Frascati, 7 April '04

# Neutrino Masses as a Probe of Grand Unification

G. Altarelli
CERN

Some recent work by our group

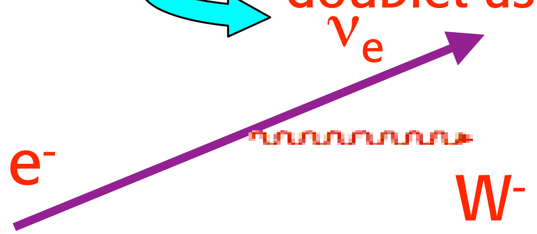
G.A., F. Feruglio, I. Masina, hep-ph/0210342 (Addendum: v2 in Nov. '03), hep-ph/0402121.

Reviews:

G.A., F. Feruglio, hep-ph/0206077/0306265

# $\nu$ Oscillations Imply Different $\nu$ Masses

$\nu_e$: same weak isospin doublet as $e^-$




flavour — mass

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

U: mixing matrix

$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

e.g 2 flav.

$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$

$\nu_\mu = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2$

Stationary source:

Stodolsky

$\nu_{1,2}$: different mass, different x-dep:

$\nu_a(x) = e^{ip_a x}\, \nu_a \qquad p_a^2 = E^2 - m_a^2$

$$P(\nu_e <-> \nu_\mu) = |< \nu_\mu(L)|\, \nu_e>|^2 = \sin^2(2\theta)\cdot\sin^2(\Delta m^2 L/4E)$$

At a distance L, $\nu_\mu$ from $\mu^-$ decay can produce $e^-$ via charged weak interact's

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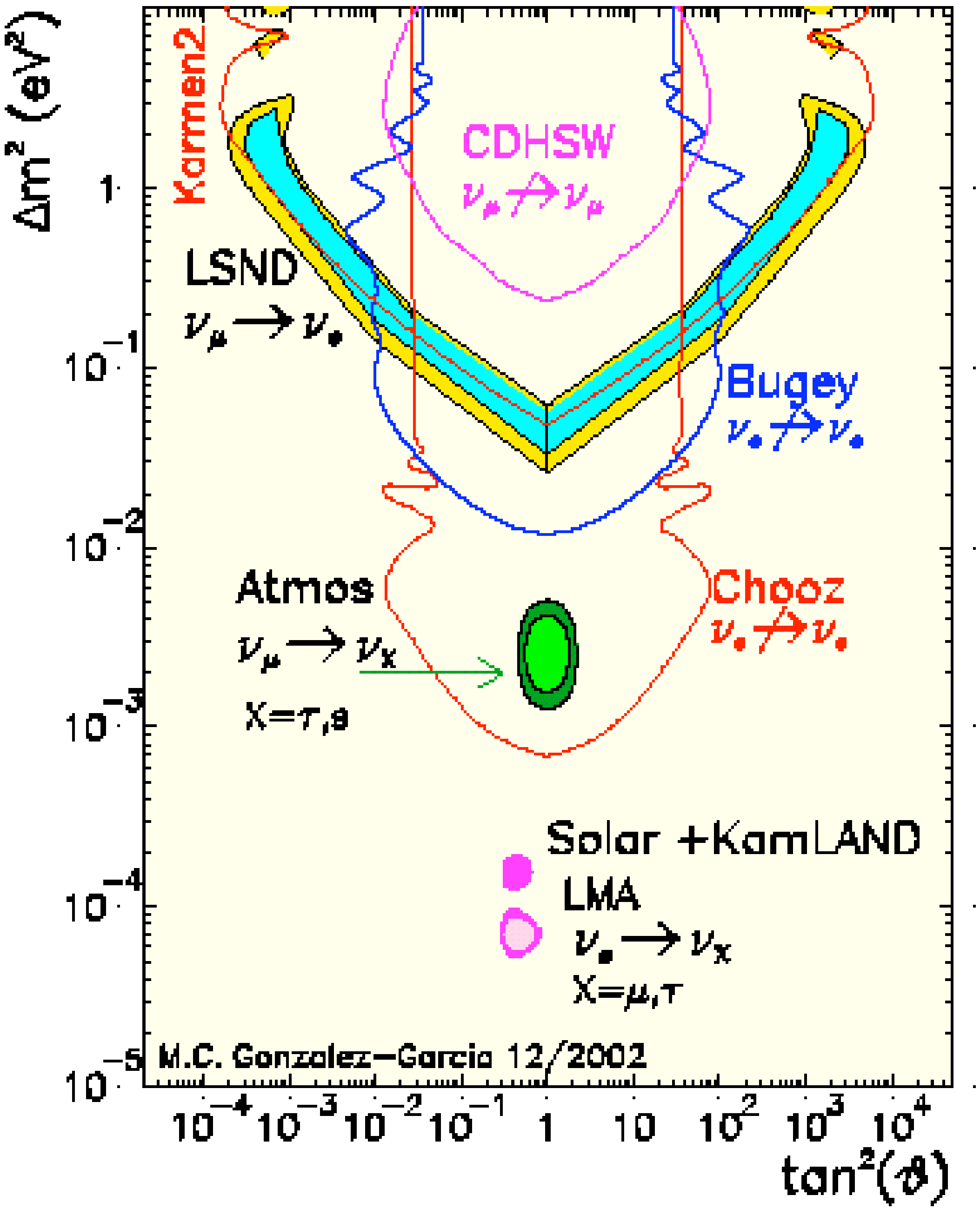
Solid evidence for
ν oscillations
(+LSND unclear)

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,
$\Delta m^2_{sol} \sim 7\ 10^{-5}\ eV^2$
($\Delta m^2_{LSND} \sim 1\ eV^2$)

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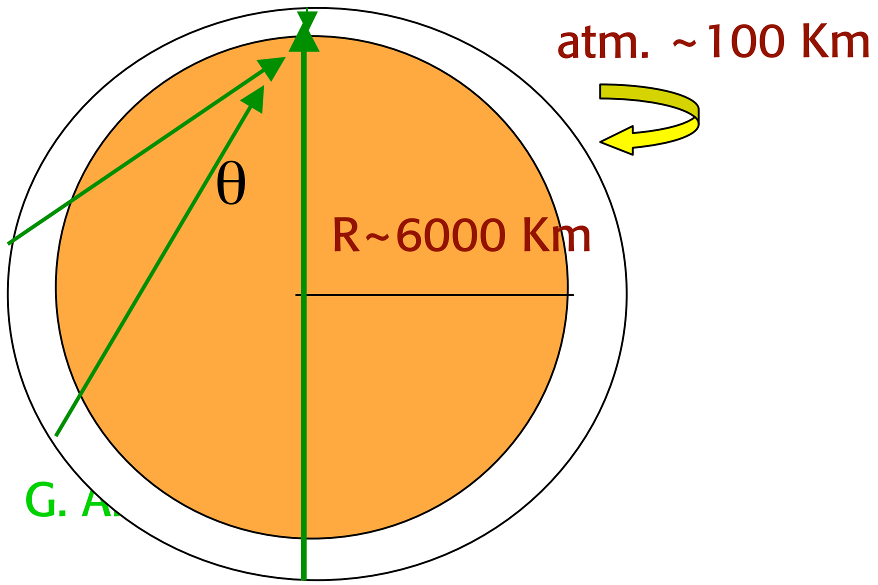
## Solar $\nu$'s


$\nu_e \rightarrow \nu_{\mu,\tau}$

For the LA solution the oscill's
occur inside the sun thru the MSW effect

Mikhaev and Smirnov; Wolfenstein

## Atmospheric $\nu$'s




$\nu_\mu \rightarrow \nu_\tau$

atmospheric $\nu$'s traverse
different L depending on
azimuth $\theta$
(up-down asymm.)

## Evolution in vacuum and in matter

$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$
$$\nu_\mu = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2$$

$\Delta m^2 = m_2^2 - m_1^2 > 0$

$$i\frac{d}{dt}\begin{bmatrix}\nu_e \\ \nu_\mu\end{bmatrix} = H_{eff}\begin{bmatrix}\nu_e \\ \nu_\mu\end{bmatrix}$$

$$H_{eff} = \frac{\Delta m^2}{4E}\begin{bmatrix}-\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$

In vacuum, 2 flavours, apart from multiples of the identity

In matter CC int's on electrons introduce a flavour dep.
(coherent forward scattering on electrons)

$$H_{eff} = \frac{\Delta m^2}{4E}\begin{bmatrix}-\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{bmatrix} + \begin{bmatrix}\sqrt{2}G_F N_e & 0 \\ 0 & 0\end{bmatrix}$$

$N_e$: n. of e per unit V

The mixing angle is changed
A resonance can appear (MSW)

$$\tan 2\theta_m = \frac{\tan 2\theta}{1 - \frac{2\sqrt{2}EG_F N_e}{\Delta m^2 \cos 2\theta}}$$

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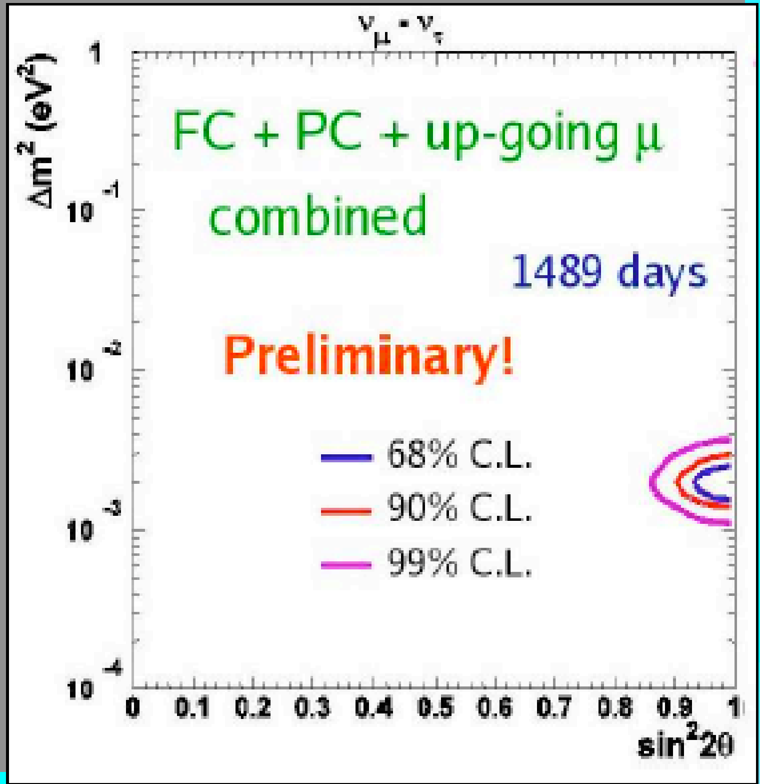
Mikhaev and Smirnov; Wolfenstein

# Atmospheric Neutrinos

Smirnov, Aachen'03

SuperKamiokande:




$\Delta m_{32}^2 = (1.3 - 3.0)\ 10^{-3}\ \mathrm{eV}^2$
$\sin^2 2\theta_{23} > 0.9$
(90 % C.L.)

Confirmed by
MACRO,
SOUDAN
K2K

Combined analysis of CHOOZ, atmospheric (SK) and solar data:

$\sin^2 2\theta_{13} < 0.067\ (3\sigma)$

G.L. Fogli et al, hep-ph/p0308055

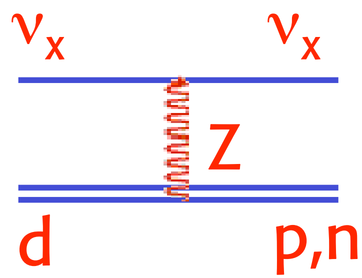
Best fit point: $\sin^2 2\theta_{23} = 1.0$
$\Delta m_{32}^2 = 2.0\ \ 10^{-3}\ \mathrm{eV}^2$

# $\nu$ Reactions in SNO

April '02

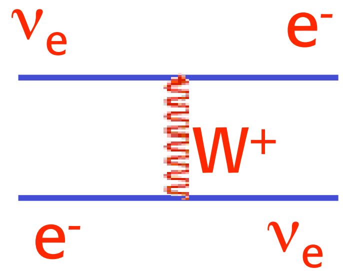
**CC** $\nu_e + d \Rightarrow p + p + e^-$

- Good measurement of $\nu_e$ energy spectrum
- Weak directional sensitivity $\propto$ 1-1/3cos($\theta$)
- $\nu_e$ only.




**NC** $\nu_x + d \Rightarrow p + n + \nu_x$

- Equal cross section for all $\nu$ types
- Measure total $^8$B $\nu$ flux from the sun.




**ES** $\nu_x + e^- \Rightarrow \nu_x + e^-$

- Low Statistics
- Mainly sensitive to $\nu_e$, some sensitivity to $\nu_\mu$ and $\nu_\tau$
- Strong directional sensitivity

$$\frac{\Phi_{cc}}{\Phi_{es}} = \frac{\nu_e}{\nu_e + 0.154(\nu_\mu + \nu_\tau)} = 1\ ?$$

$$\frac{\Phi_{cc}}{\Phi_{nc}} = \frac{\nu_e}{\nu_e + \nu_\mu + \nu_\tau} = 1\ ?$$

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# Results of April '02

Signal Extraction in $\Phi_{CC}$, $\Phi_{NC}$, $\Phi_{ES}$. $E_{Theshold} > 5$ MeV

$$\Phi_{cc}(\nu_e) = 1.76^{+0.06}_{-0.05}\,(\text{stat.})\,^{+0.09}_{-0.09}\,(\text{syst.})\ \text{x}10^6\ \text{cm}^{-2}\text{s}^{-1}$$

$$\Phi_{es}(\nu_x) = 2.39^{+0.24}_{-0.23}\,(\text{stat.})\,^{+0.12}_{-0.12}\,(\text{syst.})\ \text{x}10^6\ \text{cm}^{-2}\text{s}^{-1}$$

$$\Phi_{nc}(\nu_x) = 5.09^{+0.44}_{-0.43}\,(\text{stat.})\,^{+0.46}_{-0.43}\,(\text{syst.})\ \text{x}10^6\ \text{cm}^{-2}\text{s}^{-1}$$

Signal Extraction in $\Phi_e$, $\Phi_{\mu\tau}$.

$$\Phi_e = 1.76^{+0.05}_{-0.05}\,(\text{stat.})\,^{+0.09}_{-0.09}\,(\text{syst.})\ \text{x}10^6\ \text{cm}^{-2}\text{s}^{-1}$$

$$\Phi_{\mu\tau} = 3.41^{+0.45}_{-0.45}\,(\text{stat.})\,^{+0.48}_{-0.45}\,(\text{syst.})\ \text{x}10^6\ \text{cm}^{-2}\text{s}^{-1}$$

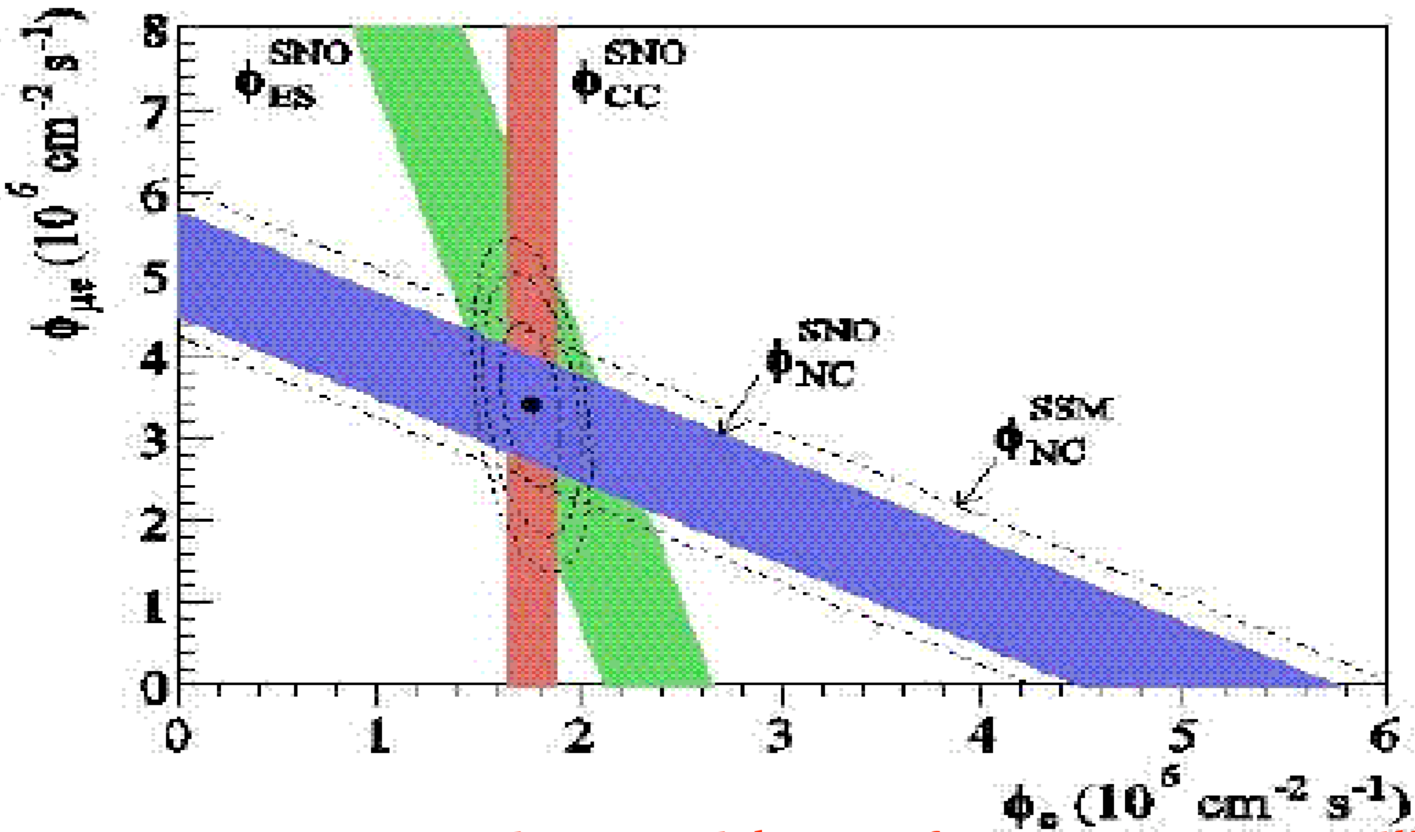
Note: $\Phi_{\mu,\tau} \sim 2\ \Phi_e$

(We receive an equal amount of $\nu_e$, $\nu_\mu$, $\nu_\tau$)

The measured total $\nu$ flux is in perfect agreement with the Solar Standard Model!!

But: $\Phi_e \sim 1/3\ (\Phi_e + \Phi_\mu + \Phi_\tau)$

$\Phi_{ssm} = 5.05^{+1.01}_{-0.81} \quad \Phi_{sno} = 5.09^{+0.44}_{-0.43}{}^{+0.46}_{-0.43}$

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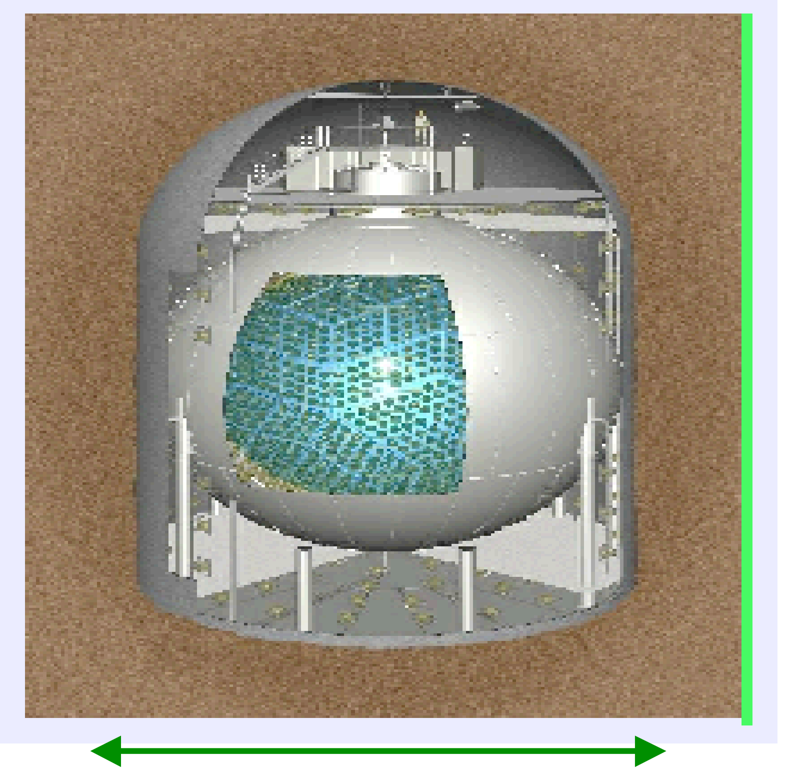
Direct evidence for $\nu_e \to \nu_{\mu,\tau}$ oscill's as solution of the solar $\nu_e$ deficit!

# Recent important results from KamLAND

Dec'02

Kamioka
Liquid
scintillator
AntiNeutrino
Detector



1 kton

13 m

Reactor $\bar{\nu}_e$ (E>2.6 MeV) detected 180 Km away at Kamiokande site

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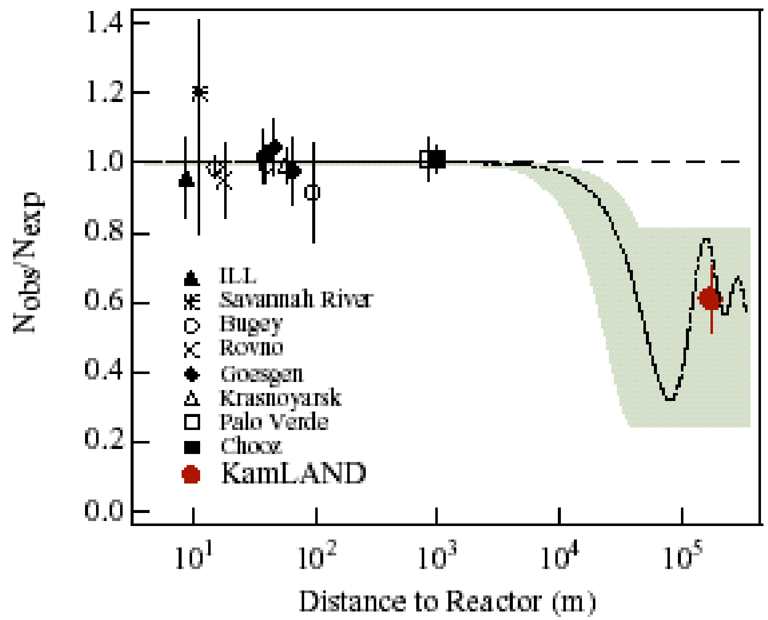
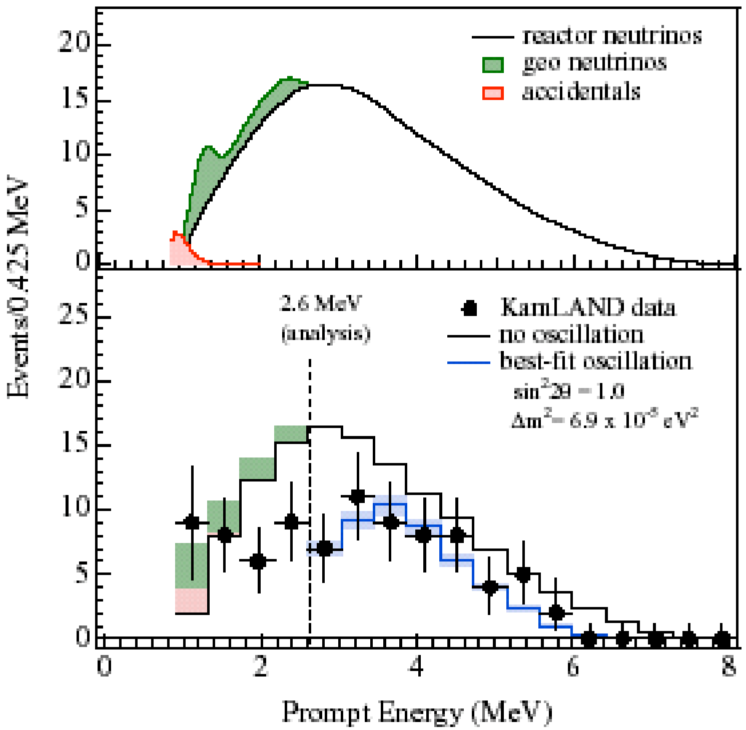
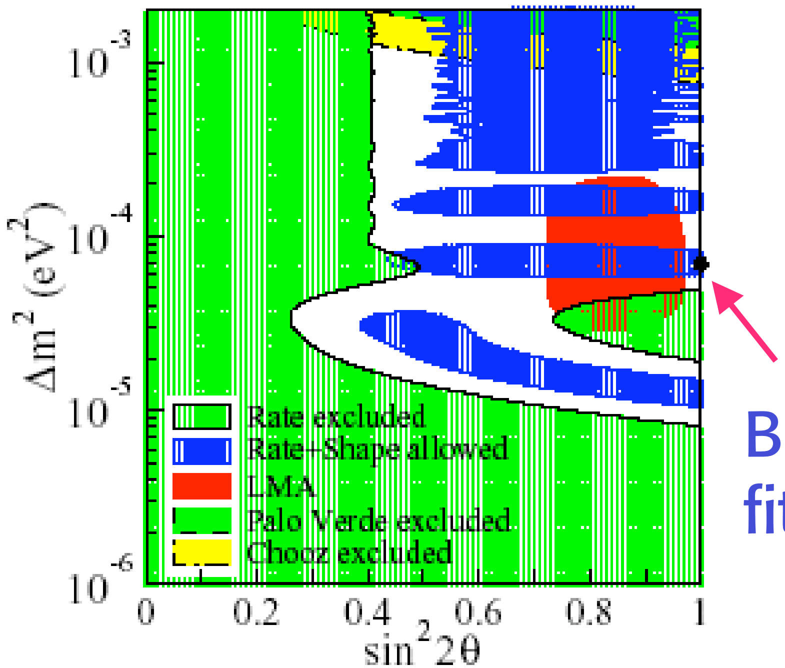
# First results from KamLAND

- Solar oscill.'s confirmed on earth
- Large angle sol. established
Best fit: $\Delta m^2 \sim 7.10^{-5}$ eV$^2$, $\sin^2 2\theta = 1$
- $\overline{\nu}_e$ from reactors behave as $\nu_e$ from sun:
Constraint on ~~CPT~~ models



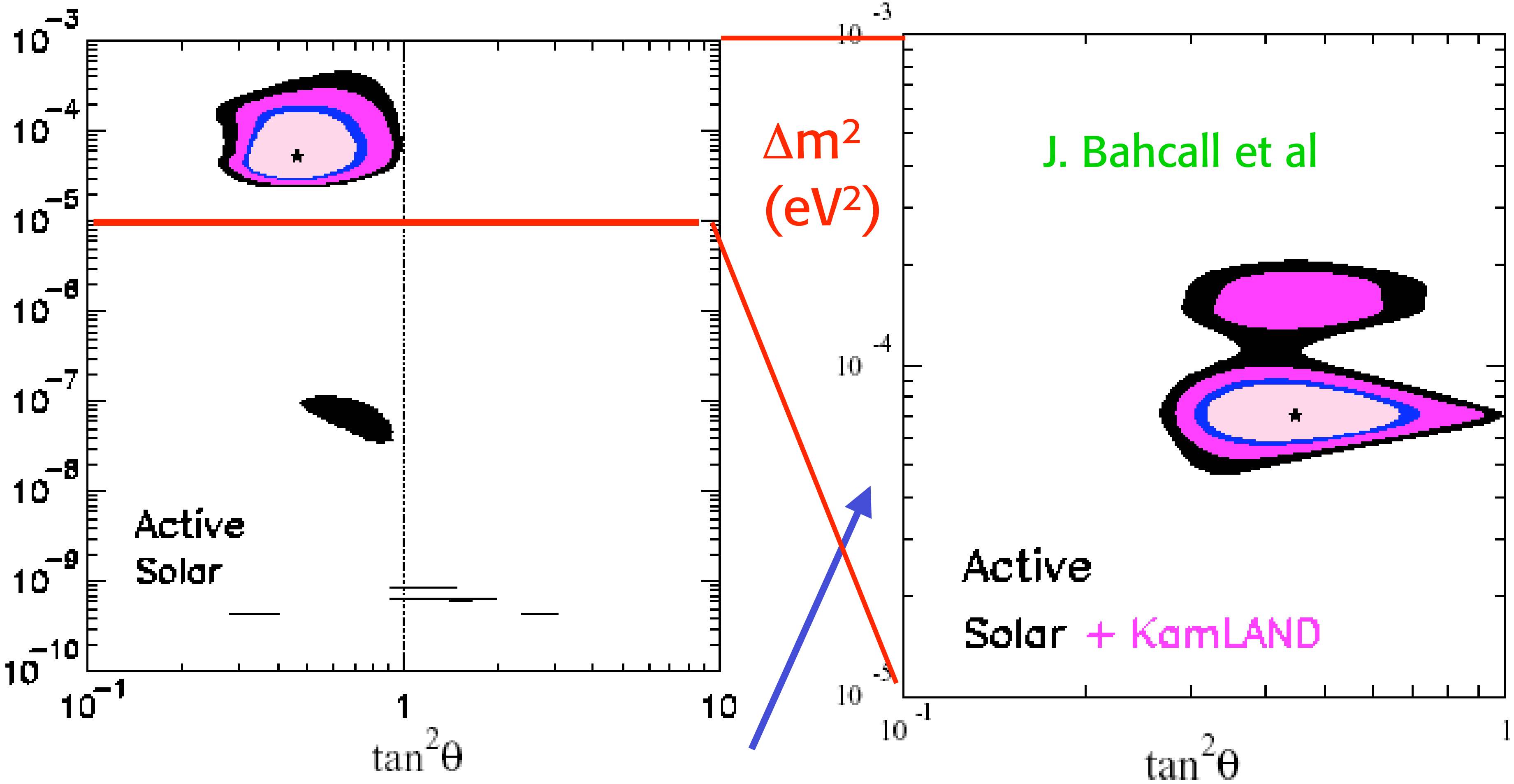




Best fit

# In summary for solar ν's:

Before Kamland

After Kamland

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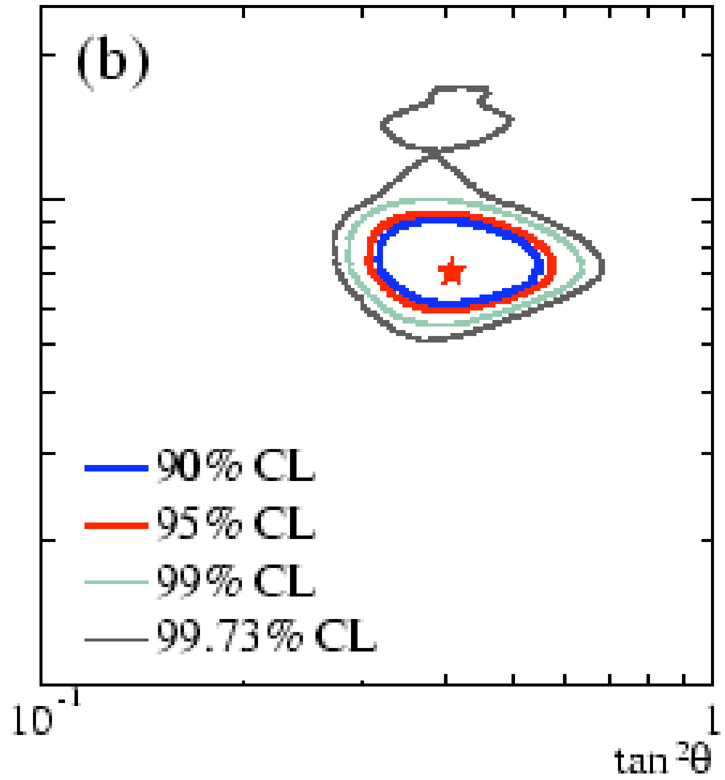
Note the change of scale

New!

# Sept.'03: SNO new results

All data now




Salt added to $D_2O$:
Better NC sensitivity

- Previous results confirmed
- More precision
- The upper $\Delta m^2$ part of the LA sol. now disfavoured
- $\theta_{12}$ is now $5.4\sigma$ from maximal

# $\nu$ Oscillations: Summary of Exp. Facts

Homestake, Gallex, Sage, (Super)Kamiokande, Macro... GNO,K2K,..

after KAMLAND, SNO-salt

## Atmospheric:

$\Delta m^2_{atm} \sim 2.5 \cdot 10^{-3}$ eV$^2$
$\sin^2\theta_{23} \sim 1/2$

$\nu_\mu \to \nu_\tau$ dominant
$\nu_\mu \to \nu_e$ small
(Chooz $|U_{13}| < \sim 0.2$)
$\nu_\mu \to \nu_{sterile}$ small

## Solar:

The MSW-LA solution selected
$\Delta m^2 \sim 7\ 10^{-5}$ eV$^2$, $\sin^2\theta_{12} \sim 0.3$

$\nu_e \to \nu_\mu, \nu_\tau$ dominant
$\nu_e \to \nu_{sterile}$ small

## LSND:

true or false?
MINIBOONE (in progress)

$\Delta m^2 \sim 1$ eV$^2$, $\sin^2\theta \sim$ small

$\nu_\mu \to \nu_e, \nu_{sterile}$
CPT violation?

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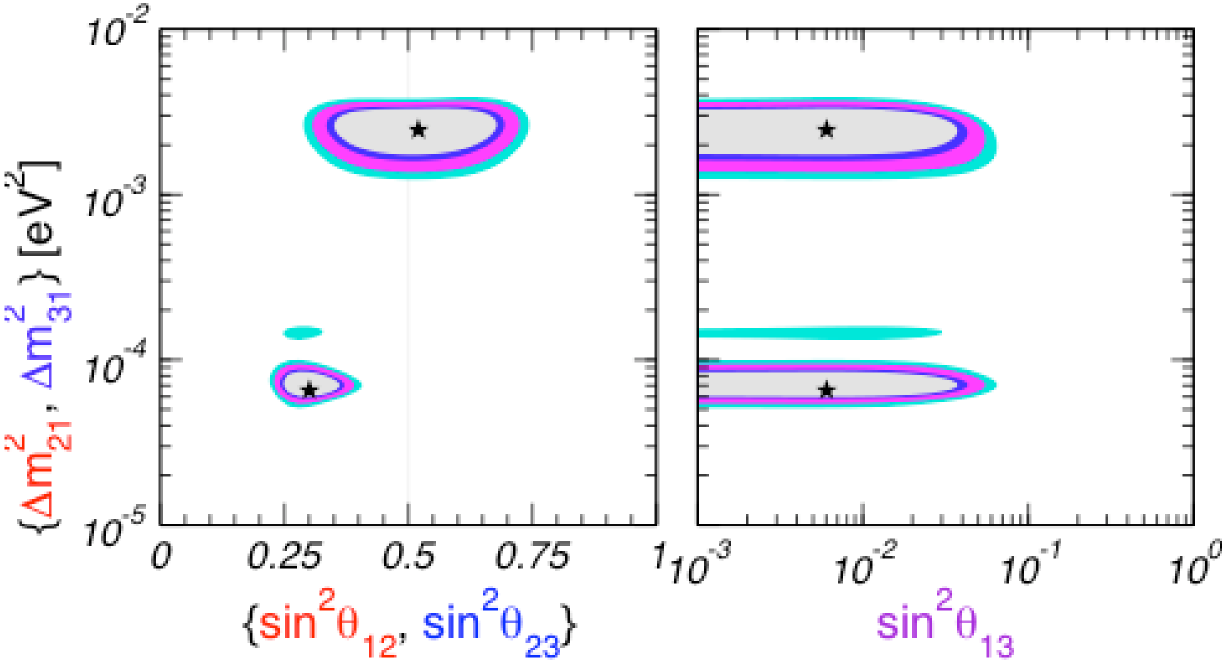
Maltoni et al

| parameter | best fit | $2\sigma$ | $3\sigma$ | $5\sigma$ |
|---|---|---|---|---|
| $\Delta m^2_{21}$ $[10^{-5}\text{eV}^2]$ | 6.9 | 6.0–8.4 | 5.4–9.5 | 2.1–28 |
| $\Delta m^2_{31}$ $[10^{-3}\text{eV}^2]$ | 2.6 | 1.8–3.3 | 1.4–3.7 | 0.77–4.8 |
| $\sin^2\theta_{12}$ | 0.30 | 0.25–0.36 | 0.23–0.39 | 0.17–0.48 |
| $\sin^2\theta_{23}$ | 0.52 | 0.36–0.67 | 0.31–0.72 | 0.22–0.81 |
| $\sin^2\theta_{13}$ | 0.006 | $\lesssim 0.035$ | $\lesssim 0.054$ | $\lesssim 0.11$ |

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# $\nu$ oscillations measure $\Delta m^2$. What is $\overline{m}$?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} \sim 7\ 10^{-5}\ eV^2$

- Direct limits (PDG '02)

End-point tritium $\beta$ decay (Mainz)

$m_{"\nu e"} < 2.8$ eV
$m_{"\nu\mu"} < 170$ KeV
$m_{"\nu\tau"} < 18.2$ MeV

- $0\nu\beta\beta$

- Cosmology $\Omega_\nu\ h^2 \sim \Sigma_i m_i\ /94eV$ ($h^2 \sim 1/2$)

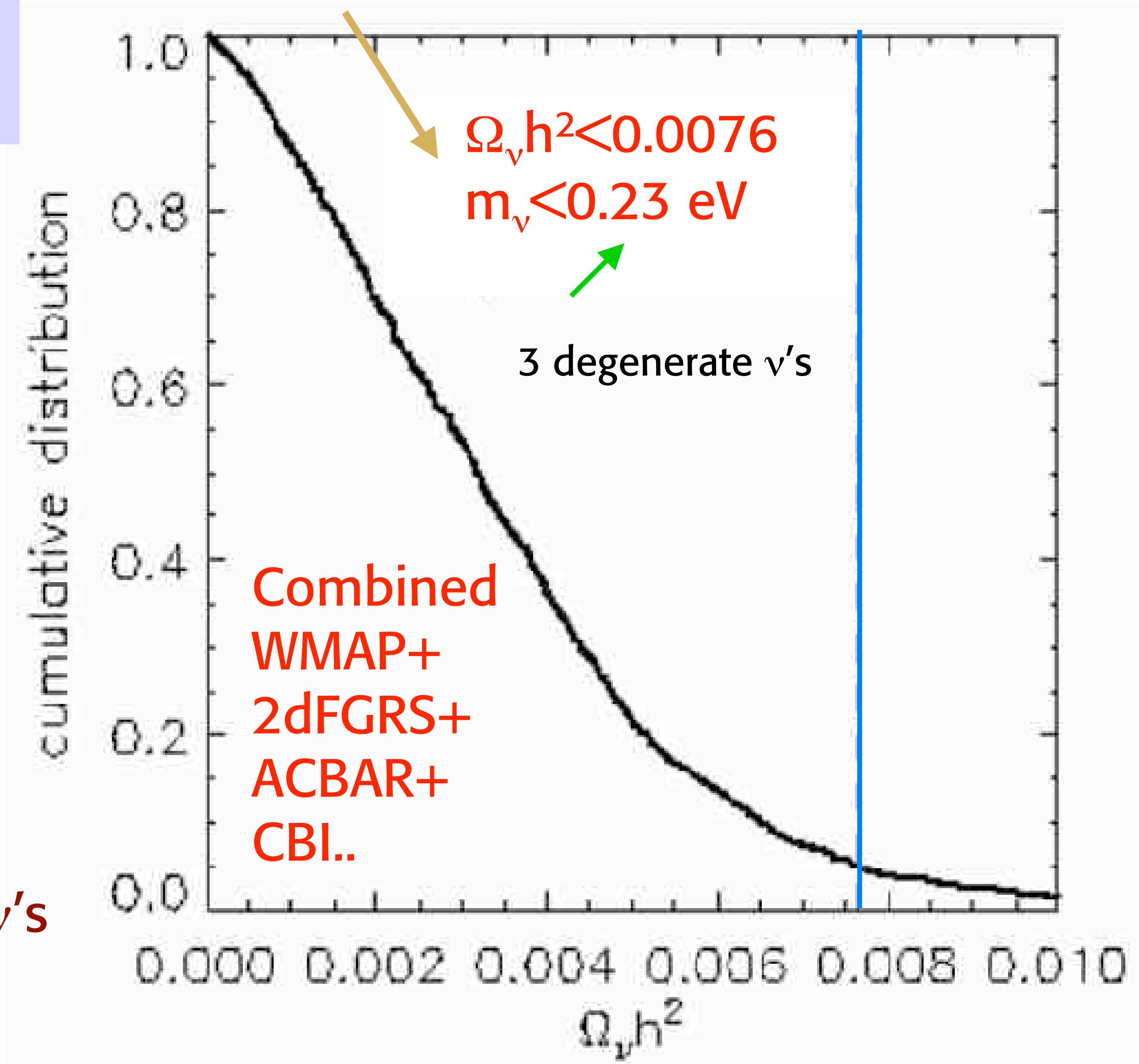
$\Sigma_i m_i\ \sim 0.69$ eV (95%) $[\Omega_\nu \sim 0.014]$ WMAP

→ Any $\nu$ mass 0.23-1eV →

**Why $\nu$'s so much lighter than quarks and leptons?**

New powerful cosmological limit

Assumes some priors!
Could be somewhat relaxed




All info on the absolute scale of ν mass is very important!

Finding $0\nu\beta\beta$ would also prove Majorana ν's

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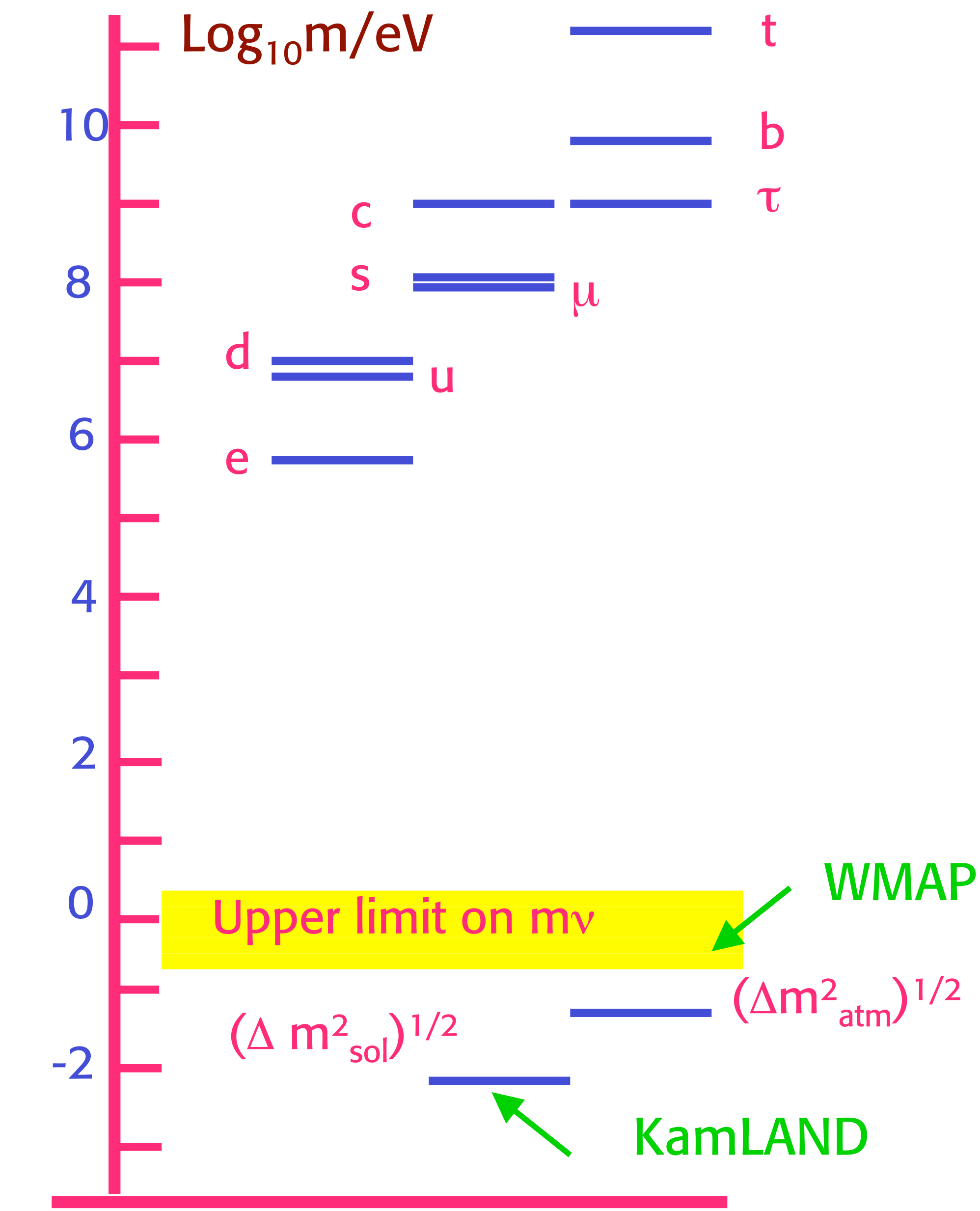


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# Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

**Massless ν's?**

- no $\nu_R$
- L conserved

**Small ν masses?**

- $\nu_R$ very heavy
- L not conserved

# How to guarantee a massless neutrino?

1) $\nu_R$ does not exist

→ No Dirac mass

$$\bar{\nu}_L \nu_R + \nu_R \bar{\nu}_L$$

and

2) Lepton Number is conserved

→ No Majorana mass

$$\bar{\nu}^c \nu -> \nu^T{}_R C \nu_R \text{ or } \nu^T{}_L C \nu_L$$

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$C = i\gamma^0\gamma^2$

## Neutrinos:

**Dirac mass:** $\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L$

(needs $\nu_R$)

$\nu$'s have no electric charge. Their only charge is lepton number L

**IF** L is not conserved (not a good quantum number)

**$\nu$ and $\bar{\nu}$ are not really different**

TCP, "Lorentz"

$| \nu, h= -1/2>$ → $| \bar{\nu}, h= +1/2>$

**Majorana mass:** $\nu^T_R \nu_R$ or $\nu^T_L \nu_L$

(we omit the charge conj. matrix C)

**Violates L, B-L by $|\Delta L| = 2$**

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## Weak isospin I

$\nu_L \Rightarrow I = 1/2, I_3 = 1/2$

$\nu_R \Rightarrow I = 0, I_3 = 0$

Dirac Mass:

$\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L$ $|\Delta I| = 1/2$

Can be obtained from Higgs doublets: $\nu_L \bar{\nu}_R H$

Majorana Mass ($\Delta L = 2$):

- $\nu^T{}_L \nu_L$ $|\Delta I| = 1$

Non ren., dim. 5 operator: $\nu^T{}_L \nu_L HH$

- $\nu^T{}_R \nu_R$ $|\Delta I| = 0$

Directly compatible with SU(2)xU(1)!

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# See-Saw Mechanism

Yanagida; Glashow;
Gell-Mann, Ramond , Slansky;
Mohapatra, Senjanovic

$M\nu^T{}_R\nu_R$ allowed by SU(2)xU(1)

Large Majorana mass M (as large as the cut-off)

$m_D\bar{\nu}_L\nu_R$

Dirac mass m from Higgs doublet(s)

$$\begin{array}{c c} & \begin{array}{cc}\nu_L & \nu_R\end{array} \\ \begin{array}{c}\nu_L \\ \nu_R\end{array} & \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix} \end{array}$$

$M >> m_D$

Eigenvalues

$$\nu_{light} = \frac{-m_D^2}{M} \quad , \quad \nu_{heavy} = M$$

sign conventional for fermions

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In general $\nu$ mass terms are:

$$\mathcal{L}_{\nu} = \bar{L} h \nu_R H + \text{h.c.} + \nu_R^T M_R \nu_R + + \nu_L^T \frac{\lambda}{M_L} \nu_L H H$$

Dirac
$m_D = hv$
$v = \langle 0|H|0 \rangle$

Maiorana

$$m = \frac{\lambda v^2}{M_L}$$

More general see-saw mechanism:

| | $\nu_L$ | $\nu_R$ |
|---|---|---|
| $\nu_L$ | $\lambda v^2/M_L$ | $m_D$ |
| $\nu_R$ | $m_D$ | $M_R$ |

$m_{light} \sim \frac{m_D^2}{M_R}$ and/or $\frac{\lambda v^2}{M_L}$

$m_{heavy} \sim M_R$

$m_{eff} = \nu^T_L m_{light} \nu_L$

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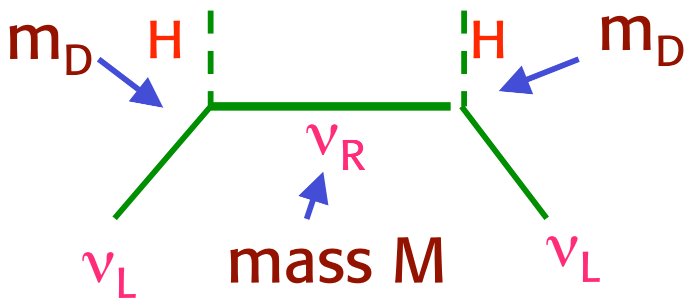
Neutrinos are (probably) Majorana particles: $\nu_L^T m_\nu \nu_L$

See-saw

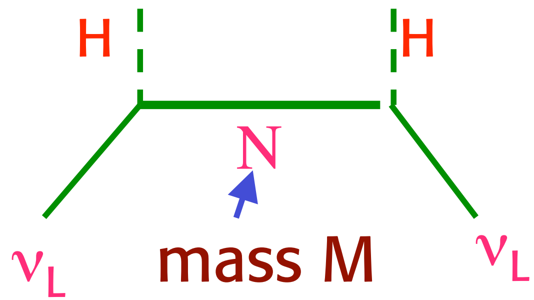



$m_\nu = m_D^T M^{-1} m_D$

connection with $m_D$

More in general: non ren. $O_5$ operator $\lambda/M\ \nu_L^T H H^T \nu_L$

e.g from




N: new particle $I_w$=0,1

A very natural and appealing explanation:

$\nu$'s are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{GUT}$

$$m_\nu \sim \frac{m^2}{M}$$

$m \quad m_t \sim v \sim 200$ GeV

M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV

$m \sim v \sim 200$ GeV

$M \sim 10^{15}$ GeV

**Neutrino masses are a probe of physics at $M_{GUT}$ !**

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# B and L conservation in SM:

"Accidental" symmetries: in SM there is no dim.4 gauge invariant operator that violates B and/or L (if no $\nu_R$, otherwise $M\, \nu^T_R\, \nu_R$ is dim-3 $|\Delta L|=2$)
The same is true in SUSY with R-parity cons.

e. g. for the $\Delta B = \Delta L = -1$ transition $u + u \to e^+ + \bar{d}$

all good quantum numbers are conserved:
e.g. colour $u \sim 3$, $\bar{d} \sim \bar{3}$ and $3 \times 3 = 6 + \bar{3}$ but

$$\frac{\lambda}{M^2}\, \overline{d^c}\Gamma u\, \overline{e^c}\Gamma u \longrightarrow \text{dim-6}$$

SU(5): $p \to e^+ \pi^0$

Once $\nu_R$ is introduced (Dirac mass) large Majorana mass is naturally induced → see-saw

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## Dark Matter

WMAP

Most of the Universe is not made up of atoms: $\Omega_{tot} \sim 1$, $\Omega_b \sim 0.044$, $\Omega_m \sim 0.27$
Most is Dark Matter and Dark Energy

Most Dark Matter is Cold (non relativistic at freeze out)
Significant Hot Dark matter is disfavoured
Neutrinos are not much cosmo-relevant: $\Omega_\nu < 0.015$ (WMAP)

SUSY has excellent DM candidates: Neutralinos
Also Axions are still viable

For 3 neutrinos: $\Omega_\nu < 0.015$ -> $m_\nu < 0.23$ eV ~ $5(\Delta m^2_{atm})^{1/2}$

the exact value depends on the cosmological model: can be somewhat relaxed

$m_\nu < \sim 1$ eV

**Baryogenesis** A most attractive possibility:

**BG via Leptogenesis near the GUT scale**

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al

Only survives if $\Delta$(B-L) is not 0
(otherwise is washed out at $T_{ew}$ by instantons)

Main candidate: decay of lightest $\nu_R$ ($M \sim 10^{12}$ GeV)

L non conserv. in $\nu_R$ out-of-equilibrium decay:
B-L excess survives at $T_{ew}$ and gives the obs. B asymm.

Quantitative studies confirm that the range of $m_i$ from $\nu$ oscill's is compatible with BG via (thermal) LG

In particular the bound was derived $m_i < 10^{-1}$ eV Close to WMAP

Buchmuller, Di Bari, Plumacher
Giudice et al

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# The scale of the cosmological constant is a big mystery.

$\Omega_\Lambda \sim 0.65$ → $\rho_\Lambda \sim (2\ 10^{-3}\ \text{eV})^4 \sim (0.1\text{mm})^{-4}$ → Similar to $m_\nu$!?

In Quantum Field Theory: $\rho_\Lambda \sim (\Lambda_{cutoff})^4$

If $\Lambda_{cutoff} \sim M_{Pl}$ → $\rho_\Lambda \sim 10^{123}\ \rho_{obs}$

Exact SUSY would solve the problem: $\rho_\Lambda = 0$

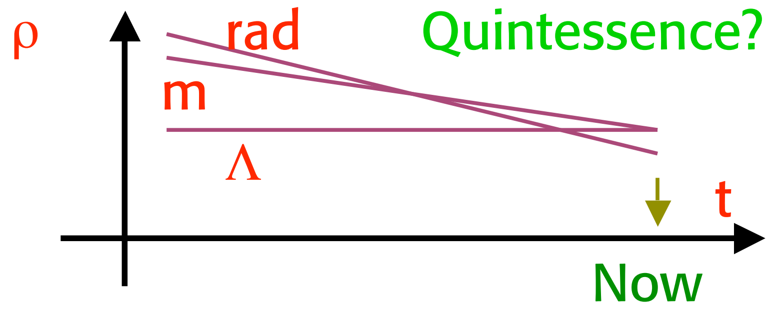
But SUSY is broken: $\rho_\Lambda \sim (\Lambda_{SUSY})^4 \sim 10^{59}\ \rho_{obs}$

It is interesting that the correct order is $(\rho_\Lambda)^{1/4} \sim (\Lambda_{EW})^2/M_{Pl}$

So far no solution:
- A modification of gravity at 0.1mm?(large extra dim.)
- Leak of vac. energy to other universes (wormholes)?

•••

Other problem:
Why now?

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## The current experimental situation is still unclear

- LSND: true or false?
- what is the absolute scale of $\nu$ masses?
- …

## Different classes of models are possible:

**If LSND true**

sterile $\nu$(s)??

CPT violat'n??

- "3-1"

$\nu_{sterile}$

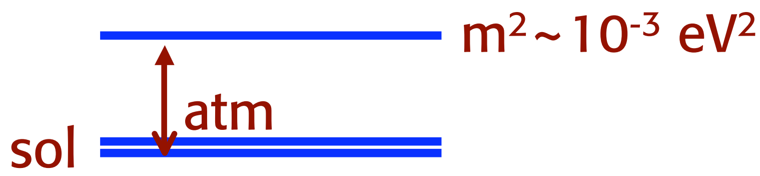

**If LSND false** → 3 light $\nu$'s are OK

We assume this case here

- Degenerate ($m^2 >> \Delta m^2$)


- Inverse hierarchy


- Normal hierarchy

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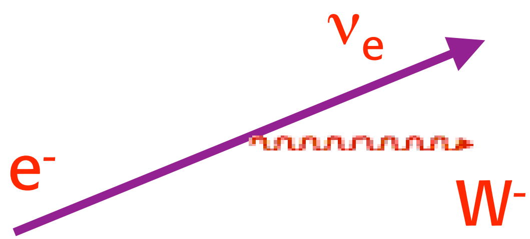
# 3-ν Models

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour mass




$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where $e^-$, $\mu^-$, $\tau^-$ are diagonal:

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

s = solar: large

$$\sim \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ \ldots & \ldots & c_{13}s_{23} \\ \ldots & \ldots & c_{13}c_{23} \end{bmatrix}$$

CHOOZ: $|s_{13}| <\sim 0.2$

atm.: ~ max

$$U \approx \begin{bmatrix} c & -s & 0 \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(some signs are conventional)

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$$m_\nu \sim U \begin{bmatrix} e^{i\phi_1} m_1 & 0 & 0 \\ 0 & e^{i\phi_2} m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U^T$$

In general 9 parameters:
3 masses, 3 angles,
3 phases

$L^T m_\nu L$

For $s_{13} \sim 0$:

$0\nu\beta\beta$

$$m_\nu \sim \begin{bmatrix} m_1 c^2 + m_2 s^2 & (m_1 - m_2) cs/\sqrt{2} & (m_1 - m_2) cs/\sqrt{2} \\ \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 & (m_1 s^2 + m_2 c^2 - m_3)/2 \\ \dots & \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 \end{bmatrix}$$

Note:
- $m_\nu$ is symmetric
- phases included in $m_i$

Relation between masses and frequencies:

$P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_e \leftrightarrow \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$P(\nu_\mu \leftrightarrow \nu_\tau) = \sin^2 \Delta_{atm} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \quad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

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In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} >$ or $< 0$

# $0\nu\beta\beta$ can tell degenerate, inverted or normal hierarchy

$$|m_{ee}|=c_{13}^2\,[m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$$

LA: ~0.3-1

**Degenerate:** $\sim|m|\,|c_{12}^2+e^{i\alpha}s_{12}^2|$

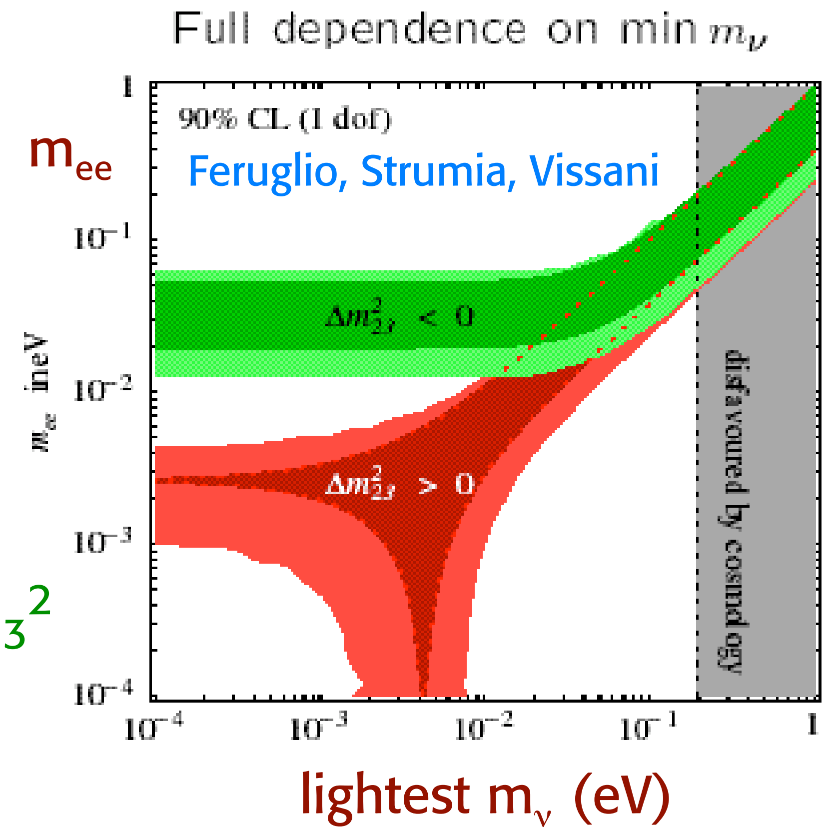
$|m_{ee}|\sim|m|\,(0.3-1)< 0.23\text{-}1$ eV

**IH:** $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim(1.6\text{-}5)\,10^{-2}$ eV

**NH:** $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim$ (few) $10^{-3}$ eV




Present exp. limit: $m_{ee}< 0.3\text{-}0.5$ eV
(and a hint of signal?????)

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# Evidence for $0\nu\beta\beta$?

Heidelberg-Moscow
Klapdor-Kleingrothaus et al

Not at all compelling!!!!

New recent ('04) paper

Iff true: (WMAP ??)

$m_{ee}/z = 0.39 \pm 0.11\,\mathrm{eV} \gg (\Delta m^2_{atm})^{1/2}$

(z~0.6-2.8
uncert. matrix element)

would clearly point
to degenerate models

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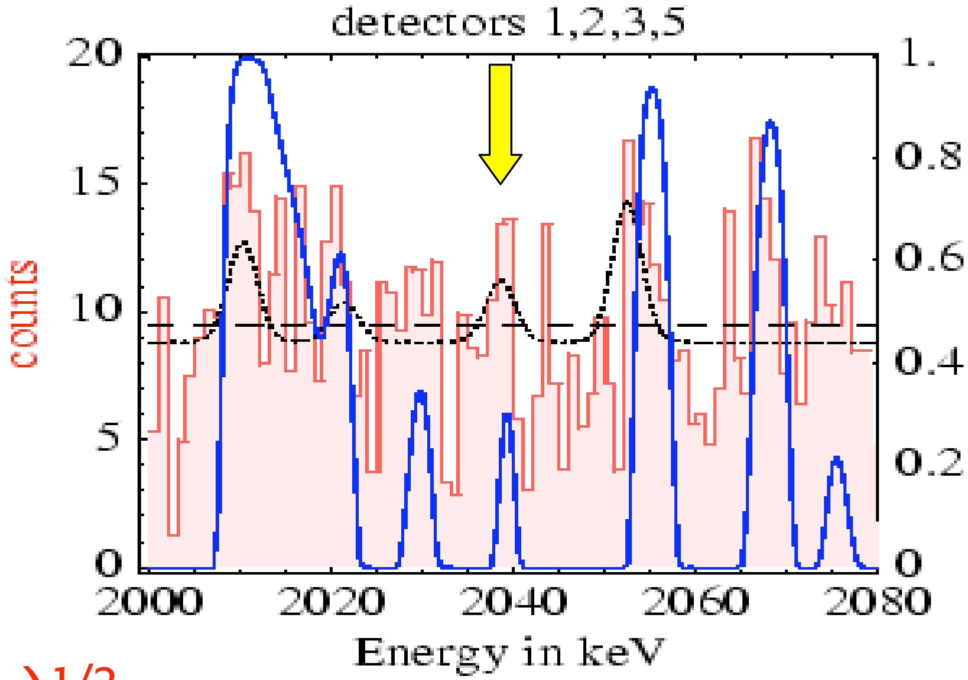
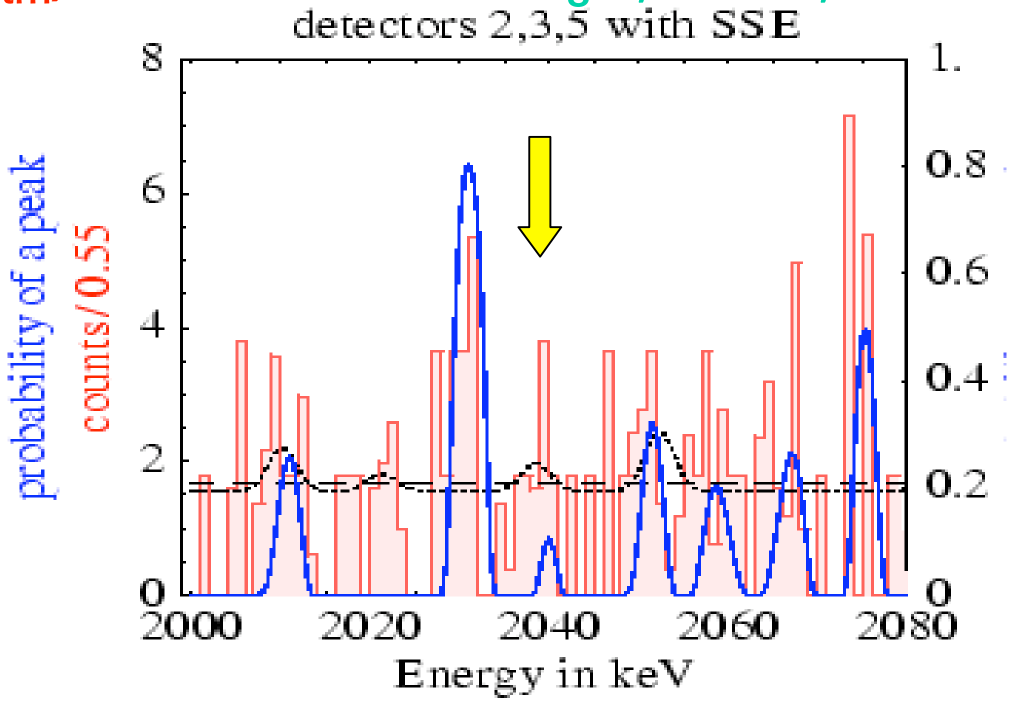



Feruglio, Strumia, Vissani



## Degenerate $\nu$'s

$m^2 >> \Delta m^2$

- Apriori compatible with hot dark matter ($m \sim 1\text{-}2$ eV)
  → was considered by many
- Limits on $m_{ee}$ from $0\nu\beta\beta$ then imply large mixing also for solar oscillations: (Vissani; Georgi,Glashow)

$m_{ee} < 0.3\text{-}0.5$ eV (Exp)

$$m_{ee} = c^2_{13}\,(m_1 c^2_{12} + m_2 s^2_{12}) + s^2_{13} m_3 \sim m_1 c^2_{12} + m_2 s^2_{12}$$

If $|m_1| \sim |m_2| \sim |m_2| \sim 1\text{-}2$ eV → $m_1 = -m_2$ and $c^2_{12} \sim s^2_{12}$

LA solution: $\sin^2\theta \sim 0.3$ → $\cos^2\theta - \sin^2\theta \sim 0.4$

a moderate suppression factor!

Trusting WMAP: $|m| < 0.23$ eV, only a moderate degeneracy is allowed: for LA, $m/(\Delta m^2_{atm})^{1/2} < 5$, $m/(\Delta m^2_{sol})^{1/2} < 30$.

Less constraints from $0\nu\beta\beta$ (both $m_1 = \pm m_2$ allowed)

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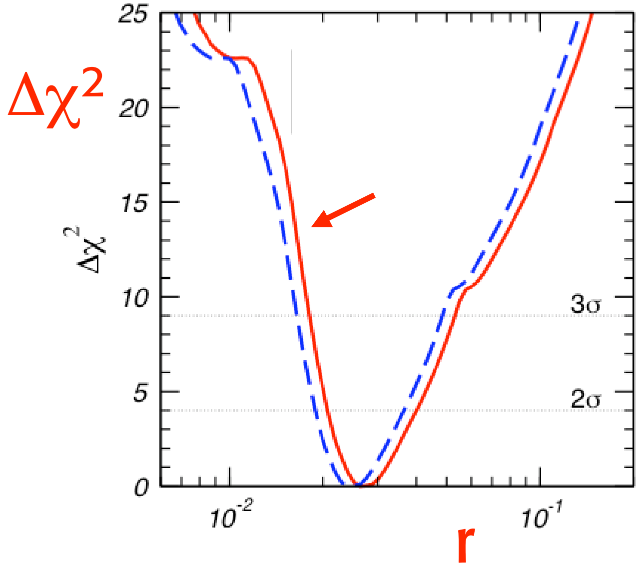
Recall: leptogenesis prefers $|m| < 0.1$ eV

After KamLAND, SNO and WMAP not too much hierarchy is needed for $\nu$ masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/40$$




Precisely at $3\sigma$: $0.018 < r < 0.053$

or

$m_{heaviest} < 1 - 0.23$ eV
$m_{next} > \sim 7\ 10^{-3}$ eV

→ Anarchical or semi-anarchical models

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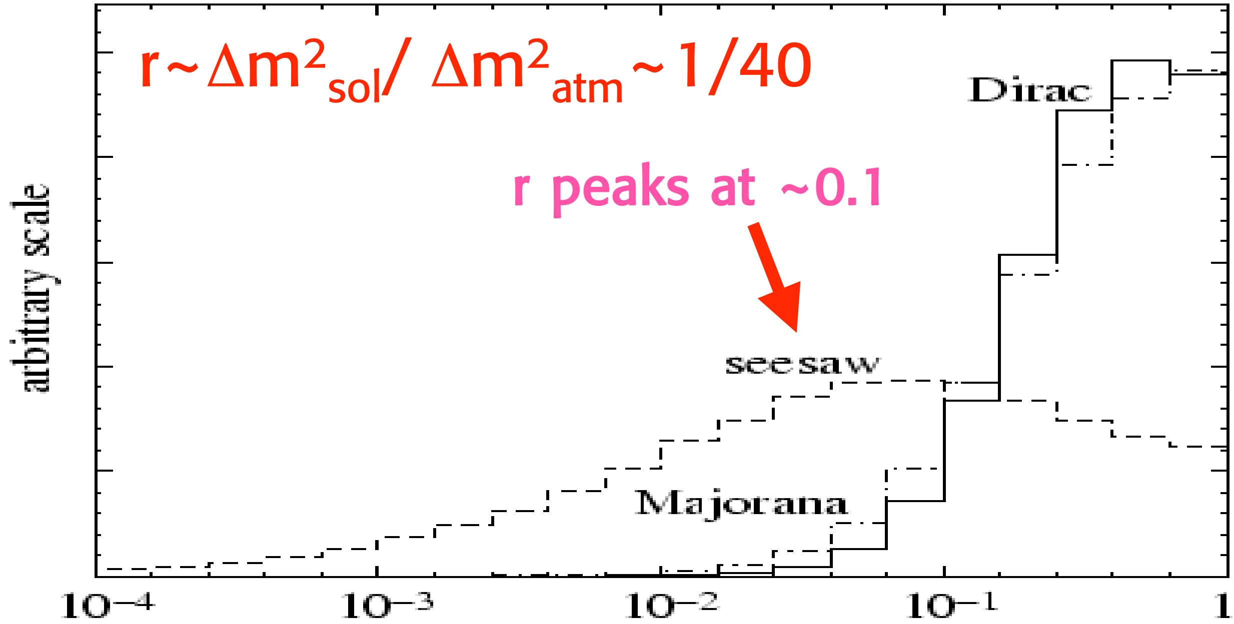
# Anarchy (or accidental hierarchy): No structure in the leptonic sector

Hall, Murayama, Weiner

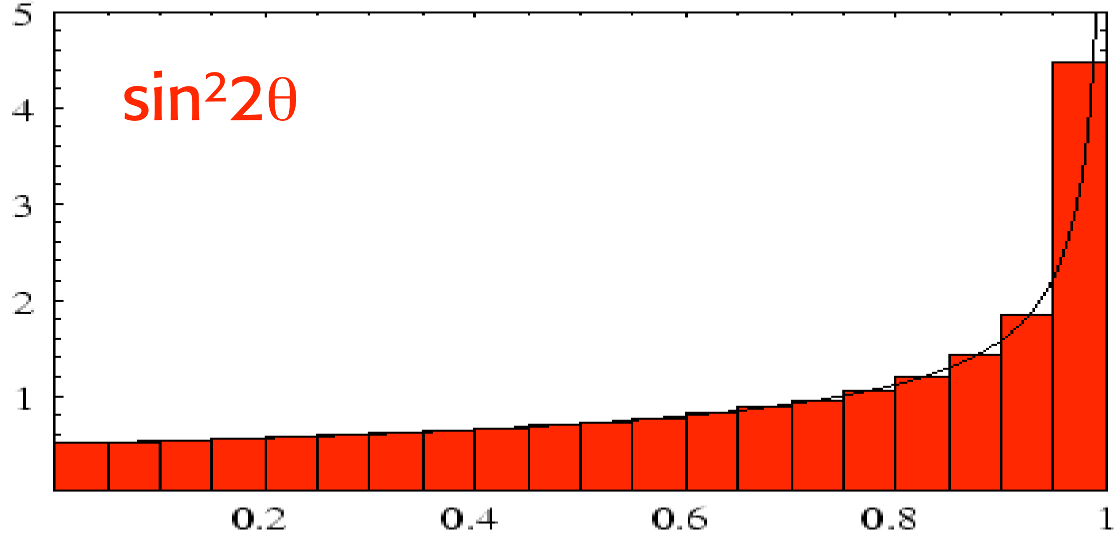
See-Saw:
$m_\nu \sim m^2/M$
produces hierarchy from random m,M

could fit LA




But: all mixing angles should be large

marginal for LA → predicts $\theta_{13}$ near bound

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# Semianarchy: no structure in 23

Consider a matrix like

$$m_\nu \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

Note: $\theta_{13} \sim \lambda$
$\theta_{23} \sim 1$

with coeff.s of o(1) and det23~o(1)
[$\lambda$~1 corresponds to anarchy]

After 23 and 13 rotations

$$m_\nu \sim \begin{pmatrix} \lambda^2 & \lambda & 0 \\ \lambda & \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Normally two masses are of o(1) and $\theta_{12} \sim \lambda$
But if, accidentally, $\eta \sim \lambda$, then the solar angle is also large.

The advantage over anarchy is that $\theta_{13}$ is small, but
the hierarchy $m^2_3 >> m^2_2$ is accidental

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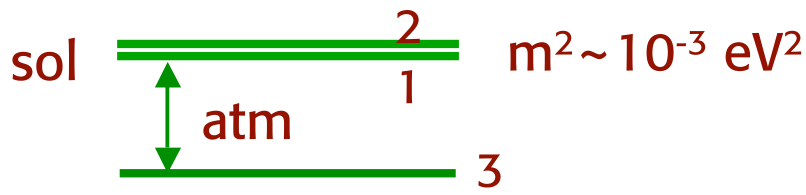
Ramond et al, Buchmuller et al

## Inverted Hierarchy




Zee, Joshipura et al;
Mohapatra et al; Jarlskog et al;
Frampton,Glashow; Barbieri et al
Xing; Giunti, Tanimoto

An interesting model for double maximal mixing (bimixing):

$$U \sim \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{bmatrix}$$

1st approximation

$$m_{\nu diag} = \begin{pmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad U m_{\nu diag} U^T = 1/\sqrt{2} \begin{pmatrix} 0 & m & m \\ m & 0 & 0 \\ m & 0 & 0 \end{pmatrix}$$

Can arise from see-saw or dim-5 $L^T H H^T L$

e.g. by approximate $L_e$-$L_\mu$-$L_\tau$ symmetry

- 1-2 degeneracy stable under rad. corr.'s

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1st approximation

$$m_{\nu diag} = \begin{bmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad U m_{\nu diag} U^T = 1/\sqrt{2} \begin{bmatrix} 0 & m & m \\ m & 0 & 0 \\ m & 0 & 0 \end{bmatrix}$$

- LA? This texture prefers $\theta_{sol}$ closer to maximal than $\theta_{atm}$
  i.e $\theta_{sol} - \pi/4$ small for $(\Delta m^2_{sol}/\Delta m^2_{atm})_{LA} \sim 1/40$

In fact: 12-> $\begin{bmatrix} 0 & m \\ m & 0 \end{bmatrix} \rightarrow$ Pseudodirac $\theta_{12}$ maximal    23-> $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \theta_{23} \sim o(1)$

With perturbations: $\begin{bmatrix} 0 & m & m \\ m & 0 & 0 \\ m & 0 & 0 \end{bmatrix} \longrightarrow m \begin{bmatrix} \delta & 1 & 1 \\ 1 & \eta & \eta \\ 1 & \eta & \eta \end{bmatrix}$

$tg^2\,\theta_{12} \sim 1 + o(\delta + \eta)$    $(\Delta m^2_{sol}/\Delta m^2_{atm})_{LA} \sim o(\delta + \eta)$

- In principle one can use the charged lepton mixing
  to go away from $\theta_{12}$ maximal.
  In practice constraints from $\theta_{13}$ small $(\delta\theta_{12} \sim \theta_{13})$

Frampton et al; GA, Feruglio, Masina '04

For the corrections to bimixing from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1-\tan^2\theta_{12})/4$

GA, Feruglio, Masina '04

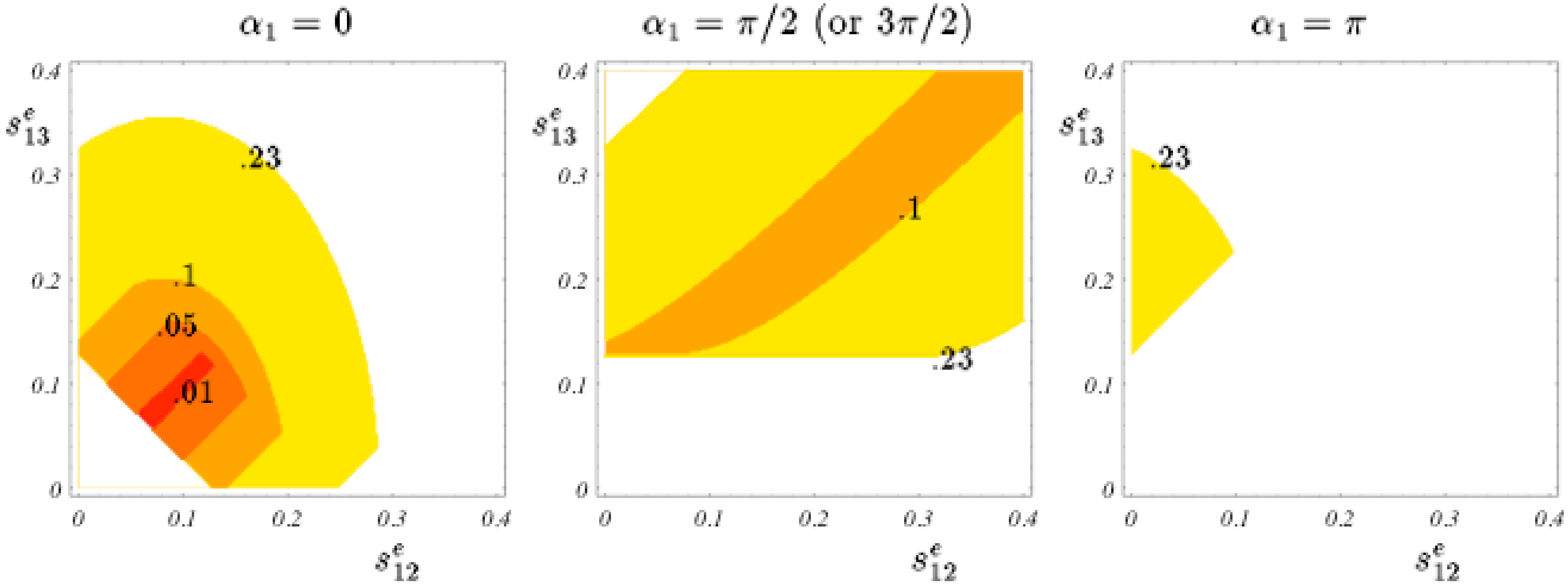


Figure 1: Taking an upper bound on $|U_{e3}|$ respectively equal to 0.23, 0.1, 0.05, 0.01, we show (from yellow to red) the allowed regions of the plane $[s^e_{12}, s^e_{13}]$. Each plot is obtained by setting $\alpha_1$ to a particular value, while leaving $\alpha_2 + \delta_e$ free. We keep the present 3 $\sigma$ window for $\delta_{sol}$ [10].

- In general more $\theta_{12}$ is close to maximal, more is IH likely

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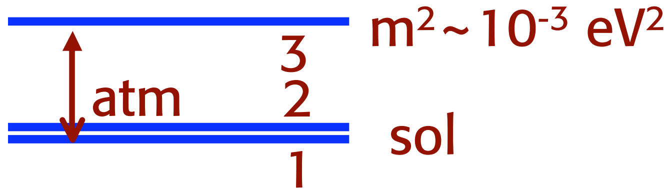
# Normal Hierarchy




- Assume 3 widely split light neutrinos.
- For u, d and $l^-$ Dirac matrices the 3rd generation eigenvalue is dominant.
- May be this is also true for $m_{\nu D}$: diag $m_{\nu D} \sim (0,0,m_{D3})$.
- Assume see-saw is dominant: $m_\nu \sim m^T_D M^{-1} m_D$
  See-saw quadratic in $m_D$: tends to enhance hierarchy
- Maximally constraining: GUT's relate q, $l^-$, $\nu$ masses!

- A crucial point: in the 2-3 sector we need both large $m_3$-$m_2$ splitting and large mixing.

  $m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}$ eV
  $m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 8\ 10^{-3}$ eV for LA

- The "theorem" that large $\Delta m_{32}$ implies small mixing (pert. th.: $\theta_{ij} \sim 1/|E_i-E_j|$) is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

  Det = 0; Eigenvl's: 0, $1+x^2$
  Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

So all we need are natural mechanisms for det[23]=0

For x~1
large splitting
and large mixing!

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# Examples of mechanisms for Det[23]~0

see-saw $m_\nu \sim m^T_D M^{-1} m_D$

1) A $\nu_R$ is lightest and coupled to $\mu$ and $\tau$

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but $m_D$ "lopsided" $\quad m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, .....

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

Caution: if 0 -> 0($\varepsilon$), det23=0 could be spoiled by suitable $1/\varepsilon$ terms in $M^{-1}$

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## An important property of SU(5)

Left-handed quarks have small mixings ($V_{CKM}$),
but right-handed quarks can have large mixings (unknown).

In SU(5):
LH for d quarks ⟷ RH for $l^-$ leptons

$\bar{5}$ → $m_d \sim \bar{d}_R d_L$ ← 10

10 → $m_e \sim \bar{e}_R e_L$ ← $\bar{5}$

$\bar{5}$ : $(\underbrace{\bar{d},\bar{d},\bar{d},}_{R}\ \underbrace{\nu,e^-}_{L})$

$$m_d = m_e^T$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models large atmospheric mixing arises (at least in part) from the charged lepton sector.

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- Hierarchical $\nu$'s and see-saw dominance

$$L^T m_\nu L \rightarrow m_\nu \sim m_D^2/M$$

allow to relate q, l, $\nu$ masses and mixings in GUT models.
For dominance of dim-5 operators -> less constraints

$$\lambda^2/M\ L^T LHH \rightarrow m_\nu \sim \lambda^2 v^2/M$$

- The correct pattern of masses and mixings, also including $\nu$'s, is obtained in simple models based on

$SU(5)xU(1)_{flavour}$

Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

- SO(10) models could be more predictive, as are non abelian flavour symmetries, eg O(3)

Albright, Barr; Babu et al; Buccella et al; Barbieri et al; Raby et al; King, Ross

- The non trivial pattern of fermion masses and mixing demands a flavour structure (symmetry)

- (SUSY) SU(5)XU(1)$_F$ models offer a minimal description of flavour symmetry

- A flexible enough framework used to realize and compare models with anarchy or hierarchy (direct or inverse) in $\nu$ sector, with see-saw dominance or not.

- On this basis we found that for LA there is still a significant preference for hierarchy vs anarchy

G.A., F. Feruglio, I. Masina, hep-ph/0210342 (v2 Nov '03)

Previous related work: Haba,Murayama; Hirsch,King; Vissani; Rosenfeld,Rosner; Antonelli et al….

## Hierarchy for masses and mixings via horizontal U(1) charges.

Froggatt, Nielsen '79

**Principle:**

A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by U(1) if $q_1+q_2+q_H$ not 0

$q_1$, $q_2$, $q_H$: U(1) charges of $\bar{R}_1$, $L_2$, H

U(1) broken by vev of "flavon" field $\theta$ with U(1) charge $q_\theta = -1$.
The coupling is allowed: if vev $\theta = w$, and $w/M = \lambda$ we get:

$\Delta_{charge}$

$$\bar{R}_1 m_{12} L_2 H\ (\theta/M)^{q1+q2+qH} \qquad m_{12} \to m_{12}\, \lambda^{q1+q2+qH}$$

Hierarchy: More $\Delta_{charge}$ -> more suppression ($\lambda$ small)

One can have more flavons ($\lambda$, $\lambda'$, ...)
with different charges (>0 or <0)etc -> many versions

With suitable charge assignments all relevant patterns can be obtained

1st fam. → 2nd → 3rd

$\Psi_{10}$: (5, 3, 0)
$\Psi_5$: (2, 0, 0) ← Equal 2,3 ch. for lopsided
$\Psi_1$: (1,-1, 0)

Recall: u ~ 10 10
$d = e^T \sim \bar{5}\, 10$
$\nu_D \sim \bar{5}\ 1$; $M_{RR} \sim 1\ 1$

| Model | $\Psi_{10}$ | $\Psi_{\bar{5}}$ | $\Psi_1$ | $(H_u, H_d)$ |
|---|---|---|---|---|
| Anarchical $(A)$ | (3,2,0) | (0,0,0) | (0,0,0) | (0,0) |
| Semi-Anarchical $(SA)$ | (2,1,0) | (1,0,0) | (2,1,0) | (0,0) |
| Hierarchical $(H_I)$ | (6,4,0) | (2,0,0) | (1,-1,0) | (0,0) |
| Hierarchical $(H_{II})$ | (5,3,0) | (2,0,0) | (1,-1,0) | (0,0) |
| Inversely Hierarchical $(IH_I)$ | (3,2,0) | (1,-1,-1) | (-1,+1,0) | (0,+1) |
| Inversely Hierarchical $(IH_{II})$ | (6,4,0) | (1,-1,-1) | (-1,+1,0) | (0,+1) |

No structure for leptons → Anarchical

No automatic det23 = 0 → Semi-Anarchical

all charges positive

Automatic det23 = 0 → Hierarchical

not all charges positive

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All entries are a given power of $\lambda$ times a free o(1) coefficient

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix}$$

In a statistical approach we generate these coeff.s as random complex numbers $\rho e^{i\phi}$ with $\phi =[0,2\pi]$ and $\rho$= [0.5,2] (default) or [0.8,1.2], or [0.95,1.05] or [0,1] (real numbers also considered for comparison)

For each model we evaluate the success rate (over many trials) for falling in the exp. allowed window:

(boundaries ~$3\sigma$ limits)

Maltoni et al, hep-ph/0309130

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

$0.018 < r < 0.053$

$|U_{e3}| < 0.23$

$0.30 < \tan^2\theta_{12} < 0.64$

$0.45 < \tan^2\theta_{23} < 2.57$

for each model the $\lambda,\lambda'$ values are optimised

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The optimised values of $\lambda$ are of the order of $\lambda_C$ or a bit larger (moderate hierarchy)

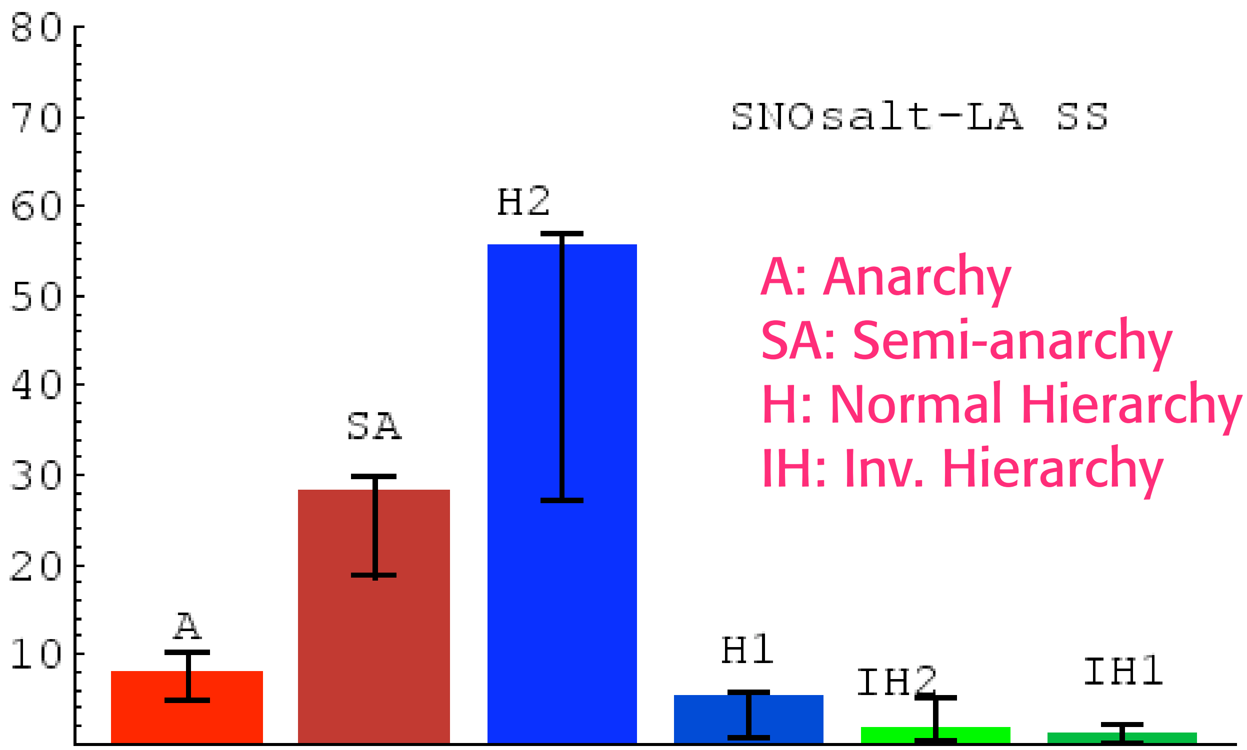
| model | $\lambda(=\lambda')$ |
|---|---|
| $A_{SS}$ | 0.2 |
| $SA_{SS}$ | 0.25 |
| $H_{(SS,II)}$ | 0.35 |
| $H_{(SS,I)}$ | 0.45 |
| $IH_{(SS,II)}$ | 0.45 |
| $IH_{(SS,I)}$ | 0.25 |

# Results with see-saw dominance (updated in Nov. '03):

1 or 2 refer to models with 1 or 2 flavons of opposite ch.

With charges of both signs and 1 flavon some entries are zero

Scale: $\Sigma$rates=100




A: Anarchy
SA: Semi-anarchy
H: Normal Hierarchy
IH: Inv. Hierarchy

Errors are linear comb. of stat. and syst. errors (varying the extraction procedure: interval of $\rho$, real or complex)

H2 is better than SA, better than A, better than IH

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# Example: Normal Hierarchy

G.A., Feruglio, Masina

Note: not all charges positive
--> det23 suppression

1st fam. 2nd 3rd

$q(10)$: $(5, 3, 0)$
$q(\bar{5})$: $(2, 0, 0)$
$q(1)$: $(1, -1, 0)$

$q(H) = 0$, $q(\bar{H}) = 0$
$q(\theta) = -1$, $q(\theta') = +1$

In first approx., with $\langle\theta\rangle/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

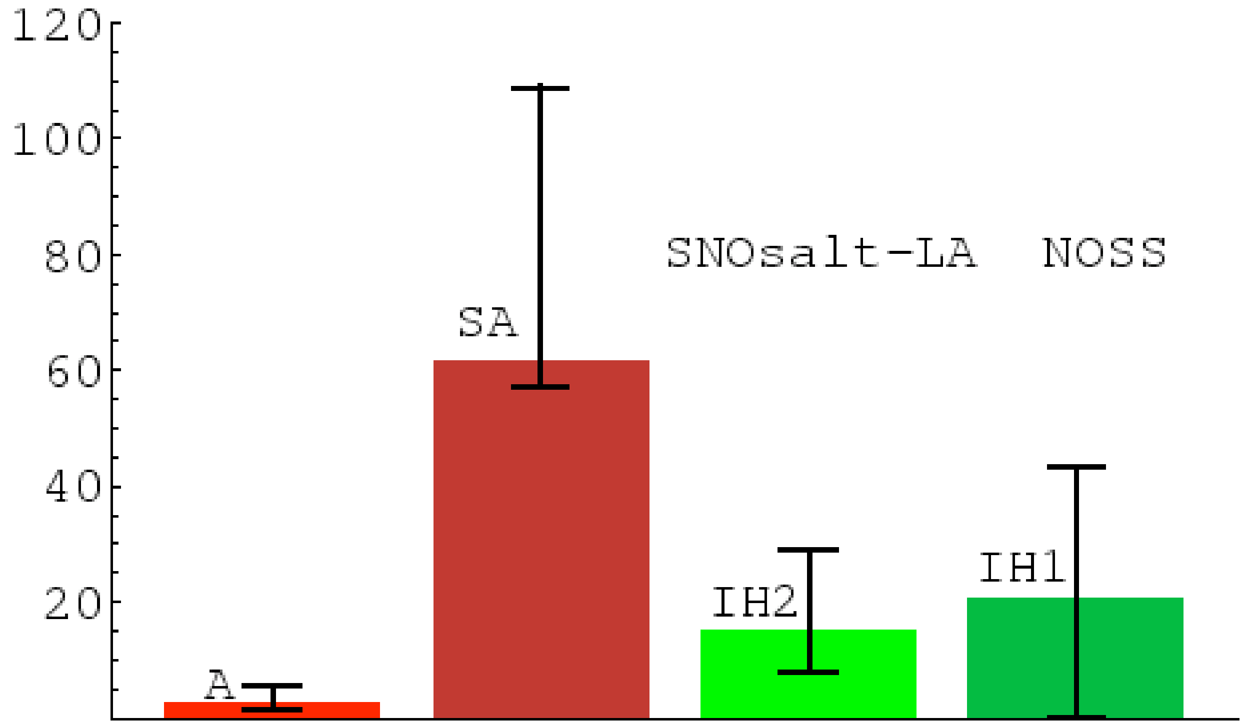
$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. 0(1) omitted, only orders of magnitude predicted

With no see-saw ($m_\nu$ generated directly from $L^T m_\nu L \sim \bar{5}\,\bar{5}\,H$) H is better than A

[With no-see-saw H coincide with SA]



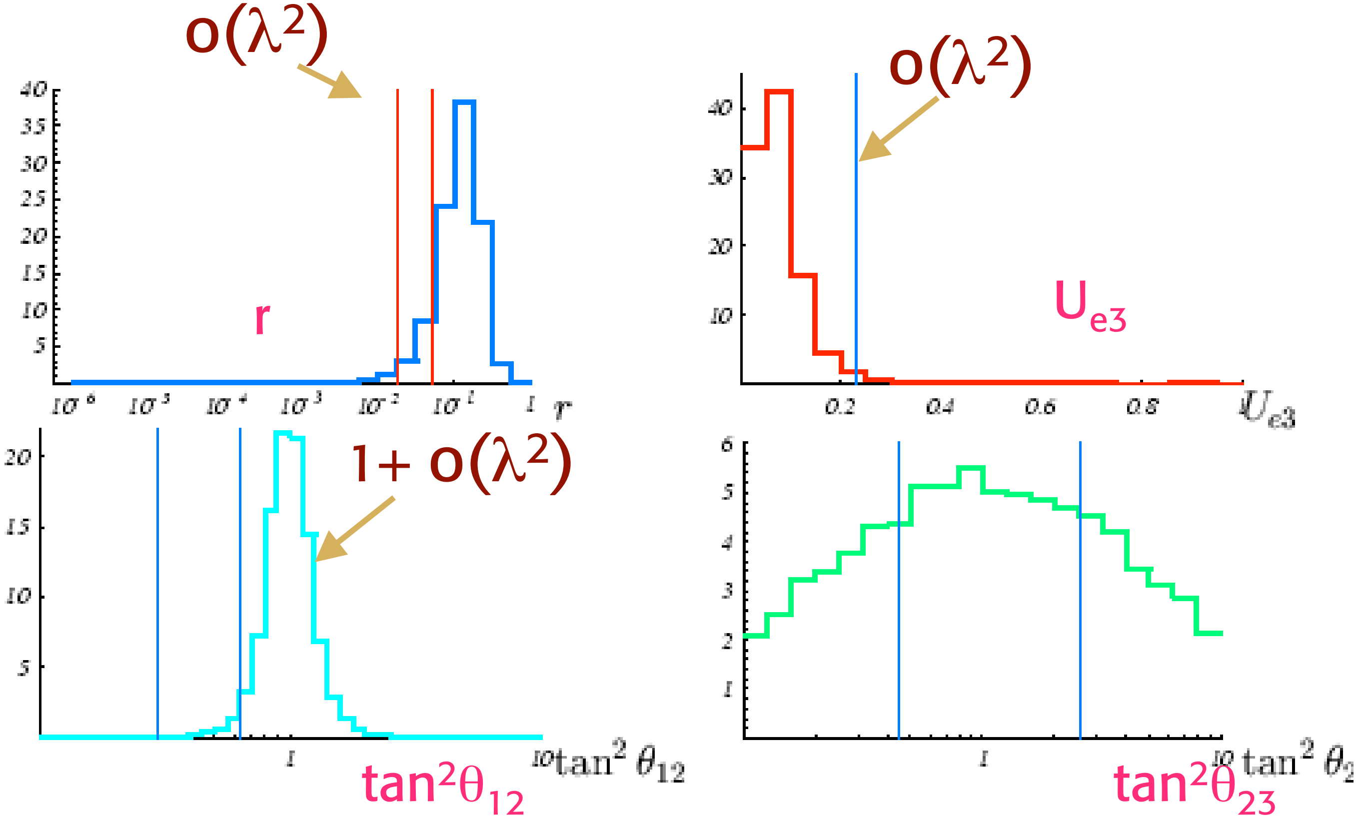

Note: we always include the effect of diagonalising charged leptons

# Some distributions

IH2 NO-SS $\lambda=\lambda'=0.3$

We see that IH tends to predict maximal solar mixing angle $\theta_{12}$

Only compatible because of ch. lepton diagonalisation




With data dritfing away from maximal $\theta_{12}$, IH is rapidly disfavoured (in U(1) models)

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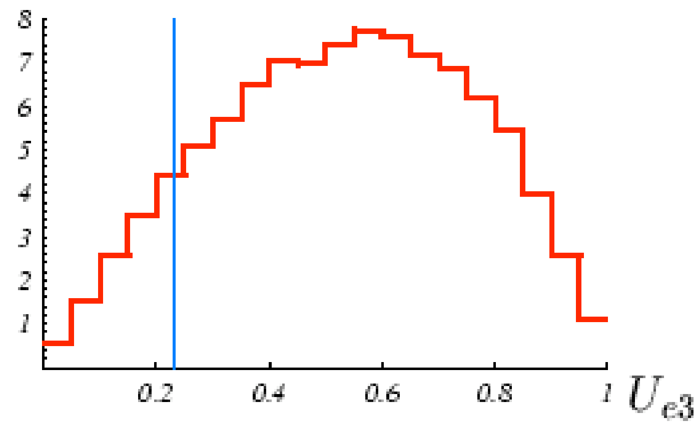
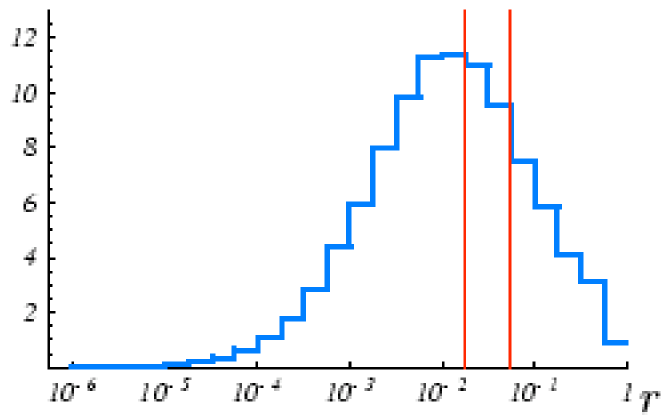
ch. lepton mixing small because $m_e$ small

## The main problem of Anarchy is $U_{e3}$ (as expected)

In all models the distr. for $\tan^2\theta_{23}$ is flat

$\lambda=\lambda'=0.2$

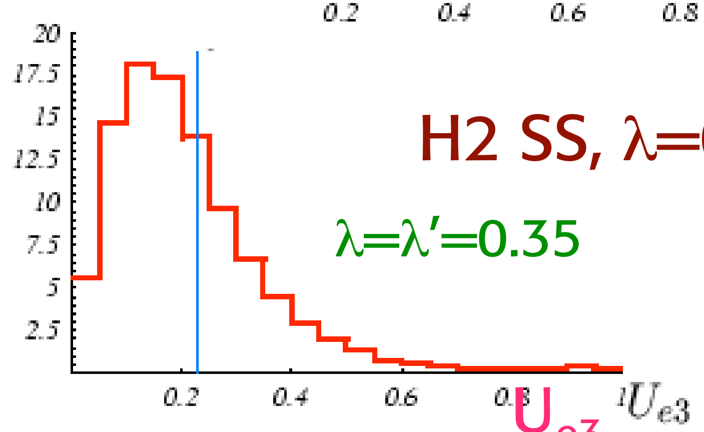


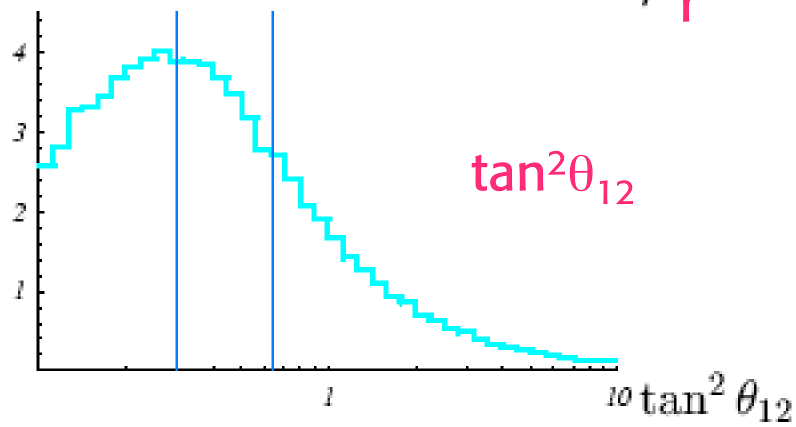

r

H2 SS, $\lambda=0.35$

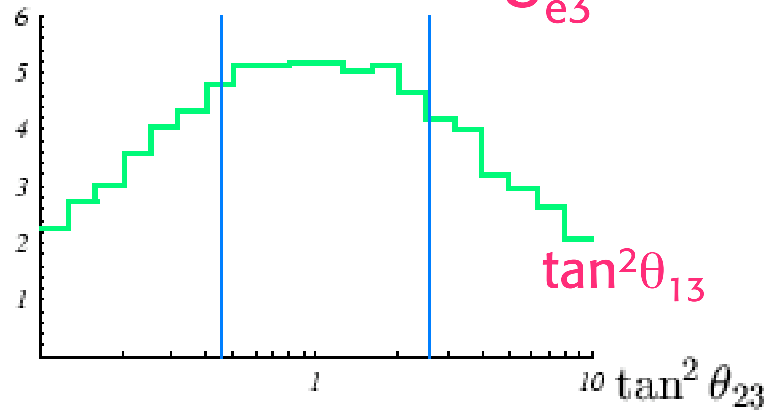
$\lambda=\lambda'=0.35$




$U_{e3}$




$\tan^2\theta_{12}$




$\tan^2\theta_{13}$

G.

# The main advantage of SA vs A is for $U_{e3}$

$$r \equiv \frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}$$

$\Psi_5 \sim (2,0,0)$ →

$$m_\nu \sim \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

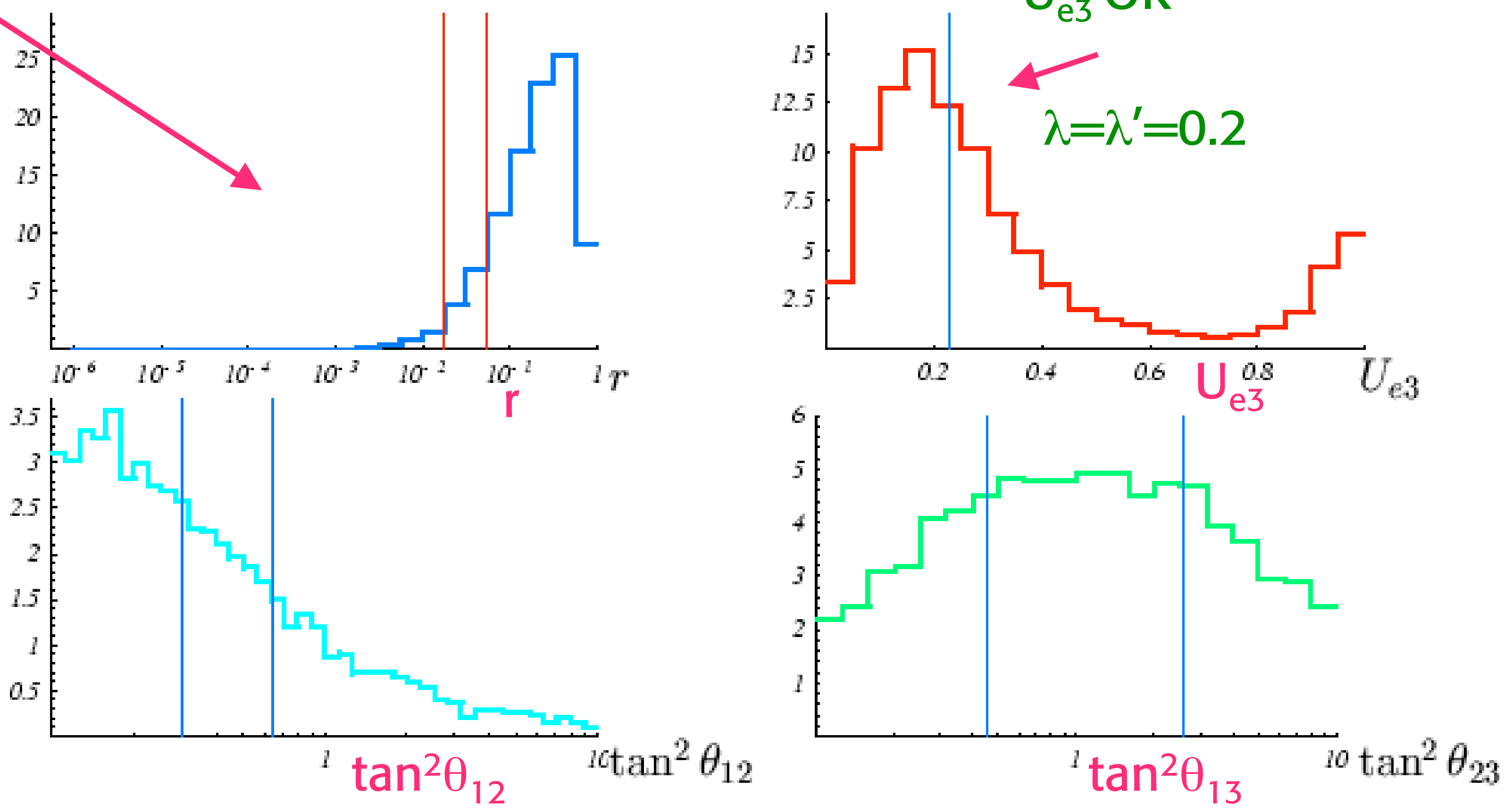
$Det_{23} \sim 0(1)$

$SA_{(NOSS)},\ \lambda = 0.2$

works when r is small enough by chance

$U_{e3}$ OK

$\lambda = \lambda' = 0.2$




G. Altarelli

## Summing up:

- $\nu$ masses very small -> Majorana $\nu$'s and see-saw mechanism
- $\nu$ masses are consistent with the standard way beyond the SM: SUSY and GUT's
- Recent exp progress:
  - $\Delta m^2_{sol}$ went closer to $\Delta m^2_{atm}$ → less hierarchy ↗ $|m_3/m_2| \sim 6$
  - smaller upper limit on absolute mass: WMAP: $\Sigma m_\nu < 0.69$ eV
- Crucial issues:
  - LSND??
  - $s_{13}$ small (how small?) disfavours anarchy
  - $s_{23} \sim$ maximal (too maximal?), $s_{12} \sim$ large not maximal disfavours inv. hierarchy
  - $0\nu\beta\beta$: near bound ? → degenerate $\nu$'s
    intermediate? → inverted hierarchy
    small ? → normal hierarchy ↘ Looks simplest and fine
  - CP violation: still in the future

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