On the gluon content of the $\eta$ and $\eta'$ mesons

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Purpose: to perform a phenomenological analysis of radiative $V \rightarrow P \gamma$ and $P \rightarrow V \gamma$ decays, with $V=\rho, K^*, \omega, \phi$ and $P=\pi, K, \eta, \eta'$, aimed at determining the gluonic content of the $\eta$ and $\eta'$ wave functions.

Outline:

- Notation
- Motivation
- A model for $V P \gamma$ M1 transitions
- Data fitting
- Comparison with other approaches
- Summary and conclusions
• **Notation**

We work in a basis consisting of the states

\[
|\eta_q\rangle \equiv \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle \quad |\eta_s\rangle = |s\bar{s}\rangle \quad |G\rangle \equiv |\text{gluonium}\rangle
\]

The physical states \(\eta\) and \(\eta'\) are assumed to be the linear combinations

\[
|\eta\rangle = X_\eta |\eta_q\rangle + Y_\eta |\eta_s\rangle + Z_\eta |G\rangle,
\]

\[
|\eta'\rangle = X_{\eta'} |\eta_q\rangle + Y_{\eta'} |\eta_s\rangle + Z_{\eta'} |G\rangle,
\]

with 

\[
X_{\eta(\eta')}^2 + Y_{\eta(\eta')}^2 + Z_{\eta(\eta')}^2 = 1 \quad \text{and thus} \quad X_{\eta(\eta')}^2 + Y_{\eta(\eta')}^2 \leq 1
\]

A significant gluonic admixture in a state is possible only if

\[
Z_{\eta(\eta')}^2 = 1 - X_{\eta(\eta')}^2 - Y_{\eta(\eta')}^2 > 0
\]

**Assumptions:**
- no mixing with \(\pi^0\) (isospin symmetry)
- no mixing with \(\eta_c\) states
- no mixing with radial excitations
• **Notation**

In absence of gluonium

\[ Z_{\eta(\eta')} \equiv 0 \]

\[ |\eta\rangle = \cos \phi_P |\eta_q\rangle - \sin \phi_P |\eta_s\rangle \]
\[ |\eta'\rangle = \sin \phi_P ||\eta_q\rangle + \cos \phi_P |\eta_s\rangle \]

with

\[ X_\eta = Y_{\eta'} \equiv \cos \phi_P \]
\[ X_{\eta'} = -Y_\eta \equiv \sin \phi_P \]

and

\[ X_{\eta(\eta')}^2 + Y_{\eta(\eta')}^2 = 1 \]

where \( \phi_P \) is the \( \eta-\eta' \) mixing angle in the quark-flavour basis related to its octet-singlet analog through

\[ \theta_P = \phi_P - \arctan \sqrt{2} \simeq \phi_P - 54.7^\circ \]

Similarly, for the vector states \( \omega \) and \( \phi \) the mixing is given by

\[ |\omega\rangle = \cos \phi_V |\omega_q\rangle - \sin \phi_V |\phi_s\rangle \]
\[ |\phi\rangle = \sin \phi_V |\omega_q\rangle + \cos \phi_V |\phi_s\rangle \]

where \( \omega_q \) and \( \phi_s \) are the analog non-strange and strange states of \( \eta_q \) and \( \eta_s \), respectively.
• Motivation

KLOE Collaboration, hep-ex/0612029

\[ \phi_P = (39.7 \pm 0.7)^\circ \]

\[ Z_{\eta'}^2 = 0.14 \pm 0.04 \]

\[ \begin{align*}
(1) \quad & \Gamma(\eta \rightarrow \gamma \gamma)/\Gamma(\omega \rightarrow \pi^0 \gamma) \\
(2) \quad & \Gamma(\eta \rightarrow \gamma \rho)/\Gamma(\pi^0 \gamma) \\
(3) \quad & \Gamma(\eta \rightarrow \omega \gamma)/\Gamma(\omega \rightarrow \pi^0 \gamma)
\end{align*} \]

\[ R_\phi \text{ with } Z^2=0 \]

\[ Y_1 = \eta' \rightarrow \gamma \gamma/\pi^0 \rightarrow \gamma \gamma \]

\[ Y_2 = \eta' \rightarrow \rho \gamma/\omega \rightarrow \pi^0 \gamma \]

\[ Y_3 = \phi \rightarrow \eta' \gamma/\phi \rightarrow \eta \gamma \]

\[ Y_4 = \eta' \rightarrow \omega \gamma/\omega \rightarrow \pi^0 \gamma \]
**Motivation**

KLOE Collaboration, PLB 541 (2002), 45

\[ Z_{\eta'}^2 = 0.06^{+0.09}_{-0.06} \]

Gluonium fraction below 15%

What are the **differences** between the two analyses?

- **improvement** in the **precision** of the new measurements
- the **use** of the **overlapping parameters** relating the **pseudoscalar** and **vector wave functions**
A model for $\text{VP}_\gamma$ $M_1$ transitions

We will work in a conventional quark model context: $P$ and $V$ are simple quark-antiquark $S$-wave bound states. All these hadrons are thus extended objects with characteristics spatial extensions fixed by their respective $P$ and $V$ wave functions.

- SU(2) limit: identical spatial extension within each isomultiplet
- SU(3) broken: constituent quark masses with $m_s > m$ and different spatial extensions for each isomultiplet

Ingredients of the model:

i) a $\text{VP}_\gamma$ magnetic dipole transition proceeding via quark or antiquark spin flip amplitude $\propto \mu_q = e_q/2m_q$

ii) spin-flip $V \rightarrow P$ conversion amplitude corrected by the relative overlap between the $P$ and $V$ wave functions.

iii) OZI-rule reduces considerably the possible transitions and overlaps

$$C_\pi \equiv \langle \pi | \omega_q \rangle = \langle \pi | \rho \rangle \quad C_K \equiv \langle K | K^* \rangle$$

$$C_q \equiv \langle \eta_q | \omega_q \rangle = \langle \eta_q | \rho \rangle \quad C_s \equiv \langle \eta_s | \phi_s \rangle$$

$U(1)_A$ anomaly
• A model for $VP\gamma$ M1 transitions

Amplitudes:

\[
\begin{align*}
    g_{\rho^0\pi^0\gamma} &= g_{\rho^+\pi^+\gamma} = \frac{1}{3}g, &
    g_{\omega\pi\gamma} &= g \cos \phi_V, &
    g_{\phi\pi\gamma} &= g \sin \phi_V, \\
    g_{K^*0K^0\gamma} &= -\frac{1}{3}g z_K \left(1 + \frac{\bar{m}_s}{m_s}\right), &
    g_{K^*+K^+\gamma} &= \frac{1}{3}g z_K \left(2 - \frac{\bar{m}_s}{m_s}\right), \\
    g_{\rho\eta\gamma} &= g z_q X_\eta, &
    g_{\rho\eta'\gamma} &= g z_q X'_\eta, \\
    g_{\omega\eta\gamma} &= \frac{1}{3}g \left(z_q X_\eta \cos \phi_V + 2 \frac{\bar{m}}{m_s} z_s Y_\eta \sin \phi_V\right), \\
    g_{\omega\eta'\gamma} &= \frac{1}{3}g \left(z_q X'_\eta \cos \phi_V + 2 \frac{\bar{m}}{m_s} z_s Y'_\eta \sin \phi_V\right), \\
    g_{\phi\eta\gamma} &= \frac{1}{3}g \left(z_q X_\eta \sin \phi_V - 2 \frac{\bar{m}}{m_s} z_s Y_\eta \cos \phi_V\right), \\
    g_{\phi\eta'\gamma} &= \frac{1}{3}g \left(z_q X'_\eta \sin \phi_V - 2 \frac{\bar{m}}{m_s} z_s Y'_\eta \cos \phi_V\right),
\end{align*}
\]

with \( g_{\omega\pi\gamma} = g \cos \phi_V = e C_\pi \cos \phi_V / \bar{m} \)
and \( z_q \equiv C_q / C_\pi, \quad z_s \equiv C_s / C_\pi, \quad z_K \equiv C_K / C_\pi \)

\[
\Gamma(V \to P\gamma) = \frac{1}{3} \frac{g_{V\gamma}^2}{4\pi} |P\gamma|^3 = \frac{1}{3} \Gamma(P \to V\gamma)
\]
Data fitting

The overlapping parameters $z_q, s$ and the mixing parameters $X_{\eta(\eta')}$ and $Y_{\eta(\eta')}$ cannot be determined independently.

Thus we start assuming $C_q = C_s = C_K = C_\pi = 1$, $z_q = z_s = z_K = 1$.

$\chi^2$/d.o.f. = 28.3/7 $\to$ 16.6/5

Then we leave the overlapping parameters free.

$g = 0.72 \pm 0.01$ GeV$^{-1}$, $\phi_V = (3.2 \pm 0.1)^\circ$,

$\frac{m_s}{m} = 1.24 \pm 0.07$, $z_K = 0.89 \pm 0.03$,

$z_q X_\eta = 0.65 \pm 0.02$, $z_q X_{\eta'} = 0.56 \pm 0.04$,

$z_s Y_\eta = -0.52 \pm 0.03$, $z_s Y_{\eta'} = 0.58 \pm 0.05$.

Then we fix the mixing parameters to the standard picture (no gluonium).

$g = 0.72 \pm 0.01$ GeV$^{-1}$, $\phi_P = (41.5 \pm 1.2)^\circ$, $\phi_V = (3.2 \pm 0.1)^\circ$,

$\frac{m_s}{m} = 1.24 \pm 0.07$, $z_K = 0.89 \pm 0.03$, $z_q = 0.86 \pm 0.03$, $z_s = 0.78 \pm 0.05$.

$\chi^2$/d.o.f. = 4.2/4

For comparison, if $z_q = z_s = z_K = 1$ then $\chi^2$/d.o.f. = 45.9/8.
Data fitting

Fixing the overlapping parameters to the values obtained under the hypothesis of no gluonium we get

\[ g = 0.72 \pm 0.01 \text{ GeV}^{-1}, \quad \phi_V = (3.2 \pm 0.1)\circ, \]
\[ \frac{m_\pi}{m} = 1.24 \pm 0.07, \quad \zeta_K = 0.89 \pm 0.03, \]
\[ X_\eta = 0.75 \pm 0.03, \quad X_{\eta'} = 0.65 \pm 0.04, \]
\[ Y_\eta = -0.66 \pm 0.04, \quad Y_{\eta'} = 0.74 \pm 0.06, \]

\[ Z^2_\eta = 0.00 \pm 0.07 \quad \text{and} \quad Z^2_{\eta'} = 0.04 \pm 0.10 \]

\[ |Z_{\eta,(\eta')}| = \sqrt{1 - X^2_{\eta,(\eta')} - Y^2_{\eta,(\eta')}} \]
\[ Y_\eta = -\frac{X_\eta X_{\eta'} Y_\eta' + \sqrt{(1 - X^2_{\eta'} - Y^2_{\eta'})(1 - X^2_\eta - X^2_{\eta'})}}{1 - X^2_{\eta'}} \]

\[ \chi^2/\text{d.o.f.} = 4.2/5 \]

The current experimental data on VP\eta transitions indicate within our model a negligible gluonic content for the \eta and \eta' mesons.
• **Data fitting**

Assuming $Z_\eta=0$ from the beginning, we get

$$g = 0.72 \pm 0.01 \text{ GeV}^{-1}, \quad \frac{m_s}{\bar{m}} = 1.24 \pm 0.07, \quad \phi_V = (3.2 \pm 0.1)^\circ,$$

$$\phi_P = (41.4 \pm 1.3)^\circ, \quad |\phi_{\eta'G}| = (12 \pm 13)^\circ,$$

$$z_K = 0.89 \pm 0.03, \quad z_q = 0.86 \pm 0.03, \quad z_s = 0.79 \pm 0.05,$$

\[\chi^2/\text{d.o.f.}=4.2/4\]

Accepting the absence of gluonium for the $\eta$ meson, the gluonic content of the $\eta'$ wave function amounts to $|\phi_{\eta'G}|=(12\pm13)^\circ$ or $(Z_\eta')^2=0.04\pm0.09$ and the $\eta$-$\eta'$ mixing angle is found to be $\phi_P=(41.4\pm1.3)^\circ$.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$g_{VP\gamma}^{\text{exp}}$(PDG)</th>
<th>$g_{VP\gamma}^{\text{th}}$(Fit 1)</th>
<th>$g_{VP\gamma}^{\text{th}}$(Fit 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^0 \rightarrow \eta\gamma$</td>
<td>$0.475 \pm 0.024$</td>
<td>$0.461 \pm 0.019$</td>
<td>$0.464 \pm 0.030$</td>
</tr>
<tr>
<td>$\eta' \rightarrow \rho^0\gamma$</td>
<td>$0.41 \pm 0.03$</td>
<td>$0.41 \pm 0.02$</td>
<td>$0.40 \pm 0.04$</td>
</tr>
<tr>
<td>$\omega \rightarrow \eta\gamma$</td>
<td>$0.140 \pm 0.007$</td>
<td>$0.142 \pm 0.007$</td>
<td>$0.143 \pm 0.010$</td>
</tr>
<tr>
<td>$\eta' \rightarrow \omega\gamma$</td>
<td>$0.139 \pm 0.015$</td>
<td>$0.149 \pm 0.006$</td>
<td>$0.146 \pm 0.014$</td>
</tr>
<tr>
<td>$\phi \rightarrow \eta\gamma$</td>
<td>$0.209 \pm 0.002$</td>
<td>$0.209 \pm 0.018$</td>
<td>$0.209 \pm 0.013$</td>
</tr>
<tr>
<td>$\phi \rightarrow \eta'\gamma$</td>
<td>$0.22 \pm 0.01$</td>
<td>$0.22 \pm 0.02$</td>
<td>$0.22 \pm 0.02$</td>
</tr>
</tbody>
</table>
**Euler angles**

In presence of gluonium,

\[
|\eta\rangle = X_\eta |\eta_q\rangle + Y_\eta |\eta_s\rangle + Z_\eta |G\rangle
\]

\[
|\eta'\rangle = X_{\eta'} |\eta_q\rangle + Y_{\eta'} |\eta_s\rangle + Z_{\eta'} |G\rangle
\]

\[
|\iota\rangle = X_\iota |\eta_q\rangle + Y_\iota |\eta_s\rangle + Z_\iota |G\rangle
\]

Glueball-like state \(\eta(1440)\)

Normalization:

\[
X_\eta^2 + Y_\eta^2 + Z_\eta^2 = 1
\]

\[
X_{\eta'}^2 + Y_{\eta'}^2 + Z_{\eta'}^2 = 1
\]

\[
X_\iota^2 + Y_\iota^2 + Z_\iota^2 = 1
\]

Orthogonality:

\[
X_\eta X_{\eta'} + Y_\eta Y_{\eta'} + Z_\eta Z_{\eta'} = 0
\]

\[
X_\eta X_\iota + Y_\eta Y_\iota + Z_\eta Z_\iota = 0
\]

\[
X_{\eta'} X_\iota + Y_{\eta'} Y_\iota + Z_{\eta'} Z_\iota = 0
\]

\[
\begin{pmatrix}
\eta \\
\eta' \\
\iota
\end{pmatrix}
= 
\begin{pmatrix}
c\phi_{\eta \eta'} c\phi_{\eta G} \\
sc\phi_{\eta \eta'} c\phi_{\eta' G} - c\phi_{\eta \eta'} s\phi_{\eta' G} s\phi_{\eta G} \\
s\phi_{\eta \eta'} s\phi_{\eta' G} + c\phi_{\eta \eta'} c\phi_{\eta' G} s\phi_{\eta G}
\end{pmatrix}
\begin{pmatrix}
- s\phi_{\eta \eta'} c\phi_{\eta G} \\
c\phi_{\eta \eta'} c\phi_{\eta' G} + s\phi_{\eta \eta'} s\phi_{\eta' G} s\phi_{\eta G} \\
c\phi_{\eta \eta'} s\phi_{\eta' G} - s\phi_{\eta \eta'} c\phi_{\eta' G} s\phi_{\eta G}
\end{pmatrix}
\begin{pmatrix}
- s\phi_{\eta G} \\
sc\phi_{\eta' G} c\phi_{\eta G} \\
sc\phi_{\eta' G} c\phi_{\eta G}
\end{pmatrix}
\begin{pmatrix}
\eta_q \\
\eta_s \\
G
\end{pmatrix}
\]

3 independent parameters: \(\phi_{\eta}, \phi_{\eta G}, \text{ and } \phi_{\eta' G}\)
• **Euler angles**

In the limit $\phi_{\eta G} = 0$:

$$
X_{\eta} = \cos \phi_P, \quad Y_{\eta} = -\sin \phi_P, \quad Z_{\eta} = 0,
$$

$$
X_{\eta'} = \sin \phi_P \cos \phi_{\eta' G}, \quad Y_{\eta'} = \cos \phi_P \cos \phi_{\eta' G}, \quad Z_{\eta'} = -\sin \phi_{\eta' G}.
$$
Data fitting

Using the latest experimental data on $(\rho, \omega, \phi) \to \eta \gamma$ (SND) and $\phi \to \eta' \gamma$ (KLOE), we get

- $\phi_P = (42.7 \pm 0.7) ^\circ$, $z_q = 0.83 \pm 0.03$, $z_s = 0.79 \pm 0.05$, $\chi^2$/d.o.f. = 4.0/5
- $\phi_P = (42.6 \pm 1.1) ^\circ$, $|\phi_{\eta'G}| = (5 \pm 21) ^\circ$, $z_q = 0.83 \pm 0.03$, $z_s = 0.79 \pm 0.05$, $\chi^2$/d.o.f. = 4.0/4

confirmation of the null gluonic content of the $\eta$ and $\eta'$ wave functions

<table>
<thead>
<tr>
<th>Transition</th>
<th>$g_{VP\gamma}^{exp}$ (latest)</th>
<th>$g_{VP\gamma}^{th}$ (Fit 3)</th>
<th>$g_{VP\gamma}^{th}$ (Fit 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^0 \to \eta \gamma$</td>
<td>0.429 $\pm$ 0.023</td>
<td>0.436 $\pm$ 0.017</td>
<td>0.437 $\pm$ 0.028</td>
</tr>
<tr>
<td>$\eta' \to \rho^0 \gamma$</td>
<td>0.41 $\pm$ 0.03 (PDG)</td>
<td>0.40 $\pm$ 0.02</td>
<td>0.40 $\pm$ 0.04</td>
</tr>
<tr>
<td>$\omega \to \eta \gamma$</td>
<td>0.136 $\pm$ 0.007</td>
<td>0.134 $\pm$ 0.006</td>
<td>0.134 $\pm$ 0.009</td>
</tr>
<tr>
<td>$\eta' \to \omega \gamma$</td>
<td>0.139 $\pm$ 0.015 (PDG)</td>
<td>0.146 $\pm$ 0.006</td>
<td>0.146 $\pm$ 0.013</td>
</tr>
<tr>
<td>$\phi \to \eta \gamma$</td>
<td>0.214 $\pm$ 0.003</td>
<td>0.214 $\pm$ 0.017</td>
<td>0.214 $\pm$ 0.012</td>
</tr>
<tr>
<td>$\phi \to \eta' \gamma$</td>
<td>0.216 $\pm$ 0.005</td>
<td>0.216 $\pm$ 0.019</td>
<td>0.216 $\pm$ 0.018</td>
</tr>
</tbody>
</table>

no gluonium

gluonium
• **Comparison with other approaches**

\[
X_\eta = -\frac{1}{\sqrt{2}} \quad Y_\eta = \frac{1}{\sqrt{3}} \quad \eta = \eta_8
\]

68% CL bands

\[
X_\eta^2 + Y_\eta^2 \leq 1
\]

democratic solution

✓ importance of $\phi \rightarrow \eta \gamma$

✓ importance of the slopes ($\phi_\nu$)
• Comparison with other approaches

\[ X_{\eta'} = \sqrt{2}Y_{\eta'} = \frac{1}{\sqrt{3}} \]

\( \eta = \eta_0 \)

\( \checkmark \) Importance of constraining even more \( \phi \rightarrow \eta'\gamma \)

More refined data for this channel will contribute decisively to clarify this issue.
• *Comparison with other approaches*

**PDG’06 data**

\[ (\phi_P, Z_{\eta'}^2) = (42.6^\circ, 0.01) \]

**Latest data**

\[ (\phi_P, Z_{\eta'}^2) = (41.4^\circ, 0.04) \]
• *Comparison with other approaches*


\[ |Z_\eta| < 0.4 \]
Comparison with other approaches


\[ R = \frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}} = 26\% \]

\[ R = \frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}} = (13 \pm 13)\% \]
**Comparison with other approaches**

\[
R_\phi \equiv \frac{\Gamma(\phi \to \eta'\gamma)}{\Gamma(\phi \to \eta\gamma)} = \cot^2 \phi_P \cos^2 \phi_{\eta'} G \left(1 - \frac{m_s}{m} \frac{z_q}{z_s} \tan \phi_V \sin 2\phi_P \right)^2 \left(\frac{p_{\eta'}}{p_\eta}\right)^3
= (4.7 \pm 0.6) \times 10^{-3}
\]

in agreement with \((4.8 \pm 0.5) \times 10^{-3}\) (PDG'06) and \((4.77 \pm 0.09 \pm 0.19) \times 10^{-3}\) (KLOE) ✔
Comparison with other approaches

KLOE Collaboration, hep-ex/0612029

\[ \phi_P = (39.7 \pm 0.7)^\circ \]

\[ Z_{\eta'}^2 = 0.14 \pm 0.04 \]
• **Summary**

We have performed a phenomenological analysis of radiative \( V \to P \gamma \) and \( P \to V \gamma \) decays with the purpose of determining the gluon content of the \( \eta \) and \( \eta' \) mesons.

The present approach is based on a conventional SU(3) quark model supplemented with two sources of SU(3) breaking, the use of constituent quark masses with \( m_s > m \) and the different overlaps between the \( P \) and \( V \) wave functions.

The use of these different overlapping parameters (a specific feature of our analysis) is shown to be of primary importance in order to reach a good agreement.

• **Conclusions**

1) The current experimental data on \( VP\gamma \) transitions indicate within our model a negligible gluonic content for the \( \eta \) and \( \eta' \) mesons,

\[
Z_{\eta}^2 = 0.00 \pm 0.07 \quad \text{and} \quad Z_{\eta'}^2 = 0.04 \pm 0.10
\]

2) Accepting the absence of gluonium for the \( \eta \) meson, the gluonic content of the \( \eta' \) wave function amounts to \( |\Phi_{\eta'G}| = (12 \pm 13) \) or \( (Z_{\eta'})^2 = 0.04 \pm 0.09 \) and the \( \eta-\eta' \) mixing angle is found to be \( \phi_P = (41.4 \pm 1.3) \)°.
**Conclusions**

3) **Imposing** the absence of gluonium for both mesons one finds $\phi_P=(41.4\pm1.3)^\circ$, in agreement with the former result.

4) The latest experimental data on $(\rho,\omega,\phi)\rightarrow\eta\gamma$ and $\phi\rightarrow\eta'\gamma$ decays confirm the null gluonic content of the $\eta$ and $\eta'$ wave functions.

5) More refined experimental data, particularly for the $\phi\rightarrow\eta'\gamma$ channel, will contribute decisively to clarify this issue.