

# Status of the $\pi^0\pi^0\gamma$ analysis

**S. Giovannella, S. Miscetti**

# Composition of the $\pi^0\pi^0\gamma$ final state

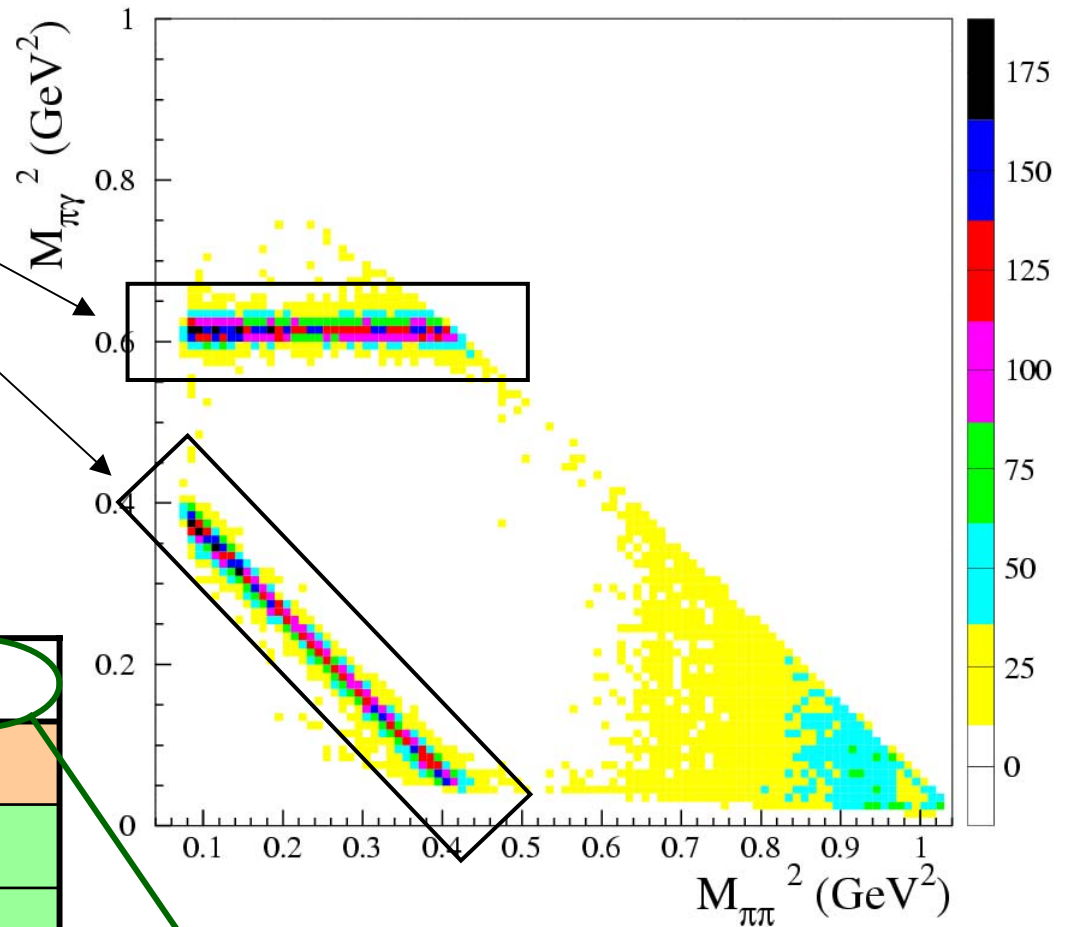
Two main contributions to  $\pi^0\pi^0\gamma$  final state @  $M_\phi$ :

1.  $e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^0\pi^0\gamma$   
 $\sigma_{\text{vis}}(M_\phi) \sim 0.5 \text{ nb}$

2.  $\phi \rightarrow S\gamma \rightarrow \pi^0\pi^0\gamma$   
 $\sigma_{\text{vis}}(M_\phi) \sim 0.3 \text{ nb}$

Backgrounds:

Process	S/B
$\phi \rightarrow a_0\gamma \rightarrow \eta\pi^0\gamma \rightarrow \gamma\gamma\pi^0\gamma$	8.51
$\phi \rightarrow \eta\gamma \rightarrow \pi^0\pi^0\pi^0\gamma$	0.06
$\phi \rightarrow \eta\gamma \rightarrow \gamma\gamma\gamma$	0.06
$\phi \rightarrow \pi^0\gamma$	0.21
$e^+e^- \rightarrow \gamma\gamma(\gamma)$	0.002



$S = \omega\pi + S\gamma$

# Data and Montecarlo samples

## DATA

2001+2002 data :  $L_{\text{int}} = 450 \text{ pb}^{-1}$

Data have some spread around the  $\phi$  peak  
+ two dedicated off-peak runs @ 1017  
and 1022 MeV  $\Rightarrow$

**Data divided in 100 keV bins of  $\sqrt{s}$**

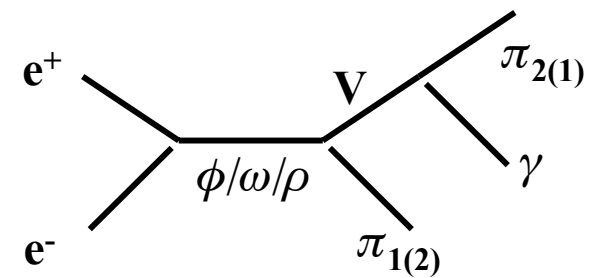
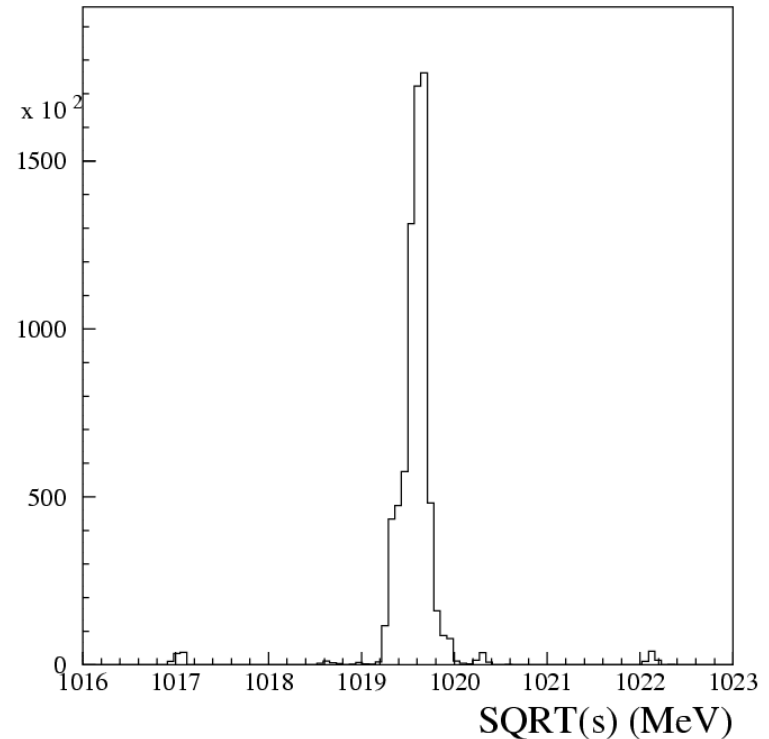
## MC

RAD04 MC production:  $5 \times L_{\text{int}}$

GG04 MC production:  $1 \times L_{\text{int}}$

**Improved  $e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^0\pi^0\gamma$  generator**

Three body phase space according to VDM  
from NPB 569 (2000), 158



# Sample preselection and kinematic fit

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1. **Acceptance** cut:

5 neutral clusters in TW with  $E > 7$  MeV and  $|\cos\theta| < 0.92$

[ TW:  $|T_{cl}-R_{cl}/c| < \text{MIN}( 5\sigma_T, 2 \text{ ns } )$  ]

2. **Kinematic fit** requiring 4-momentum conservation and the “promptness” of  $\gamma$ 's (  $T_{cl}-R_{cl}/c = 0$  )

3. **Pairing**: best  $\gamma$ 's comb. for the  $\pi^0\pi^0\gamma$  hypothesis

4. **Kinematic fit** for both  $\gamma$ 's pairing, requiring also constraints on  $\pi$  masses of the assigned  $\gamma\gamma$  pairs

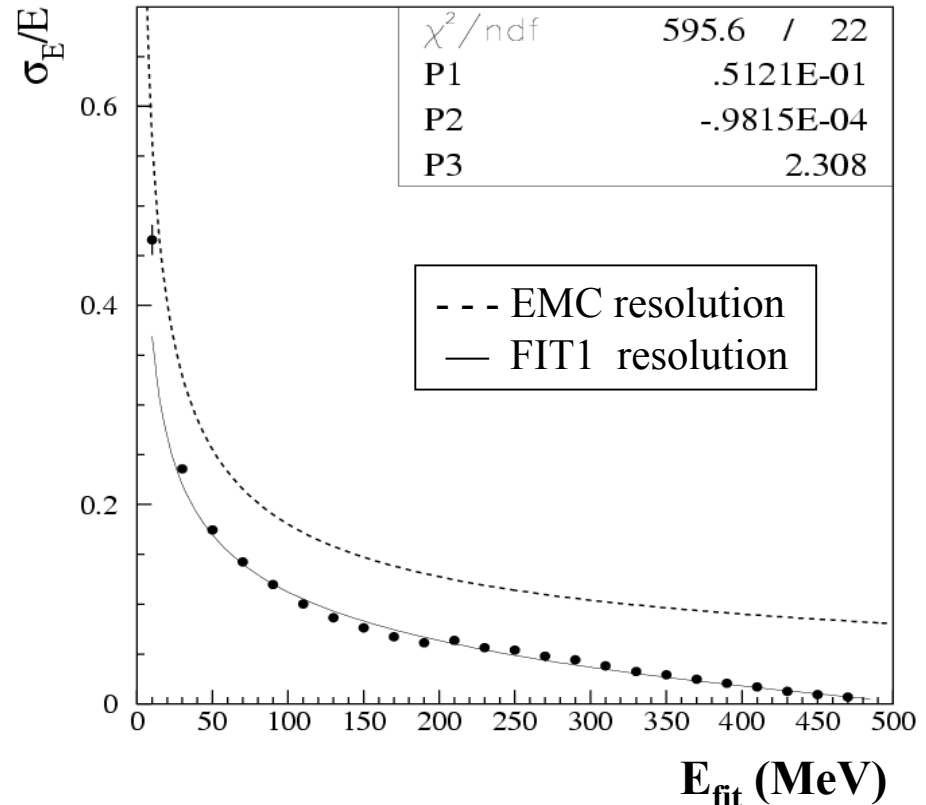
# $\gamma$ 's pairing

$\pi^0$  mass resolution parametrized as a function of the  $\gamma$ 's energy resolution after kinematic fit:

$$\sigma_M/M = 0.5 ( \sigma_{E_1}/E_1 \oplus \sigma_{E_2}/E_2 )$$

Fit function for energy resolution:

$$\sigma_E/E = ( P_1 + P_2 E ) / E[\text{GeV}]^{P_3}$$



The photon combination that minimize the following  $\chi^2$  is chosen:

$$\chi^2 = (M_{\gamma_i\gamma_j} - M_{\pi})/\sigma_{M_{ij}} + (M_{\gamma_k\gamma_l} - M_{\pi})/\sigma_{M_{kl}}$$

# Analysis cuts

1.  $e^+e^-$  ?  $\gamma\gamma$  rejection using the two most energetic clusters of the event:  $\mathbf{E_1+E_2 > 900 \text{ MeV}}$
2.  $\gamma\gamma\gamma$  + accidentals background rejection:  $\mathbf{E_\gamma(\text{Fit2}) > 7 \text{ MeV}}$
3. Cut on 2<sup>nd</sup> kinematic fit:  $\mathbf{\chi^2_{\text{Fit2}}/\text{ndf} < 3}$
4. Cut on  $\pi$  masses of the assigned  $\gamma\gamma$  pairs:  $\mathbf{|M_{\gamma\gamma} - M_\pi| < 5\sigma_M}$

Process	$\epsilon_{\text{ana}}$	S/B
$e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^0\pi^0\gamma$	50.1 %	-
$\phi \rightarrow S\gamma \rightarrow \pi^0\pi^0\gamma$	36.8 %	-
$\phi \rightarrow a_0\gamma \rightarrow \eta\pi^0\gamma \rightarrow \gamma\gamma\pi^0\gamma$	7.0 %	22.9
$\phi \rightarrow \eta\gamma \rightarrow \pi^0\pi^0\pi^0\gamma$	0.3 %	<b>8.9</b>
$\phi \rightarrow \eta\gamma \rightarrow \gamma\gamma\gamma$	$5.4 \times 10^{-4}$	50.0
$\phi \rightarrow \pi^0\gamma$	$1.5 \times 10^{-4}$	606.2
$e^+e^- \rightarrow \gamma\gamma(\gamma)$	$0.7 \times 10^{-4}$	1048.2

✓  $S = \omega\pi + S\gamma$

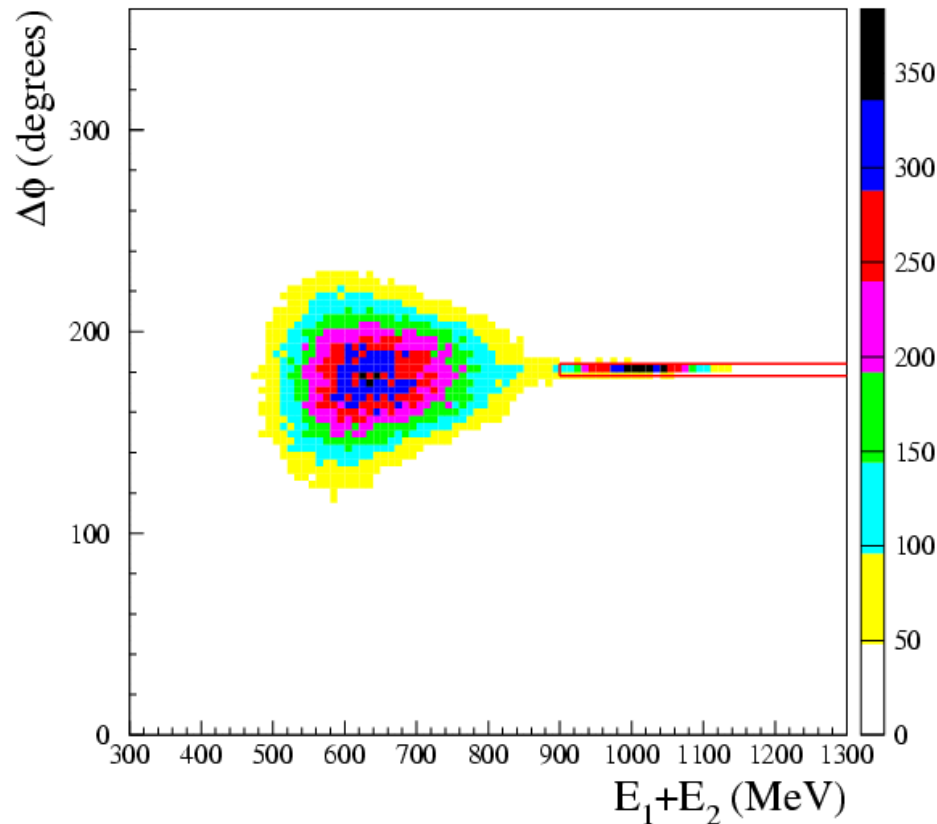
✓  $\epsilon_{\text{ana}}(S\gamma)$  obtained using the 2000 data  $M_{\pi\pi}$  shape

# $e^+e^- \rightarrow \gamma\gamma$ rejection

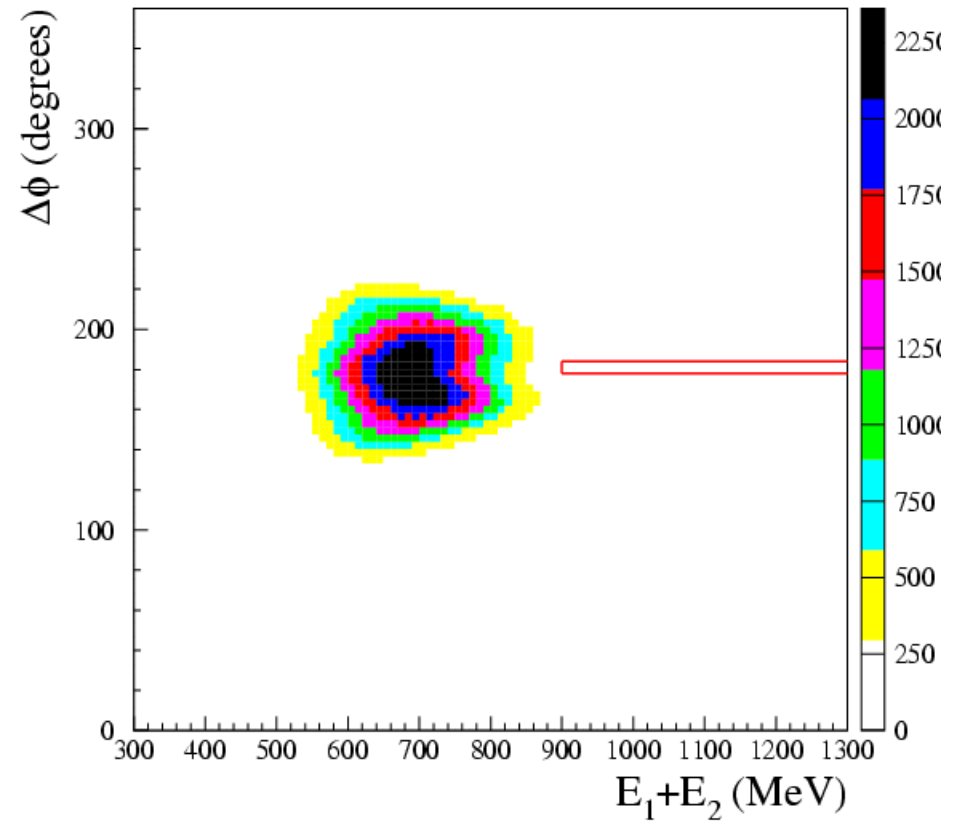
$e^+e^- \rightarrow \gamma\gamma$  rejection done using the two most energetic clusters of the event:

$E_1 + E_2 > 900$  MeV

Data

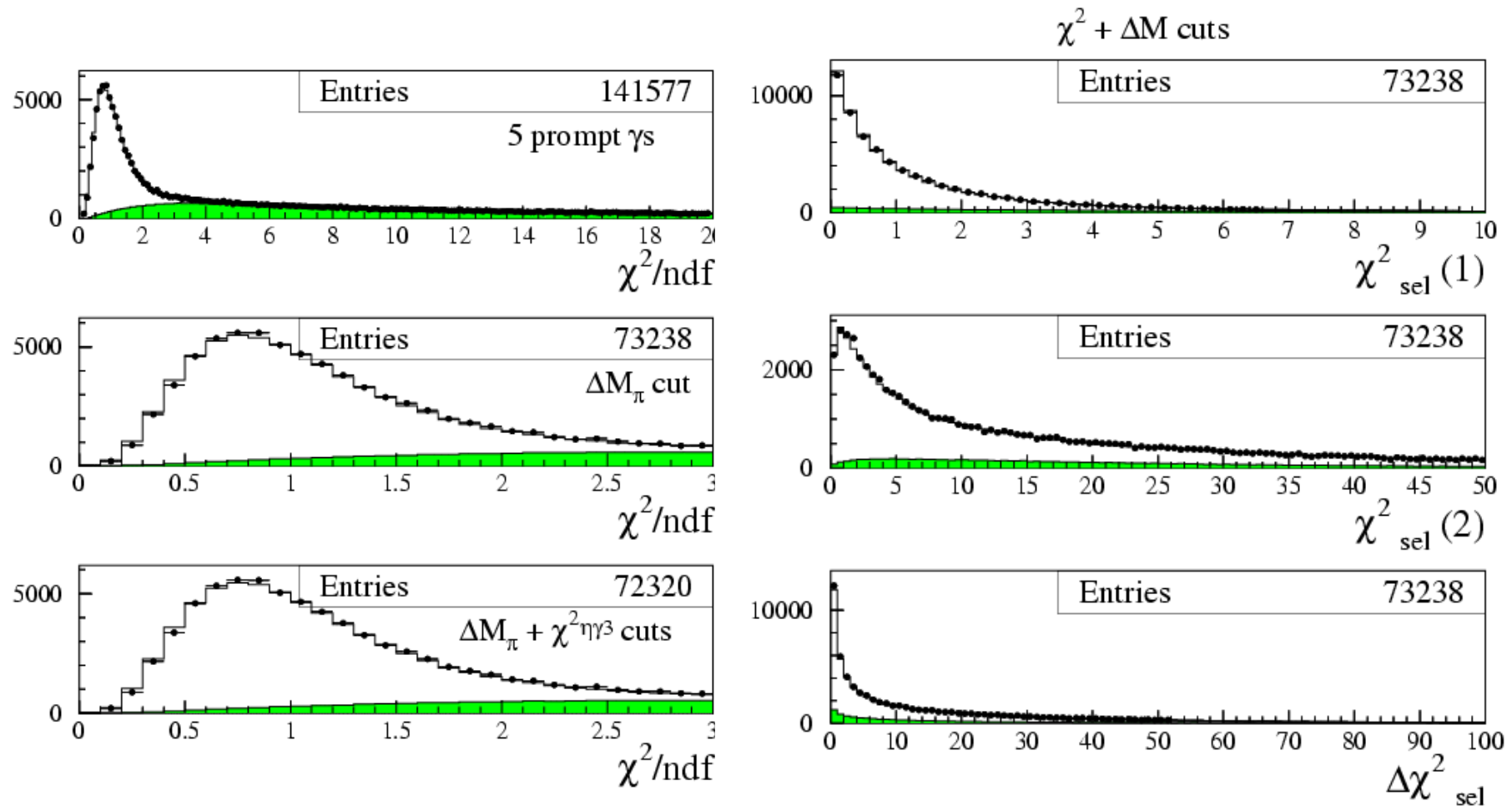


MC  $\pi^0\pi^0\gamma$  events



# Dalitz plot analysis: data-MC comparison (I)

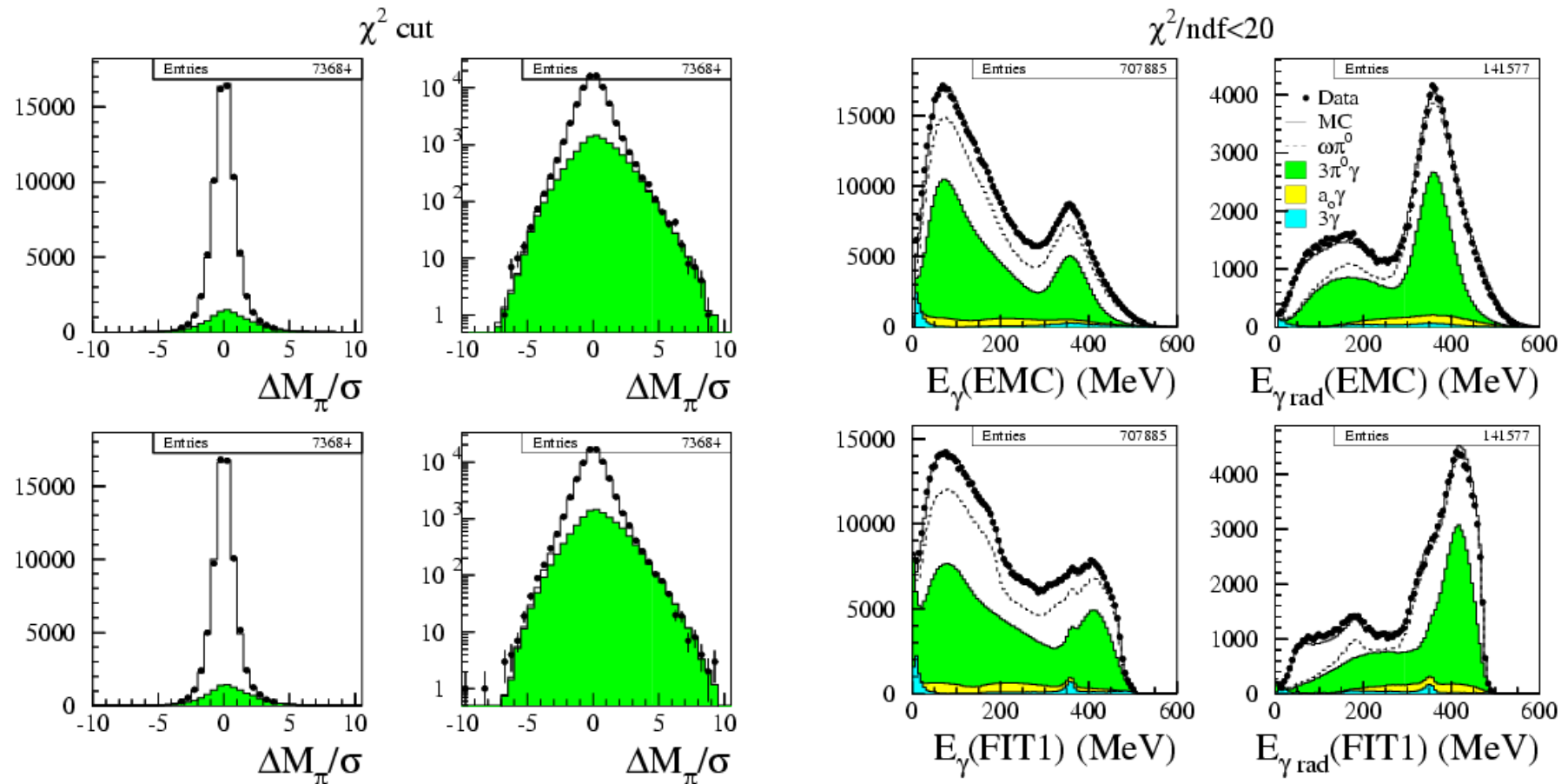
Analysis @  $\sqrt{s} = 1019.6 \text{ MeV}$  ( $L_{\text{int}} = 145 \text{ pb}^{-1}$ )





# Dalitz plot analysis: data-MC comparison (II)

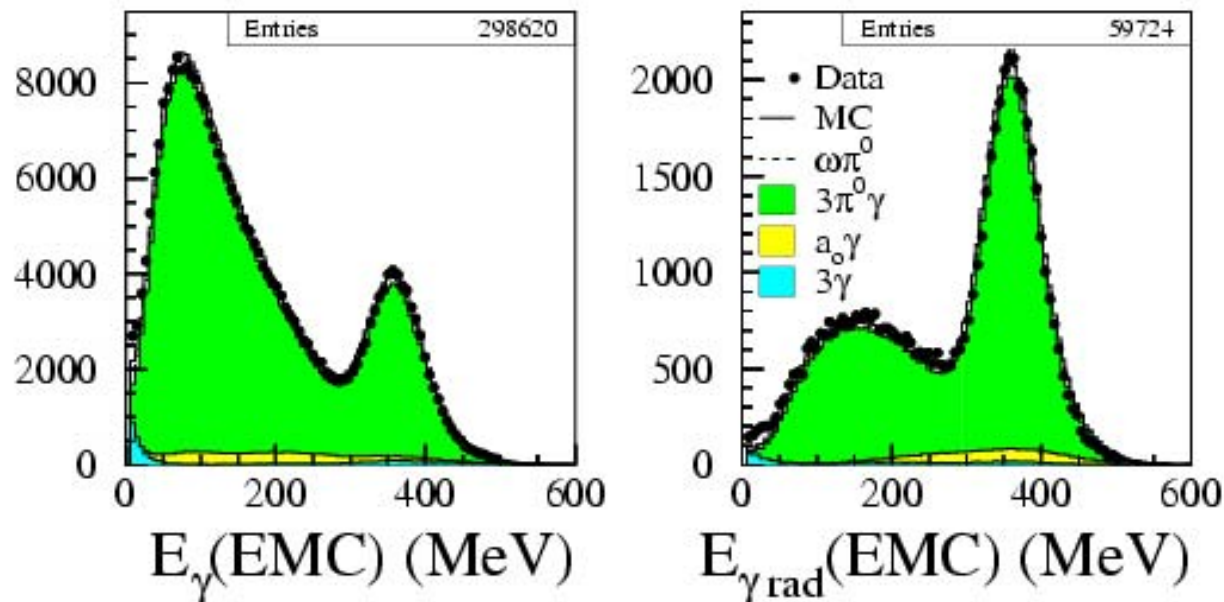
Analysis @  $\sqrt{s} = 1019.6 \text{ MeV}$  ( $L_{\text{int}} = 145 \text{ pb}^{-1}$ )



# Background study for Dalitz plot analysis (I)

In order to study the systematics connected to the background subtraction we found for each category a distribution “background dominated” to be fitted

- $\phi \rightarrow \eta \gamma \rightarrow \pi^0 \pi^0 \pi^0 \gamma$  (most relevant bckg contribution)
  - Background enriched sample :  $4 < \chi^2/\text{ndf} < 20$

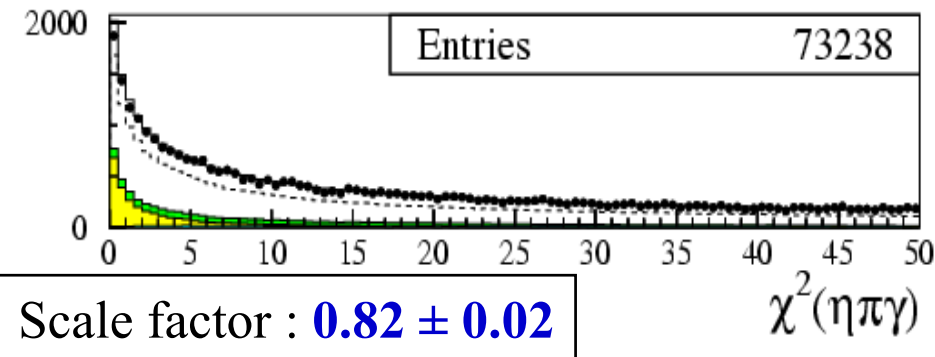
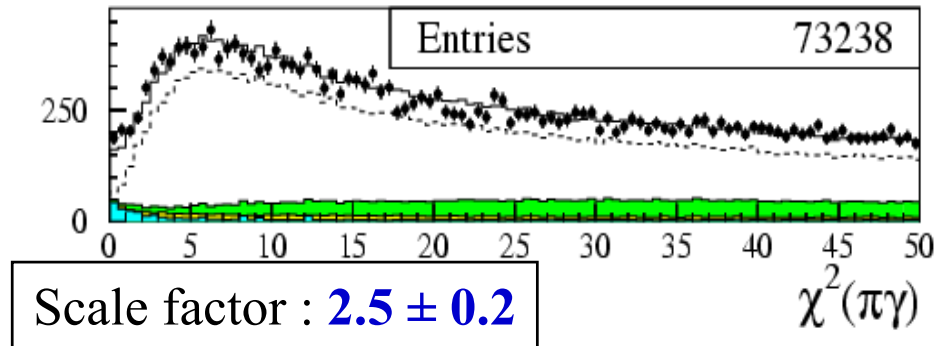
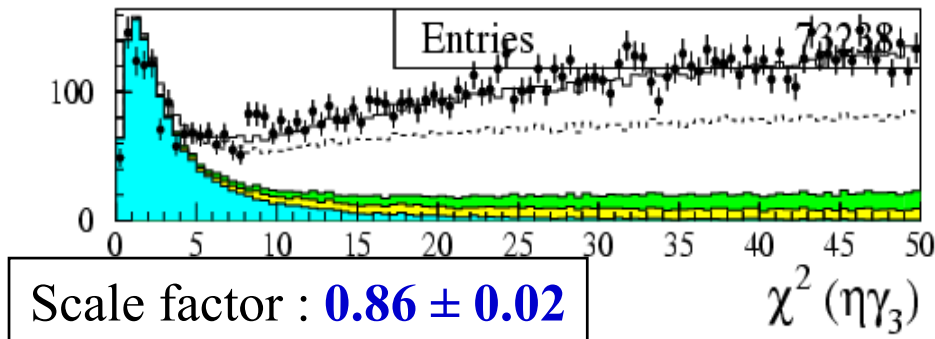


Scale factor :  
 **$1.0156 \pm 0.0002$**

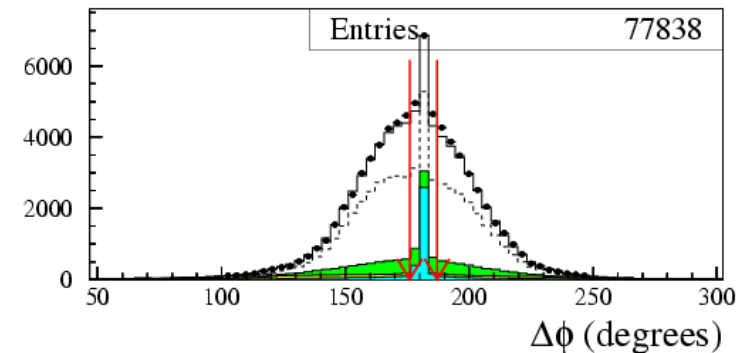
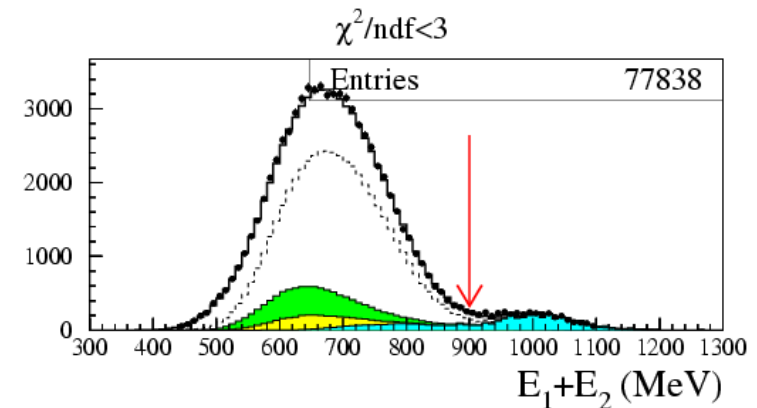
All of this fit results are used to evaluate the systematics on the background counting : **half of the difference ( 1 – scale factor )** is used

# Background study for Dalitz plot analysis (II)

For  $\phi \rightarrow \eta\gamma \rightarrow \gamma\gamma\gamma$ ,  $\phi \rightarrow \pi^0\gamma$ ,  $\phi \rightarrow a_0\gamma$   
we calculate a  $\chi^2$  in the mass hypothesis



For  $e^+e^- \rightarrow \gamma\gamma$ , we fit the  $\Delta\phi$   
distribution for  $\chi^2/\text{ndf} < 3$   
(and no  $\gamma\gamma$  rejection cut)



# Dalitz plot @ $\sqrt{s}=1019.6$ MeV

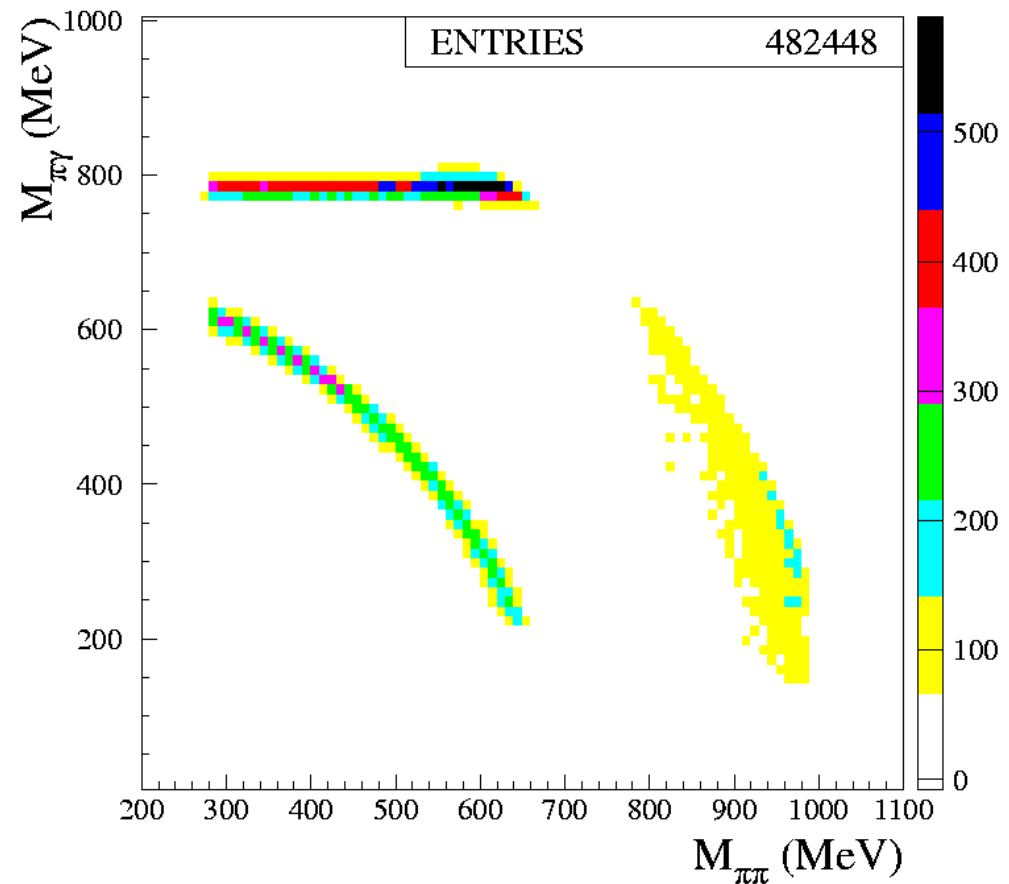
Fit to the Dalitz plot with the VDM and scalar term, including also interference

Binning: 10 MeV in  $M_{\pi\pi}$ , 12.5 MeV in  $M_{\pi\gamma}$

What is needed:

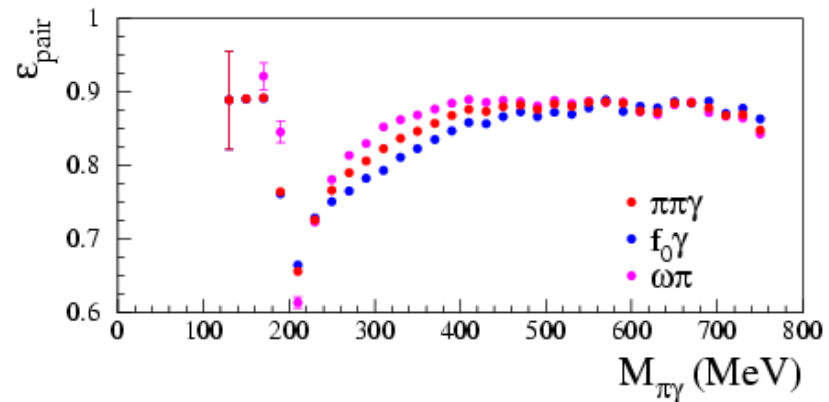
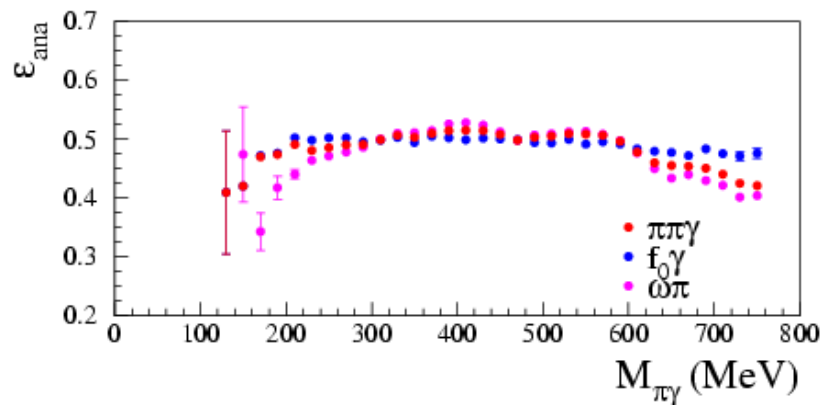
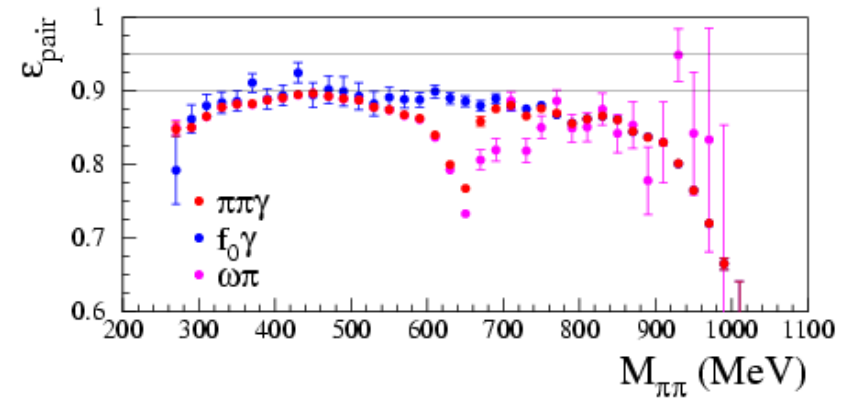
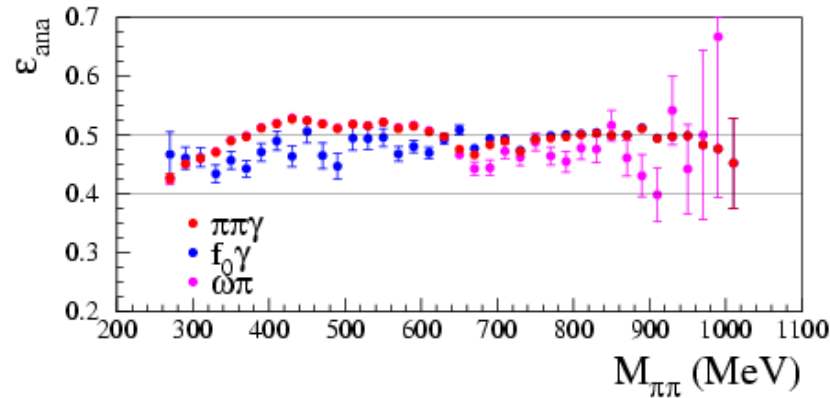
- Analysis efficiency
- Smearing matrix
- Theoretical functions
- ISR

Only statistical error and systematics on background considered for the moment



# Analysis and pairing efficiencies vs $M_{\pi\pi}$ , $M_{\pi\gamma}$

Analysis efficiency and smearing matrix evaluated from MC for each bin of the  $M_{\pi\pi} - M_{\pi\gamma}$  plane

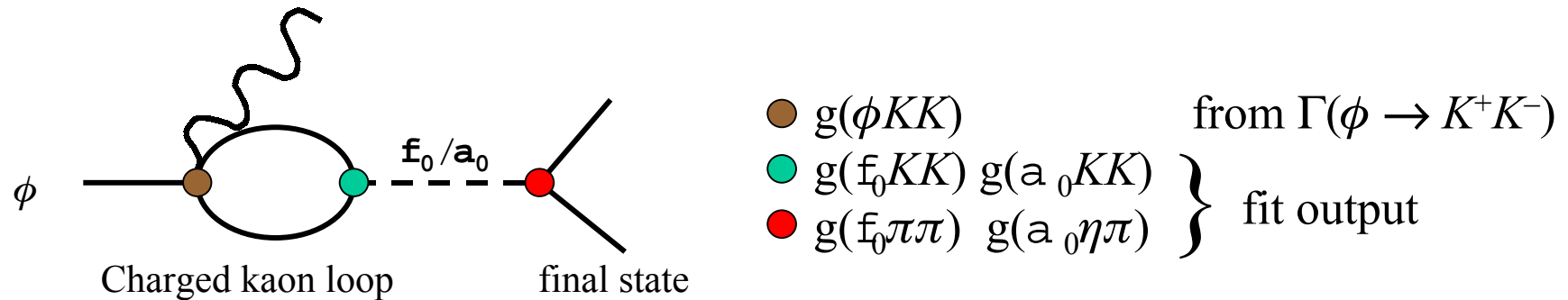


**Different for the two processes!**

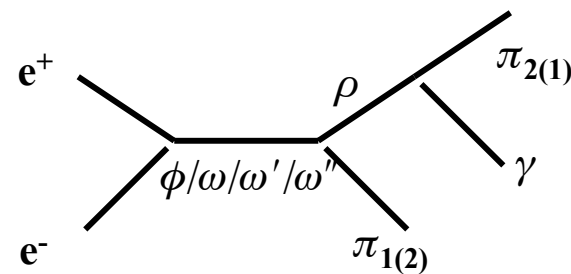
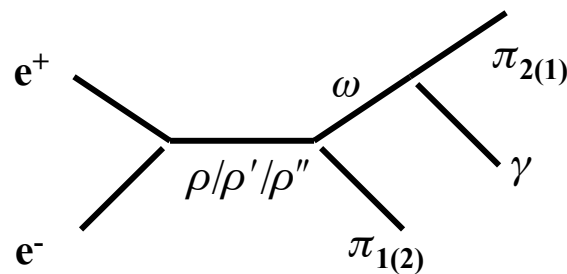
In the fit of the Dalitz different  $\epsilon_{\text{ana}}$  and smearing used for the VDM and scalar contributions. For the moment the VDM results are used also for the interference term

# Fit function: the Achasov parametrization (I)

## ➤ Scalar produced through a kaon loop



## ➤ VDM contribution from the following diagrams :



## ➤ All interferences considered

# Fit function: the Achasov parametrization (II)

$$\frac{d\sigma(e^+e^- \rightarrow \pi^0\pi^0\gamma)}{dm dm_{\pi\gamma}} = \frac{\alpha m_{\pi\gamma} m}{3(4\pi)^2 s^3} \left\{ \frac{2g_{\phi\gamma}^2}{|D_\phi(s)|^2} |g(m)|^2 \left| \frac{g_{f_0 K+K} - g_{f_0 \pi^0\pi^0}}{D_{f_0}(m)} \right|^2 + \right.$$

$f_0\gamma$

Model dependent term

$$\frac{1}{16} F_1(m^2, m_{\pi\gamma}^2) \left| \left( \frac{e^{i\phi_{\omega\phi}(m_\phi^2)} g_{\phi\gamma} g_{\phi\rho\pi} g_{\rho\pi\gamma}}{D_\phi(s)} - C_{\rho\pi} \right) \frac{e^{i\delta_{b\rho}}}{D_\rho(m_{\pi\gamma}^2)} + \frac{C_{\omega\pi^0}}{D_\omega(m_{\pi\gamma}^2)} \right|^2 +$$

$$\frac{1}{16} F_1(m^2, \tilde{m}_{\pi\gamma}^2) \left| \left( \frac{e^{i\phi_{\omega\phi}(m_\phi^2)} g_{\phi\gamma} g_{\phi\rho\pi} g_{\rho\pi\gamma}}{D_\phi(s)} + C_{\rho\pi} \right) \frac{e^{i\delta_{b\rho}}}{D_\rho(\tilde{m}_{\pi\gamma}^2)} + \frac{C_{\omega\pi^0}}{D_\omega(\tilde{m}_{\pi\gamma}^2)} \right|^2 +$$

$$\frac{1}{8} F_2(m^2, m_{\pi\gamma}^2) \text{Re} \left[ \left( \left( \frac{e^{i\phi_{\omega\phi}(m_\phi^2)} g_{\phi\gamma} g_{\phi\rho\pi} g_{\rho\pi\gamma}}{D_\phi(s)} + C_{\rho\pi} \right) \frac{e^{i\delta_{b\rho}}}{D_\rho(m_{\pi\gamma}^2)} + \frac{C_{\omega\pi^0}}{D_\omega(m_{\pi\gamma}^2)} \right) \times \right.$$

$$\left. \left( \left( \frac{e^{i\phi_{\omega\phi}(m_\phi^2)} g_{\phi\gamma} g_{\phi\rho\pi} g_{\rho\pi\gamma}}{D_\phi(s)} + C_{\rho\pi} \right) \frac{e^{i\delta_{b\rho}}}{D_\rho(\tilde{m}_{\pi\gamma}^2)} + \frac{C_{\omega\pi^0}}{D_\omega(\tilde{m}_{\pi\gamma}^2)} \right)^* \right] \mp$$

$\omega\pi/\rho\pi$

Modified in  
+  $\cos \phi$  (???)

$$\frac{1}{\sqrt{2}} \text{Re} \left[ g(m) e^{i\delta_B(m)} \frac{g_{f_0 K+K} - g_{f_0 \pi^0\pi^0}}{D_{f_0}(m)} \frac{g_{\phi\gamma}}{D_\phi(s)} \left( \right.$$

$$F_3(m^2, m_{\pi\gamma}^2) \left( \left( \frac{e^{i\phi_{\omega\phi}(m_\phi^2)} g_{\phi\gamma} g_{\phi\rho\pi} g_{\rho\pi\gamma}}{D_\phi(s)} + C_{\rho\pi} \right) \frac{e^{i\delta_{b\rho}}}{D_\rho(m_{\pi\gamma}^2)} + \frac{C_{\omega\pi^0}}{D_\omega(m_{\pi\gamma}^2)} \right)^* +$$

$$F_3(m^2, \tilde{m}_{\pi\gamma}^2) \left( \left( \frac{e^{i\phi_{\omega\phi}(m_\phi^2)} g_{\phi\gamma} g_{\phi\rho\pi} g_{\rho\pi\gamma}}{D_\phi(s)} + C_{\rho\pi} \right) \frac{e^{i\delta_{b\rho}}}{D_\rho(\tilde{m}_{\pi\gamma}^2)} + \frac{C_{\omega\pi^0}}{D_\omega(\tilde{m}_{\pi\gamma}^2)} \right)^* \left. \right] \left. \right\},$$

$f_0\gamma/VP$  interf

[N.N.Achasov, A.V.Kiselev, private communication]

**VDM parametrization:  $C_{VP}$  fixed –  $K_{VDM}$  (norm factor),  $\delta_{b\rho}$ ,  $M_V$ ,  $G_V$  free**

# Fit function: different parametrization for the scalar term

1. Point-like  $\phi S \gamma$  coupling. Corrections to a “standard” BW-like  $f_0$  (fixed  $\Gamma_S$ ) described by the  $a_0$ ,  $a_1$  parameters  
[Isidori-Maiani, private communication]

$$A_1^{\text{scal}} = \frac{e}{4F_\Phi} \frac{sM_\Phi^2}{D_\Phi(s)} \left[ \frac{g_{12}^f g_{f\gamma}^\Phi}{D_S[(1-x)s]} + \frac{a_0}{M_\Phi^2} + a_1 \frac{(1-x)s - M_S^2}{M_\Phi^4} \right]$$

2. Fit based on the hadronic scattering amplitudes  $\pi\pi \rightarrow \pi\pi$ ,  $\pi\pi \rightarrow KK$  in the  $\pi^0\pi^0\gamma$  production mechanism  
[Boglione-Pennington, Eur. Phys. J. C 30 (2003) 503]

This is implemented in our fit function with the replacement:

$$\frac{g(M_{\pi\pi}) g_{f_0 K^+ K^-} g_{f_0 \pi^+ \pi^-} e^{i\delta_B(M_{\pi\pi})}}{D_{f_0}(M_{\pi\pi})} \rightarrow (M_{\pi\pi}^2 - m_0^2) [(a_1 + b_1 M_{\pi\pi}^2 + c_1 M_{\pi\pi}^4) T(\pi\pi \rightarrow \pi\pi) + (a_2 + b_2 M_{\pi\pi}^2 + c_2 M_{\pi\pi}^4) T(KK \rightarrow \pi\pi)]$$



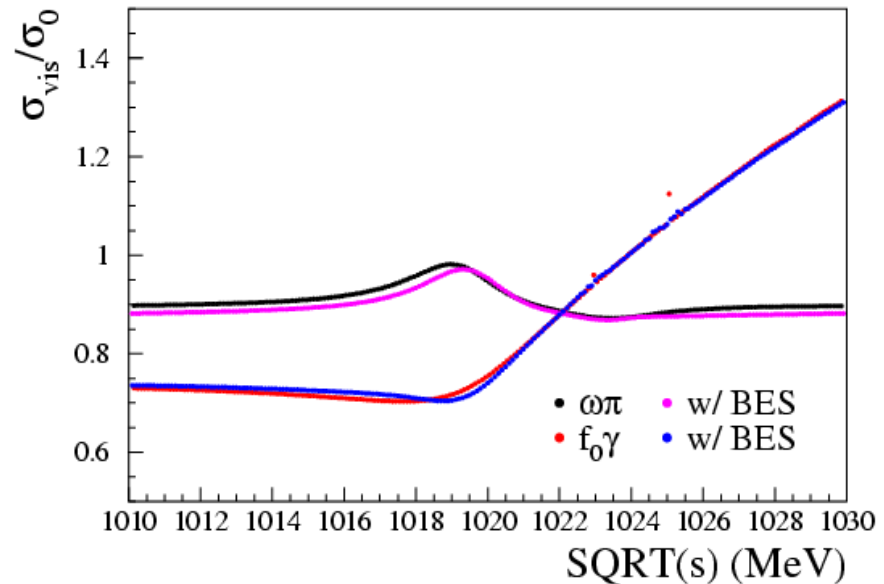
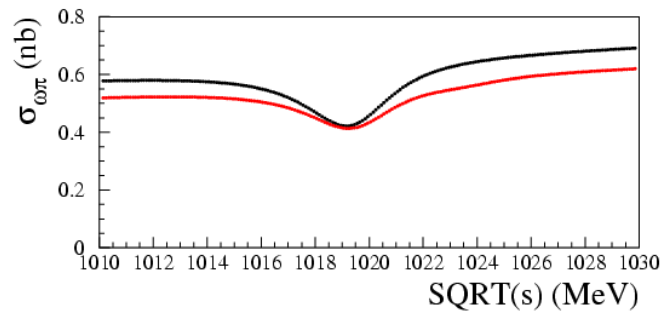
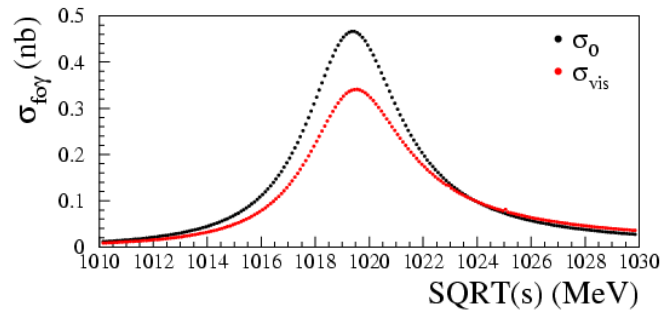
# Calculation of the radiative corrections

ISR evaluated starting from the following  $\sigma_0$  :

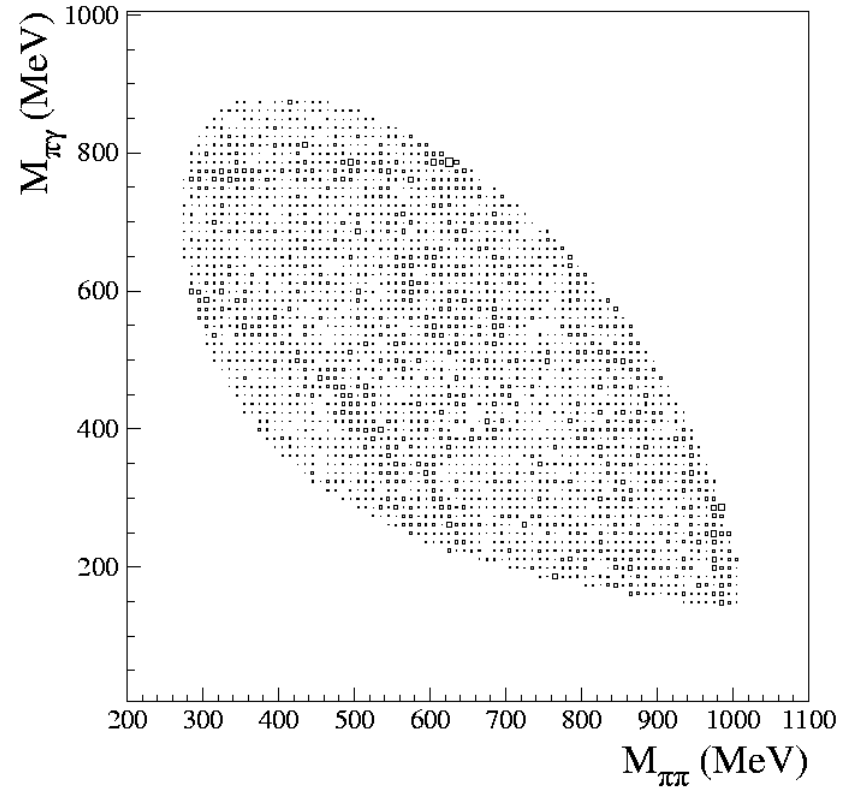
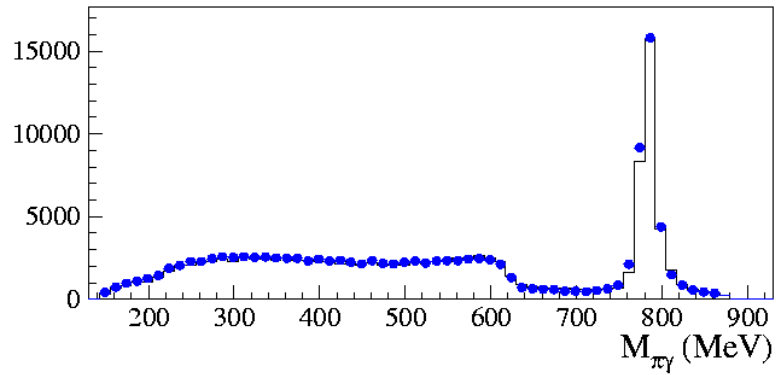
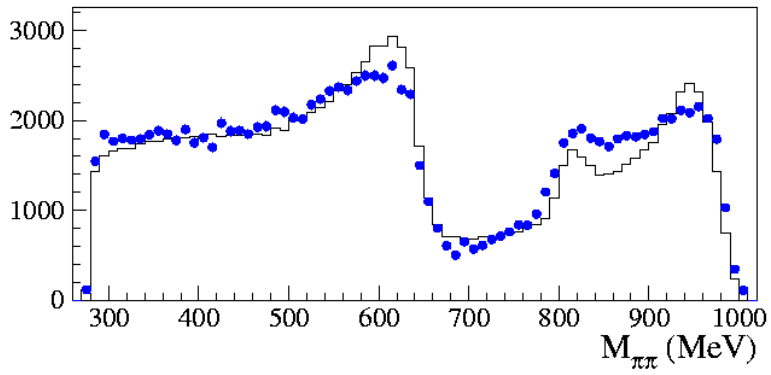
$f_0$  = “simple” BW (by integrating the Achasov scalar term)

$\omega\pi$  = SND parametrization from JETP-90 6 (2000) 927, obtained by fitting over a large  $\sqrt{s}$  range ... Proper threshold behaviour

$$\sigma_{vis} = \int_0^{4m_\pi^2} \sigma_0 [(1-x)s] H(s, x) \quad H(s,x) \text{ from Antonelli, Dreucci}$$

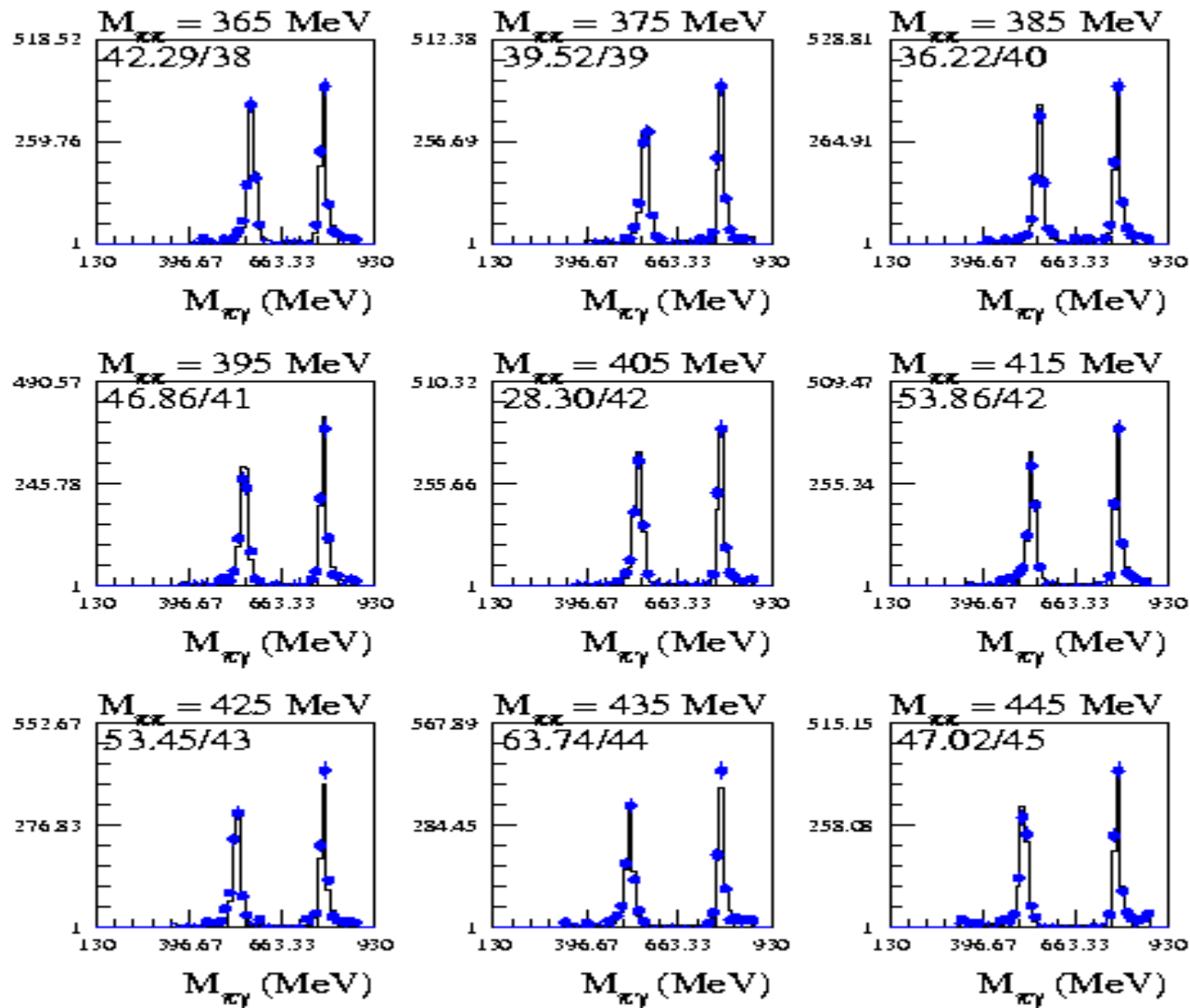


# Fit results: the Achasov parametrization

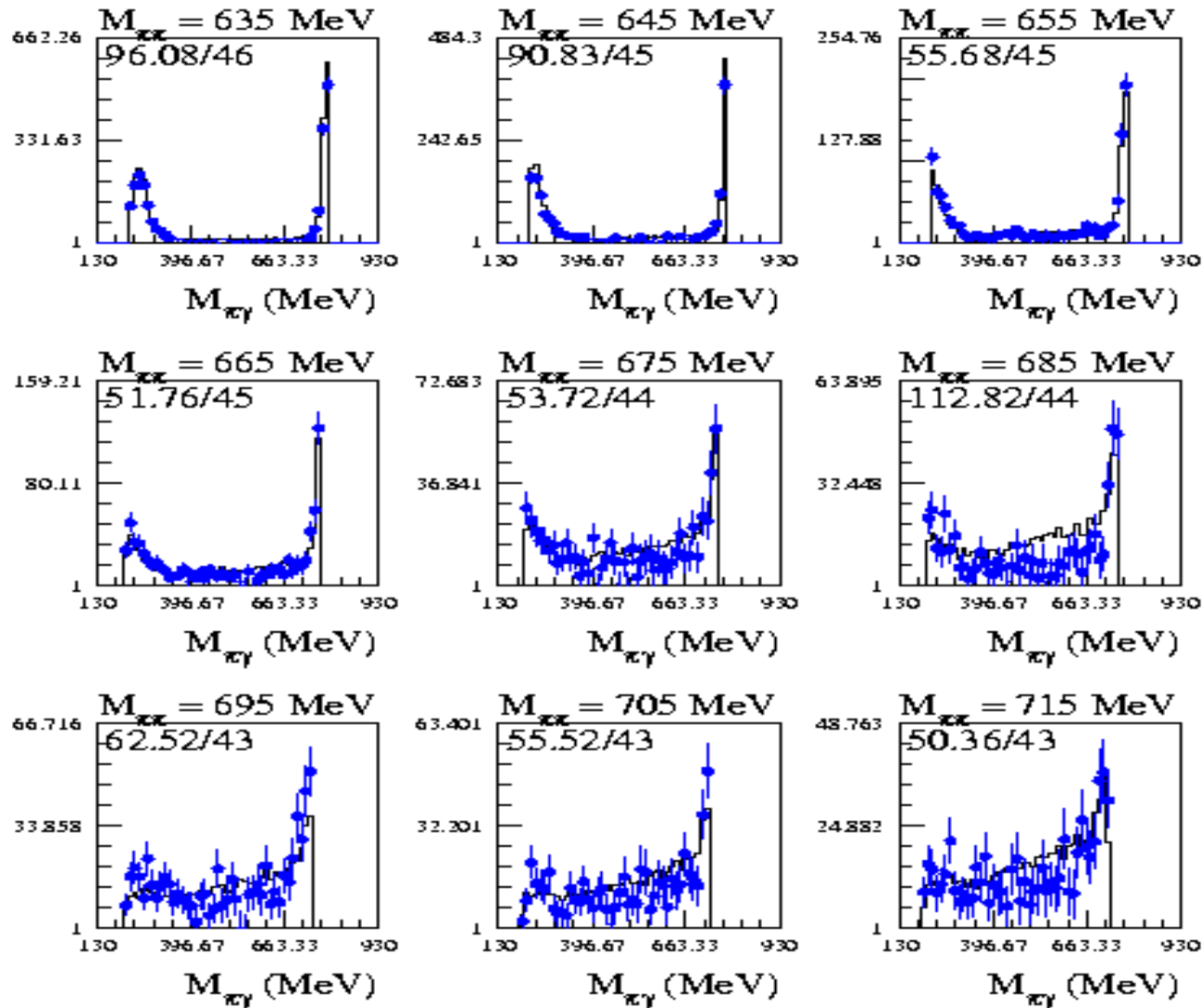


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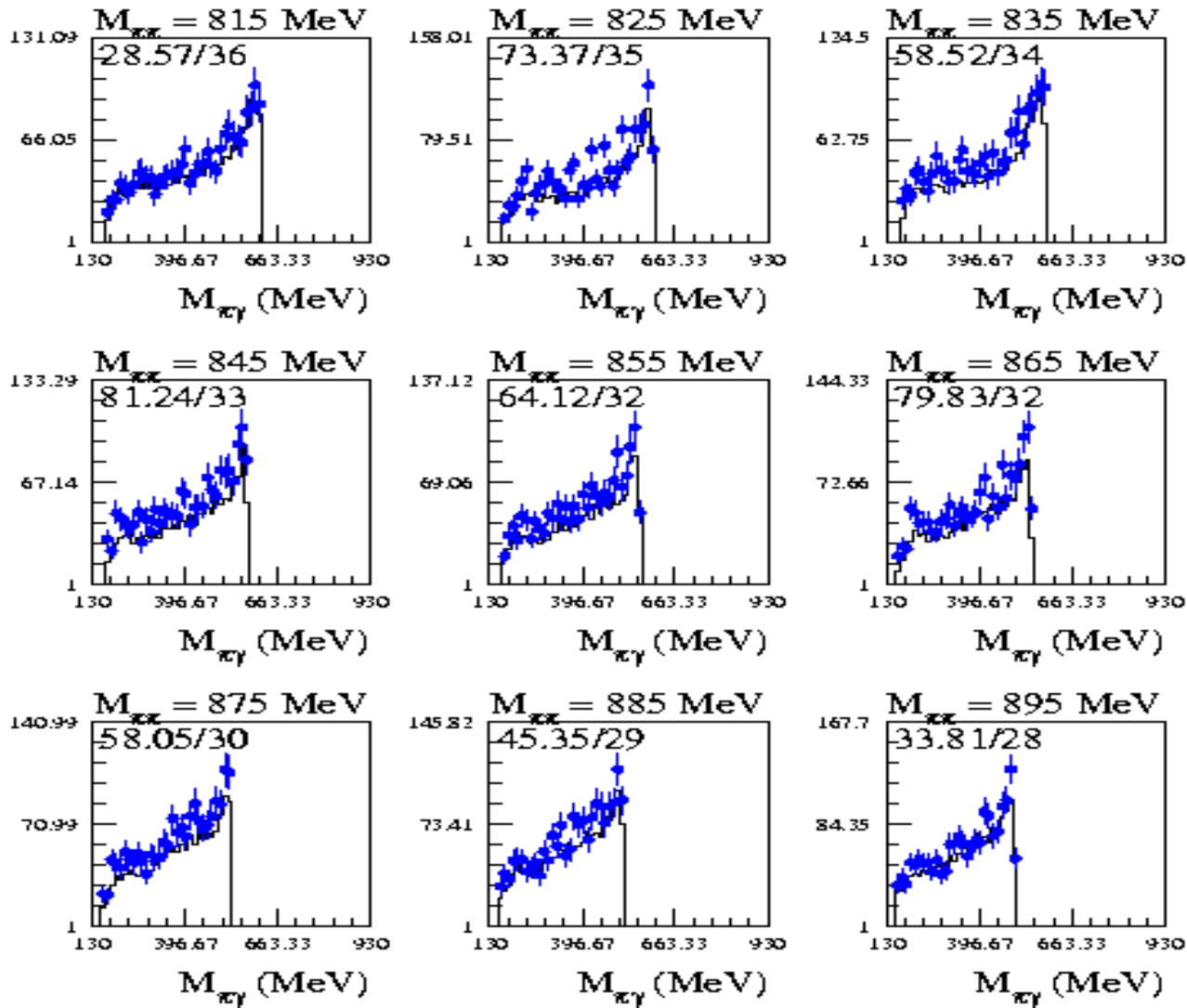
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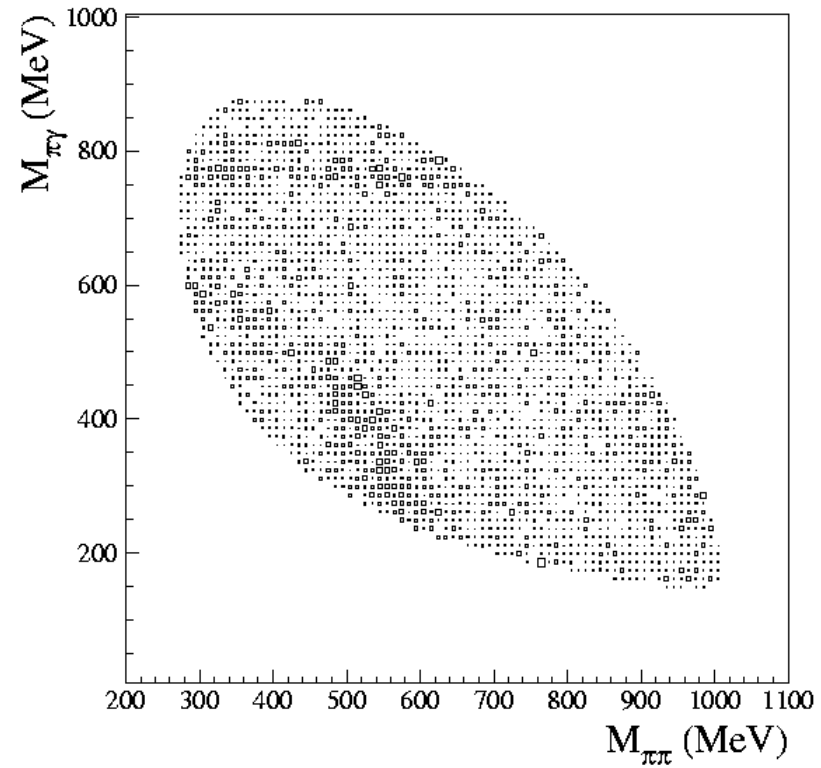
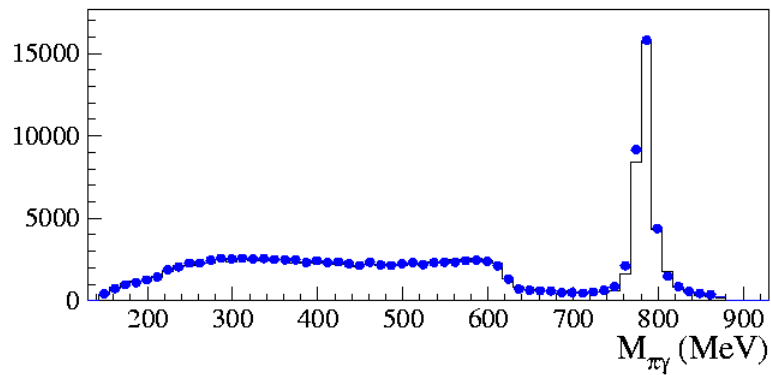
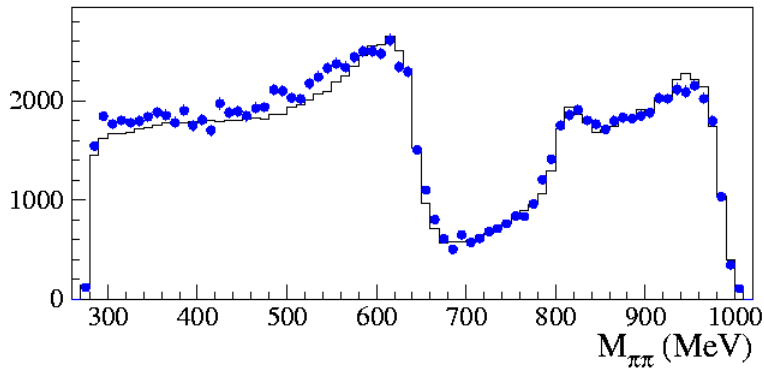
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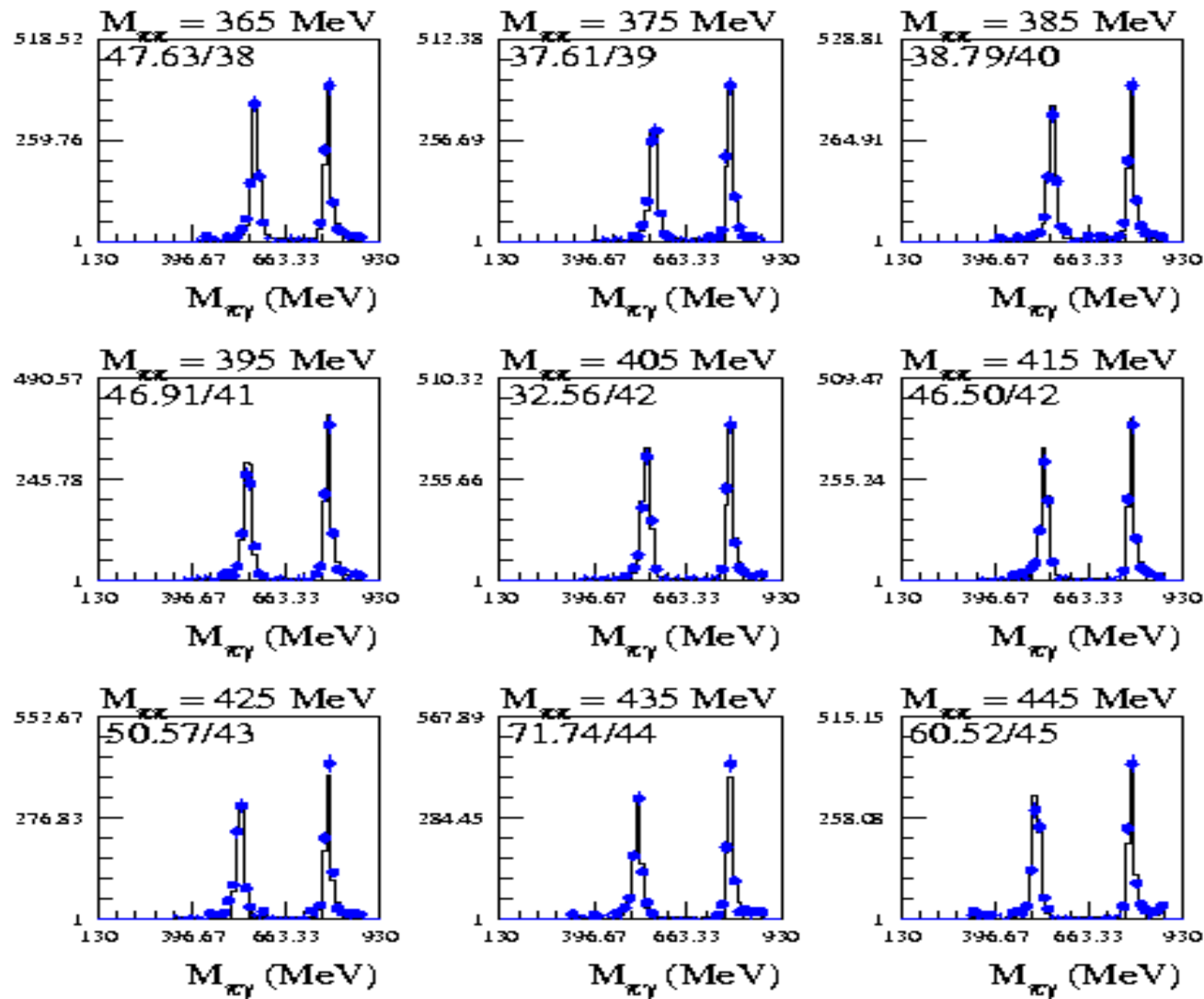
# Fit results: the Achasov parametrization



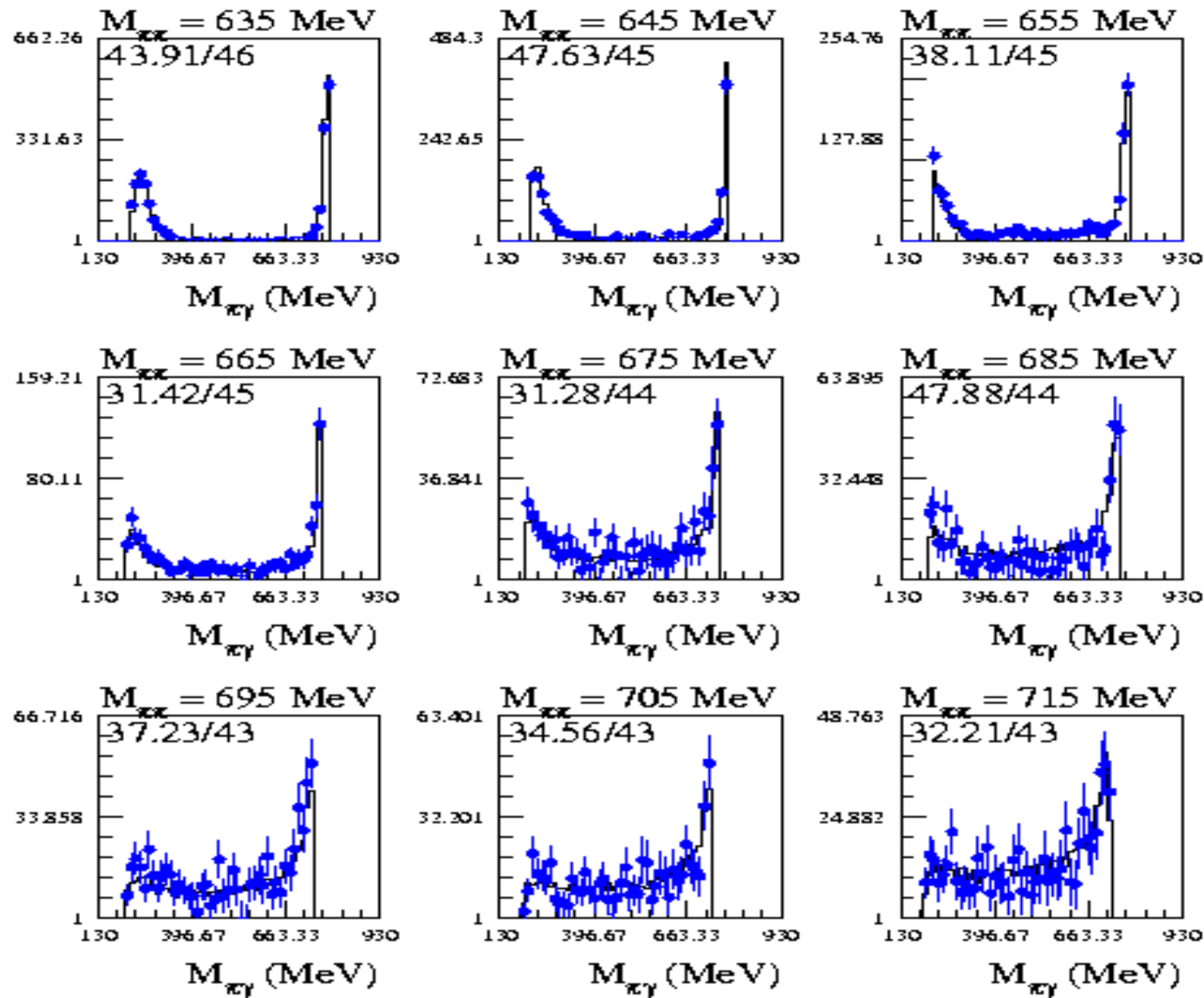
# Fit results: the Isidori-Maiani parametrization



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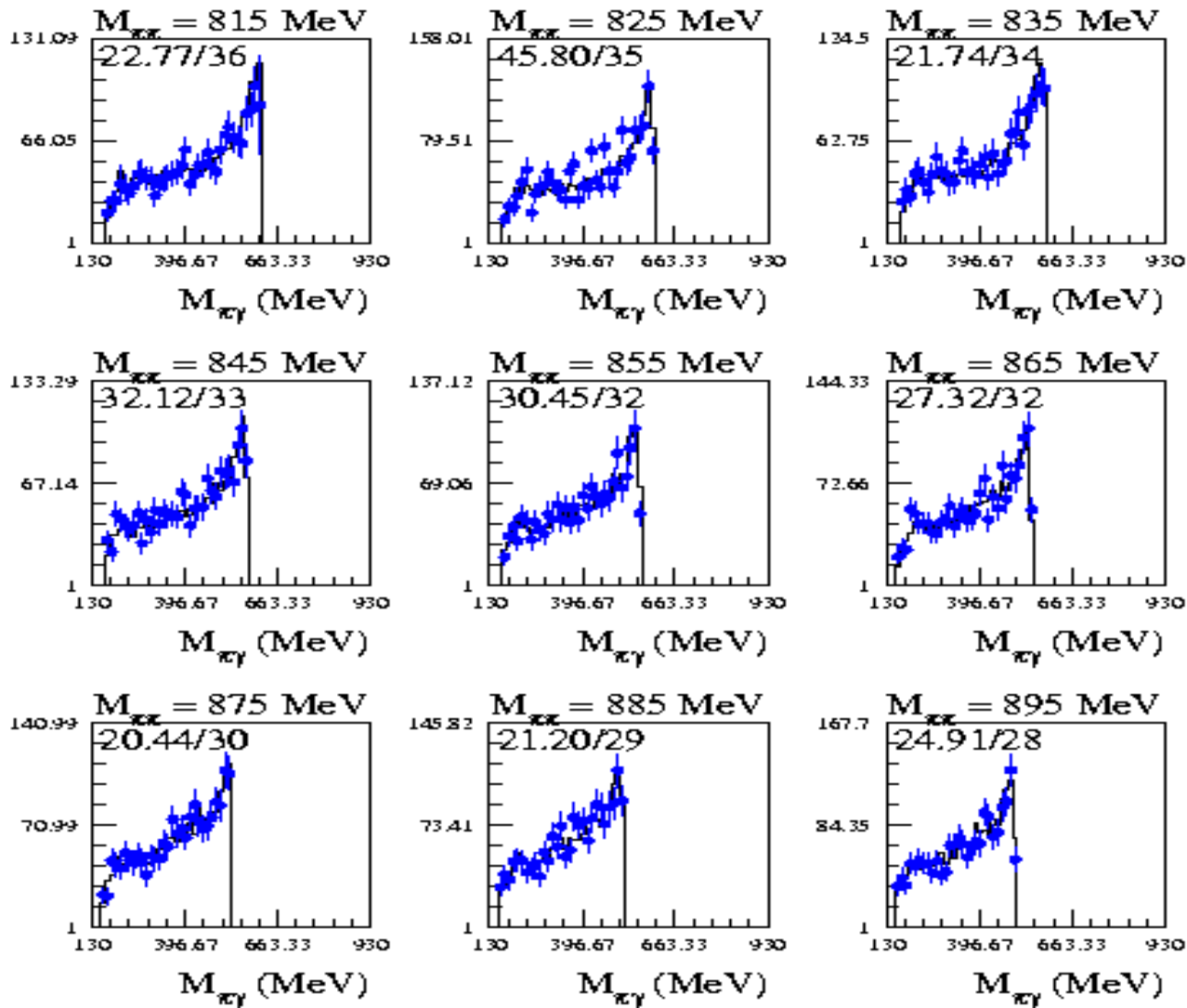


# Fit results: the Isidori-Maiani parametrization

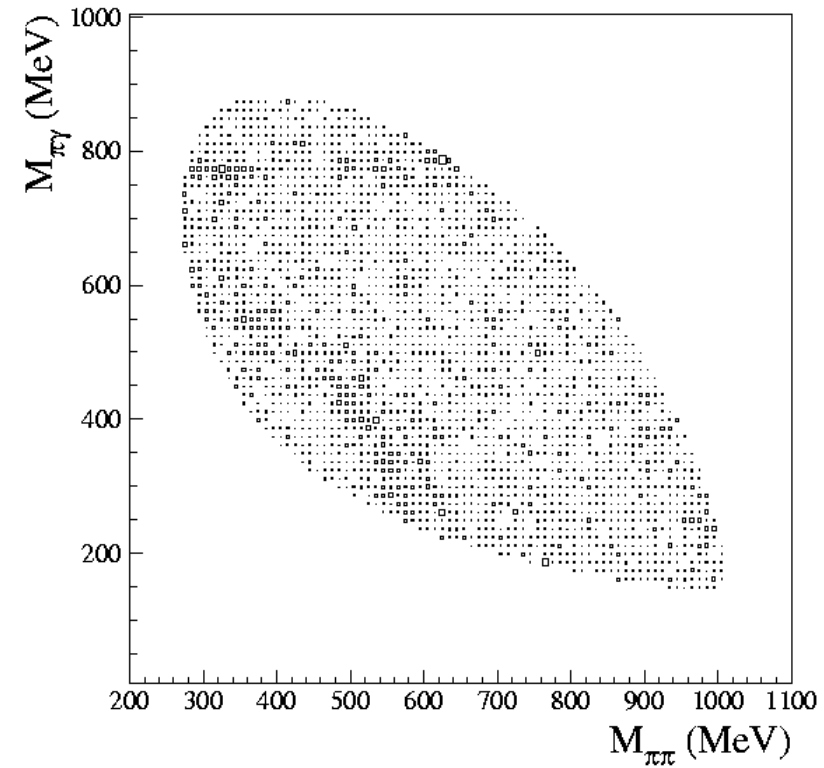
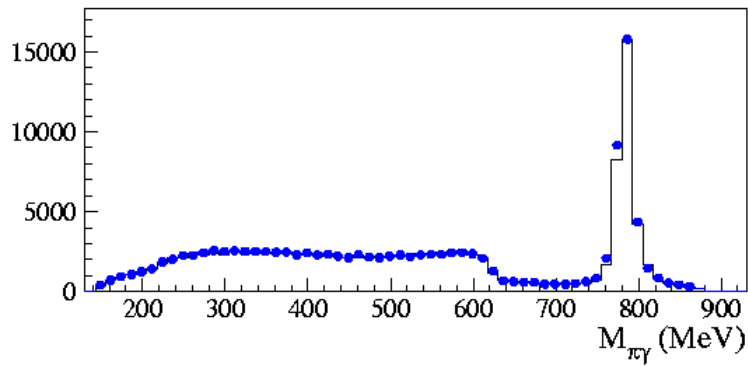
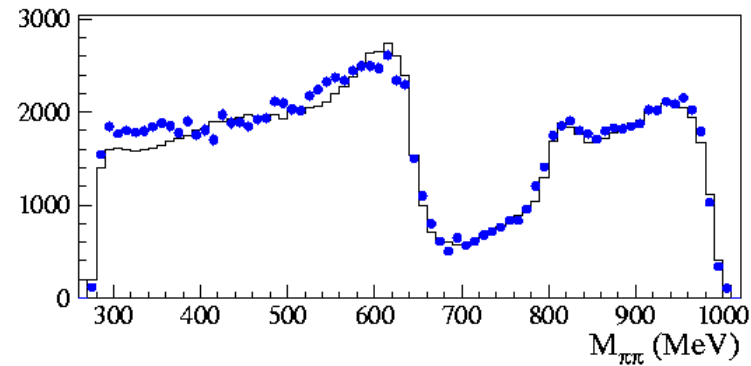




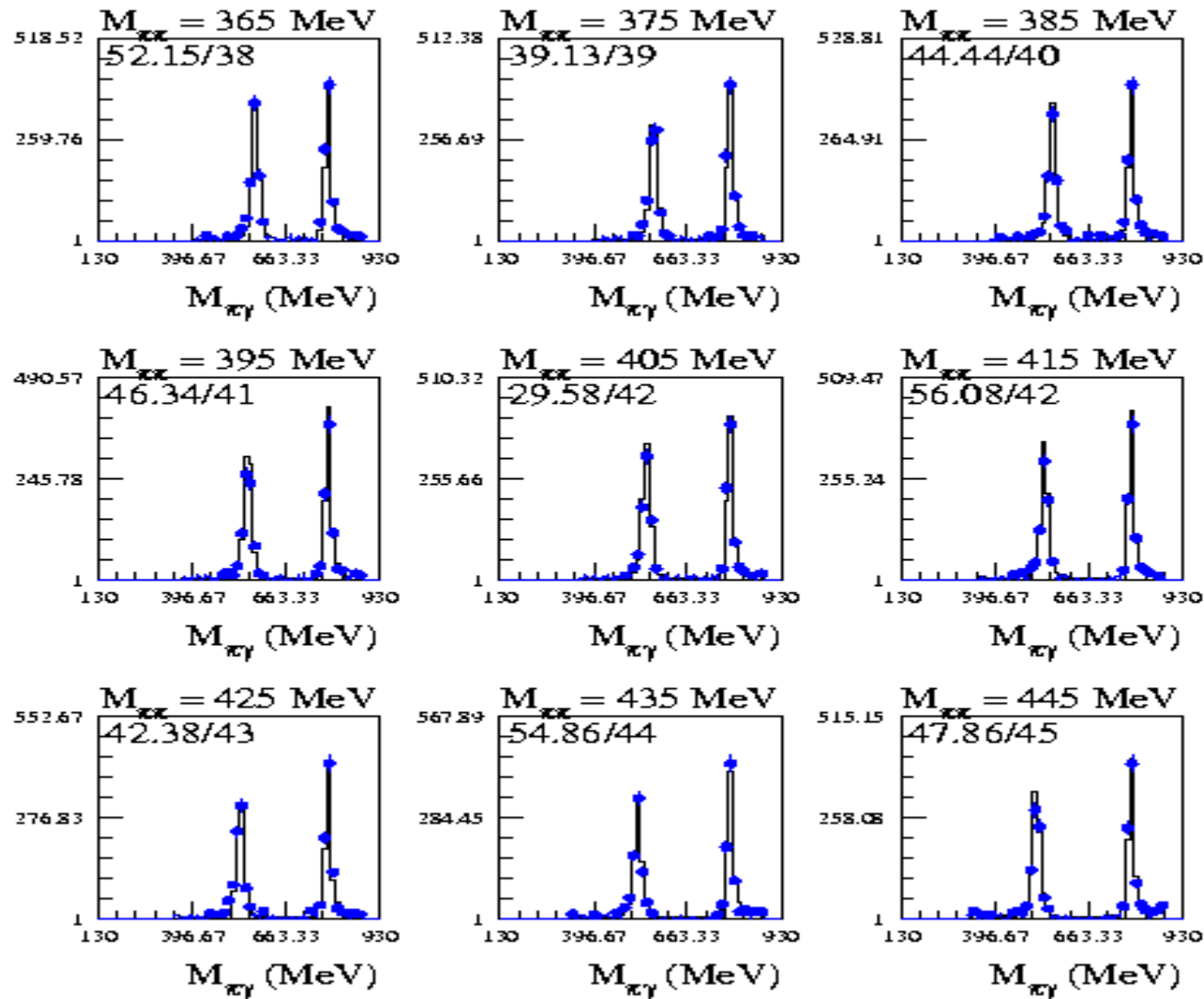
# Fit results: the Isidori-Maiani parametrization



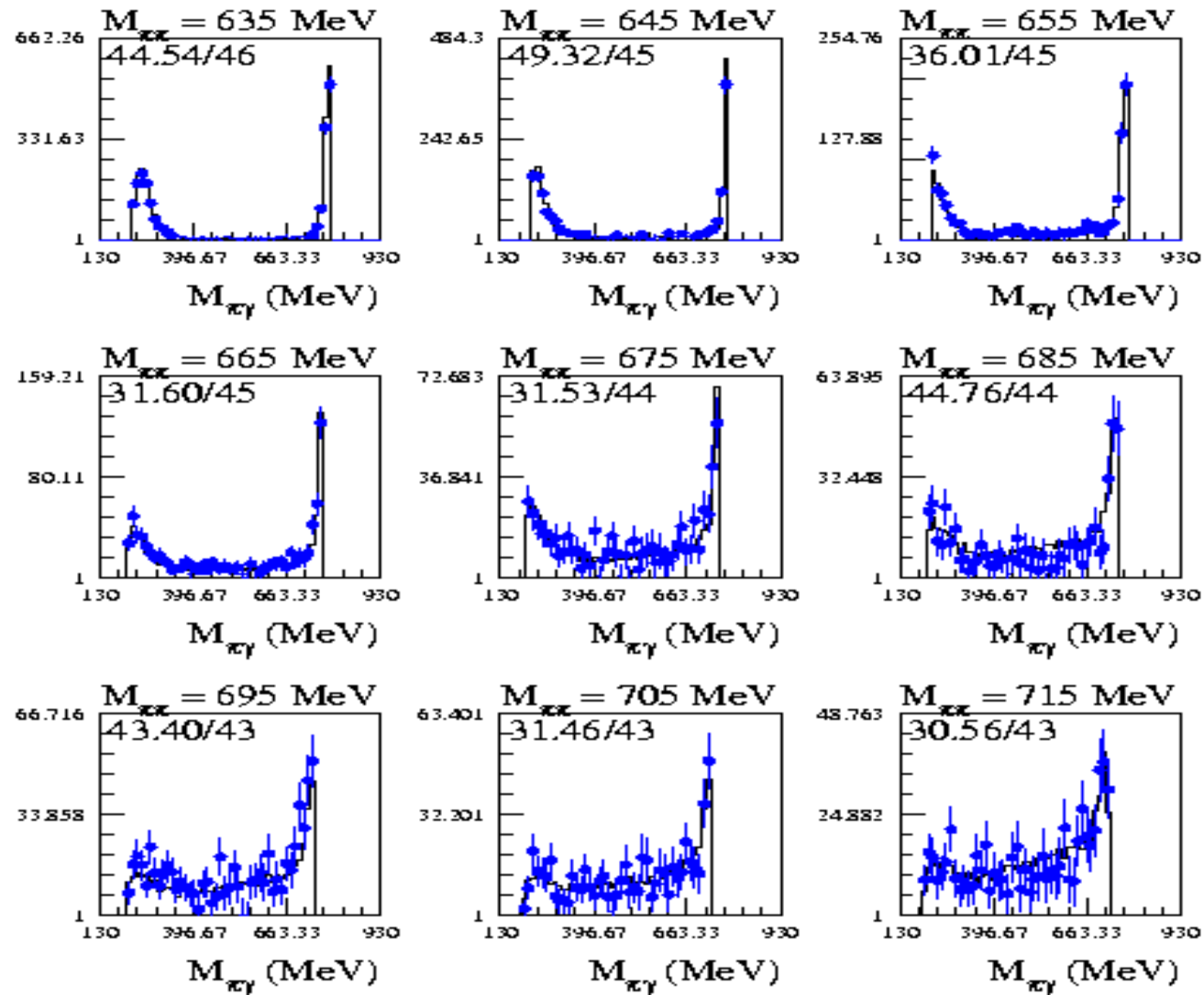
# Fit results: the Boglione-Pennington parametrization



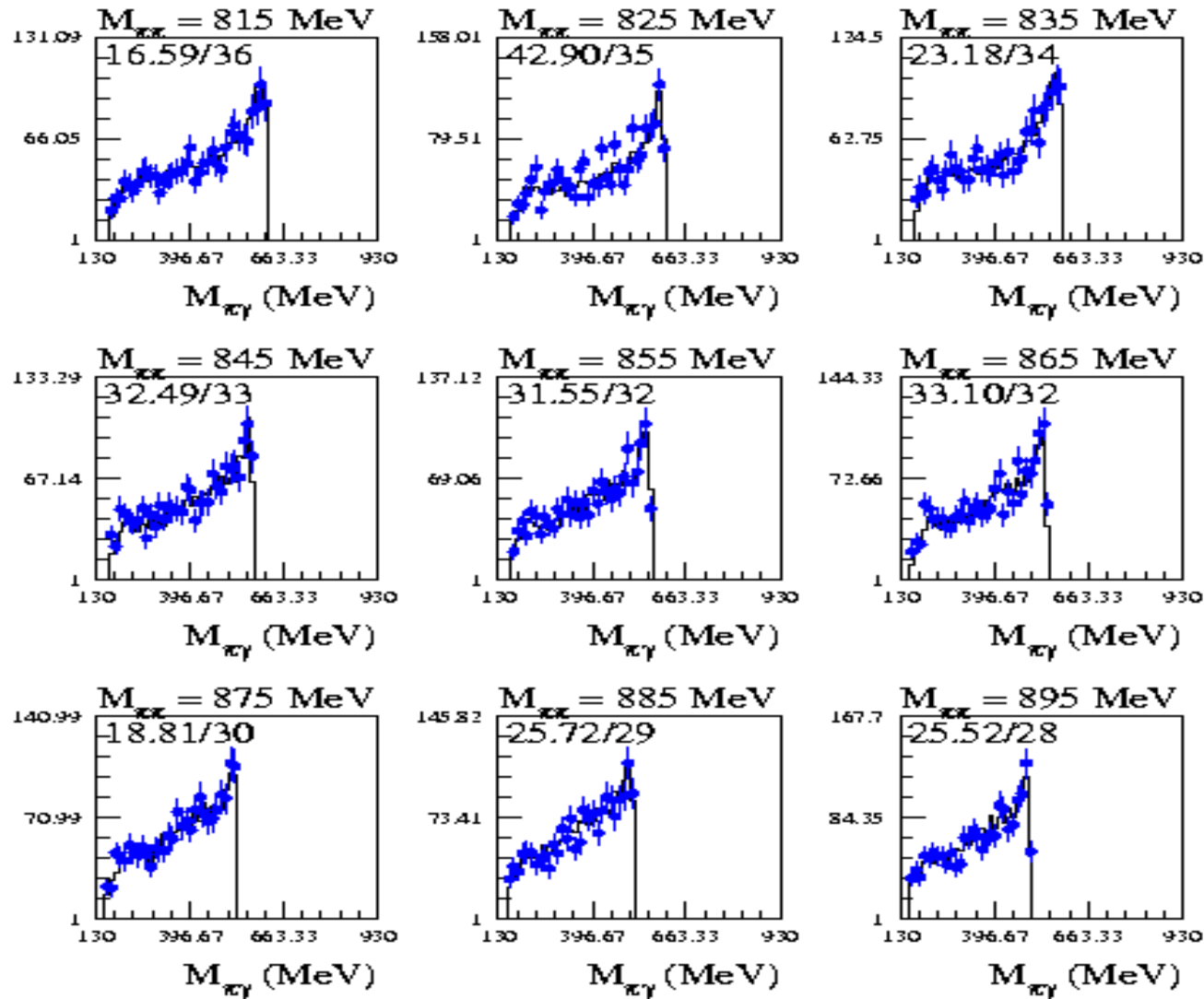
# Fit results: the Boglione-Pennington parametrization



# Fit results: the Boglione-Pennington parametrization



# Fit results: the Boglione-Pennington parametrization



# Fit results: the Achasov parametrization

	All free			$\Gamma_\omega = 8.49 \text{ MeV}$ $\Gamma_\rho = 146.4 \text{ MeV}$
vs (MeV)	1019.5	1019.6	1019.7	<b>1019.6</b>
$L_{\text{int}}$ (pb <sup>-1</sup> )	77.5	145.0	110.4	<b>145.0</b>
$M_{f_0}$ (MeV)	$962.6 \pm 0.4$	$962.2 \pm 0.2$	$964.0 \pm 0.2$	<b><math>962.3 \pm 0.6</math></b>
$g_{fK^+K^-}$ (GeV)	$4.33 \pm 0.04$	$4.42 \pm 0.03$	$4.59 \pm 0.02$	<b><math>4.44 \pm 0.05</math></b>
$g_{f\pi^+\pi^-}$ (GeV)	$2.23 \pm 0.01$	$2.28 \pm 0.01$	$2.31 \pm 0.01$	<b><math>2.29 \pm 0.01</math></b>
$\cos \phi$	$-0.06 \pm 0.04$	$0.16 \pm 0.04$	$0.02 \pm 0.04$	<b><math>0.16 \pm 0.04</math></b>
$M_\rho$ (MeV)	$780.0 \pm 0.7$	$780.0 \pm 0.2$	$780.0 \pm 0.4$	<b><math>780.0 \pm 0.2</math></b>
$\Gamma_\rho$ (MeV)	$150.0 \pm 3.3$	$150.0 \pm 1.1$	$150.0 \pm 1.1$	-
$M_\omega$ (MeV)	$781.9 \pm 0.1$	$782.25 \pm 0.07$	$781.95 \pm 0.05$	<b><math>782.2 \pm 0.1</math></b>
$\Gamma_\omega$ (MeV)	$9.00 \pm 0.01$	$9.000 \pm 0.008$	$9.000 \pm 0.006$	-
$\delta_{b\rho}$ (degree)	$78 \pm 6$	$95 \pm 2$	$94 \pm 2$	<b><math>95 \pm 3</math></b>
$K_{\text{VDM}}$	$0.84 \pm 0.02$	$0.870 \pm 0.005$	$0.861 \pm 0.005$	<b><math>0.806 \pm 0.006</math></b>
$\chi^2/\text{ndf}$	3529.3/2677 = 1.32	4188.1/2676 = 1.57	3688.6/2675 = 1.38	<b><math>4282.2/2678 = 1.60</math></b>

## Fit results: the Isidori-Maiani parametrization

	All free			$\Gamma_\omega = 8.49 \text{ MeV}$ $\Gamma_\rho = 146.4 \text{ MeV}$
vs (MeV)	1019.5	1019.6	1019.7	<b>1019.6</b>
$M_{f_0}$ (MeV)	$983.5 \pm 1.2$	$981.3 \pm 0.8$	$980.8 \pm 0.7$	<b><math>981.3 \pm 0.5</math></b>
$\Gamma_{f_0}$ (MeV)	$43.1 \pm 1.2$	$42.8 \pm 0.7$	$40.5 \pm 0.7$	<b><math>42.8 \pm 0.6</math></b>
$g_{\phi f_0} g_{f_0 \pi \pi}$	$2.11 \pm 0.07$	$1.99 \pm 0.04$	$1.91 \pm 0.03$	<b><math>2.00 \pm 0.02</math></b>
$a_0$	$3.7 \pm 0.3$	$3.2 \pm 0.1$	$2.8 \pm 0.1$	<b><math>3.22 \pm 0.05</math></b>
$a_1$	$1.0 \pm 0.3$	$0.6 \pm 0.1$	$0.1 \pm 0.1$	<b><math>0.60 \pm 0.06</math></b>
$\cos \phi$	$-0.85 \pm 0.08$	$-0.99 \pm 0.02$	$-0.88 \pm 0.05$	<b><math>-0.96 \pm 0.05</math></b>
$M_\rho$ (MeV)	$780.0 \pm 0.4$	$780.0 \pm 0.2$	$780.00 \pm 0.07$	<b><math>780.0 \pm 0.2</math></b>
$\Gamma_\rho$ (MeV)	$145.0 \pm 3.4$	$145.0 \pm 0.9$	$145.0 \pm 0.7$	-
$M_\omega$ (MeV)	$782.2 \pm 0.1$	$782.03 \pm 0.08$	$781.99 \pm 0.07$	<b><math>782.05 \pm 0.06</math></b>
$\Gamma_\omega$ (MeV)	$9.000 \pm 0.006$	$9.000 \pm 0.004$	$9.000 \pm 0.003$	-
$\delta_{b\rho}$ (degree)	$2 \pm 2$	$8 \pm 2$	$5 \pm 1$	<b><math>7 \pm 1</math></b>
$K_{\text{VDM}}$	$0.720 \pm 0.006$	$0.737 \pm 0.004$	$0.729 \pm 0.004$	<b><math>0.688 \pm 0.004</math></b>
$\chi^2/\text{ndf}$	$2613.2/2675 = 0.98$	$3081.3/2674 = 1.15$	$2917.5/2673 = 1.09$	<b><math>3355.7/2675 = 1.25</math></b>

# Fit results: the Boglione-Pennington parametrization

	All free			$\Gamma_\omega = 8.49 \text{ MeV}$ $\Gamma_\rho = 146.4 \text{ MeV}$
vs (MeV)	1019.5	1019.6	1019.7	<b>1019.6</b>
$m_0$ (MeV)	$580.2 \pm 5.1$	$345.5 \pm 0.6$	$471.6 \pm 3.2$	<b><math>547.4 \pm 3.2</math></b>
$a_1$	$11.44 \pm 0.03$	$9.345 \pm 0.001$	$6.934 \pm 0.005$	<b><math>19.49 \pm 0.05</math></b>
$b_1$	$2.08 \pm 0.01$	$-2.736 \pm 0.001$	$-18.55 \pm 0.01$	<b><math>-20.4 \pm 0.2</math></b>
$c_1$	$-11.75 \pm 0.03$	$-4.809 \pm 0.002$	$9.72 \pm 0.02$	<b><math>2.4 \pm 0.1</math></b>
$a_2$	$-15.03 \pm 0.04$	$-10.623 \pm 0.001$	$-10.148 \pm 0.007$	<b><math>-26.51 \pm 0.08</math></b>
$b_2$	$-11.85 \pm 0.01$	$-8.866 \pm 0.002$	$28.16 \pm 0.02$	<b><math>21.2 \pm 0.3</math></b>
$c_2$	$32.09 \pm 0.02$	$23.060 \pm 0.002$	$-14.39 \pm 0.01$	<b><math>10.0 \pm 0.2</math></b>
$\cos \phi$	$0.30 \pm 0.07$	$0.47 \pm 0.01$	$0.03 \pm 0.05$	<b><math>0.41 \pm 0.04</math></b>
$M_\rho$ (MeV)	$770.0 \pm 1.3$	$779.89 \pm 0.04$	$770.0 \pm 0.6$	<b><math>770.0 \pm 0.2</math></b>
$\Gamma_\rho$ (MeV)	$150.0 \pm 3.5$	$149.71 \pm 0.05$	$150.0 \pm 0.7$	-
$M_\omega$ (MeV)	$783.0 \pm 0.01$	$782.78 \pm 0.07$	$782.72 \pm 0.09$	<b><math>783.00 \pm 0.02</math></b>
$\Gamma_\omega$ (MeV)	$9.000 \pm 0.006$	$9.000 \pm 0.001$	$9.000 \pm 0.003$	-
$\delta_{b\rho}$ (degree)	$111 \pm 2$	$109 \pm 1$	$108 \pm 2$	<b><math>113 \pm 1</math></b>
$K_{\text{VDM}}$	$0.900 \pm 0.006$	$0.904 \pm 0.001$	$0.904 \pm 0.004$	<b><math>0.826 \pm 0.003</math></b>
$\chi^2/\text{ndf}$	$3056.4/2673 = 1.14$	$3211.3/2672 = 1.20$	$3483.9/2671 = 1.30$	<b><math>3984.6/2673 = 1.49</math></b>



# The parametrization with the $\sigma$ meson (I)

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The  $\sigma$  is introduced in the scalar term as in ref. PRD 56 (1997) 4084.

- The two resonances are not described by the sum of two BW but with the matrix of the inverse propagators  $G_{R_1 R_2}$ .
- Non diagonal terms on  $G_{R_1 R_2}$  are the transitions caused by the resonance mixing due to the final state interaction which occurred in the same decay channels  $R_1 \rightarrow ab \rightarrow R_2$

$$\frac{g_{f_0 K^+ K^-} g_{f_0 \pi^+ \pi^-}}{D_{f_0}(M_{\pi\pi})} \rightarrow \sum g_{R_1 k k} G_{RR}^{-1} g_{R_2 \pi\pi}$$

Where

$$G_{R_1 R_2} = \begin{pmatrix} D_{f_0} & -\Pi_{f_0 \sigma} \\ -\Pi_{\sigma f_0} & D_{\sigma} \end{pmatrix}$$

$$\Pi_{R_1 R_2} = \sum_{ab} g_{R_2 ab} P_{R_1}^{ab}(m) + C_{R_1 R_2}$$

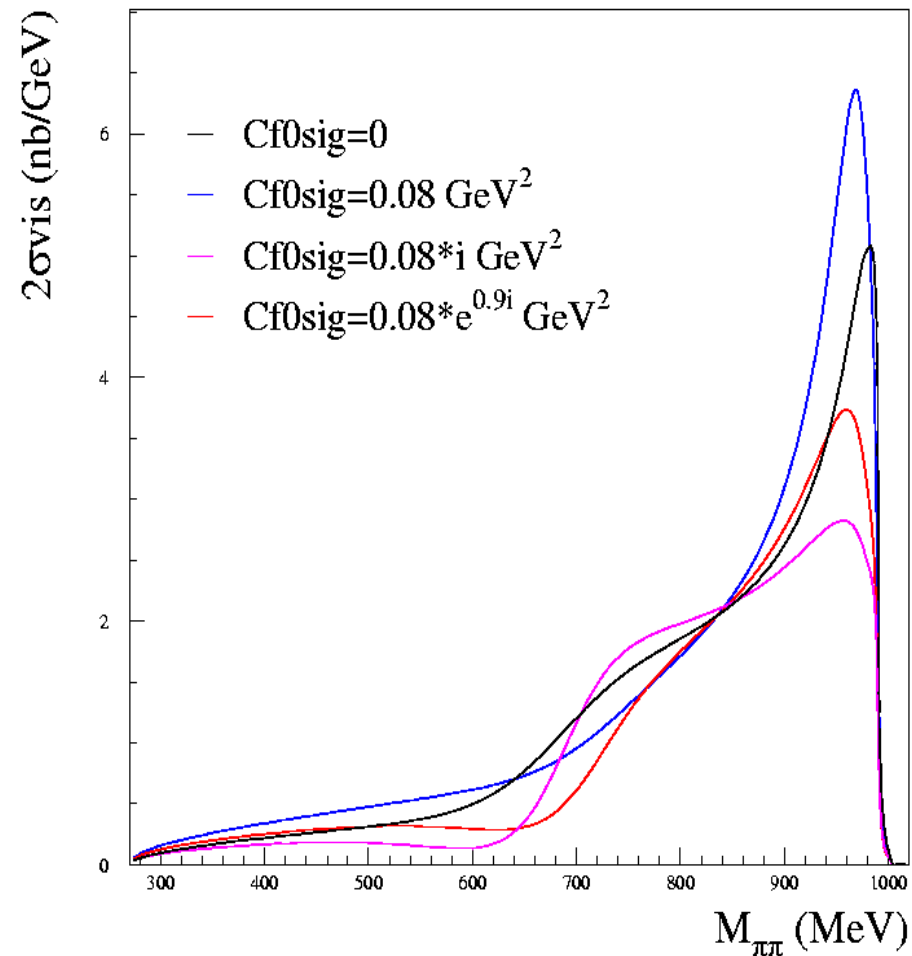
$C_{R_1 R_2} = C_{f_0 \sigma}$  takes into account the contributions of VV, 4 pseudoscalar mesons and other intermediate states. In the 4q,2q models there are free parameters

# The parametrization with the $\sigma$ meson (II)

Extensive tests have been done on the formula used.

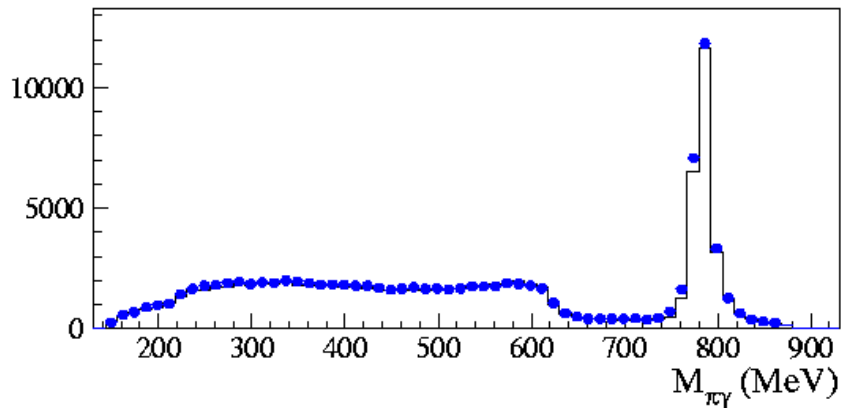
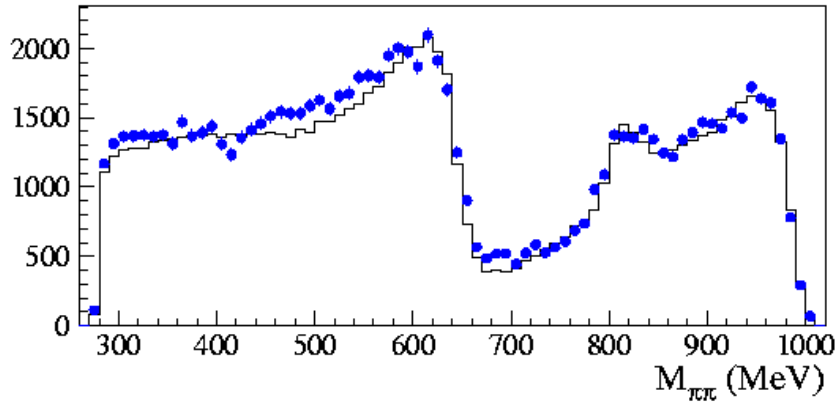
- Good agreement found between our coding and the one of Cesare we agreed that there is a mistype in the PRD
- We have asked also the help of G.Isidori-S.Pacetti to check this

The effect of the free term  $C_{f0\sigma}$  and of its phase is large



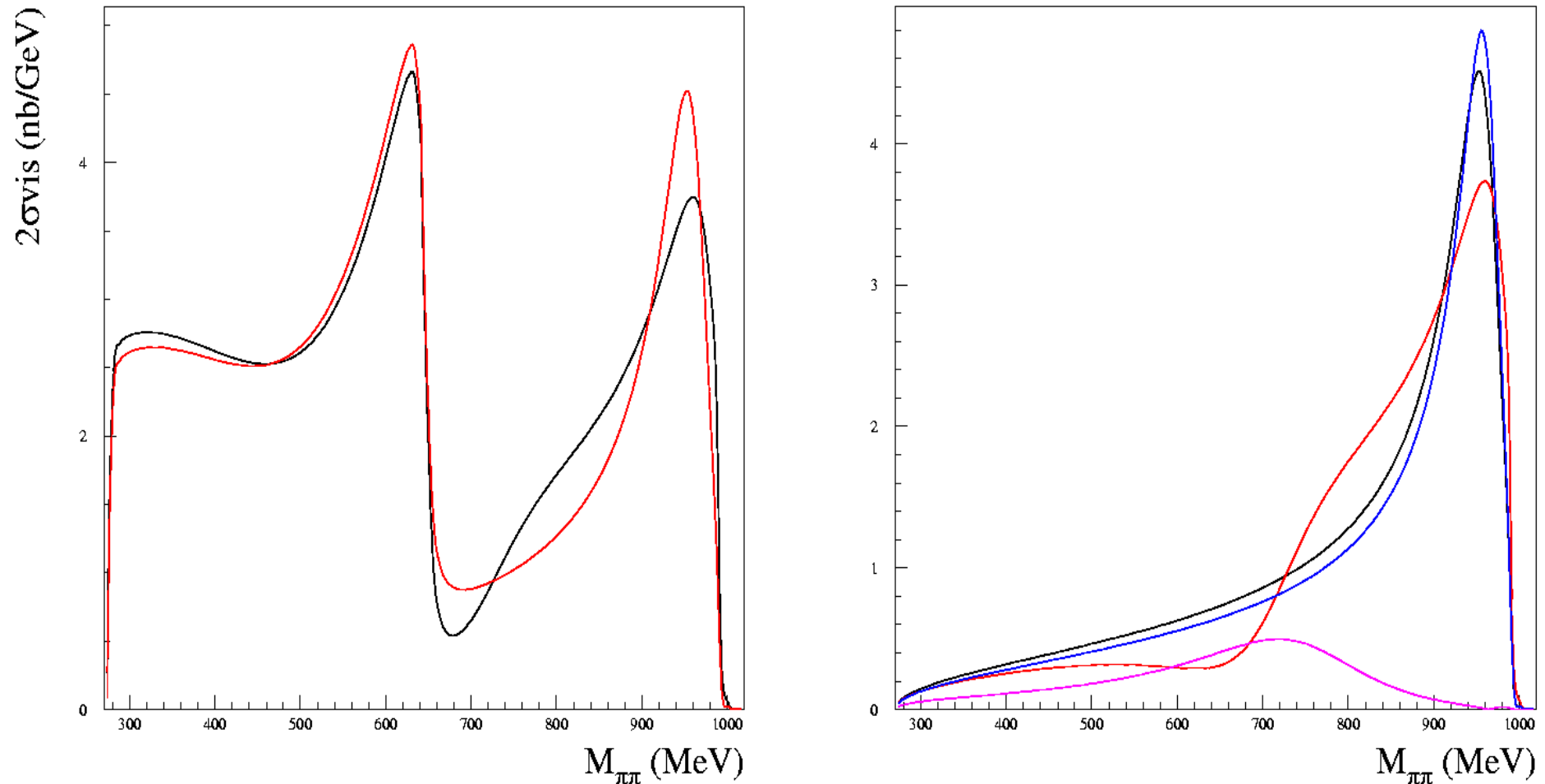
# Fit results: the Achasov parametrization with $\sigma$ (I)

Fit @ 1019.7 MeV  
SIMPLEX only



	$f_0 + \sigma$	$f_0$ only
$M_{f_0}$ (MeV)	963.7	$964.0 \pm 0.2$
$g_{fK^+K^-}$ (GeV)	4.41	$4.59 \pm 0.02$
$g_{f\pi^+\pi^-}$ (GeV)	2.18	$2.31 \pm 0.01$
$M_\sigma$ (MeV)	741.2	-
$G_{\sigma\pi\pi}$ (GeV)	2.35	-
$ C_{f_0\sigma} $ (GeV)	81.1	-
$\phi_{f_0\sigma}$	0.68	-
$\cos \phi$	0.28	$0.02 \pm 0.04$
$M_\rho$ (MeV)	780.0	$780.0 \pm 0.4$
$\Gamma_\rho$ (MeV)	150.0	$150.0 \pm 1.1$
$M_\omega$ (MeV)	781.91	$781.95 \pm 0.05$
$\Gamma_\omega$ (MeV)	8.91	$9.000 \pm 0.006$
$\delta_{b\rho}$ (degree)	98	$94 \pm 2$
$K_{\text{VDM}}$	0.848	$0.861 \pm 0.005$
$\chi^2/\text{ndf}$	$3072.1/2679 = 1.15$	$3688.6/2675 = 1.38$

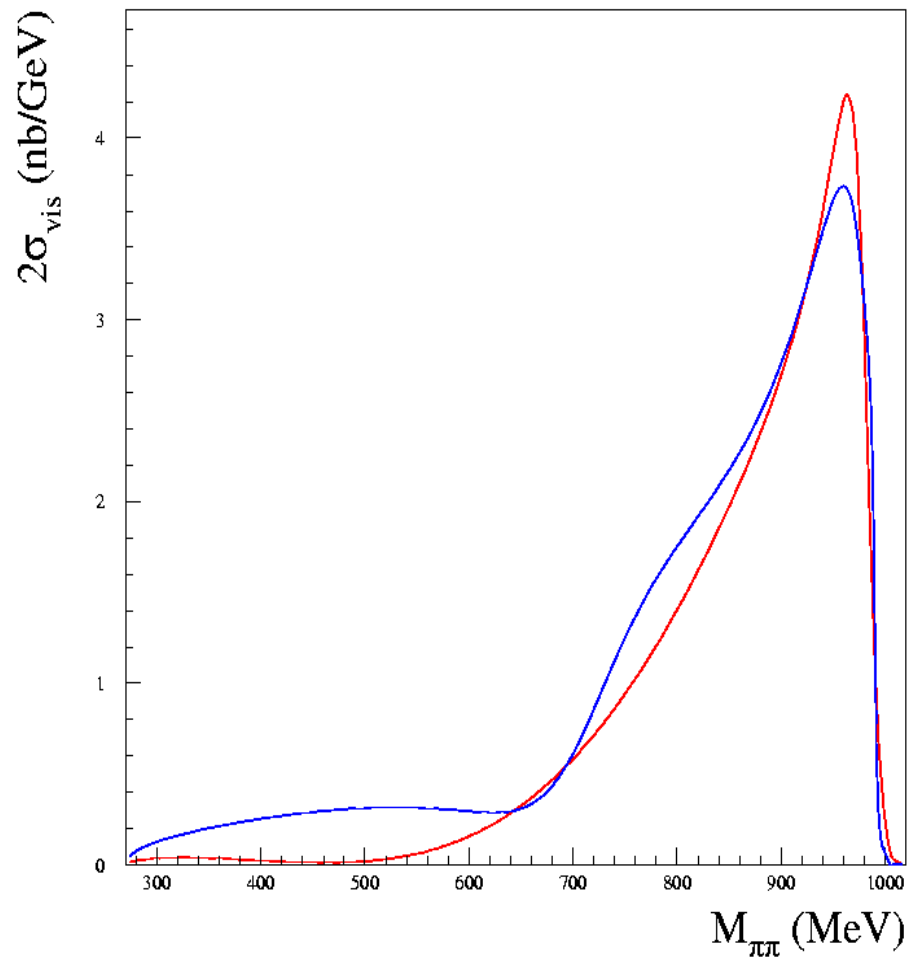
## Fit results: the Achasov parametrization with $\sigma$ (II)



- Black (red) curve are ACH model with (without) the inclusion of the  $\sigma$  meson
- Blue (purple) curve are the contribution due to the  $f_0$  ( $\sigma$ ) meson only with the ACH model when including the  $\sigma$  meson

# Comparison between ACH-IM for the scalar term

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Without the inclusion of the  $\sigma$  meson the agreement between ACH model and IM is not excellent although the integrals do not differ more than 20% above 700 MeV . Including the  $\sigma$  the agreement is better!

## Conclusions

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- ❖ Fit results start giving reasonable results. Improvement due to: better binning, reduced free VDM parameters (overall scale factor +  $\rho$ ,  $\omega$  masses). Is the interference phase added in the right way?
- ❖ Systematics still to be included in the fit
- ❖ Achasov model without sigma does not provide a good fit to data  
Parameters in agreement with our old analysis
- ❖ The Isidori-Maiani function better describes the data  
Still some doubts in the use of the  $\pi\pi$  scattering phase
- ❖ The Boglione-Pennington parametrization provide a very unstable fit, with very different parameters for different  $v_s$
- ❖ A preliminary test including  $\sigma$  in the kaon loop model shows an improvement of the fit