# Studying FSR at threshold for $\pi \pi \gamma$ events 

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Disclaimer: everything is preliminary!!!

## Generalization of FSR amplitude: we need 3 form factors $f_{i}$.

$$
\begin{array}{ll}
<\pi^{+} \pi^{-} \gamma\left|j^{\mu}\right| 0>=i e^{2} \varepsilon_{v}^{*} M_{F}^{\mu \nu} \\
M_{F}^{\mu \nu}=f_{1} \tau_{1}^{\mu \nu}+f_{2} \tau_{2}^{\mu \nu}+f_{3} \tau_{3}^{\mu \nu}
\end{array}
$$

Each form factor depends by 3 independent variables (one is $\boldsymbol{s}$ )
$\lim _{k \rightarrow 0} f_{1}=f_{1}^{s Q E D}=\frac{2 k \cdot Q F_{\pi}(s)}{(k \cdot Q)^{2}-(k \cdot l)^{2}}$
$\lim _{k \rightarrow 0} f_{2}=f_{2}^{s Q E D}=\frac{2 F_{\pi}(s)}{(k \cdot Q)^{2}-(k \cdot l)^{2}}$
Limit of soft photon (what we call sQED)
$\lim _{k \rightarrow 0} f_{3}=f_{3}^{s \text { SED }}=0$
At threshold (very hard photon) this approximation could not work

## A model for FSR, based on $\chi$ PT


-The model is a first step in the direction of a generalization of the FSR amplitude.
However, the calculation (at the moment) is approximate since:

- it doesn't take into account other mesons (like $\omega$ and $\rho^{\prime}$ )
- it doesn't include the $\phi \rightarrow \pi^{+} \pi^{-} \gamma$ decay
- ....

What about $\phi \rightarrow \pi^{+} \pi^{-} \gamma$ decay?
Recently H. Czyz et al. (hep-ph/0412239) showed that this decay can be important also at low $\mathrm{Q}^{2}$ region. This contribution can be quite different depending by the model. Charge asymmetry can help.


How to include $\phi \rightarrow \pi^{+} \pi^{-} \gamma$ decay in our calculation?

- $\phi \rightarrow \pi^{+} \pi^{-} \gamma$ is related to $\phi \rightarrow \pi^{0} \pi^{0} \gamma$ by the same matrix element (a part a factor $1 / 2$ ). We use the Achasov 4 q parametrization with the parameters of the model taken from the fit of the kloe data $\phi \rightarrow \pi^{0} \pi^{0} \gamma$.
$f\left(Q^{2}\right)=\frac{g_{\varphi \kappa^{+} \kappa^{-}} g_{f_{0} \kappa^{+} \kappa^{-}} g_{f_{0} \pi^{+} \pi^{-}}}{2 \pi^{2} m_{K}^{2}} I\left(\frac{m_{\varphi^{2}}}{m_{K}^{2}}, \frac{Q^{2}}{m_{K}^{2}}\right) \frac{e^{i \delta_{B}\left(Q^{2}\right)}}{\left(m_{f_{0}}^{2}-Q^{2}+\operatorname{Re}_{f_{0}}\left(m_{f_{0}}^{2}\right)-\Pi_{f_{0}}\left(Q^{2}\right)\right)}$
For the moment we consider only the contribution of $f_{0}$ (no $\sigma$ meson). This could be too crude at low $\mathrm{Q}^{2}$ !!!

(Analitical) Comparisons (at $\mathrm{s}=\mathrm{m}_{\phi}{ }^{2}$ ):


Since $\mathrm{M}_{\mathrm{FSR}} * \mathrm{M}_{\phi} \sim\left|\mathrm{M}_{\phi}\right|^{2}$ at low $\mathrm{Q}^{2}$ $\Delta f$ can be relevant only for destructive. interference (we will consider only this case in the following)
$\mathrm{Q}^{2}\left(\mathrm{GeV}^{2}\right)$
What happens for $s<m_{\phi}{ }^{2}$ ?

The comparison at $\mathrm{s}=1 \mathrm{GeV}^{2}$ :


In this case the interference $\mathrm{M}_{\mathrm{FSR}} * \mathrm{M}_{\phi}$ is expected to be $\gg\left|\mathrm{M}_{\phi}\right|^{2}$

The following matrix element has been introduced in a MC, for $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \gamma$ (based on EVA structure):

$$
\begin{aligned}
& d \sigma \approx\left|M_{I S R}+M_{F S R_{\chi P T}}+M_{\varphi}\right|^{2} \cong\left|M_{I S R}+M_{F S R_{X P T}}\right|^{2}+\left|M_{\varphi}\right|^{2}+ \\
& 2 \operatorname{Re}\left(\left(M_{I S R}+M_{F S R_{s Q E D}}\right) \bullet M_{\varphi}^{*}\right)
\end{aligned}
$$

$\cdot$ We neglect the contributions from $\gamma^{*} \rightarrow \rho \pi \rightarrow \pi^{+} \pi \gamma$ (found negligible in hep-ph/0411113)
-We consider destructive interference between $\mathrm{FSR}_{\mathrm{sQED}}$ and $\phi$
-We consider large angle analysis: $50^{\circ}<\theta_{\gamma}<130^{\circ}, 50^{\circ}<\theta_{\pi}<130^{\circ}$

$$
\begin{array}{cl}
\text { Numerical results: } & 50^{\circ}<\theta_{\pi}<130^{\circ} \\
\text { differential cross section... } & 50^{\circ}<\theta_{\gamma}<130^{\circ}
\end{array}
$$






Effect al low $\mathrm{Q}^{2}$...however the contribution of $\phi$ is not much accurate (no interference with $\sigma$ has been taken into account)

## And asymmetry....




## Zoom on the threshold region:




Up to $30 \%$ of contribution beyond sQED at the threshold

## And asymmetry...



## Conclusions and outlook

-First MC results on a generalization of FSR using $\chi$ PT have been presented.

- A sizeable effect can be seen on the cross section (at low $\mathrm{Q}^{2}$ ).
-The situation on the asymmetry is less clear, but it strongly depends on the parametrization of the $\phi$ direct decay
-For the near future:
-Improve the simulation:
-better parametrization of $\phi$ (including also the $\sigma$ meson)
-consistency between the various parameters of the models in MC
- Try to disentangle the various contributions:
-Improve the knowledge on the phi decay (in particular at low $\mathrm{Q}^{2}$ ):
-Constraint fit with the neutral channel
-asymmetry, and other kinematical variables (angular distributions)
- Work off resonance
- Model independent analysis of $f_{i}$ ?

