Fit to the η,η' mixing angle and the η' gluonium content.

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Antefact

First 3σ hint of η' gluonium content by KLOE

Phys. Lett. B648 (2007) 267

$$|\eta'\rangle = X_{\eta'}|q\bar{q}\rangle + Y_{\eta'}|s\bar{s}\rangle + Z_{\eta'}|G\rangle$$

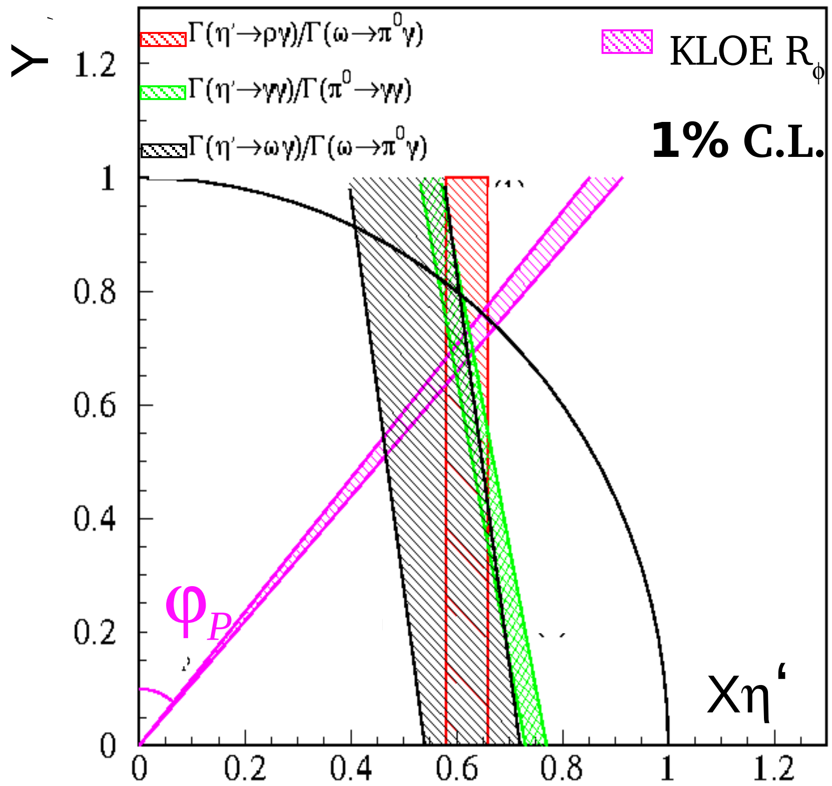
$$|\eta\rangle = \cos\phi_P|q\bar{q}\rangle - \sin\phi_P|s\bar{s}\rangle$$

$\phi_P = (39.7 \pm 0.7_{tot})^\circ$

$|\phi_G| = (22 \pm 3)^\circ$

$\sin^2\phi_G = Z^2 = 0.14 \pm 0.04$

Assuming no gluonium (Z=0)



In order to fit the gluonium we use our measurement of

$$R_\phi = \frac{Br(\phi \to \eta' \gamma)}{Br(\phi \to \eta \gamma)} = \cot^2 \phi_P \cdot \cos^2 \phi_G \left(1 - \frac{m_s}{\overline{m}} \frac{Zq}{Z_S} \cdot \tan \frac{\phi_V}{\sin 2\phi_P}\right)^2 \cdot \left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

And the following width ratios

$$\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3 \left(5 X_{\eta'} + \sqrt{2}\frac{f_q}{f_s} Y_{\eta'}\right)^2$$

$$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{Z_q}{\cos\phi_V} \cdot 3 \left(\frac{m_{\eta'}^2 - m_\rho^2}{m_\omega^2 - m_\pi^2} \frac{m_\omega}{m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{3}\left(\frac{m_{\eta'}^2 - m_\omega^2}{m_\omega^2 - m_\pi^2} \frac{m_\omega}{m_{\eta'}}\right)^3 \left[Z_q X_{\eta'} + 2\frac{m_s}{\overline{m}} Z_s \cdot \tan\phi_V \cdot Y_{\eta'}\right]^2$$

$$X_{\eta'} = \sin\phi_P \cos\phi_G$$
$$Y_{\eta'} = \cos\phi_P \cos\phi_G$$
$$Z_{\eta'} = \sin\phi_G$$

Z_q[(1)] and Z_S were taken from A. Bramon, R. Escribano, M.D. Scadron, Phys. Lett. B503 (2001) 271

From a fit without gluonium content.

ϕ_V $\omega - \phi$ **mixing**

[(1)] **Note: Z_q and Z_s are called C_{NS} and C_S in our paper.**

In Escribano et al. (JHEP 0705:006,2007) different conclusions were found:

KLOE	**Escribano**
$\phi_P = (39.7 \pm 0.7_{tot})^\circ$	$\phi_P = (41.4 \pm 1.3)^\circ$
$\lvert\phi_G\rvert = (22 \pm 3)^\circ$	$\lvert\phi_G\rvert = (12 \pm 13)^\circ$
$\sin^2\phi_G = Z^2 = 0.14 \pm 0.04$	$\sin^2\phi_G = Z^2 = 0.04 \pm 0.09$

The difference was attributed to the choice to fix the Z_q and Z_s parameter from a fit without gluonium.

Differences between Escribano's fit and ours.

Our fit

Only ϕ_p and Z^2 are left free

The ratios of Γ's are used in the fit.

$$\Gamma\,(P\rightarrow V\,\gamma)=\frac{g^2}{4\,\pi}\,|p_\gamma|^3$$

4 measured quantities are used in the fit **including** $\eta'\rightarrow\gamma\gamma\,/\,\pi^0\rightarrow\gamma\gamma$

DATA from PDG '06 + KLOE R_ϕ '07

Escribano

All theoretical parameters are left free

The couplings are used in the fit.

$$\Gamma\,(V\rightarrow P\,\gamma)=\frac{1}{3}\,\frac{g^2}{4\,\pi}\,|p_\gamma|^3$$

12 measured quantities are used in the fit **excluding** $\eta'\rightarrow\gamma\gamma\,/\,\pi^0\rightarrow\gamma\gamma$

DATA from PDG '06

To understand the dicrepancy:

1) redo the fit leaving free Z_q and Z_s

2) fit the couplings

3) Studying the effect of $\eta' \to \gamma\gamma$ / $\pi^0 \to \gamma\gamma$

Escribano's couplings

$$g_{\rho^0\pi^0\gamma} = g_{\rho^+\pi^+\gamma} = \tfrac{1}{3}g \ , \quad g_{\omega\pi\gamma} = g\cos\phi_V \ , \quad g_{\phi\pi\gamma} = g\sin\phi_V \ ,$$

$$g_{K^{*0}K^0\gamma} = -\tfrac{1}{3}g\, z_K\left(1+\frac{\bar{m}}{m_s}\right) \ , \quad g_{K^{*+}K^+\gamma} = \tfrac{1}{3}g\, z_K\left(2-\frac{\bar{m}}{m_s}\right) \ ,$$

$$g_{\rho\eta\gamma} = g\, z_q\, X_\eta \ , \quad g_{\rho\eta'\gamma} = g\, z_q\, X_{\eta'}$$

$$g_{\omega\eta\gamma} = \tfrac{1}{3}g\left(z_q\, X_\eta\cos\phi_V + 2\frac{\bar{m}}{m_s}z_s\, Y_\eta\sin\phi_V\right) \ ,$$

$$g_{\omega\eta'\gamma} = \tfrac{1}{3}g\left(z_q\, X_{\eta'}\cos\phi_V + 2\frac{\bar{m}}{m_s}z_s\, Y_{\eta'}\sin\phi_V\right) \ ,$$

All couplings are proportional to g

$$g_{\phi\eta\gamma} = \tfrac{1}{3}g\left(z_q\, X_\eta\sin\phi_V - 2\frac{\bar{m}}{m_s}z_s\, Y_\eta\cos\phi_V\right) \ ,$$

$$g_{\phi\eta'\gamma} = \tfrac{1}{3}g\left(z_q\, X_{\eta'}\sin\phi_V - 2\frac{\bar{m}}{m_s}z_s\, Y_{\eta'}\cos\phi_V\right) \ ,$$

In order to use the same experimental informations, but using our way we normalise all quantities to $\Gamma(\omega \to \pi^0\gamma)$.

In this way the fit is independent from the coupling **g.**

Discrepancy between our definition of $\Gamma(\eta' \to \rho\gamma)/\Gamma(\omega \to \pi^0\gamma)$ and Escribano definition of g's

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)} = \frac{\frac{g^2_{\eta'\to\rho\gamma}}{4\pi}|p_\gamma^{\eta'}|^3}{\frac{g^2_{\omega\to\pi^0\gamma}}{12\pi}|p_\gamma^{\omega}|^3} = 3\frac{|p_\gamma^{\eta'}|^3}{|p_\gamma^{\omega}|^3}\frac{g^2 Z_q^2 X_{\eta'}^2}{g^2\cos^2\phi_V} = 3\frac{|p_\gamma^{\eta'}|^3}{|p_\gamma^{\omega}|^3}\frac{Z_q^2}{\cos^2\phi_V}\sin^2\phi_P\cos^2\phi_G$$

while in our paper:

$$\Gamma(\eta' \to \rho\gamma)/\Gamma\left(\omega \to \pi^0\gamma\right) = \frac{C_{NS}}{\cos\varphi_V} \cdot 3\left(\frac{m_{\eta'}^2 - m_\rho^2}{m_\omega^2 - m_\pi^2}\frac{m_\omega}{m_{\eta'}}\right)^3 \cos^2\phi_G \sin^2\varphi_P$$

Paper measurements

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3\left(5X_{\eta'}+\sqrt{2}\frac{f_q}{f_s}Y_{\eta'}\right)^2$$

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{Z_q}{\cos\phi_V}\cdot 3\left(\frac{m_{\eta'}^2-m_\rho^2\, m_\omega}{m_\omega^2-m_\pi^2\, m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta'\to\omega\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{1}{3}\left(\frac{m_{\eta'}^2-m_\omega^2\, m_\omega}{m_\omega^2-m_\pi^2\, m_{\eta'}}\right)^3\left[Z_q X_{\eta'}+2\frac{m_s}{\bar{m}}Z_s\cdot\tan\phi_V\cdot Y_{\eta'}\right]^2$$

Not enough to leave free Z_q and Z_s

3 measurements with 4 parameters.

$$\frac{\Gamma(\omega\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{1}{9}\left[Z_q\cos(\phi_p)-2\frac{m_s}{\bar{m}}Z_s\tan(\phi_V)\sin(\phi_p)\right]^2(1-z_G^2)\left(\frac{m_\omega^2-m_\eta^2}{m_\omega^2-m_{\pi^0}^2}\right)^3$$

$$\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=Z_q^2\frac{\cos^2(\phi_p)}{\cos^2(\phi_V)}\left(\frac{m_\rho^2-m_\eta^2}{m_\omega^2-m_\pi^2}\frac{m_\omega}{m_\rho}\right)^3$$

$$\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{1}{9}\left[Z_q\tan(\phi_v)\cos(\phi_p)+2\frac{\bar{m}}{m_s}Z_s\sin(\phi_p)\right]^2\left(\frac{m_\phi^2-m_\eta^2}{m_\omega^2-m_\pi^2}\frac{m_\omega}{m_\phi}\right)^3$$

Using Escribano couplings we compute the ratios

$$\frac{\Gamma(\phi\to\pi^0\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\tan^2\phi_V\cdot\left(\frac{m_\phi^2-m_{\pi^0}^2}{m_\omega^2-m_{\pi^0}^2}\cdot\frac{m_\omega}{m_\phi}\right)^3,\quad \frac{\Gamma(K^{+*}\to K^+\gamma)}{\Gamma(K^{*0}\to K^0\gamma)}=\left(\frac{2\frac{m_s}{\bar{m}}-1}{1+\frac{m_s}{\bar{m}}}\right)^2\cdot\left(\frac{m_{K^{*+}}^2-m_{K^{+0}}^2}{m_{K^{*0}}^2-m_{K^0}^2}\cdot\frac{m_{K^{*0}}}{m_{K^{*+}}}\right)^3$$

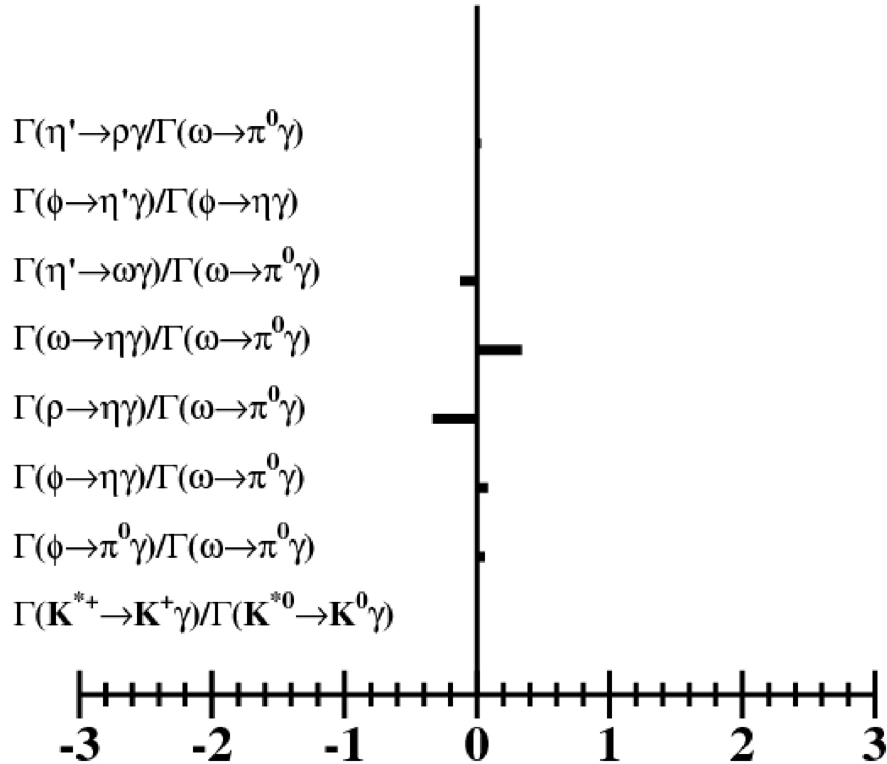
The fit is performed taking into account all the experimental correlations among the Br, and the correlations coming from the use of the $\Gamma(\omega \to \pi^0\gamma)$.

off-diagonal covariance matrix elements used in the fit.

$P \to \gamma\gamma$ is not used.

$$Pulls = \frac{Measure - Fit}{\sigma_{Measure}}$$

	This fit	Escribano
	χ^2/ndf = 0.27/2	χ^2/ndf = 4.0/4
z^2	0.10 ± 0.05	0.04 ± 0.09
ϕ_p	$(40.7 \pm 0.8)°$	$(41.4 \pm 1.3)°$
C_{NS}	0.84 ± 0.03	0.86 ± 0.03
C_S	0.79 ± 0.05	0.79 ± 0.05
ϕ_V	$(3.17 \pm 0.10)°$	$(3.2 \pm 0.1)°$
$\frac{m_s}{\bar{m}}$	1.24 ± 0.07	1.24 ± 0.07



Using the relation from Escribano couplings

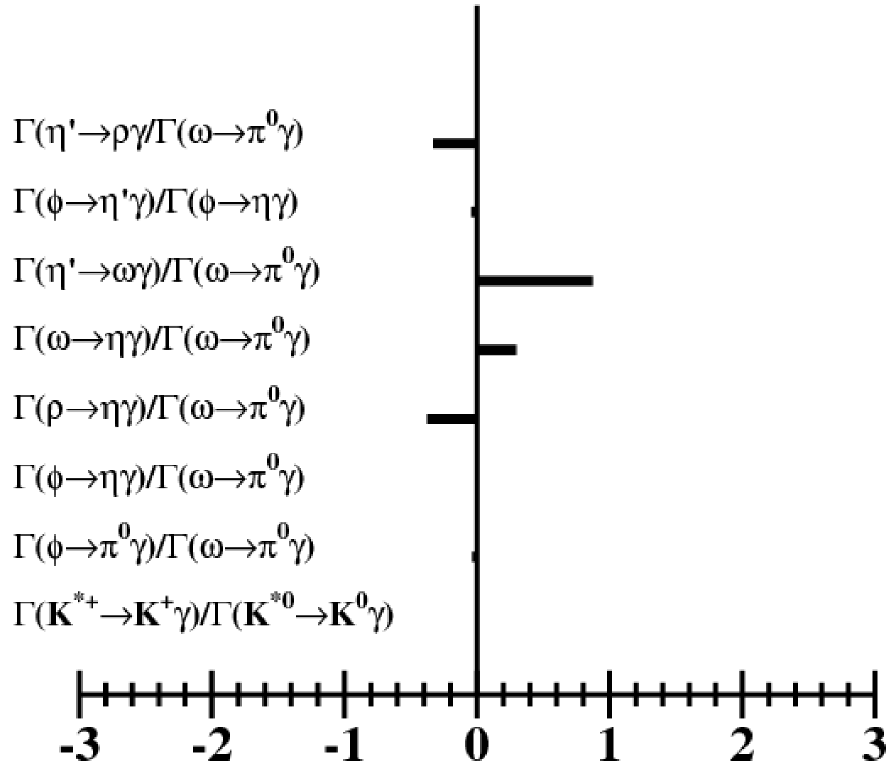
$$\frac{\Gamma(\eta' \rightarrow \rho\gamma)}{\Gamma(\omega \rightarrow \pi^0 \gamma)} = 3 \frac{|p_\gamma^{\eta'}|^3}{|p_\gamma^{\omega}|^3} \frac{Z_q^2}{\cos^2 \phi_V} \sin^2 \phi_P \cos^2 \phi_G$$

instead of our paper relation:

$$3 \frac{Z_q}{\cos \phi_V} \left(\frac{m_{\eta'}^2 - m_\rho^2 m_\omega}{m_\omega^2 - m_\pi^2 m_{\eta'}} \right)^3 \sin^2 \phi_P \cos^2 \phi_G$$

	This fit	Escribano
	$\chi^2/\mathrm{ndf} = 1.8/2$	$\chi^2/\mathrm{ndf} = 4.0/4$
Z^2	0.03 ± 0.06	0.04 ± 0.09
ϕ_P	$(41.6 \pm 0.8)°$	$(41.4 \pm 1.3)°$
C_{NS}	0.85 ± 0.03	0.86 ± 0.03
C_S	0.78 ± 0.05	0.79 ± 0.05
ϕ_V	$(3.16 \pm 0.10)°$	$(3.2 \pm 0.1)°$
$\frac{m_s}{\bar{m}}$	1.24 ± 0.07	1.24 ± 0.07

$$Pulls = \frac{Measure - Fit}{\sigma_{Measure}}$$

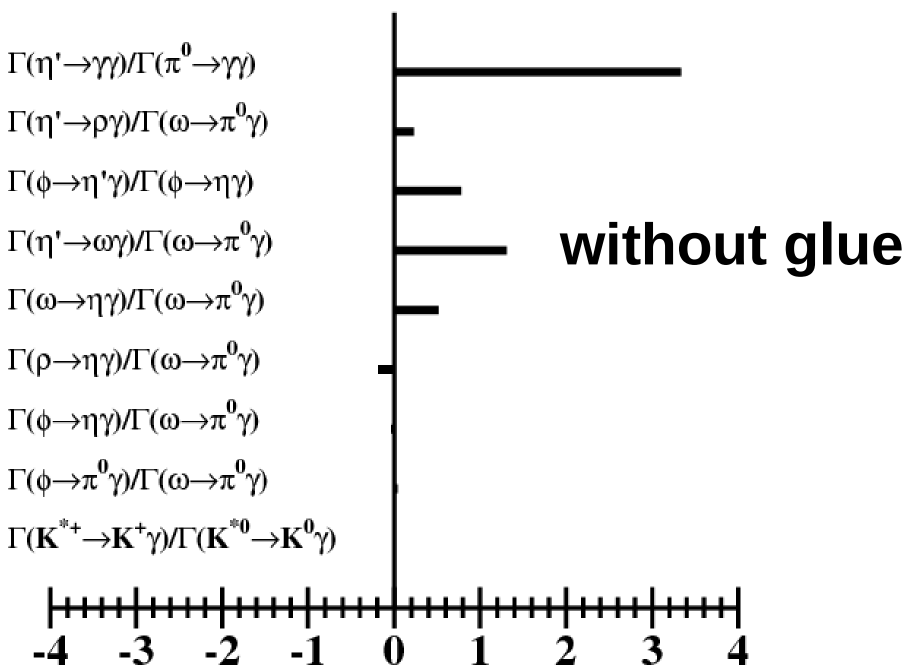


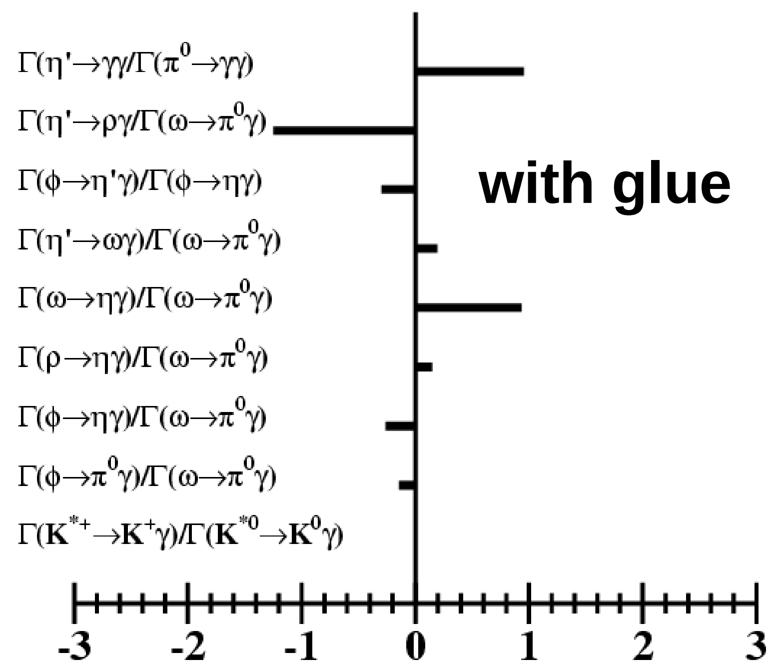
If we include the P→γγ constraint

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3\left(5\sin(\phi_P)\cos(\phi_G)+\sqrt{2}\frac{f_q}{f_s}\cos(\phi_P)\cos(\phi_G)\right)^2$$

	without glue χ^2/ndf = 13/4 (1.1%)	with glue χ^2/ndf = 5/3
z^2	fixed 0	0.105 ± 0.037
ϕ_p	$(41.6 \pm 0.5)°$	$(40.7 \pm 0.7)°$
Z_q	0.863 ± 0.024	0.866 ± 0.025
Z_s	0.78 ± 0.05	0.79 ± 0.05
ϕ_V	$(3.17 \pm 0.10)°$	$(3.15 \pm 0.10)°$
$\frac{m_s}{\bar{m}}$	1.24 ± 0.07	1.24 ± 0.07

$$Pulls=\frac{Measure-Fit}{\sigma_{Measure}}$$





In order to complete the excercise we have done the fit using Escribano method. The fit is done directly on the coupling "g" that are derived from PDG06 using the formula:

$$\Gamma(P \to V\gamma) = \frac{g^2}{4\pi}|p_\gamma|^3 \qquad \Gamma(V \to P\gamma) = \frac{1}{3}\frac{g^2}{4\pi}|p_\gamma|^3$$

$$g_{\rho^0\pi^0\gamma} = g_{\rho^+\pi^+\gamma} = \tfrac{1}{3}g \ , \quad g_{\omega\pi\gamma} = g\cos\phi_V \ , \quad g_{\phi\pi\gamma} = g\sin\phi_V \ ,$$

$$g_{K^{*0}K^0\gamma} = -\tfrac{1}{3}g\, z_K\left(1+\frac{\bar{m}}{m_s}\right) \ , \quad g_{K^{*+}K^+\gamma} = \tfrac{1}{3}g\, z_K\left(2-\frac{\bar{m}}{m_s}\right) \ ,$$

$$g_{\rho\eta\gamma} = g\, z_q\, X_\eta \ , \quad g_{\rho\eta'\gamma} = g\, z_q\, X_{\eta'}$$

$$g_{\omega\eta\gamma} = \tfrac{1}{3}g\left(z_q\, X_\eta\cos\phi_V + 2\frac{\bar{m}}{m_s}z_s\, Y_\eta\sin\phi_V\right) \ ,$$

$$g_{\omega\eta'\gamma} = \tfrac{1}{3}g\left(z_q\, X_{\eta'}\cos\phi_V + 2\frac{\bar{m}}{m_s}z_s\, Y_{\eta'}\sin\phi_V\right) \ ,$$

$$g_{\phi\eta\gamma} = \tfrac{1}{3}g\left(z_q\, X_\eta\sin\phi_V - 2\frac{\bar{m}}{m_s}z_s\, Y_\eta\cos\phi_V\right) \ ,$$

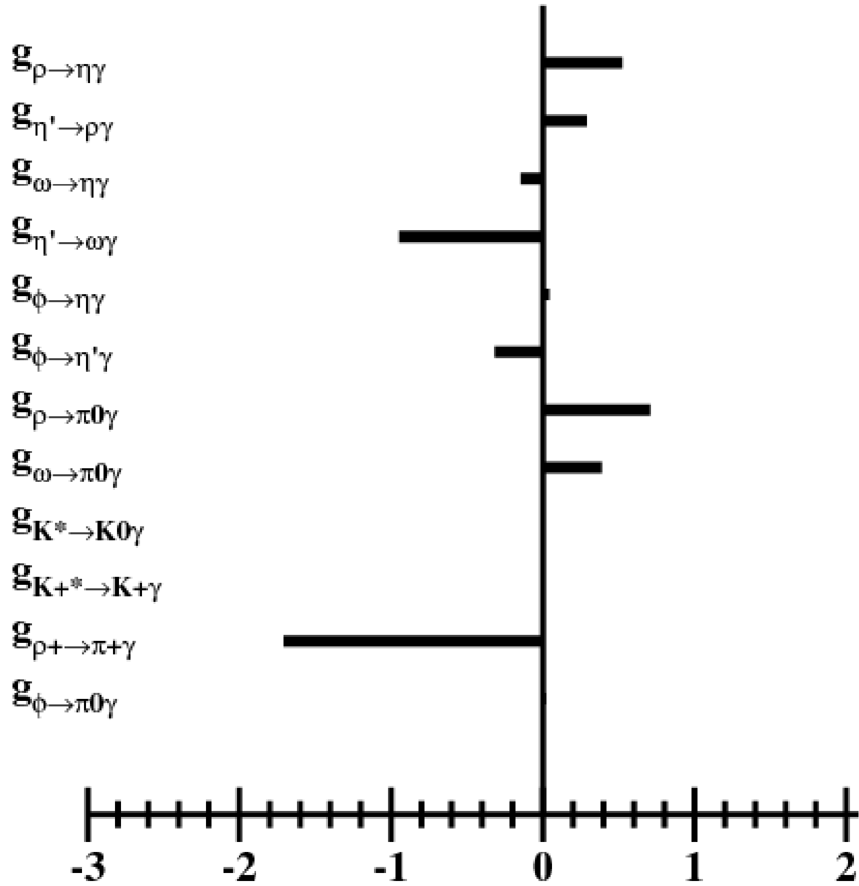
$$g_{\phi\eta'\gamma} = \tfrac{1}{3}g\left(z_q\, X_{\eta'}\sin\phi_V - 2\frac{\bar{m}}{m_s}z_s\, Y_{\eta'}\cos\phi_V\right) \ ,$$

1) No correlation taken into account

2) 12 measured quantities with 8 parameters to fit.

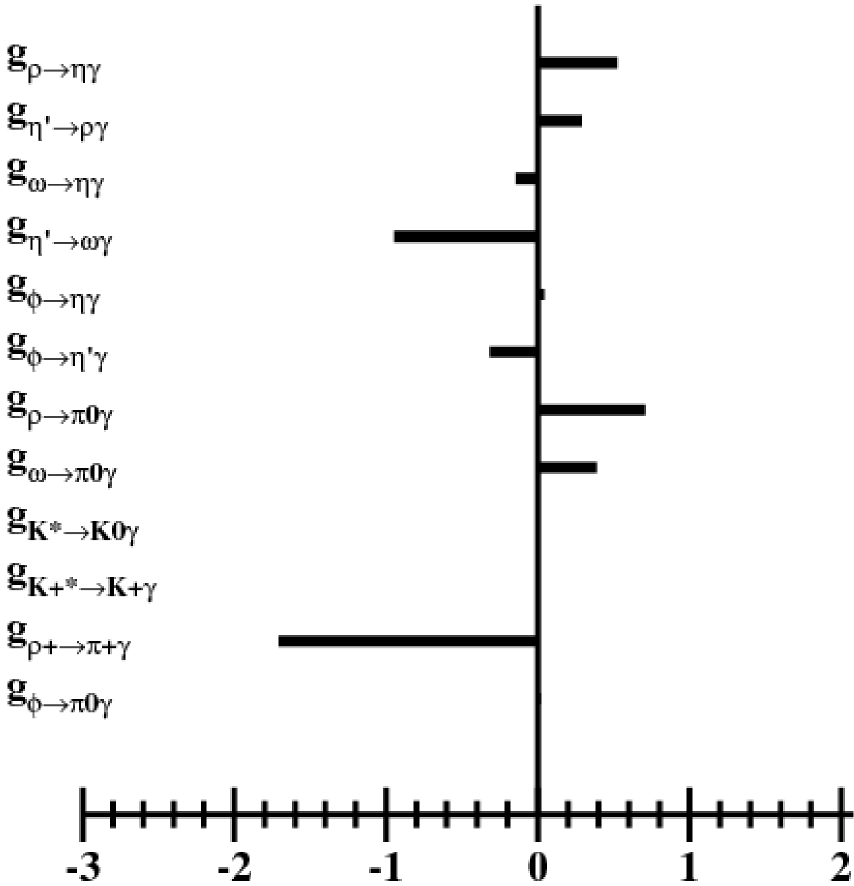
Note: No $P \to \gamma\gamma$ measurement. Fit done without gluonium.

	this fit	escribano fit
	$\chi^2/ndf = 5/5$	$\chi^2/ndf = 4.4/5$
ϕ_G	fixed 0	fixed 0
ϕ_p	$(41.5 \pm 1.0)°$	$(41.5 \pm 1.2)°$
Z_q	0.852 ± 0.024	0.86 ± 0.03
Z_s	0.78 ± 0.05	0.78 ± 0.05
ϕ_V	$(3.18 \pm 0.10)°$	$(3.2 \pm 0.1)°$
$\frac{m_s}{\bar{m}}$	1.24 ± 0.07	1.24 ± 0.07
Z_K	0.89 ± 0.03	0.89 ± 0.03
g	0.719 ± 0.010	0.72 ± 0.01



Note: No P→γγ measurement. Fit done with gluonium.

	this fit	escribano fit
	$\chi^2/ndf = 4.7/4$	$\chi^2/ndf = 4.4/5$
ϕ_G	$(11\pm11)^\circ$	$(12\pm13)^\circ$
ϕ_p	$(41.5 \pm 1.1)^\circ$	$(41.4 \pm 1.3)^\circ$
Z_q	0.86 ± 0.03	0.86 ± 0.03
Z_s	0.78 ± 0.05	0.79 ± 0.05
ϕ_V	$(3.18 \pm 0.10)^\circ$	$(3.2\pm 0.1)^\circ$
$\frac{m_s}{\bar{m}}$	1.24 ± 0.07	1.24 ± 0.07
Z_K	0.89 ± 0.03	0.89 ± 0.03
g	0.719 ± 0.010	0.72 ± 0.01



Note: $P \to \gamma\gamma$ measurement included as:

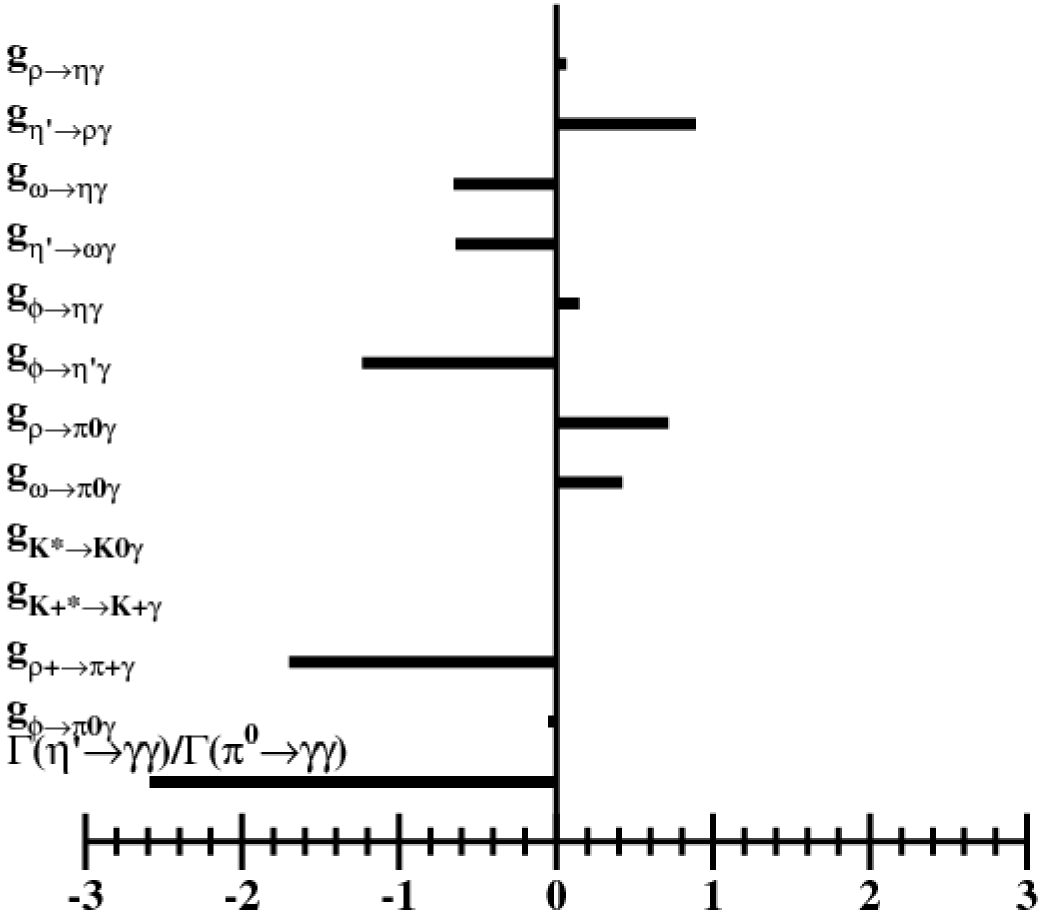
$$\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3 \left(5\sin(\phi_P)\cos(\phi_G) + \sqrt{2}\frac{f_q}{f_s}\cos(\phi_P)\cos(\phi_G)\right)^2$$

	without glue	**with glue**
	$\chi^2/\text{ndf} = 13/5$ (2.3%)	$\chi^2/\text{ndf} = 7.2/4$
ϕ_G	fixed at 0	$(20 \pm 4)^\circ$
ϕ_p	$(40.1 \pm 0.9)^\circ$	$(41.2 \pm 1.1)^\circ$
Z_q	0.85 ± 0.024	0.88 ± 0.03
Z_S	0.80 ± 0.05	0.79 ± 0.05
ϕ_V	$(3.2 \pm 0.1)^\circ$	$(3.18 \pm 0.10)^\circ$
$\frac{m_s}{\bar{m}}$	1.24 ± 0.07	1.24 ± 0.07
Z_K	0.89 ± 0.03	0.89 ± 0.03
g	0.72 ± 0.01	0.719 ± 0.010

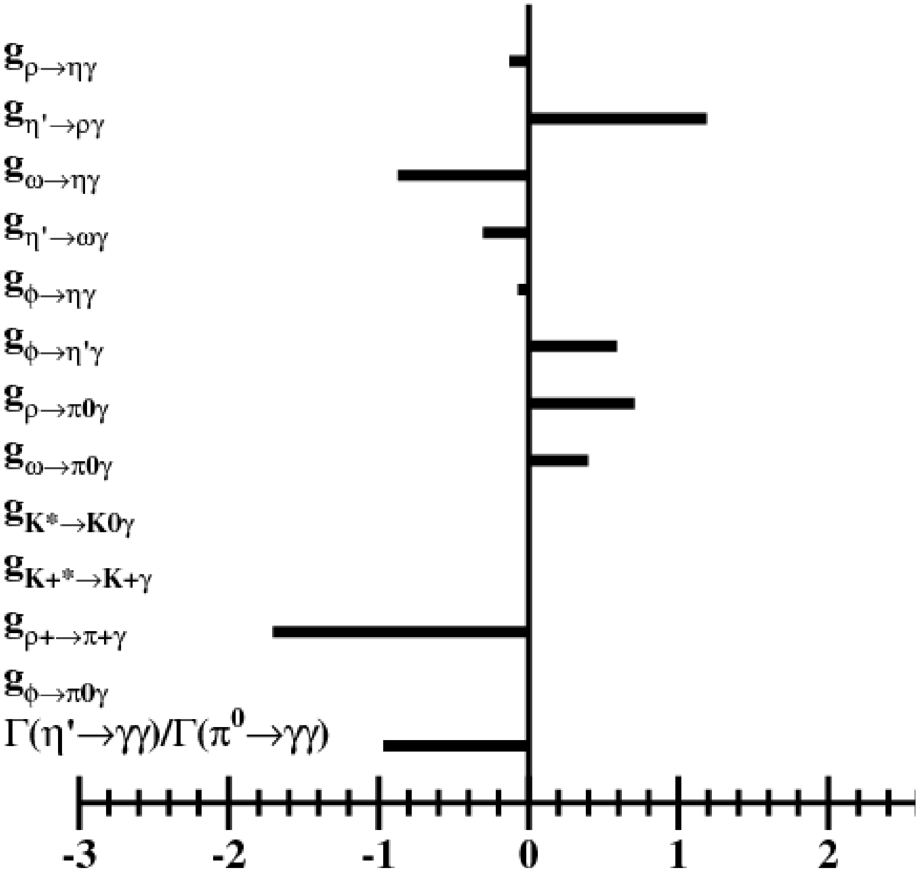
$Z_{\eta'}^2 = 0.11 \pm 0.05$

Pulls comparison

without glue



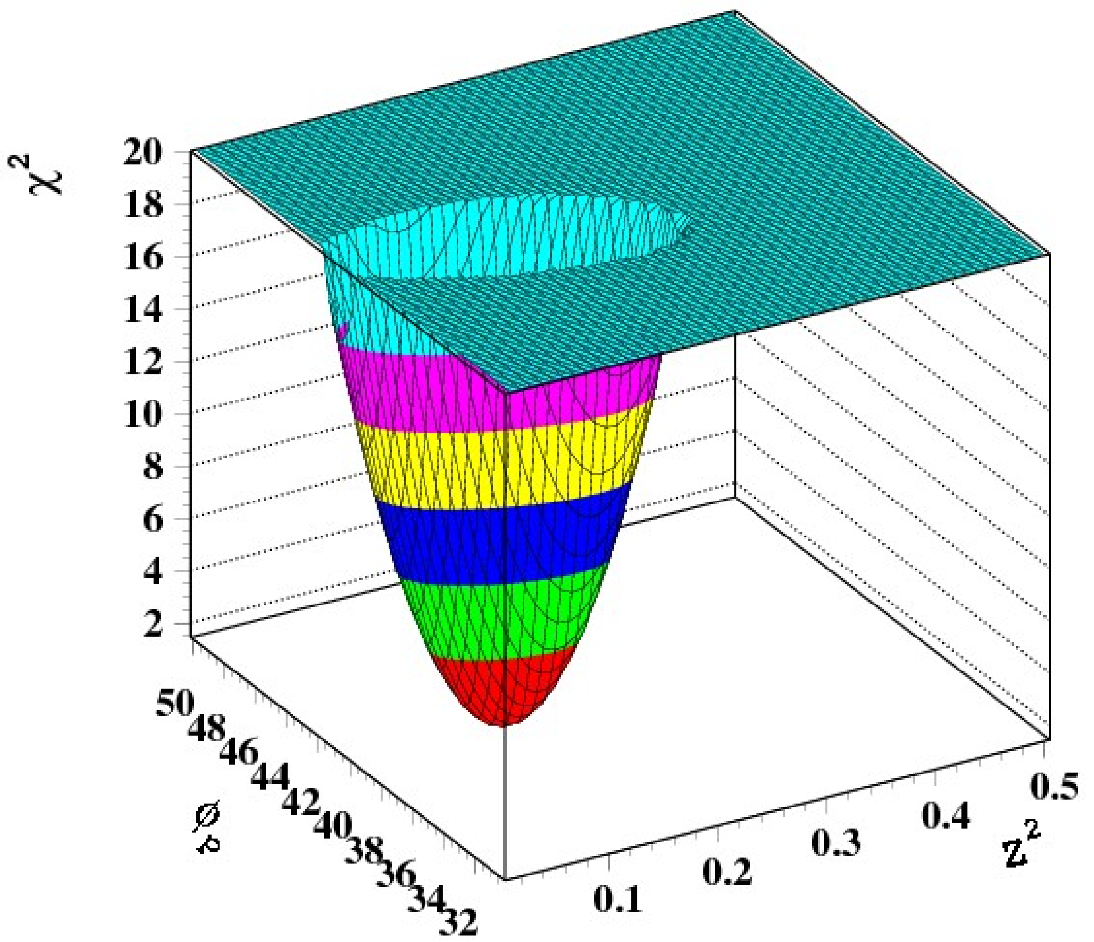
with glue

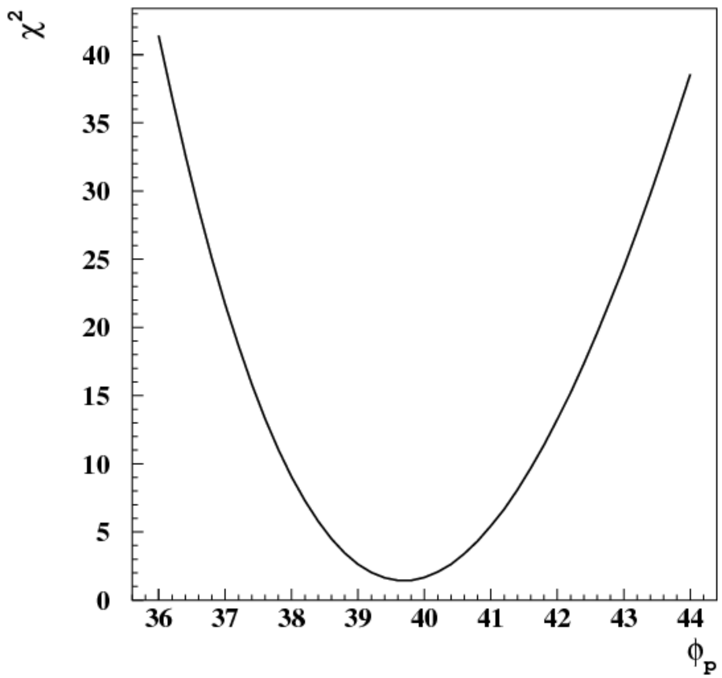


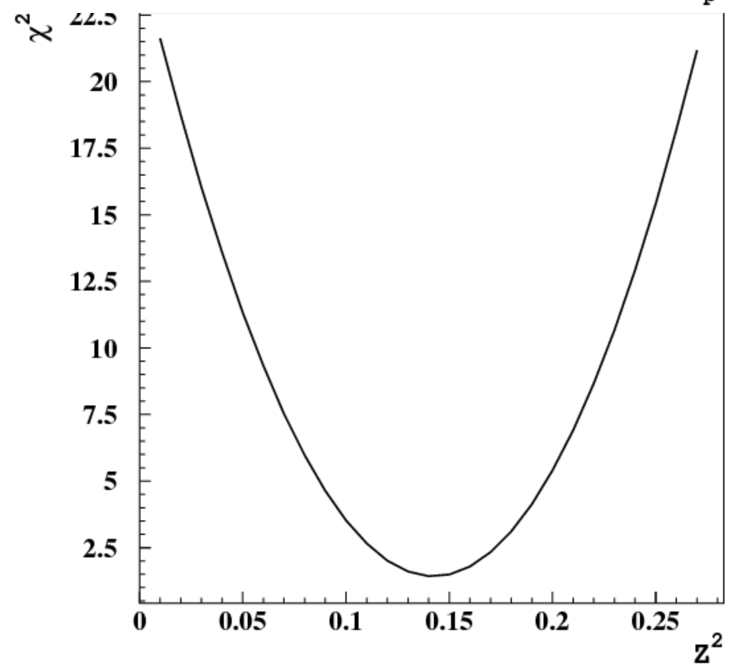
Conclusions

- The difference between our result and Escribano result is mainly due to the $P \to \gamma\gamma$
- Leaving free Z_q and Z_s parameter doesn't have important effects
- The different parametrization of $\Gamma(\eta' \to \rho\gamma)$ is also important, it brings the discrepancy at 2.8 σ ($>$ 3 σ in our paper)

Check of the χ^2 behaviour







Only one minimum in the whole parameters' domain.

Check of the Escribano hypothesis

Fit redone using Escribano fit parameters:

C_{NS} = 0.86 ± 0.03 C_S=0.78± 0.05

	Fit	Paper
z^2	0.12 ± 0.03	0.14 ± 0.04
ϕ_p	$(40.0\pm0.7)^\circ$	$(39.7\pm0.7)^\circ$

The fit is very stable respect to the overlapping parameters

Differences between Escribano and our fit

Our fit **Escribano**

Differences between Escribano and our fit

Our fit

Only ϕ_p and Z^2 are left free

Escribano

All theoretical parameters are left free

Differences between Escribano and our fit

Our fit

Only ϕ_p and Z^2 are left free

The ratios of Γ's are used in the fit.

Escribano

All theoretical parameters are left free

The Γ's are used in the fit.

Differences between Escribano and our fit

Our fit	Escribano
Only ϕ_p and Z^2 are left free	All theoretical parameters are left free
The ratios of Γ's are used in the fit.	The Γ's are used in the fit.
4 measured quantities are used in the fit	11 measured quantities are used in the fit

Differences between Escribano and our fit

Our fit	Escribano
Only ϕ_p and Z^2 are left free	All theoretical parameters are left free
The ratios of Γ's are used in the fit.	The Γ's are used in the fit.
4 measured quantities are used in the fit	11 measured quantities are used in the fit
DATA from PDG '06 + KLOE R_ϕ '07	DATA from PDG '06

Escribano amplitudes

$$g_{\omega\eta\gamma} = \frac{1}{3} g \left(z_q \, X_\eta \cos\phi_V + 2 \frac{\bar{m}}{m_s} z_s \, Y_\eta \sin\phi_V \right) \ ,$$

$$g_{\omega\eta'\gamma} = \frac{1}{3} g \left(z_q \, X_{\eta'} \cos\phi_V + 2 \frac{\bar{m}}{m_s} z_s \, Y_{\eta'} \sin\phi_V \right) \ ,$$

$$g_{\phi\eta\gamma} = \frac{1}{3} g \left(z_q \, X_\eta \sin\phi_V - 2 \frac{\bar{m}}{m_s} z_s \, Y_\eta \cos\phi_V \right) \ ,$$

$$g_{\phi\eta'\gamma} = \frac{1}{3} g \left(z_q \, X_{\eta'} \sin\phi_V - 2 \frac{\bar{m}}{m_s} z_s \, Y_{\eta'} \cos\phi_V \right) \ ,$$

$z_q = C_{NS}$

$z_s = C_s$

Constrain

ϕ_p, Z_G

Escribano amplitudes

$$g_{\omega\eta\gamma} = \frac{1}{3}g\left(z_q\, X_\eta \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s\, Y_\eta \sin\phi_V\right) \ ,$$

$$g_{\omega\eta'\gamma} = \frac{1}{3}g\left(z_q\, X_{\eta'} \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s\, Y_{\eta'} \sin\phi_V\right) \ ,$$

$$g_{\phi\eta\gamma} = \frac{1}{3}g\left(z_q\, X_\eta \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s\, Y_\eta \cos\phi_V\right) \ ,$$

$$g_{\phi\eta'\gamma} = \frac{1}{3}g\left(z_q\, X_{\eta'} \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s\, Y_{\eta'} \cos\phi_V\right) \ ,$$

$Z_q = C_{NS}$
$Z_s = C_s$

Constrain ϕ_p, Z_G

$$g_{\rho^0\pi^0\gamma} = g_{\rho^+\pi^+\gamma} = \frac{1}{3}g \ , \quad g_{\omega\pi\gamma} = g\cos\phi_V \ , \quad g_{\phi\pi\gamma} = g\sin\phi_V \ ,$$

$$g_{K^{*0}K^0\gamma} = -\frac{1}{3}g\, z_K\left(1+\frac{\bar{m}}{m_s}\right) \ , \quad g_{K^{*+}K^+\gamma} = \frac{1}{3}g\, z_K\left(2-\frac{\bar{m}}{m_s}\right) \ ,$$

Fix the parameters $m_s/\bar{m}$, ϕ_V, g

Escribano amplitudes

$$g_{\omega\eta\gamma} = \tfrac{1}{3} g \left(z_q\, X_\eta \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s\, Y_\eta \sin\phi_V \right) \ ,$$

$$g_{\omega\eta'\gamma} = \tfrac{1}{3} g \left(z_q\, X_{\eta'} \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s\, Y_{\eta'} \sin\phi_V \right) \ ,$$

$$g_{\phi\eta\gamma} = \tfrac{1}{3} g \left(z_q\, X_\eta \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s\, Y_\eta \cos\phi_V \right) \ ,$$

$$g_{\phi\eta'\gamma} = \tfrac{1}{3} g \left(z_q\, X_{\eta'} \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s\, Y_{\eta'} \cos\phi_V \right) \ ,$$

$z_q = C_{NS}$

$z_s = C_s$

Constrain
ϕ_p, Z_G

$\tan\phi_V \ll 1$ ($\phi_V = 3.2°$)

$$R_\phi = \frac{Br(\phi\rightarrow\eta'\gamma)}{Br(\phi\rightarrow\eta\gamma)} = \cot^2\phi_P \cdot \cos^2\phi_G \left(1 - \frac{m_s}{\bar{m}}\frac{C_{NS}}{C_S} \cdot \tan\frac{\phi_V}{\sin 2\phi_P}\right)^2 \cdot \left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

Escribano amplitudes

$$g_{\omega\eta\gamma} = \frac{1}{3} g \left(z_q\, X_\eta \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s\, Y_\eta \sin\phi_V \right) \ ,$$

$$g_{\omega\eta'\gamma} = \frac{1}{3} g \left(z_q\, X_{\eta'} \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s\, Y_{\eta'} \sin\phi_V \right) \ ,$$

$$g_{\phi\eta\gamma} = \frac{1}{3} g \left(z_q\, X_\eta \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s\, Y_\eta \cos\phi_V \right) \ ,$$

$$g_{\phi\eta'\gamma} = \frac{1}{3} g \left(z_q\, X_{\eta'} \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s\, Y_{\eta'} \cos\phi_V \right) \ ,$$

$z_q = C_{NS}$

$z_s = C_s$

Constrain ϕ_p, Z_G

$\tan\phi_V << 1$ ($\phi_V = 3.2°$)

$$R_\phi = \frac{Br(\phi \to \eta' \gamma)}{Br(\phi \to \eta \gamma)} = \cot^2\phi_P \cdot \cos^2\phi_G \left(1 - \frac{m_s}{\bar{m}} \frac{C_{NS}}{C_S} \cdot \tan\frac{\phi_V}{\sin 2\phi_P} \right)^2 \cdot \left(\frac{p_{\eta'}}{p_\eta} \right)^3$$

Contains terms up to $\tan^2\phi_V$, but it is not $o(\tan^2\phi_V)$

Escribano amplitudes

$$g_{\omega\eta\gamma} = \frac{1}{3} g \left(z_q X_\eta \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s Y_\eta \sin\phi_V \right) ,$$

$$g_{\omega\eta'\gamma} = \frac{1}{3} g \left(z_q X_{\eta'} \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s Y_{\eta'} \sin\phi_V \right) ,$$

$$g_{\phi\eta\gamma} = \frac{1}{3} g \left(z_q X_\eta \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s Y_\eta \cos\phi_V \right) ,$$

$$g_{\phi\eta'\gamma} = \frac{1}{3} g \left(z_q X_{\eta'} \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s Y_{\eta'} \cos\phi_V \right) ,$$

$z_q = C_{NS}$

$z_s = C_s$

Constrain

ϕ_p, Z_G

tan ϕ_V << 1 (ϕ_V = 3.2°)

$$R_\phi = \frac{Br(\phi\to\eta'\gamma)}{Br(\phi\to\eta\gamma)} = \cot^2\phi_P \cdot \cos^2\phi_G \left(1 - \frac{m_s}{\bar{m}} \frac{C_{NS}}{C_S} \cdot \tan\frac{\phi_V}{\sin 2\phi_P}\right)^2 \cdot \left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

$$+ \left(\frac{C_{NS}}{C_S} \frac{m_S}{\bar{m}} \tan\phi_V\right)^2 \left(1 + 2\cos^2\phi_P\right) \left(\frac{p_{\eta'}}{p_\eta}\right)^3 + o\left(\tan^2\phi_V\right)$$

KLOE fit with full formula

$$R_\phi = \frac{Br(\phi \to \eta' \gamma)}{Br(\phi \to \eta \gamma)} = \cot^2 \phi_P \cdot \cos^2 \phi_G \left(1 - \frac{m_s}{\bar{m}} \frac{C_{NS}}{C_S} \cdot \tan \frac{\phi_V}{\sin 2\phi_P}\right)^2 \cdot \left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

$$+ \left(\frac{C_{NS}}{C_S} \frac{m_S}{\bar{m}} \tan \phi_V\right)^2 \left(1 + 2\cos^2 \phi_P\right)$$

	Fit	Paper
z^2	0.14 ± 0.03	0.14 ± 0.04
ϕ_p	$(39.9 \pm 0.7)^\circ$	$(39.7 \pm 0.7)^\circ$

Freeing the overlapping parameters

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3\left(5X_{\eta'}+\sqrt{2}\frac{f_q}{f_s}Y_{\eta'}\right)^2$$

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{C_{NS}}{\cos\phi_V}\cdot 3\left(\frac{m_{\eta'}^2-m_\rho^2}{m_\omega^2-m_\pi^2}\frac{m_\omega}{m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta'\to\omega\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{1}{3}\left(\frac{m_{\eta'}^2-m_\omega^2}{m_\omega^2-m_\pi^2}\frac{m_\omega}{m_{\eta'}}\right)^3\left[C_{NS}X_{\eta'}+2\frac{m_s}{\bar{m}}C_s\cdot\tan\phi_V\cdot Y_{\eta'}\right]^2$$

Not enough constraint to leave free C_{NS} and C_S

We add to the fit:

$$\frac{\Gamma(\omega\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)},\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)},\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$$

Fit result

```
FCN=   1.406759     FROM MINOS     STATUS=SUCCESSFUL   187 CALLS       277
TOTAL
                 EDM=  0.34E-06    STRATEGY= 1      ERROR MATRIX ACCURATE

 EXT PARAMETER                  PARABOLIC        MINOS ERRORS
 NO.   NAME        VALUE          ERROR       NEGATIVE      POSITIVE
  1     z2        0.11720       0.41464E-01  -0.41484E-01   0.41463E-01
  2     PHIP       40.056       0.97108      -0.94271       1.0031
  3     CNSP       0.86965      0.31043E-01  -0.31096E-01   0.31044E-01
  4     CSPA       0.79071      0.47577E-01  -0.44712E-01   0.50965E-01
```

	Fit with free C_{NS} C_S	Paper	Escribano
z^2	0.12 ± 0.04	0.14 ± 0.04	$(0.04 \pm 0.09)^\circ$
ϕ_p	$(40.1 \pm 1.0)^\circ$	$(39.7 \pm 0.7)^\circ$	$(41.4 \pm 1.3)^\circ$
C_{NS}	0.87 ± 0.03		0.86 ± 0.03
C_S	0.79 ± 0.05		0.78 ± 0.05

Fit result

```
FCN=   1.406759     FROM MINOS     STATUS=SUCCESSFUL   187 CALLS       277
TOTAL
                  EDM=  0.34E-06    STRATEGY= 1      ERROR MATRIX ACCURATE

 EXT PARAMETER                      PARABOLIC         MINOS ERRORS
 NO.   NAME       VALUE           ERROR       NEGATIVE       POSITIVE
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  2    PHIP        40.056       0.97108      -0.94271        1.0031
  3    CNSP        0.86965      0.31043E-01  -0.31096E-01   0.31044E-01
  4    CSPA       0.79071       0.47577E-01  -0.44712E-01   0.50965E-01
```

	Fit with free C_{NS} C_S	Paper	Escribano
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ϕ_p	$(40.1 \pm 1.0)^\circ$	$(39.7 \pm 0.7)^\circ$	$(41.4 \pm 1.3)^\circ$
C_{NS}	0.87 ± 0.03		0.86 ± 0.03
C_S	0.79 ± 0.05		0.78 ± 0.05

←→ **Perfect agreement**

Fit results

Gluonium still at 3σ

Fit with free C_{NS} C_S		Paper	Escribano
Z^2	0.12 ± 0.04	0.14 ± 0.04	$(0.04 \pm 0.09)^\circ$
ϕ_p	$(40.1 \pm 1.0)^\circ$	$(39.7 \pm 0.7)^\circ$	$(41.4 \pm 1.3)^\circ$
C_{NS}	0.87 ± 0.03		0.86 ± 0.03
C_S	0.79 ± 0.05		0.78 ± 0.05

Perfect agreement

Fit results

TO BE CONTINUED

C[illegible]ll at 3σ

	Fit wit[illegible] S	Paper	Escribano
Z^2	[illegible].12±0.04	0.14±0.04	$(0.04 \pm 0.09)^\circ$
ϕ_p	$(40.1 \pm 1.0)^\circ$	$(39.7 \pm 0.7)^\circ$	$(41.4 \pm 1.3)^\circ$
C_{NS}	0.87±0.03		0.86±0.03
C_S	0.79 ± 0.05		0.78±0.05

Perfect agreement

The $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ constraint

Removing this constraint we obtain:

Fit with free C_{NS} C_S no $P\to\gamma\gamma$ constraint		Fit with free C_{NS} C_S		Escribano
z^2	0.09 ± 0.06	z^2	0.12 ± 0.04	$(0.04 \pm 0.09)^\circ$
ϕ_p	$(40.2 \pm 1.0)^\circ$	ϕ_p	$(40.1 \pm 1.0)^\circ$	$(41.4 \pm 1.3)^\circ$
C_{NS}	0.86 ± 0.03	C_{NS}	0.87 ± 0.03	0.86 ± 0.03
C_S	0.79 ± 0.05	C_S	0.79 ± 0.05	0.78 ± 0.05

The $\eta' \to \gamma\gamma / \pi^0 \to \gamma\gamma$ constraint

Removing this constraint we obtain:

	Fit with free C_{NS} C_S no $P \to \gamma\gamma$ constraint	Fit with free C_{NS} C_S	Escribano
Z^2	0.09 ± 0.06	0.12 ± 0.04	$(0.04 \pm 0.09)^\circ$
ϕ_P	$(40.2 \pm 1.0)^\circ$	$(40.1 \pm 1.0)^\circ$	$(41.4 \pm 1.3)^\circ$
C_{NS}	0.86 ± 0.03	0.87 ± 0.03	0.86 ± 0.03
C_S	0.79 ± 0.05	0.79 ± 0.05	0.78 ± 0.05

The $P \to \gamma\gamma$ constraint is important!! It moves the central value and reduce the error (Here someone is cheating..)

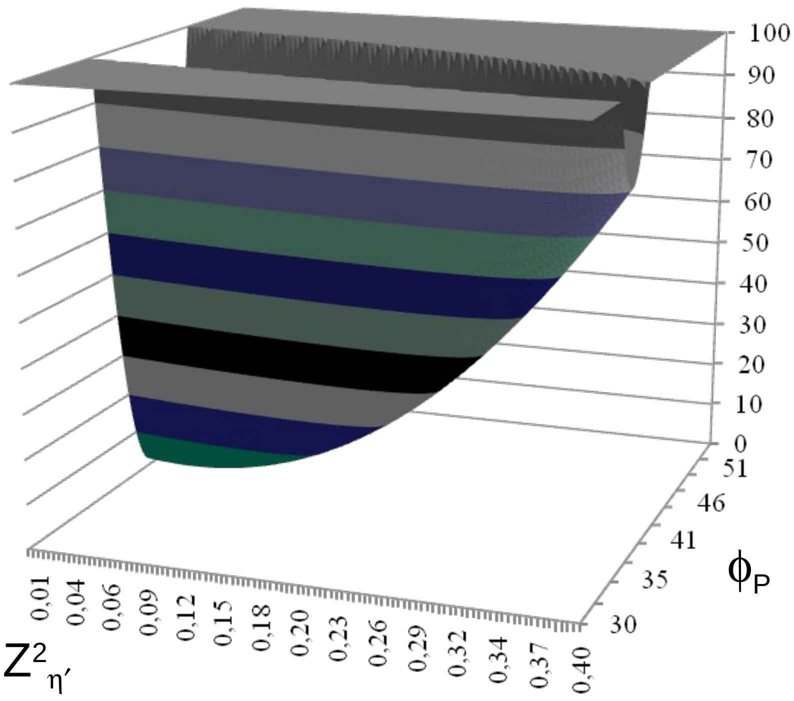
Fitting the Width (using KLOE and last SND results)

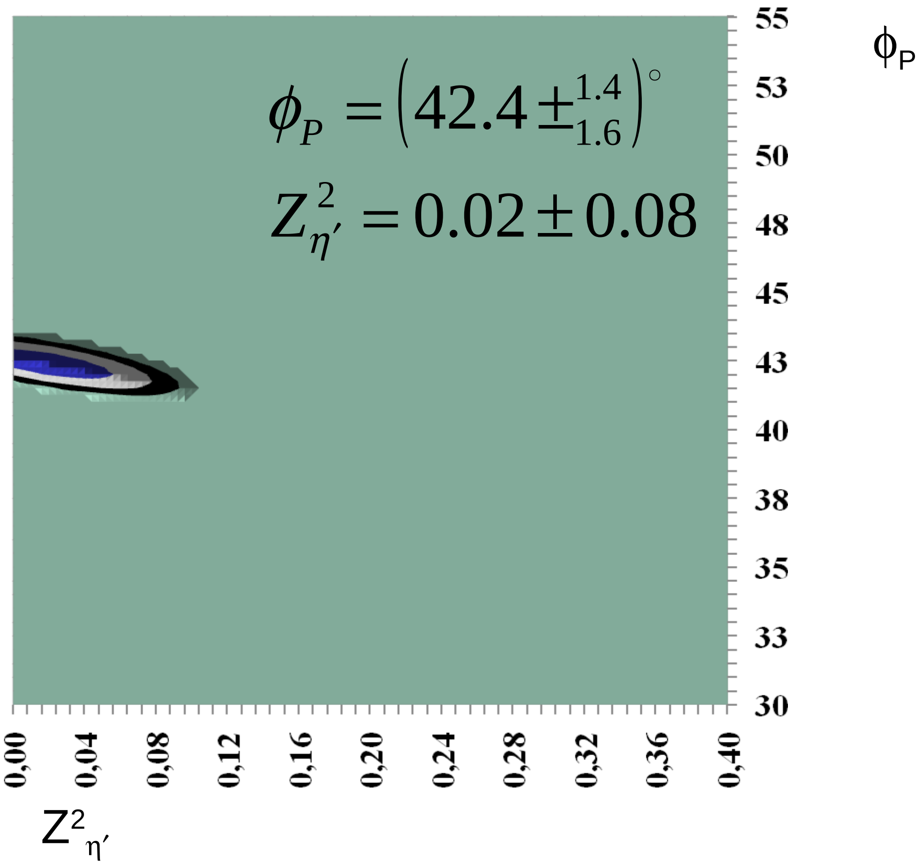
Slide from Camilla

We fit as Escribano - constraints from partial width
with our method - only $\cos\phi_P$, $\cos\phi_G$ left free
We find the following results to compare with Rafel's ones

Escribano fit

$$\left(\phi_P, Z^2_{\eta'}\right) = \left(42.6^\circ, 0.01\right)$$





Conclusions and outlook

- All the objections to our paper have been rejected by the check performed;
- To complete the study we have to implement 4 further constraints and fit with all free parameters;

- From the preliminary study we can say:
  - The gluonium is at 3σ whatever we use for the overlapping parameters or include them in the fit;
  - The $P\rightarrow\gamma\gamma$ is proved to be an important constraint: increases the gluonium component and reduces the error by 33%
  - The fit to the Γ's looks promising
- We would like to write a short answer to Escribano and Thomas at the end of the work.