4$ provide "which way" information on this meter photon, which translates into the corresponding information for its entangled, object partner. Joint detection of these photon pairs shows therefore no interference. By contrast, "clicks" on detector $D2$ or $D3$, which require the cancelation of that "which way" information when the two possible paths coincide on the central beam splitter BS in Fig. 2, lead to the expected, complementary interference patterns for jointly detected two–photon events [10].

For neutral kaons, a piece of matter is permanently inserted into both beams. The one for the object kaon has to be moved along the left hand path in order to scan a certain t_l–range. The other strangeness detector for the meter system is fixed on the right hand path point corresponding to a fix t_r^0, see Fig. 3(b). The experimenter has to observe the region from the source to this piece of matter at the right hand side. In this way the kaon moving to the right —the meter system— takes the choice to show either "which width" information if it decays during its free propagation before t_r^0 or not. In this latter case, it can be absorbed at time t_r^0 by the piece of matter. Therefore the lifetime or strangeness of the meter kaon are measured *actively*, i.e., distinguishing prompt and late decay events or $S = \pm 1$ kaon–nucleon interactions in matter. The choice whether the "wave–like" property or the "particle–like" property is observed on the meter kaon is naturally given by the instability of the kaons. It is "partially active", because the experimenter can choose at which fixed time t_r^0 the piece of matter is inserted thus making more or less likely the measurement of lifetime or strangeness. This is analogous to the optical case where the experimenter can choose the transmittivity of the two beam–splitters BSA and BSB in Fig. 2. Note that it is not necessary to identify K_S versus K_L for demonstrating the quantum marking principle. The fact that this information is somehow available is enough to prevent any interference effects. These are recovered and oscillations reappear if this lifetime mark is erased and joint events are properly classified according to the measured strangeness of each kaon.

(c) Passive eraser with "*passive*" measurements on the meter

Next we consider the setup in Fig. 3(c). We take advantage —and this is specific for kaons— of the *passive* measurement. Again the strangeness content of the object system —the kaon moving to the left hand side— is *actively* measured by inserting a piece of matter into the beam and thus scanning a given t_l interval. In the beam of the meter no material piece is inserted and the kaon moving to the right propagates freely in space. This corresponds to a *passive* measurement of either strangeness or lifetime on the meter by recording the different decay *modes* of neutral kaons. If a semileptonic decay mode is found, the strangeness content is measured and the lifetime mark is erased. The distributions of the jointly detected events will show the characteristic interference fringes and antifringes. By contrast, if a $\pi\pi$ or a $\pi\pi\pi$ decay is observed, the lifetime is measured and thus "which width" information on the object system is obtained and no interference is seen in the joint events. Clearly we have a completely passive erasing operation on the meter, the experimenter has no control whether the lifetime mark is going to be read out or not.

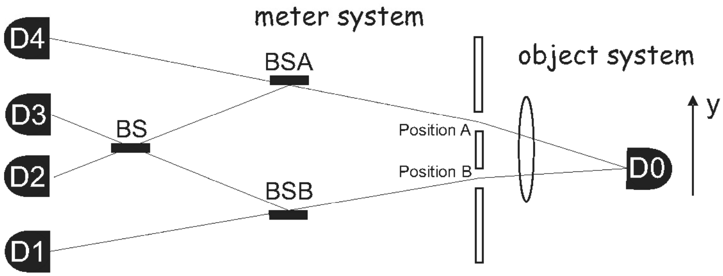


Figure 2: *Sketched setup of a 'partially active' quantum eraser. An entangled photon pair can be produced either in region A or in region B. If the detectors D1 or D4 click, one knows the production region A or B, i.e. one has full which way information also for its photon partner. Clicks of the detectors D2 or D3 cannot contain this information which has been erased at the central beam splitter. Interference is observed only in this latter case. It is a 'partially active' eraser, because the mark is erased by a probabilistic law, however, the experimenter has still partially control over the erasure, he can choose the ratio of transmittivity to reflectivity of the beam splitter BSA and BSB.*

This experiment is conceptually different from any other considered two–level quantum system.

(d) Passive eraser with "*passive*" measurements

Fig. 3(d) sketches a setup where both kaons evolve freely in space and the experimenter observes *passively* their decay modes and times. The experimenter has no control over individual pairs neither on which of the two complementary observables is measured on each kaon, nor when it is measured.

This setup is totally symmetric, thus it is not clear which side plays the role of the meter. In this sense, one could claim that this experiment should not be considered as a quantum eraser. But one could also claim that this experiment reveals the true essence of the erasure phenomenon: until the two measurements (one in each side) are completed, the factual situation is undefined; once one has the measurement results on both sides, the whole set of joint events can be classified in two subsets according to the kind of information (on strangeness or on lifetime) that has been obtained. The lifetime subset shows no interference, whereas fringes and antifringes appear when sorting the strangeness subset events according to the outcome, K^0 or $\bar{K}^0$, of the meter kaon.

Summarizing, we have discussed four experimental setups combining *active* and *passive* measurement procedures which lead to the same observable probabilities. This is even true regardless of the temporal ordering of the measurements, as follows immediately from the fact that the Δt–dependent functions in eq.(17), eq.(18) and in eq.(19), which govern the shape of the interference pattern, are even in this variable $\Delta t = t_l - t_r$. Thus kaonic erasers can also be operated in the "*delayed choice*" mode as already described in Ref. [18]. In our view this adds further light to the very nature of the quantum eraser working principle: the way in which joint detected events are classified according to the available information. In the 'delayed choice' mode, a series of strangeness measurements is performed at different times t_l on the object kaons and the corresponding outcomes are recorded. Later one can measure either lifetime or strangeness on the corresponding meter partner and, only now, full information allowing for a definite sorting of each pair is available. If we choose to perform strangeness measurements on the meter kaons and classify the joint events according the K^0 or $\bar{K}^0$ outcomes, we complete the information on each pair in such a way that oscillations and complementary anti–oscillations appear in the corresponding subsets. The alternative choice of lifetime measurements on meter kaons, instead, does not give the suitable information to classify the events in oscillatory subsets as before.

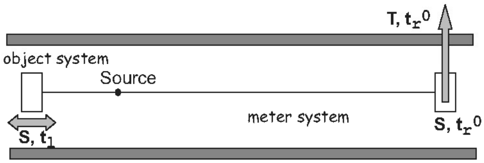
4 Conclusions

We have discussed the possibilities offered by neutral kaon states, such as those copiously produced by ϕ–resonance decays at the DAΦNE machine, to investigate two fundamental issues of quantum mechanics: quantitative Bohr's complementarity and quantum eraser phenomena. In both cases, the use of neutral kaons allows for a clear conceptual simplification and to obtain the relevant formulae in a transparent and non–controversial way.

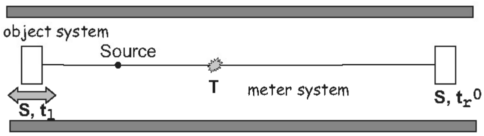
A key point is that neutral kaon propagation through the K_S and K_L components automatically parallels most of the effects of double slit devices. Thanks to this, Bohr's complementarity principle can be quantitatively discussed in the most simple and transparent way. Similarly, the relevant aspects of quantum marking and the quantum eraser admit a more clear treatment

Kaonic quantum erasers

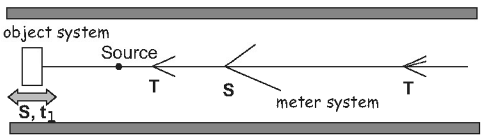
(a) Active eraser with *active* measurements (S: *active/active*; T: *active*)



(b) Partially active eraser with *active* measurements (S: *active/active*; T: *active*)



(c) Passive eraser with *passive* measurements on the meter (S: *active/passive*; T: *passive*)



(d) Passive eraser with *passive* measurements (S: *passive/passive*; T: *passive/passive*)

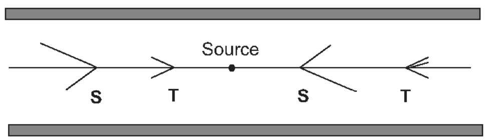


Figure 3: *The figure shows four different setups for a quantum marking and quantum erasing experiment. The first three, (a), (b) and (c), have the object system on the left hand side on which the strangeness is always actively measured at time t_l. The setups (a) and (b) are analogous to existing quantum eraser experiments with entangled photons, see Fig. 1 and Fig. 2. The setup (c) has no analog, because only for kaons a passive measurement is possible. For the last setup, (d), it is not so clear which side plays the meter/object role as it is totally symmetric and it involves only* passive *measurements.*

with neutral kaons than with other physical systems. This is particularly true when the eraser is operated in the 'delayed choice' mode and contributes to clarify the eraser's working principle. Moreover, the possibility of performing passive measurements, a specific feature of neutral kaons not shared by other systems, has been shown to open new options for the quantum eraser. In short, we have seen that, once the appropriate neutral kaon states are provided as in the DAΦNE machine, most of the additional requirements to investigate fundamental aspects of quantum mechanics are automatically offered by Nature for free.

The CPLEAR experiment [19] did only part of the job (*active* strangeness–strangeness measurements), but the KLOE 2 experiment could do the full program!

Acknowledgement: The authors thank the projects SGR–994, FIS2005–1369 and EURIDICE HPRN-CT-2002-00311. The latter allowed Beatrix Hiesmayr to work as a postdoc together with Albert Bramon and Gianni Garbarino in Barcelona, where the idea of the kaonic quantum eraser was born. We would also like to thank Antonio Di Domenico for inviting us to the very interesting Frascati–Workshop "Neutral kaon interferometry at a Φ–Factory: from Quantum Mechanics to Quantum Gravity".

References

1. R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics*, Vol. 3, (Addison-Wesley, 1965), p. 1-1, p. 11-20.
2. D. Greenberger and A. Yasin, Phys. Lett. A **128**, 391 (1988).
3. B.-G. Englert, Phys. Rev. Lett. **77**, 2154 (1996).
4. A. Bramon, G. Garbarino and B. C. Hiesmayr, Phys. Rev. A **69**, 022112 (2004).
5. B.C. Hiesmayr and V. Vedral, "*Interferometric wave-particle duality for thermodynamical systems*", quant-ph/0501015.
6. M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. Van der Zouw and A. Zeilinger, Nature **401**, 680682 (1999).

7. M. O. Scully and K. Drühl, Phys. Rev. A **25**, 2208 (1982).

8. S. Dürr and G. Rempe, Opt. Commun. **179**, 323 (2000).

9. T.J. Herzog, P.G. Kwiat, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett **75**, 3034 (1995).

10. Y.-H. Kim, R. Yu, S.P. Kuklik, Y. Shih and M.O. Scully, Phys. Rev. Lett. **84**, 1 (2000).

11. R.A. Bertlmann and B.C. Hiesmayr, Phys. Rev. A **63**, 062112 (2001).

12. T. Tsegaye, G. Björk, M. Atatüre, A.V. Sergienko, B.W.A. Saleh and M.C. Teich, Phys. Rev. A **62**, 032106 (2000).

13. S.P. Walborn, M.O. Terra Cunha, S. Padua and C.H. Monken, Phys. Rev. A **65**, 033818 (2002).

14. A. Trifonov, G. Björk, J. Söderholm and T. Tsegaye, Eur. Phys. J. D **18**, 251 (2002).

15. H. Kim, J. Ko and T. Kim, Phys. Rev. A **67**, 054102 (2003).

16. Y. Aharonov and M.S. Zubairy, Science **307**, 875 (2005).

17. A. Bramon, G. Garbarino and B. C. Hiesmayr, Phys. Rev. Lett. **92**, 020405 (2004).

18. A. Bramon, G. Garbarino and B. C. Hiesmayr, Phys. Rev. A **68**, 062111 (2004).

19. A. Apstolakis et.al., Phys. Lett. B **422**, 339 (1998).

KAON INTERFEROMETRY AT CPLEAR

Apollo Go
National Central University
Chung-Li, Taiwan

Abstract

The CPLEAR result of the possible loss of quantum coherence and the non-separability of the neutral kaon pair wave function are discussed. In addition, a new idea of testing Bell's Inequality with two regenerators at KLOE2 is proposed.

1 Introduction

Neutral kaon has several remarkable properties (strangeness oscillation, small mass difference between K_L and K_S, CP non-conservation etc) that provides a unique opportunity for testing fundamental physical laws. In this paper, we will summarise two important contribution of CPLEAR to these fundamental measurements: possible loss of quantum coherence and the non-separability of the neutral kaon pair's wave function.

2 CPLEAR experiment

The CPLEAR detector [1)] (fig.1) is located at the Low Energy Antiproton Ring (LEAR) at CERN. The continue and intense ($10^6 \bar{p}/s$) 200MeV antiproton beam is extracted from LEAR and stopped inside a 27-bar hydrogen gas target. A cylindrical tracking detector was located inside a solenoid (1m radius, 3.6m long) providing a 0.44T magnetic field parallel to the beam. It consisted of two layers of MWPCs (PC1, PC2), six layers of drift chambers and two layers of streamer tubes. A hodoscope of 32 threshold Cherenkov counters sandwiched between two swcintillator hodoscopes (S1, S2) provided charged particle identification (Cherenkov light, time of flight and energy loss). The cylindrical target (11mm radius) was surrounded by a small cylindrical proportional chamber PC0 (15mm radius, 1mm pitch, $> 99.5\%$ efficiency). A thin

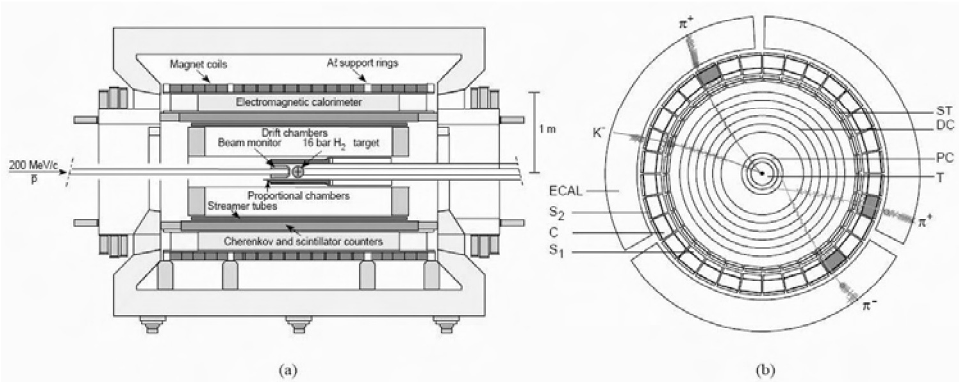


Figure 1: *CPLEAR detector: (a) longitudinal view, and (b) transverse view and display on an event* $\bar{p}p \to K^-\pi^+K^0$ *with the neutral kaon decaying into* $\pi^+\pi^-$.

silicon detector in front of the target entrance window ensured the presence of an incoming antiproton, thus rejecting background events resulting from interactions in the target support structure. A multi-level trigger optimized to select the initial K^0 or $\bar{K}^0$ via reactions

$$\bar{p}p \to K^-\pi^+K^0$$

$$\bar{p}p \to K^+\pi^-\bar{K}^0$$

(each having a branching ratio of $\approx 2 \times 10^{-3}$). By reconstructing the charged kaon and the opposite charged pion, the production point, the momentum and the initial strangeness of the neutral kaon can be measured. This detector was originally design to measure CP and T violation parameters in the neutral kaon system but it is capable of doing many other physics. The summary of the all physics output of CPLEAR experiment can be found elsewhere [2].

3 Loss of quantum coherence

The formalism of time evolution of the kaon normally used is according to a QM closed system description. Some approaches to quantum gravity [3] suggest that topologically non-trivial space-time fluctuations (space-time foam) entail an intrinsic, fundamental information loss, and therefore transitions from pure to mixed state [4]. The $K^0 - \bar{K}^0$ system is then described by a 2x2 density matrix ρ, which obeys

$$\dot{\rho} = -\mathrm{i}[\Lambda\rho - \rho\Lambda^\dagger] + \not{\delta}\Lambda\rho, \qquad (1)$$

where Λ is the time-dependent 2x2 matrix ($\Lambda = M - \frac{i}{2}\Gamma$, the mass and decay matrices) and the term $\not{\!\partial}\Lambda\rho$ induces a loss of quantum coherence in the observed system. In this context, the time evolution of the $K^0 - \bar{K}^0$ system allows for another 9 parameters, in addition to the usual seven. The CPLEAR experiment has measured 3 (α, β and γ) of the 9 parameters (the rest 6 are assumed to be zero). If different from zero, α, β and γ would point to a loss of coherence of the wave function (and also to CPT violation). The decay-rate asymmetries from the $\pi^+\pi^-$ decay channel,

$$A_{2\pi}(\tau) = \frac{N_{\bar{K}^0\to\pi^+\pi^-}(\tau) - N_{K^0\to\pi^+\pi^-}(\tau)}{N_{\bar{K}^0\to\pi^+\pi^-}(\tau) + N_{K^0\to\pi^+\pi^-}(\tau)}, \quad (2)$$

and for $e\pi\nu$ decay channel,

$$A_{\Delta m}(\tau) = \frac{[N_{\bar{K}^0\to e^-\pi^+\bar{\nu}}(\tau) + N_{K^0\to e^+\pi^-\nu}(\tau)] - [N_{\bar{K}^0\to e^+\pi^-\nu}(\tau) + N_{K^0\to e^-\pi^+\bar{\nu}}(\tau)]}{[N_{\bar{K}^0\to e^-\pi^+\bar{\nu}}(\tau) + N_{K^0\to e^+\pi^-\nu}(\tau)] + [N_{\bar{K}^0\to e^+\pi^-\nu}(\tau) + N_{K^0\to e^-\pi^+\bar{\nu}}(\tau)]}, \quad (3)$$

were fitted to data with the constrait of $|\eta_{+-}|$ and δ_l measured at long lifetimes (fig.2). It was obtained from the fit, as 90% CL limit [5]:

$$\alpha < 4.0 \times 10^{-17}\text{GeV},$$

$$\beta < 2.3 \times 10^{-19}\text{GeV},$$

$$\alpha < 3.7 \times 10^{-21}\text{GeV},$$

to be compared with a possible order of magnitude of

$$O(m_K^2/m_{\text{Planck}}) = 2 \times 10^{-20}\text{GeV}$$

for such effects. The result was consistent with no loss of quantum coherence.

4 Non-separability of the $K^0\bar{K}^0$ wave function

The strangeness of the pair of $K^0\bar{K}^0$ produced in the $\bar{p}p$ annihilation, $\bar{p}p \to K^0\bar{K}^0$ in $J^{PC} = 1^{--}$ state (92.6% of the case in CPLEAR [6]) is entangled:

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle_a \otimes |\bar{K}^0\rangle_b - |\bar{K}^0\rangle_a \otimes |K^0\rangle_b], \quad (4)$$

This is analogous [7] to the polarization in a two-photon system, more commonly used in EPR-type experiments [8]. Even though due to $K^0 - \bar{K}^0$ oscillations the individual kaon's strangeness varies with time, the measurement of the

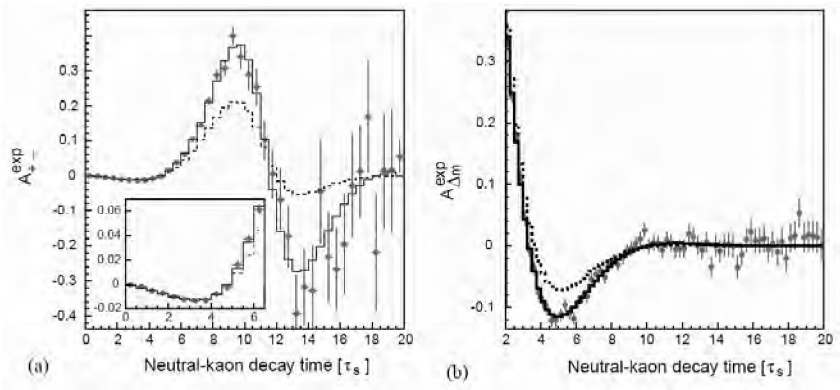


Figure 2: *The measured decay-rate asymmetries (a) $A_{2\pi}$ and (b) $A_{\Delta m}$ analysed for a possible loss of coherence. The solid lines are the result of the fit. The dashed lines represent the expected asymmetries with positive values of α, β, γ, which are 10 times larger than the limits obtained.*

strangeness of one kaon at a given time predicts with certainty the strangeness state of the other unmeasured kaon at equal proper time. There is a perfect strangeness anti-correlation at a distance. The expected QM intensities for the like-strangeness (K^0K^0 or $\bar{K}^0\bar{K}^0$) and unlike-strangeness ($K^0\bar{K}^0$ or $\bar{K}^0K^0$) final states a and b, observed at times t_a and t_b respectively are:

$$I_{like}(t_a, t_b) = \frac{1}{8}[e^{-i\gamma_L t_a - i\gamma_S t_b} + e^{-i\gamma_S t_a - i\gamma_L t_b} - 2e^{-\frac{\gamma_S+\gamma_L}{2}(t_a+t_b)}\cos(\Delta m \Delta t)], \tag{5}$$

$$I_{unlike}(t_a, t_b) = \frac{1}{8}[e^{-i\gamma_L t_a - i\gamma_S t_b} + e^{-i\gamma_S t_a - i\gamma_L t_b} + 2e^{-\frac{\gamma_S+\gamma_L}{2}(t_a+t_b)}\cos(\Delta m \Delta t)] \tag{6}$$

where $\Delta m = m_L - m_S$ and $\Delta t = t_a - t_b$. They are shown in fig. 3a. For an experiment, it is easier to measure the asymmetry:

$$A_{(ta,tb)} = \frac{I_{unlike}(t_a, t_b) - I_{like}(t_a, t_b)}{I_{unlike}(t_a, t_b) + I_{like}(t_a, t_b)} = \frac{2e^{-\frac{\gamma_S+\gamma_L}{2}(t_a+t_b)}\cos(\Delta m \Delta t)}{e^{-i\gamma_L t_a - i\gamma_S t_b} + e^{-i\gamma_S t_a - i\gamma_L t_b}}. \tag{7}$$

as shown in fig. 3b.

The strangeness was identified by product of the strong interaction in two absorbers near the target, fig. 4a, via the observation in the same event, at two different times, of a Λ and a K^+ (unlike strangeness) or a Λ and a K^- (like strangeness) [9]. The asymmetries for unlike- and like-strangeness

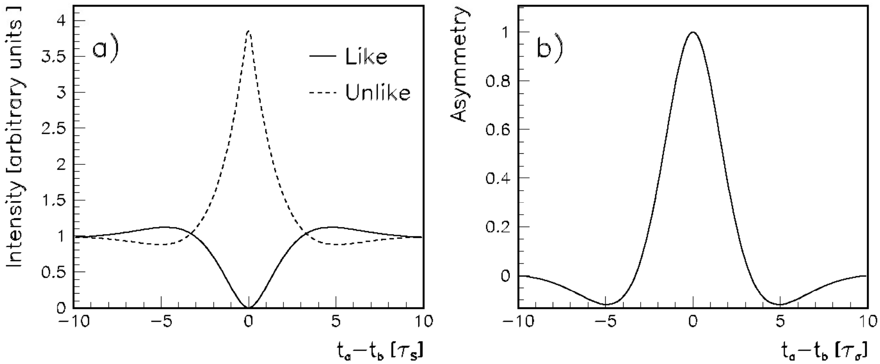


Figure 3: QM prediction for (a) intensity of the like- and unlike-strangeness as a function of Δt and (b) the asymmetry

events (ΛK^+ and ΛK^-) were measured for two experimental configurations C(0) and C(5) (fig.4b) corresponding to $\Delta t \approx 0$ and $\Delta t \approx 1.2\tau_S$ proper time differences between the two strangeness measurements, or path difference Δl of ≈ 0 and 5cm. As shown in fig. 5, these asymmetries are consistent with the values predicted from QM, and therefore consistent with the non-separability hypothesis of the $K^0\bar{K}^0$ wave funtion. The non-separability hypothesis is also strongly favoured by the yield of $\Lambda\Lambda$ events. The probability of satisfying the separability hypothesis of Furry is $< 10^{-4}$.

If after the production of the $K^0\bar{K}^0$ pair, a spontaneous decoherence takes place, i.e. a fraction of the two neutral kaon are separated and evolve independently, then the asymmetry above would be different. This can be parametrized by a factor $(1 - \xi)$, which multiphy the QM interference term in equations (5) and (6). This decoherence could happen either in the $K_L - K_S$ basis or in the $K^0 - \bar{K}^0$ basis. Bertlmann, Grimus and Hiesmayr [10] has measured this decoherence based on the CPLEAR result (fig. 5) and the result is $0.13^{+0.16}_{-0.15}$ and 0.4 ± 0.7 respectively. An improved measurement was done at KLOE [11].

5 Testing Bell Inequality at KLOE2

Having shown that the neutral kaon pair wave function is entangled, a more interesting measurement would be to test the Bell Inequality [12]. However,

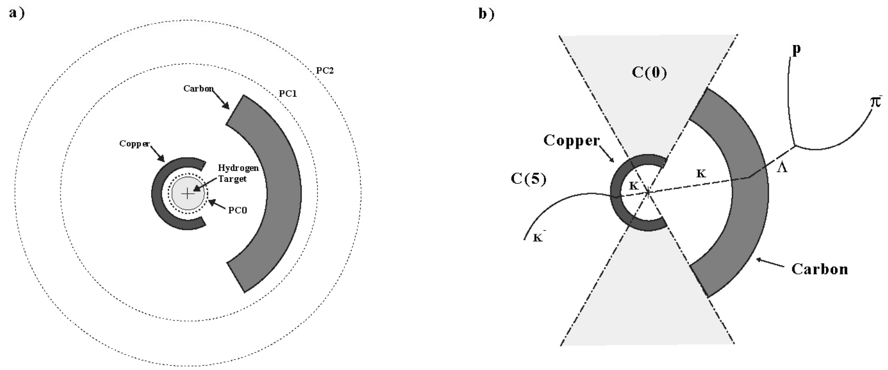


Figure 4: (a) Central region of the CPLEAR detector with the two absorbers and (b) conceptual sketch of the experiment with a ΛK event.

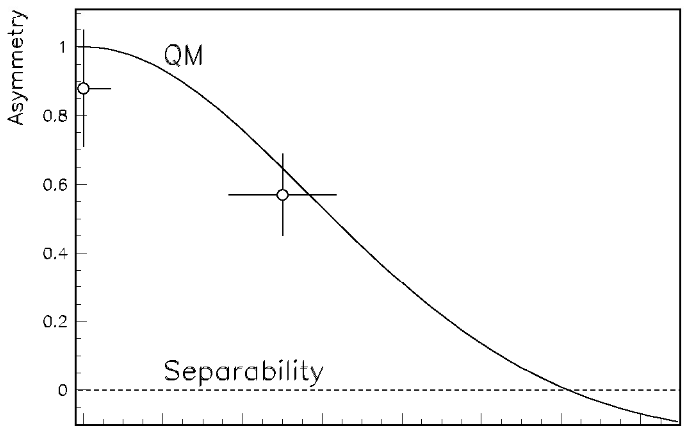


Figure 5: Asymmetry of the measured $\Lambda K^{\pm}$ for the two experimental configurations. The solid curve is the QM prediction. The dashed line is the prediction for a separate wave function hypothesis by Furry.

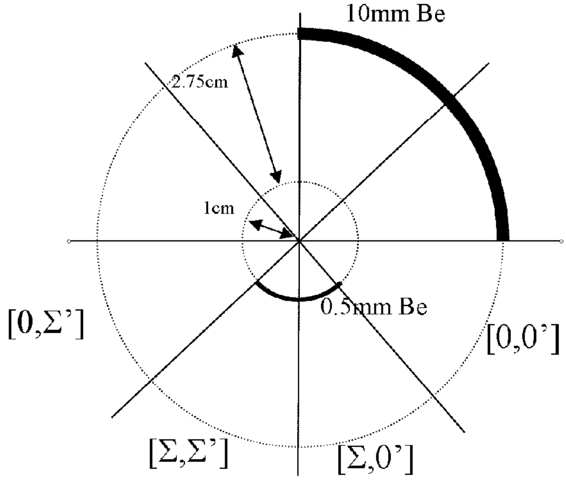


Figure 6: Tranverse view of the proposed two thin regenerator position for the 4 possible configurations to be placed near interation point at KLOE2 detector.

it was proven that due to the rapid decay of the kaon wavefunction, it is not possible to violate Bell Inequality in the $K^0\bar{K}^0$ maximally entangle state [13]. Nevertherless, Bell Inequality can be violated in the non-maximally entangled state [14]. Coherent regeneration in a thin material could be used to create such state from the initial $K^0\bar{K}^0$ pair.

Following the idea by Eberhard [15], originally proposed for asymmetric kaon factory, 4 set-ups with 2 regenerators are used. Translating into a symmetric machine, we proposse to have two partial rings of regenerators which correspond to 4 possible configuration: [0,0'], [Σ,0'], [0,Σ'] and [Σ,Σ'] (fig. 6). Measuring 4 K_LK_L probabilities (by the interation in the calorimeter), the Wigner's form of Bell Inequality can be used:

$$\mathrm{P_{K_L,K_L}}(\Sigma,\Sigma') \geq \mathrm{P_{K_L,K_L}}(0,0') + \mathrm{P_{K_L,K_L}}(\Sigma,0') + \mathrm{P_{K_L,K_L}}(0,\Sigma'). \qquad (8)$$

Back of envelop calculations using the regeneration parameters from Di Domenicc show that for configuraion [Σ,0'] and [0,Σ'], a statistics of 114 events/fb^{-1} and 100 events/fb^{-1} respectively can be achieved at KLOE2. Alternatively, one can also measure K_SK_S decays (into $\pi^+\pi^-$), this gives more statistics of 218 events/fb^{-1} and 346 events/fb^{-1}. Therefore, only around $<$ 5fb^{-1} of data are needed, certainly feasible at the new proposed KLOE2 detector with minor modification of introducing two regenerators.

6 Conclusion

Neutral Kaon is a rich system to testing fundamental QM issues. Here we have presented the test of coherence loss due to quantum gravity and a test of EPR entangled kaon pair. A further test of Bell Inequality should be possible at KLOE2.

References

1. R. Adler *et al.*, CPLEAR Collaboration, Nucl. Instr. Meth. **A379** 76 (1996).
2. A. Angelopoulos *et al.*, CPLEAR Collaboration, Physics Reports **374** 165 (2003).
3. S. Hawking, Commun. Math. Phys. **87**, 395 (1982).
4. J. Ellis, J.S. Hagelin, D.V. Nanopoulos, M. Srednicki, Nucl. Phys. **B241**, 381 (1984).
5. R. Adler *et al.*, CPLEAR Collaboration, Phys. Lett. **B364**, 239 (1995).
6. R. Adler *et al.*, CPLEAR Collaboration, Phys. Lett. **B403**, 383 (1997).
7. N. Gisin and A. Go, Am. J. Phys. **69**, 264 (2001).
8. A. Einstein, B. Podolski and N. Rosen, Phys. Rev. **47**, 777 (1935).
9. A. Apostolakis *et al.*, CPLEAR Collaboration, Phys. Lett. **B422**, 339 (1998).
10. R.A. Bertlmann, W. Grimus and B.C. Hiesmayr, Phys. Rev. **D60**, 114032 (1998).
11. A. Di Domenico, Contribution to this handbook.
12. J.S. Bell, Physics, **1**, 195 (1964).
13. G.C. Ghiraldi, R. Grassi and R. Ragazzon (ed. L. Maiani *et al.* 1992), DAΦNE Physics Handbook, LNF, Frascati, 283 (1992).
14. B.C. Hiesmayr, Eur. Phys. J. **C50** (2007) 73.
15. P. Eberhard, Nucl. Phys. **B398**, 155 (1993).
16. A. Di Domenico, Nucl. Phys. **B450**, 293 (1995).

FRASCATI PHYSICS SERIES VOLUMES

Volume I
Heavy Quarks at Fixed Target
Eds.: S. Bianco and F.L. Fabbri
Frascati, May 31–June 2, 1993
ISBN—88–86409–00–1

Volume II – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste – Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, March 5 –11, 1995
ISBN—88–86409–03–6

Volume III
Heavy Quarks at Fixed Target
Ed.: B. Cox
University of Virginia, Charlottesville
October 7–10, 1994, 11
ISBN—88–86409–04–4

Volume IV
Workshop on Physics and Detectors for DAΦNE
Eds.: R. Baldini, F. Bossi, G. Capon, G. Pancheri
Frascati, April 4–7, 1995
ISBN—88–86409–05–2

Volume V – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste – Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, March 3–9, 1996
ISBN—88–86409–07–9

Volume VI
Calorimetry in High Energy Physics
Eds.: A. Antonelli, S. Bianco, A. Calcaterra, F.L. Fabbri
Frascati, June 8–14, 1996
ISBN—88–86409–10–9

Volume VII
Heavy Quarks at Fixed Target
Ed.: L. Kőpke
Rhinefels Castle, St. Goar, October 3–6, 1996
ISBN—88–86409–11–7

Volume VIII
ADONE a milestone on the particle way
Ed.: V. Valente 1997
ISBN—88–86409–12–5

Volume IX – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste – Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, March 2–8, 1997
ISBN–88–86409–13–3

Volume X
Advanced ICFA Beam Dynamics Workshop on Beam Dynamics Issue for e^+e^- Factories
Eds.: L. Palumbo, G. Vignola
Frascati, October 20–25, 1997
ISBN—88–86409–14–1

Volume XI
Proceedings of the XVIII International Conference on Physics in Collision
Eds.: S. Bianco, A. Calcaterra, P. De Simone, F. L. Fabbri
Frascati, June 17–19, 1998
ISBN—88–86409–15–X

Volume XII – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste – Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, March 1–7, 1998
ISBN—88–86409–16–8

Volume XIII
Bruno Touschek and the Birth of thee$^{+}e^{-}$
Ed.: G. Isidori
Frascati, 16 November, 1998
ISBN—88–86409–17–6

Volume XIV – **Special Issue**
Les Rencontres de Physique de la Vallée d'Aoste –
Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, February 28–March 6, 1999
ISBN—88–86409–18–4

Volume XV
Workshop on Hadron Spectroscopy
Eds.: T. Bressani, A. Feliciello, A. Filippi
Frascati, March 8 2 1999
ISBN—88–86409–19–2

Volume XVI
Physics and Detectors for DAΦNE
Eds.: S. Bianco, F. Bossi, G. Capon, F.L. Fabbri,
P. Gianotti, G. Isidori, F. Murtas
Frascati, November 16 –19, 1999
ISBN—88–86409–21–4

Volume XVII – **Special Issue**
Les Rencontres de Physique de la Vallée d'Aoste –
Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, February 27 March 4, 2000
ISBN—88–86409–23–0

Volume XVIII
LNF Spring School
Ed.: G. Pancheri
Frascati 15–20 May, 2000
ISBN—88–86409–24–9

Volume XIX
XX Physics in Collision
Ed.: G. Barreira
Lisbon June 29–July1st. 2000
ISBN—88–86409–25–7

Volume XX
Heavy Quarks at Fixed Target
Eds.: I. Bediaga, J. Miranda, A. Reis
Rio de Janeiro, Brasil, October 9–12, 2000
ISBN—88–86409–26–5

Volume XXI
IX International Conference on Calorimetry in High Energy Physics
Eds.: B. Aubert, J. Colas, P. Nédélec, L. Poggioli
Annecy Le Vieux Cedex, France, October 9–14, 2000
ISBN—88–86409–27–3

Volume XXII – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste – Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, March 4–10, 2001
ISBN—88–86409–28–1

Volume XXIII
XXI Physics in Collision
Ed.: Soo–Bong Kim
Seoul, Korea, June 28 30, 2001
ISBN—88–86409–30–3

Volume XXIV
International School of Space Science – 2001 Course on: Astroparticle and Gamma–ray Physics in Space
Eds.: A. Morselli, P. Picozza
L'Aquila, Italy, August 30 September 7, 2000
ISBN—88–86409–31–1

Volume XXV
TRDs for the 3rd Millennium Workshop on Advanced Transition Radiation Detectors for Accelerator and Space Applications
Eds. N. Giglietto, P. Spinelli
Bari, Italy, September 20–23, 2001
ISBN—88–86409–32–X

Volume XXVI
KAON 2001 International Conference on CP Violation
Eds.: F. Costantini, G. Isidori, M. Sozzi
Pisa Italy, June 12th 17th, 2001
ISBN—88–86409–33–8

Volume XXVII – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste – Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, March 3–9, 2002
ISBN—88–86409–34–6

Volume XXVIII
Heavy Quarks at Leptons 2002
Eds.: G. Cataldi, F. Grancagnolo, R. Perrino, S. Spagnolo
Vietri sul mare (Italy), May 27th June 1st, 2002
ISBN—88–86409–35–4

Volume XXIX
Workshop on Radiation Dosimetry: Basic Technologies, Medical Applications, Environmental Applications
Ed.: A. Zanini
Rome (Italy), February 56, 2002
ISBN—88–86409–36–2

Volume XXIX – Suppl.
Workshop on Radiation Dosimetry: Basic Technologies, Medical Applications, Environmental Applications
Ed.: A. Zanini
Rome (Italy), February 56, 2002
ISBN—88–86409–36–2

Volume XXX – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste –
Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, March 9–15, 2003
ISBN—88–86409–39–9

Volume XXXI
Frontier Science 2002 – Charm, Beauty and CP,
First International Workshop on Frontier Science
Eds.: L. Benussi, R. de Sangro, F.L. Fabbri, P. Valente
Frascati, October 6–11, 2002
ISBN—88–86409–37–0

Volume XXXII
19th International Conference on x–ray and Inner–Shell Processes
Eds.: A. Bianconi, A. Marcelli, N.L. Saini
Universitá di Roma La Sapienza June 24–28, 2002
ISBN—88–86409–39–07

Volume XXXIII
Bruno Touschek Memorial Lectures
Ed.: M. Greco, G. Pancheri
Frascati, May 11, 1987
ISBN—88–86409–40–0

Volume XXXIV – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste –
Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, February 29 – March 6, 2004
ISBN—88–86409–42–7

Volume XXXV
Heavy Quarks And Leptons 2004
Ed.: A. López
San Juan, Puerto Rico, 1–5 June 2004
ISBN—88–86409–43–5

Volume XXXVI
DAΦNE 2004: Physics At Meson Factories
Eds.: F. Anulli, M. Bertani, G. Capon, C. Curceanu–Petrascu, F.L. Fabbri, S. Miscetti
Frascati, June 7–11, 2004
ISBN—88–86409–53–2

Volume XXXVII
Frontier Science 2004, Physics and Astrophysics in Space
Eds.: A. Morselli, P. Picozza, M. Ricci
Frascati, 14–19 June, 2004
ISBN—88–86409–52–4

Volume XXXVIII
II Workshop Italiano sulla Fisica di ATLAS e CMS
Eds.: Gianpaolo Carlino and Pierluigi Paolucci
Napoli, October 13 – 15, 2004
ISBN—88–86409–44–3

Volume XXXIX – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste – Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, February 27 – March 5, 2005
ISBN—88–86409–45–1

Volume XL
Frontier Science 2005 – New Frontiers in Subnuclear Physics
Eds.: A. Pullia, M. Paganoni
Milano, September 12 - 17, 2005
ISBN—88–86409–46–X

Volume XLI
Discoveries in Flavour Physics at e^+e^- Colliders
Eds.: L. Benussi, S. Bianco, C. Bloise, R. de Sangro, C. Gatti, G. Isidori, M. Martini, F. Mescia, S. Miscetti
Frascati, February 28th - March 3rd, 2006
ISBN—88–86409–51–6

Volume XLII – Special Issue
Les Rencontres de Physique de la Vallée d'Aoste –
Results and Perspectives in Particle Physics
Ed.: M. Greco
La Thuile, Aosta Valley, March 5 March 11, 2006
ISBN—88–86409–47–8