Nonperturbative Methods in Chiral Theories

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A minimization procedure for the extraction of $f_0(980)$ parameters for different models: some results

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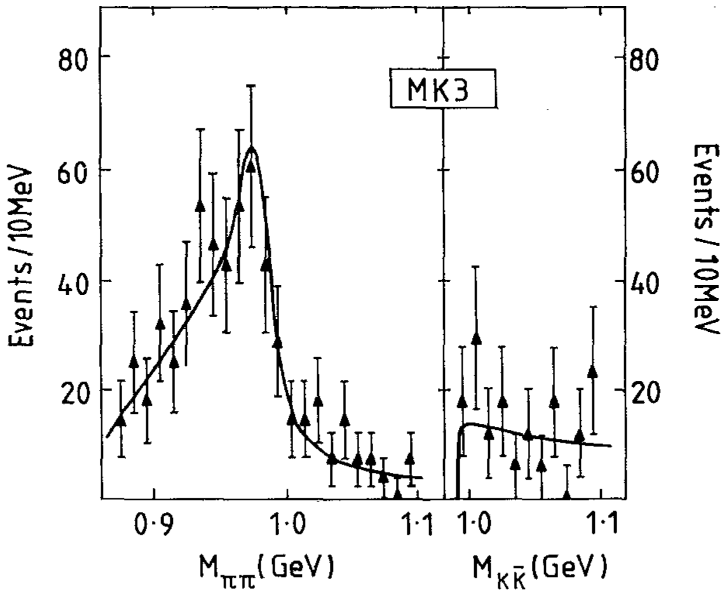
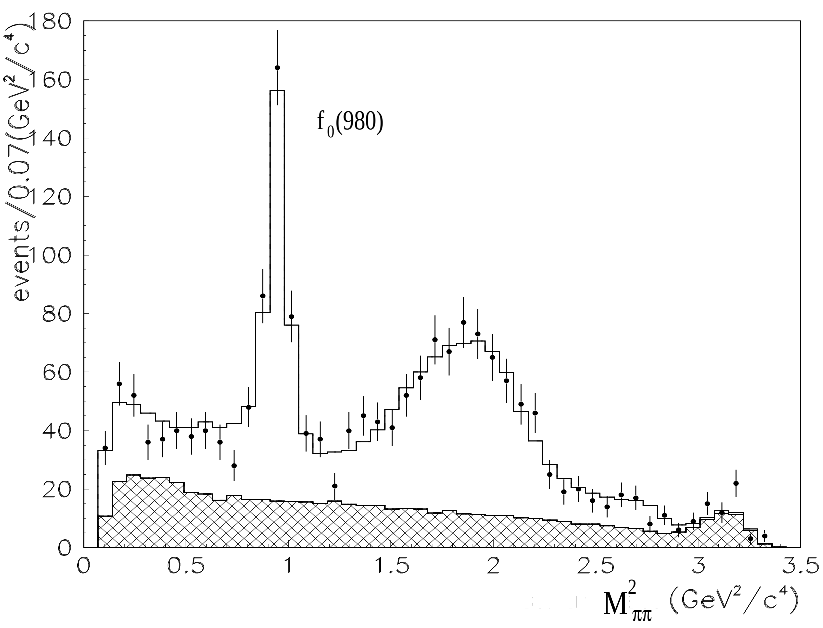
- **Conclusions and work in progress.**

The $f_0(980)$ in J/ψ, $D_s^\pm$ decays

- $J/\psi \to \phi\,\pi\,\pi,\ \phi\,K\,\bar{K}$ **analysed by Morgan *et al.* PRD48,1185**



- $D_s^\pm \to \pi^+\,\pi^-\,\pi^\pm$ **analysed by van Beveren *et al.* PLB495,300**



The transition $\phi \to f_0(980)\,\gamma \to \pi\,\pi\,\gamma$

hypothesis	$\mathrm{BR}(\phi \to \gamma\, f_0(980))$
$qq\bar{q}\bar{q}$	$\simeq \mathcal{O}(10^{-4})$
$s\bar{s}$	$\simeq \mathcal{O}(10^{-5})$
$K\bar{K}$ molecule	$\leq 10^{-5}$
$(u\bar{u}+d\bar{d})/\sqrt{2}$	$\simeq \mathcal{O}(10^{-6})$
glueball	$< 10^{-6}$

$$\mathcal{M}_\phi = \frac{e^3}{s}\frac{g_{\phi\gamma}}{D_\phi(s)} f_{\phi f_0}(Q^2)\bar{v}(p_2)\gamma^\mu u(p_1)\,[(pq)g_{\mu\nu} - q_\mu p_\nu]\,\epsilon^\nu$$

☹ you can't rely on pQCD

☹ $\mathcal{O}(p^2)$ in χPT is not enough

$$\Downarrow$$

$\phi \to f_0(980)\,\gamma \to \pi\,\pi\,\gamma$ has been implemented in the MC code EVA according to different models for $f_{\phi f_0}(Q^2)$

Pointlike ϕ-γ-f_0 coupling

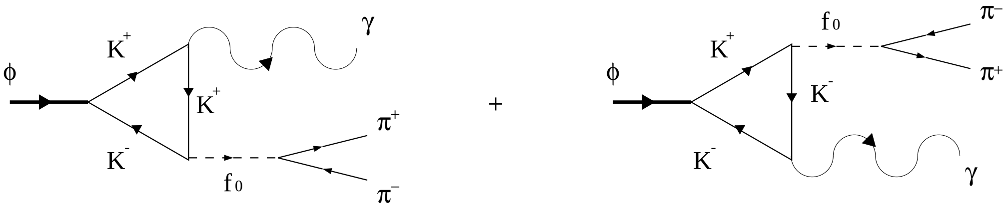
Bramon *et al.*

- $f_0(980) =$ intermediate scalar between ϕ and $\pi^+\pi^-$
- the decay has to be damped @ $Q^2 \ll m^2_{f_0}$

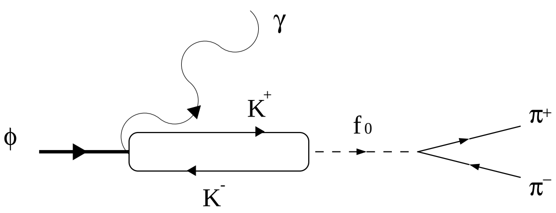
$$f_{\phi f_0}(Q^2) = \exp\left(-\frac{m^2_{f_0} - Q^2}{2\,\Delta^2}\right)\frac{g_{\phi f_0 \gamma}\, g_{f_0 \pi^+\pi^-}}{m^2_{f_0} - Q^2 - i m_{f_0}\Gamma_{f_0}}$$

K^+K^- loop and const. Γ_{f_0}

Close, Lucio *et al.*



a)



b)

$$f_{\phi f_0}(Q^2) = \frac{g_{\phi K^+K^-}\, g_{f_0\pi^+\pi^-}\, g_{f_0 K^+K^-}}{2\pi^2 m^2_{K^+}}\;\frac{I(Q^2)}{m^2_{f_0} - Q^2 - i m_{f_0}\Gamma_{f_0}}$$

Finite width corrections

Achasov *et al.*

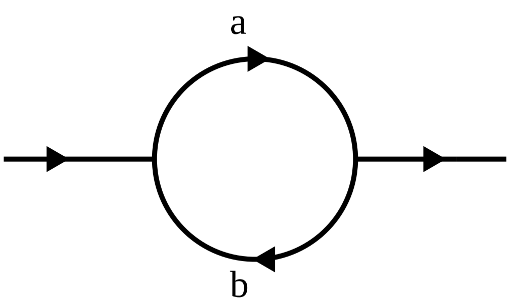
$qq\bar{q}\bar{q}$ structure

if $|f_0(980)\rangle \sim s\bar{s}(u\bar{u}+d\bar{d})/\sqrt{2}$:

- $\star$ mass close to the $K\bar{K}$ threshold
- $\star$ strong coupling to $K\bar{K}$ channels

$$\Downarrow$$

$$\Pi_{f_0}(Q^2) = \sum_{ab} \Pi^{ab}_{f_0} = \sum_{ab}$$



$$ab = \pi\pi, \quad K^+K^-, \quad K^0\bar{K}^0, \quad \eta\eta$$

$$f_{\phi f_0}(Q^2) = \frac{g_{\phi K^+K^-}\, g_{f_0\pi^+\pi^-}\, g_{f_0K^+K^-}}{2\pi^2 m^2_{K^+}} \, \frac{I(Q^2)}{m^2_{f_0} - Q^2 + \Re e\Pi_{f_0}(m^2_{f_0}) - \Pi_{f_0}(Q^2)}$$

$K\bar{K}$ molecule Achasov *et al.*

two $q\bar{q}$ mesons with a characteristic range $R_{K\bar{K}} \gtrsim 1.2\,\mathrm{fm}$

$$\mathcal{A}(\text{diagr.}) \sim \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \phi(\mathrm{k})\ \text{(internal lines)}$$

$$\phi(\mathrm{k}) = \frac{\mu^4}{(\mathrm{k}^2+\mu^2)^2}, \quad \mu = \frac{\sqrt{3}}{2R_{K\bar{K}}}$$

$$f_{\phi f_0}(Q^2) = 2\,\frac{g_{f_0\pi^+\pi^-}\, g_{f_0 K^+K^-}\, g_{\phi K^+K^-}}{M_\phi^2 - Q^2}\,\frac{|G_{f_0}(Q^2)|^2}{|D_{f_0}(Q^2)|^2}$$

D_{f_0} takes explicitly account of the analyticity at $K\bar{K}$ thresholds

Chiral unitary approach Oset *et al.*

- $\mathcal{L}_{\chi PT}\left[\mathcal{O}(p^2)\right] \Rightarrow \phi$-$K^+$-$K^-$, ϕ-γ-K^+-K^-, γ-K^+-K^-
- dynamically generated $f_0(980)$ in $K^+K^- \rightarrow \pi\pi$ scattering

troubles with $\mathcal{L}_{\chi PT}\left[\mathcal{O}(p^2)\right]$ in describing $K^+K^- \rightarrow \pi\pi$:

☹ bad approximation @ $\sqrt{s} \sim 1$ GeV

☹ it doesn't yield $f_0(980)$ pole

but Unitarity enables to include orders $> \mathcal{O}(p^2)$:

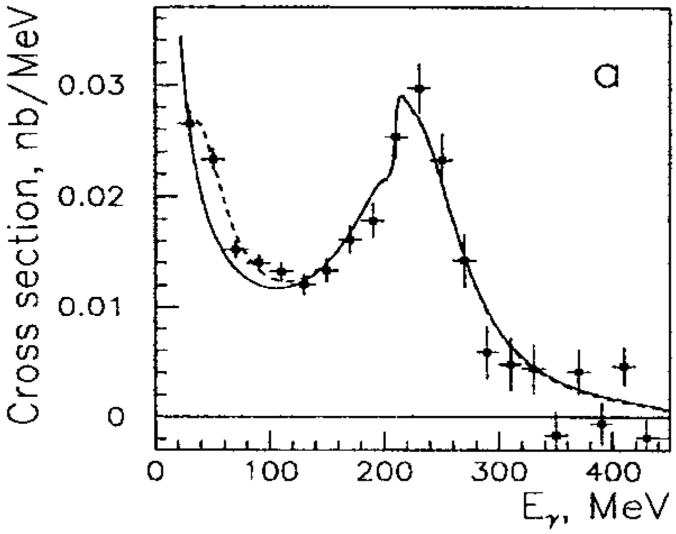
$$t_{i,j} = t^{(2)}_{i,j} + t^{(2)}_{i,k}\, G_{k,k}\, t_{k,j}\,, \qquad i,j,k = \pi\pi, K\bar{K}$$

$$t^{(2)}\, G\, t \sim \int \frac{\mathrm{d}^4 q}{(2\pi)^4} t^{(2)}(q)\, G(q)\, t(q)$$ $\Rightarrow$ only parameter: q_{max}

$$f_{\phi f_0}(Q^2) = \frac{t_{K^+K^-,\pi^+\pi^-}}{2\sqrt{2}\pi^2 m^2_{K^+} f^2_\pi}\Big[g_{\phi K^+K^-} M_\phi I(a,b) + \frac{4\pi^2 m^2_{K^+}}{M_\phi}\left(\frac{g_{\phi\gamma}}{2} - g_{\phi K^+K^-}\right) G_{K,K}\Big]$$

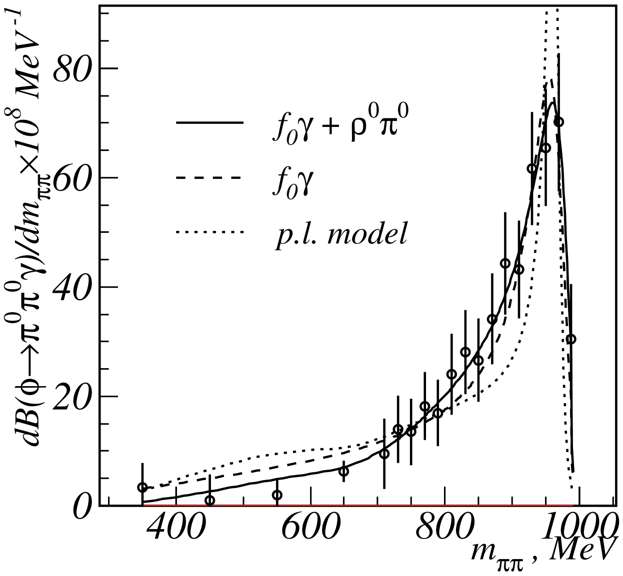
$\phi \to \pi\,\pi\,\gamma$: previous observations

CMD–2: $e^+ e^- \to \pi^+ \pi^- \gamma$



$$\sqrt{s} \sim M_\phi \begin{cases} \sigma_{ISR}(s) \propto (m_\phi/\sqrt{s})^{13} \\ \sigma_{FSR}(s) \propto (m_\phi/\sqrt{s})^{9} \end{cases}$$

SND: $\phi \to \pi^0 \pi^0 \gamma$



$$m_{f_0} = (969.8 \pm 4.5)\ \text{MeV}$$

$$g^2_{f_0 K^+K^-}/(4\pi) = 2.47^{+0.73}_{-0.51}\ \text{GeV}^2$$

$$g^2_{f_0 K^+K^-}/g^2_{f_0\pi^+\pi^-} = 4.6 \pm 0.8$$

$$\chi^2_{min}/\text{d.o.f.} = 3/14$$

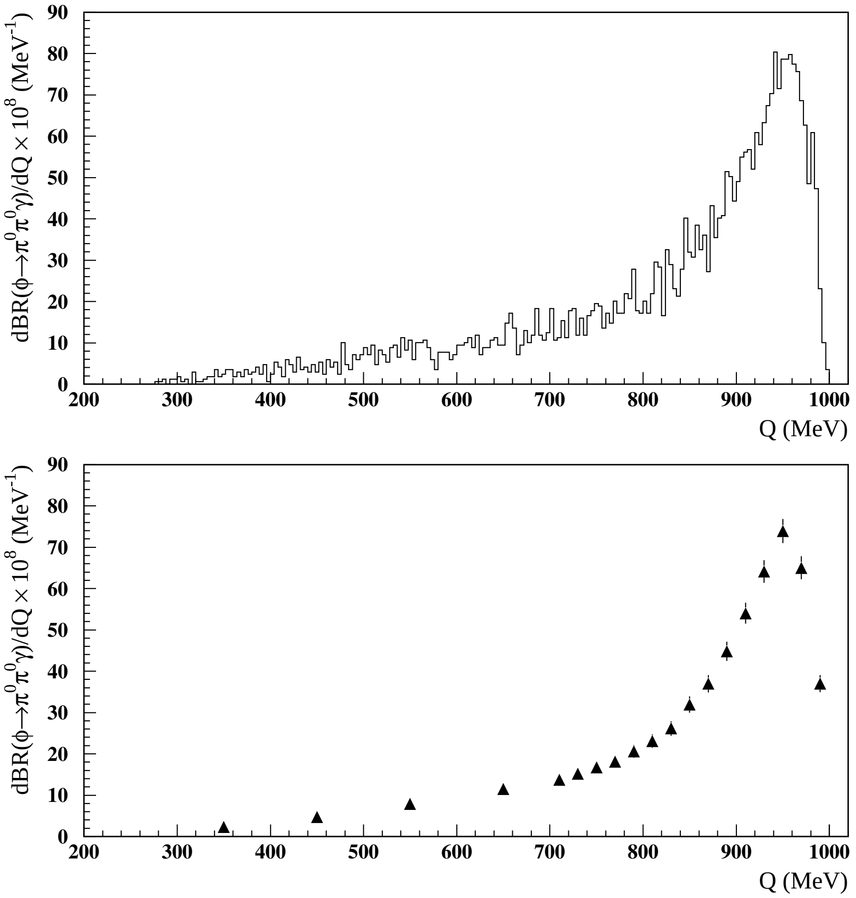
$$\text{dBR} \sim |\mathcal{M}_{f_0}|^2 + |\mathcal{M}_{\rho^0\pi^0}|^2 + 2|\mathcal{M}_{f_0}||\mathcal{M}_{\rho^0\pi^0}|\cos\theta$$

A numerical procedure

☺ no approximation in the matrix elements

☺ inclusion of efficiency/resolution effects

$$\forall\,\vec{\theta} = (\theta_1, \ldots, \theta_n) \xrightarrow{\texttt{EVA}} \begin{cases} \mathrm{BR}(\phi \to \pi^0\pi^0\gamma) \\ \frac{\mathrm{d}}{\mathrm{d}Q}\mathrm{BR}(\phi \to \pi^0\pi^0\gamma) = S^{th} \end{cases}$$



m_{f_0} = 974 MeV

Γ_{f_0} = 94 MeV

$g_{f_0 K^+K^-}\, g_{f_0 \pi^+\pi^-}$

= 7 GeV^2

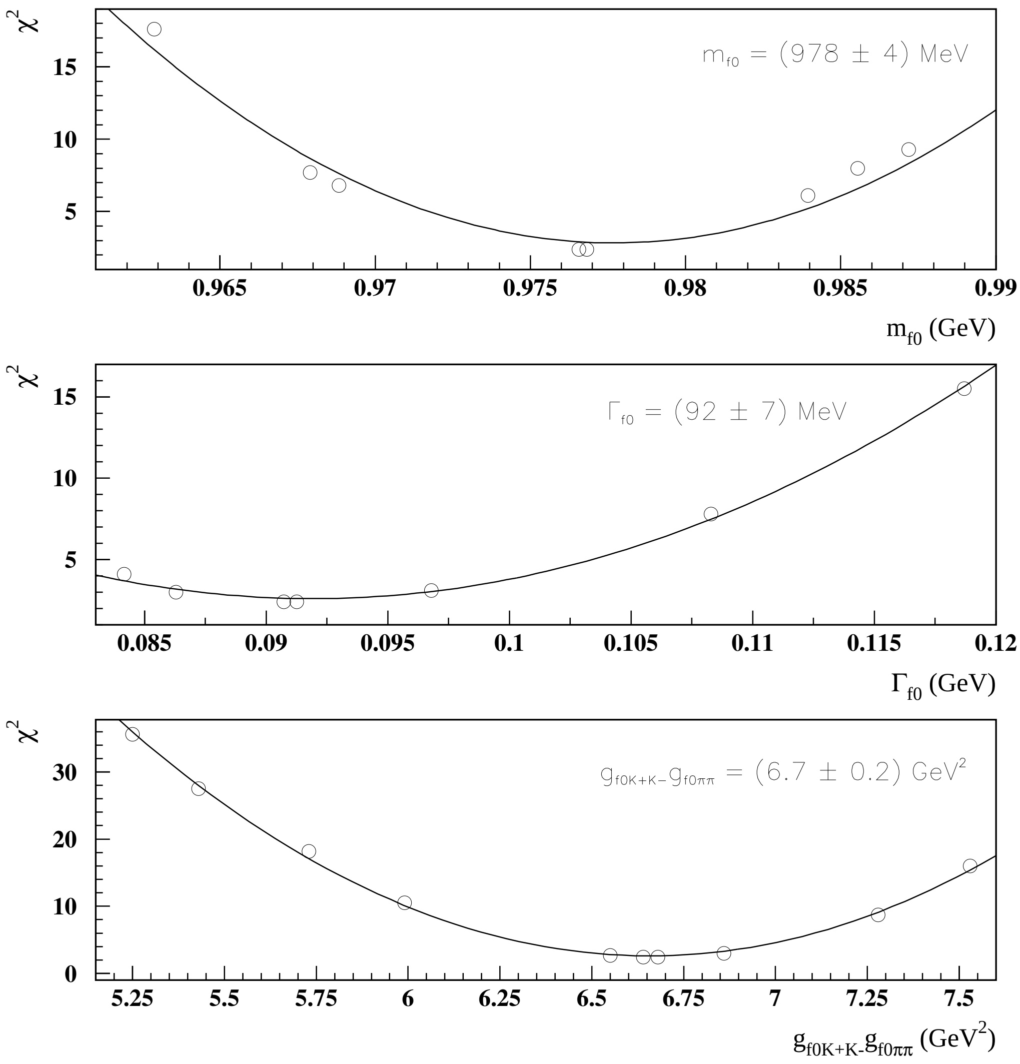
$$S_i^{cor} = \epsilon_{i,i-1} S_{i-1}^{th} + \epsilon_{i,i} S_i^{th} + \epsilon_{i,i+1} S_{i+1}^{th}$$

Estimation of parameters

$$\chi^2 = \sum_i \frac{\left(S_i^{cor}(\vec{\theta}) - S_i^{exp}\right)^2}{\sigma_i^2}$$

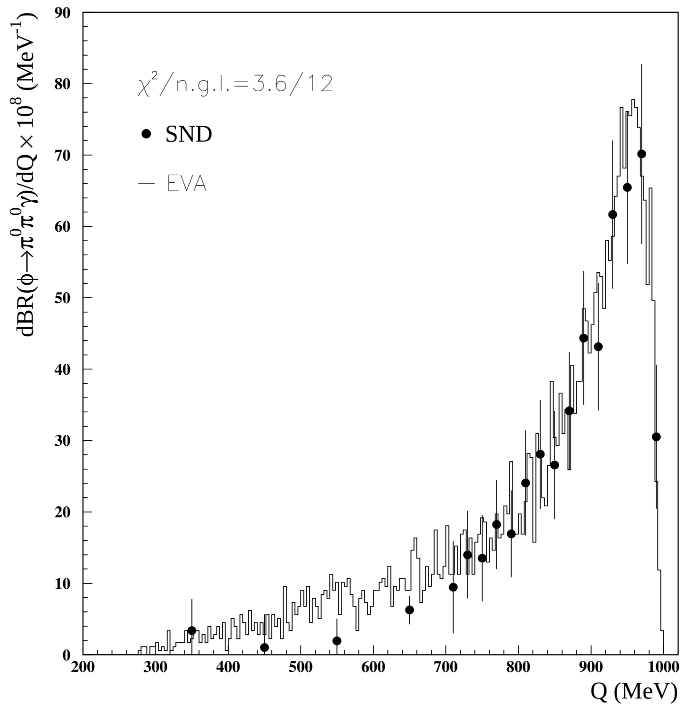
$$\chi^2(\theta_j) \simeq \chi^2_{min} + \frac{1}{\sigma_j^2}\,(\theta_j - \theta_{j,min})^2\Big|_{\theta_{i\neq j}=\theta_{i,min}}$$

an example with 3 parameters: K^+K^- loop and const. Γ_{f0}



Some results ($Q > 700$ MeV):

$qq\bar{q}\bar{q}$ structure

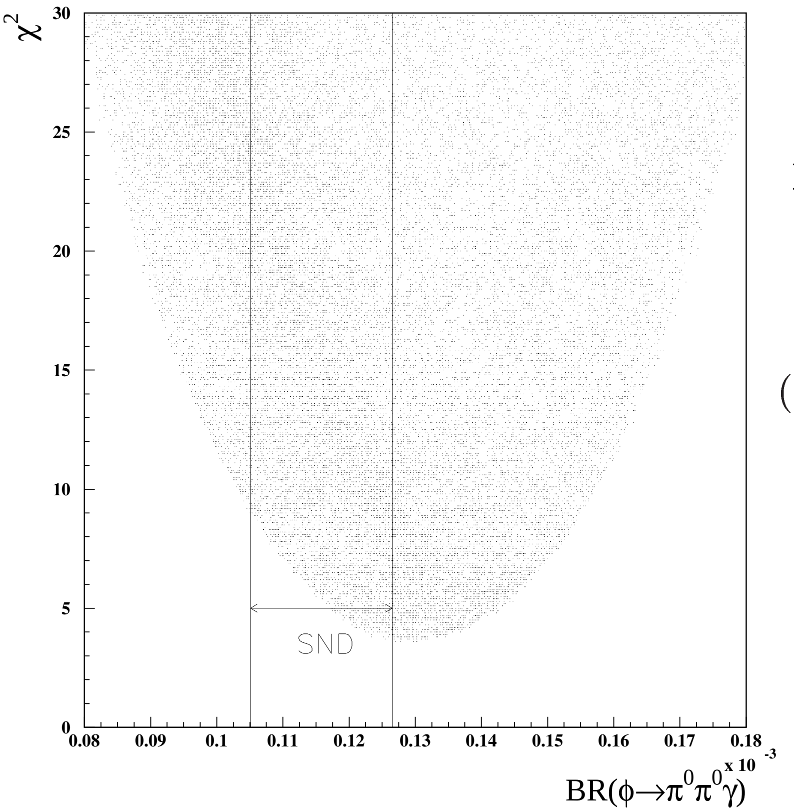


$$m_{f_0} = (965 \pm 3) \text{ MeV}$$

$$g^2_{f_0 K^+ K^-}/4\pi = (2.3 \pm 0.3) \text{ GeV}^2$$

$$g^2_{f_0 K^+ K^-}/g^2_{f_0 \pi^+ \pi^-} = 5.0 \pm 0.5$$

$$\chi^2_{min}/\text{d.o.f.} = 3.6/12$$



$$\text{BR}(\phi \to \pi^0\pi^0\gamma) \simeq 1.3 \times 10^{-4}$$

SND: $\text{BR}(\phi \to \pi^0\pi^0\gamma) =$

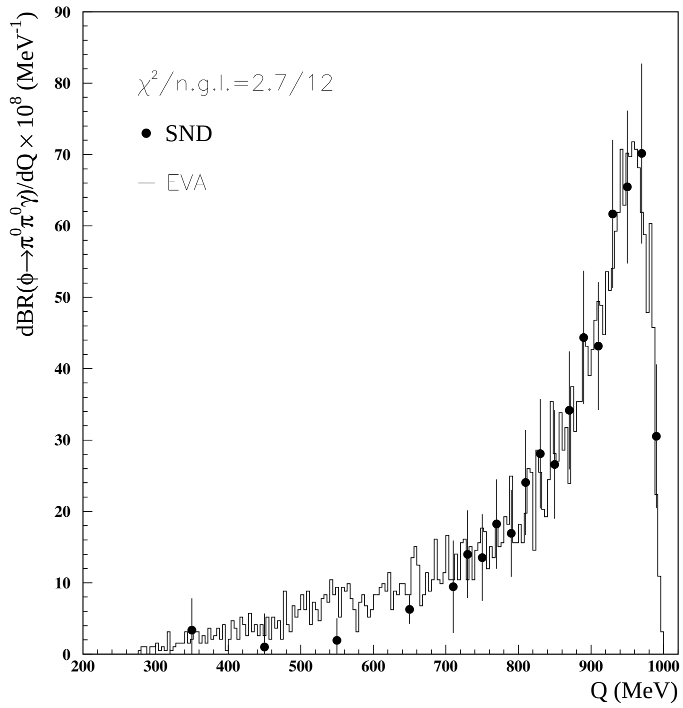
$(1.158 \pm 0.093 \pm 0.052) \times 10^{-4}$

(1) $Q \in [700, 1000]$ MeV

(2) $\phi \to \rho^0\,\pi^0 \to \pi^0\,\pi^0\,\gamma$

Some results ($Q > 700$ MeV):

K^+K^- loop and constant Γ_{f_0}



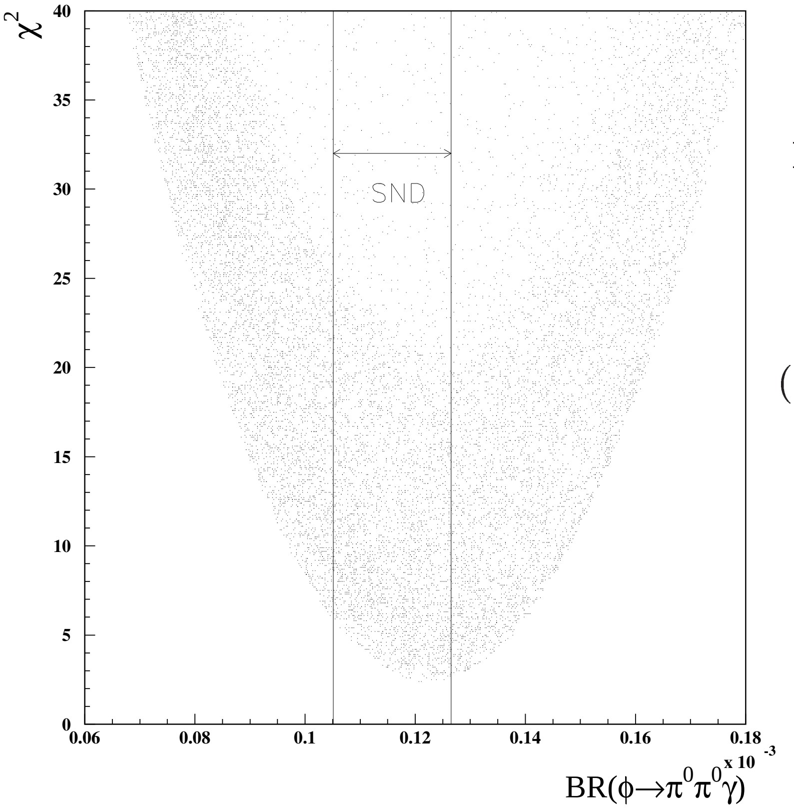
$m_{f_0} = (978 \pm 4)$ MeV

$\Gamma_{f_0} = (98 \pm 7)$ MeV

$g_{f_0 K^+K^-} g_{f_0 \pi^+\pi^-} =$

(6.7 ± 0.2) GeV^2

χ^2_{min}/d.o.f. $= 2.7/12$



$\mathrm{BR}(\phi \to \pi^0\pi^0\gamma) \simeq 1.2 \times 10^{-4}$

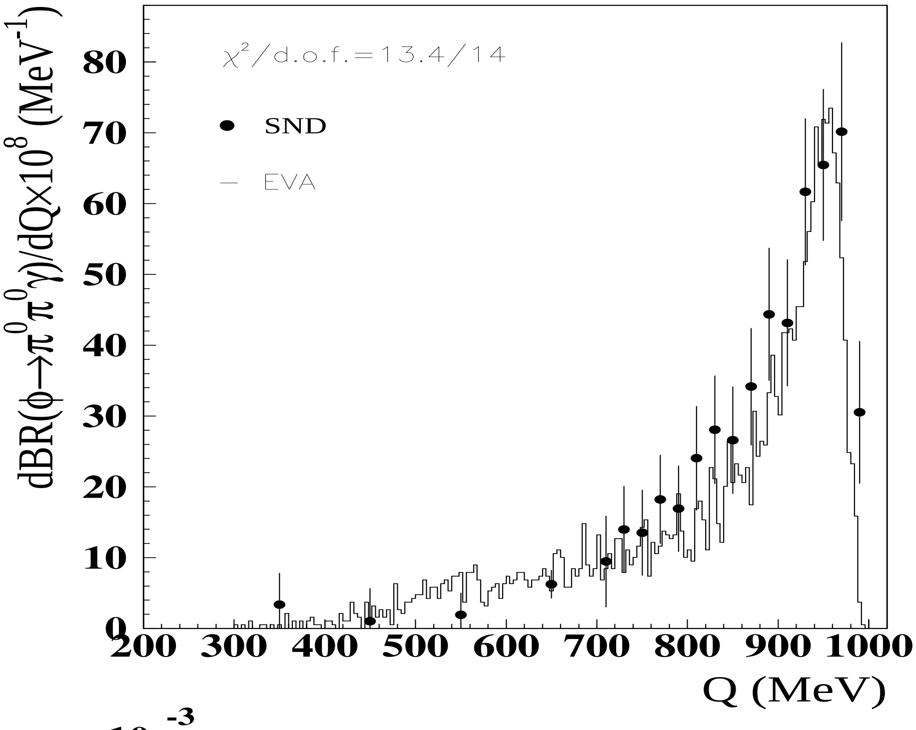
SND: $\mathrm{BR}(\phi \to \pi^0\pi^0\gamma) =$

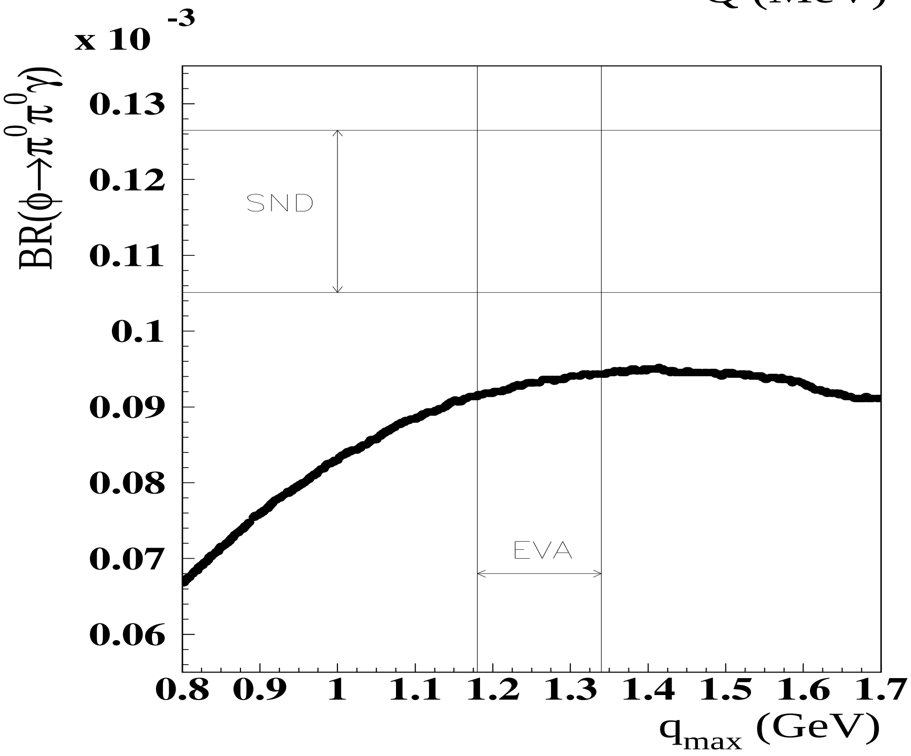
$(1.158 \pm 0.093 \pm 0.052) \times 10^{-4}$

Some results ($Q > 700$ MeV):

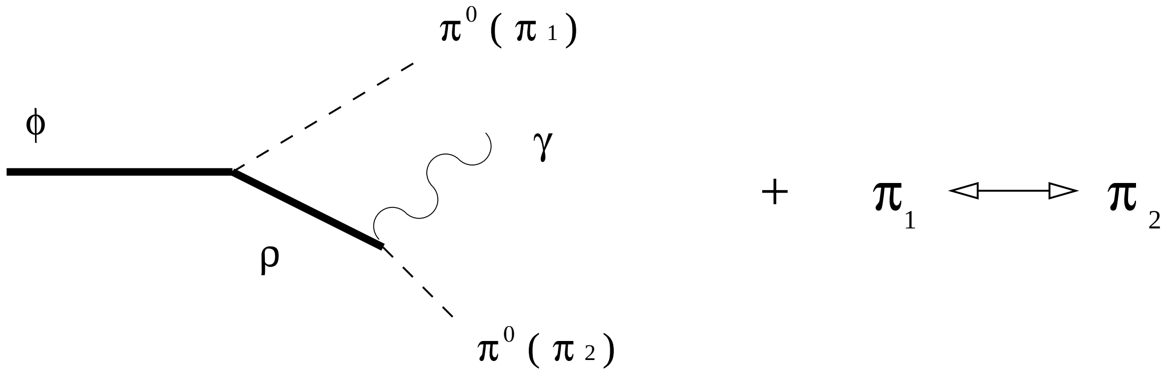
chiral unitary approach

$q_{max} = 1.26 \pm 0.08$ GeV, $\chi^2_{min}/$d.o.f. $= 13.4/14$





The decay $\phi \to \rho^0\,\pi^0 \to \pi^0\,\pi^0\,\gamma$



the decay has been included in EVA according to the VMD parametrization of Bramon *et al.*, together with the analytical expression for the interfering term:

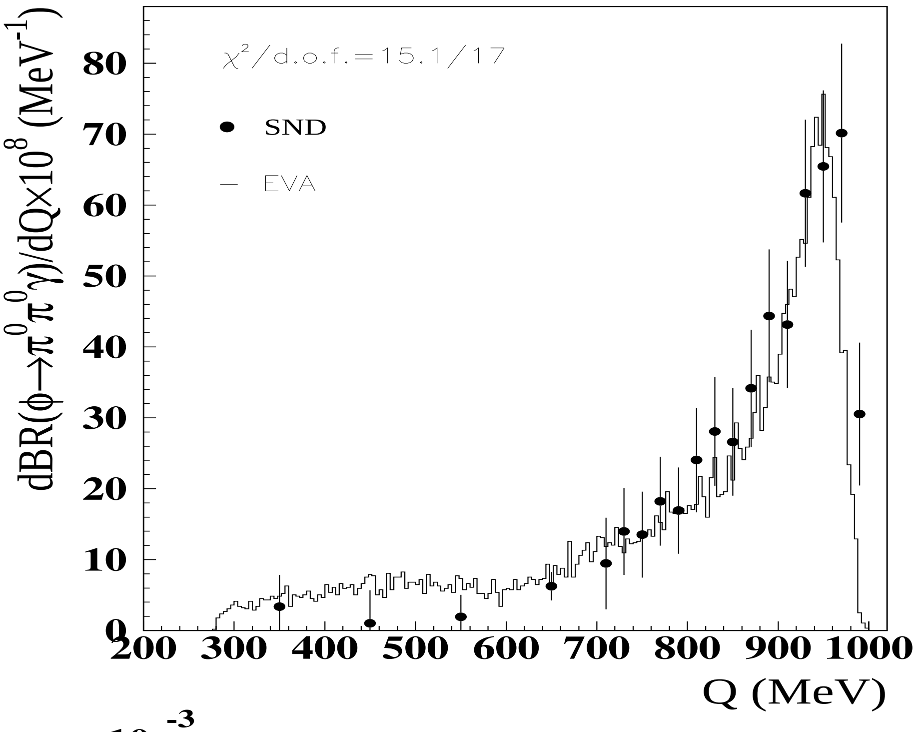
$$\boxed{|\mathcal{M}_{f_0}|^2 + |\mathcal{M}_{\rho^0\pi^0}|^2 + 2\,e^{i\varphi}\,\Re e\{\mathcal{M}^*_{f_0}\mathcal{M}_{\rho^0\pi^0}\}}$$

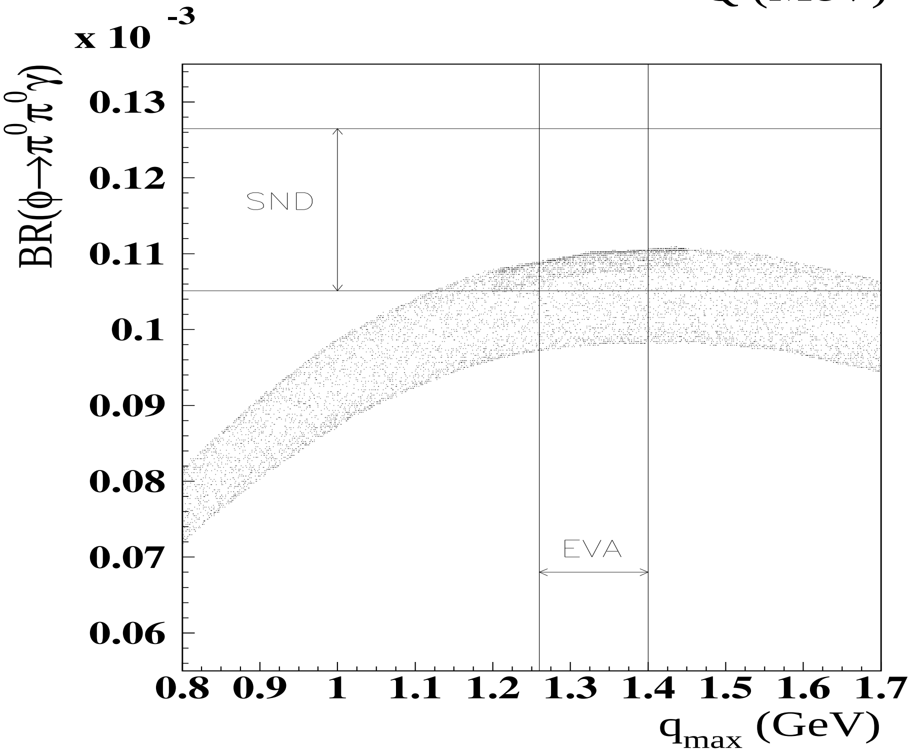
where φ is considered as a free parameter as the ones related to a given model, *e.g.* m_{f_0}, $g_{f_0K^+K^-}$ etc...

More results: χ un. approach

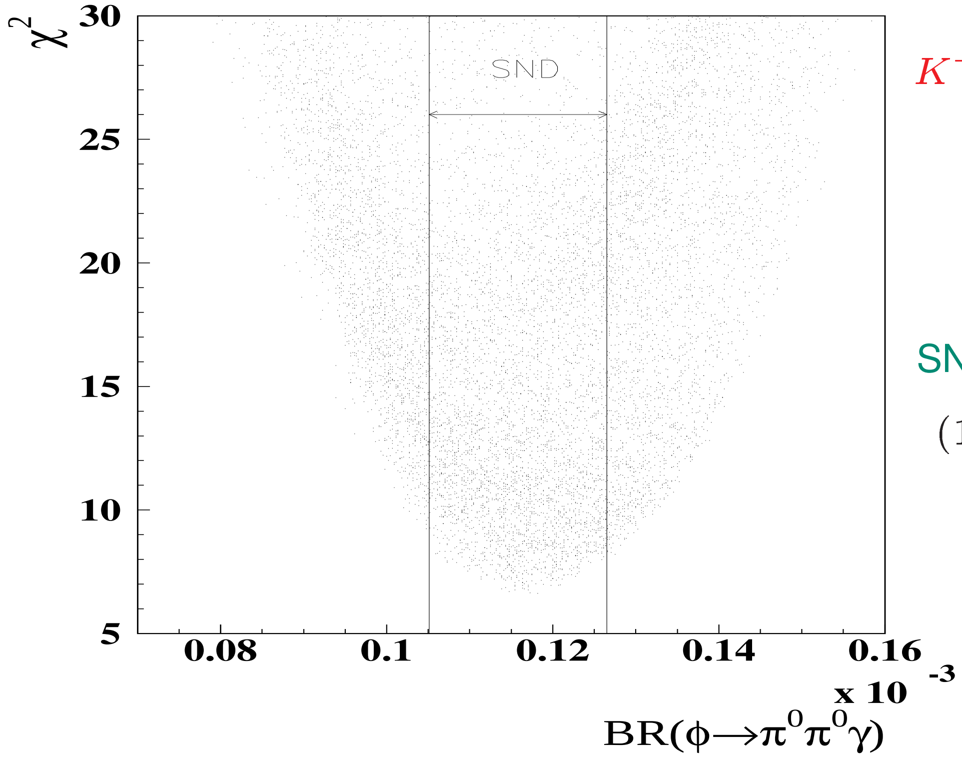
the whole Q spectrum, $\rho^0\pi^0$ improved

$q_{max} = 1.33 \pm 0.07$ GeV, $\varphi = -0.55 \pm 0.44$ rad





More χ^2 vs. $BR(\phi \to \pi^0\pi^0\gamma)$

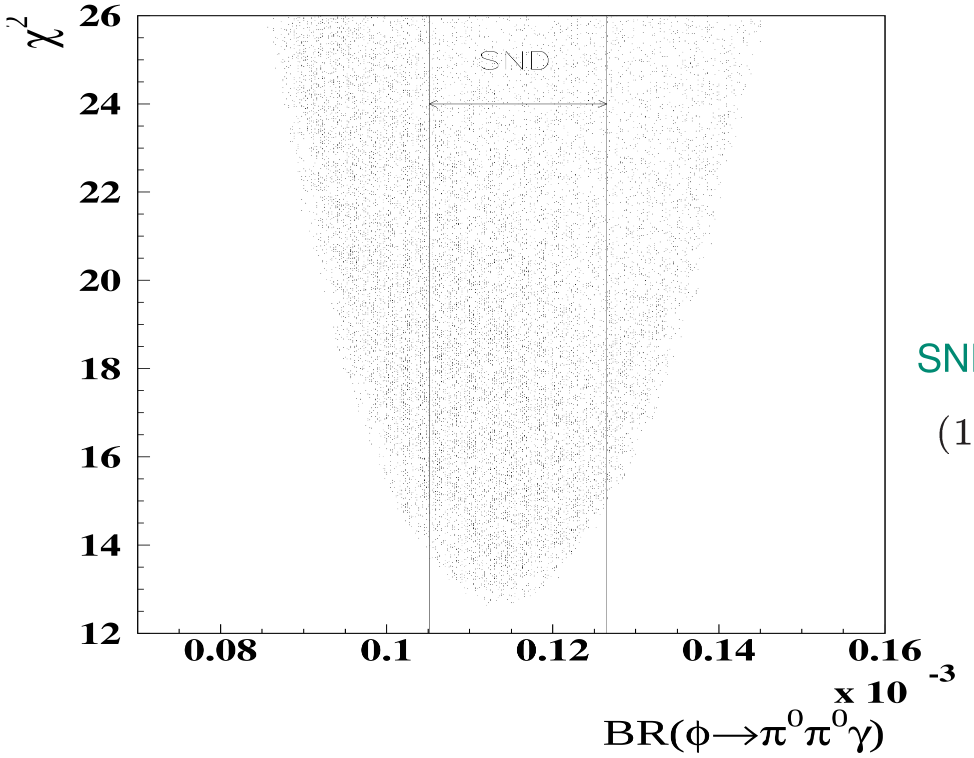


K^+K^- loop + const. Γ_{f_0}

the whole Q spectrum

$\rho^0\pi^0$ improved

SND: $BR(\phi \to \pi^0\pi^0\gamma) = (1.158 \pm 0.093 \pm 0.052) \times 10^{-4}$



$qq\bar{q}\bar{q}$ structure

the whole Q spectrum

$\rho^0\pi^0$ improved

SND: $BR(\phi \to \pi^0\pi^0\gamma) = (1.158 \pm 0.093 \pm 0.052) \times 10^{-4}$

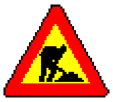
Conclusions

- The decay $\phi \to f_0(980)\,\gamma \to \pi\,\pi\,\gamma$ has been implemented with $\phi \to \rho^0\,\pi^0 \to \pi^0\,\pi^0\,\gamma$ for generating $e^+\,e^- \to \pi\,\pi\,\gamma$.
- Different models have been included: $qq\bar{q}\bar{q}$, $K\bar{K}$, the chiral unitary approach.
- A minimization procedure for extracting $f_0(980)$ parameters has been presented.
- By applying the procedure to SND data:

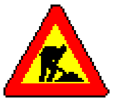
  (i) we obtained the best estimate for q_{max}

  (ii) our method gave us as similar as SND results in $qq\bar{q}\bar{q}$ structure

  (iii) we found K^+K^- loop with constant Γ_{f_0} is an excellent trial function in simulating $\phi \to f_0(980)\,\gamma$ decay.



We are going to write a paper on these results.



We plan to apply the procedure to KLOE data.