4. Hadronic cross sections $e^+e^- \to q\,\bar{q}$

4.1. Motivation

Determination of the hadronic vacuum polarisation

a) Measurement of g - 2 of muons (Brookhaven *E821*)

b) running electromagnetic coupling constant $\alpha_{Sommerfeld}(M_Z)$

$\Rightarrow$ prediction of m_{Higgs}, corrections to *electroweak data*

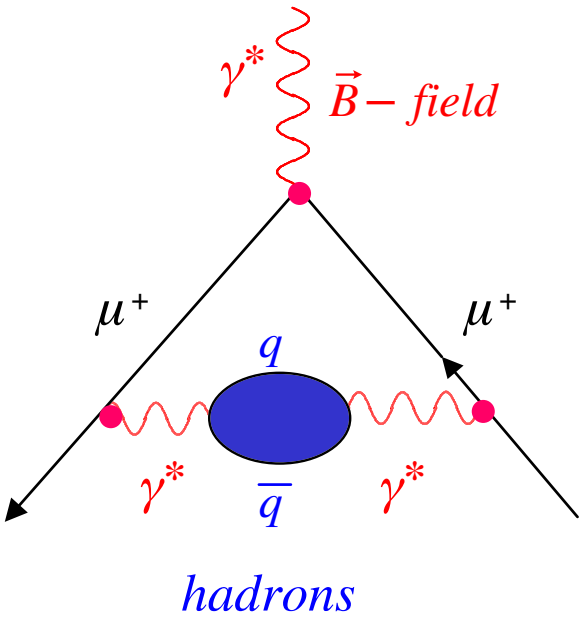
a) Anomalous magnetic moment of muons

$$a_\mu = (g_\mu - 2)/2 = \alpha/2\pi + \ldots$$

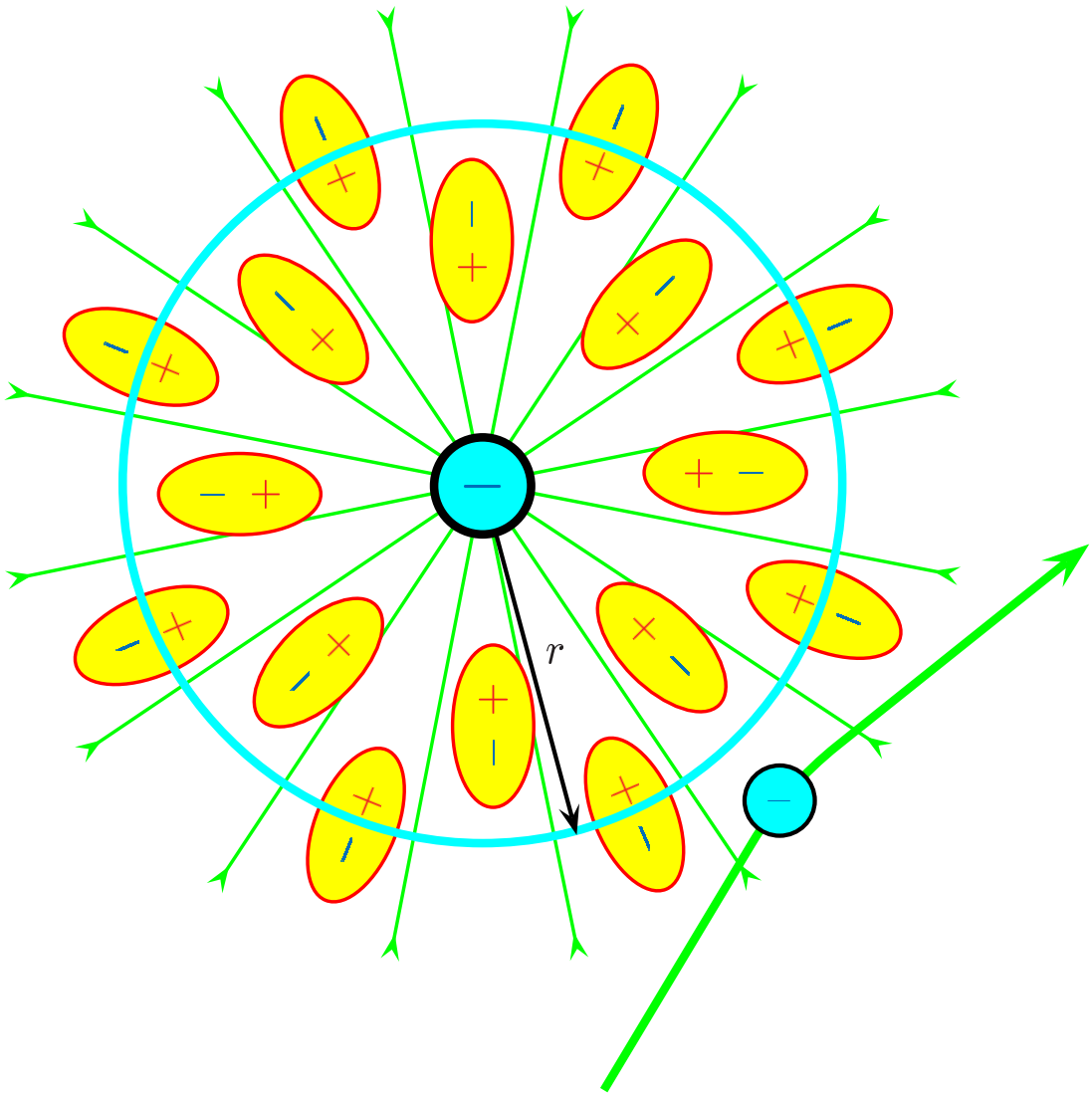
$$a_\mu^{theor} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{weak} + a_\mu^{new}$$

One of the best measured numbers of physics, calculated within *QED* up to $O(\alpha^5)$

- Sensitive to hadronic vacuum polarisation a_μ^{had}
- Sensitive to weak corrections (due to *W, Z, Higgs*) a_μ^{weak}
- might reveal effects beyond $\mathcal{SM}$ a_μ^{new}



Vacuum polarization and charge screening



In a collision of impact energy E the effective charge is the charge contained in the sphere of radius r due to vacuum polarization is larger than the classical charge seen in a large shere ($r \to \infty$)

$\Downarrow$

charge screening

(charge renormalization).

Hadronic contribution to vacuum polarisation dominated by low energy effects which cannot be obtained by perturbative *QCD* for low s

but rather by experimental data of $e^+ e^-$ annihilation into hadrons calculating the dispersion integral

$$a_\mu^{had} = \left(\frac{\alpha\, m_\mu}{3\,\pi} \right)^2 \int_{4\, m_\pi^2}^{\infty} ds\, \frac{R(s)\, \hat{K}(s)}{s^2}$$

up to some sufficiently high energies, typically *2…10 GeV*, and by *pQCD* for higher energies

$$R(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to hadrons)}{\sigma_{tot}(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

$\tilde{K}(s)$ being a smooth function ($\hat{K}(s) = 0.63$ *at* $s = 4m_\pi^2$ *and* $\hat{K}(s) = 1.0$ *at* $s \Rightarrow \infty$)

Energy denominator $1/s^2 \approx 1/E^4$ enhances dramatically low energy (non perturbative) contributions

Present Numbers:

$a_\mu^{exp} \approx (11\,659\,230 \pm 84) \cdot 10^{-10}$ *CERN 1977*

$a_\mu^{exp} \approx (11\,659\,191 \pm 59) \cdot 10^{-10}$ *BNL 1998 , ± 5 ppm*

$a_\mu^{exp} \approx (11\,659\,205 \pm 46) \cdot 10^{-10}$ *CERN 1977 +BNL 1998*

$a_\mu^{exp} \approx (11\,659\,202 \pm 16) \cdot 10^{-10}$ *BNL 2001, ± 1.3 ppm*

$a_\mu^{exp} \approx (\quad ??? \quad \pm 4) \cdot 10^{-10}$ *BNL 200?, ± 0.35 ppm*

QED contribution $a_\mu^{QED} = (11\,658\,470.57 \pm 0.29\ ?) \cdot 10^{-10}$

A. Czarnecki, W. J. Marciano 2000
up to 5-loops, up to order $(\alpha / \pi)^5$

weak correction: $a_\mu^{weak} \approx (15.2 \pm 0.4) \cdot 10^{-10}$

electroweak vacuum polarisation up to 2-loops

hadronic correction: $a_\mu^{had} \approx (692.4 \pm 6.2) \cdot 10^{-10}$

Davier, Höcker 1999
τ-decays

$a_\mu^{had} \approx (697.4 \pm 10.5) \cdot 10^{-10}$

Jegerlehner März 2001, DESY 01-028
without τ-decays, new *CMD* and *BES* data

hadronic vacuum polarisation up to *3*-loops, including light-by-light scattering

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{weak} = (11\,659\,159.7 \pm 6.7)\cdot 10^{-10}$$

DH 1999

$$= (11\,659\,164.7 \pm 10.4)\cdot 10^{-10}$$

FJ 2001

$$a_\mu^{exp} - a_\mu^{SM} \approx (43 \pm 16)\cdot 10^{-10}$$ *2.5 ... 2.9 σ difference*

by reducing the error δa_μ^{had} it is possible to isolate a_μ^{weak}

(and to look for new physics, *SUperSYmmetry* effects around the corner ?)

Meaning of precision tests and ‚new physics'

- a_μ^{exp} is 350 times less precise than a_e^{exp}
- But, much more sensitive to hadronic and electroweak quantum loop as well as ‚new physics' effects, since such contributions are generally proportional to m_ℓ^2

The $m_\mu^2 / m_e^2 \approx 40\,000$ enhancement compensates for the reduced experimental precision and makes a_μ^{exp} a much better probe of short-distance phenomena

Hadronic light by light scattering

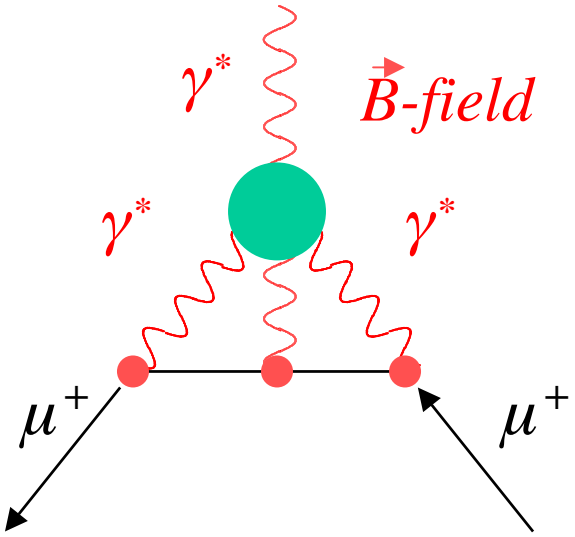
$$a_\mu^{light\ by\ light} = \begin{cases} -(9.2 \pm 3.2) \\ -(7.9 \pm 1.5) \cdot 10^{-10} \\ -(8.5 \pm 2.5) \end{cases}$$

Bijnens, Pallante, Prades1996

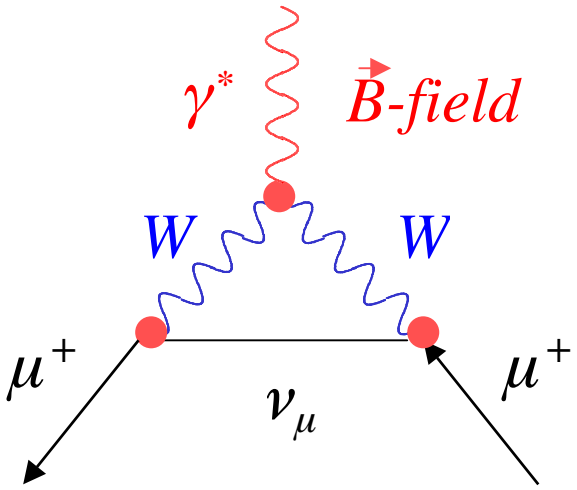
Hayakawa, Kinoshita 1998

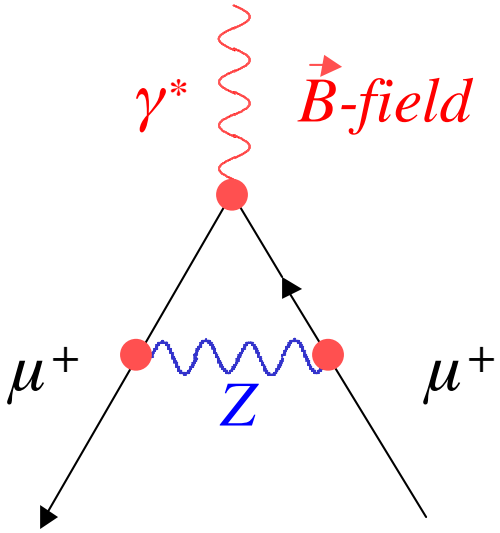
Czarnecki, Marciano 1998

Question: is the sign correct ?

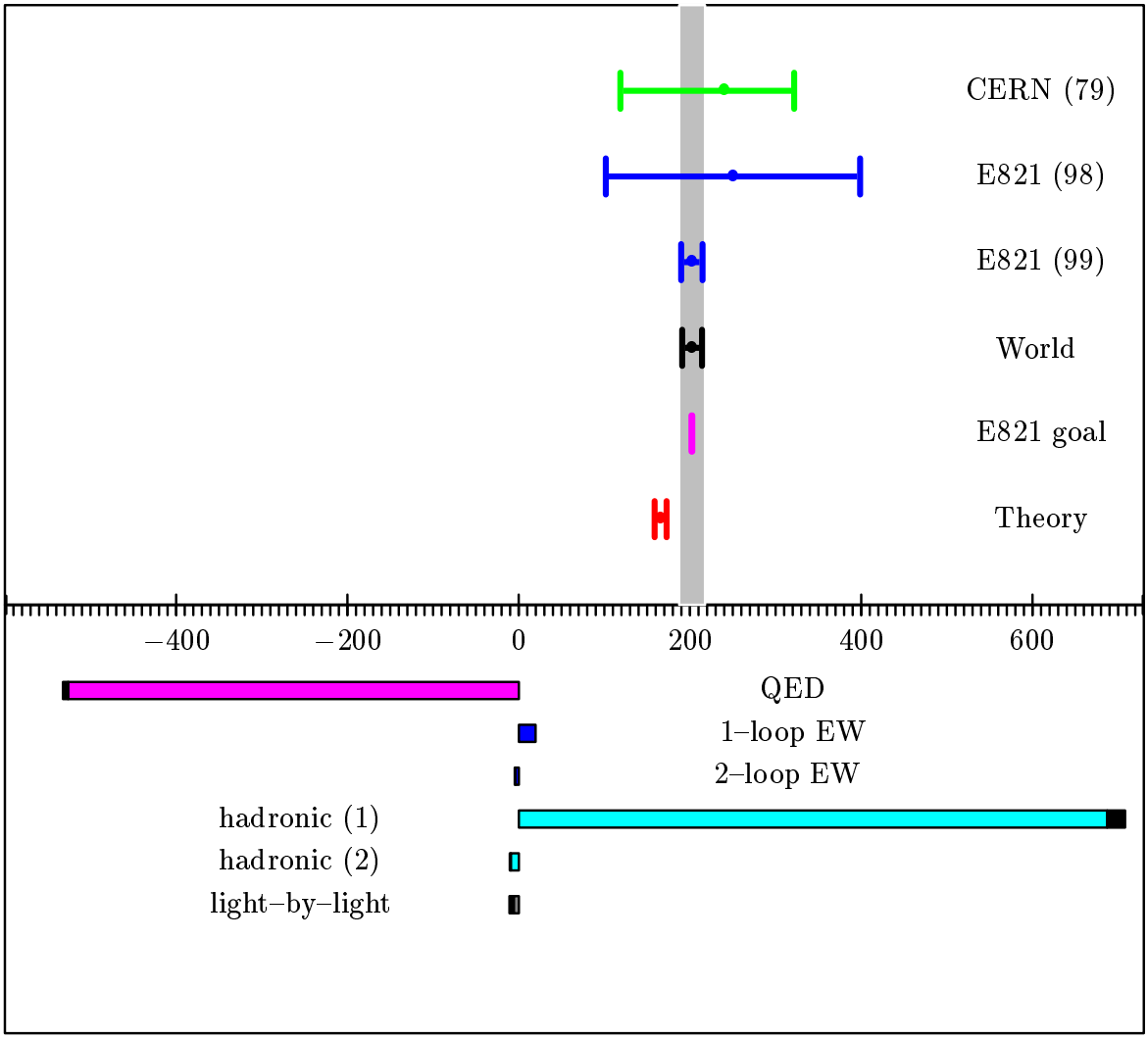


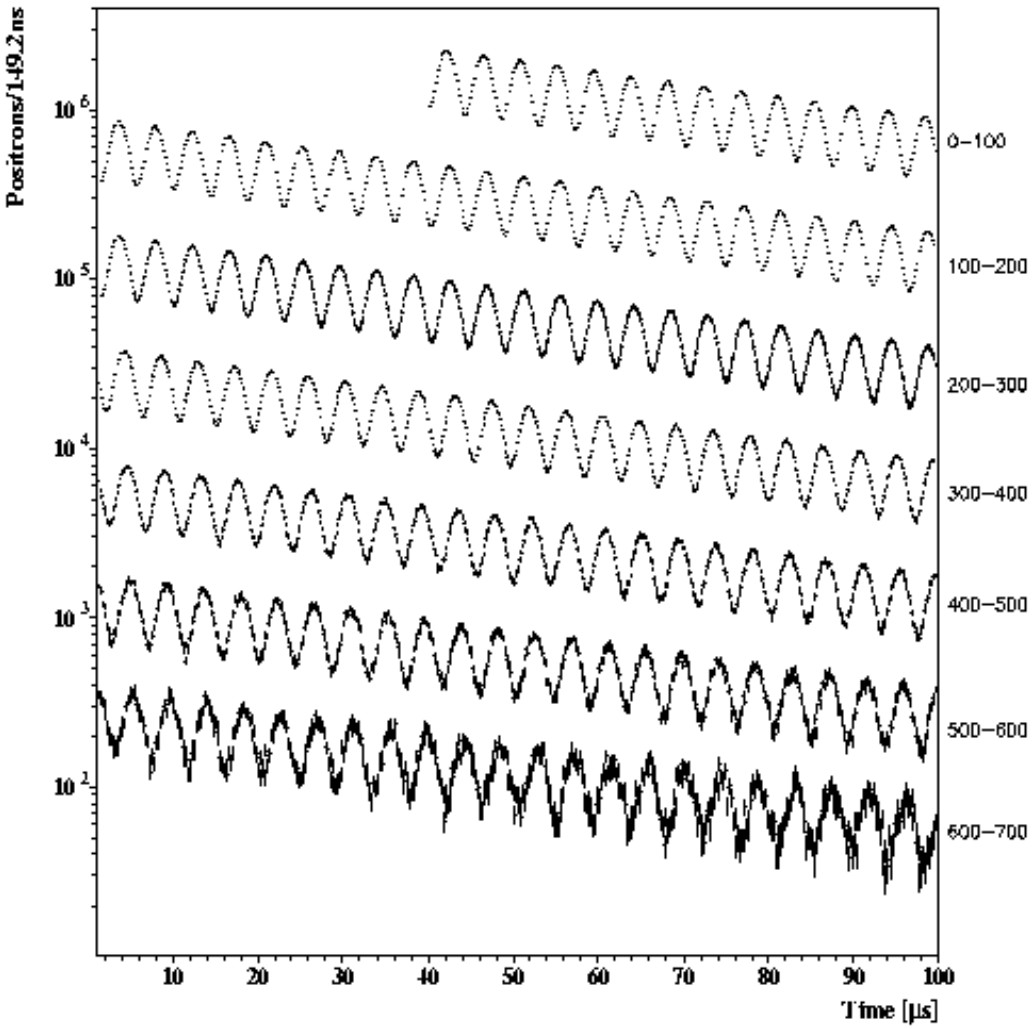
Weak contribution (1^{st} order)





of the muon. The new E821 expriment at Brookhaven revield a 2.6 σ deviation from the theory. The various types of SM contributions are shown in the lower part of the figure.




Positrons/149.2ns
10^6
10^5
10^4
10^3
10^2
0–100
100–200
200–300
300–400
400–500
500–600
600–700
10
20
30
40
50
60
70
80
90
100
Time [μs]

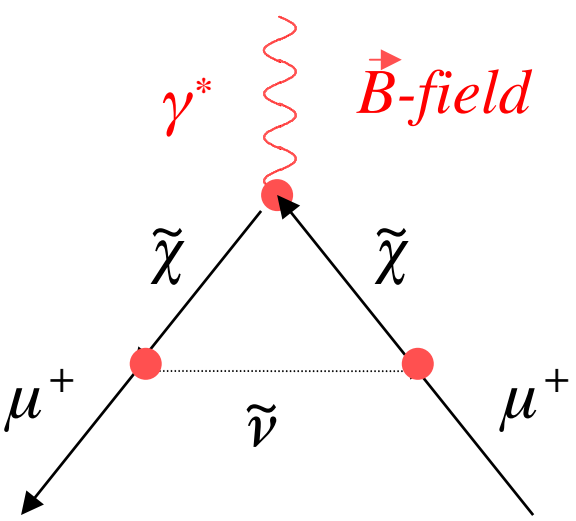
Supersymmetry

a_μ is a sensitive test for *SUSY* models with large tan β

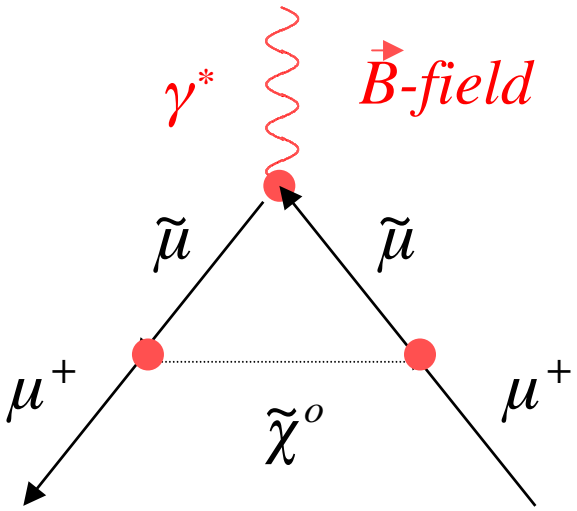
$$\alpha_\mu(SUSY) \approx \frac{\alpha}{8\pi \sin^2\theta_W}\frac{m_\mu^2}{\tilde{m}^2}\tan\beta$$

$$\approx 140\cdot 10^{-11}\left(\frac{100\,GeV}{\tilde{m}}\right)^2 \tan\beta$$

($\tilde{m}$ being some typical *SUSY* loop mass)



sneutrino-chargino



smuon-neutralino

CERN –Recherche zu '*g-2*' vom 15. Mai 2001

1. Supersymmetric contributions to muon *g-2* and the electroweak precision measurements
Cho, G C ; Hagiwara, K ; hep-ph/0105037, 4 May 2001 - 12 p.

2. Muon Anomalous Magnetic Moment and $\tau \to \mu \gamma$ in a Realistic String-Inspired Model of Neutrino Masses
Blazek, T ; King, S F ; hep-ph/0105005 ; 1 May 2001. 16 p.

3: Two-loop Barr-Zee type Contributions to $(g\text{-}2)_\mu$ in the MSSM
Arhrib, A ; Baek, S ; hep-ph/0104225 ; 23 Apr 2001. - 10 p.

4. Muon Anomalous *g -2* and Gauged $L_\mu - L_\tau$ Models
Baek, S ; Deshpande, N G ; He, X G ; Ko, P ; hep-ph/0104141, 14 Apr 2001. - 13 p.

5. CP Violation from Noncommutative Geometry
Hinchliffe, I ; Kersting, N ; hep-ph/0104137 ; 13 Apr 2001. - 16 p.

6. Transport Coefficients in Large N_f Gauge Theory : Testing Hard Thermal Loops
Moore, G D ; hep-ph/0104121 ; 11 Apr 2001. - 30 p.

7. The reach of a future Linear Collider after the *g-2* result
Richard, F ; hep-ph/0104106 ; 11 Apr 2001. - 8 p.

8. The check of QCD based on the τ-decay data analysis in the complex q^2-plane
Geshkenbein, B V ; Ioffe, B L ; Zyablyuk, K N ; hep-ph/0104048 ; 4 Apr 2001. - 24 p.

9. Anomalous Magnetic Moment of the Muon in a Composite Model
Das, P ; Rai, S K ; Raychaudhuri, S ; hep-ph/0102242 ; 19 Feb 2001. - 10 p.

10. *g-2* in composite models of leptons
Dai, Y B ; Huang, C S ; Zhang, A ; hep-ph/0103317 ; 29 Mar 2001. - 16 p.

11. A very light CP-odd scalar in the Two-Higgs-Doublet Model
Larios, F ; Tavares-Velasco, G ; Yuan, C P ; hep-ph/0103292 ; 27 Mar 2001. - 28 p. –

12. Charged-Lepton-Flavour Violation in the *CMSSM* in View of the Muon Anomalous Magnetic Moment
Carvalho, D F ; Ellis, J ; Gomez, M E ; Lola, S ; hep-ph/0103256 ; 23 Mar 2001. 14 p.

13. The new (*g-2*) for muon measurement and limits on the light Higgs bosons in 2HDM (II)
Krawczyk, M ; hep-ph/0103223 ; IFT-2001-5. - 20 Mar 2001. - 10 p.

14. Muon and tau anomalies updated
Narison, S ; hep-ph/0103199, 19 Mar 2001. - 11 p.

15. Muon Anomalous Magnetic Moment $(g\text{-}2)_\mu$ and the Randall-Sundrum Model
Kim, C S ; Kim, J D ; Song, J ; hep-ph/0103127, 13 Mar 2001. - 12 p.

16. Neutrino-Lepton Masses, Zee Scalars and Muon *g-2*
Dicus, D A ; He, H J ; Ng, J N ; hep-ph/0103126 ; 12 Mar 2001. - 4 p.

17. Scalar scenarios contributing to $(g\text{-}2)_\mu$ with enhanced Yukawa couplings
De Pires, C A S ; Silva, P S R; hep-ph/0103083 ; 8 Mar 2001. - 6 p.

18. Implications of the muon anomalous magnetic moment and Higgs-mediated flavor changing neutral currents
Kang, S K ; Lee, K Y ; hep-ph/0103064 ; 6 Mar 2001. - 6 p.

19. Effective supersymmetric theory and $(g\text{-}2)_\mu$ with R parity violation
Kim, J E ; Kyae, B ; Lee, H M ; hep-ph/0103054 ; 6 Mar 2001. - 12 p.

20. Muon *g-2*, Dark Matter Detection and Accelerator Physics
Arnowitt, R ; Dutta, B ; Hu, B ; Santoso, Y ; hep-ph/0102344 ; 28 Feb 2001. - 10 p. –

21. Constraints on Explicit *CP* Violation from the Brookhaven Muon *g-2* Experiment
Ibrahim, T ; Chattopadhyay, U ; Nath, P ; hep-ph/0102324 ; 27 Feb 2001. - 22 p. -

22. Nearly Bi-Maximal Neutrino Mixing, Muon *g-2* Anomaly and Lepton-Flavor-Violating Processes / Xing, Z ; hep-ph/0102304 ; 24 Feb 2001. - 6 p.

23. Can the Higgs sector contribute significantly to the muon anomalous magnetic moment?
Dedes, A ; Haber, H E ; hep-ph/0102297, 23 Feb 2001. - 15 p.

24. Constraining new physics in the *TeV* range by the recent *BNL* measurement of $(g\text{-}2)_\mu$
Mahanta, U ; hep-ph/0102211 ; 16 Feb 2001. - 4 p.

25. Recent Result from *E821* Experiment on Muon *g-2* and Unconstrained Minimal Supersymmetric Standard Model
Komine, S ; Moroi, T ; Yamaguchi, M ; hep-ph/0102204 ; 16 Feb 2001. - 10 p.

26. Muon Anomalous Magnetic Moment and Lepton Flavor Violation
Huang, T ; Lin, Z H ; Shan, L Y ; Zhang, X ; hep-ph/0102193 ; 15 Feb 2001. - 3 p.

27. Upper Limits on Sparticle Masses from *g-2* and the Possibility for Discovery of *SUSY* at Colliders and in Dark Matter Searches
Chattopadhyay, U ; Nath, P ; hep-ph/0102157 ; 13 Feb 2001. - 10 p.

28. Implications of Muon *g-2* for Supersymmetry and for Discovering Superpartners Directly
Everett, L ; Kane, G L ; Rigolin, S ; Wang, L T ; hep-ph/0102145 ; 12 Feb 2001. - 4 p.

Why *DAΦNE* is important for a_μ?

DAΦNE covers about *77%* of a_μ^{had} in its energy region

$$2\, m_\pi < E_{c.m.} < 1.4\ GeV$$

$$a_\mu^{had}\ (DA\Phi NE) \approx 615 \cdot 10^{-10}$$

An accuracy of *1%* of $\sigma(e^+e^- \to hadrons)$ up to *1.4 GeV*
results in $\delta a_\mu^{had} = \pm\ 7 \cdot 10^{-10}$

An accuracy of *1%* of $\sigma(e^+e^- \to hadrons)$ up to *3.1 GeV*
results in $\delta a_\mu^{had} = \pm\ 1.5 \cdot 10^{-10}$

Resonances at *DAΦNE* energies: $e^+\, e^- \to \rho, \omega, \phi \to hadrons$
(ρ resonance covers *72 %* of the hadronic contribution to a_μ)

Main multiparticle channels: $e^+\, e^- \to continuum \to hadrons$

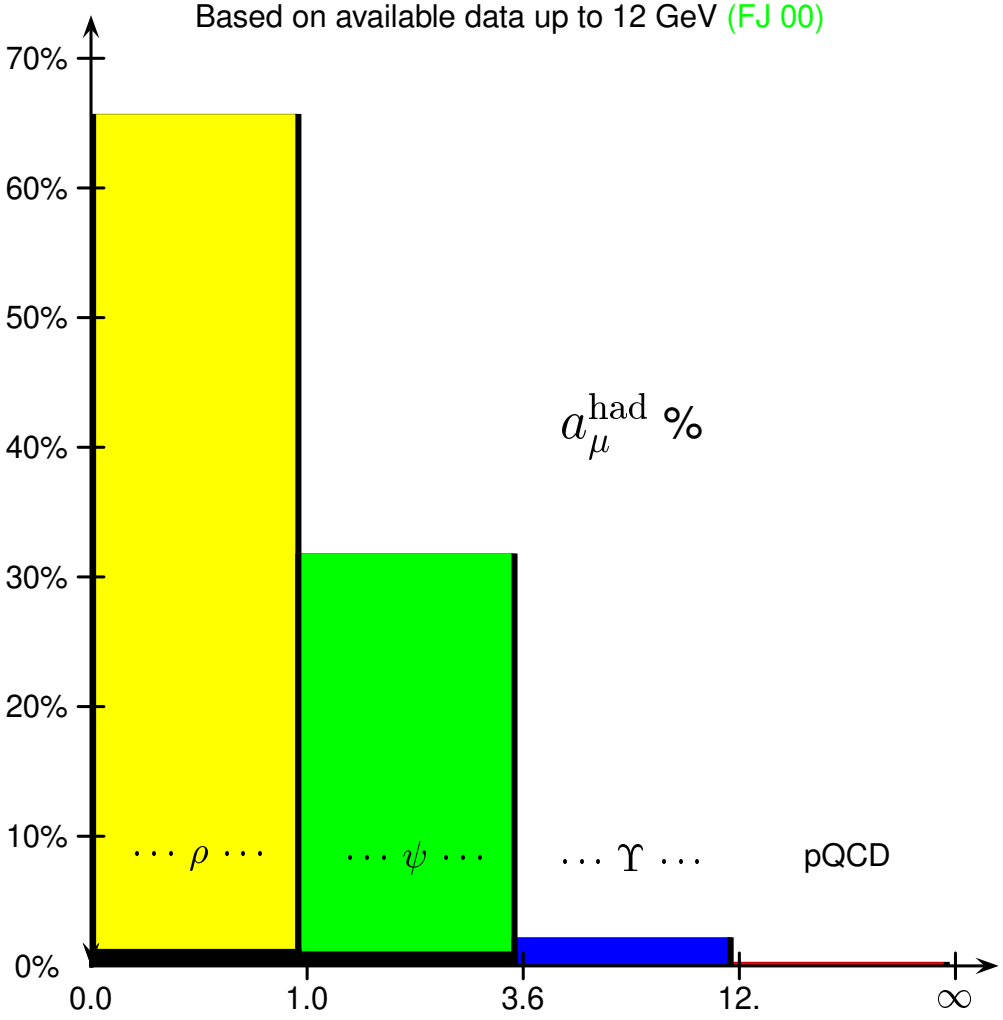
$e^+\, e^- \to \pi^+\, \pi^-$ $\quad$ $e^+\, e^- \to \pi^+\, \pi^-\, \pi^o$

$e^+\, e^- \to K^+\, K^-$ $\quad$ $e^+\, e^- \to K_L\, K_S$

Distribution of hadronic contributions to $a_\mu^{\rm had}$

e^+e^- data based approach:

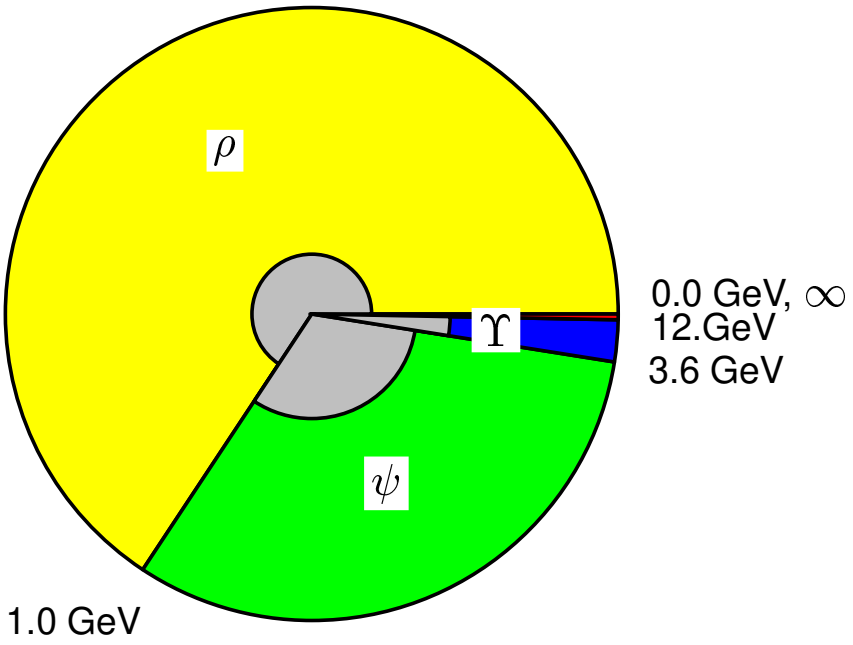
Based on available data up to 12 GeV (FJ 00)



Distribution of hadronic contributions to $a_\mu^{\rm had}$

e^+e^- data based approach:

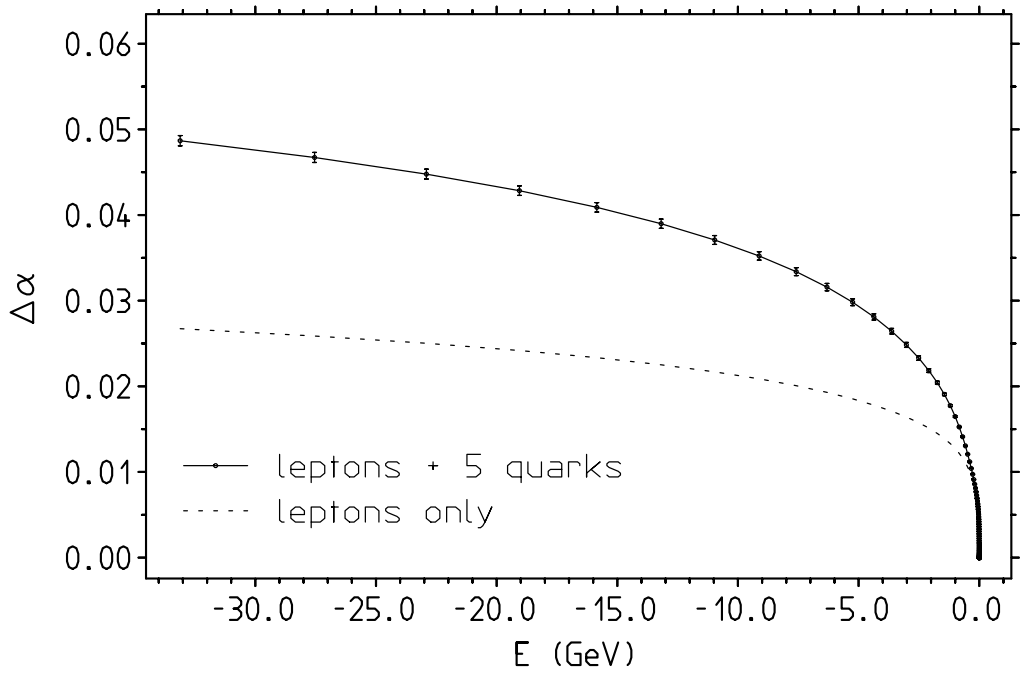
Based on available data up to 12 GeV (FJ 00)



Shaded area: 10 $\times$ error (scaled up)

Effective finestructure "constant" $\alpha(E)$

appears in many places in physics! **large EWRC !**



- Definition $\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$

Leptons (calculable in perturbation theory) $\Delta\alpha_{leptons}(M_Z^2) = 0.03142$

Quarks (not calculable in perturbation theory, using unitarity and analyticity constraints for the photon vacuum polarisation function)

$$\Delta\alpha_{had}^{(5)}(s) = -\frac{\alpha s}{3\pi}\wp\left(\int_{4m_\pi^2}^{E_{cm}^2} ds' \frac{R^{exp}(s')}{s'(s'-s)} + \int_{E_{cm}^2}^{\infty} ds' \frac{R^{pQCD}(s')}{s'(s'-s)}\right)$$

$$\Delta\alpha_{hadrons}^{(5)}(M_Z^2) = 0.0280 \pm 0.0007$$

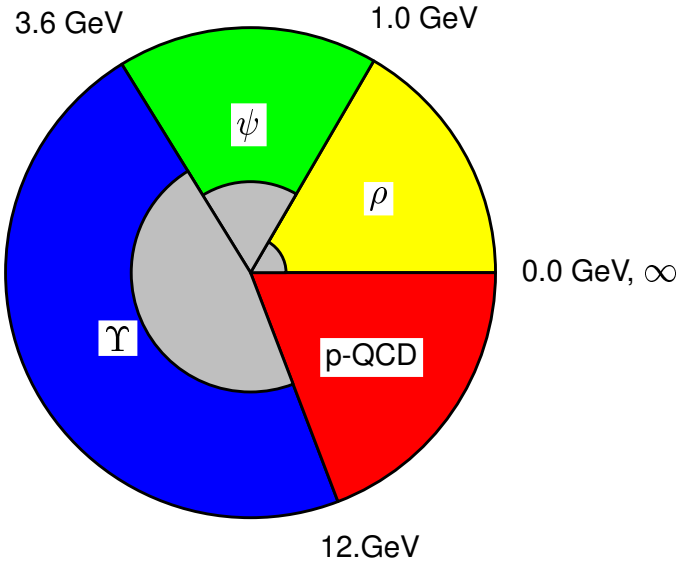
$$\Rightarrow \qquad \alpha^{-1}(M_Z) = 128.89 \pm 0.09$$

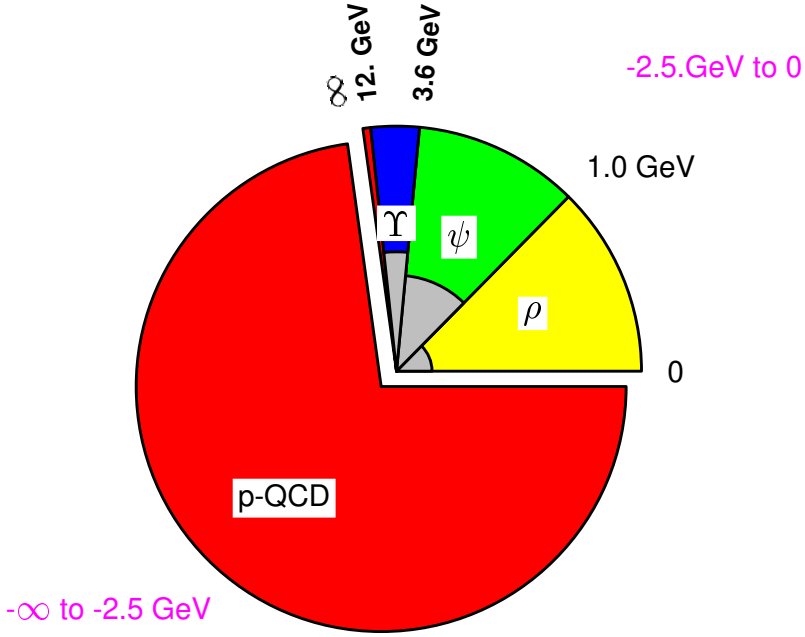
DAΦNE -energies: $50 \cdot 10^{-4}$

all energies: $280 \cdot 10^{-4}$ *(to be discussed)*

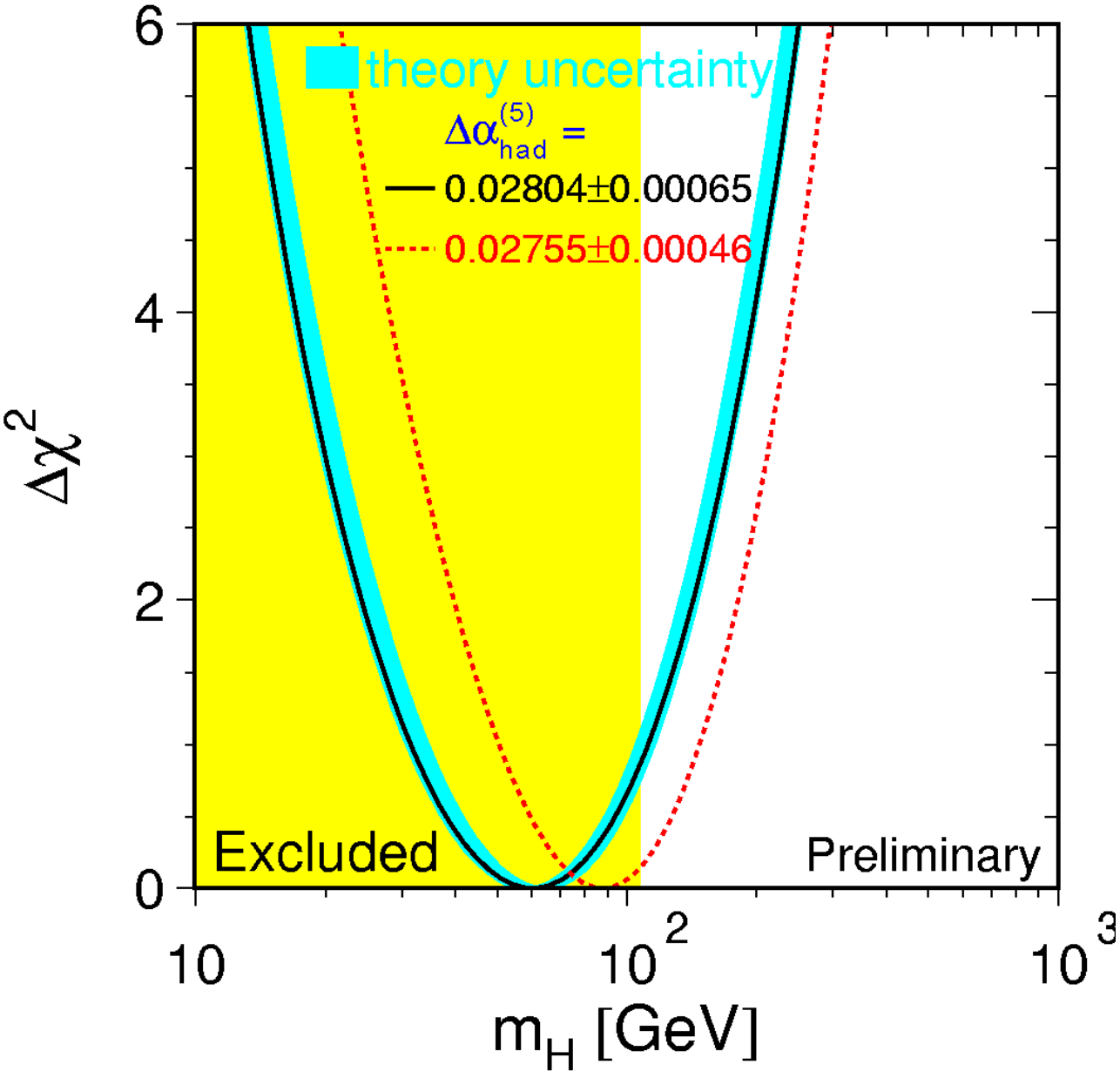
- results from *BES* indicate significance
- new investigations (*Adler* function) show that *pQCD* can give results down to ca. *2.5* Eidelman, Jegerlehner et al. 1999
- higher order *QCD* calculations Kühn, Steinhauser 1998

KLOE can significantly contribute to reduce the error of $\Delta\alpha_{had}^{(5)}$

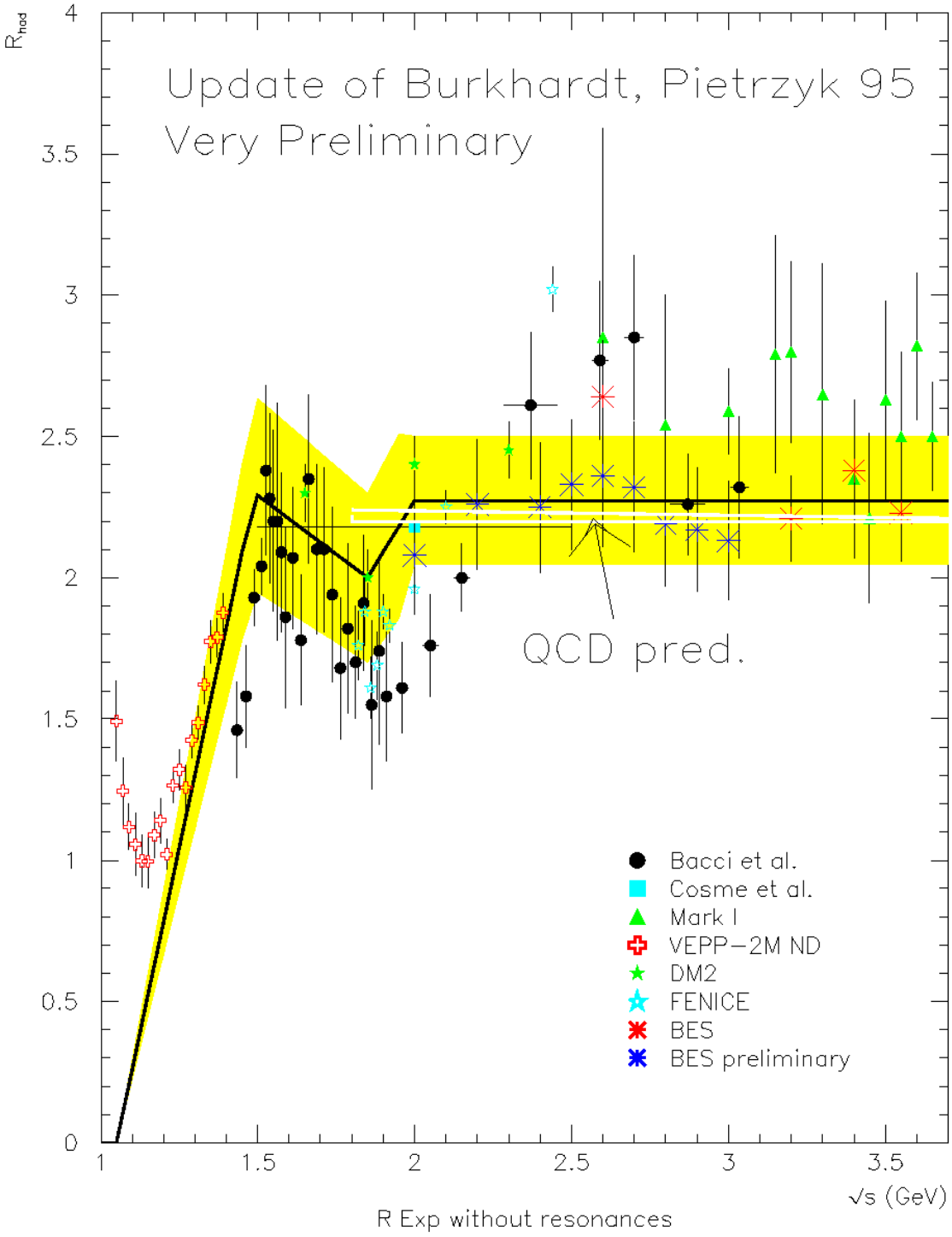




Shaded area: 10 $\times$ error (scaled up)



B. Pietrzyk, Osaka 2000



B. Pietrzyk, Osaka 2000