

# **MC generators for radiative kaon decays**

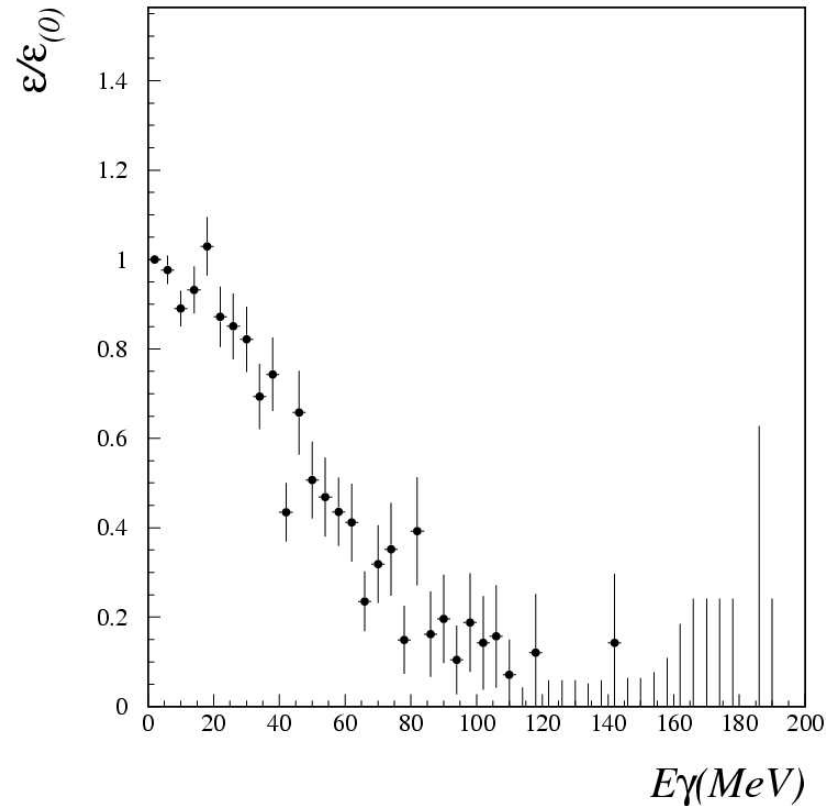
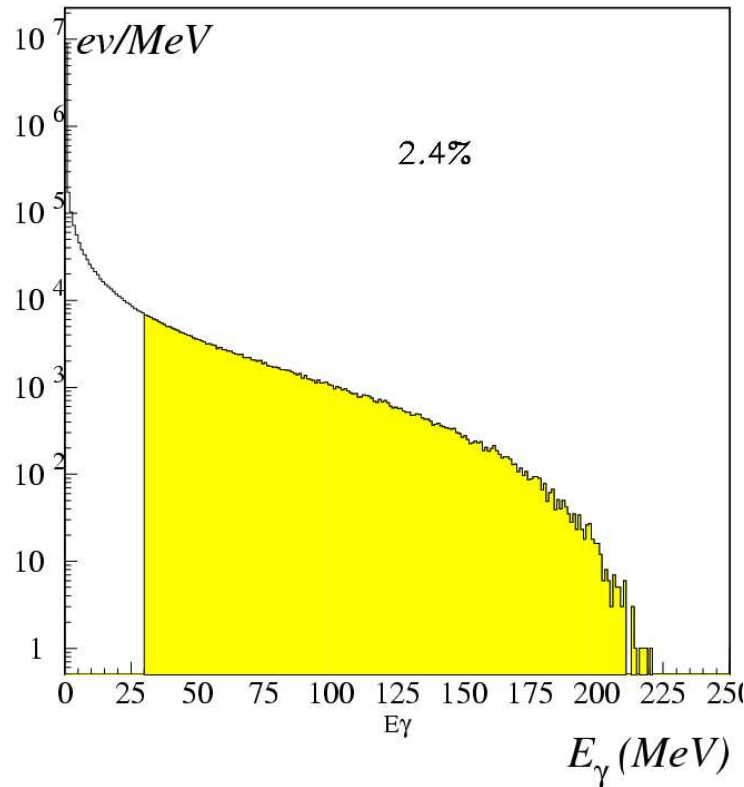
**C. Gatti**

**Universita' Roma 1 and INFN**

- **Motivations: Acceptance; Event counting.**
- **PHOTOS?**
- **The KLOE MC generators.**
- **Theoretical and technical problems.**
- **Comparisons with order  $\alpha$  calculations.**
- **Conclusions**

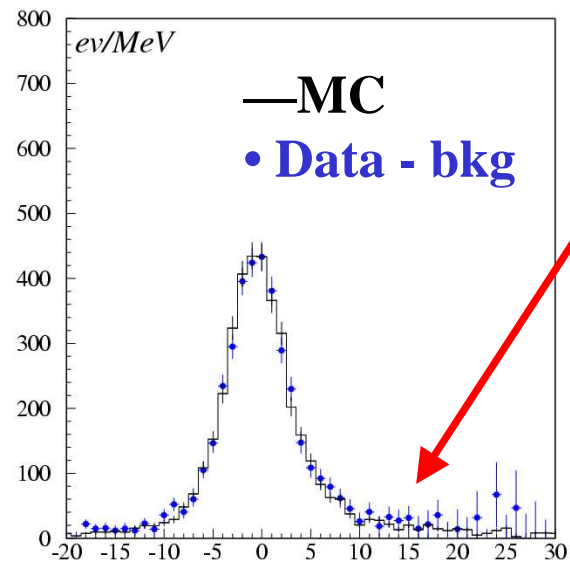
# Motivations: Acceptance

MC photon spectra for Ke3 decays and the corresponding efficiency for  $K_S$  decays in KLOE.



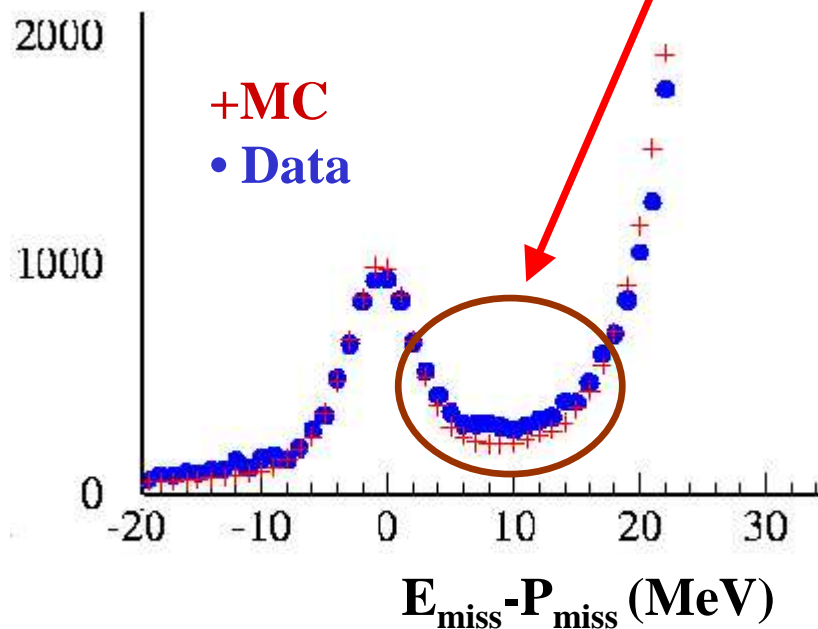
“The simulation without IB changes the acceptance by 2-3% for the Ke3 mode”  
[KTeV hep-ex/0406002](#)

# Motivations: Event counting

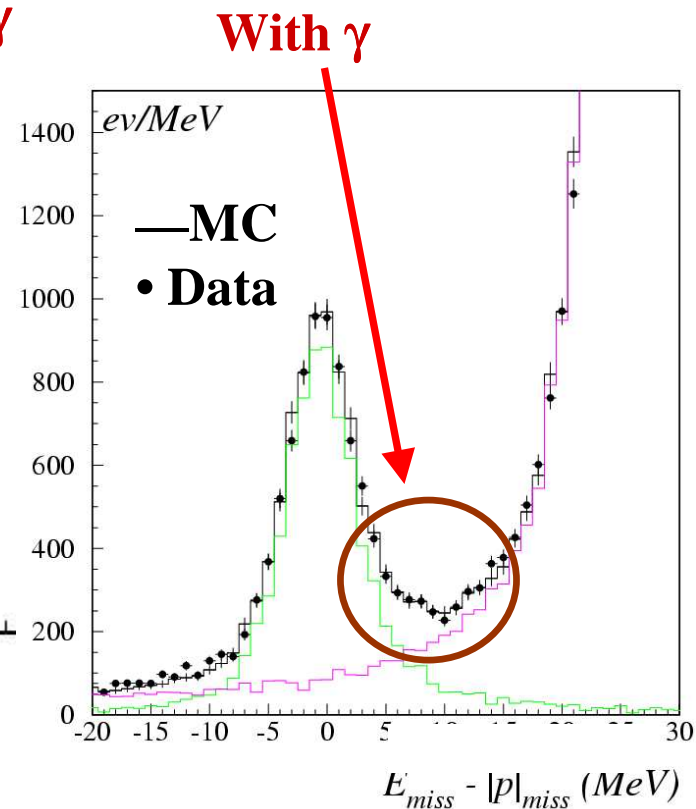


The  $E_{\text{miss}} - P_{\text{miss}}$  distribution for  $K_S \rightarrow \pi e \nu \gamma$  is asymmetric because of the presence of the photon.

The event counting is wrong ( $\sim 1-2\%$ ) if the fit is done neglecting the radiation.



Without  $\gamma$

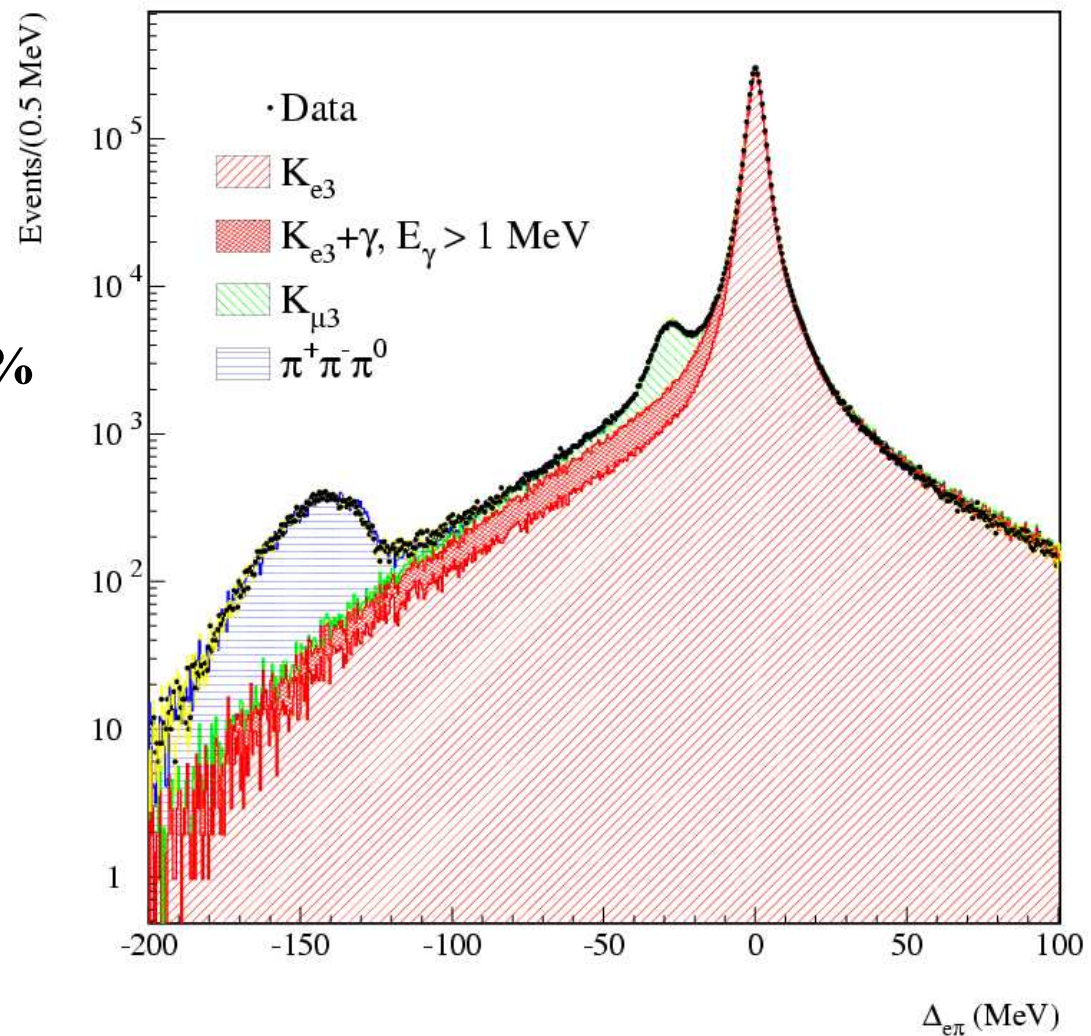


With  $\gamma$

# Motivations: Event counting

Another example:  
 $K_L$  decays at KLOE.

The  $Ke3$  BR increases by  $\sim 2\%$   
introducing the radiation.



$P_{\text{miss}} - E_{\text{miss}}$  distribution in a  $Ke3$ -enriched sample

# PHOTOS

T. Andre hep/ph 0406006

“[...] complete matrix element treatment is very difficult or even impossible. This treatment may not be necessary if the required experimental precision is not very high.”

Barberio and Was, Comp. Phys. Comm. 79 (1994) 291.

PHOTOS is a very nice program: allows an easy interfacing with any program generating decays of any particle.

But...

- Uses soft photon approximation (Low theorem) instead of full amplitude.
- Photons added incoherently to any charged particle: the interference is missing.

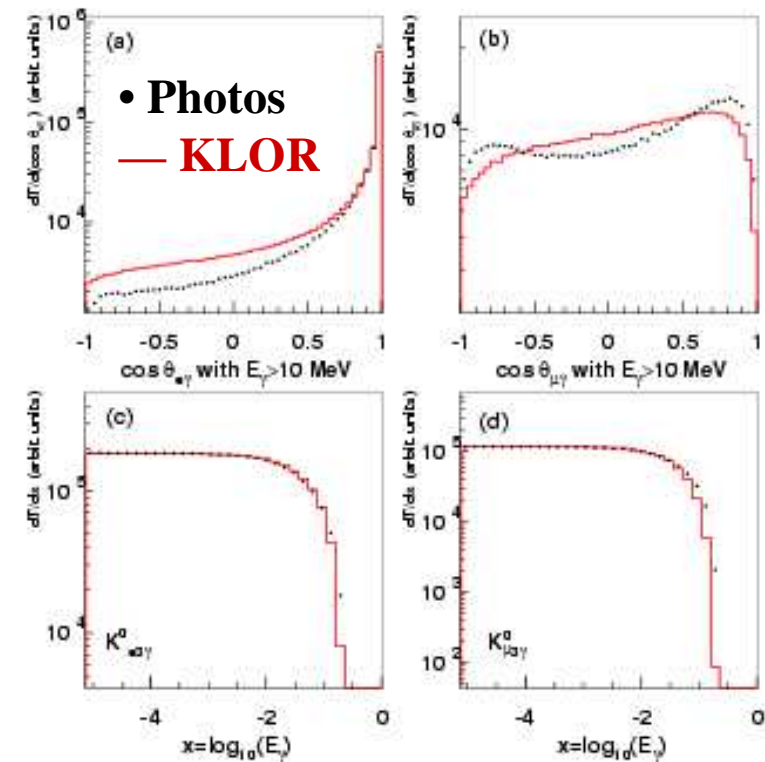
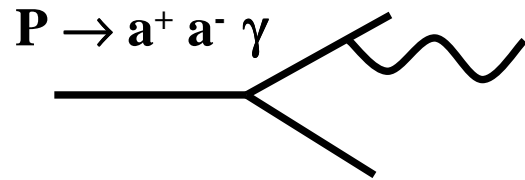


FIG. 8: Comparison of the PHOTOS (dots) and the KLOR (line) Monte Carlo generators. (a) and (b) plot the cosine of the angle between the charged lepton and the photon ( $\cos \theta_{l\gamma}$ ) for the  $K_{e3}^0$  and the  $K_{\mu3}^0$  decay modes, respectively. In plots (a) and (b) the photons radiated photons are required to an energy greater than 10 MeV in the kaon center of mass. Plots (c) and (d) compare the log of the radiated photon energy ( $\log_{10}(E_\gamma)$ ), where  $E_\gamma$  is in GeV. In PHOTOS the IR cutoff is taken to be 1 keV and in KLOR the photon mass was taken to be  $1 \text{ eV}/c^2$ .

# MC generators for radiative kaon decays

There are two problems in the simulation of radiative k decays:

- Theoretical problem:



the decay width obtained from this term is infinite.

- Technical problem: the production of  $\sim 5 \times 10^8$  kaon decays requires a short time for the generation of a single event ( $t < 1$  ms). A simple 'Hit or Miss' procedure is very ineffective if the differential decay width is not 'reasonably' flat.

See Kloe note 194

[www.lnf.infn.it/kloe/pub/knote/kn194.ps](http://www.lnf.infn.it/kloe/pub/knote/kn194.ps)

# Theoretical problem

For  $P \rightarrow a^+ a^-$  decays ...

$$M^{Brem} = e \left( \frac{p^+ \cdot \epsilon}{p^+ \cdot k} - \frac{p^- \cdot \epsilon}{p^- \cdot k} \right) M^0$$

where  $p^+$  and  $p^-$  are the momenta of the charged particles, and  $k$  and  $\epsilon$  are the momentum and polarization of the photon. The differential decay width is related to the decay width for the process without photon emission by:

$$d\Gamma_{Brem} = \Gamma_0 \frac{\alpha}{(2\pi)^2} \sum_{pol} \left| \frac{p^+ \cdot \epsilon}{p^+ \cdot k} - \frac{p^- \cdot \epsilon}{p^- \cdot k} \right|^2 \frac{d^3k}{k}. \quad (1)$$

The total decay width for 'soft' photon emission is obtained by integrating the photon energy up to a cutoff value  $E_\gamma$ , and integrating the solid angle in the limit  $\mathbf{p}^+ = -\mathbf{p}^- = \mathbf{p}$ :

$$\Gamma_{Brem} = \Gamma_0 b \int_0^{E_\gamma} \frac{dk}{k} \quad (2)$$

where  $b$  is:

$$b = \left( \frac{-2\alpha}{\pi} \right) \left[ 1 - \frac{1 + \beta^2}{2\beta} \ln \frac{1 + \beta}{1 - \beta} \right];$$

and  $\beta$  is the particle velocity in the limit of  $k = 0$ :

$$\beta = \frac{p}{E}.$$

This decay width is infinite. A finite value is obtained only by summing the decay widths for the radiative and non-radiative processes calculated at the same order in  $\alpha$  [3]:

$$\Gamma_{TOT} = \Gamma_0 + \Gamma_{Brem} = \Gamma_0 \left( 1 - b \ln \frac{E_{CM}}{\lambda} + c_0 \right) + \Gamma_0 b \ln \frac{E_\gamma}{\lambda} = \Gamma_0 \left( 1 + b \ln \frac{E_\gamma}{E_{CM}} + c_0 \right) \quad (3)$$



$$\Gamma_{TOT} = \Gamma_0 \left( 1 + b \ln \frac{E_\gamma}{E_{CM}} + \frac{1}{2!} \left( b \ln \frac{E_\gamma}{E_{CM}} \right)^2 + \dots \right) (1 + \tilde{c}_0) = \Gamma_0 \left( \frac{E_\gamma}{E_{CM}} \right)^b (1 + \tilde{c}_0).$$

The differential decay width is obtained by deriving with respect to  $E_\gamma$ :

$$\frac{d\Gamma_{TOT}}{dE_\gamma} = \Gamma_0 b (1 + \tilde{c}_0) \left( \frac{E_\gamma}{E_{CM}} \right)^{b-1} = \frac{d\Gamma_{Brem}}{dE_\gamma} \left( \frac{E_\gamma}{E_{CM}} \right)^b \quad (4)$$

where in the last step the term  $\tilde{c}_0$  is neglected with respect to 1. Eq. (4) describes the correct energy spectrum for the radiated soft photon, and shows that it is obtained from the decay width ( $d\Gamma_{Brem}/dE_\gamma$ ) in Eq. (2) by multiplying for the factor  $(E_\gamma/E_{CM})^b$ .

A more general and rigorous derivation of Eq. (4) is given in Ref. [4]. The factor  $b$  is given by:

$$b = -\frac{1}{8\pi^2} \sum_{mn} \eta_m \eta_n e_m e_n \beta_{mn}^{-1} \ln \frac{1 + \beta_{mn}}{1 - \beta_{mn}}, \quad (5)$$

where  $m$  and  $n$  run on all the external particle,  $e_n$  is the charge of the particle  $n$ ,  $\eta = +1$  or  $-1$  for an outgoing or incoming particle, and  $\beta_{mn}$  is the relative velocity of the particles  $n$  and  $m$  in the rest frame of either:

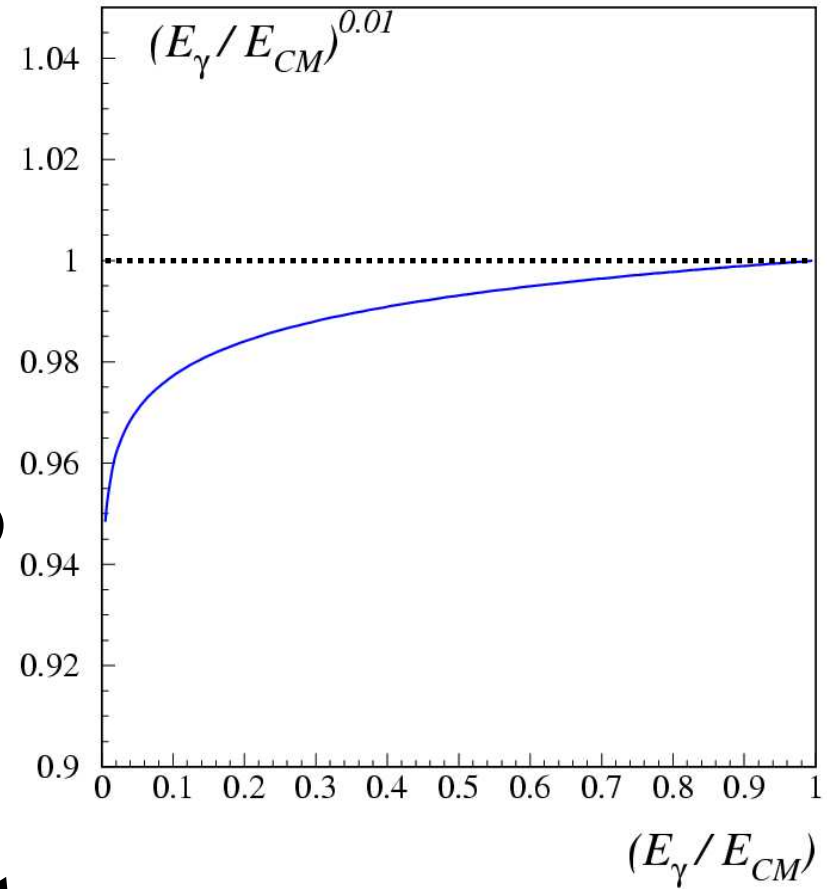
$$\beta_{mn} = \left[ 1 - \frac{m_n^2 m_m^2}{(p_n \cdot p_m)^2} \right]^{1/2}. \quad (6)$$

**[4] S.Weinberg, Phys.Rev. 140 (1965) 516**



## Final recipe

$$\left(\frac{d\Gamma}{dE_\gamma}\right) = \left(\frac{d\Gamma(o(\alpha))}{dE_\gamma}\right)_{IR} \times \left(\frac{E_\gamma}{m_K}\right)^b \quad (b > 0)$$



**Complete  $O(\alpha)$  amplitude.**  
**Mainly from  $O(p^2)$  Bijnens et al.**  
**Dafne handbook**

$$b = \frac{-1}{8\pi^2} \sum_{mn} \eta_m \eta_n e_m e_n \beta_{mn} \ln \frac{1 + \beta_{mn}}{1 - \beta_{mn}}$$

**$\beta$  relative velocity of particles m and n in the rest frame of either.**

## Technical problem

$$|A|^2 = \frac{1}{q^{1-b}} \frac{1}{E - p \cos \theta} \times F(q, p, \cos \theta)$$

↙
↘

**Divergent terms**
**Flat distribution**

### 1) Photon energy

$$\left(\frac{M}{E}\right)^{1-b} \Rightarrow F = \int dE \Rightarrow E = E_{\max} \cdot F^{1/b} \quad (0 \leq F \leq 1)$$

### 2) Collinearity

$$\frac{1}{(p+q)^2 - m^2} = \frac{1}{2p \cdot q} = \frac{1}{2q(E - p \cos \theta)} \quad \text{Relevant for } m = m_e$$

- Divergent terms are extracted using the inverse-transform method
- A 'Hit-or-Miss' on the remaining 'flat' part is used to weight the kinematical variables extracted from the previous step.

# Comparison with $\mathcal{O}(\alpha)$ calculations

$\theta/E(\text{MeV})$	10	20	30	40
$0^\circ$	$4.90 \times 10^{-2}$	$3.25 \times 10^{-2}$	$2.36 \times 10^{-2}$	$1.78 \times 10^{-2}$
$10^\circ$	2.44	1.65	1.22	0.93
$20^\circ$	1.85	1.26	0.93	0.71
$30^\circ$	1.51	1.02	0.76	0.58
$40^\circ$	1.26	0.86	0.63	0.49
$50^\circ$	1.06	0.72	0.54	0.41

Table 1: Ratios  $R^0$  for  $\text{Ke}3$  decays obtained in the MC simulation.

$\theta^\circ/E(\text{MeV})$	10	30
$0^\circ$	4.93 %	2.36 %
$20^\circ$	1.89 %	0.956 %

T. Andre hep/ph 0406006

$\theta/E(\text{MeV})$	10	20	30	40
$0^\circ$	$4.99 \times 10^{-2}$	$3.28 \times 10^{-2}$	$2.37 \times 10^{-2}$	$1.79 \times 10^{-2}$
$10^\circ$	2.51	1.69	1.25	0.96
$20^\circ$	1.92	1.30	0.96	0.74
$30^\circ$	1.57	1.06	0.79	0.61
$40^\circ$	1.31	0.89	0.67	0.51
$50^\circ$	1.11	0.76	0.57	0.44

**M.G. Doncel, Phys.Lett. 32B (1970) 623**

Table 2: Ratios  $R^0$  for  $\text{Ke}3$  decays listed in Ref. [8].

$$R = \text{BR}(\text{Ke}3\gamma E, \theta) / \text{BR}(\text{ke}3\gamma)$$

$\theta/E(\text{MeV})$	20	30	40
$0^\circ$	$3.30 \times 10^{-2}$	$2.39 \times 10^{-2}$	$1.80 \times 10^{-2}$

**H.W. Fearing et al., Phys.Rev.Lett. 24 (1970) 189**

Table 3: Ratios  $R^0$  for  $\text{Ke}3$  decays obtained from Ref. [9].

# Conclusion

- MC generators for neutral and charged kaons into leptonic, semileptonic and into two and three pions have been written and inserted into the KLOE official library.
- The sampling time for an event is  $<1$  ms/evt (except for  $k^+ \rightarrow e\nu$   $t \sim 1.5$  ms).
- MC generators use full  $O(\alpha)$  amplitude, including also interference terms.
- the extension of the soft-photon approximation to the whole energy range gives a satisfactory description for the kaon radiative decays.
- Absolute difference between MC and order  $\alpha$  calculations are  $< \sim 10^{-3} \div 10^{-4}$ .
- KLOE vs KTeV: different low energy behavior  $(1/E)^{1-b} \forall E$  and  $(1/E)$  down to few KeV respectively.