Measurement of the hadronic cross section with KLOE/KLOE-2 and its impact on $a_{\mu}^{HLO}$

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* Motivation

* KLOE measurements of $\sigma(e^+e^- \rightarrow \pi^+\pi^-(\gamma))$
  and Evaluation of $a_{\pi\pi}$

  Normalized to Bhabha events:
  - Small (photon) angle measurements (KLOE05, KLOE08)
  - Large (photon) angle measurement (KLOE10)

  Normalized to $\mu\mu\gamma$ events (NEW!):
  - Small (photon) angle measurements (KLOE11, preliminary)

  Comparison btw KLOE11 and KLOE08, KLOE10

* Conclusion
Motivation

See Passera's Talk

\[ a_\mu \] Experiment and SM prediction discrepancy about \( \sim 3\sigma \)

The main uncertainty in the estimate of theoretical prediction is due to low energy hadron contribution

Leading order adronic contribution is not calculable by perturbative QCD, but it has to be estimated by a dispersion integral using measured adronic cross section.

So, this uncertainty can be substantially reduced by hadronic cross section measurements in electron-positron annihilation at low energy.
$a_{\mu}$ comparison Exp & Theo

Standard model theory and experiment comparison [in units $10^{-11}$].

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED incl. 4-loops + LO 5-loops</td>
<td>116 584 718.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Leading hadronic vacuum polarization</td>
<td>6 903.0</td>
<td>52.6</td>
</tr>
<tr>
<td>Subleading hadronic vacuum polarization</td>
<td>$-100.3$</td>
<td>1.1</td>
</tr>
<tr>
<td>Hadronic light-by-light</td>
<td>116.0</td>
<td>39.0</td>
</tr>
<tr>
<td>Weak incl. 2-loops</td>
<td>153.2</td>
<td>1.8</td>
</tr>
<tr>
<td>Theory</td>
<td>116 591 790.0</td>
<td>64.6</td>
</tr>
<tr>
<td>Experiment</td>
<td>116 592 080.0</td>
<td>63.0</td>
</tr>
<tr>
<td>Exp. - Theo. 3.2 standard deviations</td>
<td>290.0</td>
<td>90.3</td>
</tr>
</tbody>
</table>

$$(27.7 \pm 8.4) \times 10^{-10}$$

$8.4 \approx 5_{\text{HLO}} \oplus 3_{\text{LBL}} \oplus 6_{\text{BNL}}$

$a_{\mu}^{\text{Exp}} - a_{\mu}^{\text{Theo}} = (27.6 \pm 8.7) \times 10^{-10} \sim 3.4 \sigma$

Is an hint of new physics?
Cross section data:

Two approaches:

Energy scan (CMD2, SND, BES, CLEO):

- energy of colliding beams is changed to the desired value
- “direct” measurement of cross sections
- needs dedicated accelerator/physics program
- needs to measure luminosity and beam energy for every data point

Radiative return (KLOE, BABAR, BELLE, BESIII?):

- runs at fixed-energy machines (meson factories)
- use initial state radiation process to access lower lying energies or resonances
- data come as by-product of standard physics program
- requires precise theoretical calculation of the radiator function
- luminosity and beam energy enter only once for all energy points
- needs larger integrated luminosity
Measured cross section for $e^+e^- \rightarrow \pi^+ \pi^-$
IN 2005 KLOE has published the first precision measurement of $\sigma(e^+e^-\rightarrow\pi^+\pi^-)$ with ISR using 2001 data (140pb$^{-1}$) PLB606(2005)12 ~3$\sigma$ discrepancy btw $a_\mu^{SM}$ and $a_\mu^{exp}$
**DAΦNE: A Φ-Factory**

$e^+e^- - collider$ with $\sqrt{s} = m_\Phi \approx 1.0195$ GeV

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**Integrated Luminosity**

- **Total KLOE int. Luminosity:**
  - $\int L \, dt \sim 2500 \, pb^{-1} (2001 - 05)$

- **Peak Luminosity:**
  - $L_{\text{peak}} = 1.5 \times 10^{32} \text{cm}^{-2}\text{s}^{-1}$

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**KLOE detector**

**KLOE05 measurement**

(PLB606(2005)12) was based on 140pb$^{-1}$ of 2001 data!

**KLOE08 measurement**

(PLB670(2009)285) was based on 240pb$^{-1}$ from 2002 data!

**KLOE10 measurement**

(PLB700(2011)102) based on 233 pb$^{-1}$ of 2006 data (at 1 GeV, different event selection)

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**2006:**

- Energy scan (4 points around m$\Phi$-peak)
- 240 pb$^{-1}$ at $\ = 1000$ MeV (off-peak data)
KLOE Detector

Drift chamber

Full stereo geometry, 4m diameter, 52,140 wires 90% Helium, 10% iC4H10

\[ \sigma_p / p = 0.4\% \text{ (for 90° tracks)} \]
\[ \sigma_{xy} \approx 150 \text{ mm}, \sigma_z \approx 2 \text{ mm} \]

Excellent momentum resolution
KLOE Detector

Electromagnetic Calorimeter

Pb / scintillating fibres (**4880 PMT**)

Endcap - Barrel - Modules

\[ \sigma_{E/E} = 5.7\% / \sqrt{E(\text{GeV})} \]

\[ \sigma_T = 54 \text{ ps} / \sqrt{E(\text{GeV})} \oplus 100 \text{ ps} \]

(Bunch length contribution subtracted from constant term)

**Excellent timing resolution**
SA Event Selection (KLOE08)

a) 2 tracks with $50^\circ < \theta_{\text{track}} < 130^\circ$

b) small angle (not detected) $\gamma$

c) $(\theta_{\pi\pi} < 15^\circ \text{ or } > 165^\circ)$
   - high statistics for ISR
   - low relative FSR contribution
   - suppressed $\phi \rightarrow \pi^+\pi^-\pi^0$ wrt the signal

statistics: 240pb$^{-1}$ of 2002 data 3.1 Mill.
Events between 0.35 and 0.95 GeV$^2$
### SA Kloe result (KLOE08)

**Systematic errors on** $a_\mu^{\pi\pi}$:

<table>
<thead>
<tr>
<th>Source</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reconstruction Filter</td>
<td>negligible</td>
</tr>
<tr>
<td>Background</td>
<td>0.3%</td>
</tr>
<tr>
<td>Trackmass/Miss. Mass</td>
<td>0.2%</td>
</tr>
<tr>
<td>p/e-ID and TCA</td>
<td>negligible</td>
</tr>
<tr>
<td>Tracking</td>
<td>0.3%</td>
</tr>
<tr>
<td>Trigger</td>
<td>0.1%</td>
</tr>
<tr>
<td>Acceptance ($\theta_{\pi\pi}$)</td>
<td>0.1%</td>
</tr>
<tr>
<td>Acceptance ($\theta_{\pi}$)</td>
<td>negligible</td>
</tr>
<tr>
<td>Unfolding</td>
<td>negligible</td>
</tr>
<tr>
<td>Software Trigger</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\sqrt{s}$ dep. Of H</td>
<td>0.2%</td>
</tr>
<tr>
<td>Luminosity (0.1$<em>{th}$ $\oplus$ 0.3$</em>{exp}$)%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

**Experimental fractional error on** $a_\mu = 0.6\%$

<table>
<thead>
<tr>
<th>Source</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSR resummation</td>
<td>0.3%</td>
</tr>
<tr>
<td>Radiator H</td>
<td>0.5%</td>
</tr>
<tr>
<td>Vacuum polarization</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

**Theoretical fractional error on** $a_\mu = 0.6\%$

$$a_\mu^{\pi\pi} = \int_{x_1}^{x_2} \sigma_{ee\rightarrow\pi\pi}(s)K(s)ds$$

$$a_\mu^{\pi\pi}(0.35-0.95\text{GeV}^2) = (387.2 \pm 0.5_{\text{stat}} \pm 2.4_{\text{sys}} \pm 2.3_{\text{theo}}) \cdot 10^{-10}$$
LA Event Selection (KLOE10)

2 pion tracks at large angles
50° < θπ < 130°

Photons at large angles
50° < θγ < 130°
- independent complementary analysis
- threshold region (2mπ^2) accessible
- γISR photon detected
  (4-momentum constraints)

- lower signal statistics
- larger contribution from FSR events
- larger φ → π^+π^-π^0 background contamination
- irreducible background from φ decays (φ → f^0 γ → ππ γ)

Threshold region non-trivial
due to irreducible FSR-effects, which have to be estimated from MC using phenomenological models (interference effects unknown)

At least 1 photon with 50° < θγ < 130°
and E_γ > 20 MeV => photon detected
2 pion tracks at large angles
50° < θπ < 130°

Photons at large angles
50° < θγ < 130°

- independent complementary analysis
- threshold region (2mπ)² accessible
- γISR photon detected
  (4-momentum constraints)

- lower signal statistics
- larger contribution from FSR events
- larger φ → π⁺π⁻π⁰ background contamination
- irreducible background from φ decays (φ → f0 γ → ππ γ)

At least 1 photon with 50° < θγ < 130° and Eγ > 20 MeV -> photon detected

Use data sample taken at √s ≈ 1000 MeV, 20 MeV below the φ-peak

statistics: 233 pb⁻¹ of 2006 data
600 kEvents
KLOE10: Pion Form Factor


Table of systematic errors on $a_{\mu}^{\pi\pi}(0.1\text{--}0.85 \text{ GeV}^2)$:

<table>
<thead>
<tr>
<th>Source</th>
<th>Fractional Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reconstruction Filter</td>
<td>negligible</td>
</tr>
<tr>
<td>Background</td>
<td>0.5%</td>
</tr>
<tr>
<td>$f_0+\rho\pi$</td>
<td>0.4%</td>
</tr>
<tr>
<td>$\Omega$ cut</td>
<td>0.2%</td>
</tr>
<tr>
<td>Trackmass</td>
<td>0.5%</td>
</tr>
<tr>
<td>p/e-ID and TCA</td>
<td>negligible</td>
</tr>
<tr>
<td>Tracking</td>
<td>0.3%</td>
</tr>
<tr>
<td>Trigger</td>
<td>0.2%</td>
</tr>
<tr>
<td>Acceptance</td>
<td>0.5%</td>
</tr>
<tr>
<td>Unfolding</td>
<td>negligible</td>
</tr>
<tr>
<td>Software Trigger</td>
<td>0.1%</td>
</tr>
<tr>
<td>Luminosity(0.1th + 0.3 exp)%</td>
<td>0.3%</td>
</tr>
<tr>
<td>experimental fractional error</td>
<td>1.0 %</td>
</tr>
</tbody>
</table>

experimental fractional error on $a_{\mu} = 1.0\%$

<table>
<thead>
<tr>
<th>Source</th>
<th>Fractional Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSR treatment</td>
<td>0.8%</td>
</tr>
<tr>
<td>Radiator H</td>
<td>0.5%</td>
</tr>
<tr>
<td>Vacuum polarization</td>
<td>0.1%</td>
</tr>
<tr>
<td>theoretical fractional error</td>
<td>0.9 %</td>
</tr>
</tbody>
</table>

Theoretical fractional error on $a_{\mu} = 0.9\%$

$$a_{\mu}^{\pi\pi}(0.1\text{--}0.85 \text{ GeV}^2) = (478.5 \pm 2.0 \text{ stat} \pm 5.0 \text{ sys} \pm 4.5 \text{ theo}) \cdot 10^{-10}$$
Comparison of results: KLOE10 vs KLOE08

KLOE08 result compared to KLOE10:

Fractional difference:

Excellent agreement with KLOE08, especially above 0.5 GeV

Combination of KLOE08 and KLOE10:

\[ a_{\mu}^{\pi\pi}(0.1-0.95 \text{ GeV}^2) = (488.6 \pm 6.0) \cdot 10^{-10} \]

KLOE covers ~70% of total \( a_{\mu}^{HLO} \) with a fractional total error of 1.2%
CMD and SND results compared to KLOE10: Fractional difference

Below the $\rho$ peak good agreement with CMD-2/SND.
Above the $\rho$ peak KLOE10 slightly lower (as KLOE08)
Comparison of results: KLOE10 vs BaBar

BaBar results compared to KLOE10: Fractional difference

Agreement within errors below 0.6 GeV; BaBar higher by 2-3% above
New $\sigma_{\pi\pi}$ measurement from $\pi/\mu$

An alternative way to obtain $|F_{\pi}|^2$ is the bin-by-bin ratio of pion over muon yields (instead of using absolute normalization with Bhabhas).

$$|F_{\pi}(s')|^2 \approx \frac{4(1 + 2m_{\mu}^2/s')\beta_{\mu}}{\beta_{\pi}^3} \frac{d\sigma_{\pi\pi\gamma}/ds'}{d\sigma_{\mu\mu\gamma}/ds'}$$

Many radiative corrections drop out:
- radiator function
- int. luminosity from Bhabhas
- Vacuum polarization

Separation btw $\pi\pi\gamma$ and $\mu\mu\gamma$ using $M_{\text{Trk}}$
- muons: $M_{\text{Trk}} < 115$ MeV
- pions: $M_{\text{Trk}} > 130$ MeV

Very important control of $\pi/\mu$ separation in the $\rho$ region! ($\sigma_{\pi\pi} \gg \sigma_{\mu\mu}$)
KLOE11: analysis of $\pi\pi\gamma/\mu\mu\gamma$

- 239.2 pb$^{-1}$ of 2002 data sample (the same used in KLOE08 analysis), with photon at small angle: 0.87 Million $\mu\mu\gamma$ events (compared to 3.4 Million for $\pi\pi\gamma$).

- Careful work to achieve a control of $\sim$1% in the muon selection, especially in the $\rho$ region where $\pi\pi\gamma/\mu\mu\gamma \sim 10$. $\pi/\mu$ separation crosschecked in three different methods (MTRK fit, Kinematic fit, cut on $\sigma_{\text{MTRK}}$).

- $\mu\mu\gamma$ (and $\pi\pi\gamma$) Efficiencies (Trk, Trg, PID) done on data.

- Excellent data/MC agreement for many kinematic variables: $M_{\text{TRK}}$, track and $\gamma$ polar angle, etc...

![Graphs and plots related to $\pi\pi\gamma/\mu\mu\gamma$ analysis](image-url)
**μμγ cross section: data/MC comparison**

\[
\frac{d\sigma_{\mu\mu\gamma}^{obs}}{dM_{\mu\mu}^2} = \frac{\Delta N_{\text{Obs}} - \Delta N_{\text{Bkg}}}{\Delta M_{\mu\mu}^2} \cdot \frac{1}{\varepsilon_{\text{Sel}}} \cdot \frac{1}{\int L dt}
\]

\[
\frac{d\sigma_{\mu\mu\gamma}^{\text{DATA}}}{d\sigma_{\mu\mu\gamma}^{\text{MC}}} = 0.996 \pm 0.001_{\text{stat}} \pm 0.01_{\text{syst}}
\]

The systematic error has been averaged on \(s_\mu\)

Good agreement with PHOKHARA MC

Consistency check of Radiator function, Luminosity, etc…
KLOE11 result on $|F_\pi|^2$ and comp. with KLOE08

<table>
<thead>
<tr>
<th>Syst. errors (%)</th>
<th>$\Delta^{\pi\pi}a_\mu$ abs</th>
<th>$\Delta^{\pi\pi}a_\mu$ ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reconstruction Filter</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td>Background subtraction</td>
<td>0.3</td>
<td>0.8 (0.3_{\pi\pi\gamma} + 0.7_{\mu\mu\gamma})</td>
</tr>
<tr>
<td>Trackmass</td>
<td>0.2</td>
<td>0.4 (0.2_{\pi\pi\gamma} + 0.4_{\mu\mu\gamma})</td>
</tr>
<tr>
<td>Particle ID</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td>Tracking</td>
<td>0.3</td>
<td>0.6 (0.3_{\pi\pi\gamma} + 0.5_{\mu\mu\gamma})</td>
</tr>
<tr>
<td>Trigger</td>
<td>0.1</td>
<td>0.1 (0.1_{\pi\pi\gamma})</td>
</tr>
<tr>
<td>Unfolding</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td>Acceptance ($\theta_{\pi\pi}$)</td>
<td>0.2</td>
<td>negligible</td>
</tr>
<tr>
<td>Acceptance ($\theta_{\pi}$)</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td>Software Trigger (L3)</td>
<td>0.1</td>
<td>0.1 (0.1_{\pi\pi\gamma} + 0.1_{\mu\mu\gamma})</td>
</tr>
<tr>
<td>Luminosity</td>
<td>0.3 (0.1_{th} + 0.3_{exp})</td>
<td>-</td>
</tr>
<tr>
<td>$\sqrt{s}$ dep. of $H$</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Total exp systematics</td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Vacuum Polarization</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>FSR treatment</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Rad. function $H$</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>Total theory systematics</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Total systematic error</td>
<td>0.9</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Good agreement btw the two measurements, especially in the $\rho$ region!!!

**KLOE11:** $a_\mu^{\pi\pi}(0.35-0.95 \text{ GeV}^2) = (384.1 \pm 1.2_{\text{stat}} \pm 4.0_{\text{sys}} \pm 1.2_{\text{theo}}) \cdot 10^{-10}$

**KLOE08:** $a_\mu^{\pi\pi}(0.35-0.95 \text{ GeV}^2) = (387.2 \pm 0.5_{\text{stat}} \pm 2.4_{\text{sys}} \pm 2.3_{\text{theo}}) \cdot 10^{-10}$
Comparison of results: KLOE11 vs KLOE10

KLOE11 result compared to KLOE10:

Fractional difference:

Excellent agreement between the two measurements!

Comparison with other exp. in progress

KLOE11: $a_{\mu, \pi\pi}^{(0.35-0.85 \text{ GeV}^2)} = (376.4 \pm 1.2_{\text{stat}} \pm 4.1_{\text{sys tot}}) \cdot 10^{-10}$

KLOE10: $a_{\mu, \pi\pi}^{(0.35-0.95 \text{ GeV}^2)} = (376.6 \pm 0.9_{\text{stat}} \pm 3.3_{\text{sys tot}}) \cdot 10^{-10}$
Theoretical predictions compared to the BNL result (2009)

The latest inclusion of all $e^+e^-$ data gives a discrepancy between $a_\mu^{\text{SM}}$ and $a_\mu^{\text{EXP}} \geq 3 \sigma$.

Preliminary KLOE11 in agreement with previous KLOE measurements and confirms this discrepancy!

Very important the new g-2 experiments (at FNAL and JPARC)!
Conclusion

During the last 10 years KLOE has performed a series of precision measurements with ISR which confirmed a $3\sigma$ discrepancy between $a^\text{SM}_\mu$ and the BNL measured value and allowed the measurement of $a^\pi\pi_\mu$ in the region $0.1-0.95\text{ GeV}^2$ ($70\%$ of $a^\text{HLO}_\mu$) with $1.2\%$ total error using KLOE data only.

A new (preliminary) measurement of $|F_\pi|^2$ from the $\pi\pi\gamma/\mu\mu\gamma$ ratio (based on $240\text{ pb}^{-1}$) with $1.1\%$ systematic error has been done. Preliminary results show good agreement for $\mu\mu\gamma$ cross section with PHOKHARA MC and for $|F_\pi|^2$ and $a^\pi\pi_\mu$ with previous KLOE published measurements.

Still more than $1.5\text{ fb}^{-1}$ of KLOE data on tape. This is a $\sim 4$ improvement in statistics. We plan to analyse these data to improve $\sigma^{\pi\pi}$ (and may be other channels) measurement.

In addition we expect about $25\text{ fb}^{-1}$ at KLOE-2, and that should allow to measure $\sigma^{\pi\pi}$ ($2m_\pi<s<1\text{ GeV}$) with $0.4\%$ error (now $0.7\%$ averaging CMD-2/SND/KLOE).
Spares
Cross check of $\pi/\mu$ separation

- The $\pi/\mu$ separation has been crosschecked with two different (and independent) methods:

- A kinematic fit, in the hypothesis of 2 body + $1\gamma$ (ISR) events.

- A cut on the quality of the fitted tracks, parametrized by $\sigma_{MTRK}^2$

$$\sigma_{MTRK}^2 = \begin{pmatrix} \frac{\partial M_{trk}}{\partial k_1} & \frac{\partial M_{trk}}{\partial k_2} & \frac{\partial M_{trk}}{\partial k_2} & \frac{\partial M_{trk}}{\partial \cot \theta_1} & \frac{\partial M_{trk}}{\partial \cot \theta_2} & \frac{\partial M_{trk}}{\partial \varphi_1} & \frac{\partial M_{trk}}{\partial \varphi_2} \\ \rho_1 k_1 & \rho_1 \cot \theta_1 & \rho_1 \varphi_1 & 0 & 0 & 0 & 0 \\ \rho_1 \cot \theta_1 k_1 & \rho_1 \cot \theta_1 & \rho_1 \varphi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_2 k_2 & \rho_2 \cot \theta_2 & \rho_2 \varphi_2 & \sigma_2^2 \\ 0 & 0 & 0 & \rho_2 \cot \theta_2 k_2 & \rho_2 \cot \theta_2 & \rho_2 \varphi_2 & \sigma_2^2 \\ 0 & 0 & 0 & \rho_2 \varphi_2 k_2 & \rho_2 \varphi_2 \cot \theta_2 & \rho_2 \varphi_2 & \sigma_2^2 \end{pmatrix}$$

- $\pi/\mu$ separation obtained with these methods well in agreement with the standard one.
Cross check of $\pi/\mu$ separation $\sigma_{\text{MTRK}}$

We have achieved an excellent Data/MC agreement for muons in many kinematic variables (as we did for pions).

$\pi/\mu$ separation obtained with these methods well in agreement with the standard one.
Cross check of $\pi/\mu$ separation $\sigma_{\text{MTRK}}$

The $\mu^+\mu^-\gamma$ fraction obtained by the $\sigma_{\text{MTRK}}$ cut are consistent with the standard procedure within the systematic error.

BG suppression under 10%
A cross check was realized by using an independent analysis based on a cut on the quality of the fitted tracks, parametrized by $\sigma_{\text{MTRK}}$.

The alternative analysis produced: a **BG contribution below 10%** (see figures); the $\mu^+\mu^-\gamma$ fraction determination consistent with the other procedure within the systematic error.
Extracting $\sigma_{\pi\pi}$ and $|F_\pi|^2$ from $\pi\pi\gamma$ events

a) Via absolute Normalisation to Bhabha events (KLOE05,08,10):

1) \[
\frac{d\sigma_{\pi\pi\gamma(\gamma)}}{dM^2_{\pi\pi}} = \frac{\Delta N_{\text{Obs}} - \Delta N_{\text{Bkg}}}{\Delta M^2_{\pi\pi}} \cdot \frac{1}{\varepsilon_{\text{Sel}}} \cdot \int L dt
\]

d$\sigma_{\pi\pi\gamma(\gamma)}/dM^2$ is obtained by subtracting background from observed event spectrum, divide by selection efficiencies, and \textit{int. luminosity}:

Obtain $\sigma_{\pi\pi}$ from (ISR) - radiative cross section $d\sigma_{\pi\pi\gamma(\gamma)}/dM^2$ via theoretical radiator function $H(s)$:

Relation between $|F_\pi|^2$ and the cross section $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$

b) Via bin-by-bin Normalisation to rad. Muon events (New measurement!)
Radiative Corrections

Radiator-Function \(H(s,s_\pi)\) (ISR):

- ISR-Process calculated at NLO-level
- PHOKHARA generator

Precision: 0.5\%

\[ \frac{d\sigma_{\pi\pi\gamma}}{ds_\pi} = \sigma_{\pi\pi}(s_\pi) \times H(s,s_\pi) \]

Radiative Corrections:

i) Bare Cross Section
   divide by Vacuum Polarisation \(d(s) = (a(s)/a(0))^2\)
   ➔ from F. Jegerlehner

ii) FSR
   Cross section \(s_{pp}\) must be incl. for FSR
   for use in the dispersion integral of \(a_m\)

FSR corrections have to be taken into account
in the efficiency eval. (Acceptance, \(M_{T\pi}\)) and in
the mapping \(s_\pi \rightarrow s_{\gamma*}\)

Hadronic contribution to $a_\mu$ can be estimated by means of a dispersion integral:

$$a^\text{had}_\mu = \left( \frac{\alpha}{3\pi} \frac{m_\mu}{s^2} \right)^2 \int s^2 \frac{R(s) \hat{K}(s)}{s^2} \, ds$$

$$R(s) = \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow q \bar{q} \rightarrow \text{hadrons})}{\sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)}$$

- $\hat{K}(s)$ = analytic kernel-function
- above sufficiently high energy value, typically 2...5 GeV, use $pQCD$

Input:

a) **hadronic electron-positron cross section data**

b) **hadronic $\tau$-decays**, which can be used with the help of the CVC-theorem and an isospin rotation (plus isospin breaking corrections)