Global fit to the η' gluonium content, $m_s/<m>$ and ϕ_V parameters.

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The master equations

The glue ball weight

$$|\eta'\rangle = X_{\eta'}|q\bar{q}\rangle + Y_{\eta'}|s\bar{s}\rangle + Z_{\eta'}|G\rangle$$

$$|\eta\rangle = \cos\phi_P|q\bar{q}\rangle - \sin\phi_P|s\bar{s}\rangle$$

$$|q\bar{q}\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle}{\sqrt{2}}$$

$$X_{\eta'} = \sin\phi_P \cos\phi_G$$

$$Y_{\eta'} = \cos\phi_P \cos\phi_G$$

$$Z_{\eta'} = \sin\phi_G$$

In our paper we use our measurement of

$$R_\phi = \frac{Br(\phi\to\eta'\gamma)}{Br(\phi\to\eta\gamma)} = \cot^2\phi_P \cdot \cos^2\phi_G \left(1 - \frac{m_s}{\bar{m}}\frac{C_{NS}}{C_S}\cdot\tan\frac{\phi_V}{\sin 2\phi_P}\right)^2 \cdot \left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

Together with

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)} = \frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3 \left(5X_{\eta'} + \sqrt{2}\frac{f_q}{f_s}Y_{\eta'}\right)^2$$

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)} = \frac{C_{NS}}{\cos\phi_V}\cdot 3\left(\frac{m_{\eta'}^2 - m_\rho^2\, m_\omega}{m_\omega^2 - m_\pi^2\, m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta'\to\omega\gamma)}{\Gamma(\omega\to\pi^0\gamma)} = \frac{1}{3}\left(\frac{m_{\eta'}^2 - m_\omega^2\, m_\omega}{m_\omega^2 - m_\pi^2\, m_{\eta'}}\right)^3 \left[C_{NS}X_{\eta'} + 2\frac{m_s}{\bar{m}}C_s\cdot\tan\phi_V\cdot Y_{\eta'}\right]^2$$

In order to fit the gluonium we use our measurement of

$$R_\phi = \frac{Br(\phi \to \eta' \gamma)}{Br(\phi \to \eta \gamma)} = \cot^2 \phi_P \cdot \cos^2 \phi_G \left(1 - \frac{m_s}{\overline{m}} \frac{C_{NS}}{C_S} \cdot \tan \frac{\phi_V}{\sin 2\phi_P}\right)^2 \cdot \left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

Together with

$$\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3 \left(5 X_{\eta'} + \sqrt{2}\frac{f_q}{f_s} Y_{\eta'}\right)^2$$

$$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{C_{NS}}{\cos\phi_V} \cdot 3 \left(\frac{m_{\eta'}^2 - m_\rho^2}{m_\omega^2 - m_\pi^2} \frac{m_\omega}{m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{3}\left(\frac{m_{\eta'}^2 - m_\omega^2}{m_\omega^2 - m_\pi^2} \frac{m_\omega}{m_{\eta'}}\right)^3 \left[C_{NS} X_{\eta'} + 2\frac{m_s}{\overline{m}} C_s \cdot \tan\phi_V \cdot Y_{\eta'}\right]^2$$

These parameters multiply the gluonium component

We took them from a fit to the same quantities + further constraints without assuming gluonium content

Differences between Escribano and our fit

Our fit	Escribano
Only ϕ_p and Z^2 are left free	All theoretical parameters are left free
The ratios of Γ's are used in the fit.	The Γ's are used in the fit.
4 measured quantities are used in the fit	11 measured quantities are used in the fit
DATA from PDG '06 + KLOE R_ϕ '07	DATA from PDG '06

Escribano amplitudes

$$g_{\omega\eta\gamma} = \frac{1}{3} g \left(z_q \, X_\eta \cos\phi_V + 2 \frac{\bar{m}}{m_s} z_s \, Y_\eta \sin\phi_V \right) \ ,$$

$$g_{\omega\eta'\gamma} = \frac{1}{3} g \left(z_q \, X_{\eta'} \cos\phi_V + 2 \frac{\bar{m}}{m_s} z_s \, Y_{\eta'} \sin\phi_V \right) \ ,$$

$$g_{\phi\eta\gamma} = \frac{1}{3} g \left(z_q \, X_\eta \sin\phi_V - 2 \frac{\bar{m}}{m_s} z_s \, Y_\eta \cos\phi_V \right) \ ,$$

$$g_{\phi\eta'\gamma} = \frac{1}{3} g \left(z_q \, X_{\eta'} \sin\phi_V - 2 \frac{\bar{m}}{m_s} z_s \, Y_{\eta'} \cos\phi_V \right) \ ,$$

$Z_q = C_{NS}$

$Z_s = C_s$

Constrain

ϕ_p, Z_G

Escribano amplitudes

$$g_{\omega\eta\gamma} = \frac{1}{3}g\left(z_q\, X_\eta \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s\, Y_\eta \sin\phi_V\right) \;,$$

$$g_{\omega\eta'\gamma} = \frac{1}{3}g\left(z_q\, X_{\eta'} \cos\phi_V + 2\frac{\bar{m}}{m_s} z_s\, Y_{\eta'} \sin\phi_V\right) \;,$$

$$g_{\phi\eta\gamma} = \frac{1}{3}g\left(z_q\, X_\eta \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s\, Y_\eta \cos\phi_V\right) \;,$$

$$g_{\phi\eta'\gamma} = \frac{1}{3}g\left(z_q\, X_{\eta'} \sin\phi_V - 2\frac{\bar{m}}{m_s} z_s\, Y_{\eta'} \cos\phi_V\right) \;,$$

$Z_q=C_{NS}$
$Z_s=C_s$

Constrain ϕ_p,Z_G

$$g_{\rho^0\pi^0\gamma} = g_{\rho^+\pi^+\gamma} = \frac{1}{3}g \;, \quad g_{\omega\pi\gamma} = g\cos\phi_V \;, \quad g_{\phi\pi\gamma} = g\sin\phi_V \;,$$

$$g_{K^{*0}K^0\gamma} = -\frac{1}{3}g\, z_K\left(1 + \frac{\bar{m}}{m_s}\right) \;, \quad g_{K^{*+}K^+\gamma} = \frac{1}{3}g\, z_K\left(2 - \frac{\bar{m}}{m_s}\right) \;,$$

Fix the parameters $m_s/\bar{m}$, ϕ_V,g
Not included in GM fit- now we fit also them

KLOE fit with full formula

$$R_\phi = \frac{Br(\phi \to \eta' \gamma)}{Br(\phi \to \eta \gamma)} = \cot^2 \phi_P \cdot \cos^2 \phi_G \left(1 - \frac{m_s}{\overline{m}} \frac{C_{NS}}{C_S} \cdot \tan \frac{\phi_V}{\sin 2\phi_P}\right)^2 \cdot \left(\frac{p_{\eta'}}{p_\eta}\right)^3 + \left(\frac{C_{NS}}{C_S} \frac{m_S}{\overline{m}} \tan \phi_V\right)^2 \left(1 + 2\cos^2 \phi_P\right)$$

Directly from Escribano amplitudes up to $(\tan\phi_v)^2$

KLOE PAPER CONSTRAINTS

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3\left(5X_{\eta'}+\sqrt{2}\frac{f_q}{f_s}Y_{\eta'}\right)^2$$

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{C_{NS}}{\cos\phi_V}\cdot 3\left(\frac{m_{\eta'}^2-m_\rho^2}{m_\omega^2-m_\pi^2}\frac{m_\omega}{m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta'\to\omega\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{1}{3}\left(\frac{m_{\eta'}^2-m_\omega^2}{m_\omega^2-m_\pi^2}\frac{m_\omega}{m_{\eta'}}\right)^3\left[C_{NS}X_{\eta'}+2\frac{m_s}{\bar{m}}C_s\cdot\tan\phi_V\cdot Y_{\eta'}\right]^2$$

Not enough constraint to leave free C_{NS} and C_S

KLOE PAPER CONSTRAINTS

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{m_{\eta'}}{m_\pi}\right)^3\left(5X_{\eta'}+\sqrt{2}\frac{f_q}{f_s}Y_{\eta'}\right)^2$$

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Not enough constraint to leave free C_{NS} and C_S

We added to the fit (GM status):

$$\frac{\Gamma(\omega\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)},\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)},\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$$

Fit also C_{NS} and C_S

For this presentation we add also

$$\frac{\Gamma(\phi\to\pi^0\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\tan^2\phi_V\cdot\left(\frac{m_\phi^2-m_{\pi^0}^2}{m_\omega^2-m_{\pi^0}^2}\cdot\frac{m_\omega}{m_\phi}\right)^3,\frac{\Gamma(K^{+*}\to K^+\gamma)}{\Gamma(K^{*0}\to K^0\gamma)}=\left(\frac{2\frac{m_s}{\bar{m}}-1}{1+\frac{m_s}{\bar{m}}}\right)^2\cdot\left(\frac{m_{K^{*+}}^2-m_{K^{+0}}^2}{m_{K^{*0}}^2-m_{K^0}^2}\cdot\frac{m_{K^{*0}}}{m_{K^{*+}}}\right)^3$$

Fix these parameters

The Method

The χ^2 is defined as follows:

$$\chi^2 = \Sigma_{i,j=1,3} \left(y_i - y_i^{th}\right) \times V_{ij}^{-1} \left(y_j - y_j^{th}\right)$$

V_{ij} is the error matrix which is a function of theoretical uncertainties, as well as the experim

$$V_{ij} = [B_{ij} + (A_{ik} \times C_{kl} \times A_{lj}^T)]$$

Experimental covariance matrix

Theoretical parameters covariance matrix

B_{ij} Full covariance matrix (correlation comes from the constrained fit to η' Br)

$$; A_{ik} = \begin{pmatrix} \frac{\partial y_1^{th}}{\partial f_s} & \frac{\partial y_1^{th}}{\partial f_q} & \frac{\partial y_1^{th}}{\partial C_{NS}} & \frac{\partial y_1^{th}}{\partial C_S} & \frac{\partial y_1^{th}}{\partial \frac{m_s}{\bar{m}}} \\ \frac{\partial y_2^{th}}{\partial f_s} & \frac{\partial y_2^{th}}{\partial f_q} & \frac{\partial y_2^{th}}{\partial C_{NS}} & \frac{\partial y_2^{th}}{\partial C_S} & \frac{\partial y_2^{th}}{\partial \frac{m_s}{\bar{m}}} \\ \frac{\partial y_3^{th}}{\partial f_s} & \frac{\partial y_3^{th}}{\partial f_q} & \frac{\partial y_3^{th}}{\partial C_{NS}} & \frac{\partial y_3^{th}}{\partial C_S} & \frac{\partial y_3^{th}}{\partial \frac{m_s}{\bar{m}}} \end{pmatrix}$$

Re-evaluated at each minimization step

$$C_{kl} = \begin{pmatrix} \sigma_{f_s}^2 & 0 & 0 & 0 & 0 \\ & \sigma_{f_q}^2 & 0 & 0 & 0 \\ & & \sigma_{C_{NS}}^2 & 0 & 0 \\ & & & \sigma_{C_S}^2 & 0 \\ & & & & \sigma_{\frac{m_s}{\bar{m}}}^2 \end{pmatrix}$$

The Method

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$$\chi^2 = \Sigma_{i,j=1,3} \left(y_i - y_i^{th}\right) \times V_{ij}^{-1} \left(y_j - y_j^{th}\right)$$

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Re-evaluated at each minimization step

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Fitted parameter in this fit.

The experimental covariance matrix B contains correlation among common used quantities in the fitted relations:

$$\frac{\Gamma(\omega\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)},\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)},\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$$

→ Introduces a correlation in the fitted quantities

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{Br(\eta'\to\gamma\gamma)\Gamma_{\eta'}}{\Gamma(\pi^0\to\gamma\gamma)}$$

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{Br(\eta'\to\rho\gamma)\Gamma_{\eta'}}{\Gamma(\omega\to\pi^0\gamma)}$$

	x_1	x_2	x_3	x_4	x_5	x_6
x_2	−34					
x_3	−78	−29				
x_4	−35	−24	32			
x_5	−26	−12	26	8		
x_6	−28	−11	35	11	9	
Γ	32	−2	−24	−5	−88	−8

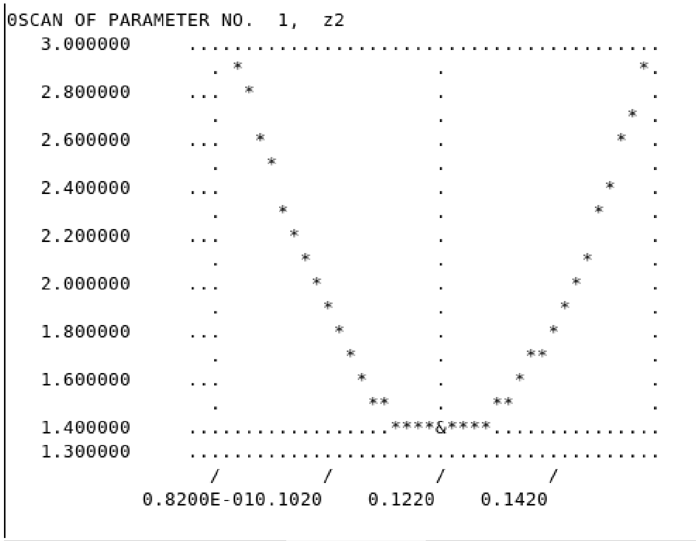
	Mode	Rate (MeV)	Scale factor
Γ_1	$\pi^+\pi^-\eta$	0.090 ±0.008	1.2
Γ_2	$\rho^0\gamma$ (including non-resonant $\pi^+\pi^-\gamma$)	0.060 ±0.005	1.2
Γ_3	$\pi^0\pi^0\eta$	0.042 ±0.004	1.6
Γ_4	$\omega\gamma$	0.0062 ±0.0008	1.2
Γ_5	$\gamma\gamma$	0.00430±0.00015	1.1
Γ_6	$3\pi^0$	(3.2 ±0.6) $\times 10^{-4}$	1.1

Br and Γ strongly correlated (above all $\Gamma(\eta'\to\gamma\gamma)$)

the Γ is measured using:

$e^+e^-\to\eta' e^+e^-$

Fit quality



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2.600000
2.400000
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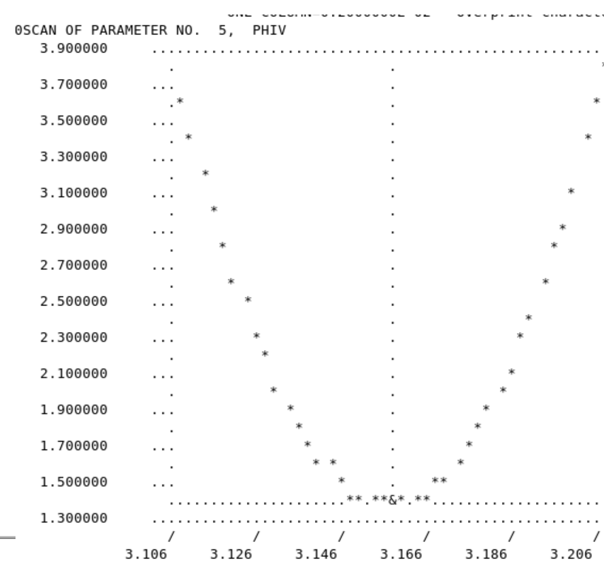

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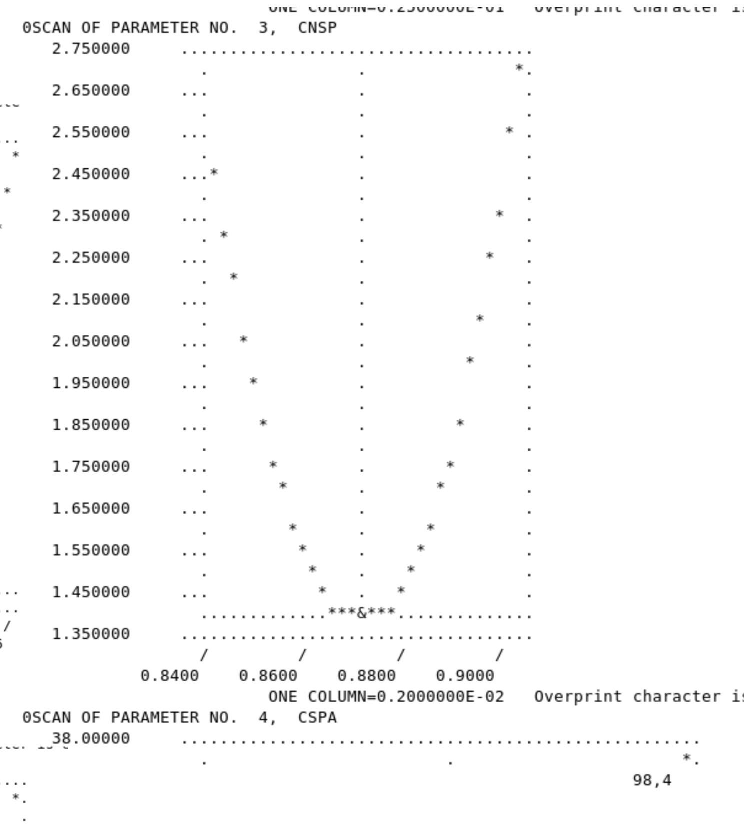

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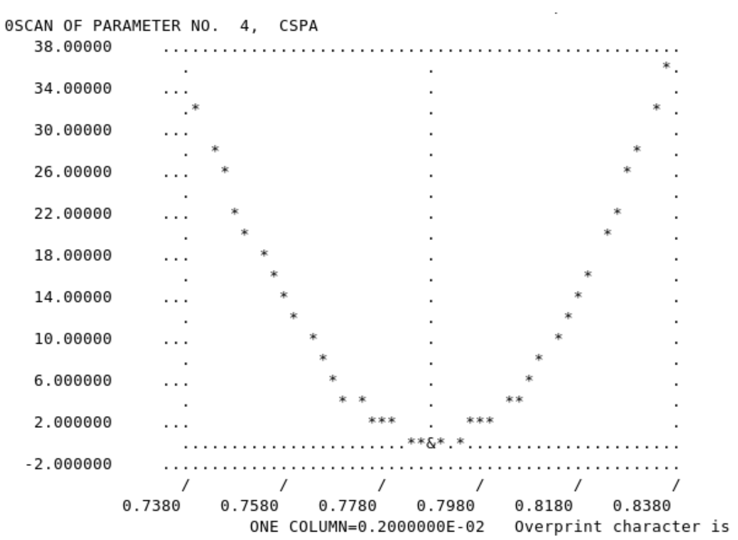

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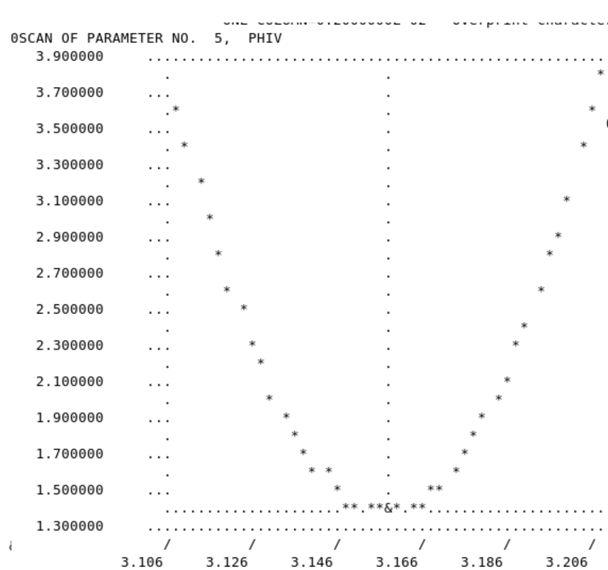

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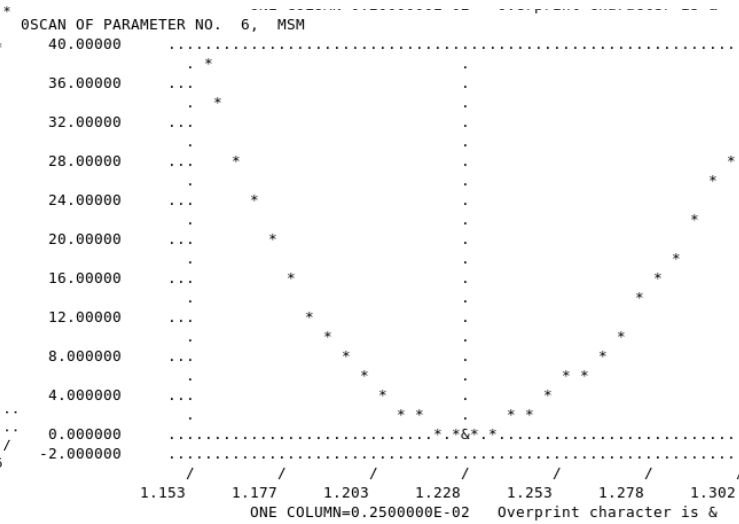

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Fit result

Fit 9 points with 6 parameters $\chi^2 = 1.41/3$ $P(\chi^2) = 70\%$

	This fit	Paper	
Z^2	0.120 ± 0.035	0.14 ± 0.04	
ϕ_p	$(40.2 \pm 0.6)^\circ$	$(39.7 \pm 0.7)^\circ$	
C_{NS}	0.87 ± 0.03	0.91 ± 0.05	Fixed parameter of the fit.
C_S	0.79 ± 0.05	0.89 ± 0.07	
ϕ_V	$(3.16 \pm 0.05)^\circ$	3.2°	
$\frac{m_s}{\bar{m}}$	1.23 ± 0.07	1.24 ± 0.07	

Fit result – imposing $Z^2_G = 0$

Fit 9 points with 5 parameters $\chi^2 = 12.9/4$ $P(\chi^2) = 1\%$

	This fit	**Paper**	
ϕ_p	$(41.3 \pm 0.5)^\circ$	41.4 ± 1.0	
C_{NS}	0.85 ± 0.03	0.91 ± 0.05	Fixed parameter of the fit.
C_S	0.78 ± 0.05	0.89 ± 0.07	Fixed parameter of the fit.
ϕ_V	$(3.18 \pm 0.05)^\circ$	3.2°	Fixed parameter of the fit.
$\frac{m_s}{\bar{m}}$	1.23 ± 0.07	1.24 ± 0.07	Fixed parameter of the fit.

Fit result- no $\eta' \to \gamma\gamma$

Fit 8 points with 6 parameters $\chi^2 = 0.94/2$ P(χ^2) = 70%

	This fit	Escribano
z^2	0.10 ± 0.05	0.04 ± 0.09
ϕ_p	$(40.5 \pm 0.8)^\circ$	$(41.4 \pm 1.3)^\circ$
C_{NS}	0.86 ± 0.03	0.86 ± 0.03
C_S	0.78 ± 0.05	0.79 ± 0.05
ϕ_V	$(3.16 \pm 0.05)^\circ$	$(3.2 \pm 0.1)^\circ$
$\frac{m_s}{\bar{m}}$	1.23 ± 0.07	1.24 ± 0.07

Something strange in Escribano fit

Transition	$g_{VP\gamma}^{\rm exp}$ (PDG)	$g_{VP\gamma}^{\rm th}$ (Fit 1)	$g_{VP\gamma}^{\rm th}$ (Fit 2)
$\rho^0 \to \eta\gamma$	0.475 ± 0.024	0.461 ± 0.019	0.464 ± 0.030
$\eta' \to \rho^0\gamma$	0.41 ± 0.03	0.41 ± 0.02	0.40 ± 0.04
$\omega \to \eta\gamma$	0.140 ± 0.007	0.142 ± 0.007	0.143 ± 0.010
$\eta' \to \omega\gamma$	0.139 ± 0.015	0.149 ± 0.006	0.146 ± 0.014
$\phi \to \eta\gamma$	0.209 ± 0.002	0.209 ± 0.018	0.209 ± 0.013
$\phi \to \eta'\gamma$	0.22 ± 0.01	0.22 ± 0.02	0.22 ± 0.02

The extrapolated error is 6 times the input from DATA

Fit increasing by a factor 12 the error on R_ϕ

$\Gamma \sim g^2$

	This fit	Escribano
z^2	0.01 ± 0.12	0.04 ± 0.09
ϕ_p	$(39.0 \pm 2)^\circ$	$(41.4 \pm 1.3)^\circ$
C_{NS}	0.83 ± 0.04	0.86 ± 0.03
C_S	0.81 ± 0.05	0.79 ± 0.05
ϕ_V	$(3.17 \pm 0.05)^\circ$	$(3.2 \pm 0.1)^\circ$
$\frac{m_s}{\bar{m}}$	1.23 ± 0.07	1.24 ± 0.07

Conclusions

1) Write to Escribano asking for explanations
(exactly know how the fit is done)

2) Write a draft