Fit of the pion form factor

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Φ Decays July 18th 2005

(18 luglio: San Federico di Utrecht, Vescovo)

- analytical functions: Kühn-Santamaria vs. Gounaris-Sakurai
- results on fitting the 2 functions
- 2 slides on recent comments on the KLOE measurement by SND Coll. and M. Davier
- outlook and perspectives

$$\sigma_{e^+e^-\to\pi^+\pi^-}(s) = \frac{\pi}{3}\,\frac{\alpha_{em}^2\beta_\pi^3}{s}\,|F_\pi(s)|^2$$

$$\beta_\pi = \sqrt{1-\frac{4\,m_\pi^2}{s}}$$

Gounaris-Sakurai

"Gounaris-Sakurai" means the same function used by CMD-2 (ALEPH used a slightly modified version):

$$F_\pi(s) = \frac{BW_\rho^{GS}(s)\left(1+\delta\frac{s}{M_\omega^2}BW_\omega(s)\right)+\beta BW_{\rho'}^{GS}(s)}{1+\beta}$$

for ρ(770) and ρ'(1450) they use the prescription given by G.J. Gounaris, J.J. Sakurai PRL21 (1968) 244

$$BW^{GS}(s) = \frac{M_V^2\left(1+d_0\frac{\Gamma_V}{M_V}\right)}{M_V^2 - s + f(s) - iM_V\Gamma_V(s)}$$

$$\Gamma_\rho(s) = \Gamma_\rho\left(\frac{p_\pi(s)}{p_\pi(M_\rho^2)}\right)^3\sqrt{\frac{M_\rho^2}{s}}$$

d_0 = constant chosen for $BW^{GS}(0)=1$

$f(s)$ = function required by analytical behaviour at $s=4m_\pi^2$ and at $s=0$

$\delta = |\delta|\, e^{i\arg\delta}$, complex interf. term

$$BW_\omega(s) = \frac{M_\omega^2}{M_\omega^2 - s - iM_\omega\Gamma_\omega}$$

Kühn-Santamaria

"Kühn-Santamaria" means the following function, used by C. Bini

$$F_\pi(s) = \frac{BW_\rho^{KS}(s)\frac{1+\alpha BW_\omega(s)}{1+\alpha} + \beta BW_{\rho'}^{KS}(s)}{1+\beta}$$

for ρ(770) and ρ'(1450) they use the prescription given by J.H. Kühn, A. Santamaria ZPC48 (1990) 445

$$BW^{KS}(s) = \frac{M_V^2}{M_V^2 - s - i\sqrt{s}\,\Gamma_V(s)}$$

$$\Gamma_\rho(s) = \Gamma_\rho\left(\frac{p_\pi(s)}{p_\pi(M_\rho^2)}\right)^3 \frac{M_\rho^2}{s}$$

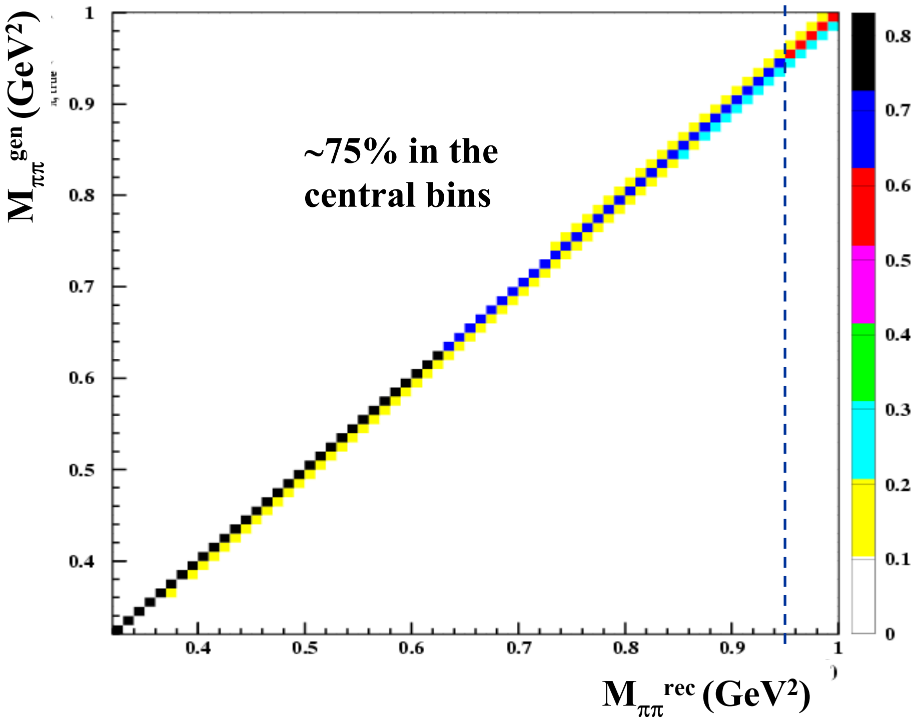
no phase (α is real) btw ρ and ω

contrary to the G.-S., <u>it has not the correct analytical behaviour for low s values</u> H. Leutwyler

$$BW_\omega(s) = \frac{M_\omega^2}{M_\omega^2 - s - iM_\omega\Gamma_\omega}$$

Definition of the χ^2

$$N_i^{exp} = \left(1 - f_i^{bkgr}\right) N_i^{obs} \qquad N_i^{th}(\vec{\theta}) = L\,\varepsilon_i^{mtrk}\varepsilon_i^{filfo} \sum_j S_{ij}\varepsilon_j^{fsr}\varepsilon_j^{geom}\varepsilon_j^{trig}\varepsilon_j^{vtx}\varepsilon_j^{trck}\,|F_{\pi,i}(\vec{\theta})|^2$$



$\underline{\theta}$ is the vector of free parameters

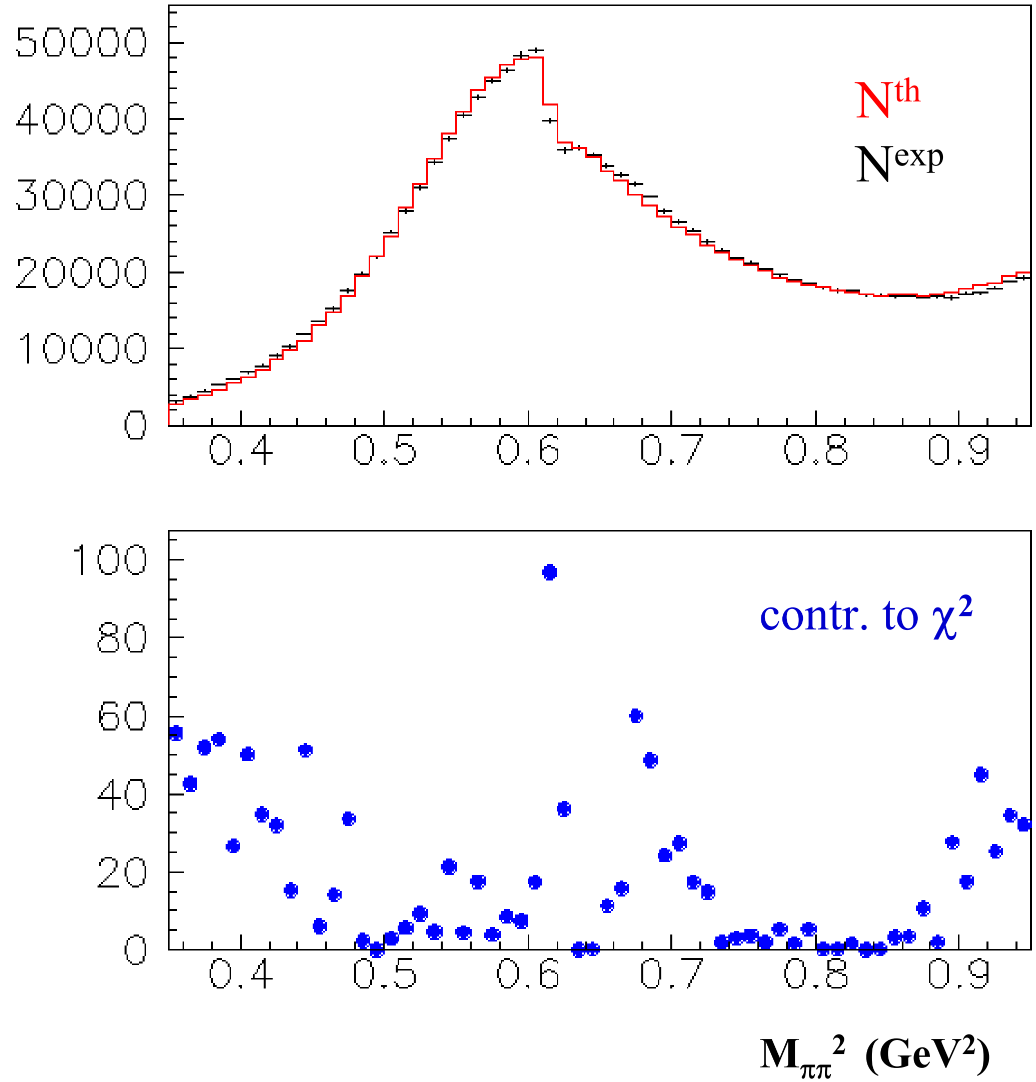
1. a χ^2 distributed function to minimize, **Prob($\chi^2 > \chi_{min}^2$, dof)**
2. statistical uncorrelated errors summed in quadrature

$$\chi^2 = \frac{\left[N_i^{exp} - N_i^{th}(\vec{\theta})\right]^2}{\sigma_{obs,\,i}^2 + \sigma_{bkgr,\,i}^2 + \sigma_{eff,\,i}^2}$$

N_i^{th} is in numerical form, because of efficiencies and smearing matrix

K-S: ρ'(1450) and ω are fixed

	this fit	C. Bini KL
M_ρ (MeV)	771.67 ± 0.15	773.1
Γ_ρ (MeV)	140.6 ± 0.2	144.0
$M_{\rho'}$ (MeV)	1465	1465
$\Gamma_{\rho'}$ (MeV)	310	310
M_ω (MeV)	782.59	782.59
Γ_ω (MeV)	8.49	8.49
α (10^{-3})	1.38 ± 0.05	1.65
β (10^{-3})	–134.0 ± 0.9	-123

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COVARIANCE MATRIX CALCULATED
SUCCESSFULLY
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χ^2/dof=   617.2 / 56

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FROM MIGRAD      STATUS=CONVERGED
EDM=  0.24E-06     STRATEGY= 1
ERROR MATRIX ACCURATE
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K-S: only ρ’(1450) is fixed

	this fit	C. Bini KL
M_ρ (MeV)	771.27 ± 0.16	773.1
Γ_ρ (MeV)	140.3 ± 0.2	144.0
$M_{\rho'}$ (MeV)	1465	1465
$\Gamma_{\rho'}$ (MeV)	310	310
M_ω (MeV)	781.4 ± 0.3	782.59
Γ_ω (MeV)	4.0 ± 0.5	8.49
α (10^{-3})	1.03 ± 0.06	1.65
β (10^{-3})	–134.2 ± 0.9	-123

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COVARIANCE MATRIX CALCULATED
SUCCESSFULLY
χ²/dof=   502.6 / 54
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EDM=  0.22E-07     STRATEGY= 1
ERROR MATRIX ACCURATE
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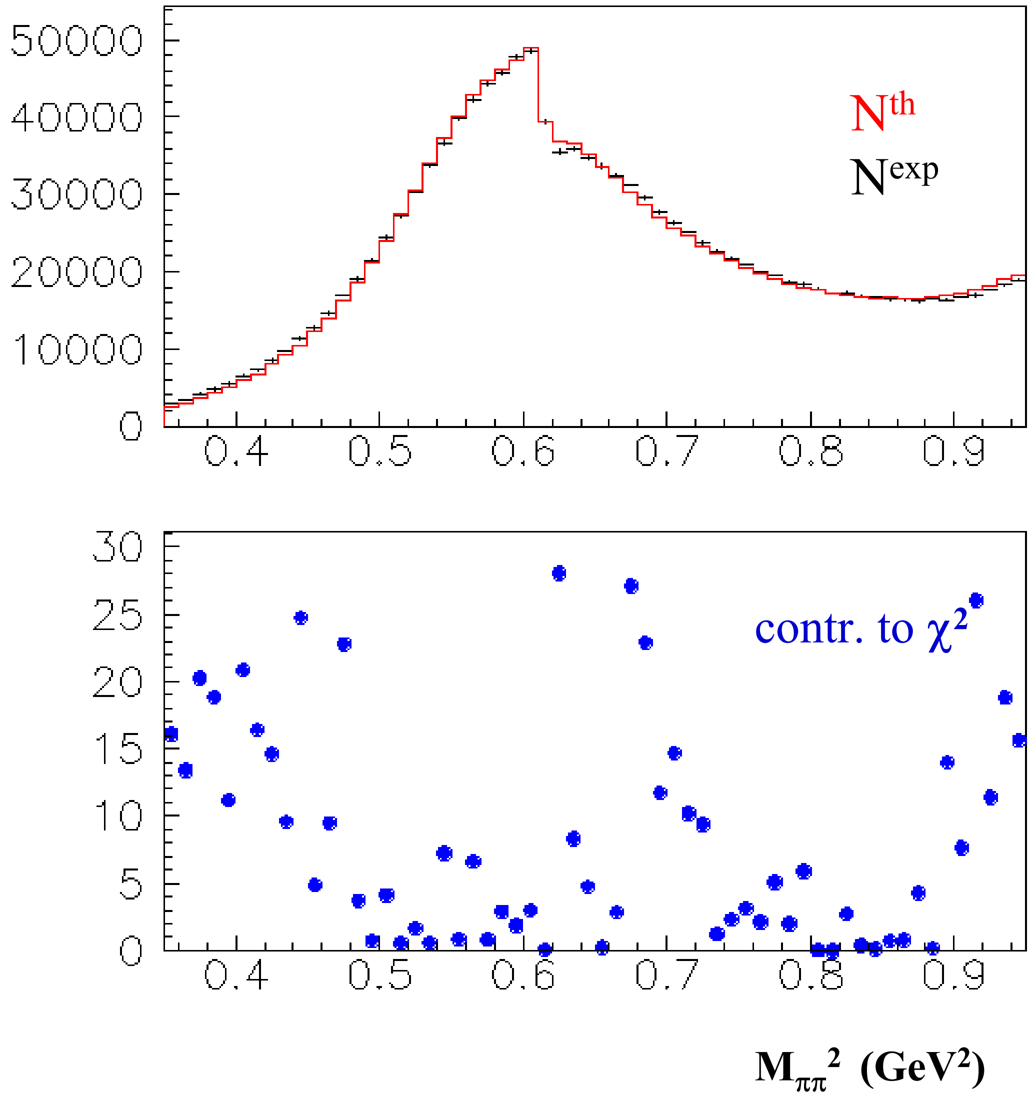
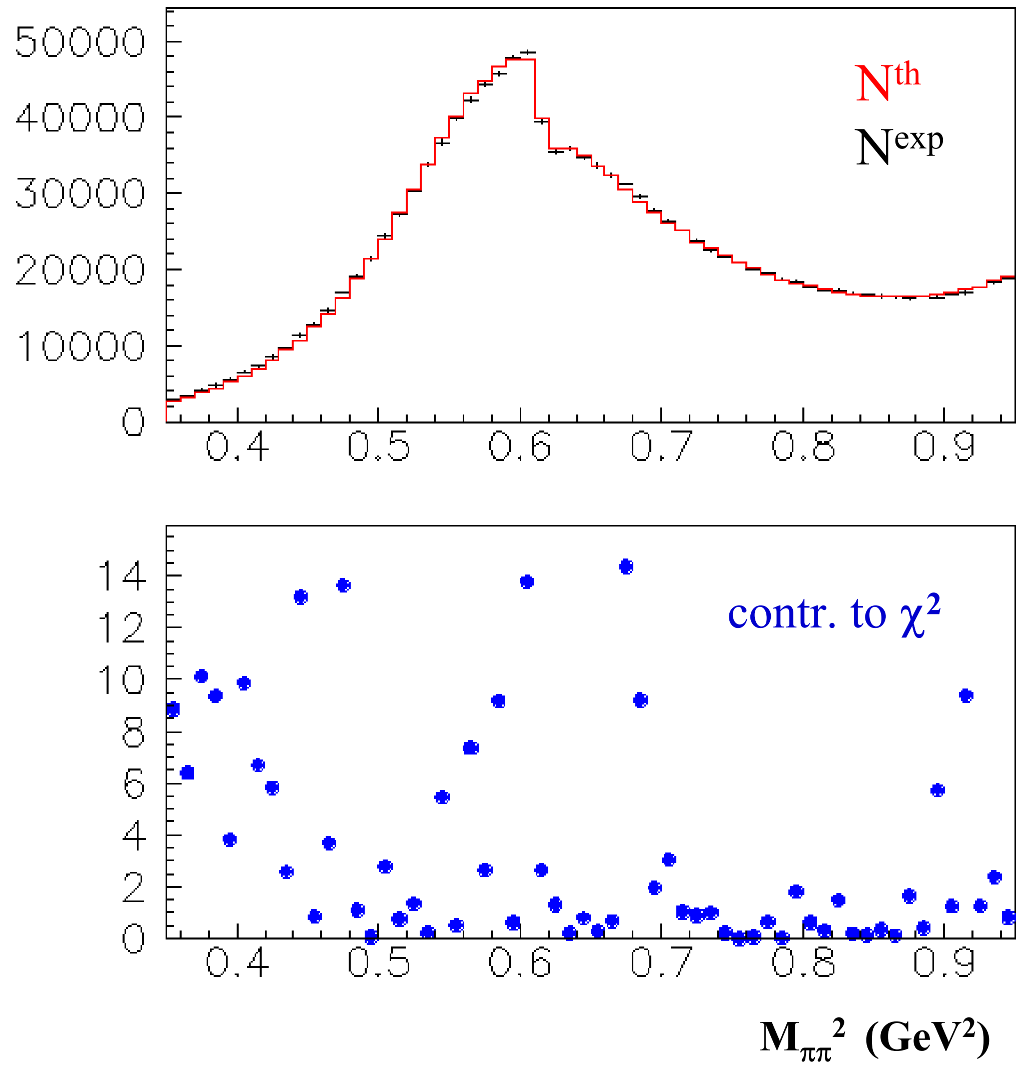


Partial conclusions with K-S

- our values are significantly different from those of Cesare, f0 effects?
- if all are left free, $\rho'(1450)$ and the ω width tend to completely unreasonable values
- a reasonable value for β is $\beta = 134 \times 10^{-3}$, despite of the χ^2
- systematic errors should be included in the χ^2 (see a couple of slides later)

G-S: ρ'(1450) and ω are fixed

M_ρ (MeV)	773.95 ± 0.16
Γ_ρ (MeV)	144.8 ± 0.3
$M_{\rho'}$ (MeV)	1465
$\Gamma_{\rho'}$ (MeV)	310
M_ω (MeV)	782.59
Γ_ω (MeV)	8.49
$\lvert\delta\rvert$ (10^{-3})	1.61 ± 0.05
arg δ (°)	13.2 ± 1.7
β (10^{-2})	–7.54 ± 0.09



COVARIANCE MATRIX CALCULATED SUCCESSFULLY

χ^2/dof= 206.8 / 55

FROM MIGRAD STATUS=CONVERGED
EDM= 0.19E-05 STRATEGY= 1
ERROR MATRIX ACCURATE

G-S: only $\rho'(1450)$ is fixed

M_ρ (MeV)	773.36 ± 0.15
Γ_ρ (MeV)	144.5 ± 0.3
$M_{\rho'}$ (MeV)	1465
$\Gamma_{\rho'}$ (MeV)	310
M_ω (MeV)	783.6 ± 0.3
Γ_ω (MeV)	1.6 ± 0.6
$\lvert\delta\rvert$ (10^{-3})	3.7 ± 1.7
arg δ (°)	73 ± 8
β (10^{-2})	–7.63 ± 0.09

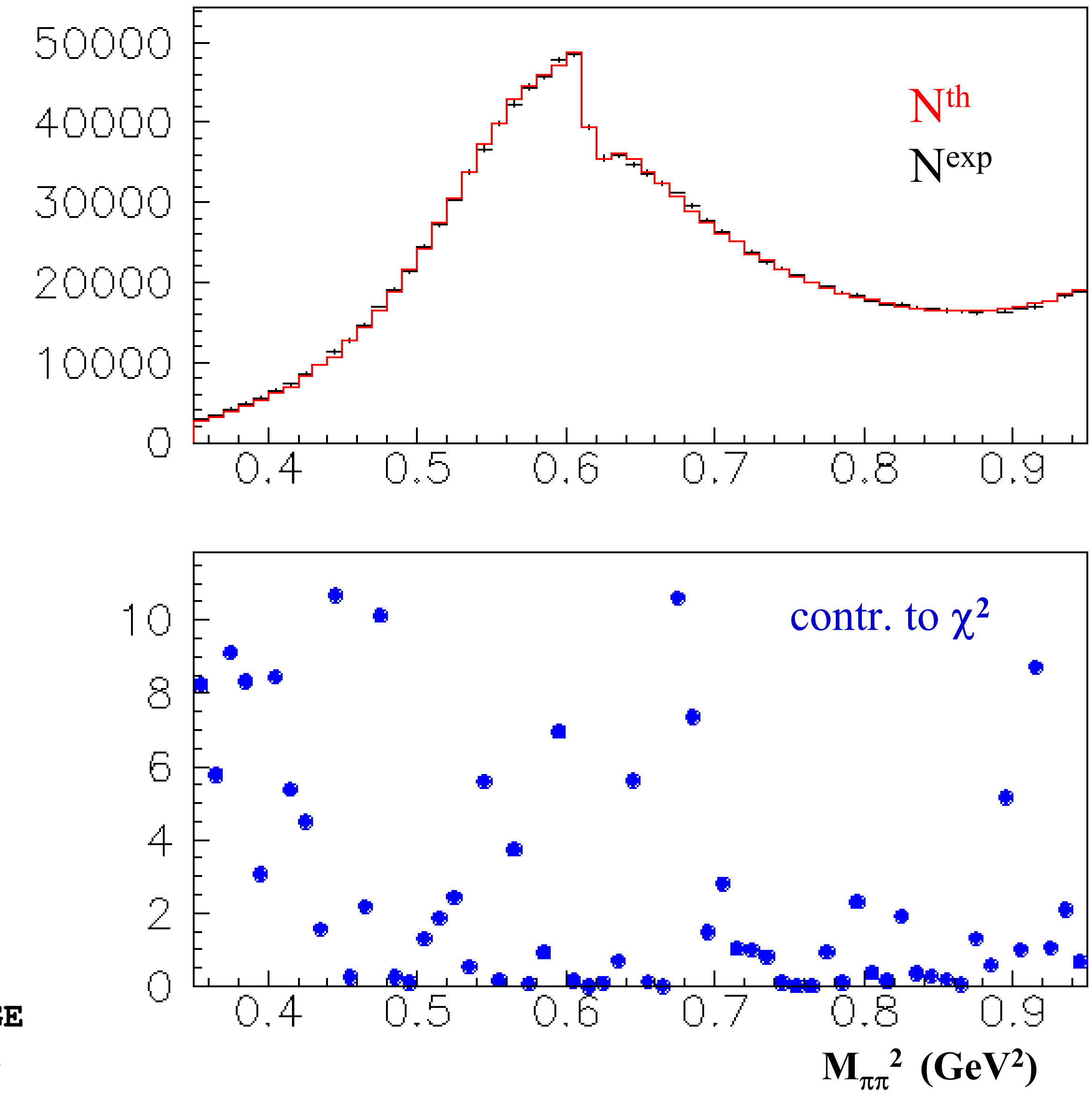
COVARIANCE MATRIX CALCULATED SUCCESSFULLY

χ^2/dof= 160.5 / 53

FROM MIGRAD STATUS=CONVERGE

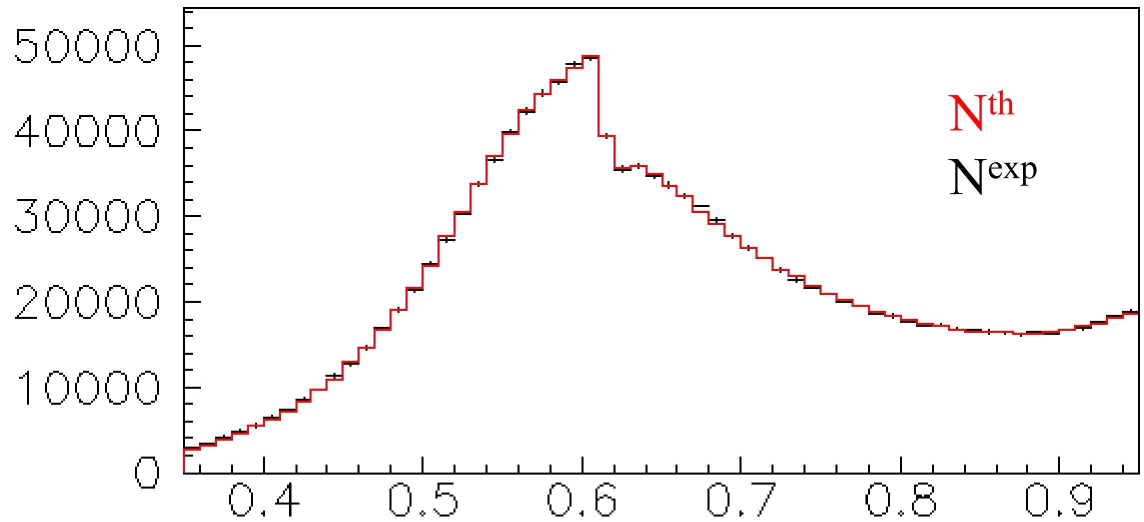
EDM= 0.94E-05 STRATEGY= 1

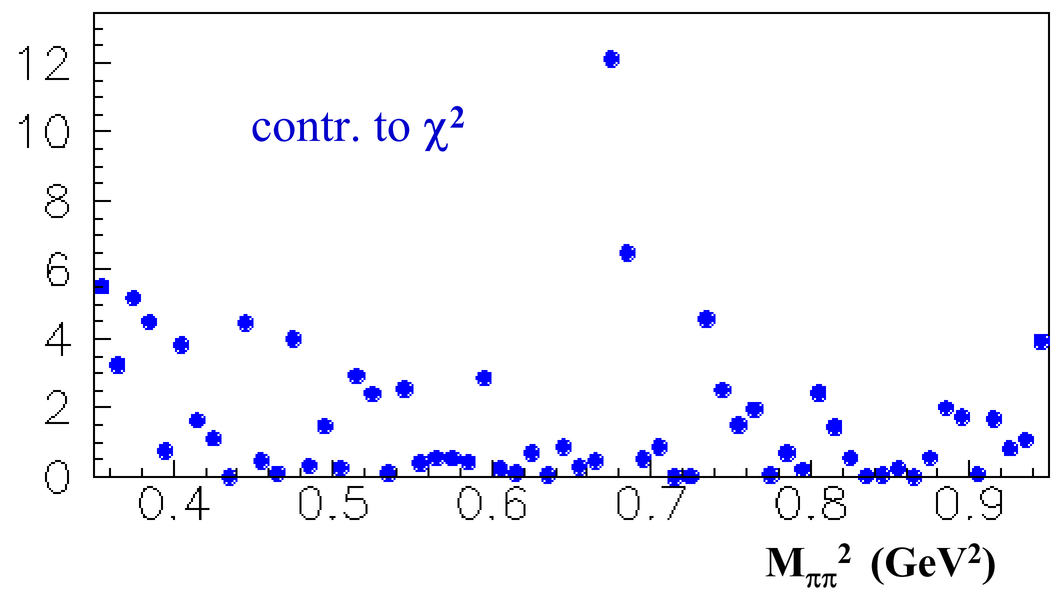
ERROR MATRIX ACCURATE



G-S: all free

M_ρ (MeV)	773.9 ± 0.2
Γ_ρ (MeV)	149.5 ± 1.0
$M_{\rho'}$ (MeV)	2200
$\Gamma_{\rho'}$ (MeV)	1.3×10^6
M_ω (MeV)	782.0 ± 0.6
Γ_ω (MeV)	5.6 ± 0.9
$\lvert\delta\rvert$ (10^{-3})	1.40 ± 0.09
arg δ ($^\circ$)	4 ± 5
β (10^{-2})	-10.4 ± 0.6





COVARIANCE MATRIX CALCULATED SUCCESSFULLY

χ^2/dof=   100.3 / 51

FROM MIGRAD   STATUS=CONVERGE
EDM= 0.86E-05   STRATEGY= 1
ERROR MATRIX ACCURATE

Systematic uncertainties

a) systematic uncertainties still to be implemented

b) three categories have been classified:

rel. syst. error

i. flat error related to a scale factor: luminosity

ii. $M_{\pi\pi}^2$ dependent error related to a $M_{\pi\pi}^2$ dependent correction: trigger

iii. errors correlated in steps of 5 bins, related to a $M_{\pi\pi}^2$ dependent correction: background estimates

Luminosity	0.6 % flat in s_π
Acceptance	0.3 % flat in s_π
Trigger	$\exp(0.43 - 4.9 s_\pi [\mathrm{GeV}^2])$ % + 0.08 %
Trackmass	0.2 % flat in s_π

s (GeV^2)	0	1	2	3	4	5	6	7	8	9
0.3...						0.8	0.7	0.6	0.6	0.5
0.4...	0.5	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
0.5...	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.6...	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.7...	0.3	0.2	0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.2
0.8...	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.9...	0.2	0.2	0.2	0.2	0.2					

covariance matrices,
penalty functions,
...work is in progress

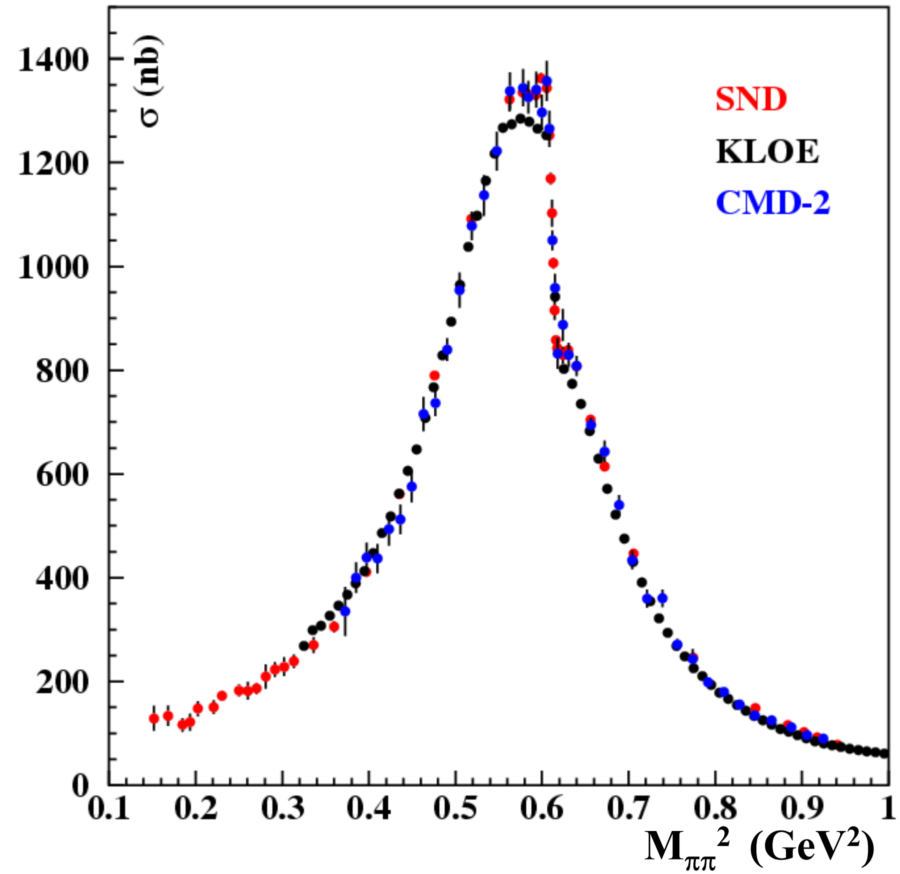
Comparisons: KLOE vs. Russia

comparison is performed through the cross section σ^0_{fsr}, undressed from vacuum polarization, but including FSR corrections

SND: data points from hep-ex/0506076 (6th column in Tab. 1), <u>not yet published</u>

KLOE: data points from our paper, inclusive in FSR, but divided by vacuum polarization

CMD-2: data points from Phys Lett, B562 (2003) 173 (2nd column in Tab. 1)

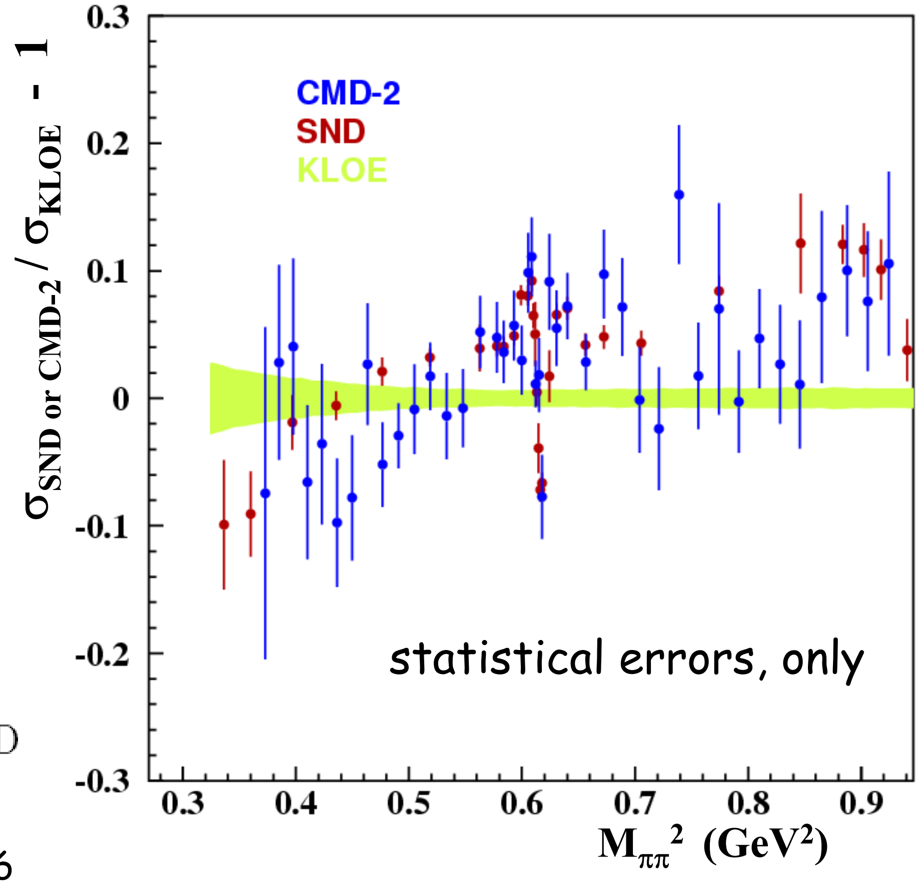


Relative comparisons

➢ fair agreement below the ρ mass among the 3 experiments (at least within ≤ 2 error bars)

➢ systematic deviation at the ρ mass btw russians and KLOE, but I would not shout loud the word discrepancy...

➢ above the ρ mass a significant difference btw SND and KLOE is out of discussion, but not btw CMD-2 and KLOE (at least within errors)



“The KLOE measurement is in conflict with the SND result as well as with the CMD2 one“

from hep-ex/0506076

“In light of the new SND data (SND-2 Coll., 2005), it seems appropriate to consider the possibility of a bias in the KLOE results,“

from hep-ph/0507078

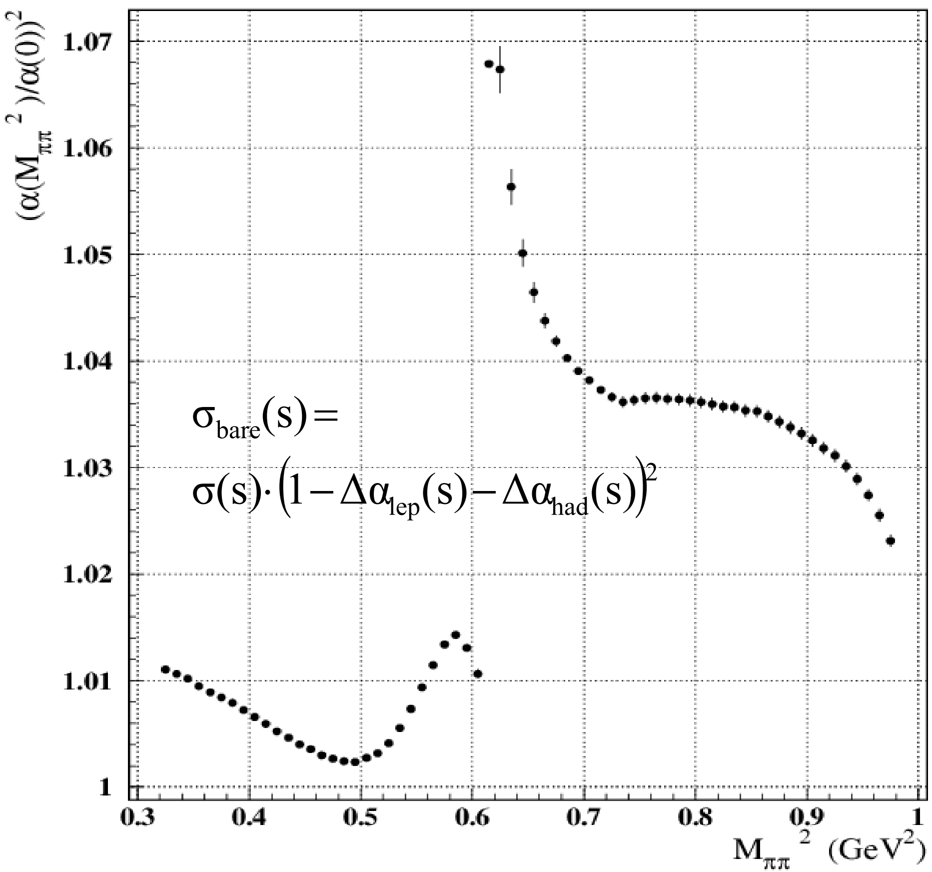
... It sounds like I believe more in SND rather than in KLOE, on which argument?

Conclusions and outlook

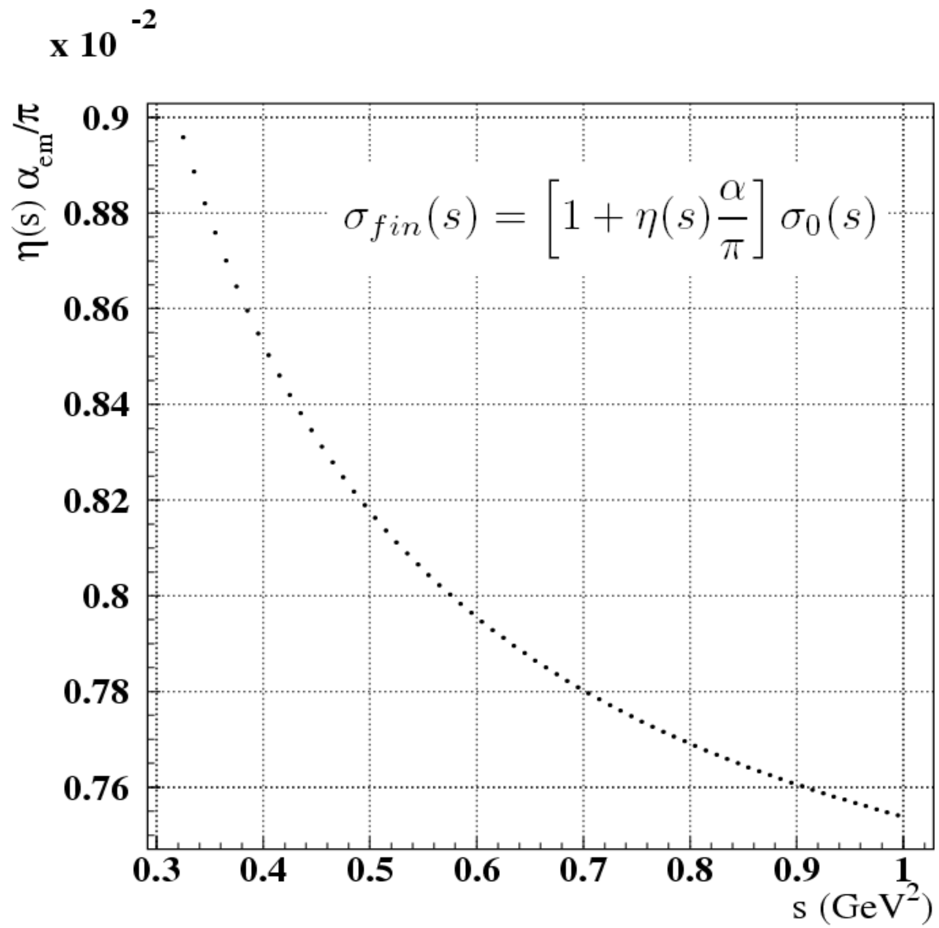
- several attempts of fitting our form factor have been performed, aside from $\rho'(1450)$, the values of the parameters are reasonable
- the functions are critical in the low mass region, above the ρ the G-S is well reproduced
- we are computing the χ^2 including the systematic errors with the proper laws of correlation
- a preliminary comparison with both SND and CMD-2 does not exhibit the "conflict", recently claimed, we are working at a more quantitative comparison in view of Lisbon
- a global fit, stat. and syst., including τ data, is planned

discussions with M. Antonelli, V. Cirigliano and G. Colangelo are acknowledged

Reminders...



$(\alpha(M_{\pi\pi}^{\ 2})/\alpha(0))^2$
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1
$\sigma_{bare}(s) =$
$\sigma(s) \cdot (1 - \Delta\alpha_{lep}(s) - \Delta\alpha_{had}(s))^2$
0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
$M_{\pi\pi}^{\ 2}$ (GeV^2)



x 10^{-2}
$\eta(s)\ \alpha_{em}/\pi$
0.9
0.88
0.86
0.84
0.82
0.8
0.78
0.76
$\sigma_{fin}(s) = \left[1 + \eta(s)\frac{\alpha}{\pi}\right] \sigma_0(s)$
0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
s (GeV^2)