Determination of $\eta \rightarrow \pi^0 \pi^0 \pi^0$ Dalitz plot slope

F.Ambrosino-T. Capussela-F.Perfetto

- Motivations
- Method
- Systematic checks
- Conclusions and outlook

$\eta \to 3\pi$ in chiral theory

The decay $\eta \to 3\pi$ occours primarily on account of the d-u quark mass differences and the result arising from lowest order chiral pertubation theory is well known:

$$A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\frac{M(s,t,u)}{3\sqrt{3}F_\pi^2}$$

With: $$Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$ And, at l.o. $$M(s,t,u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$$

A good understanding of M(s,t,u) can *in principle* lead to a very accurate determination of Q:

$$\Gamma(\eta \to 3\pi) \propto |A|^2 \propto Q^{-4}$$

…and its open questions

Still there are some intriguing questions for this decay :

•Why is it experimental width so large (270 eV) w.r.t theoretical calculation ? (Tree level: 66 eV (!!!); 1 loop : 160 eV) Possible answers:

- •Final state interaction
- •Scalar intermediate states
- •Violation of Dashen theorem

•Is the dynamics of the decay correctly described by theoretical calculations ?

$\eta \rightarrow 3\pi^0$: Dalitz plot expansion

The dynamics of the $\eta \rightarrow \pi^0 \pi^0 \pi^0$ decay can be studied analysing the Dalitz plot distribution.

The Dalitz plot density ($|A|^2$) is specified by a single parameter:

$$|A|^2 \propto 1 + 2\alpha z$$

with:

$$z = \frac{2}{3}\sum_{i=1}^{3}\left(\frac{3E_i - m_\eta}{m_\eta - 3m_{\pi^0}}\right)^2 = \frac{\rho^2}{\rho^2_{\max}}$$

$Z \in [\, 0 \quad , \quad 1\,]$

E_i = Energy of the *i*-th pion in the η rest frame.
ρ = Distance to the center of Dalitz plot.
ρ_{max} = Maximun value of ρ.

Dalitz expansion: theory vs experiment

Calculation	α
Tree	0.00
One-loop[1]	0.0015
Dispersive[2]	-0.007 - -0.014
Tree dispersive	-0.006
Absolute dispersive	-0.007

[1] Gasser,J. and Leutwyler, H., Nucl. Phys. B **250**, 539 (1985)
[2] Kambor, J., Wiesendanger, C. and Wyler, D., Nucl. Phys. B **465**, 215 (1996)

Alde (1984)	-0.022 ± 0.023
Crystal Barrel (1998)	-0.052 ± 0.020
Crystal Ball (2001)	-0.031 ± 0.004

Sample selection

The cuts used to select: $\eta \to \pi^0 \pi^0 \pi^0$ are :

- 7 and only 7 prompt neutral clusters with $21^\circ < \theta_\gamma < 159^\circ$ and $E_\gamma > 10$ MeV
- opening angle between each couple of photons > 18°
- Kinematic Fit with no mass constraint
- $P(\chi 2) > 0.01$
- 320 MeV < $E_{\gamma rad}$ < 400 MeV (after kin fit)

With these cuts the expected contribution from events other than the signal is <0.5%

Photons pairing

Recoil γ is the most energetic cluster.
In order to match every couple of photon to the right π^0 we build a χ^2-like variable for each of the 15 combinations:

$$\chi_j^{\,2} = \sum_{i=1}^{3} \left(\frac{m^j_{\pi^0_i} - M_{\pi^0}}{\sigma^j_{m_{\pi^0}}} \right)^2$$

With:

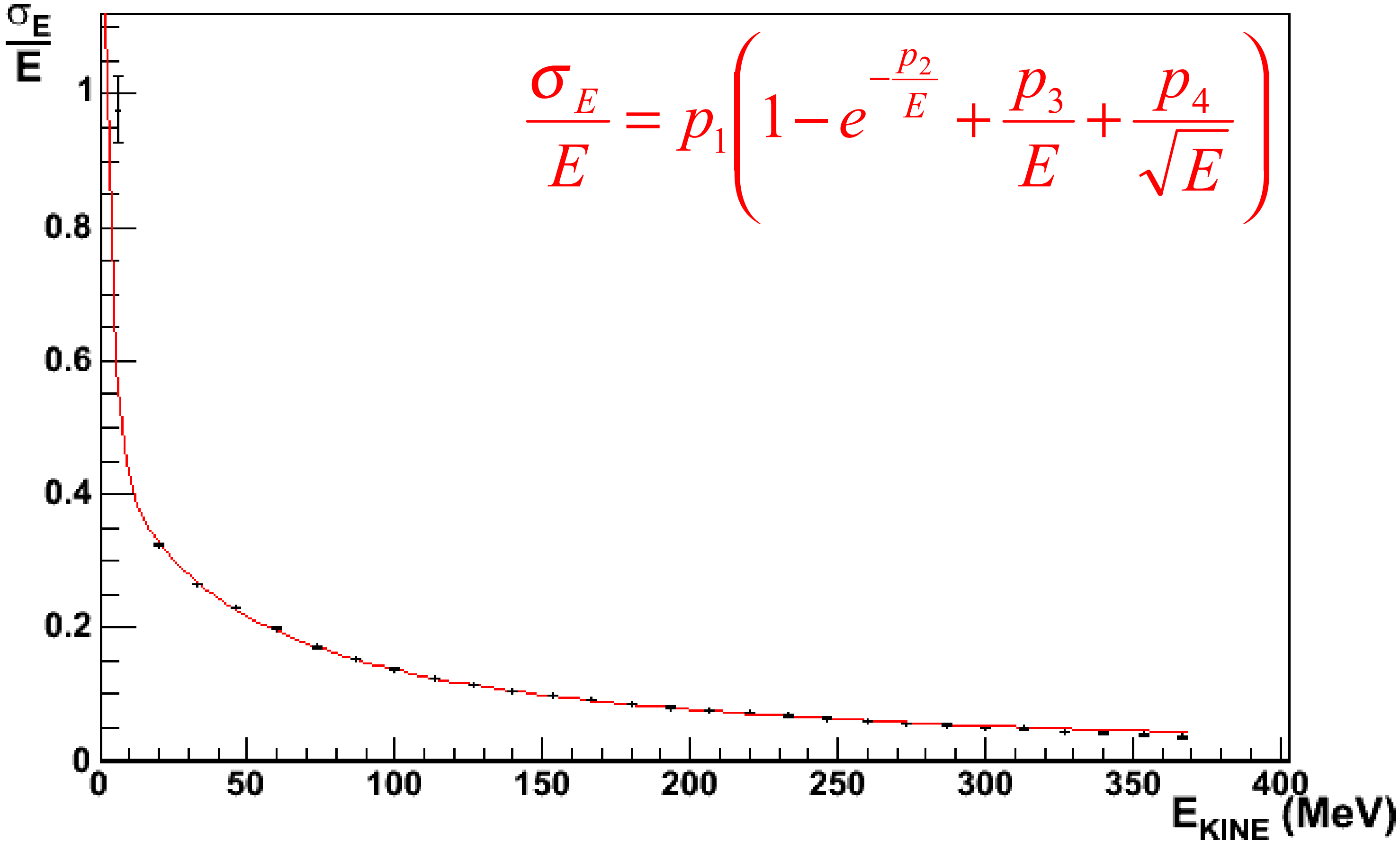
$m^j_{\pi^o_i}$ is the invariant mass of π_i^0 for j-th combination

M_{π^0} = 134.98 MeV

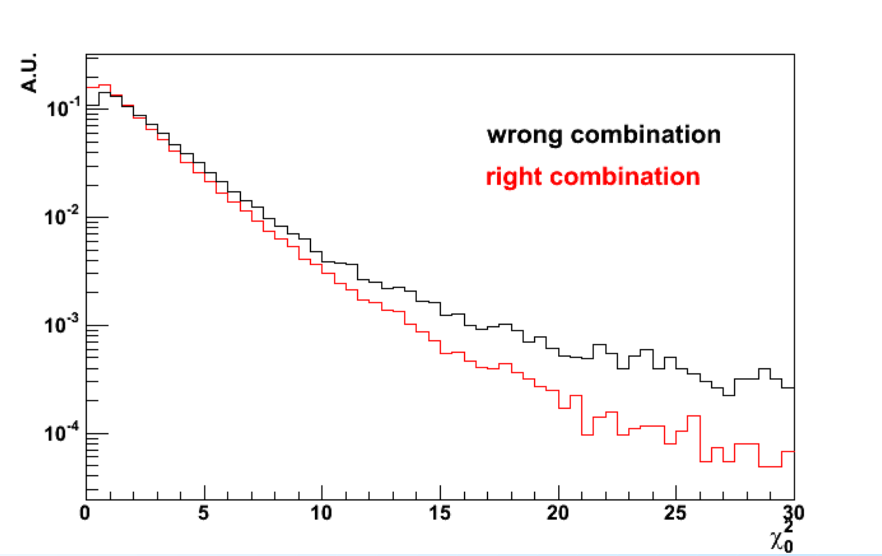
$\sigma^j_{m_{\pi^0}}$ is obtained as function of photon energies

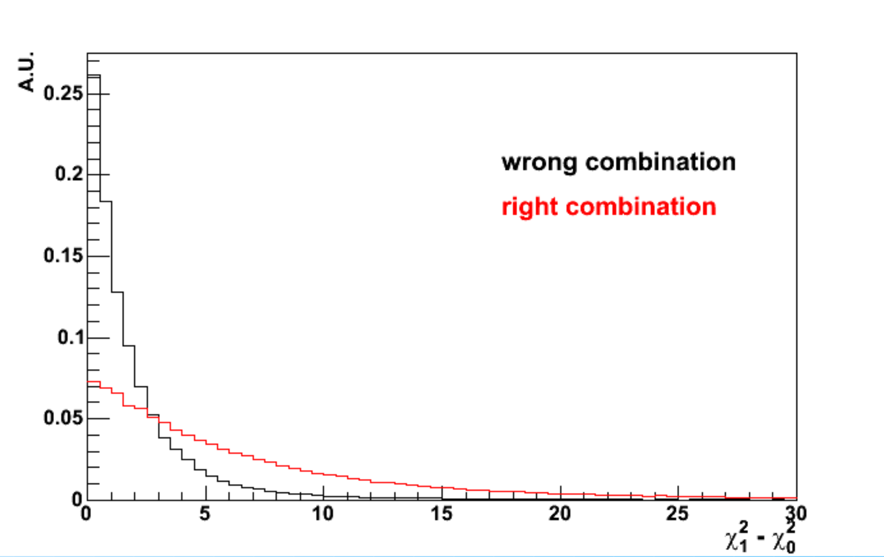
Energy resolution

$$\frac{\sigma_M}{M} = \frac{1}{2}\left(\frac{\sigma_{E_1}}{E_1} \oplus \frac{\sigma_{E_2}}{E_2}\right)$$



Combination selection





Cutting on:

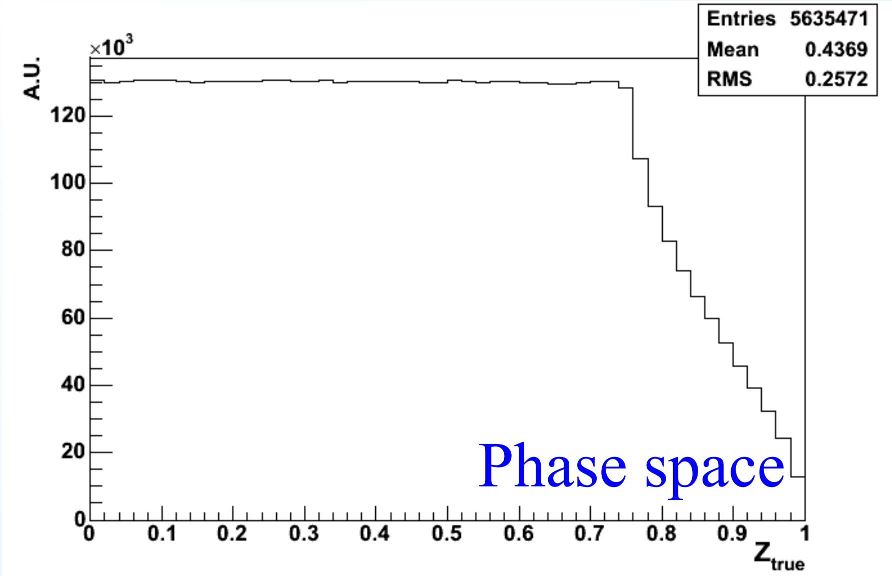
- Minimum χ^2 value
- $\Delta\chi^2$ between "best" and "second" combination

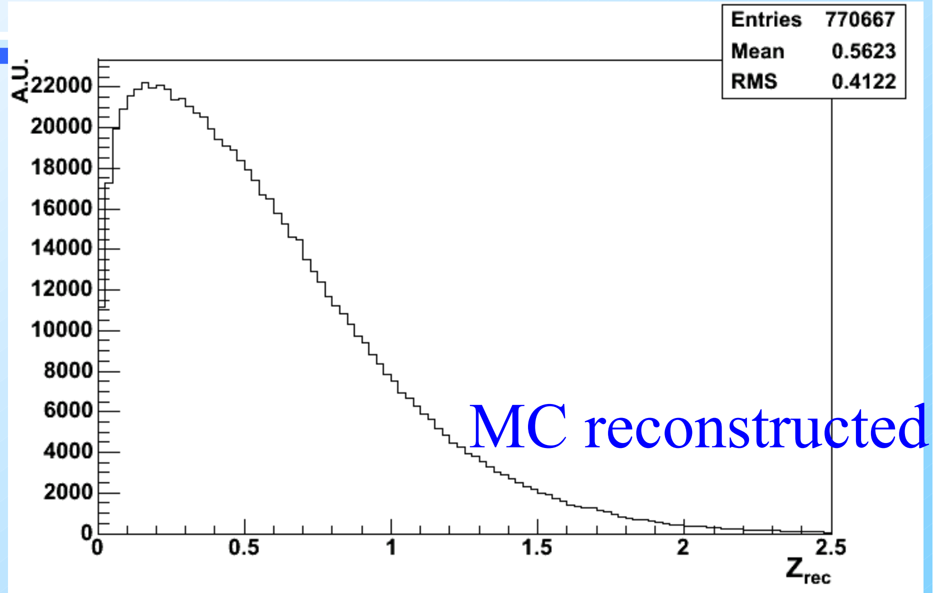
One can obtain samples with different purity-efficiency

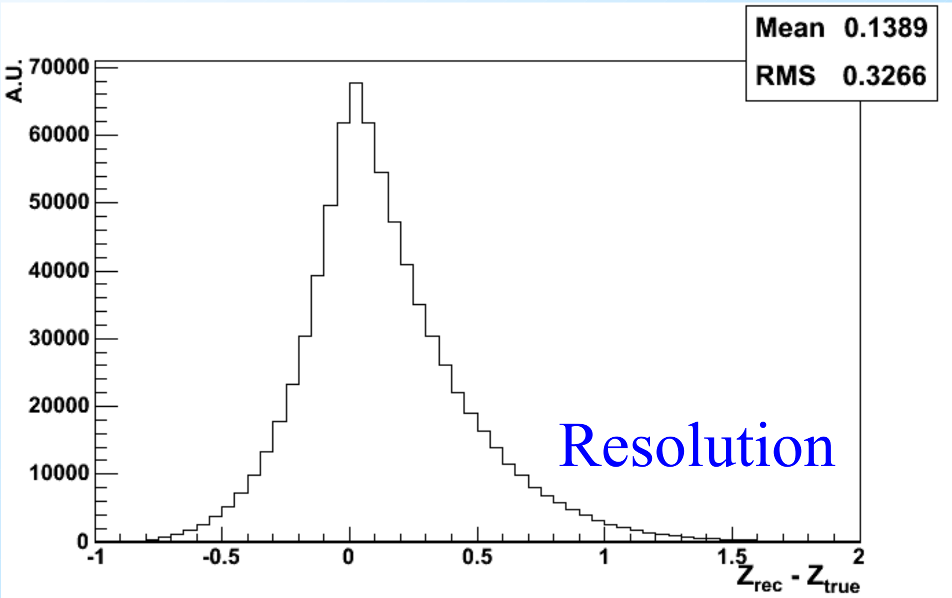
Purity= Fraction of events with all photons correctly matched to π^0 's

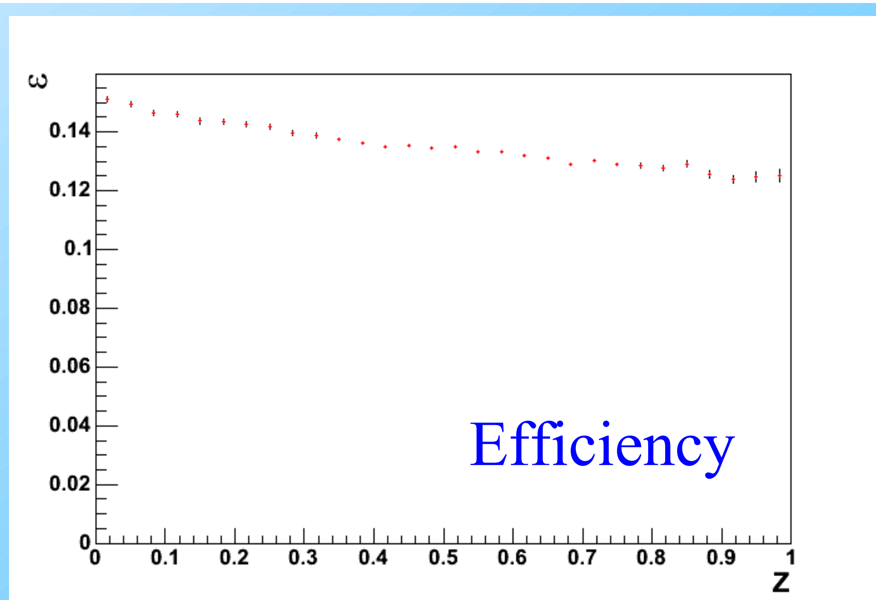
Pur ≈ 85 % Eff ≈ 22 %	Pur ≈ 92 % Eff ≈ 14 %	Pur ≈ 98 % Eff ≈ 4.5 %

The problem of resolution



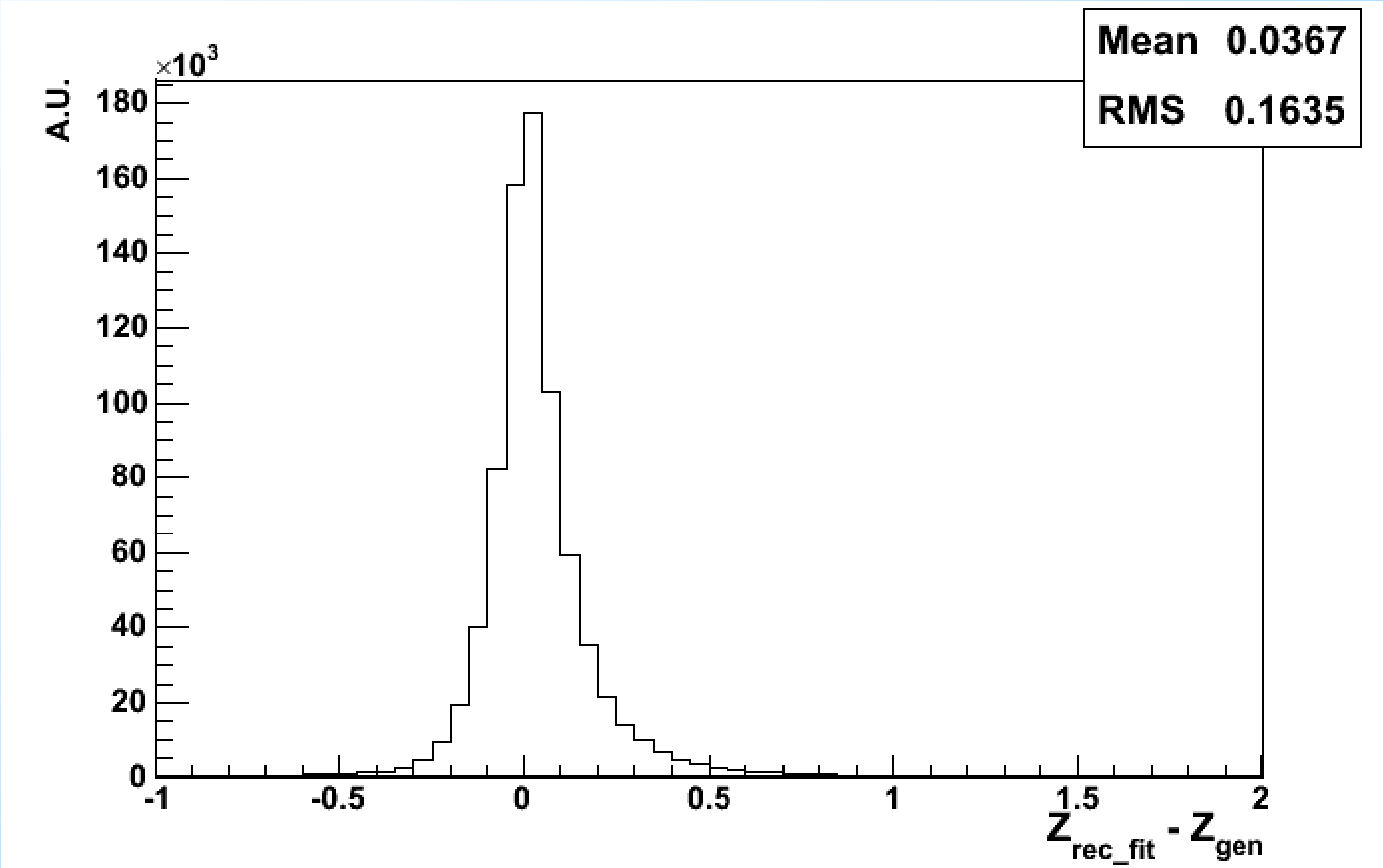


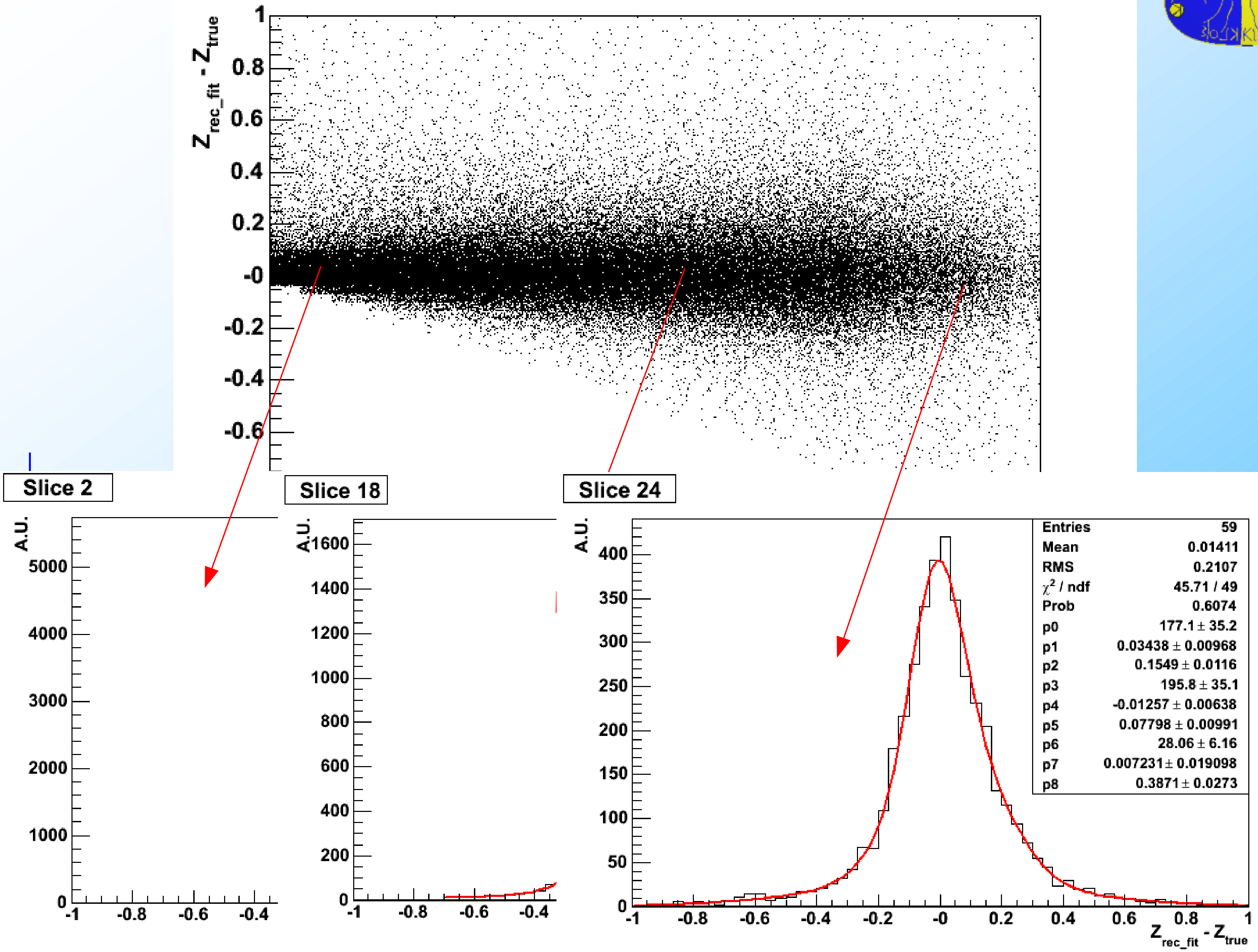




Second kinematic fit

Once a combination has been selected, one can do a second kinematic fit imposing π^0 mass for each couple of photons.





Z_rec_fit - Z_true
1
0.8
0.6
0.4
0.2
-0
-0.2
-0.4
-0.6
Slice 2
Slice 18
Slice 24
A.U.
5000
4000
3000
2000
1000
0
1600
1400
1200
1000
800
600
400
200
400
350
300
250
200
150
100
50
-1
-0.8
-0.6
-0.4
-0.2
0.2
0.4
0.6
0.8
Entries 59
Mean 0.01411
RMS 0.2107
χ2 / ndf 45.71 / 49
Prob 0.6074
p0 177.1 ± 35.2
p1 0.03438 ± 0.00968
p2 0.1549 ± 0.0116
p3 195.8 ± 35.1
p4 -0.01257 ± 0.00638
p5 0.07798 ± 0.00991
p6 28.06 ± 6.16
p7 0.007231± 0.019098
p8 0.3871 ± 0.0273

Fit procedure

The fit is done using a binned likelihood approach

We obtain an extimate $\hat{\alpha}$ by minimizing

$$-\sum_i n_i \log(\nu_i(\alpha))$$

Where:

n_i = recostructed events
ν_i = from MC truth folded with efficiency and resolution and weighted with theoretical function

Folding procedure (I)

In principle the "test histogram" can be obtained as follows

$$\frac{d\nu(j)}{dz} = \sum_{bin} \varepsilon(i) g(i,j) f_{th}(i) \left(\frac{dN(i)}{dz} \right)^{Phase-space}$$

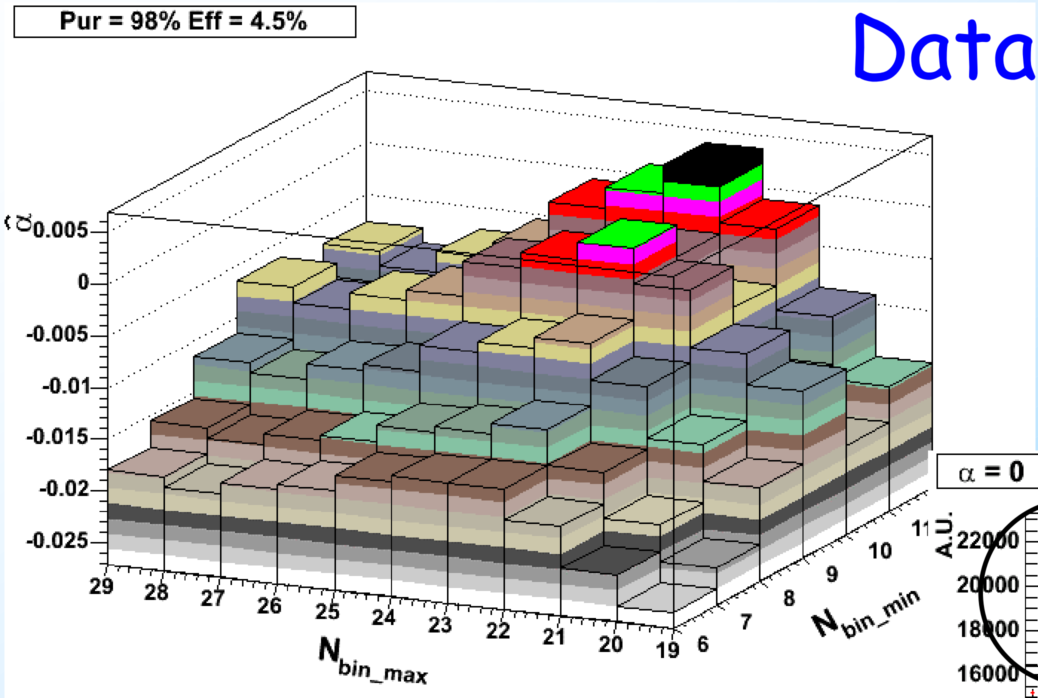
ε is for each bin: the efficiency as a function of Dalitz Plot

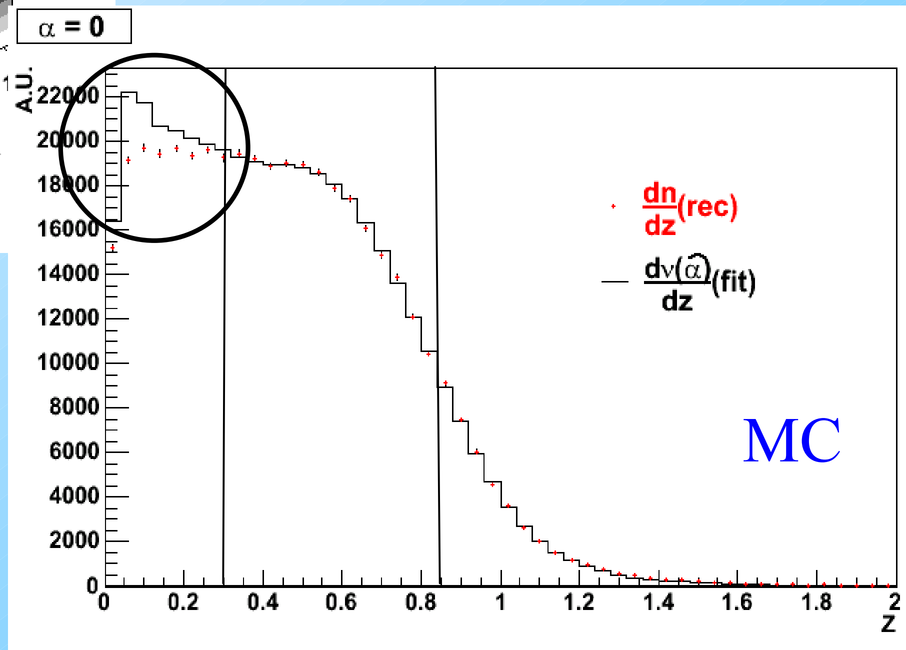
g (i,j) is the resolution function for bin i-th

$f_{th} = |M|^2 = (1 + 2\alpha z)$

dN(i)/dz = generated according to pure phase space

We got in trouble…





Folding procedure (II)

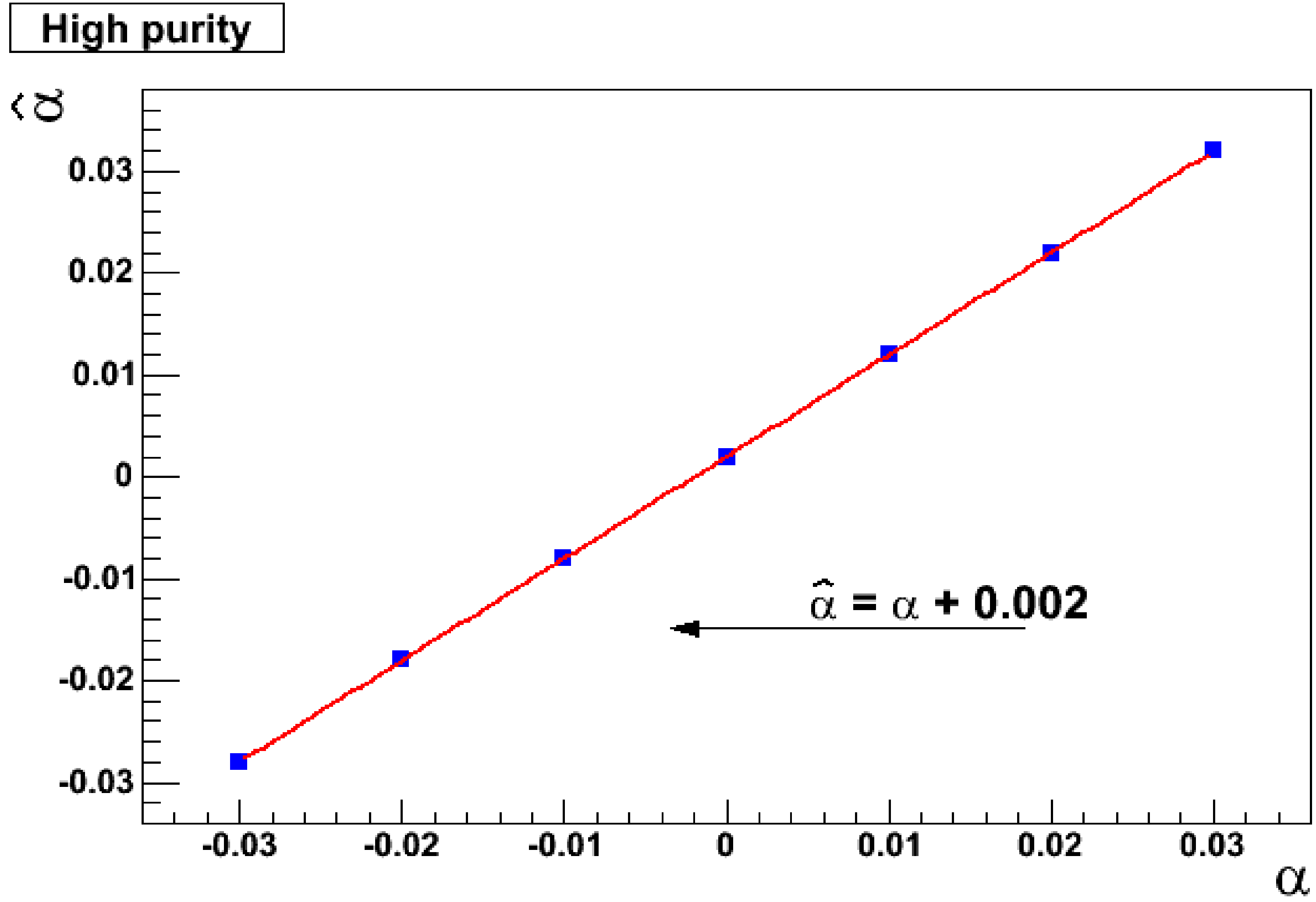
Let us free of the binning:

For each MC event (generated according to phase space)

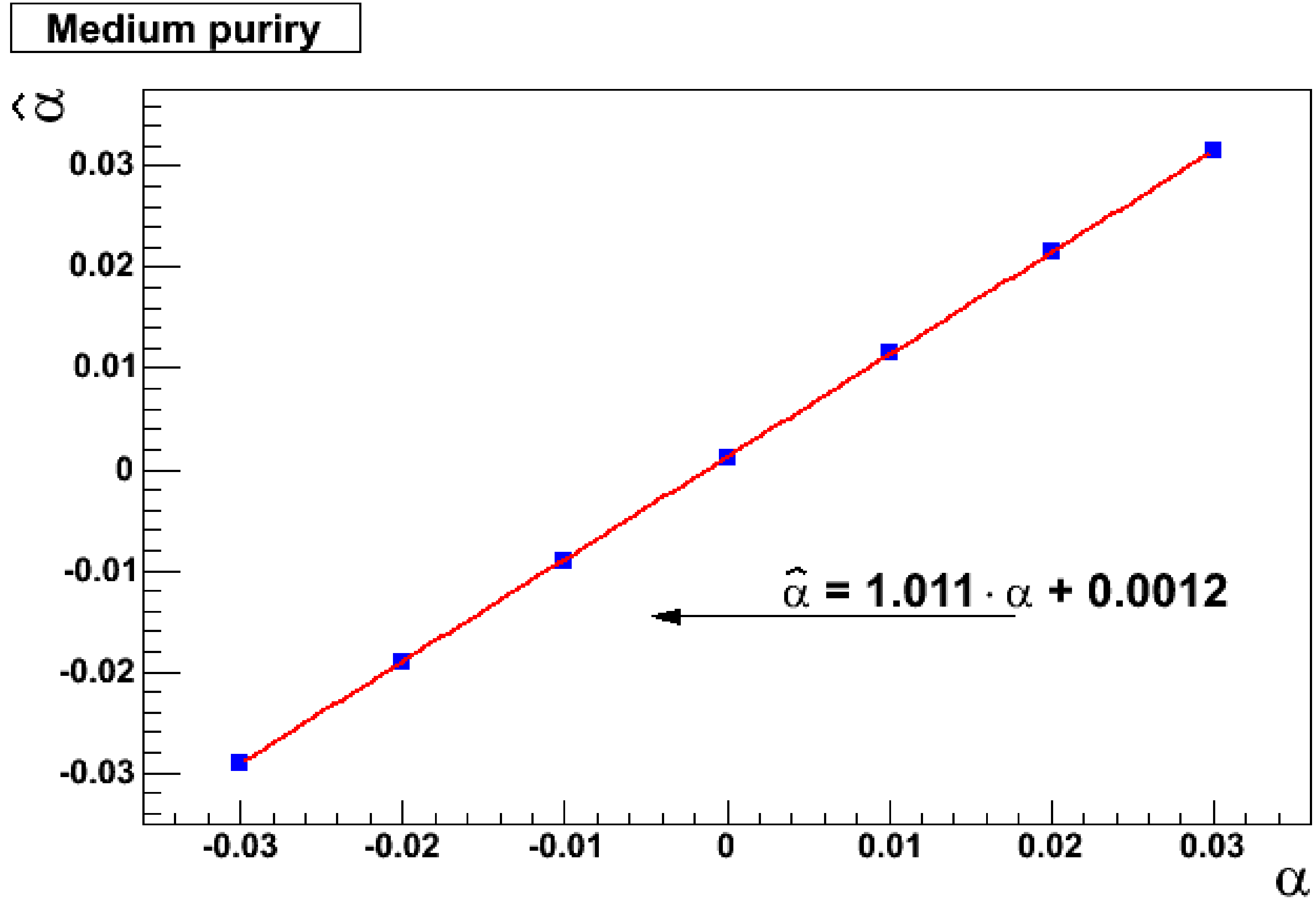
- Evaluate its z_{true} and its z_{rec} (if any!)
- Enter an histogram with the value of z_{rec}
- Weight the entry with $1 + 2\,\alpha\, z_{true}$

Then iterate procedure to find α maximizing log likelihood described before

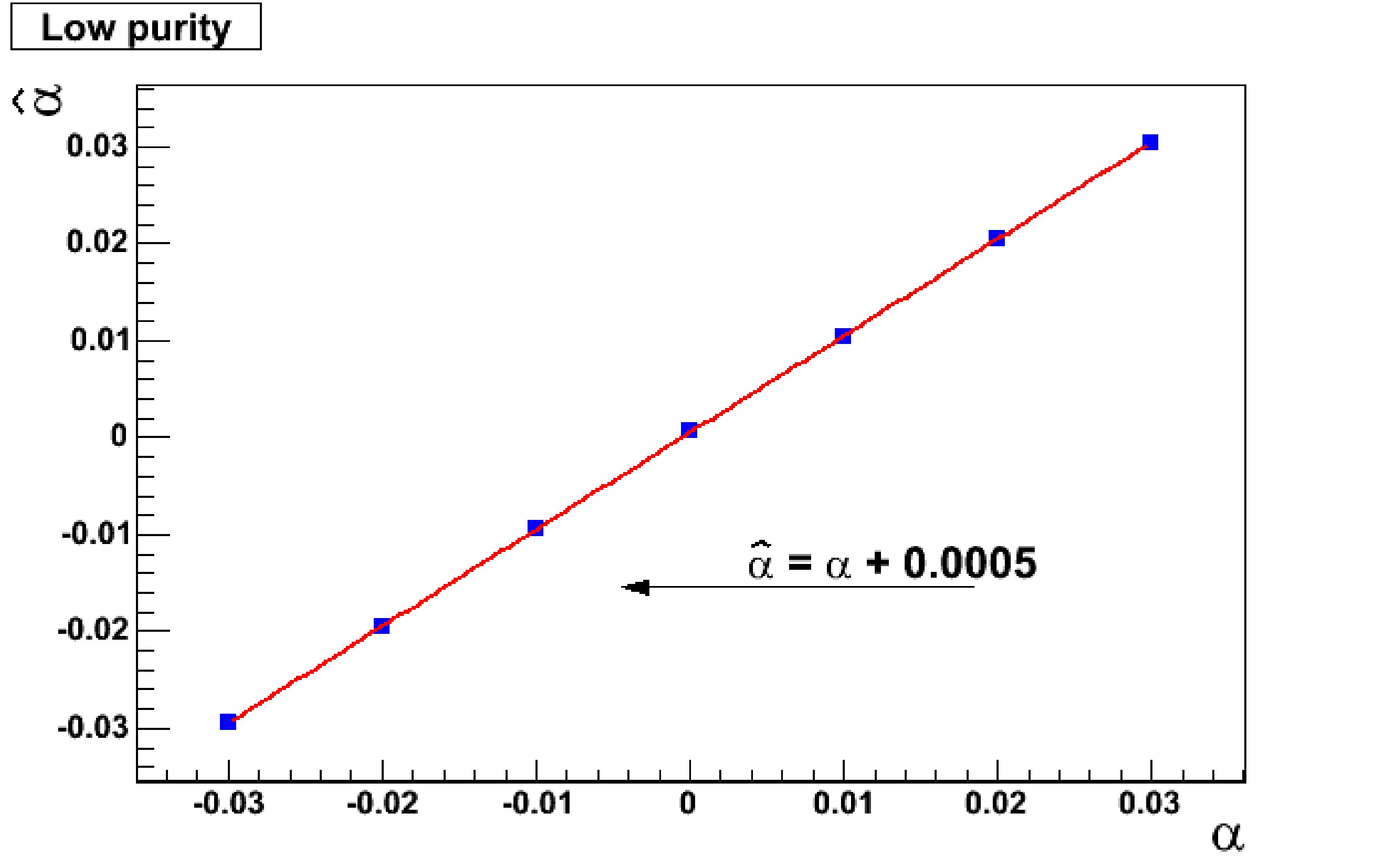
Results on MC



Results on MC



Results on MC



Systematic checks

This procedure relies heavily on MC.

The crucial checks for the analysis can be summarized in three main questions:

- Is MC correctly describing efficiencies ?
- Is MC correctly describing resolutions ?
- Is MC correctly estimating the “background” ?

Efficiency (I)

First, let us check the overall efficiency evaluing cross section. We used two “benchmarks” to check the total expected number of events:

$\sigma(\eta\gamma)_{\text{visible-peak}} = (40.2 \pm 1)$ nb (from talk by M. Dreucci on phi lineshape in Capri)

$\sigma(\phi \to \eta\gamma \to 7\gamma)_{\text{visible-peak}} = (13.8 \pm 0.5)$ nb (KN 177)

N Expected (1) = 1.35 ±0.03 Mevts
N Expected (2) = 1.48 ± 0.05 Mevts

Nfound = 1.417 ±0.001 Mevts

Efficiency (II)

Now let us look at the relative ratio between the three different samples:

N2/N1 exp. = 0.633

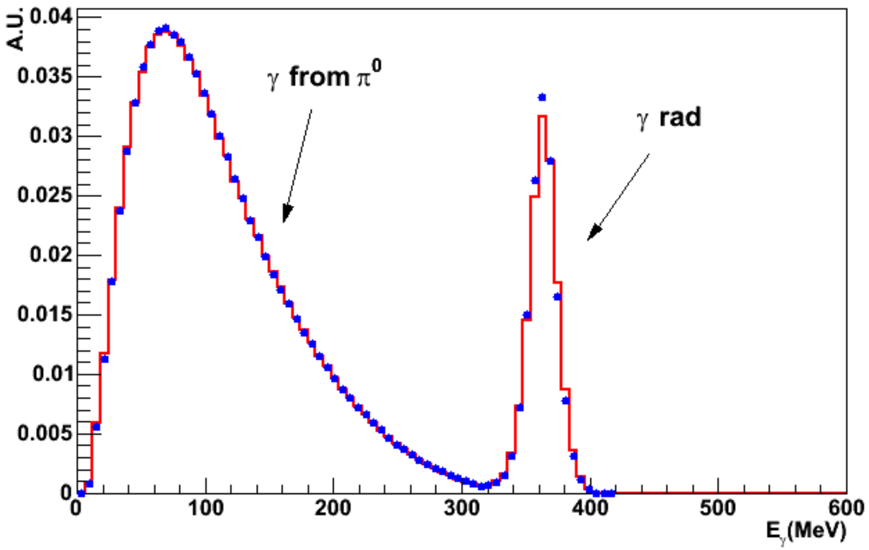
N3/N1 exp. = 0.204

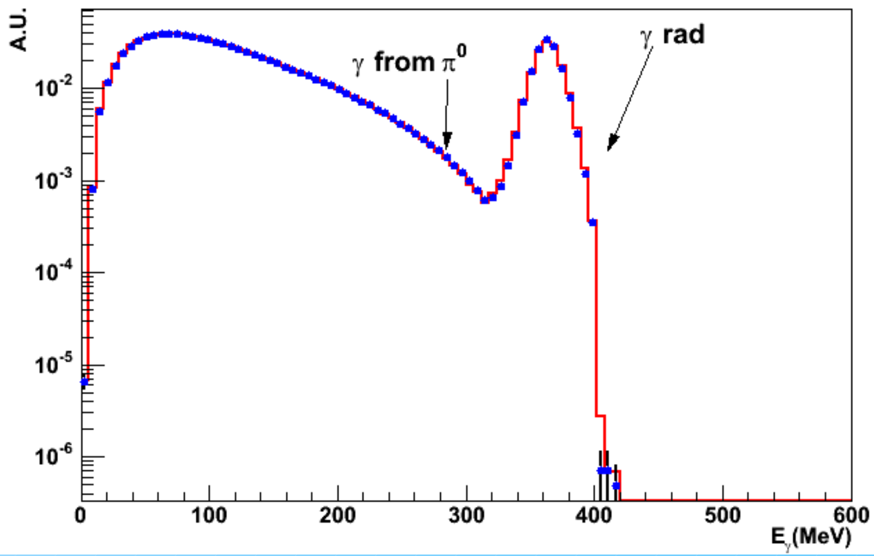
N2/N1 found = 0.628

N3/N1 found = 0.206

Efficiency (III)

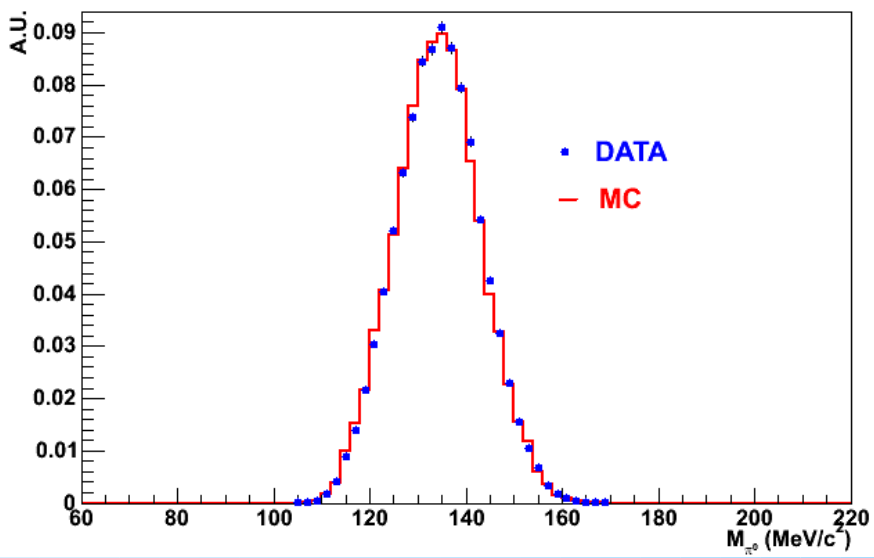
As we are mainly interested in relative efficiencies, we also check photon spectra.

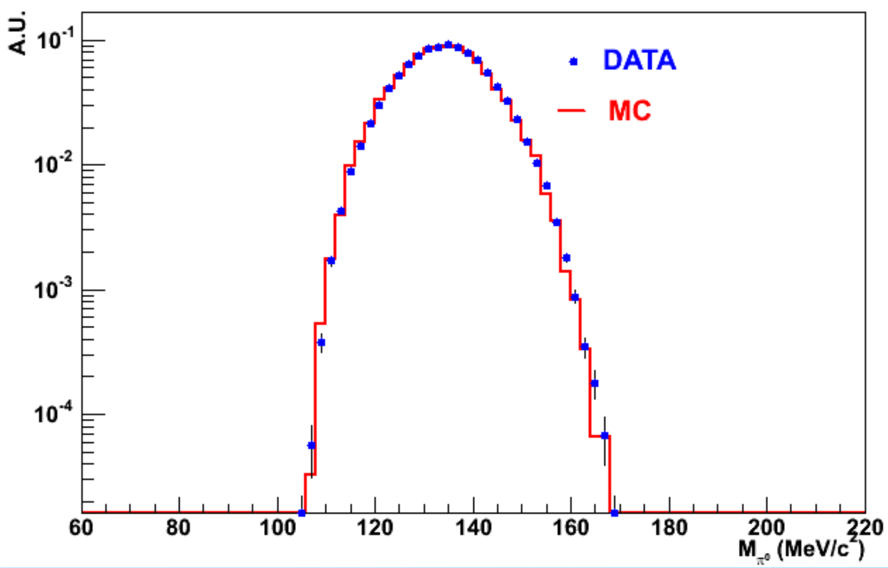




Resolution (I)

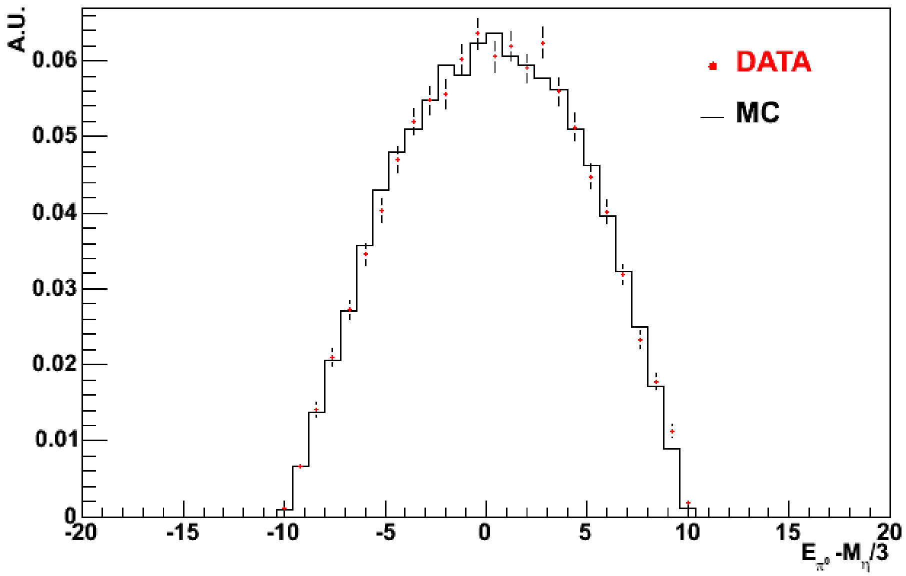
A first check on resolution is from pion mass distribution

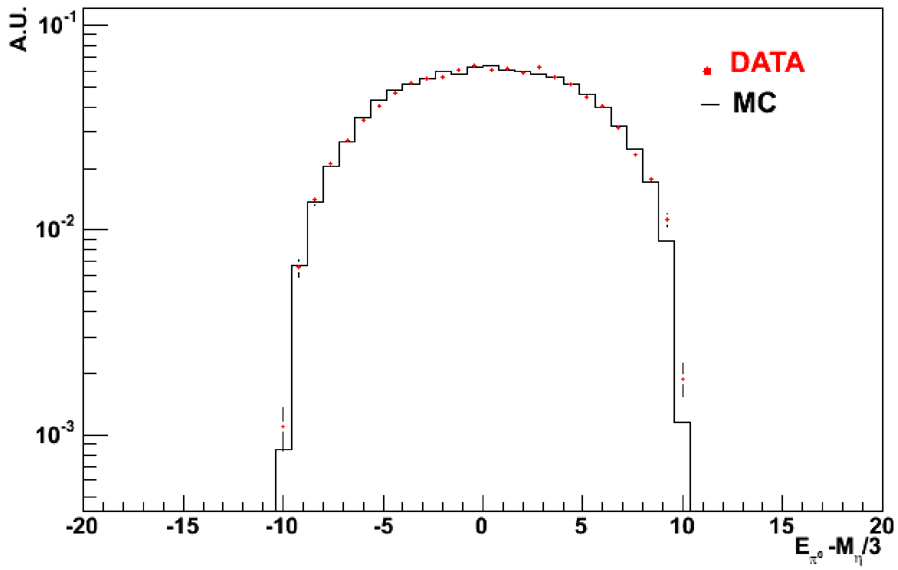




Resolution (II)

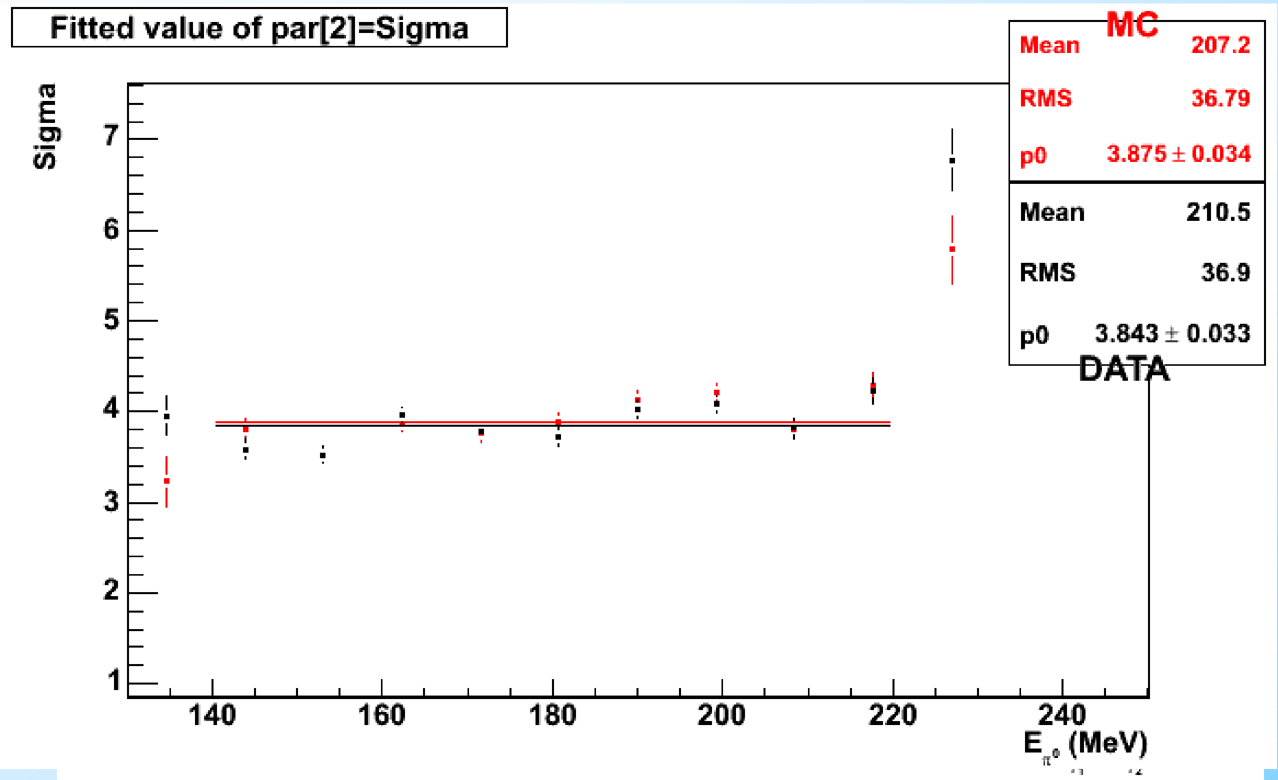
The center of Dalitz plot correspond to 3 pions with the same energy ($E_i = M_\eta/3 = 182.4$ MeV). A good check of the MC performance in evaluating the energy resolution of π^0 comes from the distribution of $E_{\pi 0} - E_i$ for $z = 0$





Resolution (III)

A further check can be done comparing the energies of the two photons in the pion rest frame as function of pion energy



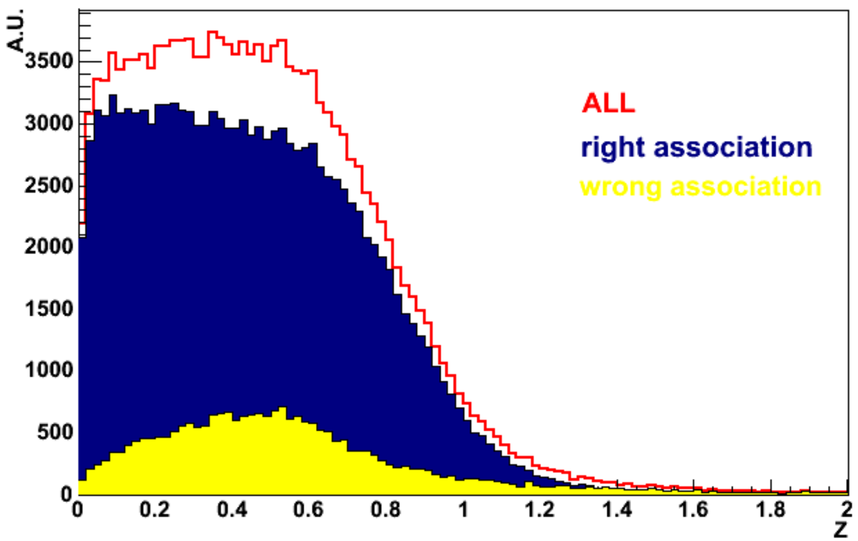
$E^*_{\gamma 1} - E^*_{\gamma 2}$

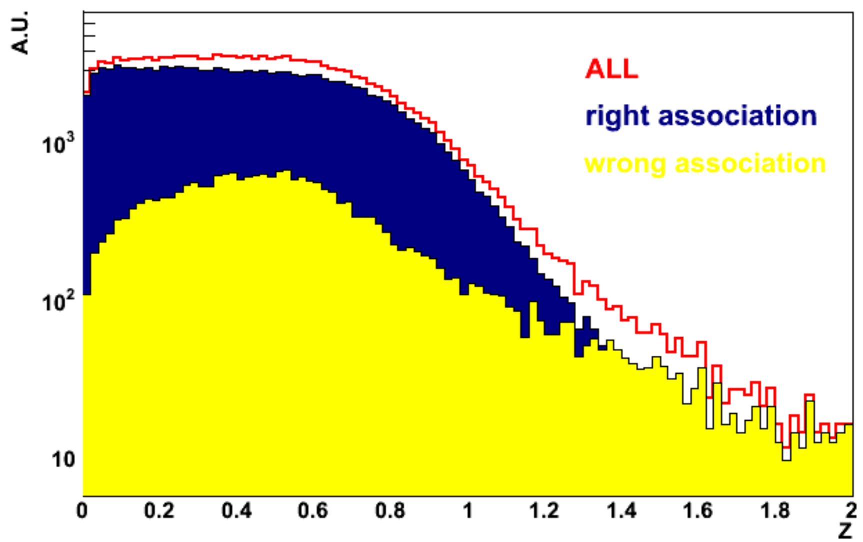
Vs.

$E\pi$

Background (I)

Background composition, low purity sample

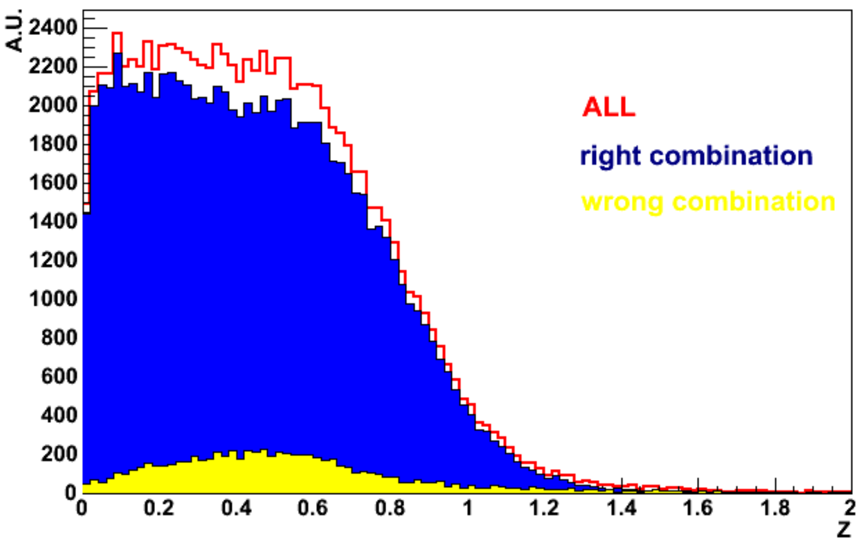


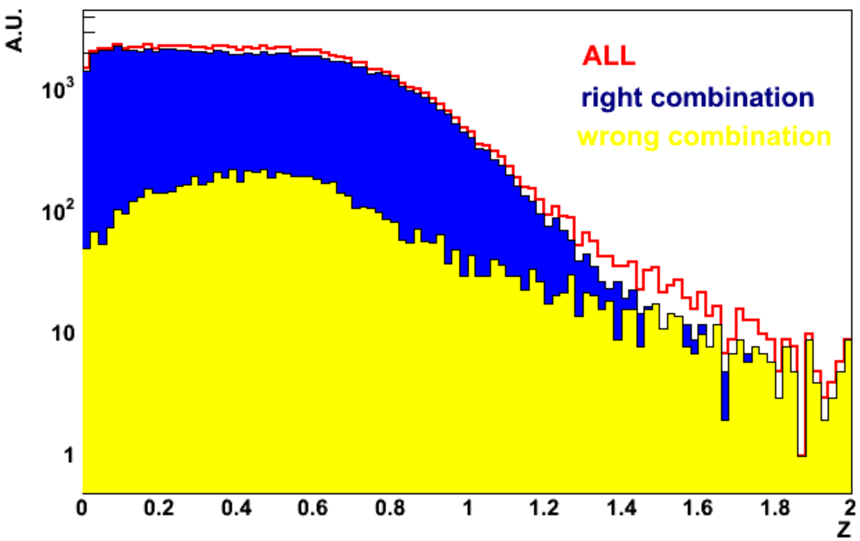


Events w accidental/total = 8.5 per mil
Events w accidental/background = 5.5%

Background (II)

Background composition, medium purity sample

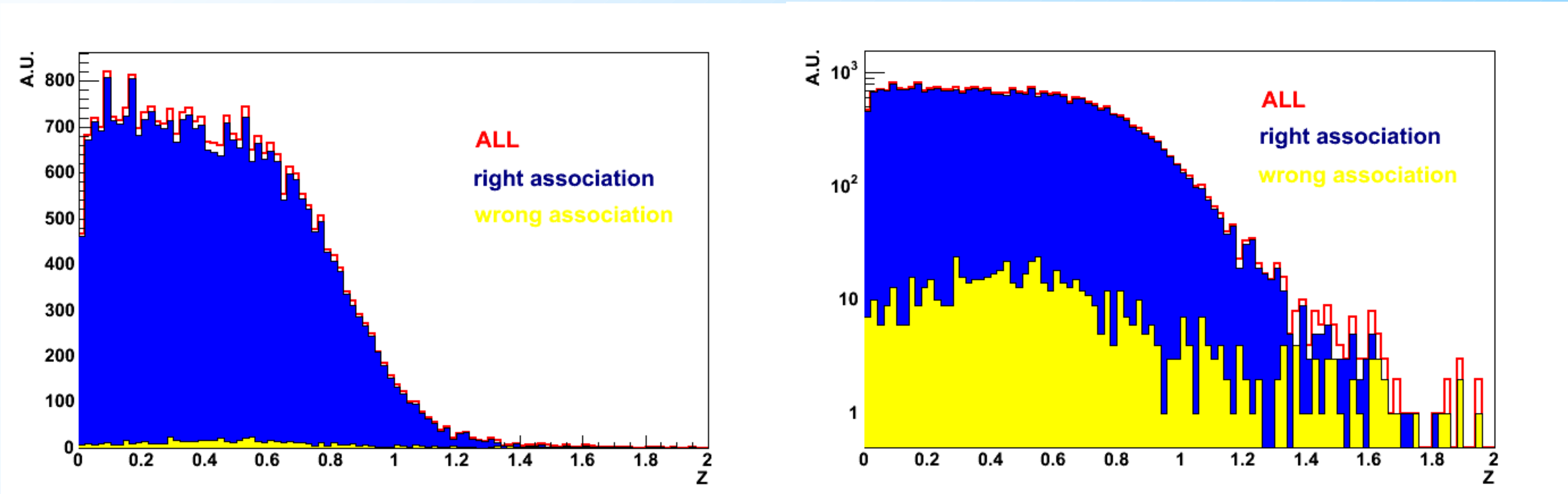




Events w accidental/total = 4.4 per mil
Events w accidental/background = 10%

Background (III)

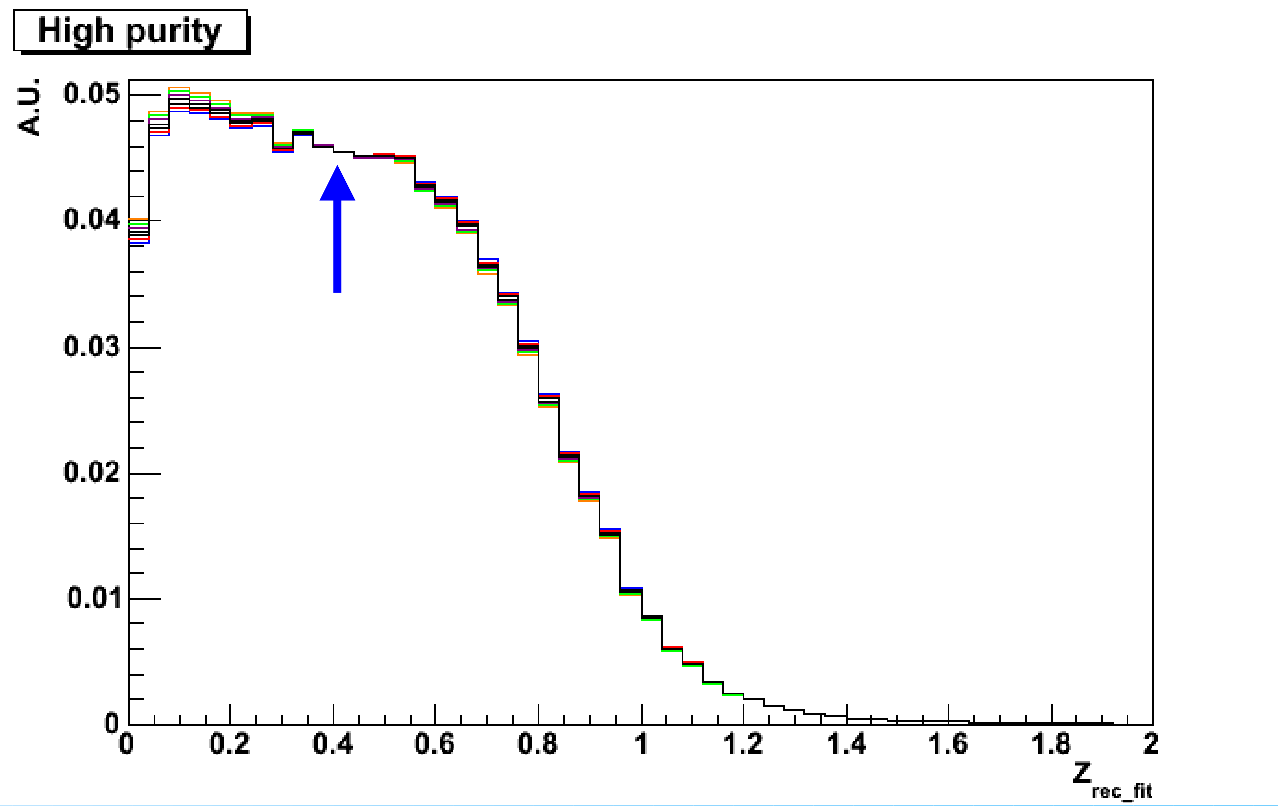
Background composition, high purity sample



Events w accidental/total = 1.4 per mil
Events w accidental/background = 32%

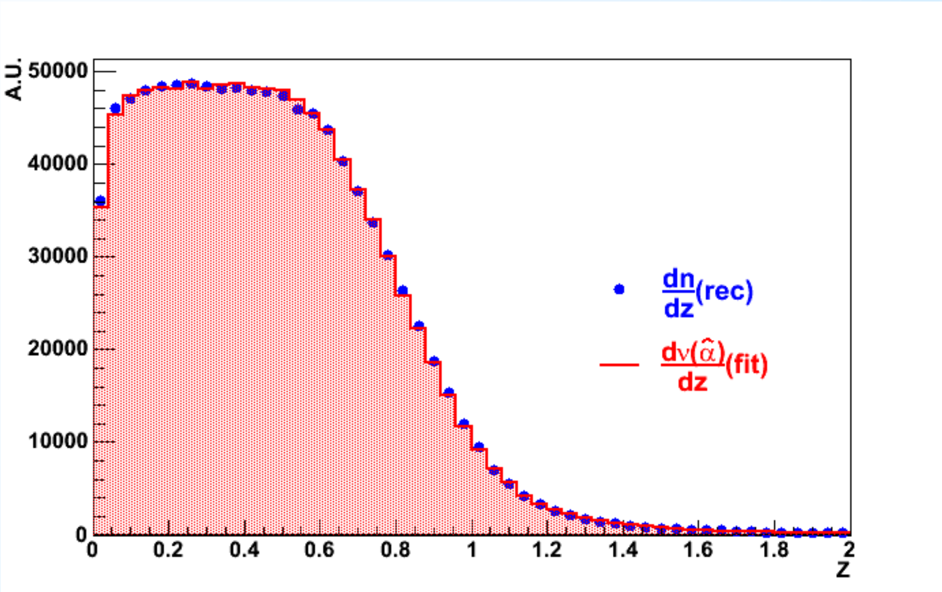
A global check

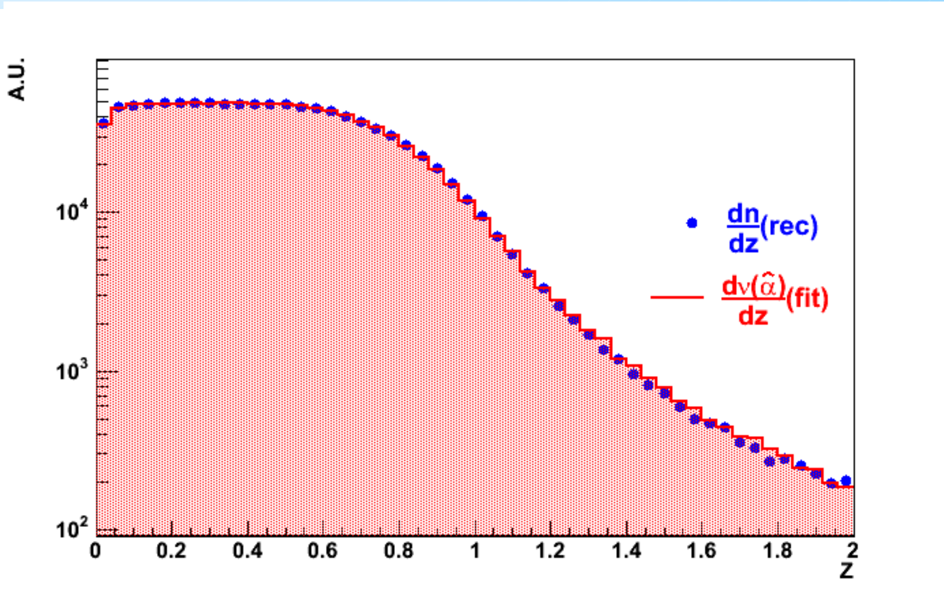
Looking at histograms generated for various α values we see that we can make a "global" check which is almost α independent.



Fitting Data version 0

Low purity

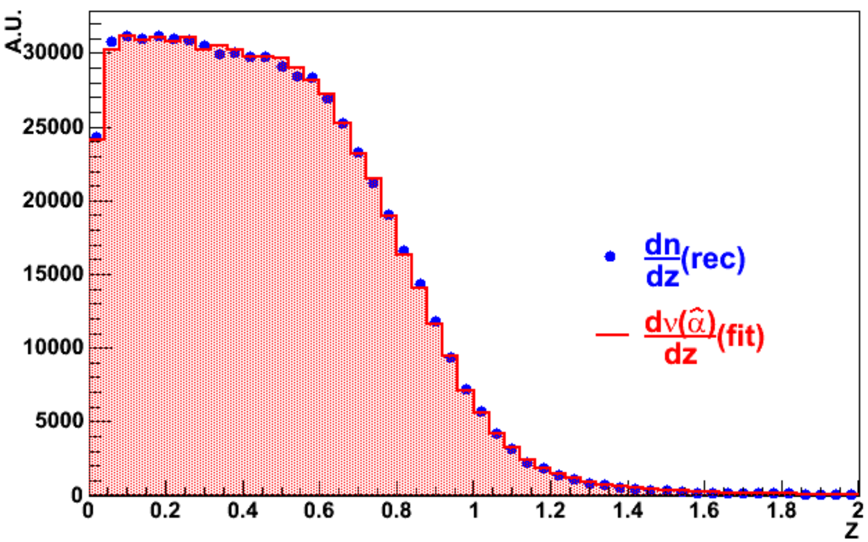


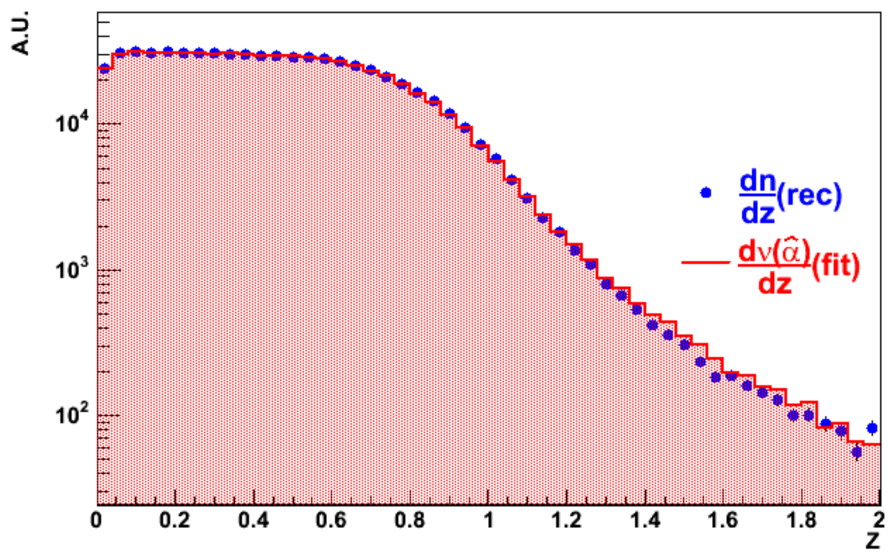


$\hat{\alpha}$ = -0.020 ± 0.002

Fitting Data version 0

Medium purity

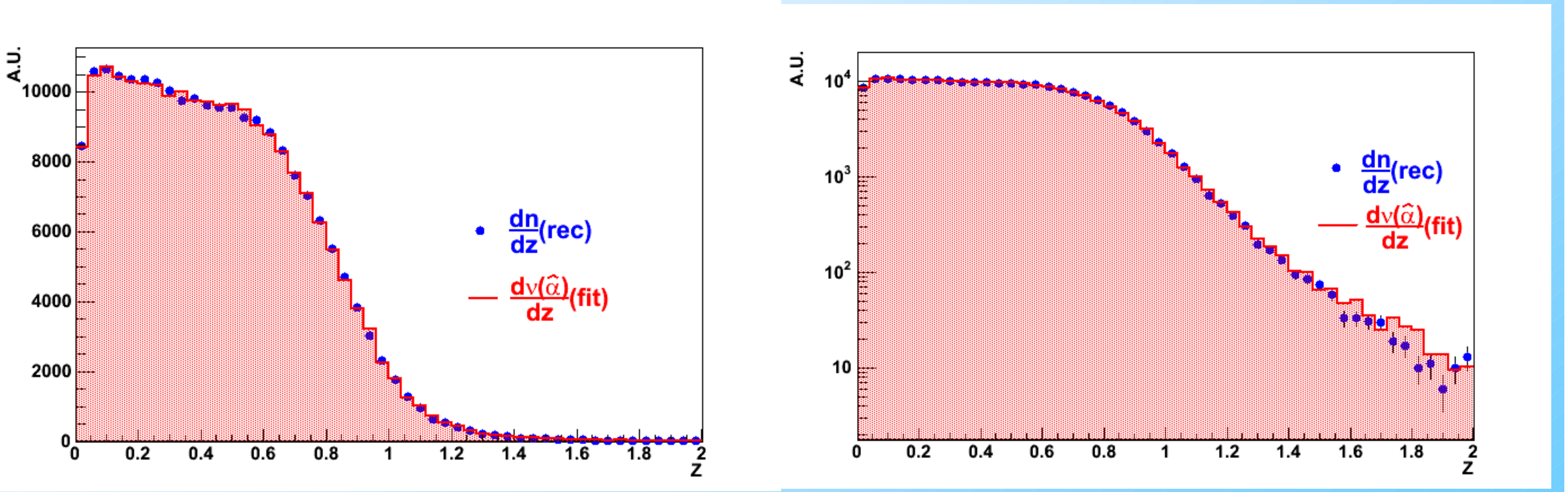




$\hat{\alpha}$ = -0.019 ± 0.003

Fitting Data version 0

High purity



$\hat{\alpha}$ = -0.013 ± 0.005

Fitting the background

Idea, try to fit background composition on DATA, neglecting α

Background fraction (MC) = 15.5 %
Background fraction (DATA) = (13.5 ± 0.5) %

Background fraction (MC) = 8.0 %
Background fraction (DATA) = (6.1 ± 0.6) %

Background fraction (MC) = 1.8 %
Background fraction (DATA) = (1.4 ± 0.5) %

Fitting the background (II)

To check procedure, fit background composition on MC, neglecting α

Background fraction (MC) = 15.5 %
Background fraction (MC fit) = (15.8 ± 0.4) %

Background fraction (MC) = 8.0 %
Background fraction (MC fit) = (8.0 ± 0.4) %

Background fraction (MC) = 1.8 %
Background fraction (MC fit) = (2.5 ± 0.4) %

Fitting the background (III)

To check how much you expect a to change your fit, try to fit background composition on MC weighted for $\alpha = -0.020$

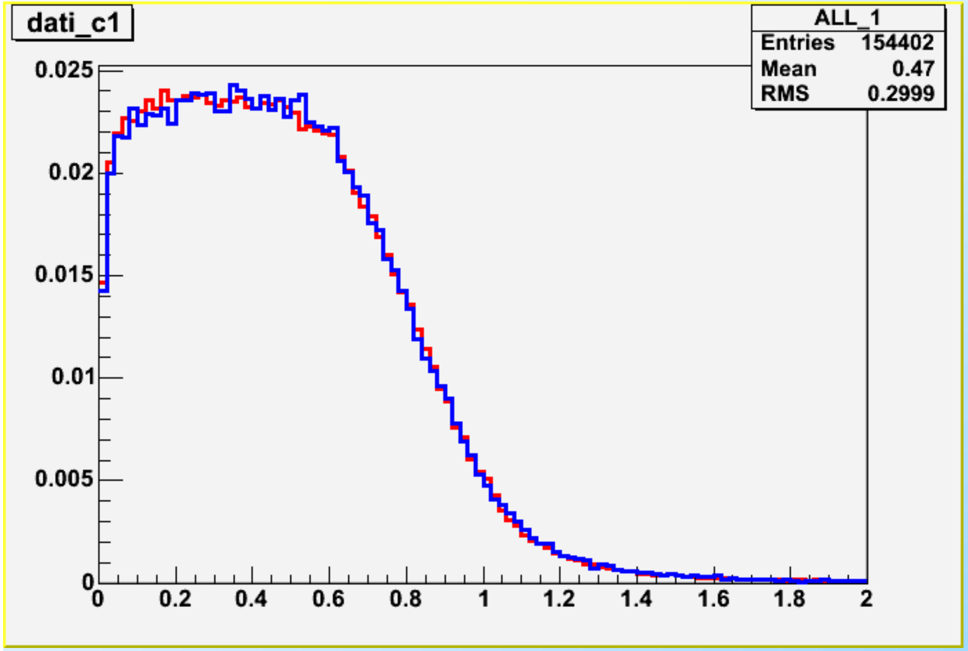
Background fraction (MC) = 15.5 %
Background fraction (MC fit) = (15.2 ± 0.3) %

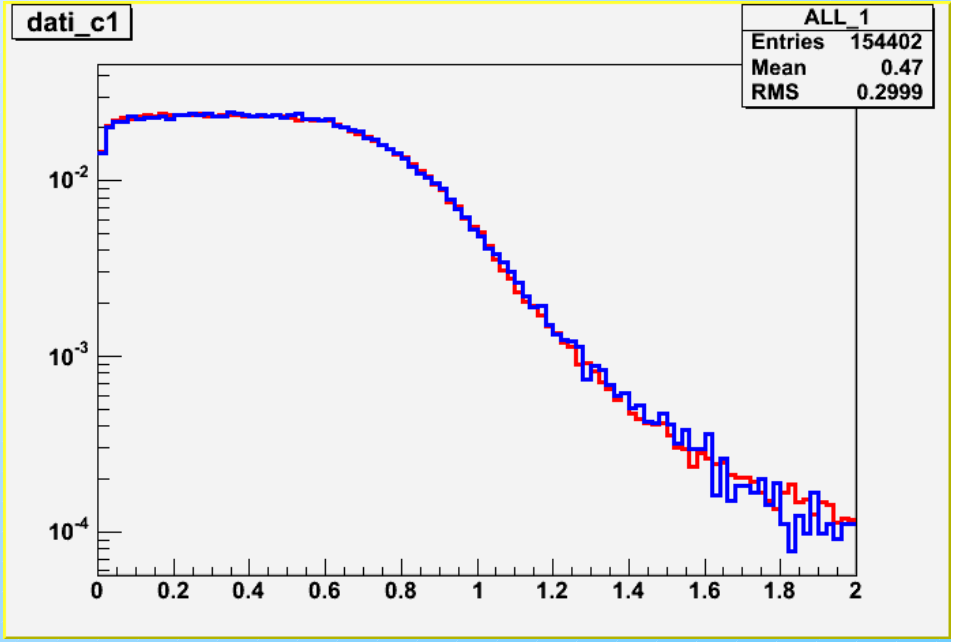
Background fraction (MC) = 8.0 %
Background fraction (MC fit) = (7.4 ± 0.3) %

Background fraction (MC) = 1.8 %
Background fraction (MC fit) = (2.5 ± 0.4) %

Fitting the background (IV)

Fits on data

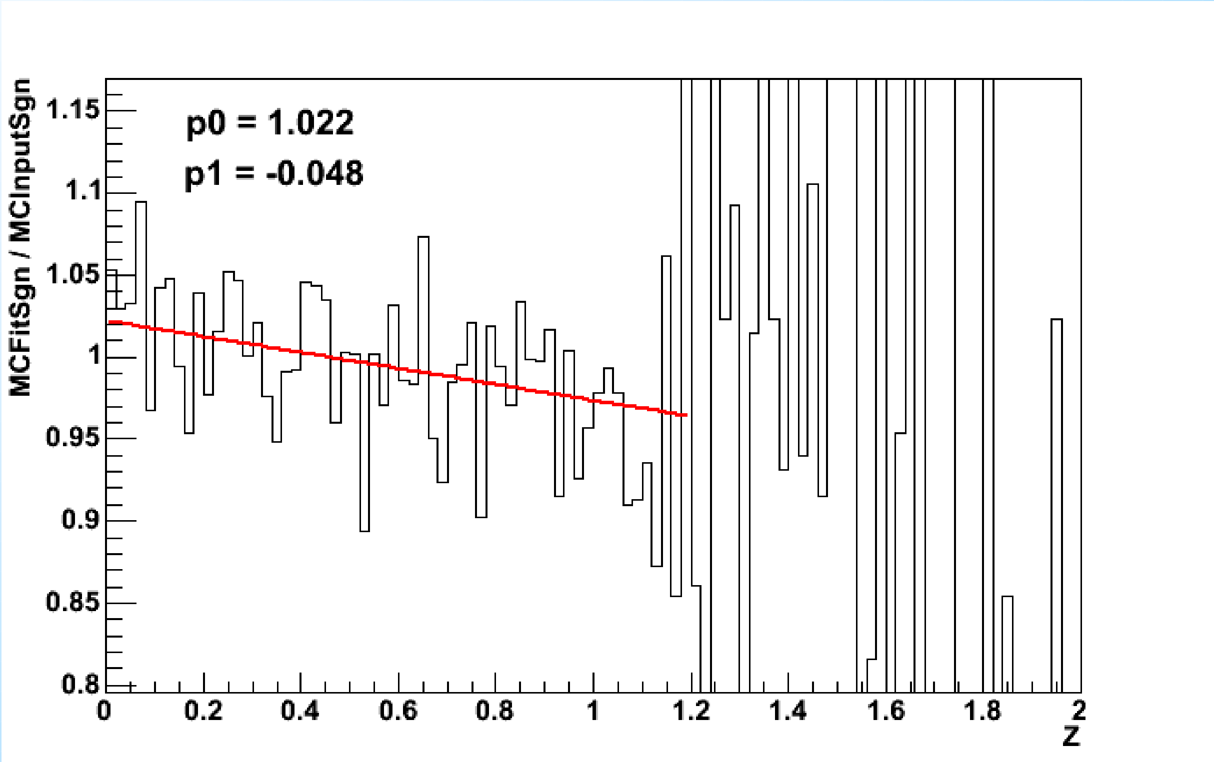




Fitting the background (V)

Fits are TOO GOOD ! After all they are done supposing $\alpha=0$!!!!
But…

$\frac{\text{MC (fit)}}{\text{MC (input)}}$



Fitting Data version 1

After reweighting the background for the ratio DATA/MC found in the previous fits we obtain:

Low purity: $\alpha = -0.015 \pm 0.002$

Medium purity: $\alpha = -0.013 \pm 0.003$

High purity: $\alpha = -0.013 \pm 0.005$

Further checks

We have checked changes for each sample for these changes:

	LP	MP	HP
Fit region (0-2) vs (0-1):	-0.002	-0.003	-0.004
Bin choice (30 vs 50)	0.0	0.0	0.0

Preliminary results

We have analyzed 352 pb-1 of 2001-2002 data and we find the preliminary results:

α = -0.015 ± 0.002 stat ± 0.002 syst   LP

α = -0.013 ± 0.003 stat -0.003 syst   MP

α = -0.013 ± 0.005 stat -0.004 syst   HP

The systematics is obtained considering the maximum variation wrt sample choice in the fit with reweighted background, and the maximum variation wrt the fitting region for the chosen sample.

These results differ by roughly 3 standard deviations from the published Crystal Ball result:

α = -0.031 ± 0.004

Conclusions

- We are analyzing an unprecedented statistics of $\eta \rightarrow 3\pi$ decays with negligible background
- The analysis is quite hard but looks also quite solid in both the fitting procedure and the control of possible systematic effects
- We obtain a result with very marginal agreement with CB one