KLOE RESULTS ON HADRONIC CROSS SECTION*

ACHIM DENIG

for the KLOE Collaboration†

Institut für Experimentelle Kernphysik, Universität Karlsruhe
Postfach 3640, 76021 Karlsruhe, Germany

(Received October 22, 2007)

The KLOE experiment at the Frascati $\phi$-factory DAΦNE has measured the hadronic cross section $\sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma_{\text{ISR}})$ using two different selection criteria for the Initial State Radiation (ISR) photon: (i) requiring the photon emission at small polar angle and (ii) detecting the photon at large polar angle in the electromagnetic calorimeter. Using a theoretical radiator function we extract the non-radiative cross section $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ and we compute the $\pi^+\pi^-$ contribution to the anomalous magnetic moment of the muon. Results presented in this paper come from the analysis of data collected in 2002 (240 pb$^{-1}$ of integrated luminosity) with an improved systematic uncertainty compared to a published KLOE analysis, which was based on 2001 data.

PACS numbers: 13.40.Em, 13.40.Gp, 13.60.Hb, 13.66.Bc

1. Motivation

One of the best known quantities in particle physics is the anomalous magnetic moment of the muon, $a_\mu = (g_\mu - 2)/2$. Recent theoretical evaluations [1–3] find a discrepancy of 3.2–3.4 standard deviations with respect to the direct measurement from the BNL-E821 collaboration at Brookhaven [4].

* Presented at the XXXI International Conference of Theoretical Physics, “Matter to the Deepest”, Ustroń, Poland, September 5–11, 2007.
A large part of the uncertainty on the theoretical standard model estimate comes from the hadronic contribution $a_{\mu}^{\text{had}}$, which at leading order is given by the hadronic vacuum polarization. At low energies this contribution is not calculable within perturbative QCD, but has to be evaluated by a dispersion integral using experimentally measured $e^+e^-$ hadronic cross section data as input. The process with $\pi^+\pi^-$ in the final state contributes with $\sim 70\%$ to $a_{\mu}^{\text{had}}$, and with $\sim 60\%$ to its uncertainty.

2. Measurement of the $\sigma_{\pi\pi}$ cross section

The measurement has been performed with the KLOE detector at the DAΦNE $e^+e^-$ collider in Frascati. As DAΦNE was designed to operate at the fixed center-of-mass energy of $\sqrt{s} = M_\phi$ an energy scan is not feasible. An alternative approach was worked out at DAΦNE using Initial State Radiation (ISR) events (so-called Radiative Return), in which the differential radiative cross section $d\sigma(e^+e^- \rightarrow \pi^+\pi^- + \gamma_{\text{ISR}})/dM_{\pi\pi}^2$ is measured as a function of the $\pi^+\pi^-$ invariant mass $M_{\pi\pi}$, and the total non-radiative cross section $\sigma_{\pi\pi} \equiv \sigma_{e^+e^- \rightarrow \pi^+\pi^-}$ is evaluated [5] using the following formula:

$$M_{\pi\pi}^2 \frac{d\sigma_{\pi\pi\gamma_{\text{ISR}}}}{dM_{\pi\pi}^2} = \sigma_{\pi\pi}(M_{\pi\pi}^2) H(M_{\pi\pi}^2).$$

$H$ is the theoretical radiator function, which we obtain from the Monte Carlo generator PHOKHARA [6]. Note that Final State Radiation (FSR) terms are neglected in Eq. 1, but are taken into account in the analysis using the model of scalar QED for photon emission off pions.

DAΦNE has delivered ca. 2.5 fb$^{-1}$ of data to the KLOE experiment up to the year 2006. In addition, about 250 pb$^{-1}$ of data have been collected at $\sqrt{s} \simeq 1$ GeV, ca. 20 MeV below the $\phi$ resonance. A first measurement of the cross section $\sigma_{e^+e^- \rightarrow \pi^+\pi^-}$ has been performed with the 2001 data set [7] using an integrated luminosity of 140 pb$^{-1}$. The results presented in this paper are obtained from 240 pb$^{-1}$ of data taken in 2002.

2.1. Event selection

Two different selection regions are considered: In the small angle analysis photons are required to be within a cone of $\theta_\gamma < 15^\circ$ around the beamline (narrow cones in Fig. 1, left), while in the large angle analysis at least one photon at a polar angle of $50^\circ < \theta_\gamma < 130^\circ$ (large central cones in Fig. 1, left) is required. In both cases the two charged pion tracks are required to be emitted in the polar angle range $50^\circ < \theta_\pi < 130^\circ$. The published result using 2001 data was obtained from an analysis using small photon angles, which is now updated with 2002 data. We also present in this paper preliminary results from a large photon angle analysis using 2002 data.
The photon is not explicitly detected in the small angle analysis, its direction is reconstructed from the tracks' momenta by closing the kinematics: \( \vec{p}_\gamma \simeq \vec{p}_{\text{miss}} = -(\vec{p}_{\pi^+} + \vec{p}_{\pi^-}) \). Since ISR-photons are mostly collinear with the beam line, a high statistics for the ISR signal events is guaranteed like this, while background processes like \( \phi \rightarrow \pi^+ \pi^- \pi^0 \) and FSR events are greatly reduced. On the other hand, a highly energetic photon emitted at small angle forces the pions also to be at small angles (and thus outside the selection cuts), resulting in a kinematical suppression of events with \( M_{\pi\pi}^2 < 0.35 \text{ GeV}^2 \). This is not the case for the large angle analysis, which allows to measure the spectrum down to the 2-pion threshold of \( 4m_\pi^2 \).

The price to pay in this case is an increased contribution from irreducible, model-dependent background processes such as events with FSR and the decay \( \phi \rightarrow f_0 \gamma \rightarrow \pi^+ \pi^- \gamma \).

Contaminations from the processes \( \phi \rightarrow \pi^+ \pi^- \pi^0 \) and \( e^+ e^- \rightarrow \mu^+ \mu^- \gamma \) are rejected by cuts in the kinematical variables trackmass \( M_{\text{Trk}} \) and missing mass (see Fig. 1, right). An algorithm for particle ID based on calorimeter information and time-of-flight is used to suppress the high rate of radiative Bhabha events.

The large angle analysis allows the closure of the kinematics by requiring the detection of the ISR-photon (with energy larger than 50 MeV) and \( 50^\circ < \theta_\gamma < 130^\circ \) in the calorimeter. A cut on the angle between the photon direction

---

1 Defined under the hypothesis that the final state consists of two charged particles with equal mass \( M_{\text{Trk}} \) and one photon.
and the missing momentum $\vec{p}_{\text{miss}}$ and a kinematic fit in the $\pi^0$ hypothesis are applied to reject the $\pi^+\pi^-\pi^0$ contamination, which is much larger than in the small angle analysis.

2.2. Luminosity measurement

The absolute normalization of the data sample is performed by measuring Bhabha events at large angles ($55^\circ < \theta < 125^\circ$), with an effective cross section of $\simeq 430\text{nb}$. To obtain the integrated luminosity, $L$, the observed number of Bhabha events is divided by the effective cross section evaluated by the Monte Carlo generator Babayaga [8], which includes QED radiative corrections with the parton shower algorithm. Recently, an updated version of the generator, Babayaga@NLO [9], has been released, in which the predicted cross section decreases by 0.7% and the theoretical uncertainty improves from 0.5% to 0.1% with respect to the older version. As a consequence the total error connected to the luminosity measurement drops from 0.6% to 0.3% and is now dominated by the experimental systematics on Bhabha events acceptance. A detailed description of the KLOE luminosity measurement can be found in [10].

2.3. Improvements with respect to the 2001 data set

The analyses of data taken since 2002 benefit from the cleaner and more stable running conditions of DAΦNE. Moreover, the following changes are applied with respect to the data taken in 2001:

- an additional third trigger level was implemented during 2002 to reduce the inefficiency on signal $\pi^+\pi^-\gamma$ events due to the KLOE detector’s cosmic muon trigger-veto, bringing this inefficiency down to few per mill. This has to be confronted with the trigger condition during 2001 data taking, in which the signal efficiency was reduced by as much as 30% due to the misidentification of pions as cosmic events,
- an improved offline background filter was used with the new data sample. This filter contributed the largest experimental systematic uncertainty to the published analysis. The implementation of a down-scaling algorithm providing an unbiased control sample greatly facilitates the evaluation of the filter efficiency, with negligible systematic uncertainty.

3. Results

From the spectrum of observed events, $N_{\text{obs}}$, the differential cross section is obtained after subtracting the residual background events, $N_{\text{bkg}}$, and dividing for the product of selection efficiencies, $\varepsilon_{\text{sel}}(M^2_{\pi\pi})$, and the integrated luminosity $L$: 
\[ \frac{d\sigma_{\pi\pi\gamma}}{dM_{\pi\pi}^2} = \frac{N_{\text{obs}} - N_{\text{bkg}}}{\Delta M_{\pi\pi}^2} \frac{1}{\varepsilon_{\text{sel}}(M_{\pi\pi}^2)} \mathcal{L}, \]

where the observed events are selected in bins of \( \Delta M_{\pi\pi}^2 = 0.01 \text{GeV}^2 \). The residual background content is obtained by fitting the \( M_{\text{Trk}} \) spectrum of the selected data sample with a superposition of Monte Carlo distributions describing the signal and background sources. The fit parameters are the relative weights of signal and background, obtained in intervals of \( M_{\pi\pi} \).

The radiator function \( H \) used in Eq. (1) is taken from the PHOKHARA Monte Carlo generator, which calculates the complete next-to-leading order ISR effects [6]. In addition, the cross section is corrected for the vacuum polarization [11] (running of \( \alpha_{\text{em}} \)), and the shift between the measured value of \( M_{\pi\pi} \) and the virtual photon mass \( M_{\gamma^*} \) for events with photons from final state radiation. Again the PHOKHARA generator, which includes FSR effects in the pointlike-pions approximation, is used to estimate the latter [12].

### 3.1. Small angle analysis

In Fig. 2 (left) the extracted spectra for the energy dependence of the cross section \( \sigma(e^+e^- \rightarrow \pi^+\pi^-) \) are shown for the 2001 and 2002 data sets. In both analyses the small angle selection cuts have been applied. In the 2001 spectrum two modifications have been taken into account compared to the published result: (i) the Bhabha reference cross section used in the luminosity evaluation has been lowered by 0.7% as predicted by the new version of the Bhabha event generator Babayaga@NLO, and (ii) a modification to the trigger efficiency correction has been applied due to an inconsistency found in the previous evaluation. The latter effect has an impact only on
the $M_{\pi\pi}$ region below the $\rho$ peak and leads to a lowering of the cross section compared to the published result. Both spectra are in good agreement. The total systematic error of the 2002 analysis has a total error of 1.1%, compared to 1.3% in the published result.

3.2. Large angle analysis

As the measurement of $\sigma_{\pi\pi}$ with large angle cuts uses an independent selection, it allows not only to cover the threshold region $M_{\pi\pi} < 600$ MeV, but it also provides an excellent overall cross check of the experimental systematics. Moreover, corrections like the relative contributions from FSR are considerably larger than in the small angle analysis and have to be subtracted using Monte Carlo. Fig. 3 shows a comparison between the small angle and large angle spectra (2002 data). The experimental systematics in the large angle case is comparable to that of the small angle case. It comes out, however, that the precision is completely limited by the model dependence in the theoretical description of the irreducible background $\phi \rightarrow f_0\gamma \rightarrow \pi^+\pi^-\gamma$, i.e. from the $\phi$ radiative decay into the scalar meson $f_0(980)$ and from the unknown interference between the FSR process and this $\phi$ radiative decay. This uncertainty is indicated by the grey error band in Fig. 3; the uncertainty is very high, especially at low and high $M_{\pi\pi}$ values and was estimated by comparing two different models for the scalar amplitude, taking the full

Fig. 3. Spectra and relative difference (inlay) of the small angle (SA) and large angle (LA) analyses for 2002 data; the grey error band indicates the model uncertainty in subtracting the irreducible $\phi \rightarrow f_0(980)\gamma$ background in the large angle analysis.

2 We do not present a detailed bin-by-bin comparison since a sophisticated unfolding procedure as done for 2001 data is not applied yet; this is not needed for the evaluation of the dispersion integral (next chapter).
difference as the error. After the $f_0(980)$ background subtraction the large angle and small angle results are in very good agreement within errors. This agreement indicates, that also the background subtraction of FSR events, which gives a large correction especially in the large angle analysis and for which the model of scalar QED was used, works reasonably well. More studies in this direction are planned in near future.

4. Evaluation of $a_{\mu}^{\pi\pi}$

The cross section measurements\(^3\) described in the previous chapter are used to determine the two-pion contribution to the hadronic contribution to the muon anomaly, $a_{\mu}^{\pi\pi}$, according to:

$$a_{\mu}^{\pi\pi} = \frac{1}{4\pi^3} \int_{s_{\text{low}}}^{s_{\text{up}}} ds \sigma_{\pi\pi}^{\text{bare}}(\gamma_{\text{FSR}})(s) K(s),$$

(3)

where the lower and upper bounds of the spectrum measured with the small angle analysis are $s_{\text{low}} = 0.35 \text{GeV}^2$ and $s_{\text{up}} = 0.95 \text{GeV}^2$, and the kernel function $K(s)$ is described in [13]. Inserting the updated 2001 small angle result into the dispersion integral in Eq. 3 yields

$$a_{\mu}^{\pi\pi}(0.35, 0.95) = (384.4 \pm 0.8_{\text{stat}} \pm 4.9_{\text{syst}}) \times 10^{-10}.$$  

This agrees well with the new preliminary small angle result based on 2002 data

$$a_{\mu}^{\pi\pi}(0.35, 0.95) = (386.3 \pm 0.6_{\text{stat}} \pm 3.9_{\text{syst}}) \times 10^{-10}.$$  

Due to the strong model dependence in describing the $\phi \to f_0(980)\gamma$ amplitude we limit the energy range in the large angle analysis to $[0.5, 0.85] \text{GeV}^2$:

2002 small angle : $a_{\mu}^{\pi\pi}(0.5, 0.85) = (255.4 \pm 0.4_{\text{stat}} \pm 2.5_{\text{syst}}) \times 10^{-10}$,  

2002 large angle : $a_{\mu}^{\pi\pi}(0.5, 0.85) = (252.5 \pm 0.6_{\text{stat}} \pm 5.1_{\text{syst}}) \times 10^{-10}.$

One finds a good agreement in $a_{\mu}^{\pi\pi}$ between the two independent measurements. Note that even restricting the comparison to the range around the mass of the $\rho$-meson, 60% of the systematic uncertainty in the large angle evaluation of $a_{\mu}^{\pi\pi}$ comes from the uncertainty on the parameters used in the $f_0(980)$ background subtraction.

A comparison of the small angle 2002 result with the most recent $a_{\mu}^{\pi\pi}$ evaluations released by the CMD-2 [14] and SND [15] experiments, in the mass range $M_{\pi\pi} \in [630, 958] \text{MeV}$, yields

---

\(^3\) Corrected for vacuum polarization effects and inclusive for FSR.
CMD-2 [14]: \( a_{\mu}^{\pi\pi}(630,958) = (361.5 \pm 1.7_{\text{stat}} \pm 2.9_{\text{sys}}) \times 10^{-10} \),

SND [15]: \( a_{\mu}^{\pi\pi}(630,958) = (361.0 \pm 2.0_{\text{stat}} \pm 4.7_{\text{sys}}) \times 10^{-10} \),

KLOE preliminary: \( a_{\mu}^{\pi\pi}(630,958) = (355.5 \pm 0.5_{\text{stat}} \pm 3.6_{\text{sys}}) \times 10^{-10} \).

While the new KLOE value is slightly lower, it agrees with the published CMD-2 and SND values within one standard deviation.

5. Conclusions and outlook

The KLOE collaboration has obtained \( \sigma_{\pi\pi} \) from the differential cross section for ISR events \( e^+e^- \rightarrow \pi^+\pi^-\gamma \), and has evaluated \( a_{\mu}^{\pi\pi} \) in the range \( M_{\pi\pi}^2 \in [0.35, 0.95] \text{ GeV}^2 \). For the small angle analysis the preliminary result from 2002 data agrees with the updated value from 2001 data. Also both preliminary 2002 results from the large and small angle analyses agree in the region \( M_{\pi\pi}^2 \in [0.5, 0.85] \text{ GeV}^2 \), where for the large angle analysis FSR and the \( f_0(980) \) contribution are much reduced. Finally, the small angle 2002 result is also in agreement within one standard deviation with the recent SND and CMD-2 values in the mass range \( M_{\pi\pi} \in [630, 958] \text{ MeV} \).

Further work is going on to refine the analyses:

- by including the bin-by-bin correlations due to the detector resolution in the small angle analysis;
- by improving the knowledge of the \( f_0(980) \) background contribution for the large angle analysis.

In addition, complementary analyses are in progress to measure the pion form factor \( |F_\pi|^2 \):

- from the bin-by-bin normalization of \( \pi^+\pi^-\gamma \) to \( \mu^+\mu^-\gamma \) spectra;
- using data taken at \( \sqrt{s} \approx 1 \text{ GeV} \) (off-peak, 250 pb\(^{-1}\)), in which background from \( \phi \)-meson decays is suppressed. We expect a further reduction of the total systematic error in this data sample, both in the small angle and large angle analyses. Moreover, it will be possible to perform a precision test of the model of scalar QED for FSR using the charge asymmetry and forward–backward asymmetry [5].
REFERENCES