Fit to the gluonium content using PDG 2008 and new KLOE measurement of Br($\omega \to \pi^0\gamma$)

Biagio Di Micco

I.N.F.N
sezione di
Roma Tre

Outline

- Model description;
- Published KLOE result respect to other estimates;
- Re-fit to all relevant measurements;
- Re-fit using PDG-2008 and new KLOE measurement of $Br(\omega \rightarrow \pi^0 \gamma)$.

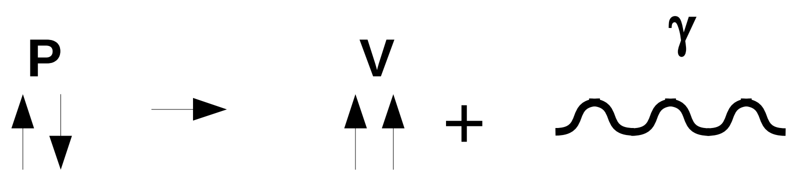
η, η': mixing and gluonium

The η,η ' mesons wave function can be decomposed in the quark mixing base as in the folowing (J. L. Rosner, Phys. Rev. D 27 (1983) 1101.).

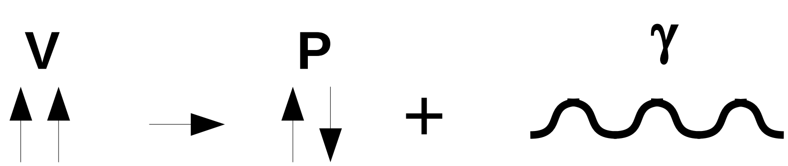
$$|\eta'\rangle = X_{\eta'}|q\bar{q}\rangle + Y_{\eta'}|s\bar{s}\rangle + Z_{\eta'}|G\rangle \qquad |\eta\rangle = \cos\varphi_P|q\bar{q}\rangle - \sin\varphi_P|s\bar{s}\rangle \qquad |q\bar{q}\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle}{\sqrt{2}}$$

$$X_{\eta'} = \sin\varphi_P \cos\varphi_G$$
$$Y_{\eta'} = \cos\varphi_P \cos\varphi_G$$
$$Z_{\eta'} = \sin\varphi_G$$

The ϕ → η,η' γ transition is modelled according a spin flip transition



$$\Gamma(P \to V\gamma) = \frac{g^2}{4\pi}|p_\gamma|^3$$



$$\Gamma(V \to P\gamma) = \frac{1}{3}\frac{g^2}{4\pi}|p_\gamma|^3$$

**Only quarks participate to the electromagnetic transition, gluonium is a spectator.
It appears in the η' decay amplitudes only through the normalisation to 1 ($Y_{\eta'} \sim \cos\varphi_G$)**

Magnetic dipole transition

Decay width:

Quark charge

$$\Gamma(V \to P\gamma) = \frac{2}{3}\alpha\omega^3 \left(\frac{E_P}{m_V}\right) \sum \left| \left\langle V | \frac{\mu_q e_q \sigma_q}{e} | P \right\rangle \right|^2$$

Pauli matrices

example: Matrix element for ρ→ηγ decay

$$|\rho\rangle = \frac{|u\bar{u}> - |d\bar{d}>}{\sqrt{2}}$$

Isospin conservation

$$\left\langle \rho | \frac{\mu_q e_q \sigma_q}{e} | \eta \right\rangle = \mu cos(\varphi_P) 2/3 \left\langle u\bar{u}_\rho | u\bar{u}_\eta \right\rangle + 1/3 \left\langle d\bar{d}_\rho | d\bar{d}_\eta \right\rangle = \mu cos(\varphi_P) \left\langle q\bar{q}_\rho | q\bar{q}_\eta \right\rangle$$

e/m_q (effective quark mass)

Magnetic dipole transition

Decay width:

Quark charge

$$\Gamma(V \to P\gamma) = \frac{2}{3}\alpha\omega^3\left(\frac{E_P}{m_V}\right)\sum\left|\left\langle V\right|\frac{\mu_q e_q \sigma_q}{e}\left|P\right\rangle\right|^2$$

Pauli matrices

example: Matrix element for ρ→ηγ decay

$$|\rho\rangle = \frac{|u\bar{u}> - |d\bar{d}>}{\sqrt{2}}$$

$$\left\langle\rho\right|\frac{\mu_q e_q \sigma_q}{e}\left|\eta\right\rangle = \mu cos(\varphi_P)2/3\left\langle u\bar{u}_\rho | u\bar{u}_\eta\right\rangle + 1/3\left\langle d\bar{d}_\rho | d\bar{d}_\eta\right\rangle = \mu cos(\varphi_P)\left\langle q\bar{q}_\rho | q\bar{q}_\eta\right\rangle$$

e/m_q (effective quark mass)

wave function overlapping

$$C_q =< \eta_q|\omega_q >=< \eta_q|\rho > \quad C_s =< \eta_s|\phi_s > \quad C_\pi =< \pi|\omega_q >=< \pi|\rho >$$

In the formulas only the ratios appear: $Z_{NS} = C_q/C_\pi \quad Z_S = C_s/C_\pi$

QCD effects reside in mixing parameters, overlapping parameters and effective quark masses.

KLOE [Phys. Lett. B648 (2007) 267] has fitted:

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{z_q^2}{\cos^2\phi_V}\cdot 3\left(\frac{m_{\eta'}^2-m_\rho^2 m_\omega}{m_\omega^2-m_\pi^2 m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta'\to\omega\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{1}{3}\left(\frac{m_{\eta'}^2-m_\omega^2 m_\omega}{m_\omega^2-m_\pi^2 m_{\eta'}}\right)^3\left[z_q X_{\eta'}+2\frac{m_s}{\bar{m}}z_s\cdot\tan\phi_V\cdot Y_{\eta'}\right]^2$$

together with the measured branching ratio:

$R_\phi = (4.77 \pm 0.09 \pm 0.19)\times 10^{-3}$

$$R_\phi=\frac{Br(\phi\to\eta'\gamma)}{Br(\phi\to\eta\gamma)}=\cot^2\phi_P\cdot\cos^2\phi_G\left(1-\frac{m_s}{\bar{m}}\frac{z_q}{z_S}\cdot\frac{\tan\phi_V}{\sin 2\phi_P}\right)^2\cdot\left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

and the ratio:

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3\left(5X_{\eta'}+\sqrt{2}\frac{f_q}{f_s}Y_{\eta'}\right)^2$$

E. Kou, Phys. Rev. D 63 (2001) 54027

V P γ and Pγγ transitions

KLOE [Phys. Lett. B648 (2007) 267] has fitted:

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{z_q^2}{\cos^2\phi_V}\cdot 3\left(\frac{m_{\eta'}^2-m_\rho^2\,m_\omega}{m_\omega^2-m_\pi^2\,m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta'\to\omega\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{1}{3}\left(\frac{m_{\eta'}^2-m_\omega^2\,m_\omega}{m_\omega^2-m_\pi^2\,m_{\eta'}}\right)^3\left[z_q X_{\eta'}+2\frac{m_s}{\bar m}z_s\cdot\tan\phi_V\cdot Y_{\eta'}\right]^2$$

Were taken from a global fit without gluonium:

A. Bramon, R. Escribano, M.D. Scadron
Phys. Lett. B503 (2001) 271

together with the measured branching ratio:

$$R_\phi = (4.77 \pm 0.09 \pm 0.19)\times 10^{-3}$$

$$R_\phi=\frac{Br(\phi\to\eta'\gamma)}{Br(\phi\to\eta\gamma)}=\cot^2\phi_P\cdot\cos^2\phi_G\left(1-\frac{m_s}{\bar m}\frac{z_q}{z_S}\cdot\frac{\tan\phi_V}{\sin 2\phi_P}\right)^2\cdot\left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

and the ratio:

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3\left(5X_{\eta'}+\sqrt{2}\frac{f_q}{f_s}Y_{\eta'}\right)^2$$

E. Kou, Phys. Rev. D 63 (2001) 54027

V P γ and Pγγ transitions

KLOE [Phys. Lett. B648 (2007) 267] has fitted:

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{z_q^2}{\cos^2\phi_V}\cdot 3\left(\frac{m_{\eta'}^2-m_\rho^2 m_\omega}{m_\omega^2-m_\pi^2 m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta'\to\omega\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{1}{3}\left(\frac{m_{\eta'}^2-m_\omega^2 m_\omega}{m_\omega^2-m_\pi^2 m_{\eta'}}\right)^3\left[z_q X_{\eta'}+2\frac{m_s}{\bar{m}} z_s\cdot\tan\phi_V\cdot Y_{\eta'}\right]^2$$

T. Feldmann, Int. J. Mod. Phys. A 15 (2000) 159

together with the measured branching ratio:

$R_\phi = (4.77 \pm 0.09 \pm 0.19)\times 10^{-3}$

$$R_\phi=\frac{Br(\phi\to\eta'\gamma)}{Br(\phi\to\eta\gamma)}=\cot^2\phi_P\cdot\cos^2\phi_G\left(1-\frac{m_s}{\bar{m}}\frac{z_q}{z_S}\cdot\frac{\tan\phi_V}{\sin 2\phi_P}\right)^2\cdot\left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

and the ratio:

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3\left(5X_{\eta'}+\sqrt{2}\frac{f_q}{f_s}Y_{\eta'}\right)^2$$

E. Kou, Phys. Rev. D 63 (2001) 54027

Fit redone leaving free all parameters

1) Leave the z's parameter free;
2) Add more constraints (needed to perform the fit with larger number of parameters);
3) Check the contribution from $\eta' \to \gamma\gamma$ / $\pi^0 \to \gamma\gamma$

$$\frac{\Gamma(\omega \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{9}\left[z_q \cos(\phi_p) - 2\frac{m_s}{\bar{m}} z_s \tan(\phi_V)\sin(\phi_p)\right]^2 (1 - z_G^2)\left(\frac{m_\omega^2 - m_\eta^2}{m_\omega^2 - m_{\pi^0}^2}\right)^3$$

$$\frac{\Gamma(\rho \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = z_q^2 \frac{\cos^2(\phi_p)}{\cos^2(\phi_V)}\left(\frac{m_\rho^2 - m_\eta^2}{m_\omega^2 - m_\pi^2}\frac{m_\omega}{m_\rho}\right)^3$$

$$\frac{\Gamma(\phi \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{9}\left[z_q \tan(\phi_v)\cos(\phi_p) + 2\frac{\bar{m}}{m_s} z_s \sin(\phi_p)\right]^2 \left(\frac{m_\phi^2 - m_\eta^2}{m_\omega^2 - m_\pi^2}\frac{m_\omega}{m_\phi}\right)^3$$

$$\frac{\Gamma(\phi \to \pi^0\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \tan^2\phi_V \cdot \left(\frac{m_\phi^2 - m_{\pi^0}^2}{m_\omega^2 - m_{\pi^0}^2} \cdot \frac{m_\omega}{m_\phi}\right)^3 , \frac{\Gamma(K^{+*} \to K^+\gamma)}{\Gamma(K^{*0} \to K^0\gamma)} = \left(\frac{2\frac{m_s}{\bar{m}} - 1}{1 + \frac{m_s}{\bar{m}}}\right)^2 \cdot \left(\frac{m_{K^{*+}}^2 - m_{K^{+0}}^2}{m_{K^{*0}}^2 - m_{K^0}^2} \cdot \frac{m_{K^{*0}}}{m_{K^{*+}}}\right)^3$$

The experimental covariance matrix B contains correlation among common used quantities in the fitted relations:

$$\frac{\Gamma(\omega\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)},\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)},\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)} \longrightarrow$$

Introduces a correlation in the fitted quantities

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{Br(\eta'\to\gamma\gamma)\Gamma_{\eta'}}{\Gamma(\pi^0\to\gamma\gamma)} \qquad \frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{Br(\eta'\to\rho\gamma)\Gamma_{\eta'}}{\Gamma(\omega\to\pi^0\gamma)}$$

	x_1	x_2	x_3	x_4	x_5	x_6
x_2	−34					
x_3	−78	−29				
x_4	−35	−24	32			
x_5	−26	−12	26	8		
x_6	−28	−11	35	11	9	
Γ	32	−2	−24	−5	−88	−8

	Mode	Rate (MeV)	Scale factor
Γ_1	$\pi^+\pi^-\eta$	0.090 ±0.008	1.2
Γ_2	$\rho^0\gamma$ (including non-resonant $\pi^+\pi^-\gamma$)	0.060 ±0.005	1.2
Γ_3	$\pi^0\pi^0\eta$	0.042 ±0.004	1.6
Γ_4	$\omega\gamma$	0.0062 ±0.0008	1.2
Γ_5	$\gamma\gamma$	0.00430±0.00015	1.1
Γ_6	$3\pi^0$	(3.2 ±0.6) × 10^{-4}	1.1

Br and Γ strongly correlated (above all Γ(η'→γγ))

the Γ is measured using:

$$e^+e^-\to\eta' e^+e^-$$

An independent measurement of the η' total width is welcome.

Fit results without η'→ γγ / π^0→ γγ

$$\Gamma(P \to V\gamma) = \frac{g^2}{4\pi}|p_\gamma|^3$$

Fit redone a la Escribano

(using couplings and not taking into account correlations)

very similar results

	Fit with width ratios	Escribano *et al.*, JHEP 0705:006 (2007)	Fit with couplings
$\chi^2/n.d.f(Prob)$	1.8/2 (41 %)	4.2/4 (38 %)	4.7/4 (32 %)
Z_G^2	0.03 ± 0.06	0.04 ± 0.09	0.04 ± 0.07
φ_G	$(10 \pm 10)^\circ$	$(12 \pm 13)^\circ$	$(11 \pm 11)^\circ$
φ_P	$(41.6 \pm 0.8)^\circ$	$(41.4 \pm 1.3)^\circ$	$(41.5 \pm 1.1)^\circ$
Z_{NS}	0.85 ± 0.03	0.86 ± 0.03	0.86 ± 0.03
Z_S	0.78 ± 0.05	0.79 ± 0.05	0.78 ± 0.05
φ_V	$(3.16 \pm 0.10)^\circ$	$(3.2 \pm 0.1)^\circ$	$(3.18 \pm 0.10)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07	1.24 ± 0.07
Z_K		0.89 ± 0.03	0.89 ± 0.03
g		0.72 ± 0.01	0.72 ± 0.01

disappear in the ratio

Table 3: Comparison among the fit results without the $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ measurement and the Escribano *et al.* results.

Adding the $\eta' \to \gamma\gamma$ / $\pi^0 \to \gamma\gamma$ constraint

	Gluonium allowed	Gluonium at zero
$\chi^2/n.d.f(Prob)$	5/3 (17.5 %)	13/4 (1.1 %)
Z_G^2	0.105 ± 0.037	0 fixed
φ_P	$(40.7 \pm 0.7)^\circ$	$(41.6 \pm 0.5)^\circ$
Z_{NS}	0.866 ± 0.025	0.863 ± 0.024
Z_S	0.79 ± 0.05	0.78 ± 0.05
φ_V	$(3.15 \pm 0.10)^\circ$	$(3.17 \pm 0.10)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

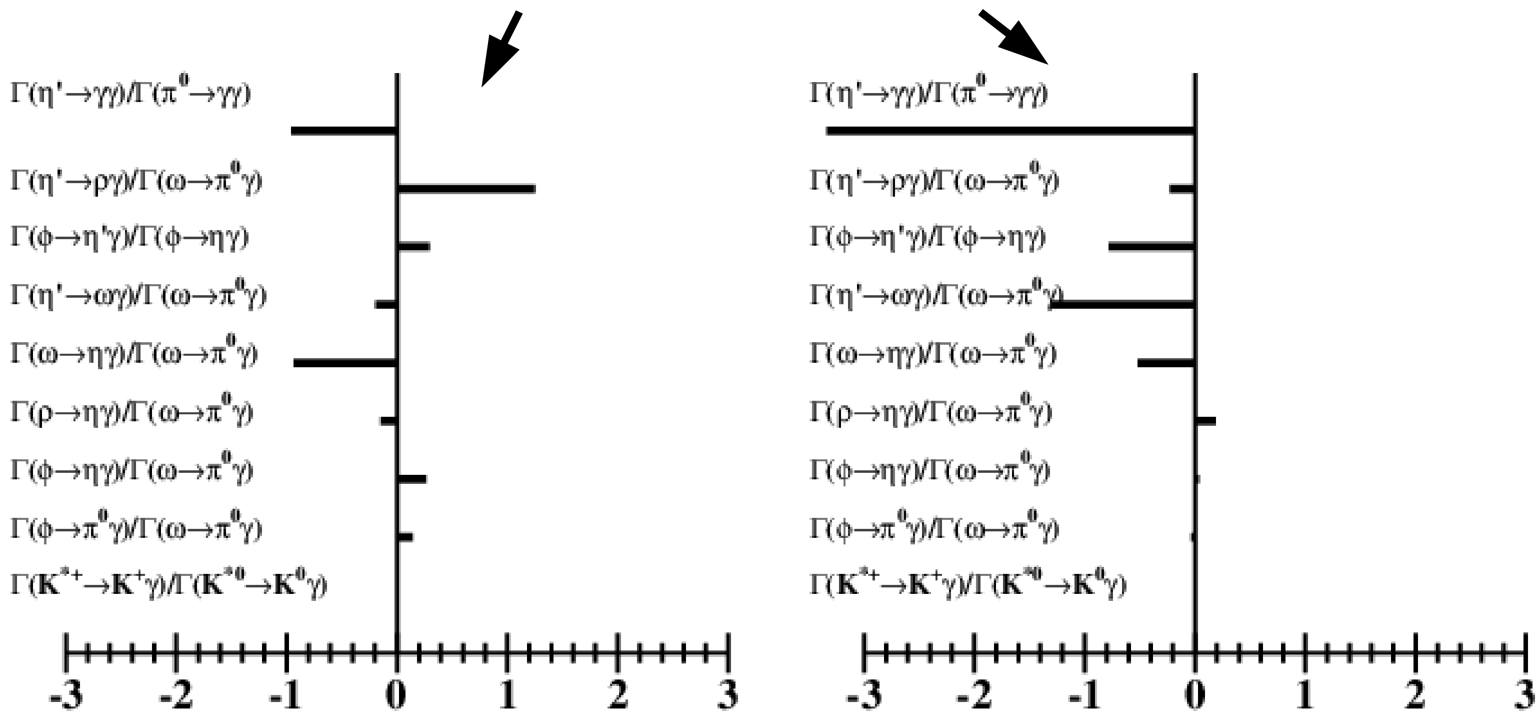
KLOE
(Phys. Lett. B648 (2007) 267)

$\phi_P = (39.7 \pm 0.7_{tot})^\circ$

$|\phi_G| = (22 \pm 3)^\circ$

$\sin^2\phi_G = Z^2 = 0.14 \pm 0.04$

Fit pulls



Update using PDG 2008

Result of the fit

PDG08

	Gluonium allowed	Gluonium at zero
$\chi^2/n.d.f(Prob)$	7.9/3 (5 %)	$15/4\ (5\times10^{-3})$)
Z_G^2	0.097 ± 0.037	0 fixed
φ_P	$(41.0 \pm 0.7)^\circ$	$(41.7 \pm 0.5)^\circ$
Z_{NS}	0.86 ± 0.02	0.858 ± 0.021
Z_S	0.79 ± 0.05	0.78 ± 0.05
φ_V	$(3.17 \pm 0.09)^\circ$	$(3.19 \pm 0.09)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

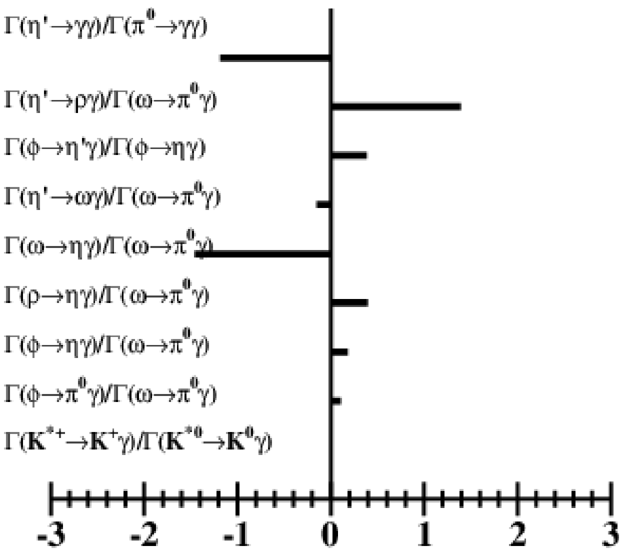
The same gluonium content but unsatisfying fit quality.

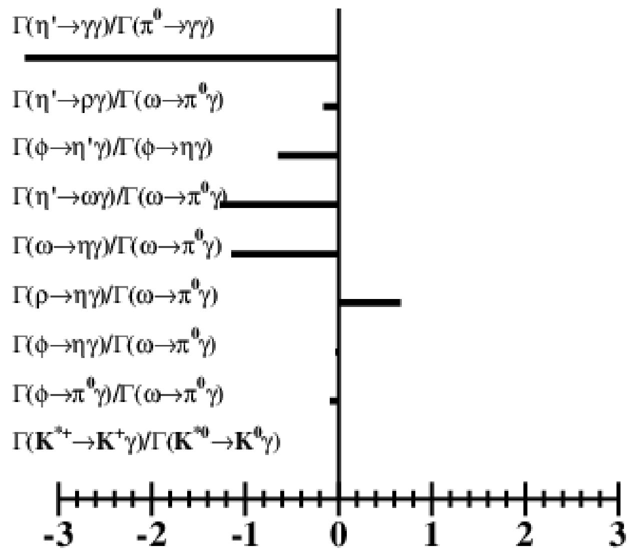
PDG06

	Gluonium allowed	Gluonium at zero
$\chi^2/n.d.f(Prob)$	5/3 (17.5 %)	13/4 (1.1 %)
Z_G^2	0.105 ± 0.037	0 fixed
φ_P	$(40.7 \pm 0.7)^\circ$	$(41.6 \pm 0.5)^\circ$
Z_{NS}	0.866 ± 0.025	0.863 ± 0.024
Z_S	0.79 ± 0.05	0.78 ± 0.05
φ_V	$(3.15 \pm 0.10)^\circ$	$(3.17 \pm 0.10)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

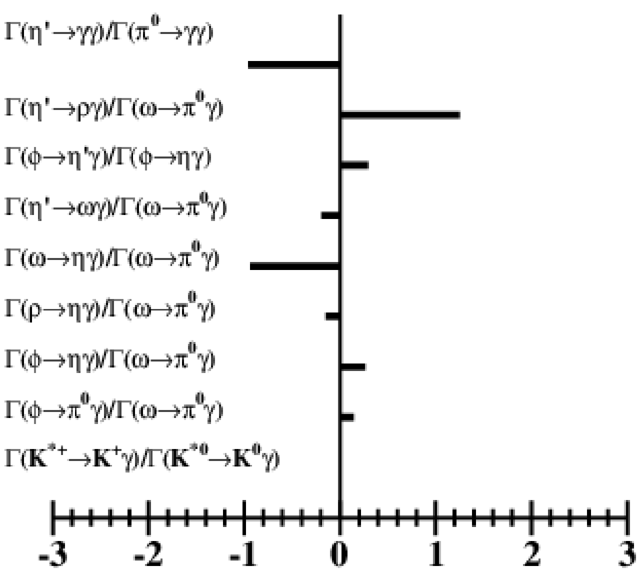
Table 2: Fit results.

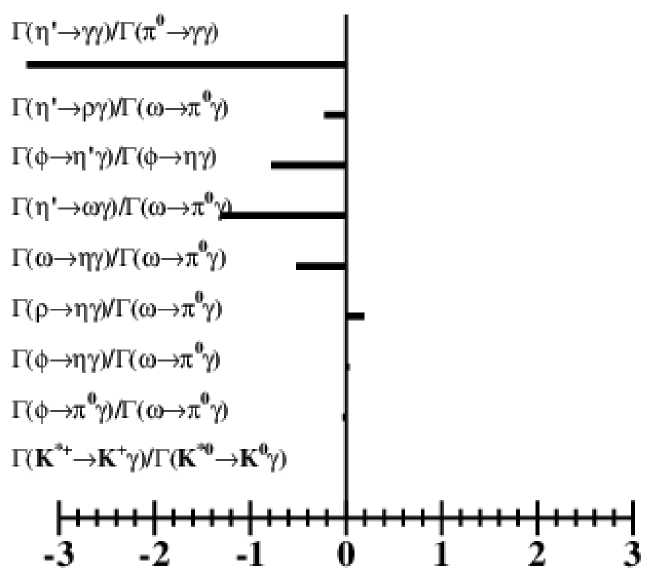
Pulls of the fit





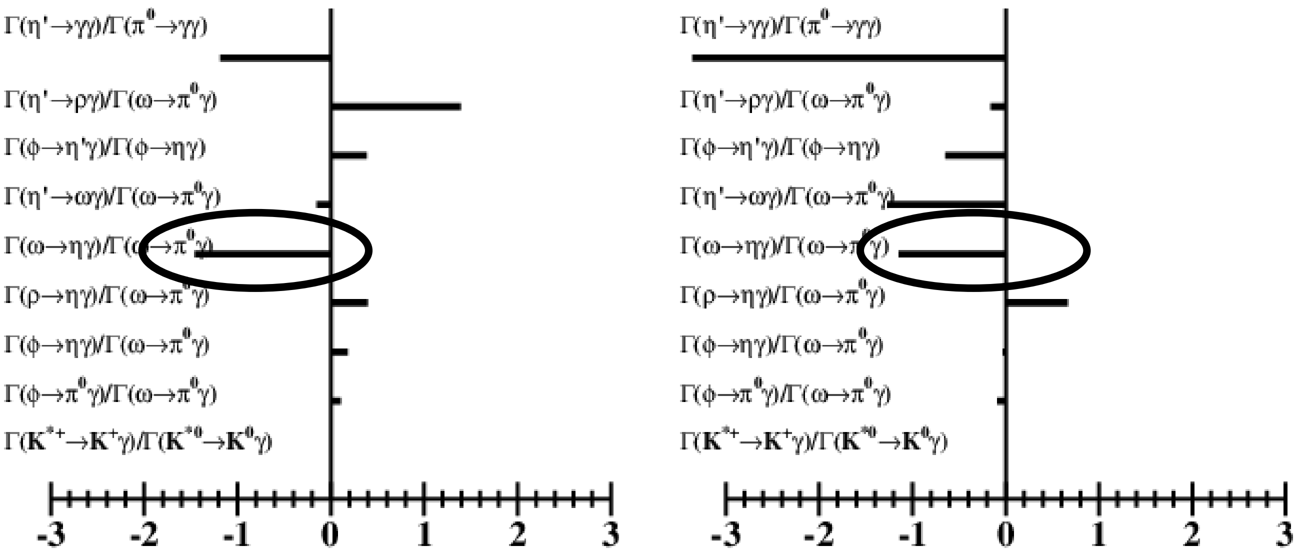
PDG08



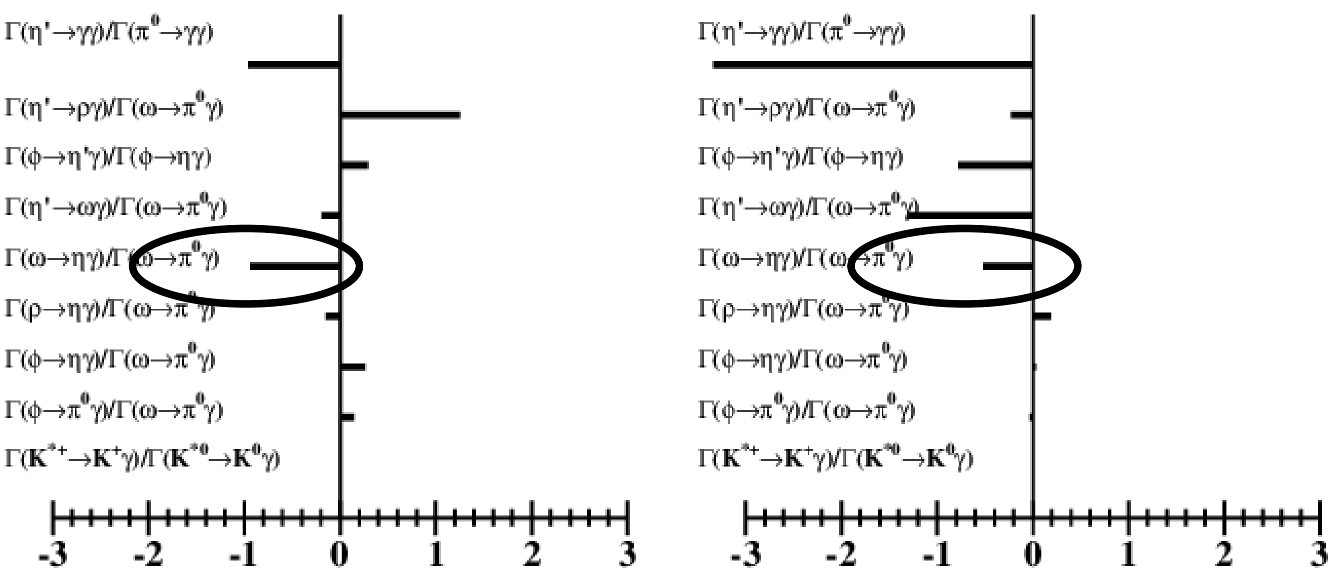


PDG06

Pulls of the fit



PDG08



PDG06

$\omega \to \eta\gamma$ pull has increased in both gluonium hypothesis

The $\omega \to \eta\gamma$ partial width changed from

$(4.9 \pm 0.5)\times10^{-4}$ to $(4.6 \pm 0.4)\times10^{-4}$

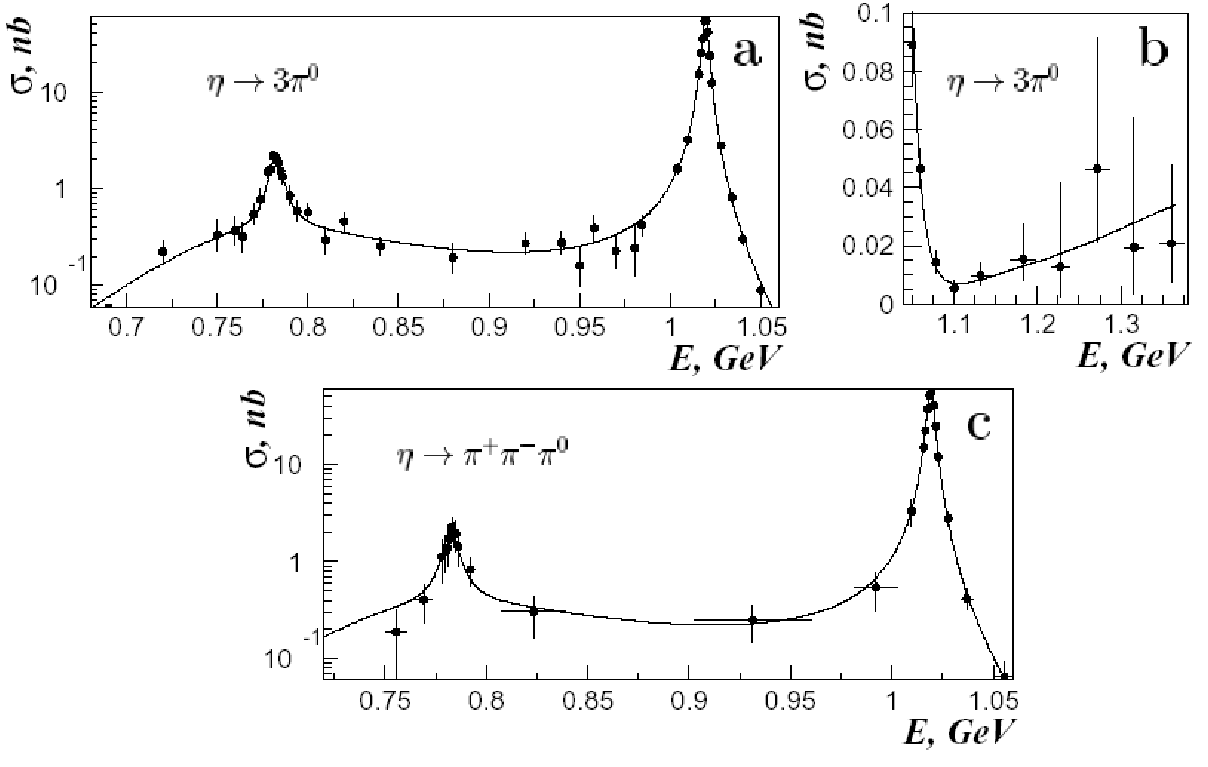
This value is determined by the global PDG fit, and it is mainly determined by:

$\Gamma(e^+e^-) \times \Gamma(\eta\gamma)/\Gamma^2_{total}$ $\Gamma_9\Gamma_5/\Gamma^2$

VALUE (units 10^{-8})	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
3.31±0.28 OUR FIT	Error includes scale factor of 1.1.				
3.18±0.28 OUR AVERAGE					
$3.10\pm0.31\pm0.11$	33k	[24] ACHASOV	07B	SND	0.6–1.38 $e^+e^- \to \eta\gamma$
$3.17^{+1.85}_{-1.31}\pm0.21$	17.4k	[25] AKHMETSHIN	05	CMD2	0.60-1.38 $e^+e^- \to \eta\gamma$
$3.41\pm0.52\pm0.21$	23k	[26,27] AKHMETSHIN	01B	CMD2	$e^+e^- \to \eta\gamma$

ACHASOV 07B: Phys. Rev. D76 (2007) 077101

The branching ratio is extracted with a global fit to the $e^+e^- \to \eta\gamma$ with a VMD model with $\rho, \omega, \phi, \rho'$ included (ρ' parameters varied to compute systematics and constrained from $e^+e^- \to \eta\rho$).



ω contribution overwhelmed by the $\rho \to \eta\gamma$ contribution
no correlation matrix is given in the paper

Direct $\omega \to \eta\gamma$ branching ratio measurement

The fit is dominated by the SND measurement

$\Gamma(\eta\gamma)/\Gamma_{total}$

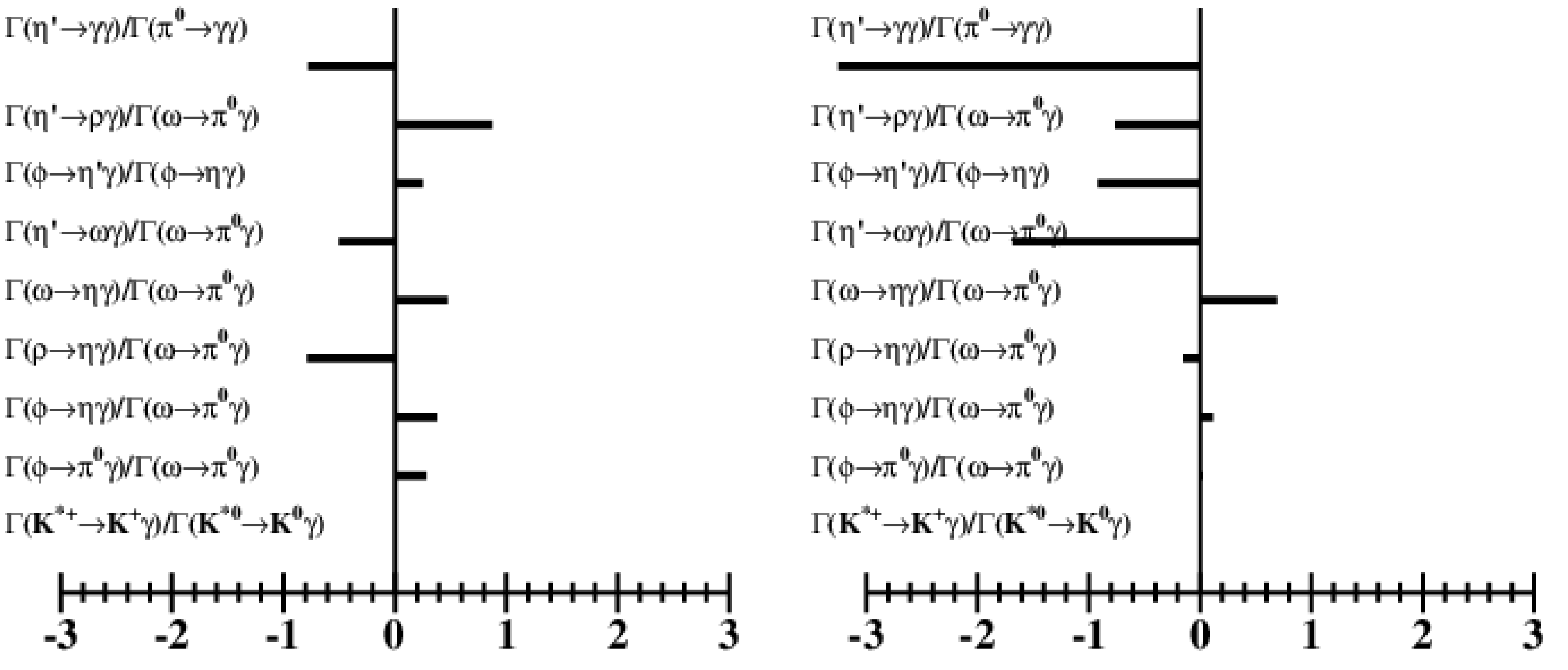
VALUE (units 10^{-4})		EVTS	DOCUMENT ID		TECN	COMMENT
4.6 ±0.4	**OUR FIT**	Error includes scale factor of 1.1.				
6.3 ±1.3	**OUR AVERAGE**	Error includes scale factor of 1.2.				
6.6 ±1.7			[53] ABELE	97E	CBAR	0.0 $\bar{p}p \to 5\gamma$
8.3 ±2.1			ALDE	93	GAM2	38 $\pi^- p \to \omega n$
3.0 $^{+2.5}_{-1.8}$			[54] ANDREWS	77	CNTR	6.7–10 γ Cu

In Crystal Barrel the channel $p\bar{p} \to \eta\,\omega$ is used that is 6 times larger than $p\bar{p} \to \eta\,\rho$, the $\omega \to \eta\gamma$ Br was normalized to the $\omega \to \pi^0\gamma$ Br

using $\omega \to \eta\gamma$ from PDG average

	Gluonium allowed	Gluonium at zero
$\chi^2/n.d.f(Prob)$	3.9/3 (27.5 %)	13/4 (1.1 %)
Z_G^2	0.111 ± 0.036	0 fixed
φ_P	$(40.6 \pm 0.7)^\circ$	$(41.5 \pm 0.5)^\circ$
Z_{NS}	0.890 ± 0.025	0.882 ± 0.023
Z_S	0.79 ± 0.05	0.78 ± 0.05
φ_V	$(3.15 \pm 0.10)^\circ$	$(3.18 \pm 0.09)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

Pulls



Γ(η'→γγ)/Γ(π0→γγ)
Γ(η'→ργ)/Γ(ω→π0γ)
Γ(φ→η'γ)/Γ(φ→ηγ)
Γ(η'→ωγ)/Γ(ω→π0γ)
Γ(ω→ηγ)/Γ(ω→π0γ)
Γ(ρ→ηγ)/Γ(ω→π0γ)
Γ(φ→ηγ)/Γ(ω→π0γ)
Γ(φ→π0γ)/Γ(ω→π0γ)
Γ(K*+→K+γ)/Γ(K*0→K0γ)
-3
-2
-1
0
1
2
3

Results using KLOE Br($\omega \to \pi^0 \gamma$)

KLOE Br($\omega \to \pi^0 \gamma$) = 8.09 ± 0.14 %

PDG08 = 8.92 ± 0.24 %

3σ away

	Gluonium allowed	Gluonium at zero
$\chi^2/n.d.f(Prob)$	4.16/3 (25 %)	13/4 (1.1 %))
Z_G^2	0.109 ± 0.036	0 fixed
φ_P	$(40.5 \pm 0.7)^\circ$	$(41.4 \pm 0.5)^\circ$
Z_{NS}	0.935 ± 0.025	0.926 ± 0.023
Z_S	0.83 ± 0.05	0.82 ± 0.05
φ_V	$(3.3 \pm 0.09)^\circ$	$(3.3 \pm 0.09)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

No big effect on Z_G but important contribution to $Z_{S,NS}$ and ω–ϕ mixing angle

Conclusion and Outlook (KLOE 2)

Memo almost ready, will be internally published next week for referee review (S. Giovanella & P. Gauzzi).

1) **Measure $\eta' \to \gamma\gamma$ branching ratio that is the only one pointing for the gluonium;**

2) Improve $\phi \to \eta'\gamma$ through the measurement of $\eta' \to \pi\pi\eta$, $\pi^+\pi^-\gamma$, $\omega\gamma$, $\gamma\gamma$, $\pi^+\pi^-\pi^0$ (upper limit), n° π;

Fit procedure.

The χ^2 is defined as follows:

$$\chi^2 = \Sigma_{i,j=1,3} \left(y_i - y_i^{th}\right) \times V_{ij}^{-1} \left(y_j - y_j^{th}\right)$$

V_{ij} is the error matrix which is a function of theoretical uncertainties, as well as the experim

$$V_{ij} = [B_{ij} + (A_{ik} \times C_{kl} \times A_{lj}^T)]$$

Experimental covariance matrix (B_{ij})

Theoretical parameters covariance matrix (C_{kl})

B_{ij} **Full covariance matrix (correlation comes from the constrained fit to η' Br)**

$$; A_{ik} = \begin{pmatrix} \frac{\partial y_1^{th}}{\partial f_s} & \frac{\partial y_1^{th}}{\partial f_q} & \frac{\partial y_1^{th}}{\partial C_{NS}} & \frac{\partial y_1^{th}}{\partial C_S} & \frac{\partial y_1^{th}}{\partial \frac{m_s}{m}} \\ \frac{\partial y_2^{th}}{\partial f_s} & \frac{\partial y_2^{th}}{\partial f_q} & \frac{\partial y_2^{th}}{\partial C_{NS}} & \frac{\partial y_2^{th}}{\partial C_S} & \frac{\partial y_2^{th}}{\partial \frac{m_s}{m}} \\ \frac{\partial y_3^{th}}{\partial f_s} & \frac{\partial y_3^{th}}{\partial f_q} & \frac{\partial y_3^{th}}{\partial C_{NS}} & \frac{\partial y_3^{th}}{\partial C_S} & \frac{\partial y_3^{th}}{\partial \frac{m_s}{m}} \end{pmatrix}$$

$$C_{kl} = \begin{pmatrix} \sigma_{f_q}^2 & 0 \\ 0 & \sigma_{f_s}^2 \end{pmatrix}$$

Re-evaluated at each minimization step (A_{ik})