



On the gluon content of the η and η' mesons

Rafel Escribano

Grup de Física Teòrica & IFAE (UAB)

March 27, 2007

Laboratori Nazionali di Frascati

in collab. with Jordi Nadal, [hep-ph/0703187](https://arxiv.org/abs/hep-ph/0703187)

Work partly supported by the EU, MRTN-CT-2006-035482, “FLAVIANet” network

Purpose: to perform a **phenomenological analysis** of radiative $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ decays, with $V = \rho, K^*, \omega, \phi$ and $P = \pi, K, \eta, \eta'$, aimed at determining the **gluonic content** of the η and η' wave functions

Outline:

- *Notation*
- *Motivation*
- *A model for $VP\gamma$ $M1$ transitions*
- *Data fitting*
- *Comparison with other approaches*
- *Summary and conclusions*

• Notation

We work in a **basis** consisting of the states

$$|\eta_q\rangle \equiv \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle \quad |\eta_s\rangle = |s\bar{s}\rangle \quad |G\rangle \equiv |\text{gluonium}\rangle$$

The **physical states** η and η' are assumed to be the linear combinations

$$\begin{aligned} |\eta\rangle &= X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle + Z_\eta|G\rangle, \\ |\eta'\rangle &= X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|G\rangle, \end{aligned}$$

with $X_{\eta(\eta')}^2 + Y_{\eta(\eta')}^2 + Z_{\eta(\eta')}^2 = 1$ and thus $X_{\eta(\eta')}^2 + Y_{\eta(\eta')}^2 \leq 1$

A **significant gluonic admixture** in a state is possible only if

$$Z_{\eta(\eta')}^2 = 1 - X_{\eta(\eta')}^2 - Y_{\eta(\eta')}^2 > 0$$

Assumptions:

- no mixing with π^0 (isospin symmetry)
- no mixing with η_c states
- no mixing with radial excitations

- **Notation**

In **absence** of **gluonium**

$$Z_{\eta(\eta')} \equiv 0$$



$$\begin{aligned} |\eta\rangle &= \cos \phi_P |\eta_q\rangle - \sin \phi_P |\eta_s\rangle \\ |\eta'\rangle &= \sin \phi_P |\eta_q\rangle + \cos \phi_P |\eta_s\rangle \end{aligned}$$

with $X_\eta = Y_{\eta'} \equiv \cos \phi_P$ and $X_{\eta(\eta')}^2 + Y_{\eta(\eta')}^2 = 1$

$$X_{\eta'} = -Y_\eta \equiv \sin \phi_P$$

where ϕ_P is the η - η' **mixing angle** in the **quark-flavour basis** related to its **octet-singlet** analog through

$$\theta_P = \phi_P - \arctan \sqrt{2} \simeq \phi_P - 54.7^\circ$$

Similarly, for the **vector states** ω and ϕ the mixing is given by

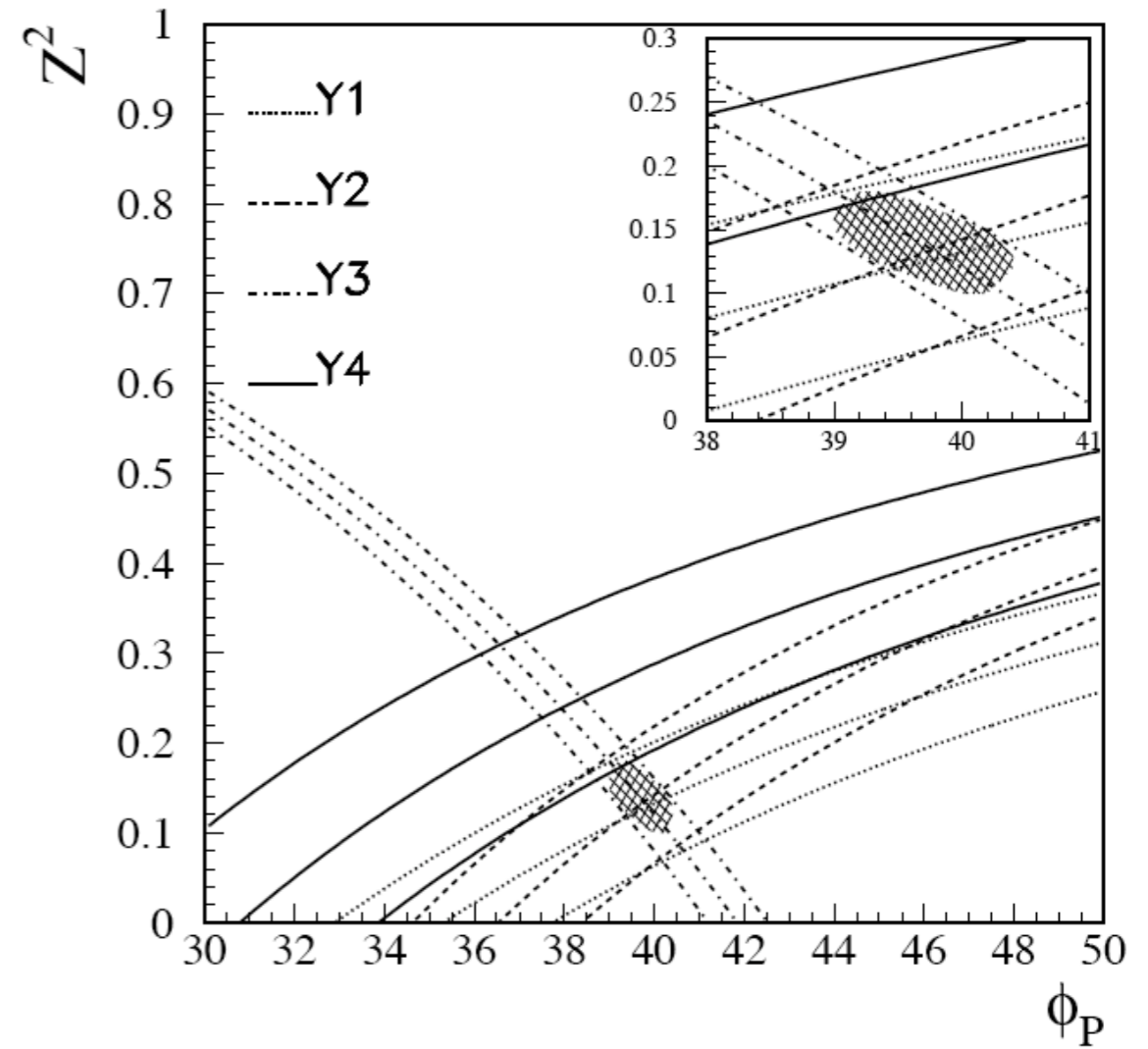
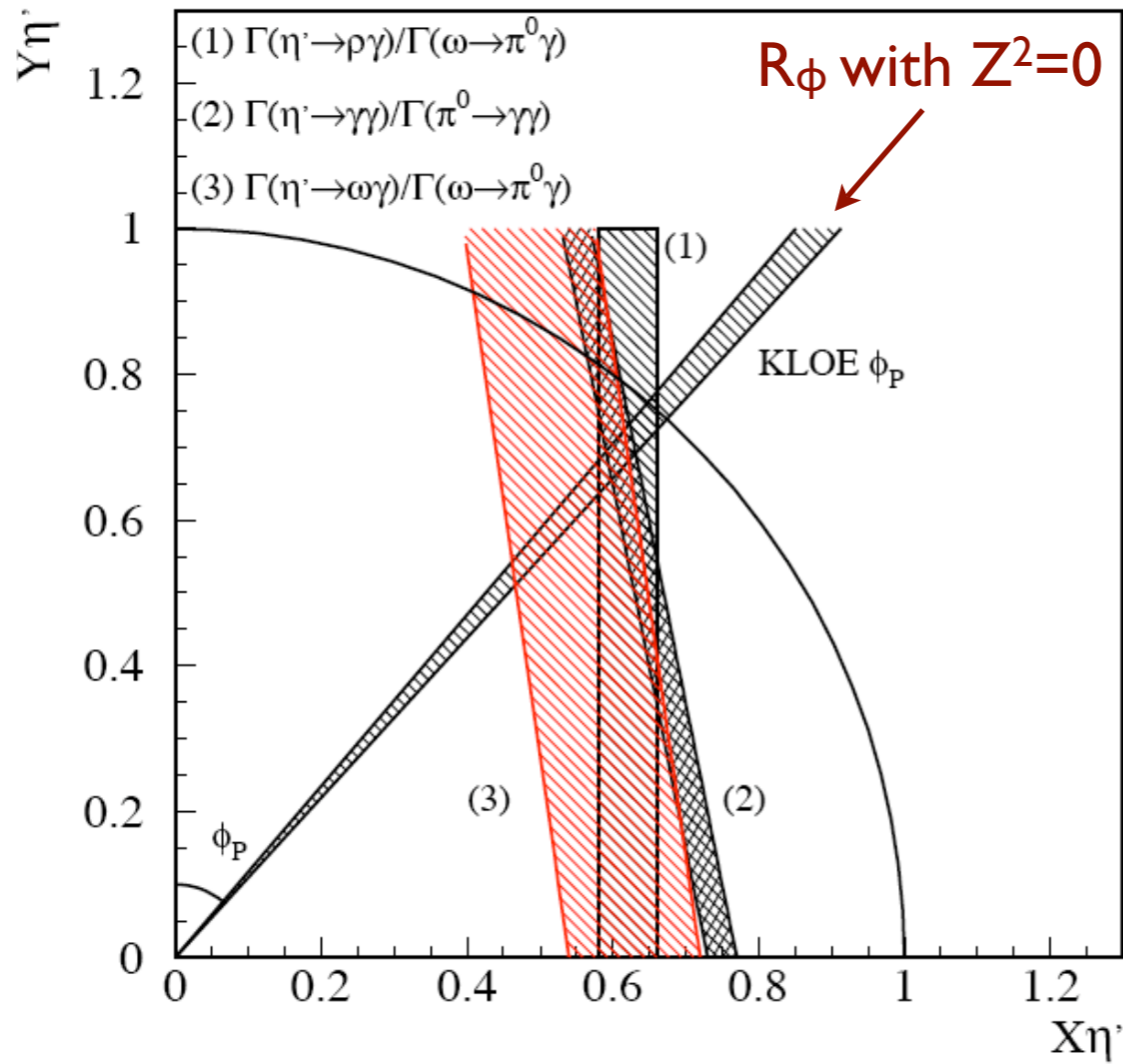
$$|\omega\rangle = \cos \phi_V |\omega_q\rangle - \sin \phi_V |\phi_s\rangle$$

$$|\phi\rangle = \sin \phi_V |\omega_q\rangle + \cos \phi_V |\phi_s\rangle$$

where ω_q and ϕ_s are the analog **non-strange** and **strange** states of η_q and η_s , respectively.

• Motivation

KLOE Collaboration, hep-ex/0612029



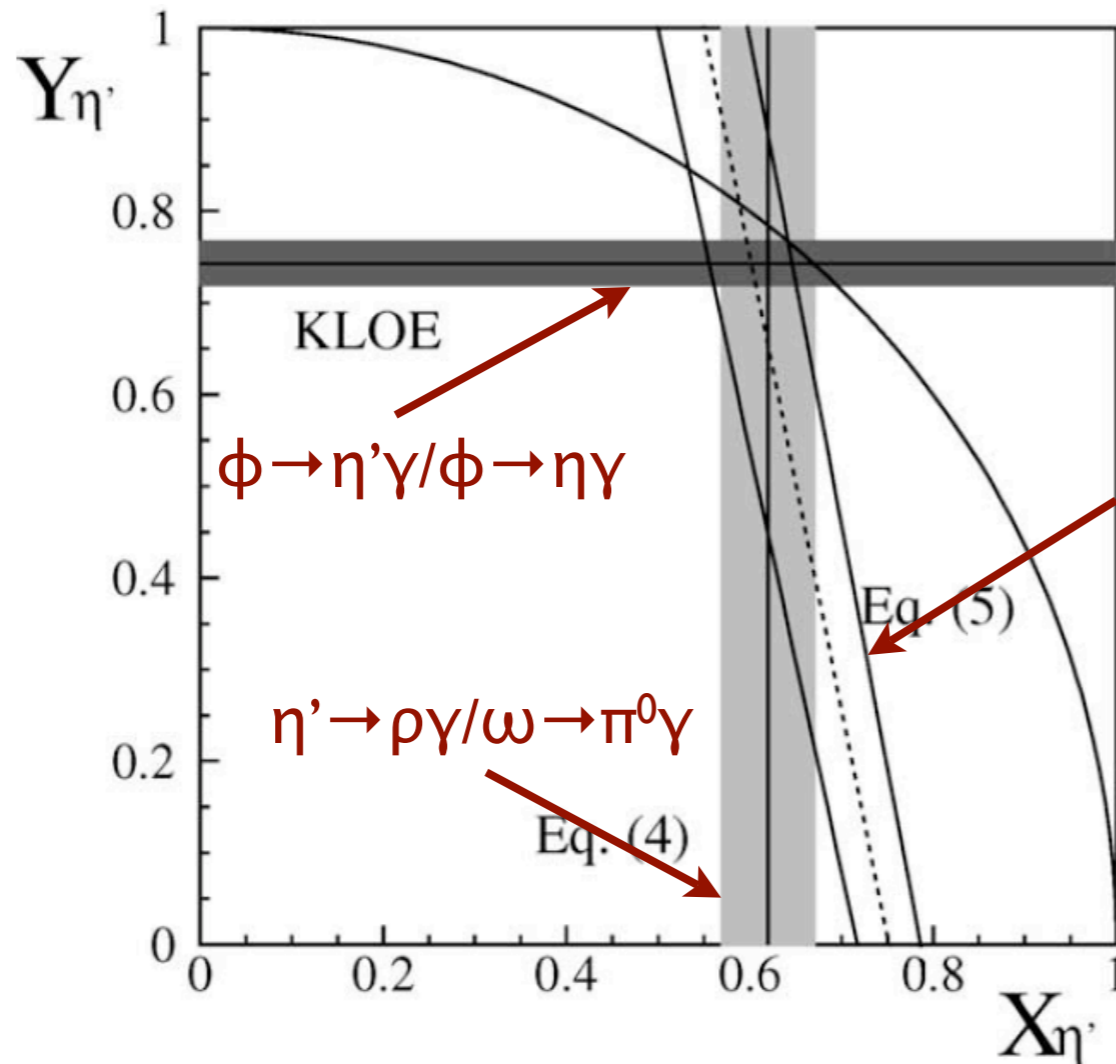
$$\phi_P = (39.7 \pm 0.7)^\circ$$

$$Z_{\eta'}^2 = 0.14 \pm 0.04$$

$Y1 = \eta' \rightarrow \gamma\gamma / \pi^0 \rightarrow \gamma\gamma$
 $Y2 = \eta' \rightarrow \rho\gamma / \omega \rightarrow \pi^0\gamma$
 $Y3 = \phi \rightarrow \eta'\gamma / \phi \rightarrow \eta\gamma$
 $Y4 = \eta' \rightarrow \omega\gamma / \omega \rightarrow \pi^0\gamma$

- *Motivation*

KLOE Collaboration, PLB 541 (2002), 45



$$Z_{\eta'}^2 = 0.06^{+0.09}_{-0.06}$$

Glueonium fraction below 15%

What are the **differences** between the two analyses?

- **improvement** in the **precision** of the **new measurements**
- the **use** of the **overlapping parameters** relating the **pseudoscalar** and **vector wave functions**

- *A model for $VP\gamma$ $M1$ transitions*

We will work in a conventional quark model context: **P** and **V** are simple quark-antiquark **S-wave bound states**

→ all these hadrons are thus **extended objects** with characteristics **spatial extensions** fixed by their respective **P** and **V** wave functions

SU(2) limit → identical spatial extension within each **isomultiplet**

SU(3) broken → constituent quark masses with $m_s > m$ and different spatial extensions for each **isomultiplet**

Ingredients of the model:

- i) a $VP\gamma$ **magnetic dipole transition** proceeding via quark or antiquark **spin flip amplitude** $\propto \mu_q = e_q/2m_q$
- ii) **spin-flip $V \rightarrow P$ conversion amplitude** corrected by the **relative overlap** between the **P** and **V** wave functions
- iii) **OZI-rule** reduces considerably the **possible transitions** and **overlaps**

$U(1)_A$ anomaly →

$$C_\pi \equiv \langle \pi | \omega_q \rangle = \langle \pi | \rho \rangle \quad C_K \equiv \langle K | K^* \rangle$$

$$C_q \equiv \langle \eta_q | \omega_q \rangle = \langle \eta_q | \rho \rangle \quad C_s \equiv \langle \eta_s | \phi_s \rangle$$

- A model for $VP\gamma$ $M1$ transitions

Amplitudes:

$$g_{\rho^0\pi^0\gamma} = g_{\rho^+\pi^+\gamma} = \frac{1}{3}g, \quad g_{\omega\pi\gamma} = g \cos \phi_V, \quad g_{\phi\pi\gamma} = g \sin \phi_V,$$

$$g_{K^{*0}K^0\gamma} = -\frac{1}{3}g z_K \left(1 + \frac{\bar{m}}{m_s}\right), \quad g_{K^{*+}K^+\gamma} = \frac{1}{3}g z_K \left(2 - \frac{\bar{m}}{m_s}\right),$$

$$g_{\rho\eta\gamma} = g z_q X_\eta, \quad g_{\rho\eta'\gamma} = g z_q X_{\eta'},$$

$$g_{\omega\eta\gamma} = \frac{1}{3}g \left(z_q X_\eta \cos \phi_V + 2 \frac{\bar{m}}{m_s} z_s Y_\eta \sin \phi_V \right),$$

$$g_{\omega\eta'\gamma} = \frac{1}{3}g \left(z_q X_{\eta'} \cos \phi_V + 2 \frac{\bar{m}}{m_s} z_s Y_{\eta'} \sin \phi_V \right),$$

$$g_{\phi\eta\gamma} = \frac{1}{3}g \left(z_q X_\eta \sin \phi_V - 2 \frac{\bar{m}}{m_s} z_s Y_\eta \cos \phi_V \right),$$

$$g_{\phi\eta'\gamma} = \frac{1}{3}g \left(z_q X_{\eta'} \sin \phi_V - 2 \frac{\bar{m}}{m_s} z_s Y_{\eta'} \cos \phi_V \right),$$

with $g_{\omega\pi\gamma} = g \cos \phi_V = e C_\pi \cos \phi_V / \bar{m}$

and $z_q \equiv C_q / C_\pi$, $z_s \equiv C_s / C_\pi$, $z_K \equiv C_K / C_\pi$

$$\Gamma(V \rightarrow P\gamma) = \frac{1}{3} \frac{g_{VP\gamma}^2}{4\pi} |\mathbf{p}_\gamma|^3 = \frac{1}{3} \Gamma(P \rightarrow V\gamma)$$

- *Data fitting*

The **overlapping parameters** $z_{q,s}$ and the **mixing parameters** $X_{\eta(\eta')}$ and $Y_{\eta(\eta')}$ cannot be determined **independently**

Thus we start assuming $C_q = C_s = C_K = C_\pi = 1$ \longrightarrow $z_q = z_s = z_K = 1$

\longrightarrow $\chi^2/\text{d.o.f.} = 28.3/7 \rightarrow 16.6/5$

Then we leave the **overlapping parameters free**

$$\begin{aligned}
 g &= 0.72 \pm 0.01 \text{ GeV}^{-1}, & \phi_V &= (3.2 \pm 0.1)^\circ, \\
 \frac{m_s}{\bar{m}} &= 1.24 \pm 0.07, & z_K &= 0.89 \pm 0.03, & \chi^2/\text{d.o.f.} &= 4.2/4 \\
 z_q X_\eta &= 0.65 \pm 0.02, & z_q X_{\eta'} &= 0.56 \pm 0.04, \\
 z_s Y_\eta &= -0.52 \pm 0.03, & z_s Y_{\eta'} &= 0.58 \pm 0.05.
 \end{aligned}$$

Then we fix the **mixing parameters** to the standard picture (**no gluonium**)

$$\begin{aligned}
 g &= 0.72 \pm 0.01 \text{ GeV}^{-1}, & \phi_P &= (41.5 \pm 1.2)^\circ, & \phi_V &= (3.2 \pm 0.1)^\circ, \\
 \frac{m_s}{\bar{m}} &= 1.24 \pm 0.07, & z_K &= 0.89 \pm 0.03, & z_q &= 0.86 \pm 0.03, & z_s &= 0.78 \pm 0.05. & \chi^2/\text{d.o.f.} &= 4.4/5
 \end{aligned}$$

For comparison, if $z_q = z_s = z_K = 1$ then $\chi^2/\text{d.o.f.} = 45.9/8$

- *Data fitting*

Fixing the **overlapping parameters** to the values obtained under the **hypothesis** of **no gluonium** we get

$$g = 0.72 \pm 0.01 \text{ GeV}^{-1}, \quad \phi_V = (3.2 \pm 0.1)^\circ,$$

$$\frac{m_s}{\bar{m}} = 1.24 \pm 0.07, \quad z_K = 0.89 \pm 0.03,$$

$\chi^2/\text{d.o.f.}=4.2/5$

$$X_\eta = 0.75 \pm 0.03, \quad X_{\eta'} = 0.65 \pm 0.04,$$

$$Y_\eta = -0.66 \pm 0.04, \quad Y_{\eta'} = 0.74 \pm 0.06,$$



$$Z_\eta^2 = 0.00 \pm 0.07$$

and

$$Z_{\eta'}^2 = 0.04 \pm 0.10$$

$$|Z_{\eta,(\eta')}| = \sqrt{1 - X_{\eta,(\eta')}^2 - Y_{\eta,(\eta')}^2}$$



$$|Z_{\eta'}| = 0.2 \pm 0.3$$

$$Y_\eta = -\frac{X_\eta X_{\eta'} Y_{\eta'} + \sqrt{(1 - X_{\eta'}^2 - Y_{\eta'}^2)(1 - X_\eta^2 - X_{\eta'}^2)}}{1 - X_{\eta'}^2}$$



The **current experimental data** on **VP γ** transitions indicate **within our model** a **negligible gluonic content** for the η and η' mesons

• Data fitting

Assuming $Z_\eta=0$ from the beginning, we get

$$g = 0.72 \pm 0.01 \text{ GeV}^{-1}, \quad \frac{m_s}{\bar{m}} = 1.24 \pm 0.07, \quad \phi_V = (3.2 \pm 0.1)^\circ,$$

$$\phi_P = (41.4 \pm 1.3)^\circ, \quad |\phi_{\eta'G}| = (12 \pm 13)^\circ,$$

$$\chi^2/\text{d.o.f.}=4.2/4$$

$$z_K = 0.89 \pm 0.03, \quad z_q = 0.86 \pm 0.03, \quad z_s = 0.79 \pm 0.05,$$



Accepting the **absence** of **gluonium** for the η meson, the **gluonic content** of the η' wave function amounts to $|\phi_{\eta'G}|=(12\pm 13)^\circ$ or $(Z_{\eta'})^2=0.04\pm 0.09$ and the η - η' **mixing angle** is found to be $\phi_P=(41.4\pm 1.3)^\circ$


Transition	$g_{VP\gamma}^{\text{exp}}$ (PDG)	$g_{VP\gamma}^{\text{th}}$ (Fit 1)	$g_{VP\gamma}^{\text{th}}$ (Fit 2)
$\rho^0 \rightarrow \eta\gamma$	0.475 ± 0.024	0.461 ± 0.019	0.464 ± 0.030
$\eta' \rightarrow \rho^0\gamma$	0.41 ± 0.03	0.41 ± 0.02	0.40 ± 0.04
$\omega \rightarrow \eta\gamma$	0.140 ± 0.007	0.142 ± 0.007	0.143 ± 0.010
$\eta' \rightarrow \omega\gamma$	0.139 ± 0.015	0.149 ± 0.006	0.146 ± 0.014
$\phi \rightarrow \eta\gamma$	0.209 ± 0.002	0.209 ± 0.018	0.209 ± 0.013
$\phi \rightarrow \eta'\gamma$	0.22 ± 0.01	0.22 ± 0.02	0.22 ± 0.02

no gluonium

gluonium

- Euler angles

In presence of gluonium,

glueball-like state $\eta(1440)$ 

$$\begin{aligned}
 |\eta\rangle &= X_\eta |\eta_q\rangle + Y_\eta |\eta_s\rangle + Z_\eta |G\rangle \\
 |\eta'\rangle &= X_{\eta'} |\eta_q\rangle + Y_{\eta'} |\eta_s\rangle + Z_{\eta'} |G\rangle \\
 |\iota\rangle &= X_\iota |\eta_q\rangle + Y_\iota |\eta_s\rangle + Z_\iota |G\rangle
 \end{aligned}$$

Normalization:

$$\begin{aligned}
 X_\eta^2 + Y_\eta^2 + Z_\eta^2 &= 1 \\
 X_{\eta'}^2 + Y_{\eta'}^2 + Z_{\eta'}^2 &= 1 \\
 X_\iota^2 + Y_\iota^2 + Z_\iota^2 &= 1
 \end{aligned}$$

Orthogonality:

$$\begin{aligned}
 X_\eta X_{\eta'} + Y_\eta Y_{\eta'} + Z_\eta Z_{\eta'} &= 0 \\
 X_\eta X_\iota + Y_\eta Y_\iota + Z_\eta Z_\iota &= 0 \\
 X_{\eta'} X_\iota + Y_{\eta'} Y_\iota + Z_{\eta'} Z_\iota &= 0
 \end{aligned}$$



3 independent parameters: ϕ_P , $\phi_{\eta G}$ and $\phi_{\eta' G}$

$$\begin{pmatrix} \eta \\ \eta' \\ \iota \end{pmatrix} = \begin{pmatrix} c\phi_{\eta\eta'} c\phi_{\eta G} & -s\phi_{\eta\eta'} c\phi_{\eta G} & -s\phi_{\eta G} \\ s\phi_{\eta\eta'} c\phi_{\eta' G} - c\phi_{\eta\eta'} s\phi_{\eta' G} s\phi_{\eta G} & c\phi_{\eta\eta'} c\phi_{\eta' G} + s\phi_{\eta\eta'} s\phi_{\eta' G} s\phi_{\eta G} & -s\phi_{\eta' G} c\phi_{\eta G} \\ s\phi_{\eta\eta'} s\phi_{\eta' G} + c\phi_{\eta\eta'} c\phi_{\eta' G} s\phi_{\eta G} & c\phi_{\eta\eta'} s\phi_{\eta' G} - s\phi_{\eta\eta'} c\phi_{\eta' G} s\phi_{\eta G} & c\phi_{\eta' G} c\phi_{\eta G} \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \\ G \end{pmatrix}$$

- *Euler angles*

In the limit $\phi_{\eta G}=0$:

$$X_{\eta} = \cos \phi_P , \quad Y_{\eta} = -\sin \phi_P ,$$

$$Z_{\eta} = 0 ,$$

$$X_{\eta'} = \sin \phi_P \cos \phi_{\eta' G} , \quad Y_{\eta'} = \cos \phi_P \cos \phi_{\eta' G} , \quad Z_{\eta'} = -\sin \phi_{\eta' G} .$$

• Data fitting

Using the latest experimental data on $(\rho, \omega, \phi) \rightarrow \eta\gamma$ (SND) and $\phi \rightarrow \eta'\gamma$ (KLOE), we get

$$\phi_P = (42.7 \pm 0.7)^\circ, \quad z_q = 0.83 \pm 0.03, \quad z_s = 0.79 \pm 0.05, \quad \chi^2/\text{d.o.f.} = 4.0/5$$

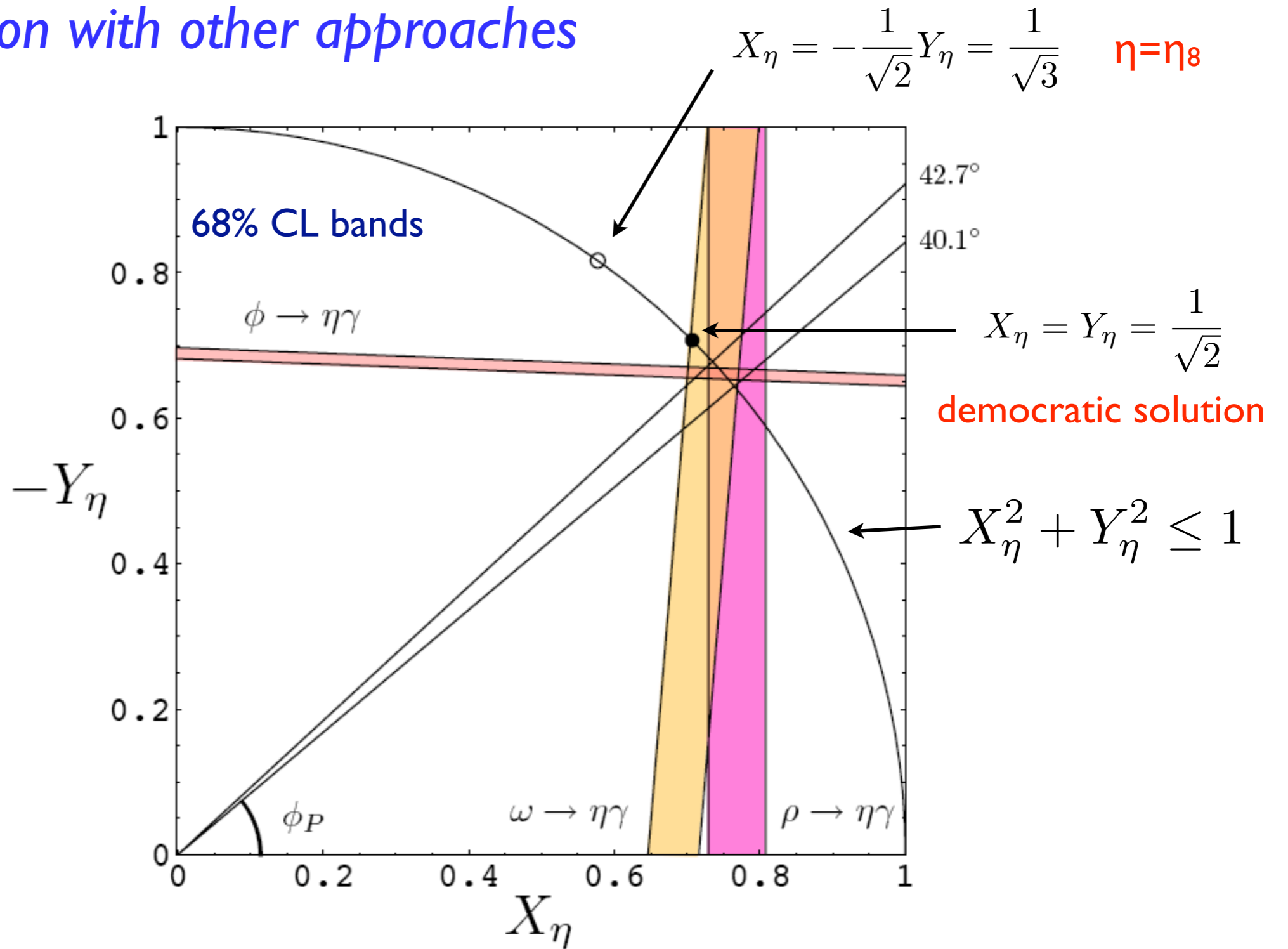
$$\phi_P = (42.6 \pm 1.1)^\circ, \quad |\phi_{\eta'G}| = (5 \pm 21)^\circ, \quad z_q = 0.83 \pm 0.03, \quad z_s = 0.79 \pm 0.05, \quad \chi^2/\text{d.o.f.} = 4.0/4$$



confirmation of the null gluonic content of the η and η' wave functions

Transition	$g_{VP\gamma}^{\text{exp}}$ (latest)	$g_{VP\gamma}^{\text{th}}$ (Fit 3)	$g_{VP\gamma}^{\text{th}}$ (Fit 4)	
$\rho^0 \rightarrow \eta\gamma$	0.429 ± 0.023	0.436 ± 0.017	0.437 ± 0.028	no gluonium
$\eta' \rightarrow \rho^0\gamma$	0.41 ± 0.03 (PDG)	0.40 ± 0.02	0.40 ± 0.04	gluonium
$\omega \rightarrow \eta\gamma$	0.136 ± 0.007	0.134 ± 0.006	0.134 ± 0.009	
$\eta' \rightarrow \omega\gamma$	0.139 ± 0.015 (PDG)	0.146 ± 0.006	0.146 ± 0.013	
$\phi \rightarrow \eta\gamma$	0.214 ± 0.003	0.214 ± 0.017	0.214 ± 0.012	
$\phi \rightarrow \eta'\gamma$	0.216 ± 0.005	0.216 ± 0.019	0.216 ± 0.018	

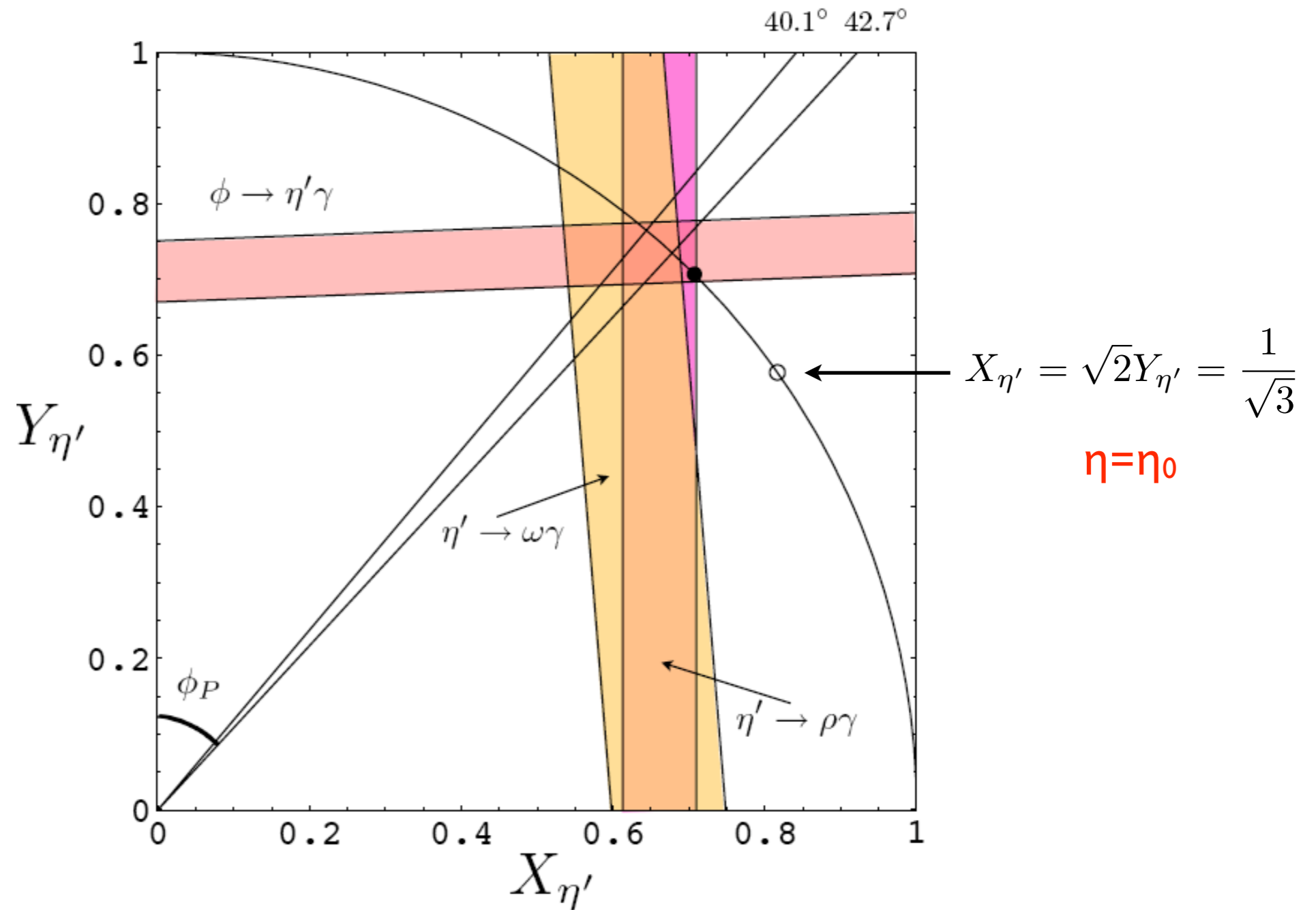
• Comparison with other approaches



✓ importance of $\phi \rightarrow \eta\gamma$

✓ importance of the slopes (ϕ_V)

- Comparison with other approaches



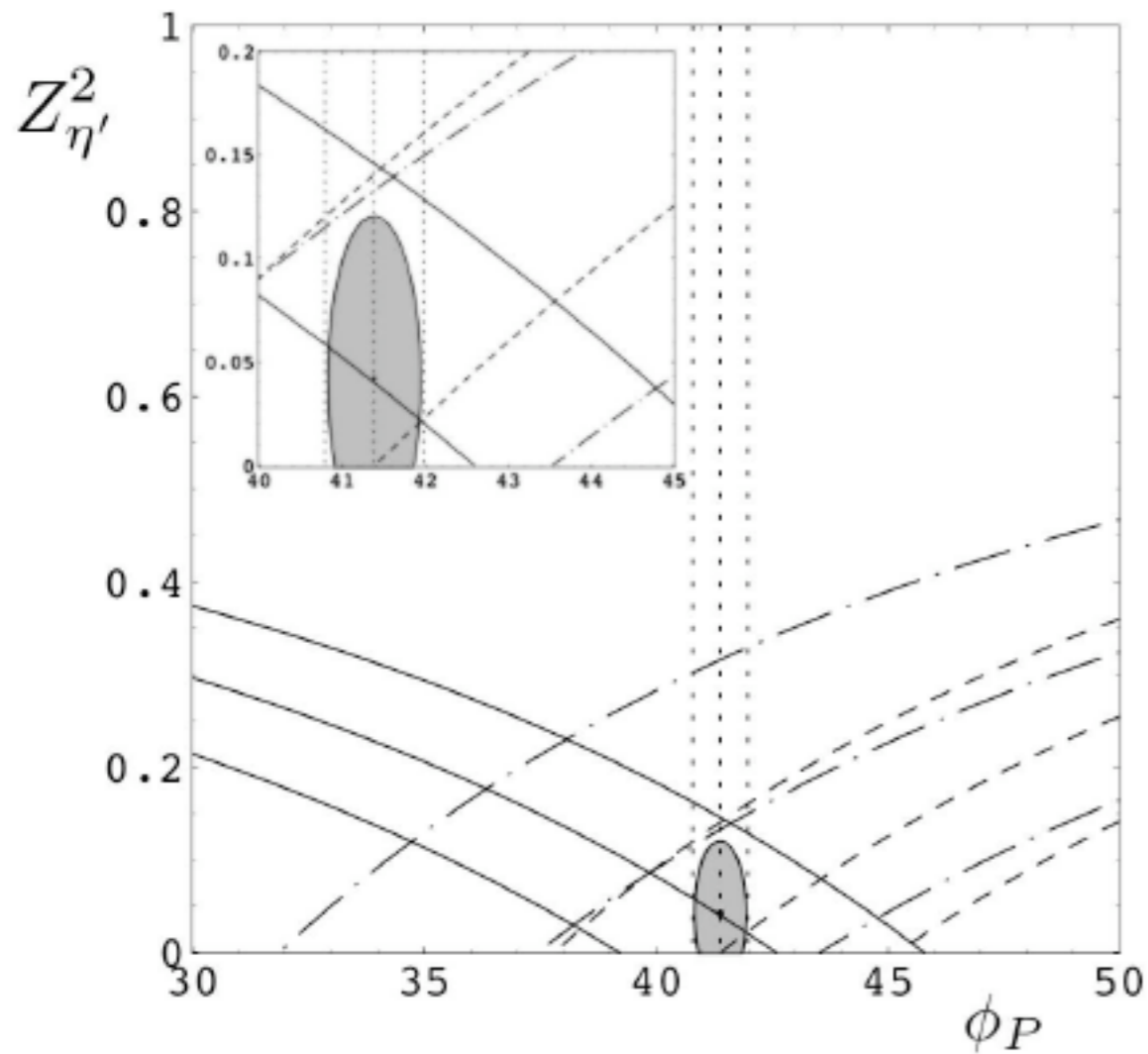
✓ importance of constraining even more $\phi \rightarrow \eta' \gamma$



More refined data for this channel will contribute decisively to clarify this issue

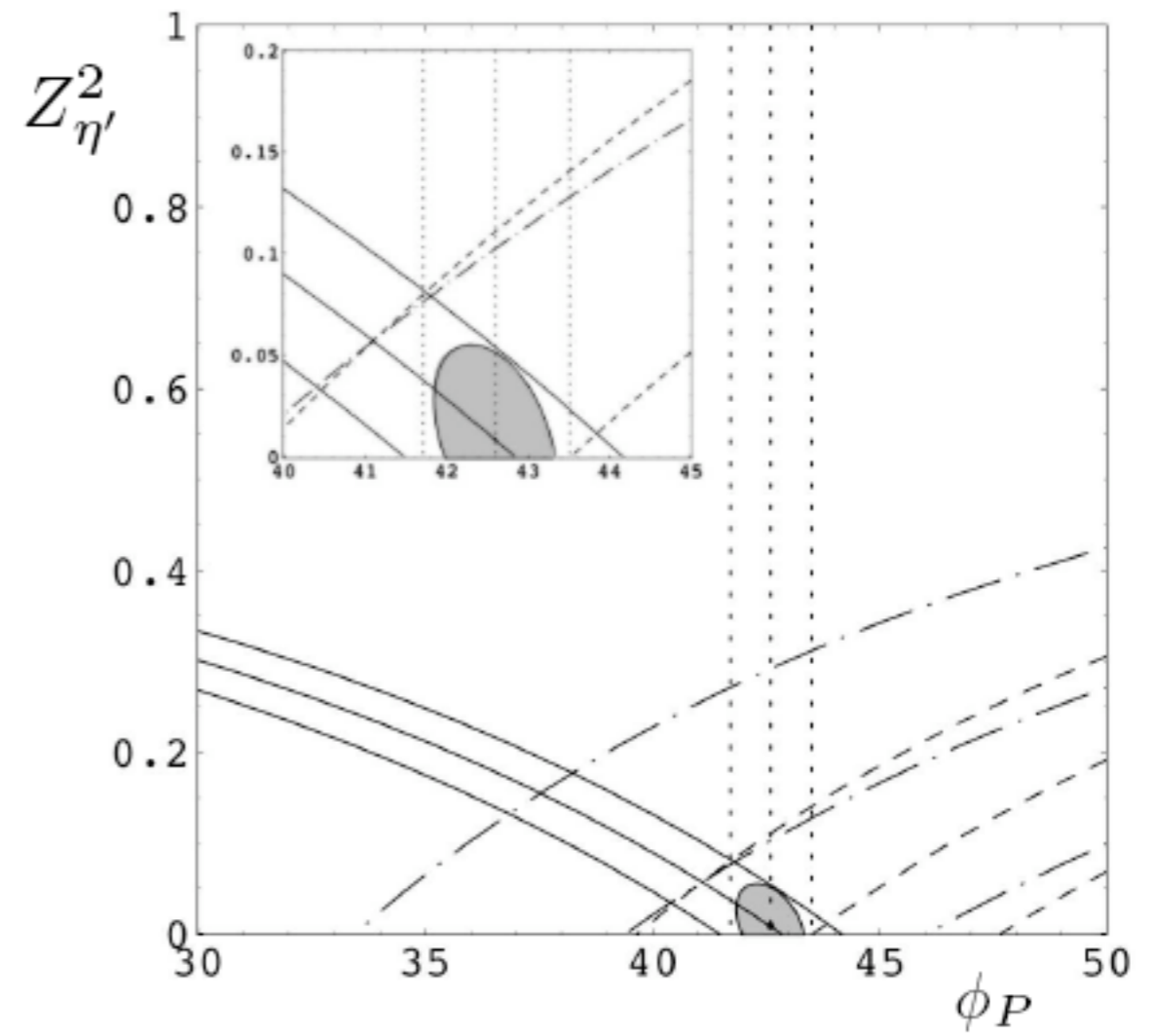
- Comparison with other approaches

PDG'06 data



$$(\phi_P, Z_{\eta'}^2) = (41.4^\circ, 0.04)$$

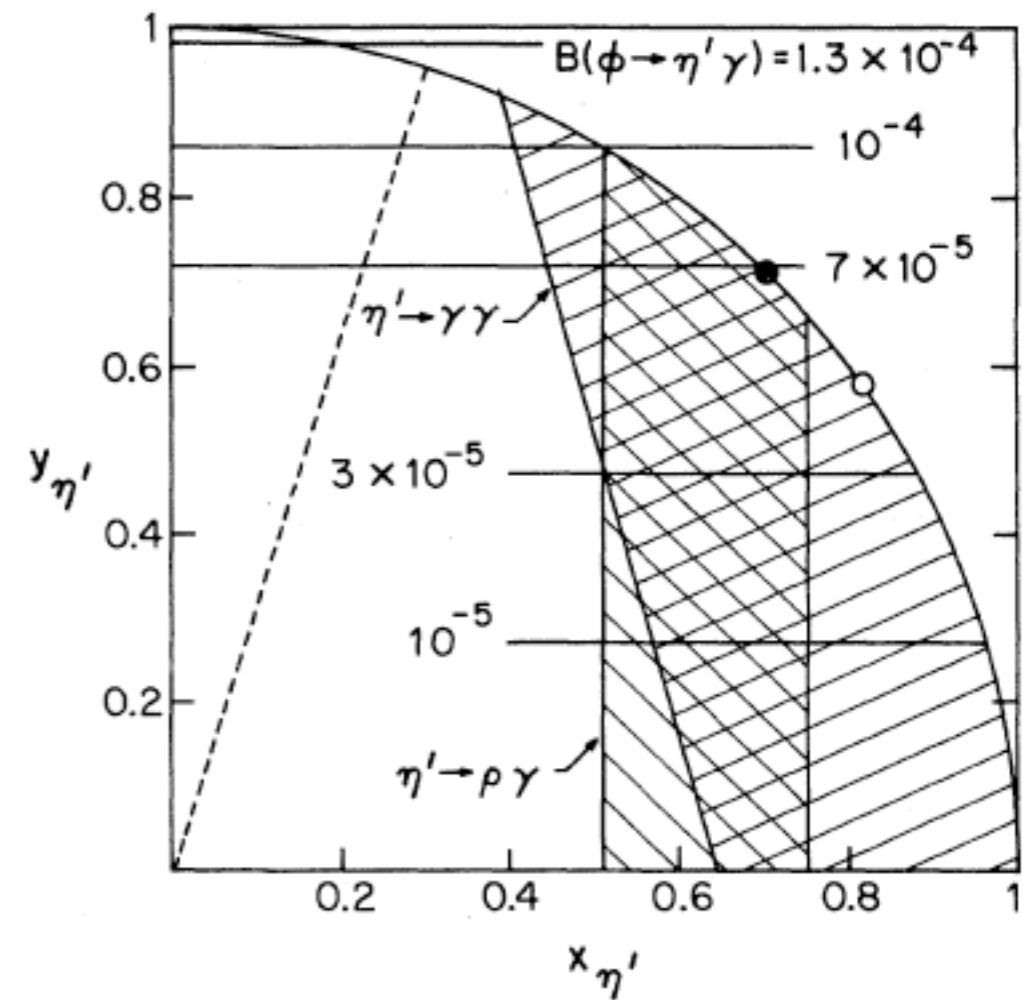
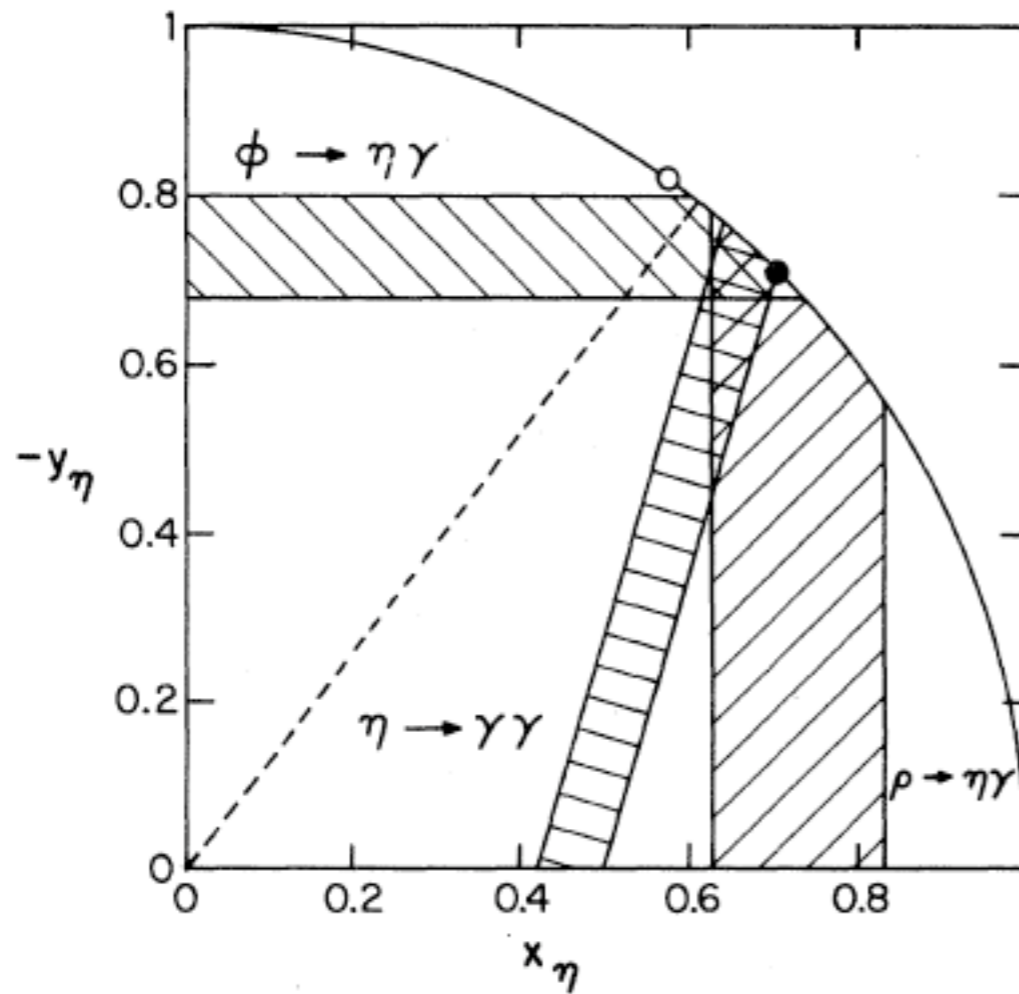
latest data



$$(\phi_P, Z_{\eta'}^2) = (42.6^\circ, 0.01)$$

- Comparison with other approaches

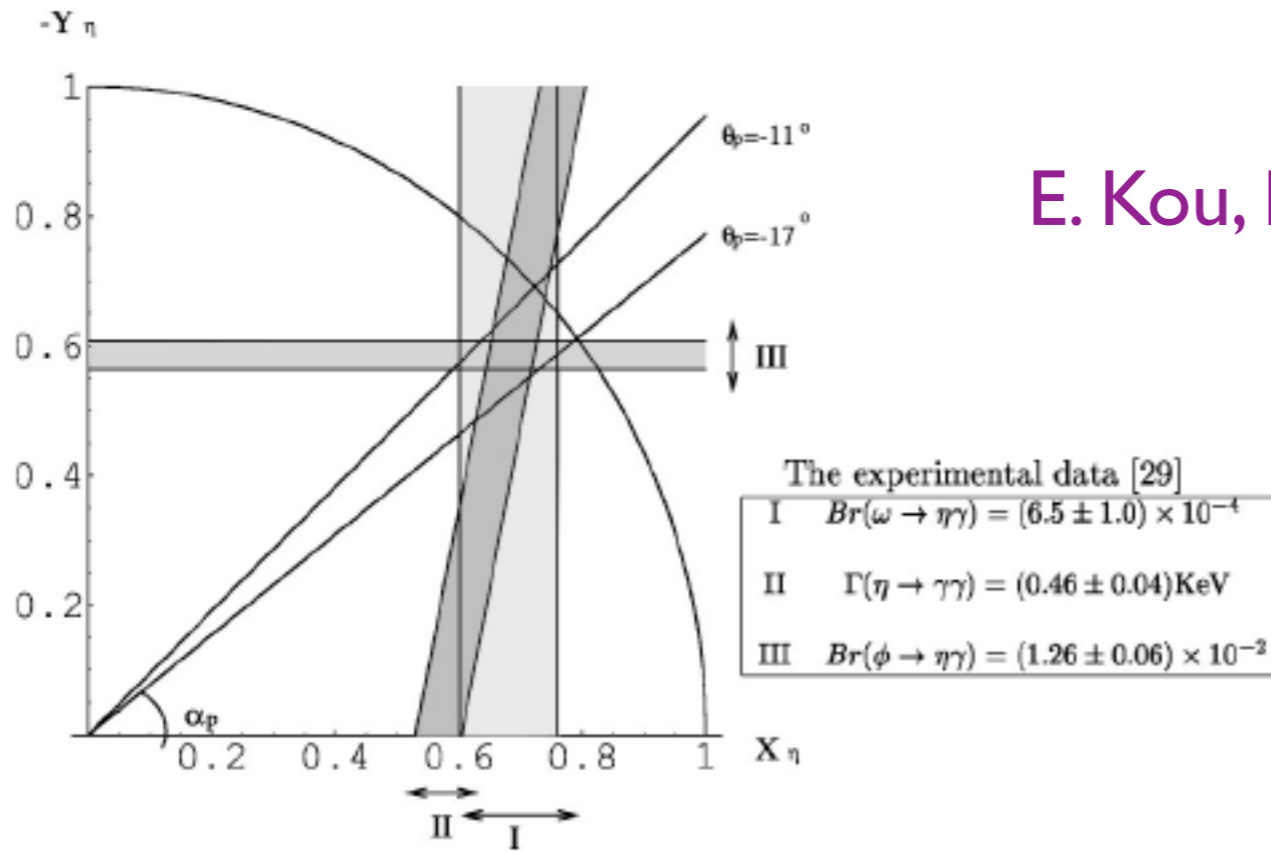
J. L. Rosner, Phys. Rev. D27 (1983), 1101



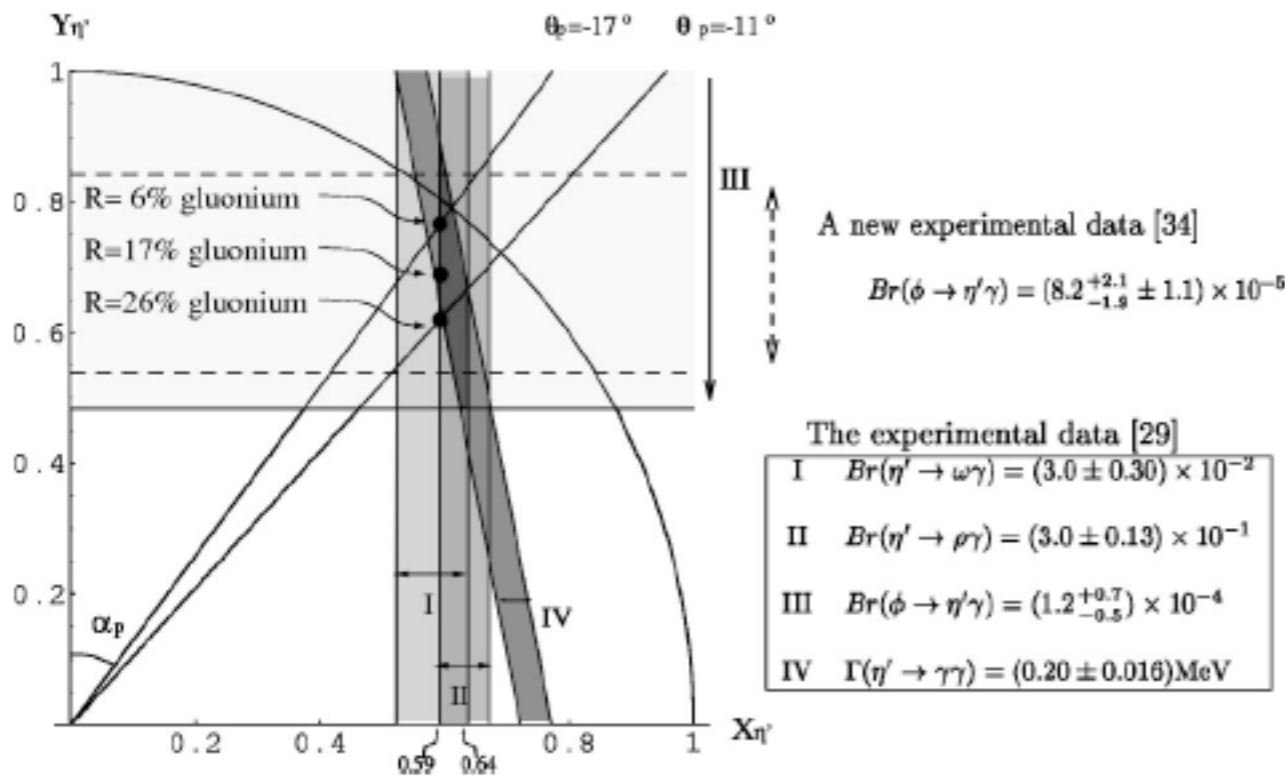
$$|Z_\eta| < 0.4$$

• Comparison with other approaches

E. Kou, Phys. Rev. D63 (2001), 054027



$$R = \frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}} = 26\%$$



$$R = \frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}} = (13 \pm 13)\%$$

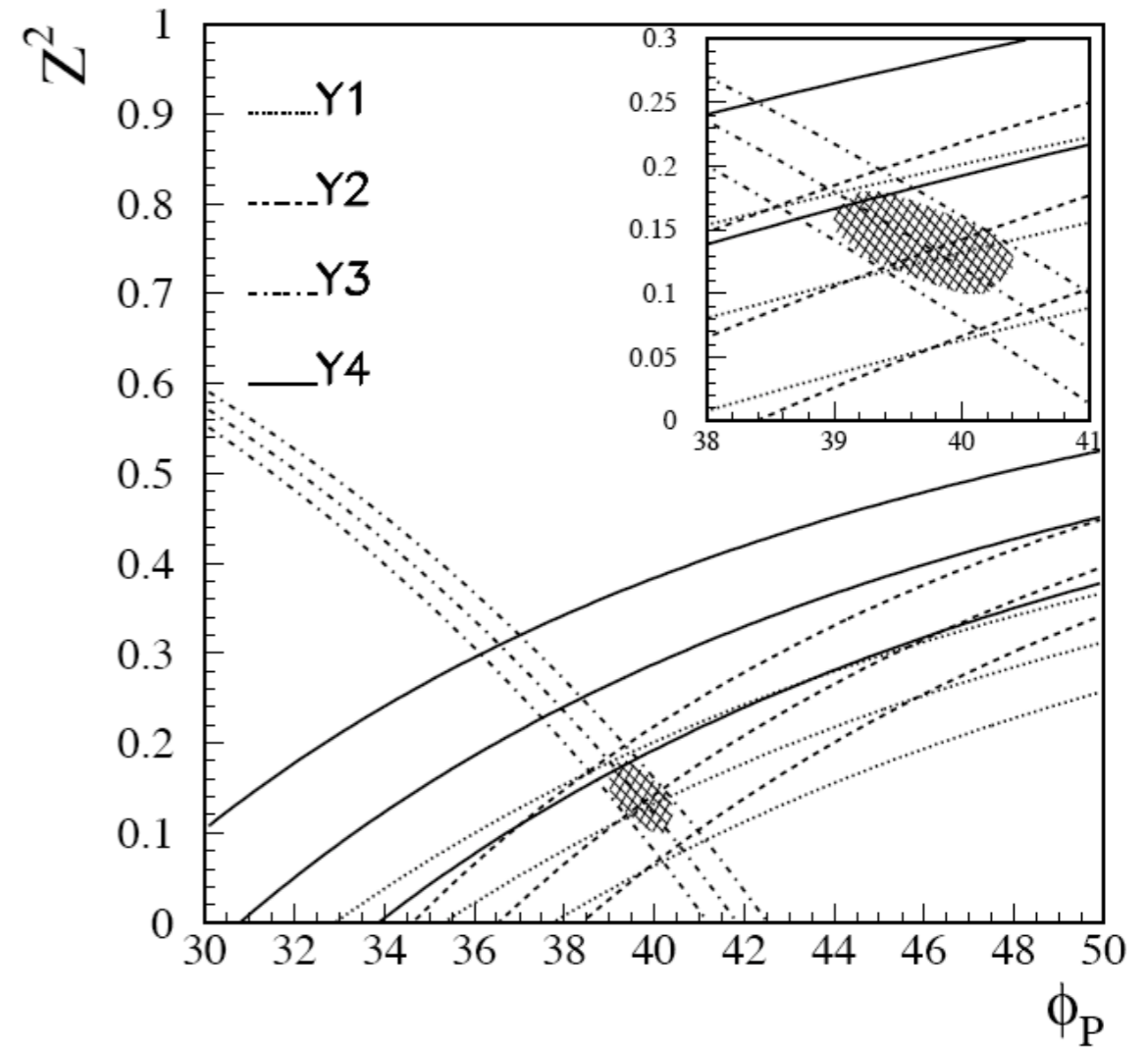
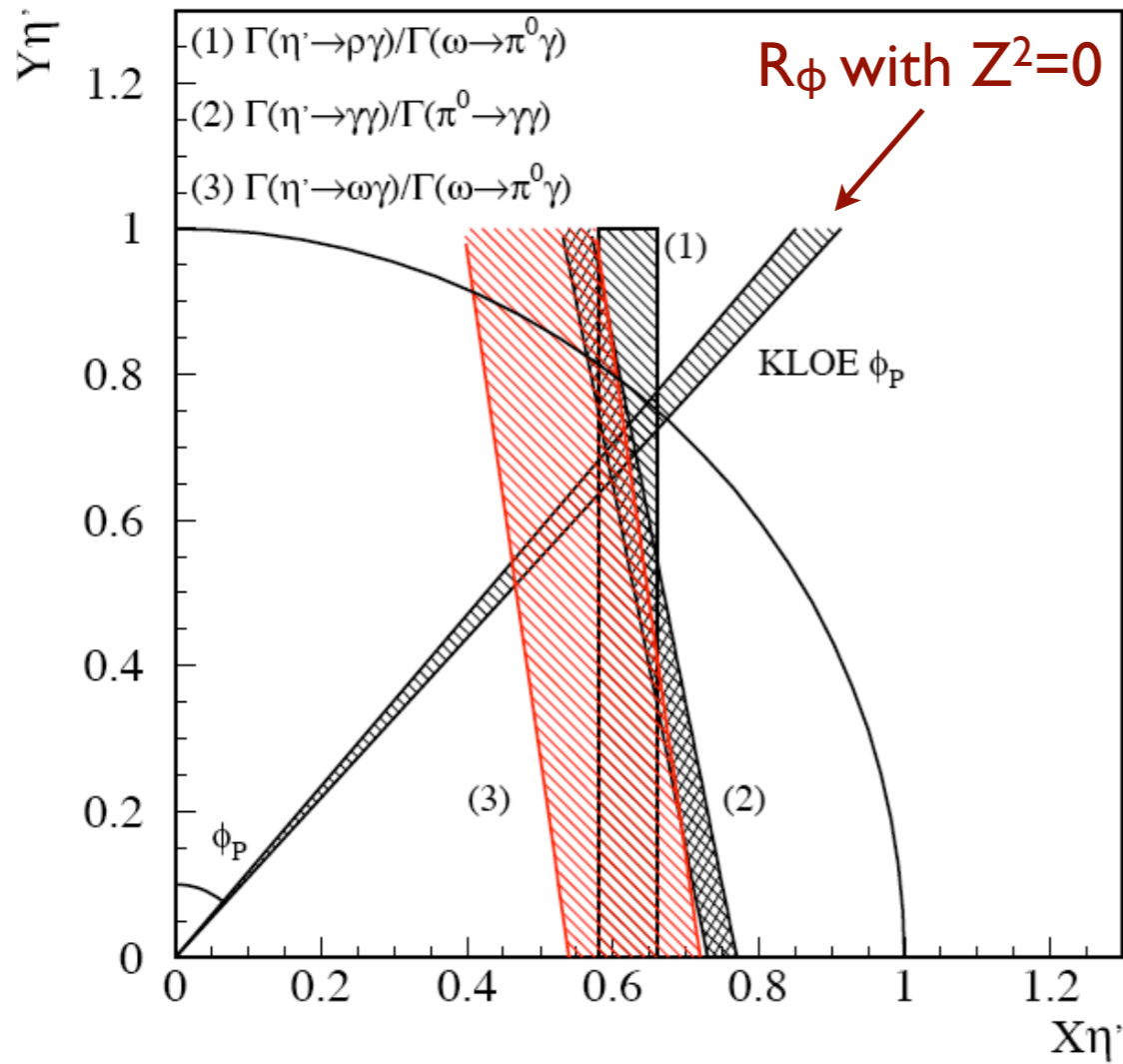
- Comparison with other approaches

$$R_\phi \equiv \frac{\Gamma(\phi \rightarrow \eta' \gamma)}{\Gamma(\phi \rightarrow \eta \gamma)} = \cot^2 \phi_P \cos^2 \phi_{\eta' G} \left(1 - \frac{m_s}{\bar{m}} \frac{z_q}{z_s} \frac{\tan \phi_V}{\sin 2\phi_P} \right)^2 \left(\frac{p_{\eta'}}{p_\eta} \right)^3 = (4.7 \pm 0.6) \times 10^{-3}$$

in agreement with $(4.8 \pm 0.5) \times 10^{-3}$ (PDG'06) and $(4.77 \pm 0.09 \pm 0.19) \times 10^{-3}$ (KLOE) ✓

- Comparison with other approaches

KLOE Collaboration, hep-ex/0612029



$$\phi_P = (39.7 \pm 0.7)^\circ$$

$$Z_{\eta'}^2 = 0.14 \pm 0.04$$

- Y1 = $\eta' \rightarrow \gamma\gamma / \pi^0 \rightarrow \gamma\gamma$
- Y2 = $\eta' \rightarrow \rho\gamma / \omega \rightarrow \pi^0\gamma$
- Y3 = $\phi \rightarrow \eta'\gamma / \phi \rightarrow \eta\gamma$
- Y4 = $\eta' \rightarrow \omega\gamma / \omega \rightarrow \pi^0\gamma$

• Summary

We have performed a **phenomenological analysis** of radiative $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ decays with the **purpose** of determining the **gluon content** of the η and η' mesons

The **present approach** is based on a **conventional SU(3) quark model** supplemented with two sources of SU(3) breaking, the use of **constituent quark masses** with $m_s > m$ and the **different overlaps** between the **P** and **V** wave functions

The use of these **different overlapping parameters** (a specific feature of our analysis) is shown to be of **primary importance** in order to reach a **good agreement**

• Conclusions

- 1) The **current experimental data** on $VP\gamma$ transitions indicate **within our model** a **negligible gluonic content** for the η and η' mesons,

$$Z_\eta^2 = 0.00 \pm 0.07$$

and

$$Z_{\eta'}^2 = 0.04 \pm 0.10$$

- 2) Accepting the **absence** of **gluonium** for the η meson, the **gluonic content** of the η' wave function amounts to $|\phi_{\eta'G}| = (12 \pm 13)^\circ$ or $(Z_{\eta'})^2 = 0.04 \pm 0.09$ and the η - η' **mixing angle** is found to be $\phi_P = (41.4 \pm 1.3)^\circ$

• Conclusions

- 3) Imposing the absence of gluonium for both mesons one finds $\phi_P = (41.4 \pm 1.3)^\circ$, in agreement with the former result
- 4) The latest experimental data on $(\rho, \omega, \phi) \rightarrow \eta\gamma$ and $\phi \rightarrow \eta'\gamma$ decays confirm the null gluonic content of the η and η' wave functions
- 5) More refined experimental data, particularly for the $\phi \rightarrow \eta'\gamma$ channel, will contribute decisively to clarify this issue