A global fit to determine the pseudoscalar mixing angle and the gluonium content of the η' meson

The KLOE collaboration

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ABSTRACT: We update the values of the η-η' mixing angle and of the η' gluonium content by fitting our measurement $R_\phi = \mathrm{BR}(\phi \to \eta'\gamma)/\mathrm{BR}(\phi \to \eta\gamma)$ together with several vector meson radiative decays to pseudoscalars ($V \to P\gamma$), pseudoscalar mesons radiative decays to vectors ($P \to V\gamma$) and the $\eta' \to \gamma\gamma$, $\pi^0 \to \gamma\gamma$ widths. From the fit we extract a gluonium fraction of $Z_G^2 = 0.12 \pm 0.04$, the pseudoscalar mixing angle $\psi_P = (40.4 \pm 0.6)^\circ$ and the $\phi - \omega$ mixing angle $\psi_V = (3.32 \pm 0.09)^\circ$. Z_G^2 and ψ_P are fairly consistent with those previously published. We also evaluate the impact on the η' gluonium content determination of future experimental improvements of the η' branching ratios and decay width.

Contents

1 Introduction

The η' meson, being almost a pure SU(3)$_{\text{flavour}}$ singlet, is considered a good candidate to host a gluon condensate. The question of a gluonium component in the η' meson has been extensively investigated in the past but it is still without a definitive conclusion [1]. We extract the η' gluonium content and the η-η' mixing angle in the constituent quark model according to the Rosner [2] approach with the modifications introduced in ref. [3] as described in the following. We use the same method of ref. [4]; in addition, we also introduce in the fit the $\pi^0 \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ branching fractions according to the prescription of ref. [5]. This method relates our measurement of the ratio $\phi \to \eta'\gamma$ and $\phi \to \eta\gamma$ branching ratio (BR), $R_\phi = \text{BR}(\phi \to \eta'\gamma)/\text{BR}(\phi \to \eta\gamma)$ [6], to the η' gluonium content and to the η, η' mixing angle. The same quantities were extracted in our previous analysis [6] with some assumptions. This has given rise to some objections from refs. [4] and [7]. Here we give an answer to these objections and we repeat the fit taking into account their comments. Then we repeat the fit with recently updated experimental results.

The η and η' states can be represented in the base $|N\rangle = (|u\bar{u}\rangle + |d\bar{d}\rangle)/\sqrt{2}$, $|S\rangle = |s\bar{s}\rangle$ and $|G\rangle = |\text{gluonium}\rangle$ as:

$$|\eta'\rangle = \cos\psi_G \sin\psi_P |N\rangle + \cos\psi_G \cos\psi_P |S\rangle + \sin\psi_G |G\rangle \qquad (1.1)$$

$$|\eta\rangle = \cos\psi_P|N\rangle - \sin\psi_P|S\rangle \qquad (1.2)$$

where ψ_P is the η-η' mixing angle and $Z_G^2 = \sin^2\psi_G$ is the gluonium fraction in the η' meson, while we neglect the gluonium content in the η meson. According to ref. [8] the state $|G\rangle$ could be the $\eta(1405)$: a pseudoscalar glue ball candidate.

The ratio $R_\phi = \text{BR}(\phi \to \eta'\gamma)/\text{BR}(\phi \to \eta\gamma)$ is related to the ψ_P and ψ_G parameters by the formula [6]:

$$R_\phi = \cot^2\psi_P \cos^2\psi_G \left(1 - \frac{m_s}{\bar{m}} \frac{Z_q}{Z_s} \frac{\tan\psi_V}{\sin 2\psi_P}\right)^2 \left(\frac{p_{\eta'}}{p_\eta}\right)^3 \tag{1.3}$$

where $p_{\eta'}$ and p_η are the momenta of the η' and η meson respectively in the ϕ reference frame, $m_s/\bar{m} = 2m_s/(m_u + m_d)$ is the constituent quark mass ratio and ψ_V is the ϕ-ω mixing angle. Following ref. [3] we define the constant $C_q = \langle q\bar{q}_\rho | q\bar{q}_\eta \rangle$ as the overlap between the spatial wave functions of the quark-antiquark pair in the ρ and the η meson. Isospin symmetry is assumed exact, so that $m_u = m_d = \bar{m}$ and the following further relations follow:

$$C_q = \langle q\bar{q}_\eta | q\bar{q}_\omega \rangle = \langle q\bar{q}_\eta | q\bar{q}_\rho \rangle, \quad C_s = \langle s\bar{s}_\eta | s\bar{s}_\phi \rangle, \quad C_\pi = \langle q\bar{q}_\pi | q\bar{q}_\omega \rangle = \langle q\bar{q}_\pi | q\bar{q}_\rho \rangle$$

where we indicate with $|q\bar{q}_\eta\rangle$ and $|q\bar{q}_\omega\rangle$ the $q\bar{q}$ spatial wave function in the η and ω mesons, and with $|s\bar{s}_\eta\rangle$ and $|s\bar{s}_\phi\rangle$ the $\bar{s}s$ spatial wave function in the η and ϕ mesons. The parameters Z_q and Z_s are the ratios: $Z_q = C_q/C_\pi$ and $Z_s = C_s/C_\pi$. In this model SU(3)$_{\text{flavour}}$ breaking effects are accounted for by the different values of the effective quark masses, $m_s > m_u = m_d = \bar{m}$, and by $Z_q \neq Z_s$.

In our previous analysis [6] the parameters Z_s, Z_q, ψ_V and $m_s/\bar{m}$ were taken from ref. [3] where $\text{BR}(\phi \to \eta'\gamma)$ and $\text{BR}(\phi \to \eta\gamma)$ were fitted together with other $V \to P\gamma$ decay rates (V indicates the vector mesons ρ, ω, ϕ and P the pseudoscalars π^0, η, η') assuming no η' gluonium content. We fitted [6] our measurement

$$R_\phi = \frac{\text{BR}(\phi \to \eta'\gamma)}{\text{BR}(\phi \to \eta\gamma)} = (4.77 \pm 0.09_{stat.} \pm 0.19_{syst.}) \times 10^{-3}$$

together with the available data [9] on $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$, $\Gamma(\eta' \to \rho\gamma)/\Gamma(\omega \to \pi^0\gamma)$ and $\Gamma(\eta' \to \omega\gamma)/\Gamma(\omega \to \pi^0\gamma)$. The dependence of these ratios on the mixing angle ψ_P and the gluonium content ψ_G is given by the following equations:

$$X_{\eta'} = \sin\psi_P \cos\psi_G, \quad Y_{\eta'} = \cos\psi_P \cos\psi_G$$

$$\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{1}{9}\left(\frac{m_{\eta'}}{m_{\pi^0}}\right)^3 \left(5\frac{f_\pi}{f_q}\cos\psi_G \sin\psi_P + \sqrt{2}\frac{f_\pi}{f_s}\cos\psi_G \cos\psi_P\right)^2 \tag{1.4}$$

$$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = 3\frac{Z_q^2}{\cos^2(\psi_V)}\left(\frac{m_{\eta'}^2 - m_\rho^2}{m_\omega^2 - m_\pi^2} \cdot \frac{m_\omega}{m_{\eta'}}\right)^3 X_{\eta'}^2 \tag{1.5}$$

$$\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{3}\left(\frac{m_{\eta'}^2 - m_\omega^2}{m_\omega^2 - m_\pi^2} \cdot \frac{m_\omega}{m_{\eta'}}\right)^3 \left[Z_q X_{\eta'} + 2\frac{\bar{m}}{m_s} Z_s \cdot \tan\psi_V Y_{\eta'}\right]^2. \tag{1.6}$$

where f_π is the pion decay constant and f_q and f_s are the decay constants of the isospin singlet states (mainly η, η' mesons) in the no-anomaly limit [5]. The fit result was $\psi_P = (39.7 \pm 0.7)^\circ$ and $Z_G^2 = \sin^2\psi_G = 0.14 \pm 0.04$, $P(\chi^2) = 49\%$. Imposing $\psi_G = 0$ the χ^2 probability of the fit decreased to 1%.

In ref. [4], a procedure similar to [3] has been adopted, but they fitted also the gluonium component in the η' wave function that was previously fixed at zero. The result $Z_G^2 = 0.04 \pm 0.09$ deviates 1σ from our value but with a larger error. In refs. [4] and [7] this difference was attributed to the use in our fit of the parameters Z_s and Z_q obtained in ref. [3] assuming no gluonium content. However further tests of the fit procedure showed that Z_G^2 and ψ_P are marginally sensitive to large variations of Z_q and Z_s [10]. Here we repeat the fit with a larger number of free parameters, including Z_s and Z_q.

2   Fit description

In order to enlarge the set of parameters used in the fit we add to the equations (1.4)–(1.6) the following further relations that can be derived directly from [4]:

$$\frac{\Gamma(\omega \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{9}\left[Z_q\cos\psi_P - 2\frac{\bar{m}}{m_s}Z_s\tan\psi_V\sin\psi_P\right]^2\left(\frac{m_\omega^2 - m_\eta^2}{m_\omega^2 - m_{\pi^0}^2}\right)^3 \tag{2.1}$$

$$\frac{\Gamma(\rho \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = Z_q^2\frac{\cos^2\psi_P}{\cos^2\psi_V}\left(\frac{m_\rho^2 - m_\eta^2}{m_\omega^2 - m_{\pi^0}^2}\frac{m_\omega}{m_\rho}\right)^3 \tag{2.2}$$

$$\frac{\Gamma(\phi \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{9}\left[Z_q\tan\psi_V\cos\psi_P + 2\frac{\bar{m}}{m_s}Z_s\sin\psi_P\right]^2\left(\frac{m_\phi^2 - m_\eta^2}{m_\omega^2 - m_{\pi^0}^2}\frac{m_\omega}{m_\phi}\right)^3 \tag{2.3}$$

$$\frac{\Gamma(\phi \to \pi^0\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \tan^2\psi_V \cdot \left(\frac{m_\phi^2 - m_{\pi^0}^2}{m_\omega^2 - m_{\pi^0}^2}\frac{m_\omega}{m_\phi}\right)^3 \tag{2.4}$$

$$\frac{\Gamma(K^{*+} \to K^+\gamma)}{\Gamma(K^{*0} \to K^0\gamma)} = \left(\frac{2\frac{m_s}{\bar{m}} - 1}{1 + \frac{m_s}{\bar{m}}}\right)^2 \cdot \left(\frac{m_{K^{*+}}^2 - m_{K^+}^2}{m_{K^{*0}}^2 - m_{K^0}^2} \cdot \frac{m_{K^{*0}}}{m_{K^{*+}}}\right)^3 \tag{2.5}$$

Notice that, differently from [4] where the VPγ couplings are fitted, we fit directly ratios of partial decay widths. This allows both to reduce the parameters involved in the fit (two of them cancel out in the ratios), and to simplify the error treatment using quantities proportional (directly or inversely) to the experimental measurements. As an example, a ratio of $\Gamma's$ is written as:

$$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{\mathrm{BR}(\eta' \to \rho\gamma)}{\mathrm{BR}(\omega \to \pi^0\gamma)}\frac{\Gamma_{\eta'}}{\Gamma_\omega}$$

In this way the correlation matrix among the η' branching ratios and the decay widths can be used directly . The fit is performed minimising the χ^2 function:

$$\chi^2 = \sum_{i,j}(y_i - y_i^{\mathrm{th}})\left(\mathbf{V}^{-1}\right)_{ij}(y_j - y_j^{\mathrm{th}})$$

where y_i are the experimental measurements of the ratios on the left side of equations (1.4)–(1.6) and (2.1)–(2.5), y_i^{th} are the theoretical predictions on the right side of the same equations and $\mathbf{V}^{-1}$ is the inverse of the covariance matrix. This last quantity is obtained summing the contribution from the experimental error on branching ratios, decay widths

$\rho\gamma$	-0.34					
$\pi^0\pi^0\eta$	-0.78	-0.29				
$\omega\gamma$	-0.35	-0.24	0.32			
$\gamma\gamma$	-0.26	-0.12	0.26	0.08		
$3\pi^0$	-0.28	-0.11	0.35	0.11	0.09	
$\Gamma_{\eta'}$	0.32	-0.02	-0.24	-0.05	-0.88	-0.08
	$\pi^+\pi^-\eta$	$\rho\gamma$	$\pi^0\pi^0\eta$	$\omega\gamma$	$\gamma\gamma$	$3\pi^0$

Table 1. Correlation matrix of the η' branching ratios from PDG-2006 fit [9].

and their correlations and the uncertainty from theoretical inputs. The method is the same of our previous fit [6], but only the parameters $f_q/f_\pi = 1.00 \pm 0.01$ and $f_s/f_\pi = 1.4 \pm 0.014$ [11] are taken as input, while in the past also Z_s, Z_q, ψ_V and $m_s/\overline{m}$ were fixed. The parameters f_q/f_π and f_s/f_π involve only the ratio $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$. The contribution from theoretical error is evaluated by standard error propagation:

$$\mathbf{V^{th} = A\,C\,A^T}$$

where $\mathbf{C}$ is the covariance matrix of the uncorrelated parameters $x_1 = f_q/f_\pi$ and $x_2 = f_s/f_\pi$ and $\mathbf{A}$ is:

$$(\mathbf{A})_{ij} = \frac{\partial y_i^{th}}{\partial x_j}$$

The covariance matrix $\mathbf{V}$ is indeed:

$$\mathbf{V = V^{exp} + V^{th}}$$

where $\mathbf{V^{exp}}$ is the covariance matrix of the data used in the fit. Particularly relevant is the correlation between the η' branching fractions and the decay widths shown in table 1.

The η' width and the $\eta' \to \gamma\gamma$ branching ratio are 88% correlated. This is because the η' width is evaluated dividing the $\Gamma(\eta' \to \gamma\gamma)$, obtained by measuring the cross section $\sigma(e^+e^- \to \eta'e^+e^-)$, by the $\eta' \to \gamma\gamma$ branching ratio. The fit results and the correlation matrix are shown in table 2. The gluonium fraction $Z_G^2 = 0.105 \pm 0.037$ is 2.8σ from zero. In order to identify the measurements which require the presence of the gluonium in the η' we have repeated the fit fixing Z_G at zero. The results of the fit are shown in table 2. The χ^2 probability is now quite low, reflecting the 2.8σ effect seen in the previous fit, while the pseudoscalar mixing angle is quite stable.

In figure 1 we show for each measurement the pulls of the two fits, defined as $p_i = (y_i - y_i^{th})/\sigma_{y_i}$. The measurement which does not fit in the *no-gluonium* hypothesis is the ratio $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$, whose pull is less than -3, bringing the χ^2 probability to 1.1%. It becomes ~ -1 when *gluonium* is allowed. We have then repeated the fit without using the $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$ information. The result is compared with ref. [4] in table 3. The results of the two fits are in agreement, making it evident that the origin of the discrepancy with ref. [4] is due to their neglecting the $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$ datum. In ref. [4] the couplings among the vectors and the pseudoscalar mesons are used

	Z_G free	$Z_G = 0$
χ^2/ndf (CL)	5/3 (17%)	13/4 (1.1%)
Z_G^2	0.105 ± 0.037	0 fixed
ψ_P	$(40.7 \pm 0.7)^\circ$	$(41.6 \pm 0.5)^\circ$
Z_q	0.866 ± 0.025	0.863 ± 0.024
Z_s	0.79 ± 0.05	0.78 ± 0.05
ψ_V	$(3.15 \pm 0.10)^\circ$	$(3.17 \pm 0.10)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

ψ_P	-0.513				
Z_q	0.003	0.041			
Z_s	0.088	-0.188	0.050		
ψ_V	-0.068	-0.019	0.150	0.077	
$m_s/\bar{m}$	0	0	0	0.935	0
	Z_G^2	ψ_P	Z_q	Z_s	ψ_V

Table 2. Fit results using the PDG-2006 data [9] and their correlation matrix.
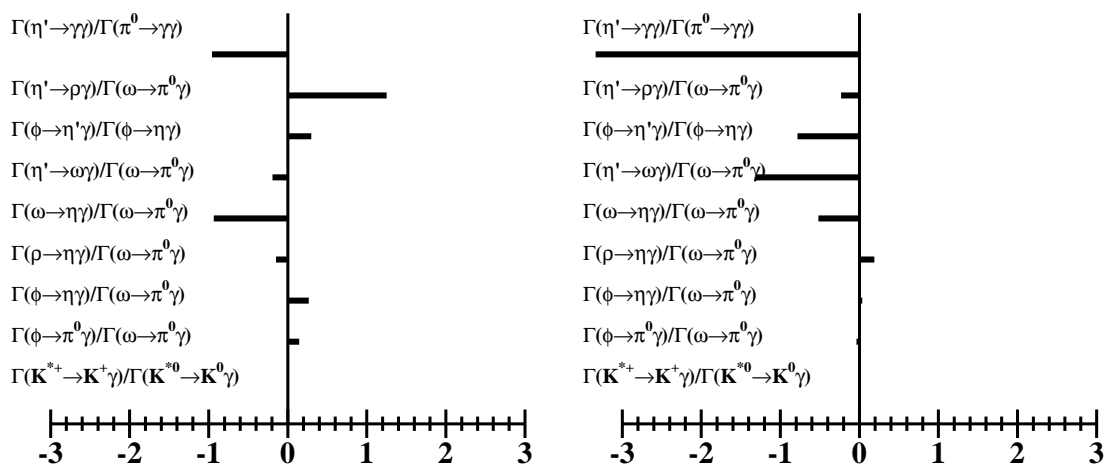

Figure 1. Pulls of the fit shown in table 2, *left:* Z_G free, *right:* $Z_G = 0$ (fixed).

in place of the width ratio. The couplings are related to the partial decay width by the following formulae:

$$\Gamma(V \to P\gamma) = \frac{1}{3}\frac{g_{VP\gamma}^2}{4\pi}|\vec{p}_\gamma|^3, \quad \Gamma(P \to V\gamma) = \frac{g_{VP\gamma}^2}{4\pi}|\vec{p}_\gamma|^3$$

In order to make a full comparison between the two methods we have performed the fit also using the couplings and we have obtain the same results [12].

	Fit with PDG-2006	Fit of ref. [4]
χ^2/ndf (CL)	1.8/2 (41%)	4.2/4 (38%)
Z_G^2	0.03 ± 0.06	0.04 ± 0.09
ψ_G	$(10 \pm 10)^\circ$	$(12 \pm 13)^\circ$
ψ_P	$(41.6 \pm 0.8)^\circ$	$(41.4 \pm 1.3)^\circ$
Z_q	0.85 ± 0.03	0.86 ± 0.03
Z_s	0.78 ± 0.05	0.79 ± 0.05
ψ_V	$(3.16 \pm 0.10)^\circ$	$(3.2 \pm 0.1)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

Table 3. Comparison among the fit results without the $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ measurements and the results of ref. [4]. PDG-2006 data [9] have been used in both fits.

	Z_G free	$Z_G = 0$ fixed
χ^2/ndf (CL)	7.9/3 (5%)	15/4 (5×10^{-3})
Z_G^2	0.097 ± 0.037	0 fixed
ψ_P	$(41.0 \pm 0.7)^\circ$	$(41.7 \pm 0.5)^\circ$
Z_q	0.86 ± 0.02	0.86 ± 0.02
Z_s	0.79 ± 0.05	0.78 ± 0.05
ψ_V	$(3.17 \pm 0.09)^\circ$	$(3.19 \pm 0.09)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

Table 4. Fit results using the PDG-2008 data.

ψ_P	-0.502				
Z_q	-0.072	0.161			
Z_s	0.081	-0.180	0.028		
ψ_V	-0.082	0.013	0.169	0.078	
$m_s/\bar{m}$	0	0	0	0.940	0
	Z_G^2	ψ_P	Z_q	Z_s	ψ_V

Table 5. Correlation matrix from the fit shown in table 4.

3 Update with the recent PDG results

In the Review of Particle Physics [13] new measurements of the ρ, ω, η and η' mesons have been included, which change slightly the partial decay widths used in the fit. Therefore we repeat the fit using these updated values together with our R_ϕ measurement. All the correlation coefficients among the measurements are taken into account in the fit. The results of the fit are shown in table 4 and the correlation matrix in table 5; the pulls of the fit are shown in figure 2.

The results in table 4 show that the gluonium hypothesis is still highly favoured with respect to the null gluonium hypothesis. Nevertheless the fit probability is quite low also in the gluonium hypothesis: it goes from 17% using PDG06 data to 5% using PDG08 data.
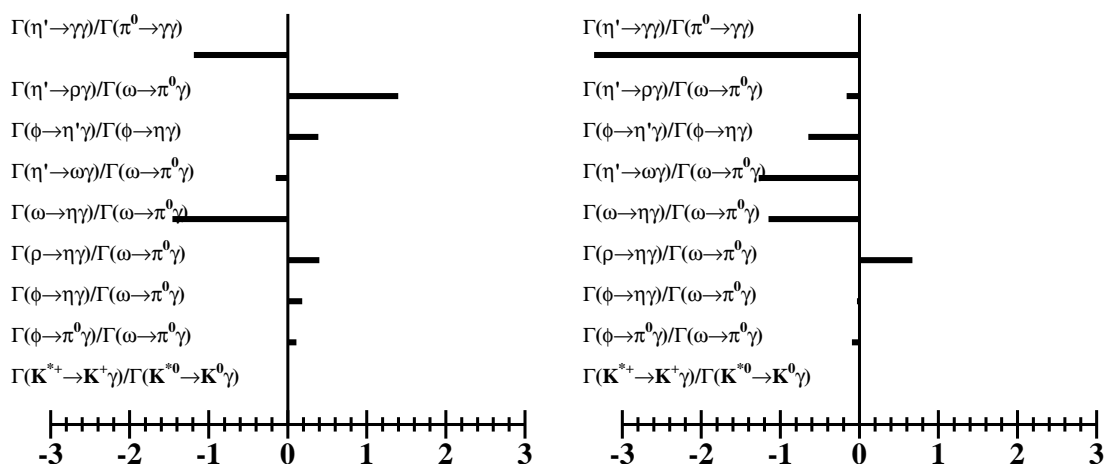
Figure 2. Pulls of the fit using PDG-2008 data, *left:* Z_G free, *right:* $Z_G = 0$ (fixed).

The reason of the worsening of the fit is found comparing the pulls of the new fit (figure 2) with the previous one (figure 1). In particular, the pull of the ratio $\Gamma(\omega \to \eta\gamma)/\Gamma(\omega \to \pi^0\gamma)$ goes from -0.93 using PDG-2006 data to -1.5 using PDG-2008 while the pull of the ratio $\Gamma(\rho \to \eta\gamma)/\Gamma(\omega \to \pi^0\gamma)$ goes from -0.14 to +0.39. This happens because the PDG estimate of the $\mathrm{BR}(\omega \to \eta\gamma)$ has changed from $(4.9 \pm 0.5) \times 10^{-4}$ to $(4.6 \pm 0.4) \times 10^{-4}$, lowering the Γ's ratio and worsening the pull.[1]

The $\mathrm{BR}(\omega \to \eta\gamma)$ and $\mathrm{BR}(\rho \to \eta\gamma)$ PDG values are dominated by the measurement of the $e^+e^- \to \eta\gamma$ cross section by SND [14] as a function of $\sqrt{s}$ in the ρ, ω, ϕ mass range. From the measured cross section they extract the ρ, ω partial decay widths assuming the Vector Meson Dominance model and a parametrisation for the ρ' resonance. Some correlation is therefore expected between the ρ and the ω partial decay widths which are not discussed in ref. [14], moreover the decay widths are model dependent. The average value reported by PDG-2008, $\mathrm{BR}(\omega \to \eta\gamma) = (6.3 \pm 1.3) \times 10^{-4}$, is dominated by a model independent measurement [15] and is 1.2σ away from the PDG fit. Using this value for $\mathrm{BR}(\omega \to \eta\gamma)$ we obtain a much better χ^2 probability: $P(\chi^2) = 28\%$ in the gluonium hypothesis and 1.1% fixing the gluonium at zero [12]. Both gluonium content and pseudoscalar mixing angle are unchanged ($Z_G^2 = 0.11 \pm 0.04$, $\psi_P = (40.6 \pm 0.7)^\circ$ in the gluonium hypothesis). Therefore we will use the average value for $\mathrm{BR}(\omega \to \eta\gamma)$ in the following.

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[1]A fit without the $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$ ratio has been performed in order to check its effect on the $\omega \to \eta\gamma$ and $\rho \to \eta\gamma$ pulls. In this case the χ^2/ndf of the fit is high also in the null gluonium hypothesis, nevertheless the $\Gamma(\omega \to \eta\gamma)/\Gamma(\omega \to \pi^0\gamma)$ pull is -0.68 while the $\Gamma(\rho \to \eta\gamma)/\Gamma(\omega \to \pi^0\gamma)$ pull is +1.1. In other words the $\mathrm{BR}(\omega \to \eta\gamma)$ fits better while the $\mathrm{BR}(\rho \to \eta\gamma)$ fits worse. The difference of the two measurements from the best fit is still $\sim 2\sigma$, therefore the poor χ^2 with PDG-2008 data is not due to $\eta' \to \gamma\gamma$ but to the inconsistency between $\Gamma(\omega \to \eta\gamma)$ and $\Gamma(\rho \to \eta\gamma)$ measurements.

	Z_G free	$Z_G = 0$ fixed
χ^2/ndf (CL)	4.6/3 (20%)	14.7/4 (0.5%)
Z_G^2	0.115 ± 0.036	0
ψ_P	$(40.4 \pm 0.6)^\circ$	$(41.4 \pm 0.5)^\circ$
Z_q	0.936 ± 0.025	0.927 ± 0.023
Z_s	0.83 ± 0.05	0.82 ± 0.05
ψ_V	$(3.32 \pm 0.09)^\circ$	$(3.34 \pm 0.09)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

Table 6. Fit results using PDG-2008 inputs, $\mathrm{BR}(\omega \to \eta\gamma)$ from PDG direct measurement average and the KLOE $\mathrm{BR}(\omega \to \pi^0\gamma)$ and R_ϕ. The equations (4.1) have been used for the f_q/f_π and f_s/f_π parameters.

ψ_P	-0.507				
Z_q	0.063	-0.018			
Z_s	0.092	-0.189	0.013		
ψ_V	-0.059	-0.012	0.045	0.028	
$m_s/\bar{m}$	-0.002	0.003	0.001	0.949	0.000
	Z_G^2	ψ_P	Z_q	Z_s	ψ_V

Table 7. Correlation matrix of the fit shown in table 6.

4 Update with the new KLOE measurement of $\mathrm{BR}(\omega \to \pi^0\gamma)$

The relations (1.5), (1.6) and (2.1)–(2.4) are dependent from the $\omega \to \pi^0\gamma$ decay rate. Recently we have improved the measurement of this branching fraction $\mathrm{BR}(\omega \to \pi^0\gamma) = (8.09 \pm 0.14)\%$ [16]. This value is about 3σ different from the PDG 2008 value: $\mathrm{BR}(\omega \to \pi^0\gamma) = (8.92 \pm 0.24)\%$. We then performed the fit using our measurement of $\mathrm{BR}(\omega \to \pi^0\gamma)$.

Moreover f_q/f_π and f_s/f_π have been fixed according to ref. [5]. In the exact isospin symmetry approximation the relations

$$f_q = f_\pi; \qquad f_s = \sqrt{2f_K^2 - f_\pi^2}$$

hold, where f_π and f_K are the π and K decay constants. Therefore $f_s/f_\pi = \sqrt{2f_K^2/f_\pi^2 - 1}$. Using f_K/f_π from lattice calculation [18] we get:

$$\frac{f_q}{f_\pi} = 1 \qquad \frac{f_s}{f_\pi} = 1.352 \pm 0.007. \tag{4.1}$$

The results of the fit are shown in table 6 and the correlation matrix in table 7.

The η-η' mixing angle and the η' gluonium content are not substantially modified, but the χ^2 probability is improved with respect to the previous fits. The $\phi - \omega$ mixing angle, ψ_V, is also slightly changed from $(3.17 \pm 0.09)^\circ$ to $(3.32 \pm 0.09)^\circ$ by our new measurement of $\mathrm{BR}(\omega \to \pi^0\gamma)$, ψ_V being directly related to the ratio $\Gamma(\phi \to \pi^0\gamma)/\Gamma(\omega \to \pi^0\gamma)$ (see eq. (2.4)).
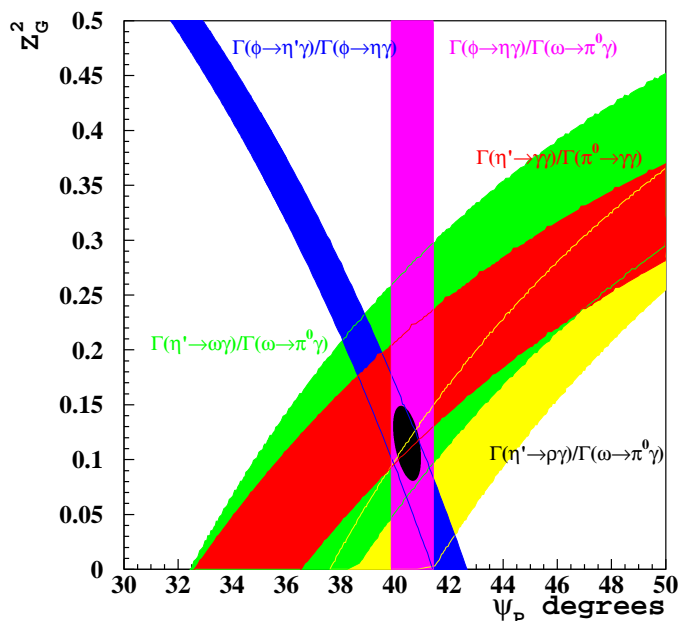
Figure 3. 68% confidence level regions of the shown measurements in the Z_G^2, ψ_P plane.

width ratio	fitted value	error	width ratio	fitted value	error
$\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)}$	570	35	$\frac{\Gamma(\rho \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.061	0.004
$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.0735	0.007	$\frac{\Gamma(\phi \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.072	0.003
$\frac{\Gamma(\phi \to \eta'\gamma)}{\Gamma(\phi \to \eta\gamma)}$	0.0047	0.0002	$\frac{\Gamma(\phi \to \pi^0\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.0079	0.0004
$\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.0087	0.0010	$\frac{\Gamma(K^{*+} \to K^+\gamma)}{\Gamma(K^{*0} \to K^0\gamma)}$	0.43	0.06
$\frac{\Gamma(\omega \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.0064	0.0015			

Table 8. Fitted values of the Γ ratios.

In figure 3 we show the 68% CL contour of the η' related measurements in the Z_G^2, ψ_P plane where the contribution of each measurement to the fit results can be appreciated. The best fit values of the width ratios are shown in table 8 together with their correlation coefficients in table 9.

5 Prospects with improved measurements

We have shown that within the precision of the available data the crucial measurement sensitive to the η' gluonium content is $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$. The theoretical framework used to describe $\eta' \to \gamma\gamma$ and $V \to P\gamma$ transition is different. In fact while in the first case we have a quark-antiquark annihilation into two photons, in the second case we have a transition among two mesons with photon emission via the spin flip of one of the two quarks. Therefore it is important both to reach a sensitivity to the gluonium independently from the $\eta' \to \gamma\gamma$ decay and to measure again the $\eta' \to \gamma\gamma$ branching ratio.

	$\eta'\gamma\gamma$	$\eta'\rho\gamma$	$\phi\eta'\gamma$	$\eta'\omega\gamma$	$\omega\eta\gamma$	$\rho\eta\gamma$	$\phi\eta\gamma$	$\phi\pi^0\gamma$
$\eta'\rho\gamma$	0.28							
$\phi\eta'\gamma$	0	0						
$\eta'\omega\gamma$	0.24	0.48	0					
$\omega\eta\gamma$	0	0.04	0	0.28				
$\rho\eta\gamma$	0	0.13	0	0.08	0.05			
$\phi\eta\gamma$	0	0.28	0	0.18	0.10	0		
$\phi\pi^0\gamma$	0	0.17	0	0.11	0.06	0	0.36	
$K^{*+}K^+\gamma$	0	0	0	0	0	0	0	0

Table 9. Correlation coefficients of the best-fitted values.

	with $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$	without $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$
Z_G^2	0.12 ± 0.03	0.11 ± 0.04
ψ_P	$(40.5 \pm 0.6)^\circ$	$(40.5 \pm 0.6)^\circ$
Z_{NS}	0.93 ± 0.02	0.93 ± 0.03
Z_S	0.83 ± 0.05	0.83 ± 0.05
ψ_V	$(3.32 \pm 0.08)^\circ$	$(3.32 \pm 0.09)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

Table 10. Fit values assuming 1% error on the η' branching fractions: *left:* using $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$, *right:* without using $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$.

The $\eta' \to \gamma\gamma$ branching ratio affects directly the η' decay width while the $\eta' \to \pi^+\pi^-\eta$ and the $\eta' \to \pi^0\pi^0\eta$ branching ratios affect the systematic errors on the R_ϕ measurement [6]. The increase of the precision on all main η' branching fractions at 1% level could lead the gluonium contribution to $\sim 4\sigma$ (see table 10). The results shown in the table are obtained assuming as central value of all measurements the width ratios shown in table 8 and assigning to all η' branching ratios a 1% error (the error on $\Gamma_{\eta'}$ and R_ϕ has been recomputed accordingly); the correlation matrix is taken equal to the present measurements. The fit gives a large η' gluonium component even without using the $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ measurement (see table 10, right column).

In figure 4 the expected 68% CL contour in the plane ψ_P, Z_G^2 is shown. Notice that the $\eta' \to \gamma\gamma$ measurement would be not needed anymore to have a significative gluonium component.

A big improvement in the gluonium content determination can be obtained by measuring the η' decay width directly through the measurement of $\sigma(e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\eta')$. The measurement of this cross section at 1% level and the $\eta' \to \gamma\gamma$ branching ratio at the same level of accuracy would allow to determine the η' width:

$$\Gamma_{\eta'} = \frac{\Gamma(\eta' \to \gamma\gamma)}{\text{BR}(\eta' \to \gamma\gamma)}$$

with a fractional error of $\sim 1.4\%$. In figure 4, right the 68% CL contours in the (ψ_P, Z_G^2)
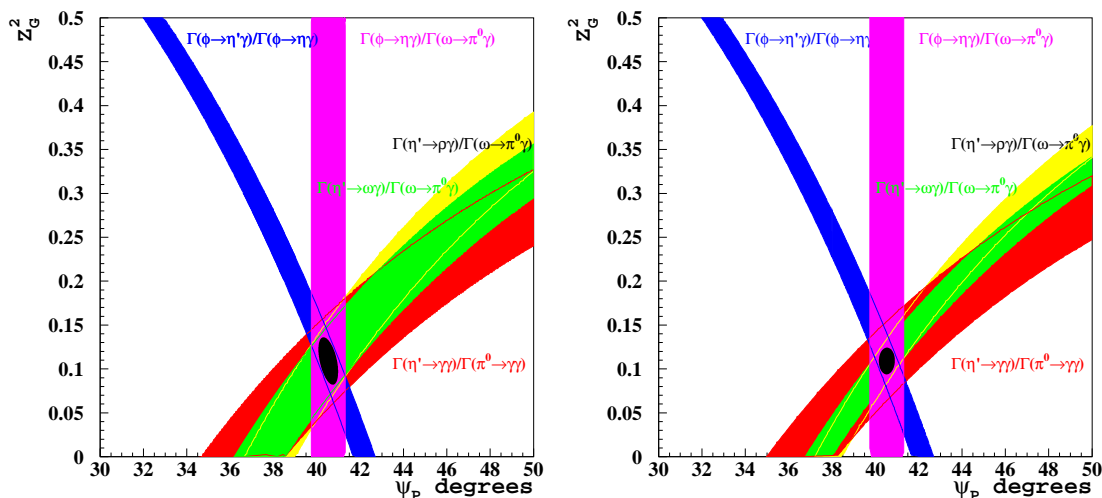
Figure 4. Future prospects: one sigma confidence regions in the (ψ_P, Z_G^2) plane assuming present fitted measurements but with reduced errors: *left:* with 1% error on η' branching ratios, *right:* as before plus 1% precision on $\Gamma(\eta' \to \gamma\gamma)$.

parameter plane are shown for all η' related measurements. The improvement in the fit accuracy is evident. The gluonium contribution would be measured at $\sim 5\sigma$.

6 Conclusions

The origin of the difference between the KLOE result [6] on the η' gluonium content and ref. [4] is the use of the $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$ measurement. A global fit to all measured $V \to P\gamma$ and $P \to V\gamma$ transitions of light mesons has been performed extracting all the relevant parameters. The new results are slightly different from our previous results but confirm the presence of a significative gluonium contribution in the η' meson. The origin of this contribution has been investigated and found to stem mainly from the $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0\gamma\gamma)$ measurement. The fit has been updated with all recent measurements from PDG [13] and some disagreement with the PDG fitted value of the BR($\omega \to \eta\gamma$) has been found. The average of the $\omega \to \eta\gamma$ branching ratio is therefore preferred to the PDG fit and a better fit is obtained while the gluonium content and the mixing angle are unaffected. The result has been updated using the recent KLOE measurement of the $\omega \to \pi^0\gamma$ branching ratio and the lattice results for f_q/f_π and f_s/f_π assuming exact isospin symmetry. The mixing angle and the gluonium content are again marginally affected by this measurement while a slightly larger $\phi - \omega$ mixing angle is obtained.

Finally, we have estimated the impact of future improvements in the measurements of the η' branching ratios and η' width in the gluonium content determination. A 5σ evidence could be obtained with a 1% precision on each measurement.

Acknowledgments

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