Measurement of the absolute branching ratio of the $K^+ \to \pi^+\pi^0\,(\gamma)$ decay with the KLOE detector

The KLOE Collaboration

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Abstract

We have measured the absolute branching ratio of the $K^+ \to \pi^+\pi^0\,(\gamma)$ decay, using ∼20 million tagged K^+ mesons collected with the KLOE detector at DAΦNE, the Frascati ϕ–factory. Signal counts are obtained from the fit of the distribution of the momentum of the charged decay particle in the kaon rest frame. The result, inclusive of final-state radiation, is BR($K^+ \to \pi^+\pi^0\,(\gamma)$)=$0.2065 \pm 0.0005_{\rm stat} \pm 0.0008_{\rm syst}$.

Key words: e+e- Experiments, Kaon decays
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1 Introduction

The branching ratio of the $K^+ \to \pi^+\pi^0\,(\gamma)$ decay ($K_{\pi 2}$) is part of the KLOE program of precise and fully inclusive kaon branching ratios (BRs) measurement. We have already measured the main K_L [1,2] and K_S [3,4] branching

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ratios. We report here our measurement of the BR($K^+ \to \pi^+\pi^0\,(\gamma)$) which together with BR($K^\pm \to \mu^\pm\nu$) [5], BR($K^\pm \to \pi^0 l^\pm \nu$) [6] and BR($K^\pm \to \pi^\pm\pi^0\pi^0$) [7] covers 95% of all charged kaon decays. The importance of this measurement is twofold: i) the most recent measurement based on 16,000 events from a sample of $\sim 10^5$ kaon decays, BR($K^+ \to \pi^+\pi^0\,(\gamma)$)=$0.2118 \pm 0.0028$ [8] , dates back to more than 30 years ago and gives no information on the radiation cut-off and ii) this BR is necessary to obtain BR(K_{l3}) from measurements normalized to BR($K_{\pi 2}$) [9,10]. The $K^+ \to \pi^+\pi^0\,(\gamma)$ branching ratio can be used, together with the $K_S \to \pi\pi$ branching ratios [3], to determine the relative phase $\delta_0 - \delta_2$ of the I=0 and I=2 s-wave $\pi\pi$-scattering amplitudes [11]. In the following we report our measurement of the absolute branching ratio BR($K^+ \to \pi^+\pi^0\,(\gamma)$) performed with the KLOE detector using an integrated luminosity $\int Ł\,\mathrm{d}\,t \sim 250$ pb^{-1} collected at DAΦNE, the Frascati ϕ–factory. DAΦNE is an e^+e^- collider operated at the energy of 1020 MeV, the mass of the ϕ meson. Equal energy positron and electron beams collide at an angle of (π−0.025) radians producing ϕ-mesons with a transverse momentum of ∼13 MeV. In its rest frame, the ϕ-meson decays into anti-collinear K^+K^- pairs of ∼127 MeV momentum and this remains approximately true in the laboratory. Detection of a $K^\pm$ (the tagging kaon) therefore signals the presence of a $K^\mp$ (the tagged kaon) of given momentum and direction. This procedure, called tagging, allows measurements of absolute BRs.

2 The KLOE detector

The KLOE detector consists of a large volume drift chamber surrounded by an electromagnetic sampling calorimeter. The entire detector is immersed in an axial magnetic field $B = 0.52$ T. The drift chamber (DC) [12], 3.3 m long and 4 m in diameter, has a stereo geometry with 12,582 drift cells arranged in 58 layers and operates with a 90% helium-10% isobutane gas mixture. Tracking in the DC provides measurements of the momentum of charged particles with $\sigma(p_\perp)/p_\perp \leq 0.4\%$ for polar angles larger than 45°. The spatial resolution is ∼150 μm in the bending plane, ∼2 mm on the z coordinate and ∼3 mm on decay vertices. The electromagnetic calorimeter (EMC) [13] consists of a cylindrical barrel and two endcaps, covering a solid angle of 98% of 4π. Particles crossing the lead-scintillator-fiber structure of the EMC, segmented into five planes in depth, are detected as local energy deposits. Deposits close in time and space are grouped into clusters. The energy and time resolution for electromagnetic showers are $\sigma_E/E = 5.7\%/\sqrt{E(\mathrm{GeV})}$ and $\sigma_t = 57\ \mathrm{ps}/\sqrt{E(\mathrm{GeV})} \oplus 100$ ps, respectively. The trigger [14] requires two isolated energy deposits in the EMC with: E > 50 MeV in the barrel and E > 150 MeV in the endcaps. Cosmic-ray muons are identified as events with two energy deposits with E > 30 MeV in the outermost EMC planes and vetoed at the trigger level (CRV). A software filter (SF), based on the topology and multiplicity of EMC clusters and DC hits, is applied to reject machine background. The effect of both CRV and SF on the BR measurement must be determined. In the following the coordinate system is defined with the z-axis along the bisectrix of the e^+e^- beams, the y-axis vertical and the x-axis toward the center of the collider rings and origin at the collision point.

3 The measurement

Tagging with $K^- \to \mu^-\nu$ ($K^-_{\mu 2}$) and $K^- \to \pi^-\pi^0$ ($K^-_{\pi 2}$) decays provides two samples of pure K^+ for signal search. These two-body decays are easily identified as peaks in the distribution of the p^*_π variable, the momentum of the charged decay particle in the kaon rest frame evaluated using the pion mass, as described in Ref. [15]. The tagging kaon is required to satisfy the trigger request by itself, minimizing the dependence of the trigger efficiency on the decay mode of the tagged kaon. The residual dependency, which we refer to as the tag bias in the following, must be determined for the BR evaluation. We choose to measure BR($K_{\pi 2}$) using K^+ mesons because for them the nuclear interaction correction is negligible, since the probability of interaction is $\sim 10^{-5}$ for K^+ and $\sim$3.4% for K^-.

The branching ratio is determined as:

$$\mathrm{BR}\ (K^+ \to \pi^+\pi^0\,(\gamma)) = \frac{N_{K^+\to\pi^+\pi^0\,(\gamma)}}{N_{\mathrm{Tag}}} \times \frac{1}{\epsilon\, C_{\mathrm{CRV}}\, C_{\mathrm{SF}}\, C_{\mathrm{TB}}} \tag{1}$$

where $N_{K^+\to\pi^+\pi^0\,(\gamma)}$ is the signal count, N_{Tag} the number of tagged events and ϵ is the overall efficiency, including the detector acceptance ϵ_{det} and the reconstruction efficiency ϵ_{rec}. The detector acceptance ($\epsilon_{\mathrm{det}}\sim$59%), entering in the final efficiency evaluation, is taken from MC and its value is related to the charged kaon lifetime τ. Consequently the BR depends on τ as:

$$\mathrm{BR}^{(\tau)}/\mathrm{BR}^{(0)} = 1 - 0.0395\ \mathrm{ns}^{-1}(\tau - \tau^{(0)}) \tag{2}$$

with $\tau^{(0)} = 12.385$ ns, the current world average value [19]. A variation of the lifetime of 0.1% changes the BR of 0.05% of its value. The corrections C_{CRV}, C_{SF} and C_{TB} account for the cosmic-ray muons veto, the software filter and tag bias effects, respectively.

The sample used for this measurement has been processed and filtered with the KLOE standard reconstruction software and event classification procedure [16]. The KLOE Monte Carlo (MC) simulation package, GEANFI, has

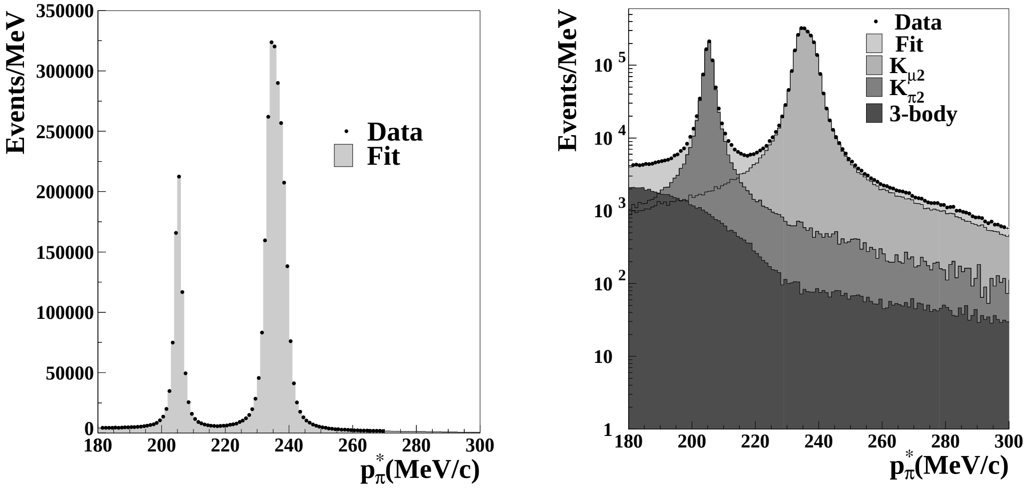

Fig. 1. Fit of the p^*_π distribution. Left: black dots are data and grey histogram is the fit output. Right: the three contributions used to fit the data are shown: $K_{\mu2}$, $K_{\pi2}$ and three-body decays.

been used to produce an event sample equivalent to the data. The different operating conditions of DAΦNE during data taking, machine parameters and background, are included in the MC on a run-by-run basis. The simulation also includes final-state radiation [17] guaranteeing correct measurement of fully inclusive BRs.

3.1 $K^-_{\mu2}$-tagged sample

The number of K^+ tagged by $K^-_{\mu2}$ decays, the $K^-_{\mu2}$-tagged sample, is $N_{\rm Tag}$ = 12,113,686. The $K^+_{\pi2}$ signal selection uses DC information only. The K^+ track is identified as a positive track with point of closest approach (PCA) to the interaction point (IP) satisfying $\sqrt{x^2_{\rm PCA}+y^2_{\rm PCA}} < 10$ cm and $|z_{\rm PCA}| < 20$ cm, and momentum $70 < p_{\rm K} < 130$ MeV. The PCA is evaluated extrapolating the K^+ track backwards to the IP taking into account energy losses. Decay vertices (V) are accepted in the fiducial volume $40 < \sqrt{x^2_{\rm v}+y^2_{\rm v}} < 150$ cm, $|z_{\rm v}|$ <150 cm. Loose cuts on p^*_π and on the difference between the momenta of the kaon and the charged secondary track, $50 < p^*_\pi < 370$ MeV and $-320 < \Delta p < -50$ MeV, reject $K \to 3\pi$ decays.

The $K^+_{\pi2}$ signal count is extracted from the fit of the p^*_π distribution (Fig. 1). The p^*_π spectrum has two peaks: the first at ∼236 MeV due to muons from $K_{\mu2}$ decays, and the second at ∼205 MeV due to pions from $K_{\pi2}$ decays. The contribution from three-body decays shows at lower p^*_π values. Having used the pion mass for the p^*_π evaluation, the $K_{\mu2}$ peak is distorted. We fit the p^*_π distribution between 180 and 400 MeV using three contributions: $K_{\mu2}$, $K_{\pi2}$ and three-body decays. The shapes of the $K_{\mu2}$ and $K_{\pi2}$ peaks are obtained from data control samples, selected using EMC information only. The $K_{\pi2}$ spectrum is obtained from the $K_{\pi 2}$-control-sample used for the efficiency evaluation and described later. The $K_{\mu 2}$ spectrum is obtained from the control sample selected for the BR($K^+ \to \mu^+ \nu$) measurement [5]. Once a tagging $K^-_{\mu 2}$ decay has been identified, we ask for only one EMC cluster with energy $E_{\rm Clu} > 80$ MeV and no clusters with energy between 20 and 80 MeV. There are no requirements on EMC clusters with energy below 20 MeV, in order to retain $K^+ \to \mu^+ \nu\, \gamma$ decays and $K^+ \to \mu^+ \nu$ decays with machine background clusters in the EMC. This high-purity sample (∼99%) is called $K_{\mu 2}$-control-sample. Bin by bin MC corrections account for small distortions induced in the p^*_π spectra by the control sample selections. The three-body component is obtained from the MC simulation, which has been tuned with the data $K_{\mu 2}$- and $K_{\pi 2}$-control-sample. Fig. 1 left shows the result of the fit of the p^*_π distribution compared to the data, while the three different contributions are visible on the right. The fit gives $N_{K^+ \to \pi^+ \pi^0 (\gamma)} = 818,347 \pm 1,912$, the error accounting for the statistics (not only data).

The reconstruction efficiency $\epsilon_{\rm rec}$ has been evaluated with data. Since the $K^+_{\pi 2}$ events are identified from DC information, the data control sample is selected using EMC information. Once a tagging $K^-_{\mu 2}$ decay has been identified, we construct by kinematics the K^+ track from the K^- track. We then search for two photons in the EMC and, using their time and energy information, we determine the K^+ decay point, the di-photon mass and momentum. The best accuracy is obtained minimizing the sum of the square of the differences between the decay time from photons and K^+ path and between the di-photon mass and the π^0 mass. Having determined the track and decay point of the K^+ and the π^0 direction, we determine the expected π^+ track using the two-body decay hypothesis. The kinematics of this hypothesis is then verified by requiring the presence of a cluster in the EMC, with a distance from the pion track $d_{\rm Clu} < 30$ cm. These events define the $K_{\pi 2}$-control-sample. The contamination from K^+ decays without a π^0 in the final state is $\sim 0.1\%$. About 5% contamination from K_{l3} decays is present and becomes about 3% after signal selection. Corrections accounting for small distortions due to the selection of the data control sample have been evaluated using MC. Defining $\epsilon_{\rm true}$ the true efficiency to reconstruct signal decays in the DC volume and $\epsilon_{\rm cs}$ the reconstruction efficiency obtained using the $K_{\pi 2}$-control-sample, the average correction to be applied to the efficiency is $\epsilon_{\rm true}/\epsilon_{\rm cs} \sim 0.99$. The efficiency to be used in eq. 1 is $\epsilon = 0.3176 \pm 0.0005$.

The corrections $C_{\rm CRV} = 1.0005 \pm 0.0003$ and $C_{\rm SF} = 1.0183 \pm 0.0003$ have been measured with data taken without the cosmic-ray muons veto and the software filter, respectively. The correction for the tag bias, $C_{\rm TB} = 1.0106 \pm 0.0005_{\rm stat}$, has been evaluated using MC. The distributions of variables used for the selection of the tagging decay have been checked with data. Table 1 left lists the statistical fractional uncertainties on the branching ratio measurement and the total value is 0.3%.

Statistical errors using $K^-_{\mu 2}$ tag	
Source	Value (%)
fit signal count	0.23
efficiency	0.12
SF and CRV	0.04
ϵ correction	0.12
TB	0.05
Total	0.30

Statistical errors using $K^-_{\pi 2}$ tag	
Source	Value (%)
fit signal count	0.27
efficiency	0.13
SF and CRV	0.03
ϵ correction	0.13
TB	0.05
Total	0.33

Table 1
Summary of fractional statistical uncertainties on BR($K^+_{\pi 2}$) measured using $K^-_{\mu 2}$- and $K^-_{\pi 2}$-tagged samples. Left $K^-_{\mu 2}$ tag, right $K^-_{\pi 2}$ tag.

3.2 $K^-_{\pi 2}$-tagged sample

The number of K^+ tagged by $K^-_{\pi 2}$ decays, the $K^-_{\pi 2}$-tagged sample, is $N_{\mathrm{Tag}} = 9,352,915$. Table 2 compares the values of the C_{CRV}, C_{SF} and C_{TB} corrections obtained for $K^-_{\mu 2}$- and $K^-_{\pi 2}$-tagged events. The two tags have very different corrections for the effect of the software filter (SF). The C_{SF} correction measured using the $K^-_{\mu 2}$ tag is $\sim$1.8% while using the $K^-_{\pi 2}$ tag is $\sim$0.1%. The same signal selection as before is applied to the sample tagged by $K^-_{\pi 2}$ decays and the fit of the p^*_π distribution determines the signal count. The spectra used for the fit have been obtained as described in the previous section, once a tagging $K^-_{\pi 2}$ decay has been identified. The signal count is $N_{K^+\to\pi^+\pi^0\,(\gamma)} = 621,612 \pm 1,678$. For the efficiency evaluation we have used the $K_{\pi 2}$-control-sample tagged by $K^-_{\mu 2}$ decays. The efficiency is $\epsilon = 0.3182 \pm 0.0005$, corrected for the control sample selection and the detector acceptance taken from MC. The total statistical fractional uncertainty on BR($K^-_{\pi 2}$) measured using the $K^-_{\pi 2}$-tagged sample is 0.33%. Table 1 right summarizes the fractional statistical uncertainties.

	$K^-_{\mu 2}$ tag	$K^-_{\pi 2}$ tag
C_{CRV}	1.0005±0.0003	1.0007±0.0003
C_{SF}	1.0183±0.0003	1.00093±0.00006
C_{TB}	1.0106±0.0005	1.009±0.0006

Table 2
Corrections to BR($K^+_{\pi 2}$) measured using $K^-_{\mu 2}$- and $K^-_{\pi 2}$-tagged samples.

Systematic errors using $K^-_{\mu 2}$ tag	
Source	Value (%)
p^*_π fit range	0.06
$K_{\mu 2}$ shape	0.12
$K_{\pi 2}$ shape	0.16
efficiency	0.30
$\rho_{\rm v}^{\rm min}$	0.17
lifetime τ	0.12
TB	0.01
Nucl. int.	< 0.02
Total	0.42

Systematic errors using $K^-_{\pi 2}$ tag	
Source	Value (%)
p^*_π fit range	0.07
$K_{\mu 2}$ shape	0.14
$K_{\pi 2}$ shape	0.17
efficiency	0.30
$\rho_{\rm v}^{\rm min}$	0.17
lifetime τ	0.12
TB	0.01
Nucl. int.	< 0.02
Total	0.43

Table 3
Summary of fractional systematic uncertainties on BR($K^+_{\pi 2}$) measured using $K^-_{\mu 2}$- and $K^-_{\pi 2}$-tagged samples. Left $K^-_{\mu 2}$ tag, right $K^-_{\pi 2}$ tag.

4 Systematic uncertainties

The systematic uncertainties on BR($K^+_{\pi 2}$) from $K^-_{\mu 2}$ and $K^-_{\pi 2}$-tagged samples are listed in tables 3 left and right, respectively. The stability of both BR measurements with respect to different data taking periods and conditions has been checked. A detailed discussion of the systematic studies follows. These studies have been done varying the selection cuts in wide intervals and checking the stability of the BR.

The lower bound of the p^*_π fit range, 180 MeV, has been moved from 165 to 190 MeV, changing by almost a factor of two the contribution from three-body decays. For each value of the lower bound, we have performed the fit of the p^*_π distribution and evaluated the overall efficiency. We observe a minimal change of the BR value in the above range. The maximum variation of the BR is taken as systematic uncertainty. The contributions to the fractional systematic uncertainty on the BR are 0.06% ($K^-_{\mu 2}$ tag) and 0.07% ($K^-_{\pi 2}$ tag).

The spectrum of the $K_{\mu 2}$ component for the fit of the p^*_π distribution is obtained from the $K_{\mu 2}$-control-sample. The $K_{\mu 2}$ spectrum is most affected by the cut at 20 MeV on the cluster energy $E_{\rm Clu}$ [15], connected to the acceptance of a photon from $K^+ \to \mu^+ \nu\ \gamma$ decays or from machine background events. The stability of the BR measurement has been checked by changing the $E_{\rm Clu}$ cut from 10 to 30 MeV, corresponding to a change in the purity of the $K_{\mu 2}$-control-sample from ∼99.3% to ∼97%. Negligible effects are observed with $E_{\rm Clu}$ values larger than 30 MeV. The maximum variation of the BR has been taken as systematic uncertainty. The fractional systematic uncertainties are 0.12% ($K^-_{\mu 2}$ tag) and 0.14% ($K^-_{\pi 2}$ tag).

The spectrum of the $K_{\pi 2}$ component for the fit of the p^*_π distribution is obtained from the $K_{\pi 2}$-control-sample. The systematic effect has been estimated performing the fit with the p^*_π spectrum obtained from a different control sample. We select K^+ decays in the DC, using the signal selection of sec. 3.1, and require the identification of a π^0, looking for two photons in the EMC fulfilling the following requests. The two photons have to be on-time: the difference between the kaon decay times, evaluated using the cluster time and the distance between the K^+ decay vertex and the cluster position, has to be within $3\sigma_{\rm t}$ (see sec. 2). The kaon decay time from the kaon path and from the photons have to be compatible within resolutions. The difference between the di-photon mass and the π^0 mass has to be within 3σ, with $\sigma\sim$18 MeV. The K_{l3} contamination of this sample is 20%, larger than the 3% contamination of the $K_{\pi 2}$-control-sample. Thus the spectrum of the $K_{\pi 2}$ component obtained from this sample needs larger MC bin by bin corrections (as large as 60%) compared to the default used (20% at maximum and for low values of p^*_π). Using this spectrum we have been performed the fit of the p^*_π distribution, also varying the fit range. The BR results are in agreement, within errors, with the values obtained using the spectrum from the $K_{\pi 2}$-control-sample. The maximum difference between the BRs obtained with the two spectra has been taken as systematic uncertainty. The fractional contribution is 0.16% ($K^-_{\mu 2}$ tag) and 0.17% ($K^-_{\pi 2}$ tag).

The reconstruction efficiency has been evaluated with data using the $K_{\pi 2}$-control-sample. The systematic uncertainty has been estimated using a different control sample, with larger K_{l3} contamination ($\sim$11% compared to $\sim$3%) and MC correction to be applied to the efficiency ($\sim$12% compared to $\sim$1%). K^+ decays with a π^0 in the final state are selected, as done for the $K_{\pi 2}$-control-sample but without the $d_{\rm Clu}$ cut. We determine the p^*_π of the charged secondary track, using the two-body hypothesis and the K^+ and π^0 momenta. Two-body decays are then selected applying the asymmetric cut $0.5\sigma < p^*_\pi - 205 < \sigma$, with $\sigma\sim$18 MeV, around the peak at 205 MeV of the p^*_π distribution. The BRs measured using the efficiencies obtained from the above sample and the $K_{\pi 2}$-control-sample agree within errors. Conservatively the difference between these two BRs is taken as the systematic uncertainty. The contribution to the fractional systematic uncertainty is 0.3%.

The K^+ decay vertex has to satisfy the requirement $40 < \rho_{\rm v} = \sqrt{x^2_{\rm v} + y^2_{\rm v}} < 150$ cm. The lower bound of the $\rho_{\rm v}$ range, $\rho^{\rm min}_{\rm v} = 40$ cm, has been moved from 38 to 42 cm with the detector acceptance changing of $\sim$6% of its value. For each $\rho^{\rm min}_{\rm v}$ value, we have performed the fit, evaluated the efficiency and measured the BR. The efficiency has been evaluated with the $K_{\pi 2}$-control-sample. The resolution on the K^+ decay point, using only the time information in the EMC, is $\sigma{\sim}1.5$ cm. Thus the above interval corresponds to a change of more than 2σ. The BR results are in agreement within the statistical error and their rms is taken as systematic uncertainty. The contribution to the fractional systematic uncertainty is 0.17%.

The BR depends on the charged kaon lifetime τ through the detector acceptance. The systematic effect has been obtained using eq. 2 and the 0.24% fractional accuracy of the KLOE measurement τ=12.347±0.030 ns [20]. The contribution to the fractional systematic uncertainty is 0.12%.

The fractional systematic uncertainty from the tag definition is 0.01%, as obtained changing separately the requirements to identify the tagging decay.

The fraction of K^+ undergoing nuclear interaction has been evaluated using the MC simulation and considered as upper bound value of the systematic uncertainty. The contribution to the fractional systematic uncertainty is <0.02%.

The fit of the p^*_π distribution providing the count for $K^+_{\pi 2}$ decays, gives the number of $K^+ \to \mu^+\nu\ (\gamma)$ decays as well. The reliability of the fit procedure is confirmed by comparing BR($K^+_{\mu 2}$) and finding agreement with our published result [5]. The criteria for signal selection and efficiency evaluation from this reference have been followed. There is therefore a correlation $\rho(K_{\mu 2}, K_{\pi 2})$ of -3.4% between our BR($K^+_{\pi 2}$) and BR($K^+_{\mu 2}$) measurements using the signal count extracted from the fit procedure. For our published BR($K^+_{\mu 2}$) result [5] we did not use the fit of the p^* distribution to extract the signal count. The number of $K^+ \to \mu^+\nu\ (\gamma)$ decays was obtained by counting the number of events with $p^* > 225$ MeV, after background subtraction. Therefore there is no correlation between the published BR($K^+_{\mu 2}$) value and our BR($K^+_{\pi 2}$) value.

When averaging the BR($K^+_{\pi 2}$) values obtained from the $K^-_{\mu 2}$- and $K^-_{\pi 2}$-tagged samples we have to account for correlations. The same data control sample for efficiency evaluation has been used for both measurements, thus giving a correlation in the statistical as well as in the systematic contribution to the BR uncertainty. The contribution to the systematic uncertainty from the charged kaon lifetime value τ is common to both measurements as well as the contribution from the $\rho_{\rm v}^{\rm min}$ value. The correlation between the two BR($K^+_{\pi 2}$) measurement is 56%.

5 Conclusions

We have measured the branching ratio of the $K^+ \to \pi^+\pi^0\,(\gamma)$ decay, fully inclusive of final-state radiation, using K^+ samples tagged by $K^-_{\mu 2}$ and $K^-_{\pi 2}$ decays. From 12,113,686 $K^-_{\mu 2}$-tagged events, we find $N_{K^+\to\pi^+\pi^0\,(\gamma)} = 818,347 \pm$

$1,912$ signal counts. Using eq. 1 we obtain the branching ratio:

$$\mathrm{BR}\ (K^+ \to \pi^+\pi^0(\gamma))\Big|_{\mathrm{K}_{\mu2}-\mathrm{tag}} = 0.20638 \pm 0.00062_{\mathrm{stat}} \pm 0.00087_{\mathrm{syst}}. \quad (3)$$

From 9,352,915 $K^-_{\pi 2}$-tagged events we have $N_{K^+\to\pi^+\pi^0\,(\gamma)} = 621,612 \pm 1,678$ signal counts corresponding to:

$$\mathrm{BR}\ (K^+ \to \pi^+\pi^0(\gamma))\Big|_{\mathrm{K}_{\pi2}-\mathrm{tag}} = 0.20668 \pm 0.00068_{\mathrm{stat}} \pm 0.00089_{\mathrm{syst}}. \quad (4)$$

The above BRs are evaluated using the current average value for the $K^\pm$ lifetime $\tau^{(0)} = 12.385$ ns (see eq.2). Averaging these two results, accounting for correlations, we obtain:

$$\mathrm{BR}(K^+ \to \pi^+\pi^0(\gamma)) = 0.2065 \pm 0.0005_{\mathrm{stat}} \pm 0.0008_{\mathrm{syst}}. \quad (5)$$

This absolute branching ratio measurement is fully inclusive of final-state radiation and has a 0.46% accuracy. Our result is 1.3% ($\sim 2\sigma$) lower than the PDG fit [19]. The global fit to all available charged kaon measurements of Ref. [18] gives $\mathrm{BR}(K^+ \to \pi^+\pi^0(\gamma)) = 0.2064 \pm 0.0008$, in agreement with our result.

We fit the six largest $K^\pm$ BRs and the lifetime τ using our measurements of τ [20], $\mathrm{BR}(K^+_{\pi2})$ (eq. 5), $\mathrm{BR}(K^+_{\mu2})$ [5], $\mathrm{BR}(K^\pm_{l3})$ [6] and $\mathrm{BR}(K^\pm \to \pi^\pm\pi^0\pi^0)$ [7], with their dependence on τ, together with $\mathrm{BR}(K^\pm \to \pi^\pm\pi^+\pi^-)$ from the PDG04 average [1] [21], with the sum of the BRs constrained to unity. The fit results, with $\chi^2/\mathrm{ndf} = 0.59/1$ (CL=44%), are shown in table 4 and confirm the validity of our measurement (eq. 5), assuming the correctness of $\mathrm{BR}(K^\pm \to \pi^\pm\pi^+\pi^-)$.

We can also evaluate $\mathrm{BR}(K^\pm \to \pi^\pm\pi^+\pi^-)$ by using our measurements of the above listed BRs and imposing the constraint $\sum \mathrm{BR}(K^\pm \to f) = 1$. With $\mathrm{BR}(K^+_{\mu2}) = 0.63660 \pm 0.00175$, $\mathrm{BR}(K^\pm \to \pi^\pm\pi^0\pi^0) = 0.01763 \pm 0.00025$ and $\mathrm{BR}(K^+_{\pi2}) = 0.20681 \pm 0.00094$, $\mathrm{BR}(K^\pm_{e3}) = 0.04972 \pm 0.00053$ and $\mathrm{BR}(K^\pm_{\mu3}) = 0.03237 \pm 0.00039$, evaluated at τ equal to our measured value 12.347±0.030 ns, we get $\mathrm{BR}(K^\pm \to \pi^\pm\pi^+\pi^-) = 0.0568 \pm 0.0022$. This result is in agreement with the PDG04 average $\mathrm{BR}(K^\pm \to \pi^\pm\pi^+\pi^-) = 0.0550 \pm 0.0010$ [21].

Using $\mathrm{BR}(K^+_{\pi2})$ from eq. 5 and our measurement of $K_S \to \pi\pi$ branching ratios [3] we determine the s-wave $\pi\pi$ scattering phase shift $\delta_0 - \delta_2 = (44.5 \pm 1.0)^\circ$, evaluated in the isospin limit without corrections from strong and electromagnetic isospin breaking [11].

[1] PDG '06 gives the result of their constrained fit but not the average of the data

Parameter	Value	Correlation coefficients					
BR($K_{\mu 2}$)	0.6376(12)						
BR($K_{\pi 2}$)	0.2071(9)	+0.48					
BR($\pi^{\pm}\pi^{+}\pi^{-}$)	0.0553(9)	−0.48	+0.21				
BR(K_{e3})	0.0498(5)	+0.37	−0.13	+0.16			
BR($K_{\mu 3}$)	0.0324(4)	+0.34	−0.12	+0.15	+0.58		
BR($\pi^{\pm}\pi^{0}\pi^{0}$)	0.01765(25)	−0.11	+0.05	−0.05	+0.04	+0.04	
τ (ns)	12.344(29)	−0.15	−0.21	−0.07	−0.06	−0.05	−0.015

Table 4
Results of the fit to $K^{\pm}$ BRs.

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