Generalized Parton Distributions and Color Transparency Phenomena

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July 19th, 2004

Motivation

- To understand the transverse structure of hadrons and its relation to Final State Interactions (problem first studied within QCD, perhaps, by Ellis, Furmanski, and Petronzio, '80s)

  $\Rightarrow$ Full-fledged subject:

  "2004 Transversity Workshop" in Trento

  http://www.lnf.infn.it/conference/transversity04

- Closely related to studies of perturbative $\Leftrightarrow$ non-perturbative transition

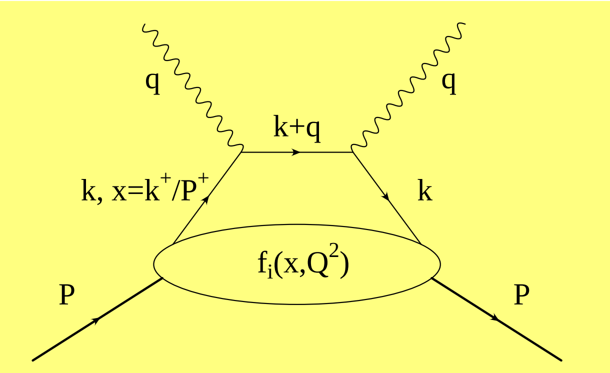
  $\Rightarrow$ Blooming subject:

  "2005 Duality Workshop" in Frascati

  Contact: duality05ioc@lnf.infn.it.

Mapping out the Quark and Gluon Structure of Matter

PDFs: $f_i(x)$

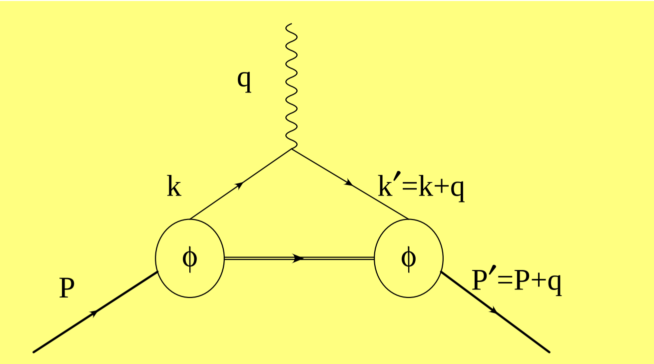


$$d\sigma_{hadron} \approx d\sigma_{parton} \times f_i(x)$$

$\Downarrow$

Hadronic Structure in Momentum Space

Form Factors: $F_{1,2}(Q^2)$



$$A_{hadron} \approx A_{parton} \times \phi_{in}(k) \times \phi_{out}(k')$$

$\Downarrow$

Hadronic Structure in Coordinate Space

PDFs

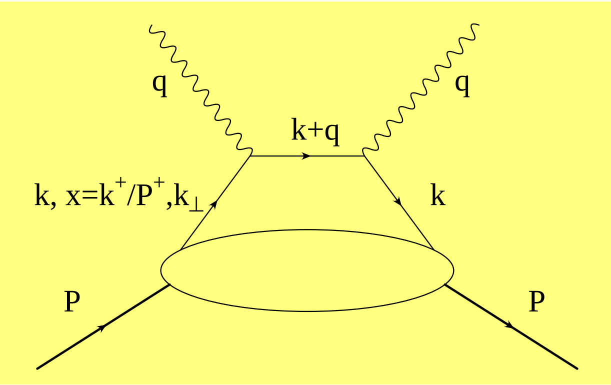
$$P^+ \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, S \mid \psi\left(-\frac{\xi^-}{2}\right) \gamma^+ \psi\left(\frac{\xi^-}{2}\right) \mid P, S\rangle = \bar{u}(P,S)\gamma^+ u(P,S) f(x)$$

FFs

$$\langle P', S' \mid \psi(0)\gamma^+\psi(0) \mid P, S\rangle = \bar{u}(P', S') \left[\gamma^+ F_1(Q^2) + \frac{i\sigma^{+\nu}q_\nu}{2M} F_2(Q^2)\right] u(P,S)$$

Direct Information on Transverse D. O. F.

Unintegrated PDFs

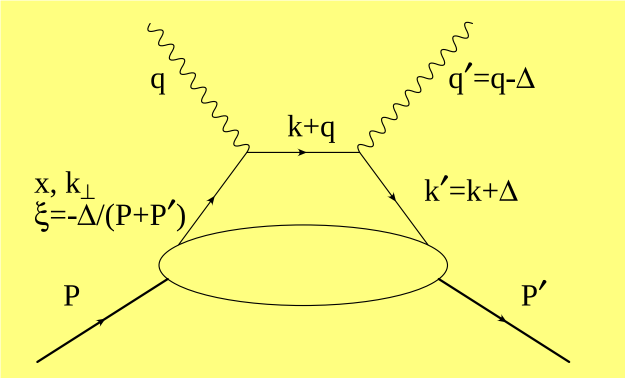


$f(x, k_\perp^2)$

$\Downarrow$

Observables:
$f_{1T}^{\perp},\ h_1^{\perp},\ g_{1T},\ h_{1T}^{\perp},\ h_{1L}.$

Impact Parameter PDFs



$q_\xi(x, b)$

$\Downarrow$

Observables:
$H,\ E,\ \widetilde{H},\ \widetilde{E}$

Unintegrated PDFs

$$\int \frac{d\xi^- d^2\xi}{(2\pi)^3} e^{ixP^+\xi^- - i\mathbf{k}_\perp\cdot\xi} \langle P, S \mid \psi\left(-\frac{\xi^-}{2}, -\frac{\xi_\perp}{2}\right) \gamma^+ \psi\left(\frac{\xi^-}{2}, \frac{\xi_\perp}{2}\right) \mid P, S\rangle = f(x, k_\perp^2)$$

Generalized Parton Distributions (GPDs)

$$\bar{P}^+ \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P', S' \mid \psi\left(-\frac{\xi^-}{2}\right) \gamma^+ \psi\left(\frac{\xi^-}{2}\right) \mid P, S\rangle =$$

$$\bar{u}(P', S') \left[\gamma^+ H(x, \xi, -\Delta^2) + \frac{i\sigma^{+\nu} q_\nu}{2M} E(x, \xi, -\Delta^2)\right] u(P, S) \equiv \mathcal{F}\left[q_\xi(x, \mathbf{b})\right]$$

Beyond naive definition ...

(Brodsky, Hoyer, Hwang, X.Ji, Schmidt ...)

Because of the different $\xi_\perp$'s in the correlation, a Wilson line (gauge link $\Rightarrow$ FSI) appears even in $A^+ = 0$.

Delicate issue: whether or not the gauge link gives a non-zero contribution depends on the integration path.

Difficult to calculate *i.e.* to render the gauge link contribution in terms of "partonic" quantities $\Rightarrow$ models ... (Burkardt)

Summary of current findings on $k_\perp$ and $S_\perp$ dependent structure functions

$\Rightarrow$ First proposed in $\approx$ 1990 (*Sivers*, PRD41 (1990) 83); 1996(*Mulders & Tangermann*, NPB461 (1996) 197)

- Observable from *i.e.* $ep^\uparrow \to e'\pi X$:

$$\frac{d\sigma}{d^2q_\perp} \propto sin\phi\, f_{1\perp} \otimes_\perp D_1$$

$\Rightarrow$ Dedicated experimental analyses since $\approx$ 2000 (SIDIS: $ep^\uparrow \to e'\pi X$):

- A_{UT} Sivers & Collins @ HERMES
- A_{UT} Collins @ COMPASS (preliminary)
- A_{LU} @ HERMES and Jlab (preliminary)
- Fragmentation Functions @ HERMES

$\Rightarrow$ More experiments , pp, Drell-Yan, etc... planned

Comparison with theory is a slow process

$\Rightarrow$ Structure functions in SIDIS appear in products with fragmentation functions (two unknowns!)

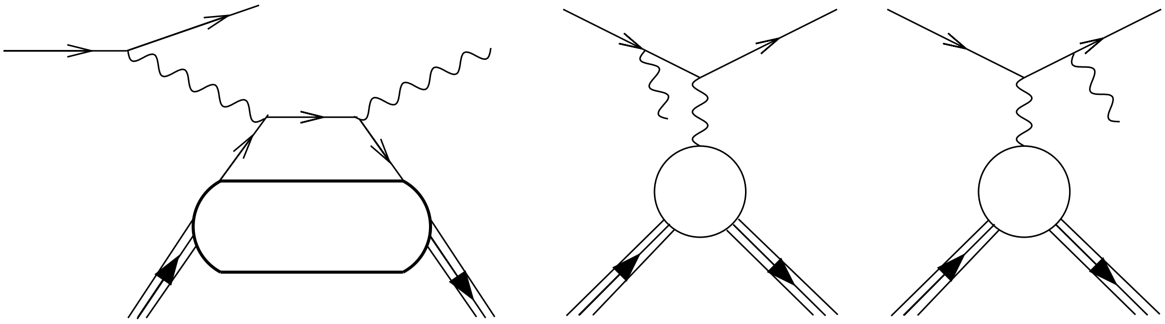
$\Rightarrow$ Transverse information encoded in $f(x, k_\perp)$ is extracted from integrals: $\int d^2k_\perp$... (jet SIDIS and weighted moments in $k_\perp$).

$\Rightarrow$ Sivers asymmetry, $\propto \langle k_\perp \rangle = \int d^2k_\perp f(x, k_\perp) k_\perp$ (with account for gauge link) is difficult to model (see however Burkardt (2004)).

Summary of current findings on GPDs

⇒ First proposed ≈ 1997 (*Radyushkin, Ji*)

⇒ Dedicated experiments since ≈ 2000: HERMES and Jlab measure GPDs through Deeply Virtual Compton Scattering (DVCS)



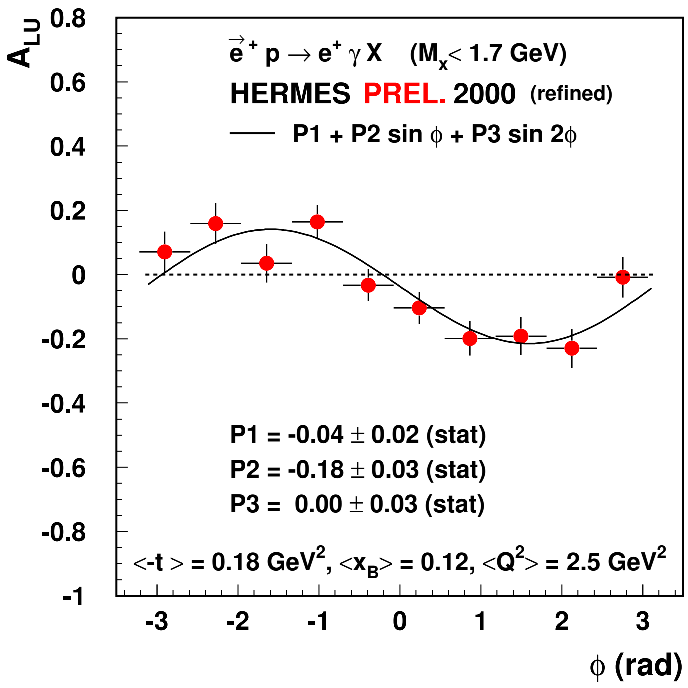
(a) (b)

• Observable given by Interference Term between DVCS (a) and BH(b):

$$d\sigma^{\rightarrow} - d\sigma^{\leftarrow} \propto sin\phi \left[F_1(\Delta^2)\mathcal{H} + \frac{x}{2-x}(F_1+F_2)\widetilde{\mathcal{H}} + \frac{\Delta^2}{M^2}F_2(\Delta^2)\mathcal{E} \right]$$

$$\mathcal{H} = \sum_q e_q^2 (H(\xi,\xi,\Delta^2) - H(-\xi,\xi,\Delta^2))$$

⇒ Experiments disentangle DVCS from BH and confirm the presence of Leading Twist type asymmetries (2002).



Comparison with theory is a slow process

⇒ Only Envelope is measured through DVCS (and related experiments)

⇒ Only constraints are given by:

$$F_1(\Delta^2) = \int dx \sum_q e_q H_q(x, \xi, -\Delta^2 \equiv t)$$

$$q(x) = H_q(x, 0, 0)$$

Impact parameter space interpretation
Burkardt, Diehl, Belitsky, Ji, and Yuan (2003)

$\Rightarrow$ Connect GPDs to Impact Parameter space dependent PDFs (IPPDFs)
Soper, PRD15 (1977) 1411

$$\boxed{\frac{dn_q}{dx d\mathbf{b}} \equiv q(x, \mathbf{b})}$$

LC variables:

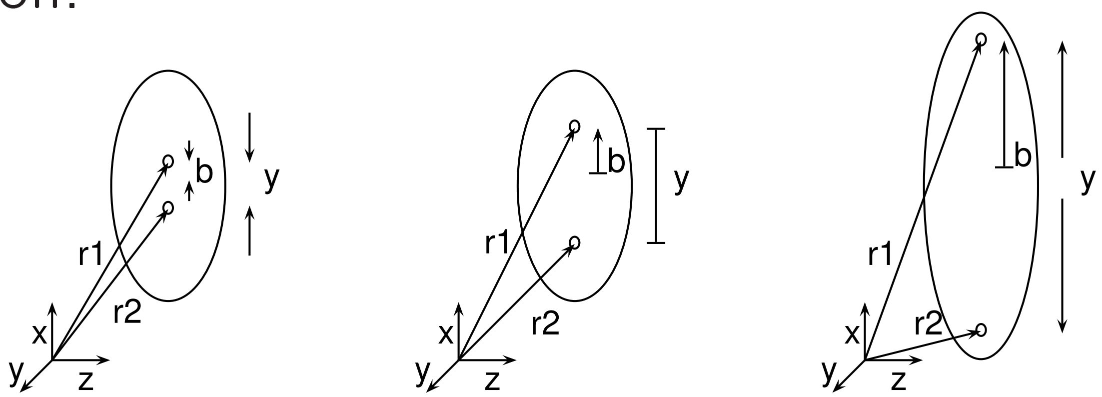
- $x^\mu \equiv (x^o; \mathbf{x} \equiv \mathbf{b}, x^3) \rightarrow$ space 4-vector
- $x^+ = 1/\sqrt{2}(x^0 + x^3) \rightarrow$ time
- $P^+ = \sum p_i^+ \rightarrow$ system's mass
- $x_i = p_i^+/P^+ \rightarrow$ Feynman x
- $\mathbf{R} = \sum x_i^+ \mathbf{b}_i \rightarrow$ center of mass
- $P^+ \mathbf{R} \rightarrow$ generator of galilean boosts.

Subgroup of Poincare' group that leaves $x^+ = const.$ invariant is isomorphic to Galileian group in two dimensions.

By definition:

$$\boxed{q(x, \mathbf{b}) = \int \frac{d^2\mathbf{\Delta}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{\Delta}} H_q(x, 0, -\mathbf{\Delta}^2)}$$

• Since $q(x, \mathbf{b})$ satisfies positivity constraints and it can be interpreted as a probability distribution, $H_q(x, 0, -\mathbf{\Delta}^2)$ is also a probability distribution.



(a) (b) (c)

Relation between Generalized Parton Distributions and Unintegrated Parton Distributions

S.L. and S.K. Taneja, hep-ph/0405014

Role of Transverse Momentum

$$
\boxed{\begin{aligned}
\nu W_2(x) &= \sum_i e_i^2 x \int d^2\mathbf{k}\ f(x,\mathbf{k}) \\
F(\boldsymbol{\Delta}) &= \sum_i e_i \int d^2\mathbf{k} \int_0^1 dx f(x,\mathbf{k},\mathbf{k}+(1-x)\boldsymbol{\Delta})
\end{aligned}}
$$

$f(x,\mathbf{k},\mathbf{k}') = \phi^*(x,\mathbf{k})\phi(x,\mathbf{k}')$ Non-diagonal $\mathbf{k}$ distribution

$f(x,\mathbf{k}) \equiv f(x,\mathbf{k},\mathbf{k}) =\mid \phi(x,\mathbf{k}) \mid^2$ Diagonal $\mathbf{k}$ distribution

Role of Transverse Coordinate

$$
\boxed{\begin{aligned}
\nu W_2(x) &= \sum_i e_i^2\, x \int d^2\mathbf{b}\, q_i(x,\mathbf{b}) \\
F_1(\Delta) &= \sum_i e_i \int d^2\mathbf{b}\, e^{i\boldsymbol{\Delta}\cdot\mathbf{b}} \int_0^1 dx\, q_i(x,\mathbf{b}),
\end{aligned}}
$$

$q(x,\mathbf{b},\mathbf{b}') = \Psi^*(x,\mathbf{b}')\Psi(x,\mathbf{b})$ Non-diagonal $\mathbf{b}$ distribution

$q(x,\mathbf{b},\mathbf{b}) =\mid \Psi(x,\mathbf{b}) \mid^2 \equiv q(x,\mathbf{b})$ Diagonal $\mathbf{b}$ distribution

Master Formula

S.L. and S.K. Taneja, hep-ph/0405014

$$\boxed{\boxed{f(x,\mathbf{k}) = \int d^2\mathbf{b} \int d^2\mathbf{b}'\, e^{i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\, q(x,\mathbf{b},\mathbf{b}')}}$$

More precisely:

$$\begin{aligned}
\nu W_2(x) &= \sum_i e_i^2 x \int d^4k\ S(k)\,\delta\left(x - \frac{k^+}{P^+}\right) \\
&= \sum_i e_i^2 x \int d^2\mathbf{k} \int dP_X^2\, S(k), \qquad (1)
\end{aligned}$$

where the integration over k^- in the last line is replaced by an integration over the invariant mass of the spectator system, $P_X^2 = (P-k)^2$.

What is the *radius* of a hadronic configuration?

Transverse interparton separation (squared)

Soper '77

$$\boxed{\langle y^2 \rangle = \frac{\langle b^2 \rangle}{(1-x)^2}}$$

Radius (squared):

$$\boxed{MAX\left\{\langle b^2 \rangle,\ \left(\frac{x}{1-x}\right)^2 \langle b^2 \rangle\right\}}$$

Observables: Color Transparency Phenomena...

Summary of current findings on CT

$\Rightarrow$ First proposed $\approx$ 1988 (*Brodsky, Mueller*)

$\Rightarrow$ A number of dedicated experiments since then
($pA \to p'p(A-1)$, *Carroll et al., 1988*;
$eA \to e'p(A-1)$, *Makins et al., 1994, O'Neill et al., 1995*; $\gamma^* A \to \rho A$ *HERMES, 2002*;
$\gamma A \to \pi^+ n(A-1)$, *Jlab, 2002* ...)

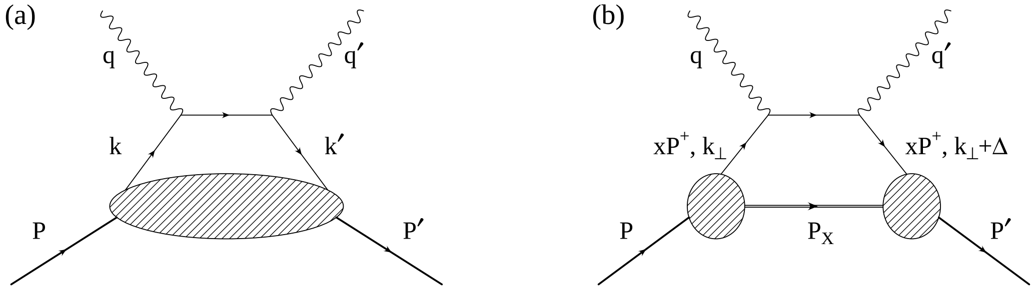
• *No systematics, no marked trend ... however, none of the experiments does rule out <u>small size configurations</u>.*

Whether or not a pQCD description of hadrons holds at the Q^2 values presently available, or at reach at future experimental programs, it has now become imperative to investigate the basic question of the existence and observability of small size hadronic configurations.

Model

with S.K. Taneja at U.Va., hep-ph/0405014

Consider a **two component** (quark-diquark) picture for the lowest Fock components of the proton.



Assumptions:

- Leading Twist ⇒ Relativistic Impulse Approximation: final states are the scattered quark with 4-momentum $k' = k + q$, and diquark with 4-momentum P_X.

- Kinematics ⇒ the struck quark is off-shell: $k^2 \neq m^2$, the diquark is on-shell with mass $P_X^2 = \mathrm{M_X}^2$.

- Dynamics ⇒ introduce quark-diquark-nucleon vertex functions.

LC wave function: $\Phi(x,\mathbf{k}) = \dfrac{\mathcal{M}(k^2, P_X^2)}{D(x,\mathbf{k})}$

$$\begin{cases} D(x,\mathbf{k}) = M_X^2\, x - \dfrac{\mathbf{k}^2}{1-x} \\ \mathcal{M} = g^2\, \dfrac{k^2 - m^2}{k^2 - \Lambda^2} \end{cases}$$

DEFINITIONS IN TERMS OF LC W.F.:

$$\boxed{H(x,0,-\mathbf{\Delta}^2) = \int d^2\mathbf{k}\,\Phi^*(x,\mathbf{k})\Phi(x,\mathbf{k}+(1-x)\mathbf{\Delta})}$$

$$\nu W_2(x) = x\int d^2\mathbf{k}\ \mid \Phi(x,\mathbf{k})\mid^2$$

$$F_1(\Delta^2) = \int d^2\mathbf{k}\int_0^1 dx\,\Phi^*(x,\mathbf{k})\Phi(x,\mathbf{k}+(1-x)\mathbf{\Delta})$$

$$q(x,\mathbf{b}) = \mathcal{F}\left[H(x,\Delta)\right]$$

ROLE OF INTRINSIC $k_\perp$ ($\equiv \mathbf{k}$):

$$f(x,\mathbf{k}) \equiv \mid \Phi(x,\mathbf{k})\mid^2$$

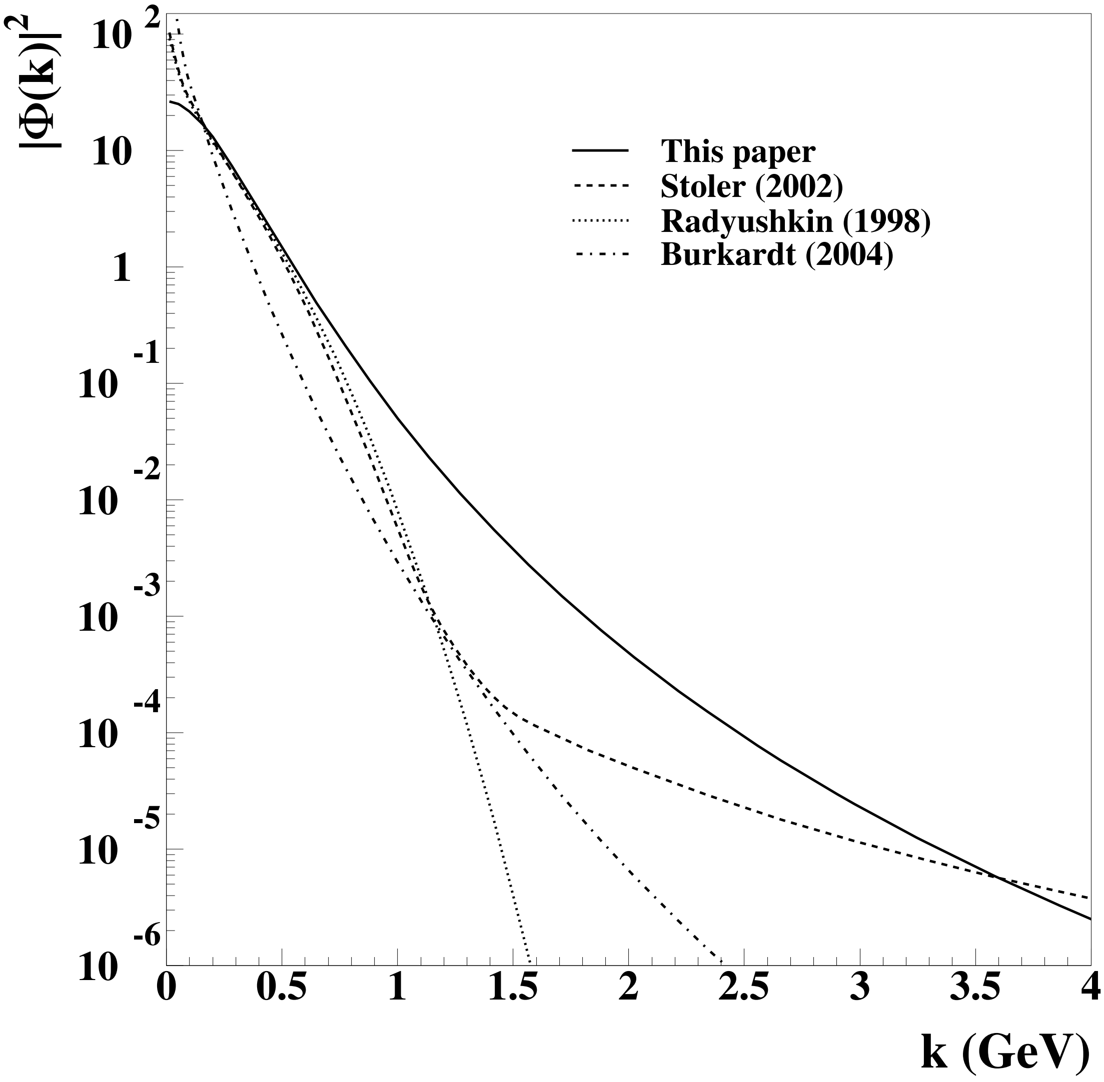
$$f(x,\mathbf{k}) = \int d^2\mathbf{b}\int d^2\mathbf{b}'\,e^{i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\,q(x,\mathbf{b},\mathbf{b}')$$

intrinsic $k_\perp$ distribution

$$q(x,\mathbf{b},\mathbf{b}') = \Psi^*(x,\mathbf{b}')\Psi(x,\mathbf{b}) \stackrel{\mathbf{b}\to\mathbf{b}'}{\equiv} q(x,\mathbf{b})$$

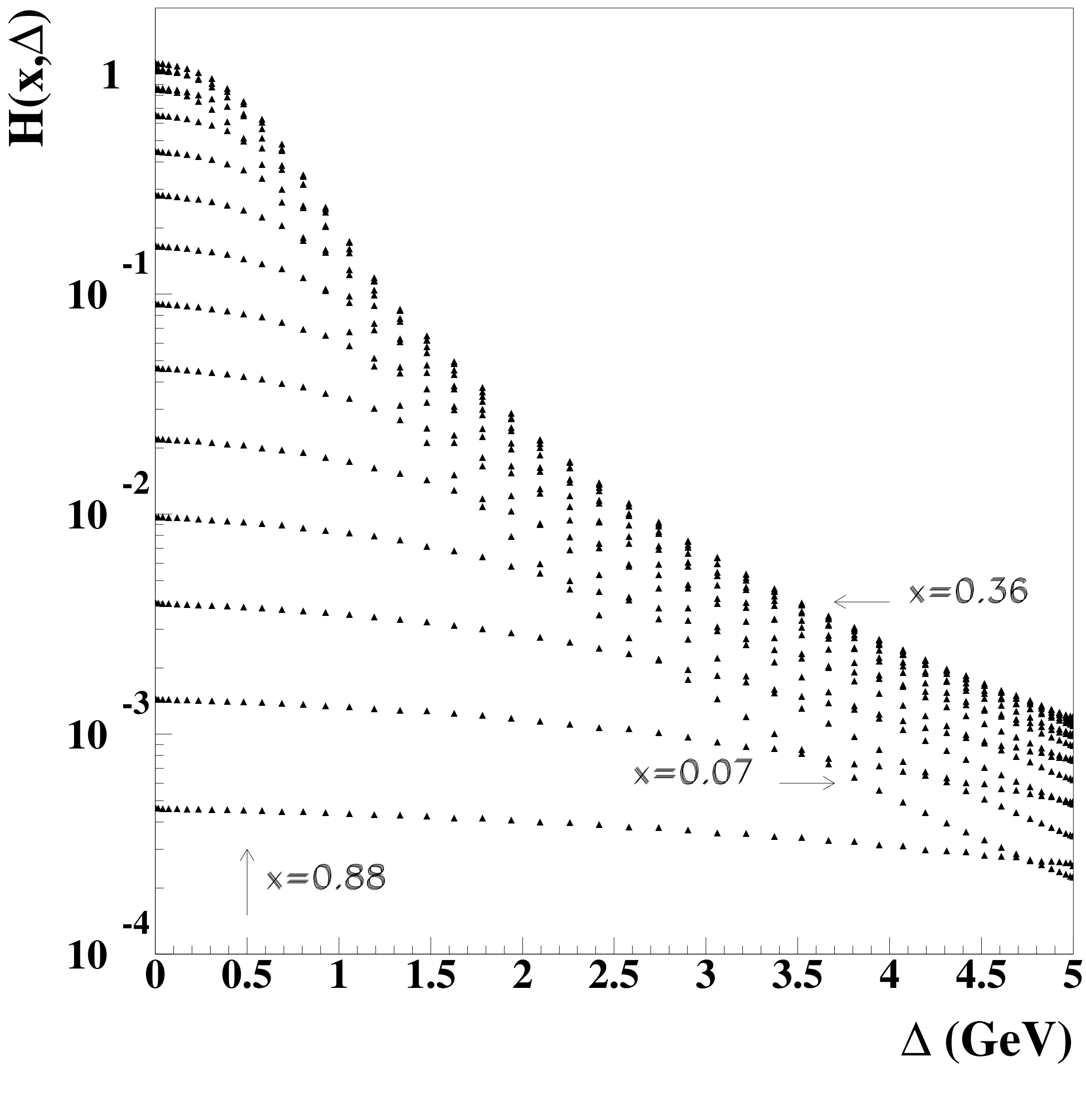
non-diagonal one-body transverse density.

Comparison with existing parametrizations



$$\Phi(\mathbf{k}) = \int dx\, \Phi(x, \mathbf{k})$$

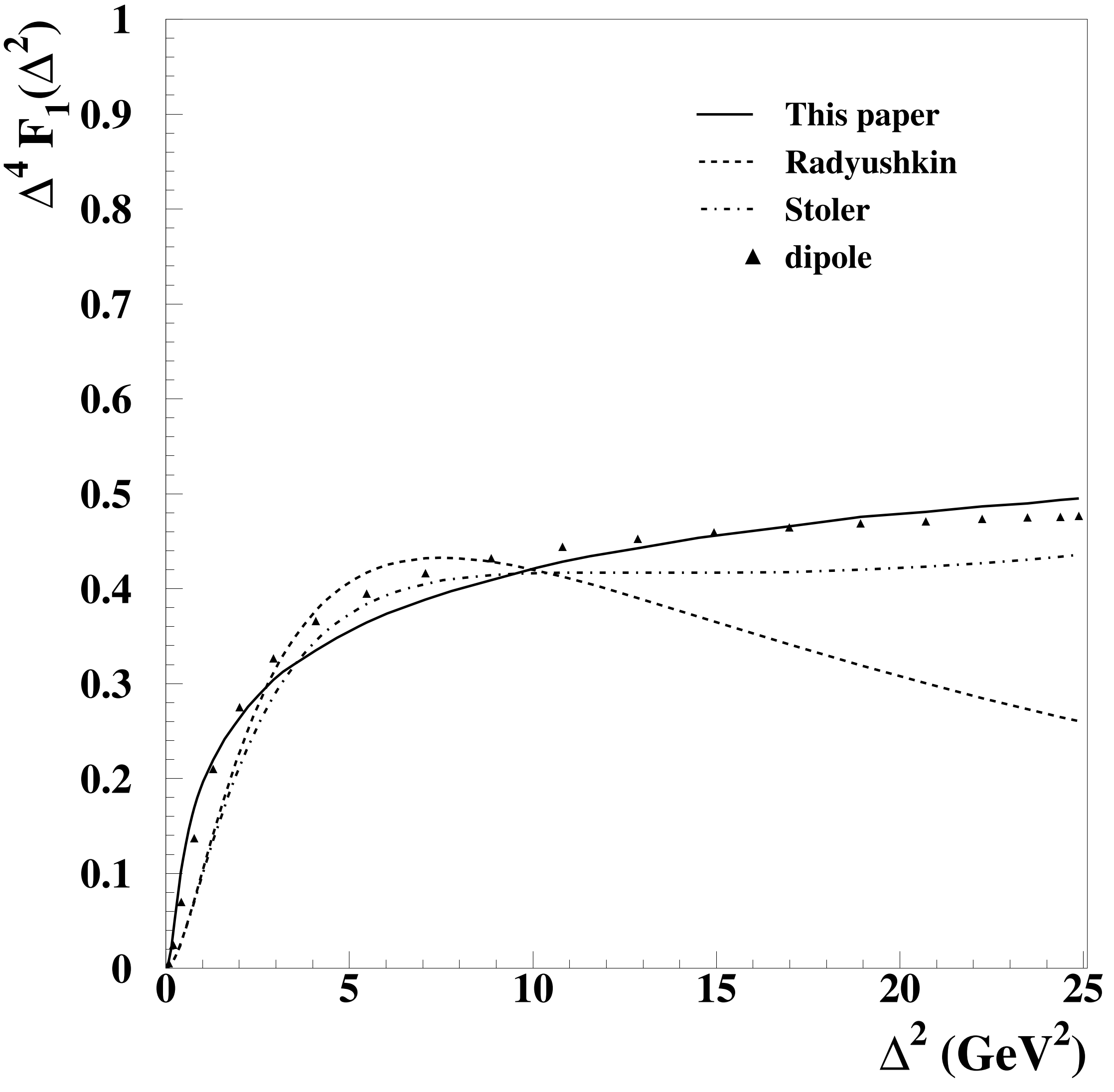
Generalized Parton Distribution



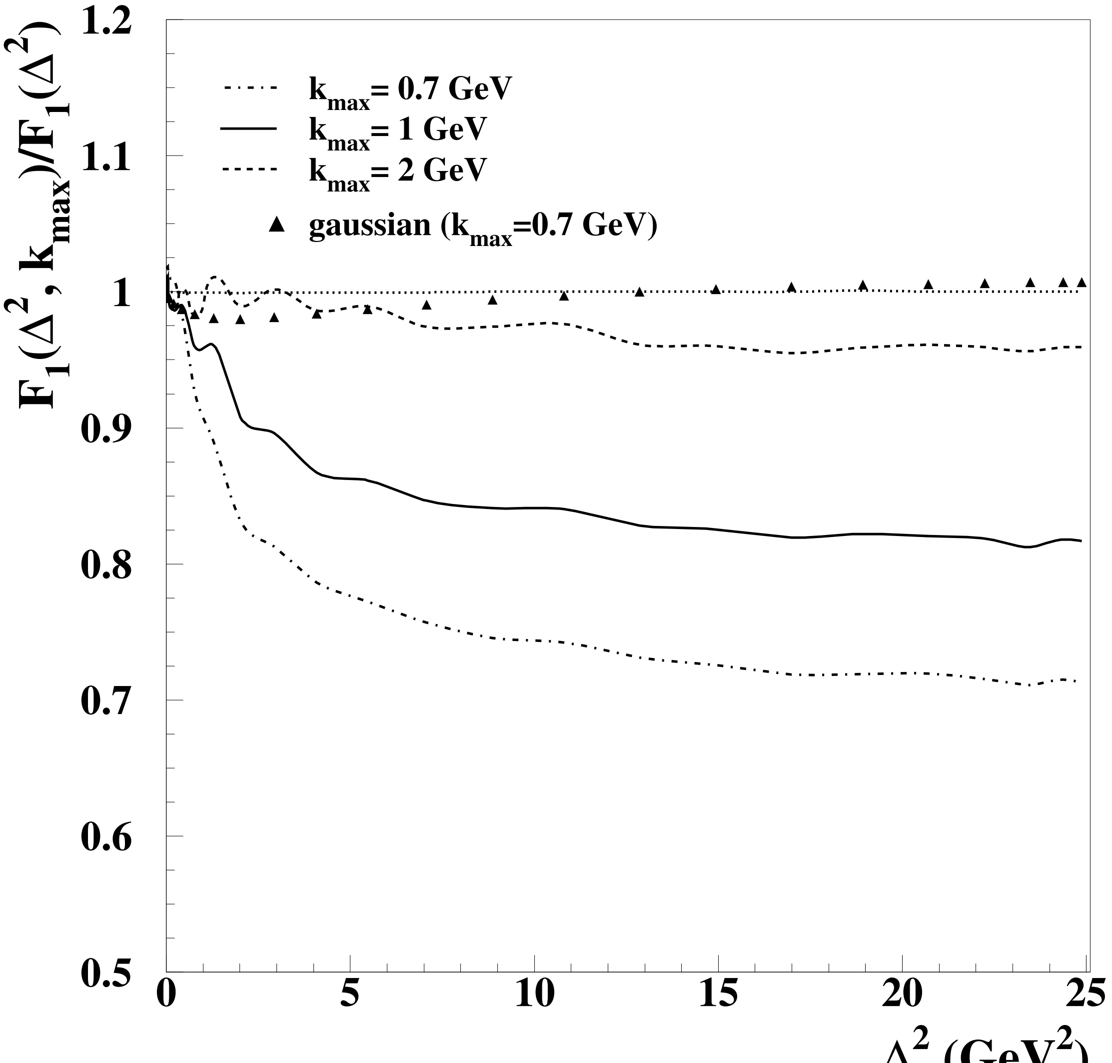
Parametrized as:

$$H(x, -\mathbf{\Delta}^2) = p_1(x) \frac{1}{p_2(x) + p_3(x)\Delta + p_4(x)\Delta^2}$$

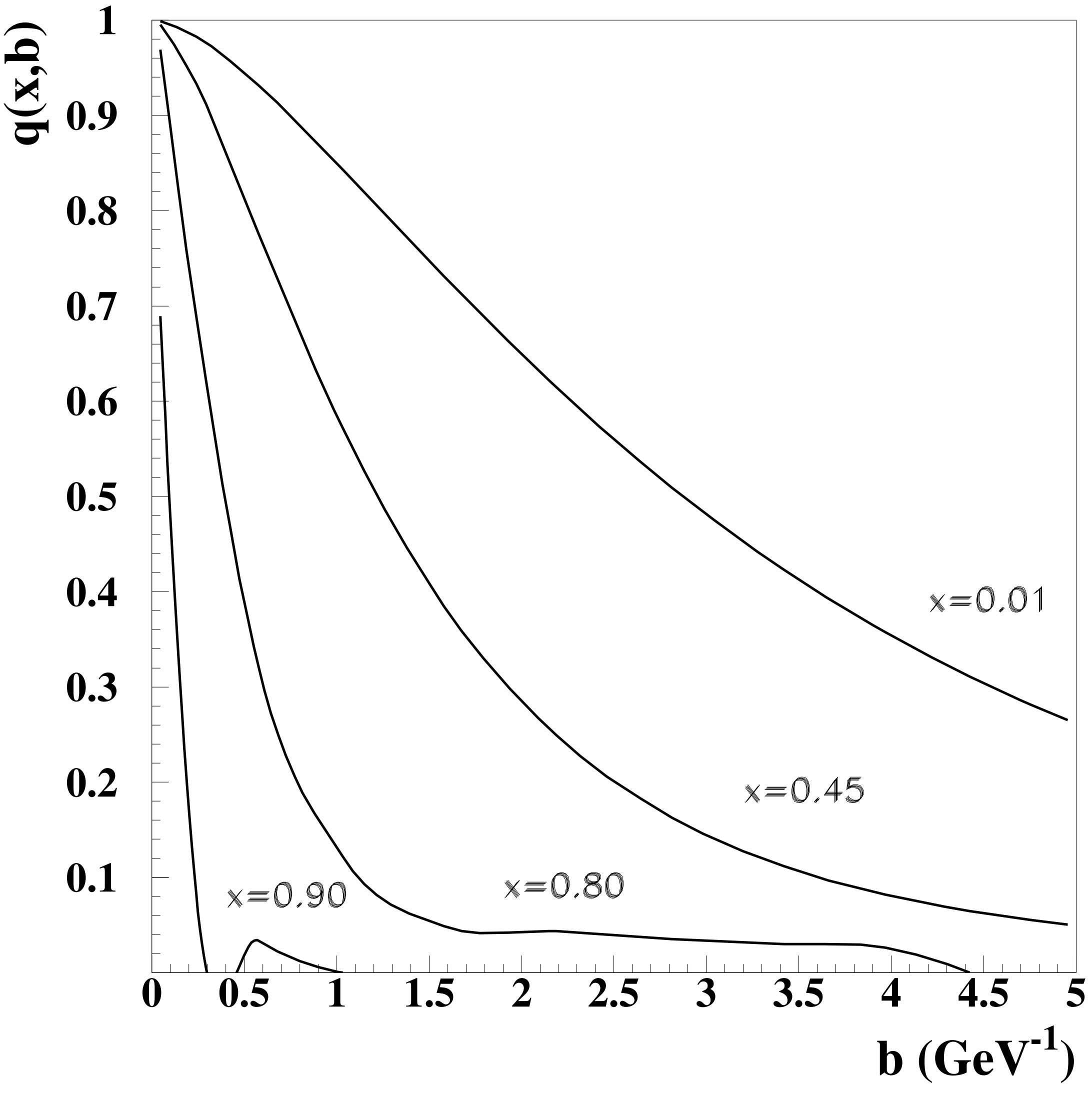
Dirac Form Factor



Dominant $k_\perp$ components in Form Factor

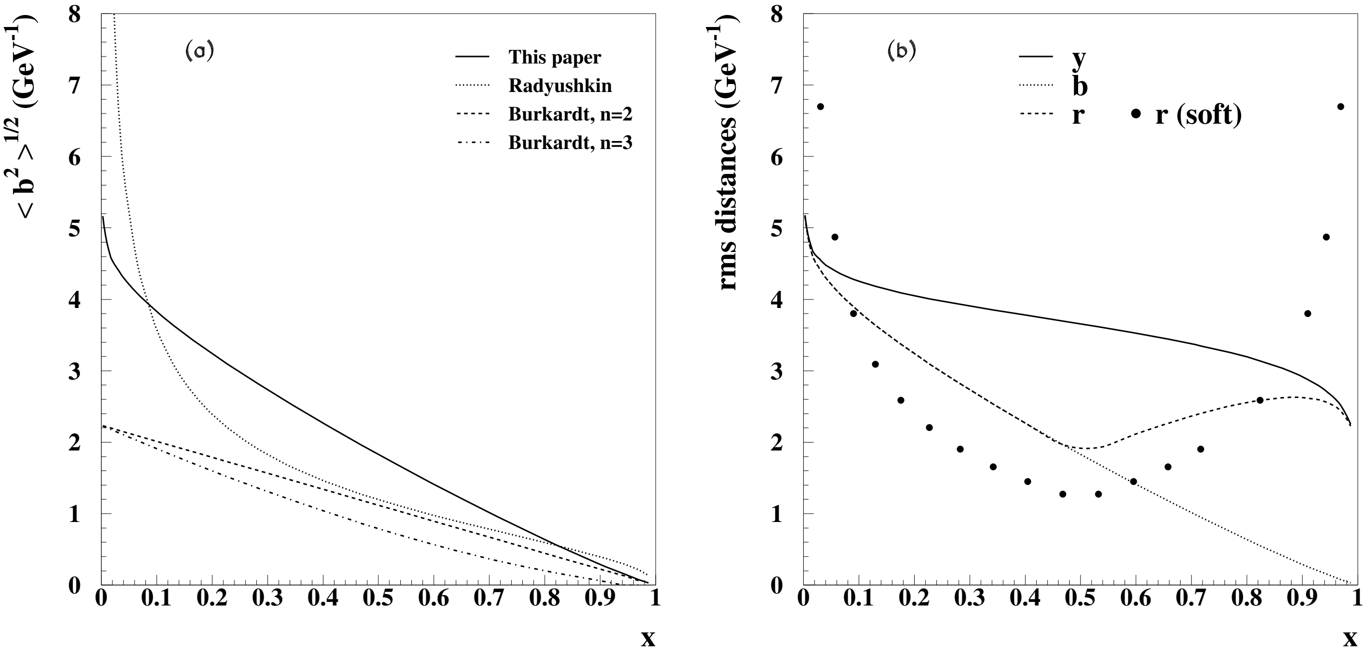


Transverse spatial dependence



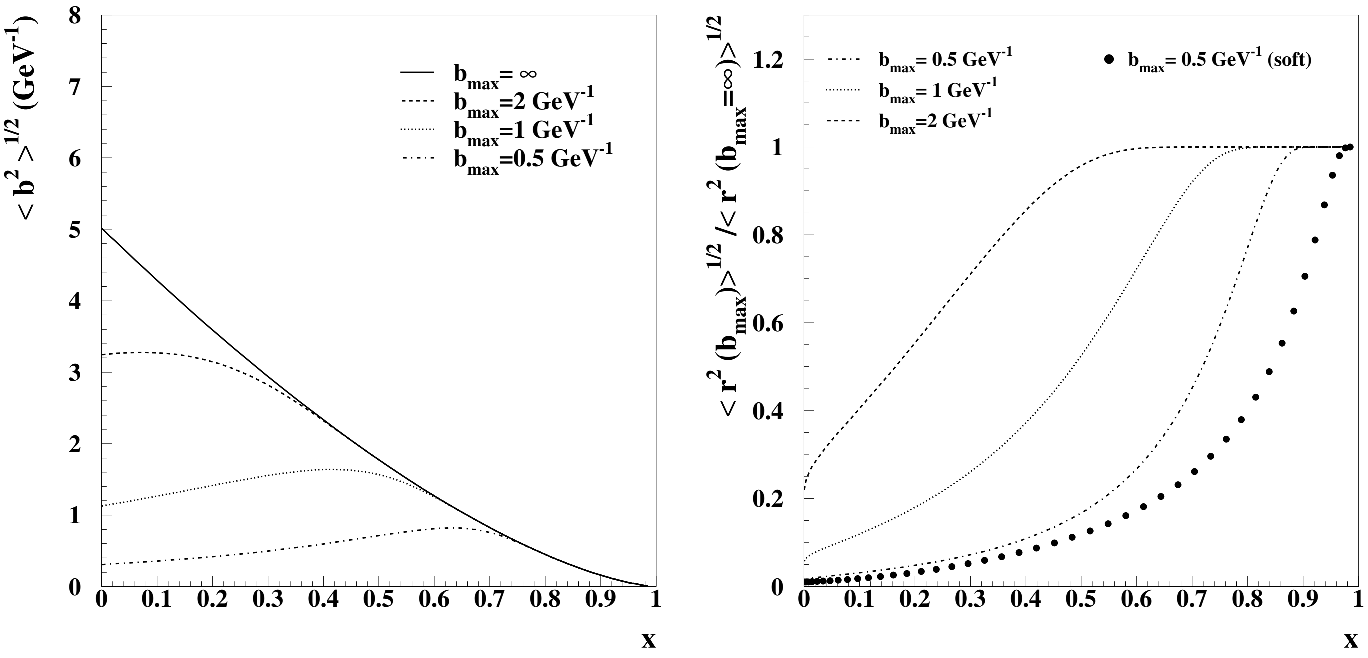
$q(x, \mathbf{b}) \rightarrow$ Hankel transform

Radius



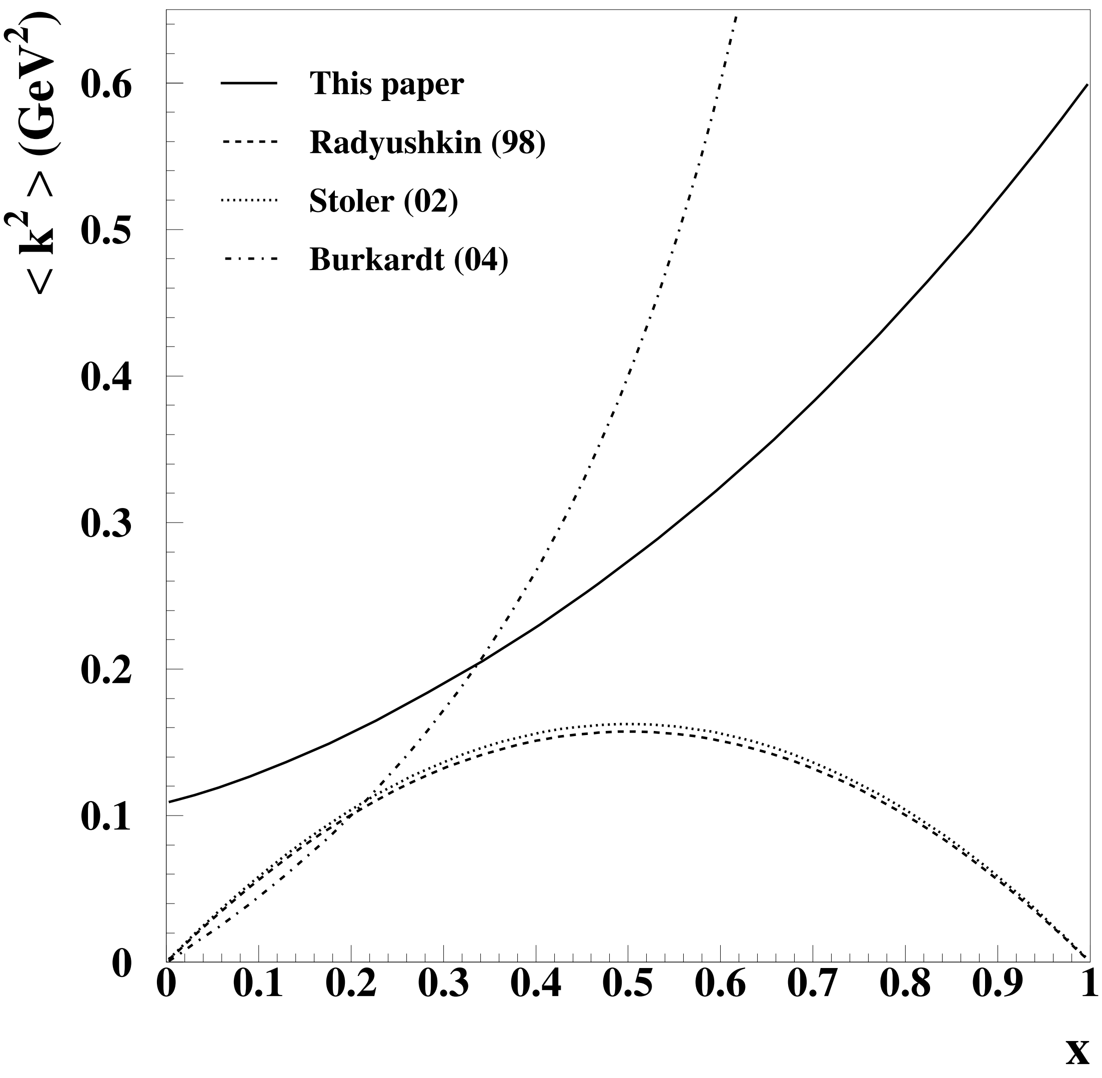
$$r_{rms} = MAX \left\{ \sqrt{\langle b^2 \rangle}, \frac{x}{1-x}\sqrt{\langle b^2 \rangle} \right\}$$

Independently from the fine tuning of our result, the radius does not blow up at large x

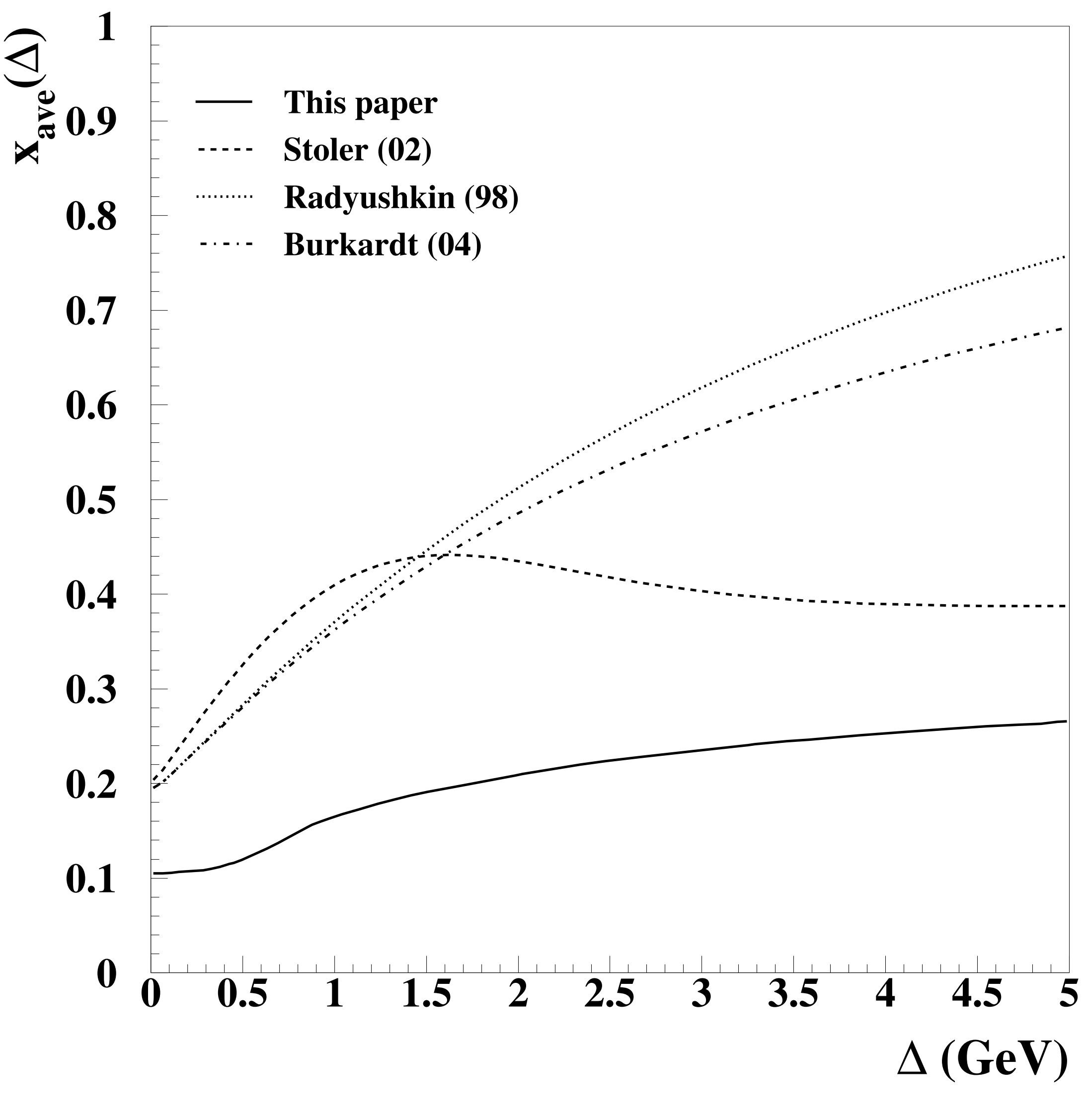


Contribution of different transverse distances to the rms impact parameter $\langle b^2\rangle^{1/2}$.

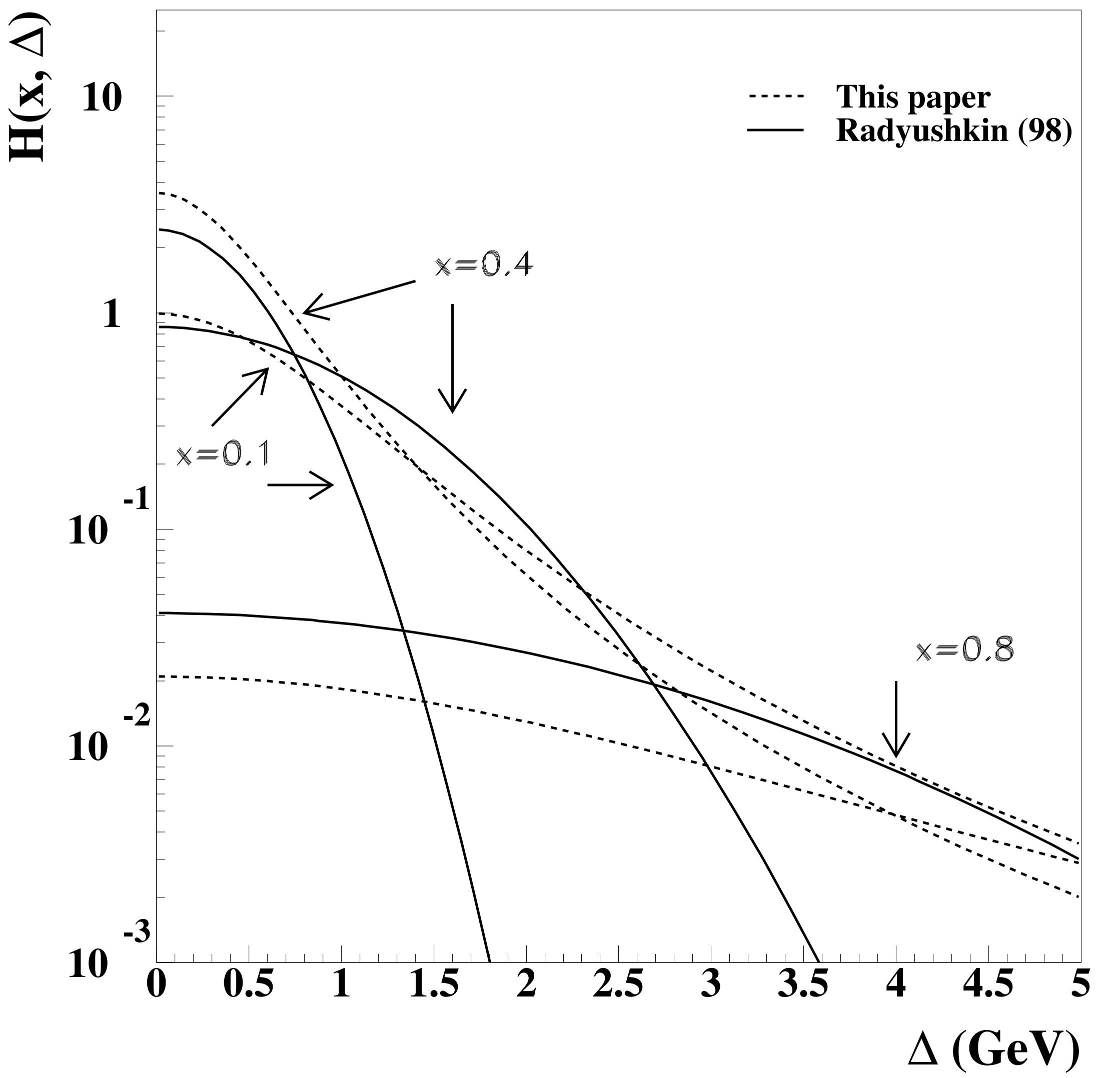
Dominant (transverse) $k_\perp$ components



Dominant (longitudinal) x components



$$x_{ave}(\Delta) = \frac{\int_0^1 dx\, x\, H(x, \Delta)}{\int_0^1 dx H(x, \Delta)}$$



H(x, Δ)
This paper
Radyushkin (98)
x=0.4
x=0.1
x=0.8
Δ (GeV)

Color Transparency and GPDs

$$T_A(Q^2) = \frac{\left[\int_0^1 dx\, H_A(x,\Delta)\right]^2}{\left[\int_0^1 dx\, H(x,\Delta)\right]^2}$$

$$H_A(Q^2) = \int_0^{b_{max}(A)} db\, b\, q(x,b)\, J_0(b\Delta)$$

$b_{max}(A) =$ size of the filter given by a square function

$\Rightarrow$ By assuming

$$q(x,b) = A(x)\exp(-\alpha(x)\, b)$$

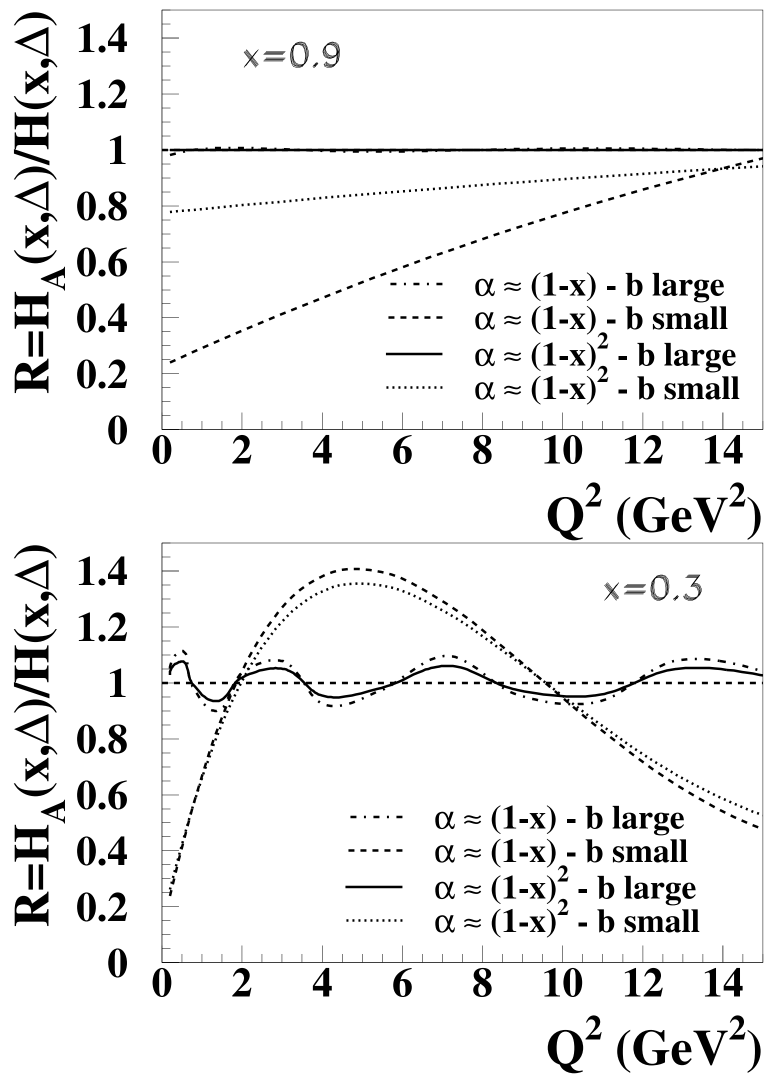
with

i) $\alpha \propto (1-x)/x$ (small $k_\perp$)

ii) $\alpha \propto (1-x)^2/x$ (large $k_\perp$)

we predict both damping and oscillations of the transparency ratio:

$$\boxed{R = 1 - \exp(-\alpha\, b_{max})\left[\alpha\, b_{max} J_0(\Delta b_{max}) + \cos(\Delta b_{max})\right]}$$



x=0.9
R=H_A(x,Δ)/H(x,Δ)
α ≈ (1-x) - b large
α ≈ (1-x) - b small
α ≈ (1-x)^2 - b large
α ≈ (1-x)^2 - b small
Q^2 (GeV^2)
x=0.3
R=H_A(x,Δ)/H(x,Δ)
α ≈ (1-x) - b large
α ≈ (1-x) - b small
α ≈ (1-x)^2 - b large
α ≈ (1-x)^2 - b small
Q^2 (GeV^2)

Discussion and Conclusions

(1) Many Challenges ...

1. Experiment: Transverse structure is much more difficult to measure (THE experts are right here in this room ...)

2. Theory/Phenomenology: Need more work on connection between transversity in momentum and coordinate space $\Rightarrow$ this will help both "visualizing" and discerning physical situations, as we have seen in the CT case.

(2)

- In order to describe the form factor, one needs high $k_\perp$ components in the LC wave function.

- These are provided naturally within a two component model.

- High $k_\perp$ components are also responsible for decreasing the hadronic transverse size as x increases.

- This mechanism can be clearly tested by comparing different parametrizations.

- The decrease predicted in $\langle b^2 \rangle$, and consistent with the form factor behavior and measured $\langle k_\perp^2 \rangle$ values, seems to be however not sufficient to guarantee the onset of Color Transparency.

- Main message: Based on a LC description, a range of configurations with varying sizes can be monitored!

(3) Outlook

- The interplay bewteen $\mathbf{k}_\perp$, x, $\mathbf{b}$ and $\boldsymbol{\Delta}$ determines the size of hadronic configurations $\Rightarrow$ another important role of GPDs (exploratory work in S.L. and S.K. Taneja, hep-ph/0405014).

- Yet, direct measurements of GPDs are a very slow process.

- Use CT phenomena, specifically <u>Nuclear Filtering</u> to extract information.

- Beyond the nuclear factorized formula:
  - "Passive" role of nucleus: description in terms of both quark and nucleon spectral functions, role of off-shellness ...
  - "Active" role of nucleus: rescattering beyond convolution might yield terms directly proportional to GPDs.

- Flavor dependence

- OAM and deformation (role of ξ variable)

- Connection with transversity functions in transverse space: start from study of E (S.L., work in progress)

- Evolution in b-space (work in progress with A. Drago)