TESTING QUANTUM MECHANICS AT DAΦNE *

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Abstract

After a brief introduction to EPR-paradox and Bell's inequality, it is shown that a Bell-like inequality can be formulated for the neutral kaon system at a ϕ-factory using the Pauli spin formalism, in our case called *K-spin*, and taking into account CP violation. Experimental methods to reveal tiny violations of this inequality by quantum mechanics are discussed. The statistical accuracy achievable at DAΦNE, the Frascati ϕ-factory, seems adequate to successfully perform such a test.

1 Introduction

One of the most disconcerting aspects of quantum theory is surely that one related to its probabilistic and non-deterministic interpretation. We might consider the subtle and clever argument advanced by Einstein, Podolsky and Rosen (EPR) in 1935,[1] as the first serious attempt of overcoming such an interpretation. Their conclusion could be summarized as follows: either quantum mechanics is incomplete in the sense that there exist some variables (e.g. hidden variables) not taken into account by the quantum wave-function description, or the locality principle is violated by quantum mechanics (i.e. there exist faster-than-light signals). Assuming

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the validity of the locality principle, the EPR conclusion could be interpreted as an argument in favour of local hidden-variables theories, which explain the stochastic nature of quantum mechanics as due to the lack of knowledge of the values that some *hidden* parameters (the exact nature of which remains unspecified) are assuming during the measurement process. In this way hidden variables could restore completeness in the theory, and presumably reconcile quantum mechanics with a deterministic and/or realistic viewpoint[1].

It is worth noting that the hidden-variable concept has nothing obscure and can be simply illustrated by the example of classical thermodynamics. In fact, the latter can be considered as an effective theory of an underlying hidden-variables theory: statistical mechanics. Thermodynamics is able to predict only averages over ensembles of microscopic *hidden* quantities, which are related, in this case, to the atomic structure of matter. Thus "hidden" does not mean "unmeasurable" but simply "not known at present".

After EPR paper, the debate on the question of completeness of quantum mechanics was carried out by many physicists, but remained quite sterile. Starting in 1965, however, the situation changed dramatically. J. Bell proved an important theorem, concerning the whole class of local hidden-variables theories, stating that such theories cannot reproduce all statistical predictions of quantum mechanics.[2)]

Let us consider, as usual, a pair of spin 1/2 particles moving in opposite directions, and described quantum mechanically by the singlet state:

$$|i\rangle = \frac{1}{\sqrt{2}}\{|A\uparrow(\vec{p})\rangle|A\downarrow(-\vec{p})\rangle - |A\downarrow(\vec{p})\rangle|A\uparrow(-\vec{p})\rangle\} \tag{1}$$

where the state $|A\uparrow(\vec{p})\rangle$ represents a particle moving in the $\vec{p}$ direction with the spin *up* along the $\vec{a}$ direction. Let us consider three different axes $\vec{a}$, $\vec{b}$, $\vec{c}$ and the corresponding spin projection operators $A = \vec{\sigma}\cdot\vec{a}$, $B = \vec{\sigma}\cdot\vec{b}$ and $C = \vec{\sigma}\cdot\vec{c}$. Let us assume that we perform a measurement of A, B or C on the particle moving in the $\vec{p}$ direction, and a measurement of A, B or C on the other particle moving in the opposite direction. Bell showed that, according to local hidden-variables theories, the following inequality can be derived for the system considered[2,3)]:

$$\begin{aligned} P[A\uparrow(\vec{p}); B\uparrow(-\vec{p})] \quad \leq \quad & P[A\uparrow(\vec{p}); C\uparrow(-\vec{p})] + P[C\uparrow(\vec{p}); B\uparrow(-\vec{p})] + \\ & + P[C\downarrow(\vec{p}); C\downarrow(-\vec{p})] \end{aligned} \tag{2}$$

where $P[A\uparrow(\vec{p}); B\uparrow(-\vec{p})]$ is the probability of finding A=+1 (spin up) for the particle moving in the $\vec{p}$ direction, and B=+1 (spin up) for the particle moving in the opposite direction. Inequality (2) is generally referred as Bell's inequality. For some axes choices, quantum mechanical predictions violate inequality (2), showing the incompatibility of the quantum theory with the local hidden-variables viewpoint.[2)]

Hence the surprising conclusion of the debate initiated by EPR seems that quantum mechanics implies an unavoidable violation of the locality principle, regardless of a possible hidden-variables completion. The importance of Bell's theorem just lies in having made possible experiments aimed at testing whether nature behaves in a non-local way, as predicted by quantum mechanics, or not.

[1] Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone

The experimental tests of inequality (2), that have been performed so far, can be divided in three main categories:
(a) tests using optical photons from atomic-cascades (or from parametric down conversion)[4,5];
(b) tests using γ-rays from positronium annihilation[6];
(c) tests using protons from p-p scattering.[7]
Experiments of category (a) were the most precise and significant ones, and most of them yielded results in excellent agreement with quantum mechanics. However the validity of any experimental test of inequality (2) relies on some *ad hoc* additional assumptions that restrict the ensemble of local hidden-variables theories for which inequality (2) holds. Hence, in practice, only a restricted class of these theories may be tested against quantum mechanics, and a very general conclusion cannot be drawn. In order to avoid or reduce the impact of possible loopholes, two approaches seem possible. On one hand, inequality (2) can be modified in such a way that additional assumptions do not limit the generality of the test. However, as shown in ref. 8, a serious experimental drawback for this kind of tests consists in the requirement of very high efficiency detectors. On the other hand, the possibility of complementing the above-mentioned classes of experiments with other tests in completely different physical contexts (e.g. involving different particles, energies and time scales as well as additional assumptions) would make loopholes quite artificial, and experimental results less subject to criticism. Here, in the spirit of this second approach, a possible test at a ϕ-factory using neutral kaons is proposed and discussed; references to other interesting work can be found elsewhere.[9]

2 The $K^0 - \overline{K^0}$ system and the K-spin formalism

There exist a straightforward formal analogy between the $K^0 - \overline{K^0}$ system and a spin 1/2 particle. In fact a *K-spin* picture can be introduced such that the the strangeness eigenstates $|K^0\rangle$ and $|\overline{K^0}\rangle$ can be regarded as a K-spin doublet and called *up* and *down* states along the z direction in the K-spin space. Pauli matrices represent the generators of an $SU(2)$ group in this space. CP eigenstates $|K_1\rangle \propto \left\{|K^0\rangle + |\overline{K^0}\rangle\right\}$ and $|K_2\rangle \propto \left\{|K^0\rangle - |\overline{K^0}\rangle\right\}$ are *up* and *down* eigenstates of the K-spin projection operator along the x axis. Let us also consider the K-spin eigenstates along a third generic direction x' and call them $|K_\alpha\rangle \propto \{|K_1\rangle - \eta^*|K_2\rangle\}$ and $|K_\beta\rangle \propto \{|K_2\rangle + \eta|K_1\rangle\}$ where η is a complex parameter that determines the x' direction. Note that for $\eta = 0$ we get $x' = x$. It is worth reminding that the physical states, i.e. the Hamiltonian eigenstates $|K_S\rangle \propto \{|K_1\rangle + \epsilon|K_2\rangle\}$ and $|K_L\rangle \propto \{|K_2\rangle + \epsilon|K_1\rangle\}$, where ϵ is the usual CP violation parameter[2], are non-orthogonal because of CP violation. Thus $|K_\alpha\rangle$ and $|K_\beta\rangle$ cannot be made equal to $|K_S\rangle$ and $|K_L\rangle$ for any choice of η; even if $\eta = \epsilon$, we get $|K_\beta\rangle = |K_L\rangle$ but $|K_\alpha\rangle \neq |K_S\rangle$.

In the following calculations $|\eta|$ is chosen to be of the same order as $|\epsilon|$, so that terms of order $O(|\eta|^2) \sim O(|\epsilon|^2) \sim 10^{-6}$ can be neglected, while terms $O(|\eta|) \sim O(|\epsilon|) \sim 10^{-3}$ are retained.

At a ϕ-factory, due to C-conservation in strong interactions, a coherent neutral kaon state

[2]Here CPT invariance in the $K^0 - \overline{K^0}$ mass matrix is assumed, for simplicity, even if it is not a necessary assumption.

is produced in $\phi \to K^0\overline{K^0}$ decays:

$$|i\rangle = \frac{1}{\sqrt{2}}\{|K^0(\vec{p})\rangle|\overline{K^0}(-\vec{p})\rangle - |\overline{K^0}(\vec{p})\rangle|K^0(-\vec{p})\rangle\} \tag{3}$$

In complete analogy to the spin case, a Bell's inequality can be written for the state (3), where spin projections along the $\vec{a}$, $\vec{b}$, $\vec{c}$ axes are substituted by K-spin projections along, for instance, the x', z, x axes[9)]:

$$\begin{aligned} P[K_\alpha(\vec{p},t_1);K^0(-\vec{p},t_2)] \;\leq\; & P[K_\alpha(\vec{p},t_1);K_1(-\vec{p},t_1)] + P[K_1(\vec{p},t_1);K^0(-\vec{p},t_2)] + \\ & + P[K_2(\vec{p},t_1);K_2(-\vec{p},t_1)] \end{aligned} \tag{4}$$

The above inequality holds for $t_1 \leq t_2$ (a similar one holds for $t_1 \geq t_2$[9)]), where t_1 and t_2 are the kaon proper times at the instant of measurement. Note that, in order to test the locality principle, t_1 and t_2 cannot be chosen arbitrarily, but have to correspond to space-like measurements, i.e. $g \leq t_1/t_2 \leq g^{-1}$, where $g = (1-\beta_K)/(1+\beta_K)$ and $\beta_K \simeq 0.22$.

Compared to inequality (2), inequality (4) shows an explicit dependence on time. This is due to the time evolution of the K-spin states and kaon decays, and could constitute an interesting difference with respect to the spin case. Taking, for simplicity, $t_1 = t_2 = t$ and substituting quantum mechanical predictions in (4) we get:

$$\frac{e^{-\Gamma t}}{4}(1 + 2\Re\eta) \leq \frac{e^{-\Gamma t}}{4} \tag{5}$$

with $\Gamma = \Gamma_S + \Gamma_L$. Inequality (5) is violated for $\Re\eta > 0$. It is worth noting that this violation is quite small ($\sim 10^{-3}$) with respect to violations expected in other classes experiments ($\sim 40\%$).

3 K-spin identification methods

In order to perform an experimental test of inequality (4), the $|K^0\rangle$, $|\overline{K^0}\rangle$, $|K_1\rangle$, $|K_2\rangle$ and $|K_\alpha\rangle$ states have to be identified.

3.1 Identification of $|K^0\rangle$, $|\overline{K^0}\rangle$, $|K_1\rangle$ and $|K_2\rangle$ states

According to quantum mechanics and assuming the validity of $\Delta S = \Delta Q$ rule[3], we expect that detection and reconstruction of $K \to \pi^-\ell^+\nu$ and $K \to \pi^+\ell^-\overline{\nu}$ decay channels unambiguously identifies, at the decay vertex position, a $|K^0\rangle$ or a $|\overline{K^0}\rangle$ state, respectively. Furthermore, as possible direct CP violation effects are completely negligible at our level of accuracy, detection and reconstruction of $K \to 2\pi$ or $K \to 3\pi^0$ unambiguously identifies, at the decay vertex position, a $|K_1\rangle$ or a $|K_2\rangle$ state, respectively. Hence, for example, the following proportionality relation holds:

$$\Gamma[2\pi(\vec{p},t_1);\pi^-\ell^+\nu(-\vec{p},t_2)] = cP[K_1(\vec{p},t_1);K^0(-\vec{p},t_2)] \tag{6}$$

where the l.h.s. term is an observable double differential rate, and

$$c = \Gamma(K_1 \to 2\pi)\Gamma(K^0 \to \pi^-\ell^+\nu) \tag{7}$$

[3]Possible violations of this rule, according to Standard Model predictions, are completely negligible at our level of accuracy.

is a constant.

In order to make relation (6) valid also for local hidden-variables theories, the following, quite reasonable, additional assumption is necessary[4]:
(i) *the decay process is a local random process, namely decay rates are time-independent quantities:* $\Gamma(K^0 \to \pi^- \ell^+ \nu) = const,\ etc.$.

3.2 Identification of $|K_\alpha\rangle$ state

Identification of the $|K_\alpha\rangle$ state is slightly more complicated, and can be performed by means of two similar methods.

3.2.1 Regenerator method

Let us consider a regenerator put on the flight path of a single neutral kaon. The η parameter could be chosen such that a $|K_\beta\rangle$ state traversing the regenerator is rotated, by coherent regeneration, into a pure $|K_2\rangle$ state. The latter cannot decay into two pions, and if we observe a $K_1 \to 2\pi$ decay immediately behind the regenerator, it means that a $|K_\alpha\rangle$ passed through the regenerator, and not a $|K_\beta\rangle$. Note that, as $O(|\eta|) \sim 10^{-3}$, a small rotation, hence a thin regenerator, is necessary.
A more detailed discussion about this method and its feasibility at DAΦNE can be found elsewhere.[9)]

3.2.2 K-spin oscillation method

Due to K-spin non-conservation by weak interactions, the time evolution of the neutral kaon state in vacuum induces "K-spin oscillation" (sometimes called "vacuum regeneration"), a generalization of the well known strangeness oscillation phenomenon. Let us define $P[K_{\alpha,\beta}(0) \Rightarrow K_{1,2}(\Delta t)]$ as the probability of finding a $|K_{1,2}\rangle$ state after a time interval Δt from an initial $|K_{\alpha,\beta}\rangle$ state. Analogously to the regenerator method, a suitable parameter $\eta = \eta(\Delta t)$ can be chosen such that, after a certain time Δt, time evolution in vacuum rotates a $|K_\beta\rangle$ state into a pure $|K_2\rangle$ state, i.e. $P[K_\beta(0) \Rightarrow K_1(\Delta t)] = 0$. Then a $K_1 \to 2\pi$ decay at time $t + \Delta t$ unambiguously identifies the presence of a $|K_\alpha\rangle$ state at time t, and the following proportionality relation (and similar ones), holds:

$$P[K_\alpha(\vec{p}, t_1); K^0(-\vec{p}, t_2)] = \kappa P[K_1(\vec{p}, t_1 + \Delta t); K^0(-\vec{p}, t_2)] \tag{8}$$

where $\kappa^{-1} = P[K_\alpha(0) \Rightarrow K_1(\Delta t)] = e^{-\Gamma_S \Delta t}$, and $P[K_1(\vec{p}, t_1 + \Delta t); K^0(-\vec{p}, t_2)]$ is a measurable probability. It is worth noting that, in this case, the time interval Δt plays the role of the polarizer setting in photon experiments.

Both regenerator and K-spin oscillation methods are based on proportionality relations like (8), which hold for quantum mechanics. In order to make them valid also for local hidden-variables theories, the following additional assumption is necessary:
(ii) *quantum mechanical predictions for single kaon state propagation in matter and in vacuum are valid, in particular those related to the four probabilities* $P[K_{\alpha,\beta}(0) \Rightarrow K_{1,2}(\Delta t)]$.

[4] A critical discussion about the following additional assumptions (i) and (ii) can be found elsewhere.[9,10)]

This assumption seems quite reasonable and is experimentally well proven in the case of strangeness oscillations and coherent regeneration.

4 A possible strategy for an experimental test at DAΦNE

Let us consider four inequalities of kind (4) with four different identifications of the $\vec{a}$, $\vec{b}$, $\vec{c}$ axes: (x', z, x), $(x', -z, x)$, (x, z, x'), $(x, -z, x')$. In order to test them a two step measurement can be performed:

(I) - quantum mechanics predicts that the probabilities $P[K_\alpha(\vec{p}, t_1); K_1(-\vec{p}, t_1)]$, $P[K_2(\vec{p}, t_1); K_2(-\vec{p}, t_1)]$ and $P[K_\beta(\vec{p}, t_1); K_\beta(-\vec{p}, t_1)]$ contribute negligibly, with respect to other terms, to the violation of the four Bell's inequalities. These predictions have to be experimentally verified. An advantage here is that upper limits, rather than precise measurements, are sufficient[5].

(II) - Once step (I) measurements are successfully performed, in order to maximize the quantum violation effect, and to reduce some systematic errors in the measurement, the four Bell's inequalities can be grouped and reduced to the equality form[9)]:

$$\Sigma = 1 \tag{9}$$

where

$$\Sigma = \frac{P[K_\alpha(\vec{p}, t_1); K^0(-\vec{p}, t_2)] / P[K_\alpha(\vec{p}, t_1); \overline{K^0}(-\vec{p}, t_2)]}{P[K_1(\vec{p}, t_1); K^0(-\vec{p}, t_2)] / P[K_1(\vec{p}, t_1); \overline{K^0}(-\vec{p}, t_2)]} \tag{10}$$

For the simple case $t_1 = t_2 = t$, the corresponding quantum mechanical prediction is $\Sigma_{QM} = 1 + 4\Re(\eta)$, violating equality (9) by the quantity $4\Re(\eta) = 10^{-3} \div 10^{-2}$.

At DAΦNE, the Frascati ϕ-factory,[11)] an integrated luminosity per year $L \sim 10^4$ pb^{-1} is expected, corresponding to $\sim 2 \cdot 10^{10}$ $\phi \to K^0\overline{K^0}$ decays. Let us consider only K-spin oscillation method for identification of $|K_\alpha\rangle$ states, which does not require any modification of the experimental set-up. Rough estimates of the expected number of events (neglecting quantum interference effects) in the region $(0 \leq t_1, t_2 \leq 6\tau_S)$ and for $L \sim 10^4$ pb^{-1} are:

$$N[2\pi, \pi\ell\nu] \sim L\sigma_\phi BR(\phi \to K_S, K_L) BR(K_S \to 2\pi) BR(K_L \to \pi\ell\nu)(1 - e^{-6\tau_S/\tau_L}) \sim 7 \cdot 10^7 \tag{11}$$

$$N[2\pi, 3\pi^0] \sim 4.5 \cdot 10^7 \tag{12}$$

$$N[2\pi, 2\pi] \sim 6 \cdot 10^5 \tag{13}$$

$$N[3\pi^0, 3\pi^0] \sim 0.1 \tag{14}$$

Concerning step (I) measurements, it can be shown that the probability $P[K_\alpha(\vec{p}, t_1); K_1(-\vec{p}, t_1)]$ can be considered negligible if we observe:

$$N[2\pi, 2\pi] \ll 1.5 \cdot 10^8 \tag{15}$$

[5]Experimental verification of $P[K_2(\vec{p}, t_1); K_2(-\vec{p}, t_1)]$ and $P[K_\beta(\vec{p}, t_1); K_\beta(-\vec{p}, t_1)]$ could be avoided making the following, quite reasonable, additional assumption:
(iii) *the singlet state perfect anti-correlation holds,*
that makes $P[K_2(\vec{p}, t_1); K_2(-\vec{p}, t_1)] = P[K_\beta(\vec{p}, t_1); K_\beta(-\vec{p}, t_1)] = 0$, but reduces the validity of the test to a less general class of local hidden-variables theories.

while probability $P[K_2(\vec{p},t_1);K_2(-\vec{p},t_1)]$ can be considered negligible if we observe:

$$N[3\pi^0,3\pi^0] \ll 25 \tag{16}$$

Making a comparison with (13) and (14), the verification of conditions (15) and (16) seems experimentally feasible at DAΦNE. Furthermore, it can be easily shown that probability $P[K_\beta(\vec{p},t_1);K_\beta(-\vec{p},t_1)]$ can be considered negligible if we observe the following proportionality relationship[6]:

$$R = \int N[2\pi(\vec{p},t+\Delta t);3\pi^0(-\vec{p},t)]dt \propto e^{-\Gamma_s \Delta t} \tag{17}$$

with a precision of the order $\delta R/R \sim 0.1\%$. This should be just the expected statistical precision achievable at DAΦNE.

Concerning step (II) measurement, the double ratio Σ can be expressed in terms of detected number of events as follows:

$$\Sigma(\Delta t) = \int \frac{N[2\pi(\vec{p},t_1+\Delta t);\pi^-\ell^+\nu(-\vec{p},t_2)]/N[2\pi(\vec{p},t_1+\Delta t);\pi^+\ell^-\overline{\nu}(-\vec{p},t_2)]}{N[2\pi(\vec{p},t_1);\pi^-\ell^+\nu(-\vec{p},t_2)]/N[2\pi(\vec{p},t_1);\pi^+\ell^-\overline{\nu}(-\vec{p},t_2)]} dt_1 dt_2 \tag{18}$$

where an integration over the allowed (t_1,t_2) region is performed. The quantity $\Sigma(\Delta t)$ is shown in Fig.1, where the statistical error expected at DAΦNE, and decay vertex resolution effects are also shown.

As a conclusion, the statistical precision achievable at DAΦNE seems adequate to reveal Bell's inequality violations. A general purpose detector like KLOE,[12,13)] primarily designed for CP and CPT violation physics, could successfully perform such a measurement. However systematic errors have to be carefully investigated, even if some of them drop identically in the double ratio Σ.

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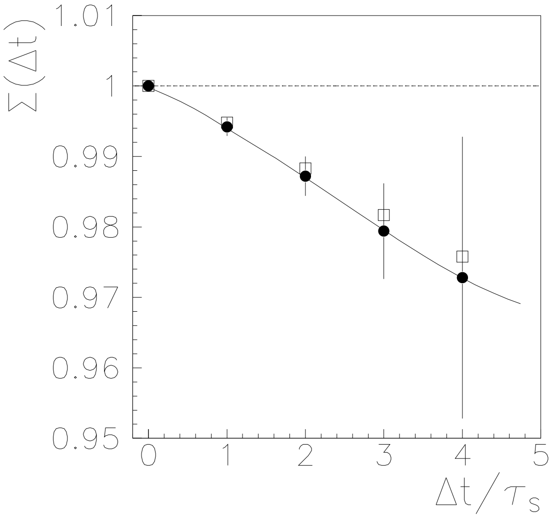

Figure 1: The double ratio Σ as a function of Δt. The full and dashed curves are predictions of quantum mechanics and local hidden-variables theories, respectively. Full circles and open squares represent possible measurements with, respectively, perfect and 3 mm decay vertex resolution. The error bars represent the expected statistical uncertainty for an integrated luminosity $L \sim 10^4$ pb^{-1}.

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