Measurement of the hadronic cross section with KLOE/KLOE-2 and its impact on a_μ^{HLO}

Giuseppe Mandaglio
(for the KLOE/KLOE-2 collaboration)
University of Messina & INFN Sez. Catania

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Outlook

* Motivation

* KLOE measurements of $\sigma(e^+e^- \rightarrow \pi^+\pi^-(\gamma))$

and Evaluation of $a_\mu^{\pi\pi}$

Normalized to Bhabha events:

- Small (photon) angle measurements (KLOE05, KLOE08)
- Large (photon) angle measurement (KLOE10)

Normalized to $\mu\mu\gamma$ events (NEW!):

- Small (photon) angle measurements(KLOE11, *preliminary*)

Comparison btw KLOE11 and KLOE08, KLOE10

* Conclusion

Motivation

See Passera's Talk

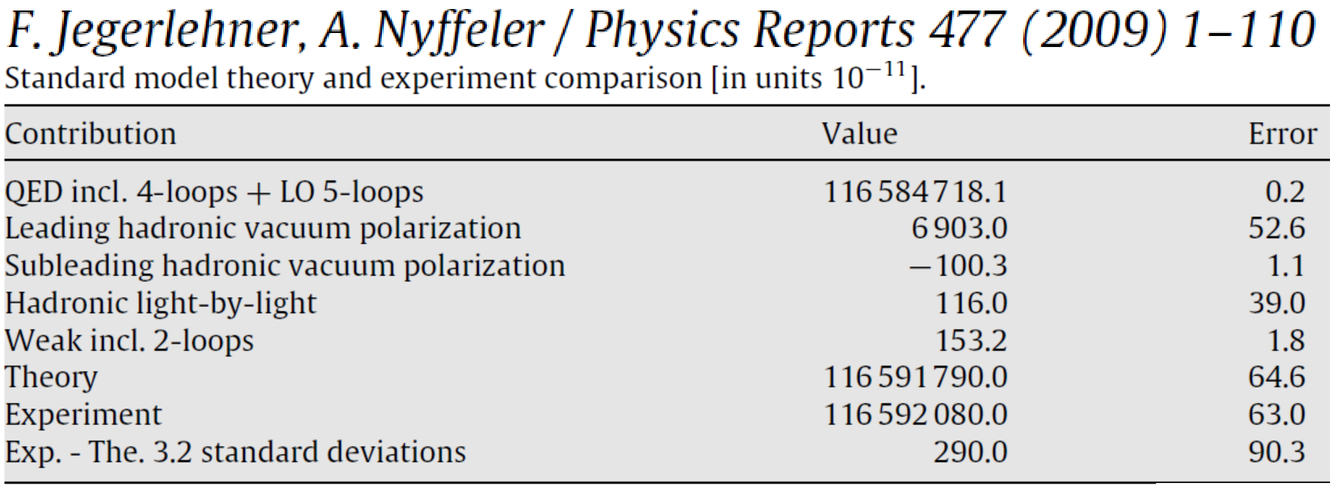
a_μ Experiment and SM prediction discrepancy about $\sim 3\sigma$

The main uncertainty in the estimate of theoretical prediction is due to low energy hadron contribution

Leading order adronic contribution is not calculable by perturbative QCD, but it has to be estimated by a dispersion integral using measured adronic cross section.

So, this uncertainty can be substantially reduced by hadronic cross section measurements in electron-positron annihilation at low energy.

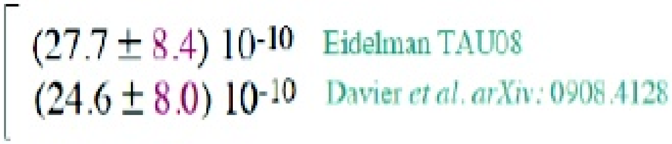
a_μ comparison Exp & Theo



F. Jegerlehner, A. Nyffeler / Physics Reports 477 (2009) 1–110

Standard model theory and experiment comparison [in units 10^{-11}].

Contribution	Value	Error
QED incl. 4-loops + LO 5-loops	116 584 718.1	0.2
Leading hadronic vacuum polarization	6 903.0	52.6
Subleading hadronic vacuum polarization	−100.3	1.1
Hadronic light-by-light	116.0	39.0
Weak incl. 2-loops	153.2	1.8
Theory	116 591 790.0	64.6
Experiment	116 592 080.0	63.0
Exp. - The. 3.2 standard deviations	290.0	90.3



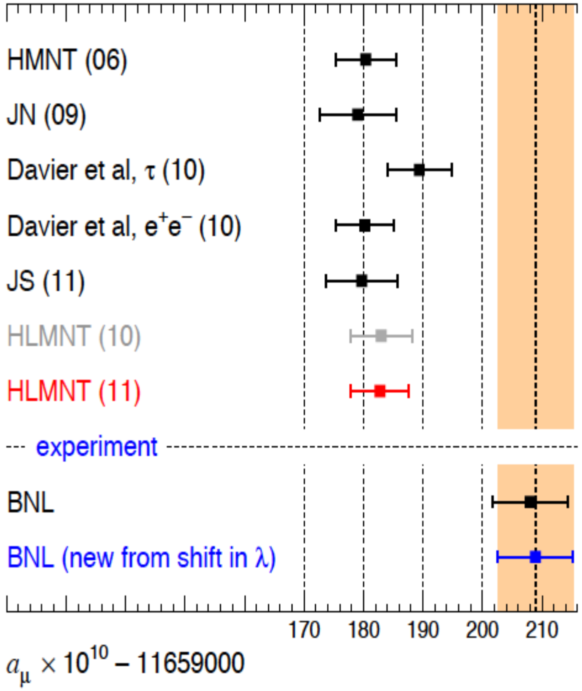
$(27.7 \pm 8.4)\ 10^{-10}$ Eidelman TAU08

$(24.6 \pm 8.0)\ 10^{-10}$ Davier *et al. arXiv:* 0908.4128

$8.4 \approx \quad 5_{HLO} \oplus 3_{LBL} \oplus 6_{BNL}$

$a_\mu^{Exp} - a_\mu^{Theo} = (\ 27.6 \pm 8.7)\ \text{x}\ 10^{-10} \sim 3.4\ \sigma$

Is an hint of new physics?



Cross section data:

Two approaches:

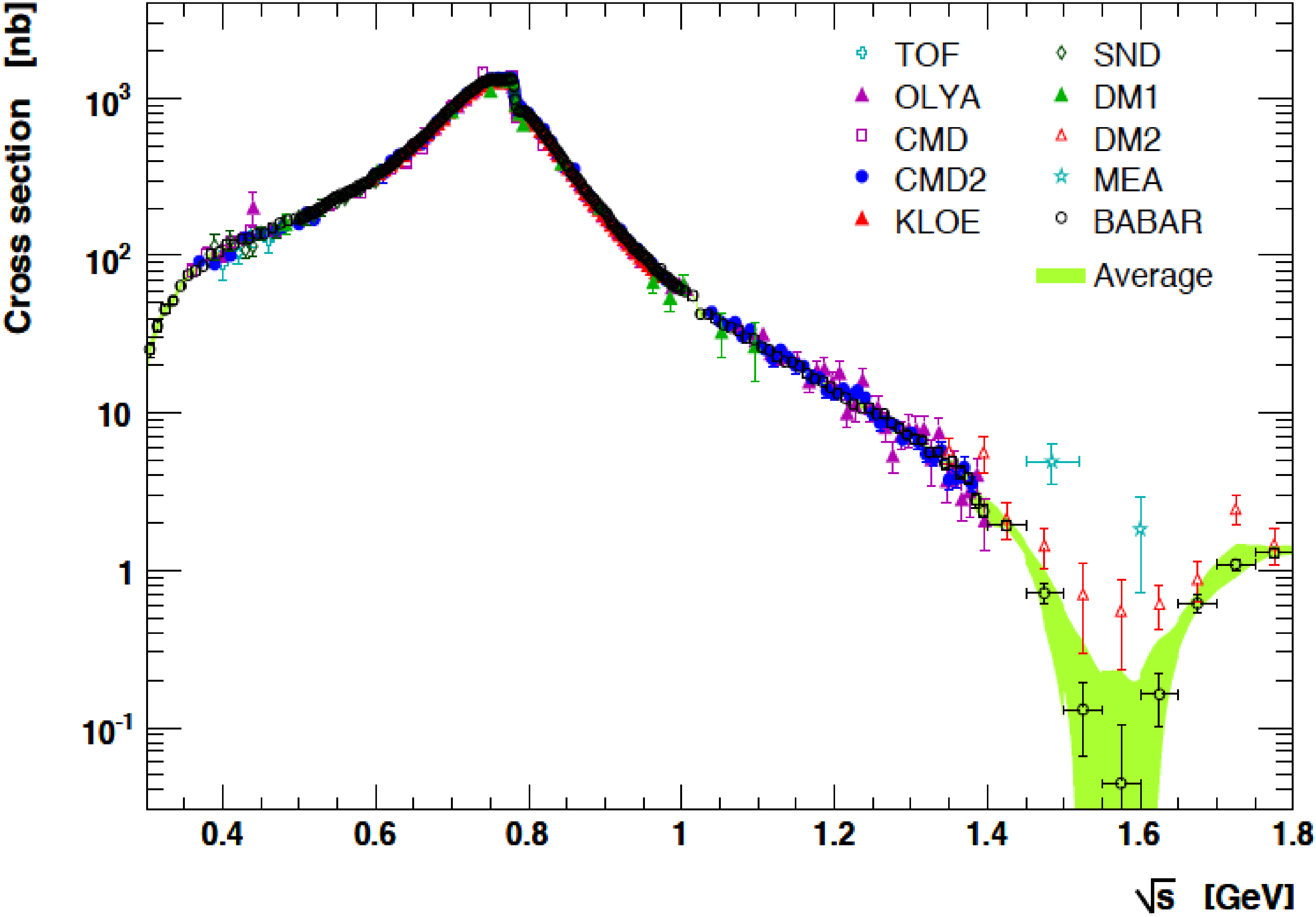
Energy scan (CMD2, SND, BES,CLEO):

- energy of colliding beams is changed to the desired value
- “direct” measurement of cross sections
- needs dedicated accelerator/physics program
- needs to measure luminosity and beam energy for every data point

Radiative return (KLOE, BABAR, BELLE,BESIII?):

- runs at fixed-energy machines (meson factories)
- use initial state radiation process to access lower lying energies or resonances
- data come as by-product of standard physics program
- requires precise theoretical calculation of the radiator function
- luminosity and beam energy enter only once for all energy points
- needs larger integrated luminosity

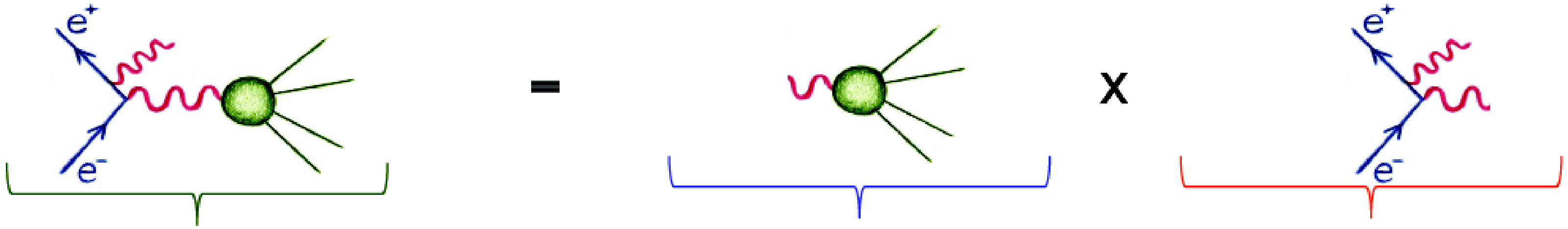
Measured cross section for $e^+e^- \to \pi^+ \pi^-$



ISR: Initial State Radiation

Neglecting final state radiation (FSR):

$$\frac{d\sigma(e^+ e^- \rightarrow hadrons + \gamma)}{dM^2_{hadr}} = \frac{\sigma(e^+ e^- \rightarrow hadrons, M^2_{hadr})}{s} H(s, M^2_{hadr})$$



Theoretical input: precise calculation of the radiation function $H(s, M^2_{hadr})$

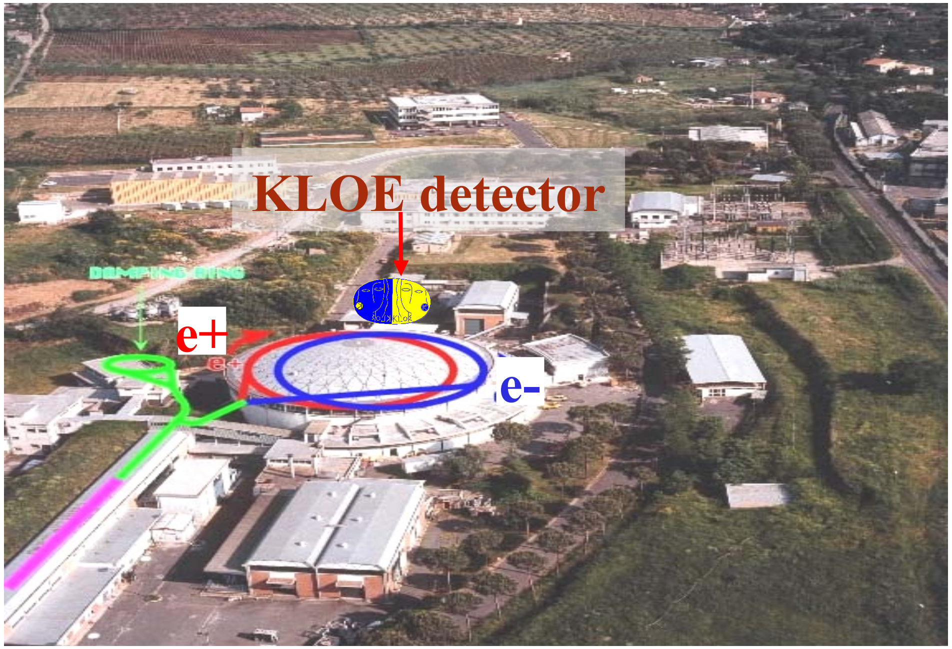
PHOKHARA MC Generator

Binner, Kühn, Melnikov; Phys. Lett. B 459, 1999
H. Czyż, A. Grzelińska, J.H. Kühn, G. Rodrigo, Eur. Phys. J. C 27, 2003
(exact next-to-leading order QED calculation of the radiator function)

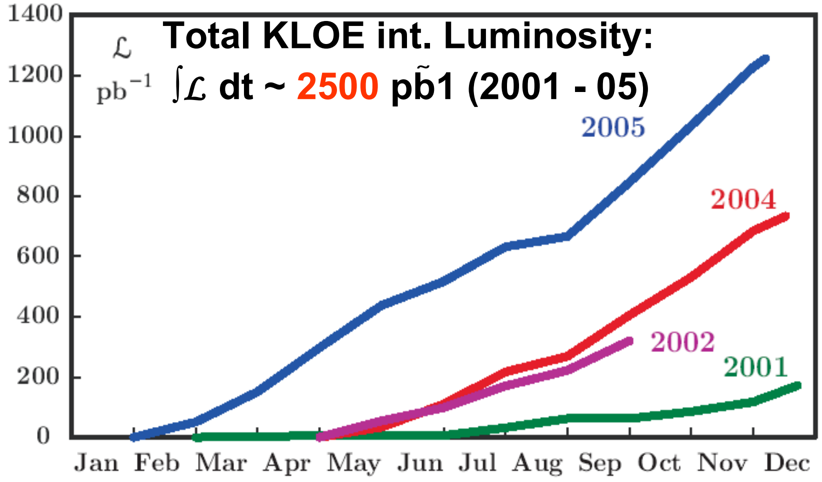
IN 2005 KLOE has published the first precision measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ with ISR using 2001 data ($140 pb^{-1}$) PLB606(2005)12 ~3σ discrepancy btw a_μ^{SM} and a_μ^{exp}

DAΦNE: A Φ-Factory

e^+e^- - collider with $\sqrt{s} = m_\Phi \approx 1.0195$ GeV



Integrated Luminosity



Peak Luminosity Lpeak= 1.5 • 10^{32}cm^{-2}s^{-1}

KLOE05 measurement (PLB606(2005)12) was based on 140pb^{-1} of 2001 data!

2006:

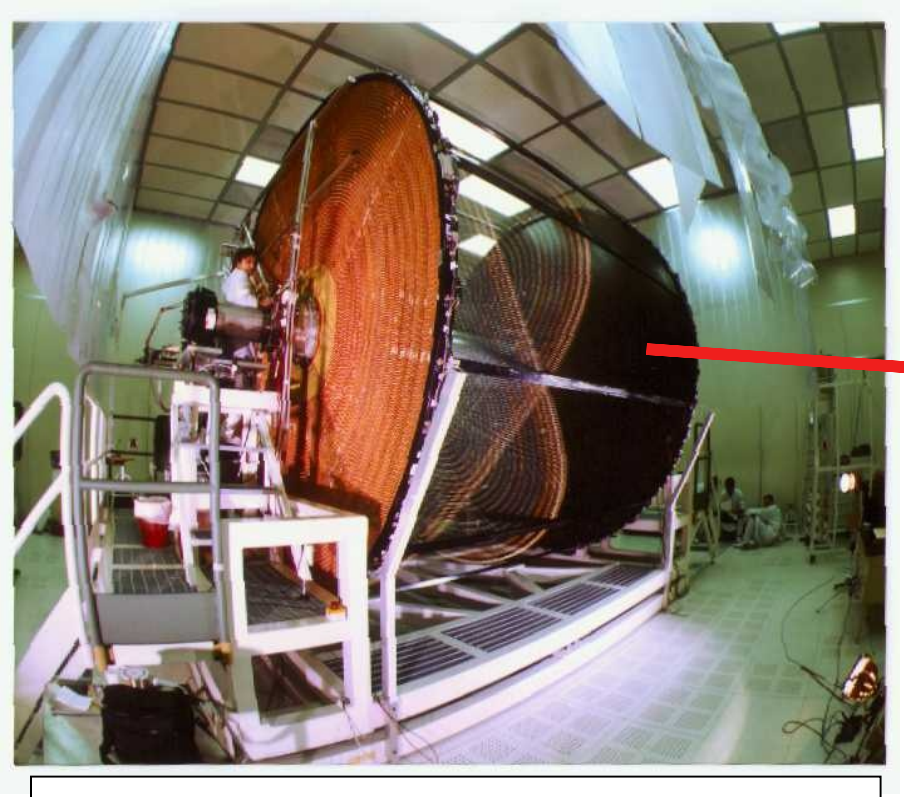
- **Energy scan (4 points around mΦ-peak)**
- **240 pb^{-1} at   = 1000 MeV (off-peak data)**

KLOE08 measurement (PLB670(2009)285) was based on 240pb^{-1} from 2002 data!

KLOE10 measurement (PLB700 (2011)102) based on 233 pb^{-1} of 2006 data (at 1 GeV, different event selection)

KLOE Detector

Drift chamber

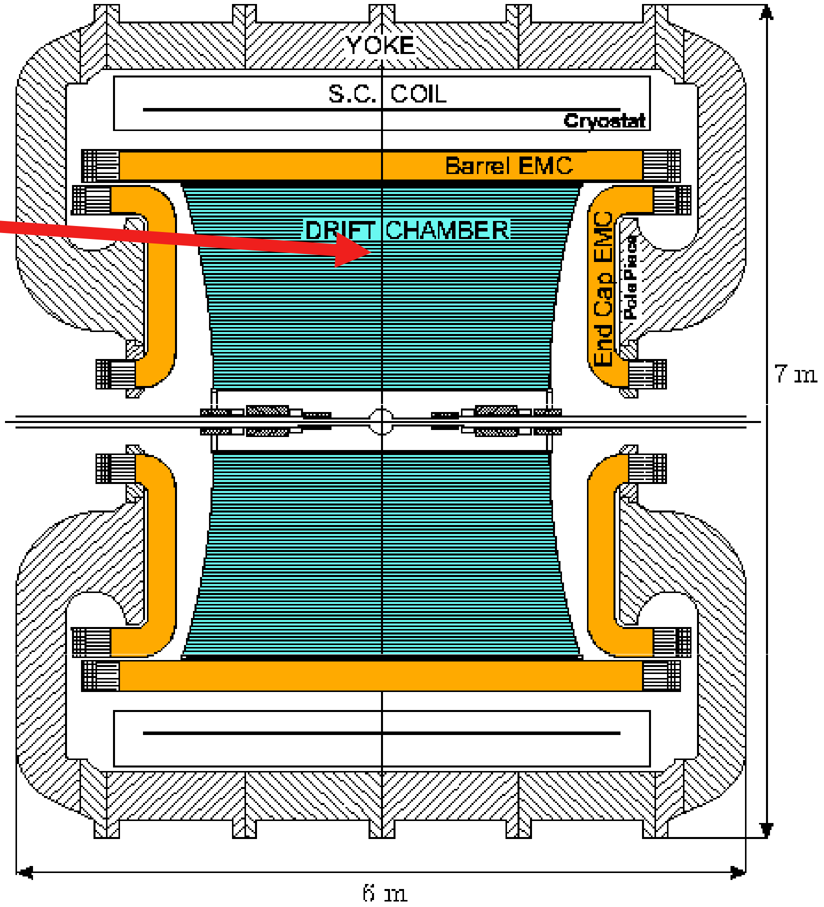


$\sigma_p/p = 0.4\%$ (for 90° tracks)

$\sigma_{xy} \approx 150$ mm, $\sigma_z \approx 2$ mm

Excellent momentum resolution

Full stereo geometry, 4m diameter, 52.140 wires 90% Helium, 10% iC4H10

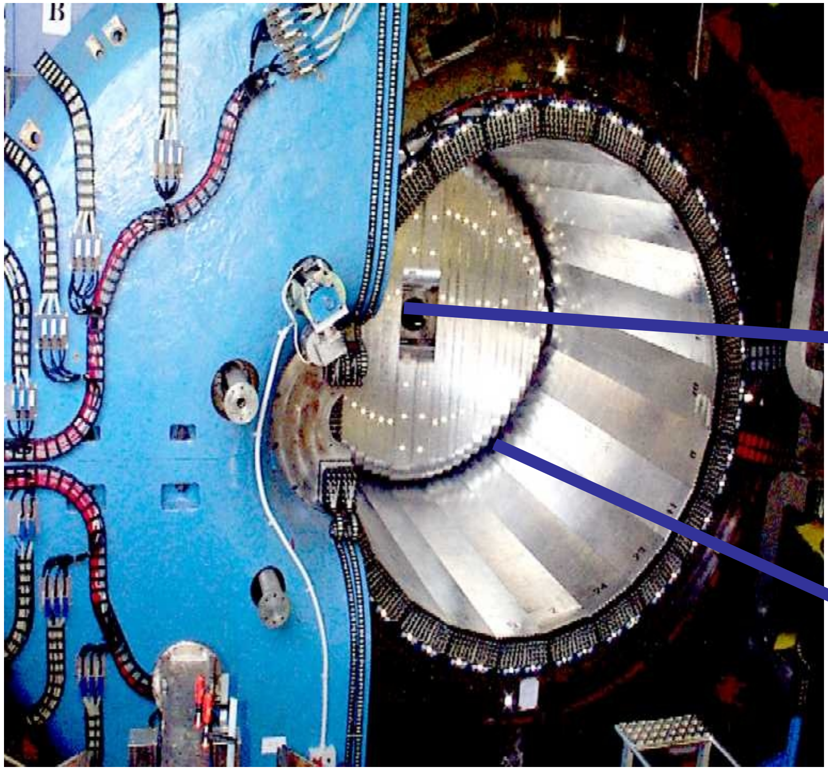


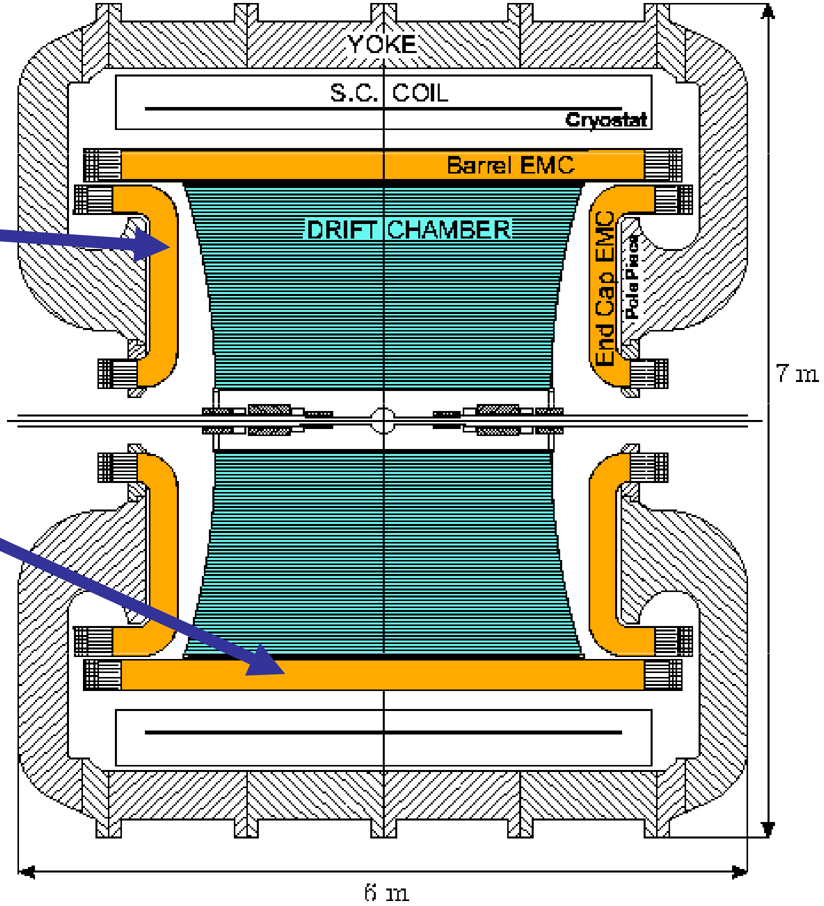
KLOE Detector

Electromagnetic Calorimeter

Pb / scintillating fibres (4880 PMT)

Endcap - Barrel - Modules





$\sigma_E/E = 5.7\% / \sqrt{E(\mathrm{GeV})}$

$\sigma_T = 54\ \mathrm{ps} / \sqrt{E(\mathrm{GeV})} \oplus 100\ \mathrm{ps}$

(Bunch length contribution subtracted from constant term)

Excellent timing resolution

SA Event Selection (KLOE08)

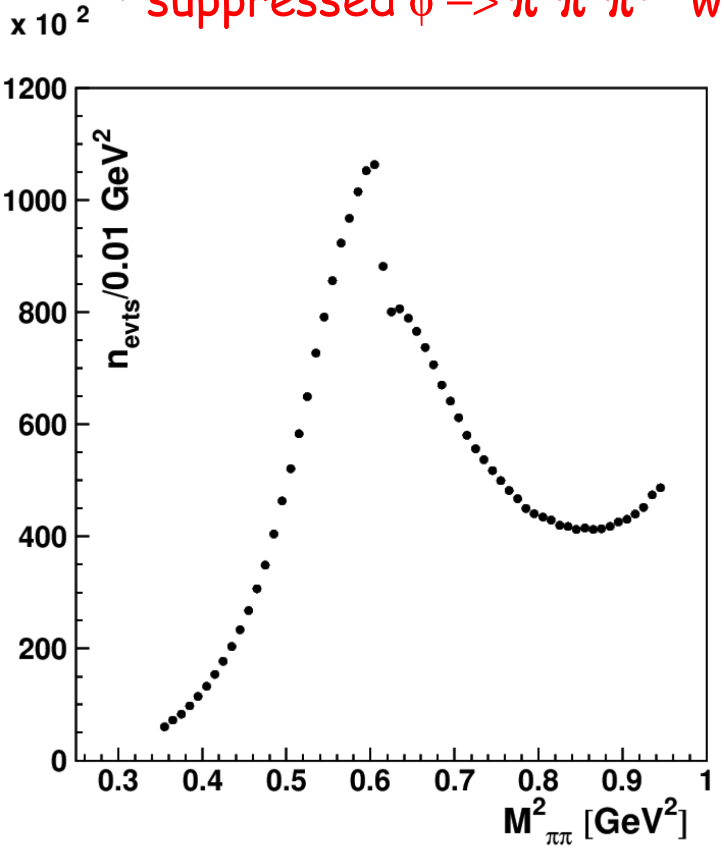
a) 2 tracks with 50° < θ_{track} < 130°

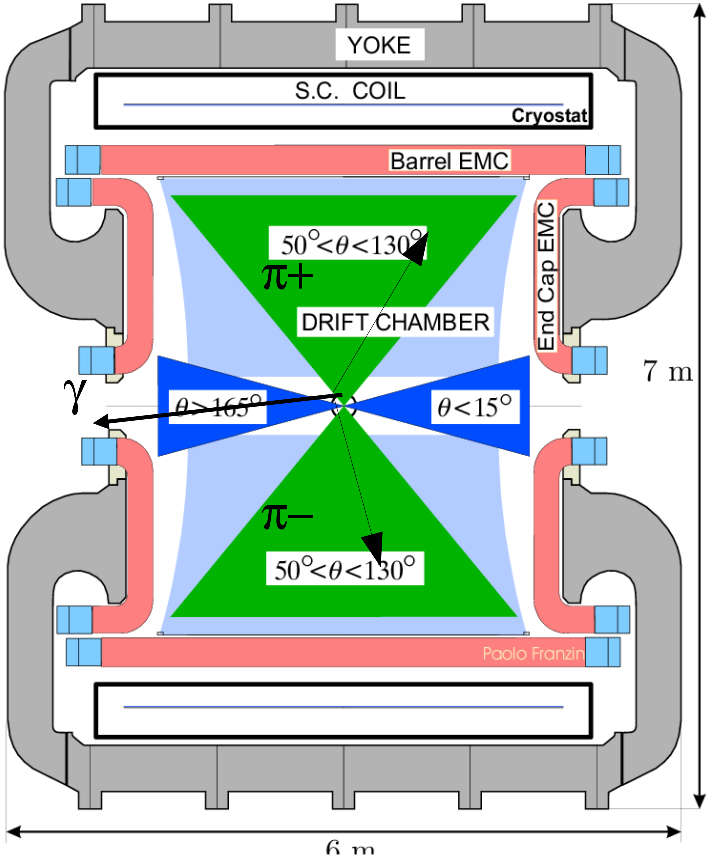
b) small angle (not detected) γ

c) ($\theta_{\pi\pi} < 15°$ or $> 165°$)

- ✓ high statistics for ISR
- ✓ low relative FSR contribution
- ✓ suppressed $\phi \rightarrow \pi^+\pi^-\pi^0$ wrt the signal

kinematics: $\vec{p}_\gamma = \vec{p}_{miss} = -(\vec{p}_+ + \vec{p}_-)$





statistics: **240pb^{-1} of 2002 data 3.1 Mill.**

Events between 0.35 and 0.95 GeV^2

SA Kloe result (KLOE08)

Systematic errors on $a_\mu^{\pi\pi}$:

Reconstruction Filter	negligible
Background	0.3%
Trackmass/Miss. Mass	0.2%
p/e-ID and TCA	negligible
Tracking	0.3%
Trigger	0.1%
Acceptance ($\theta_{\pi\pi}$)	0.1%
Acceptance (θ_{π})	negligible
Unfolding	negligible
Software Trigger	0.1%
$\sqrt{s}$ dep. Of H	0.2%
Luminosity($0.1_{th} \oplus 0.3_{exp}$)%	0.3%

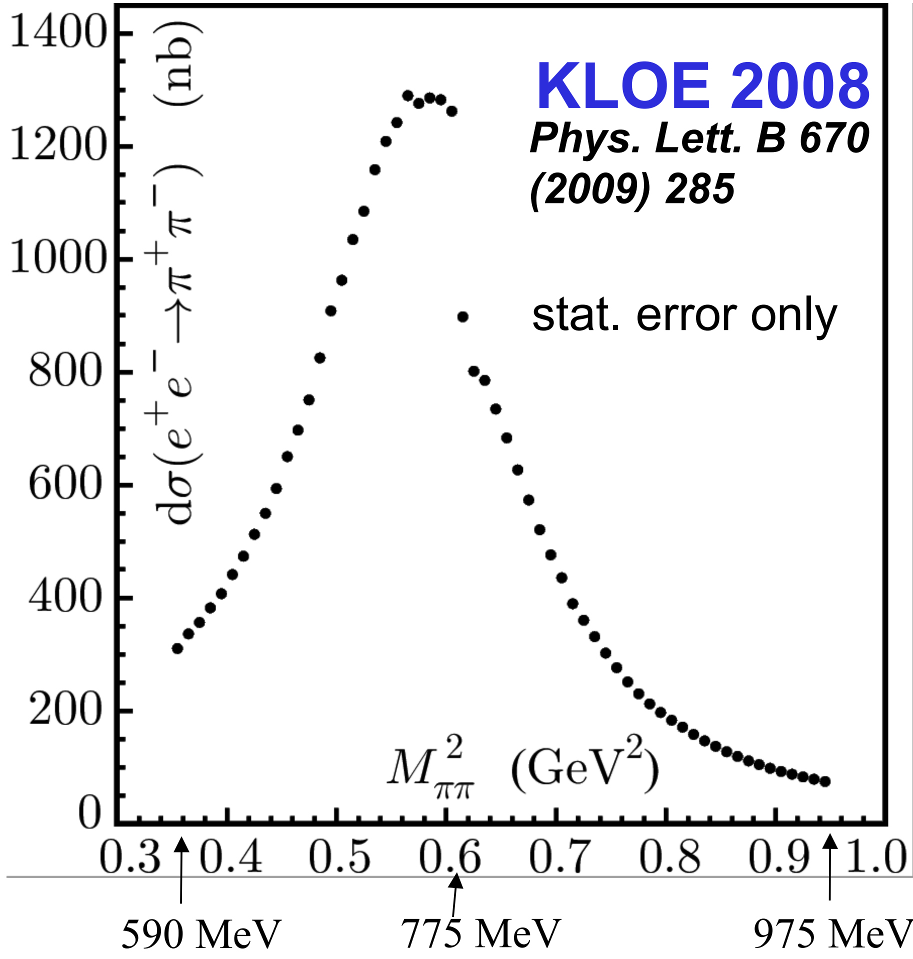
experimental fractional error on a_μ = 0.6 %

FSR resummation	0.3%
Radiator H	0.5%
Vacuum polarization	0.1%

theoretical fractional error on a_μ = 0.6 %

$$a_\mu^{\pi\pi} = \int_{x1}^{x2} \sigma_{ee\to\pi\pi}(s)K(s)ds$$

$\sigma_{\pi\pi}$, undressed from VP, inclusive for FSR as function of $(M_0^{\pi\pi})^2$



$$a_\mu^{\pi\pi}(0.35\text{-}0.95\,\text{GeV}^2) = (387.2 \pm 0.5_{stat} \pm 2.4_{sys} \pm 2.3_{theo}) \cdot 10^{-10}$$

LA Event Selection (KLOE10)

2 pion tracks at large angles
$50° < \theta_\pi < 130°$

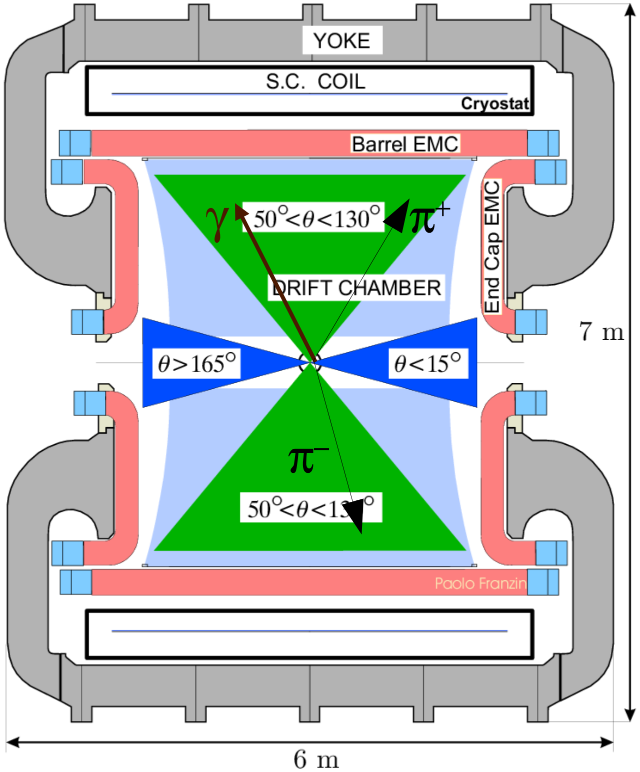
Photons at large angles
$50° < \theta_\gamma < 130°$

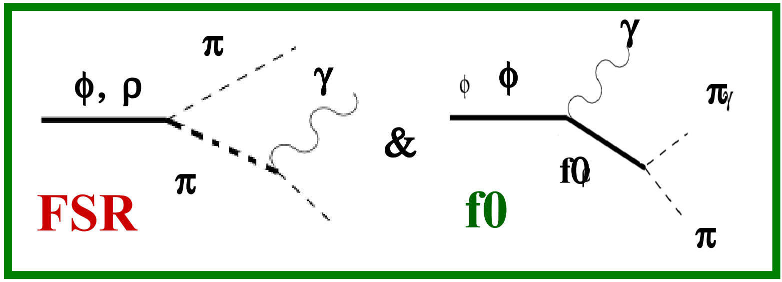
- ✓ **independent complementary analysis**
- ✓ **threshold region $(2m_\pi)^2$ accessible**
- ✓ **γISR photon detected (4-momentum constraints)**

- ✓ **lower signal statistics**
- ✓ **larger contribution from FSR events**
- ✓ **larger $\phi \rightarrow \pi^+\pi^-\pi^0$ background contamination**
- ✓ **irreducible background from ϕ decays ($\phi \rightarrow f^0\,\gamma \rightarrow \pi\pi\,\gamma$)**

Threshold region non-trivial due to irreducible FSR-effects, which have to be estimated from MC using phenomenological models (interference effects unknown)

At least 1 photon with $50° < \theta_\gamma < 130°$ and E_γ >20 MeV => photon detected





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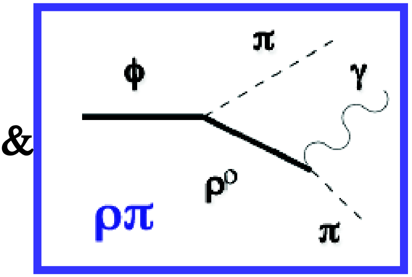


LA Event Selection (KLOE10)

2 pion tracks at large angles

50° < θπ <130°

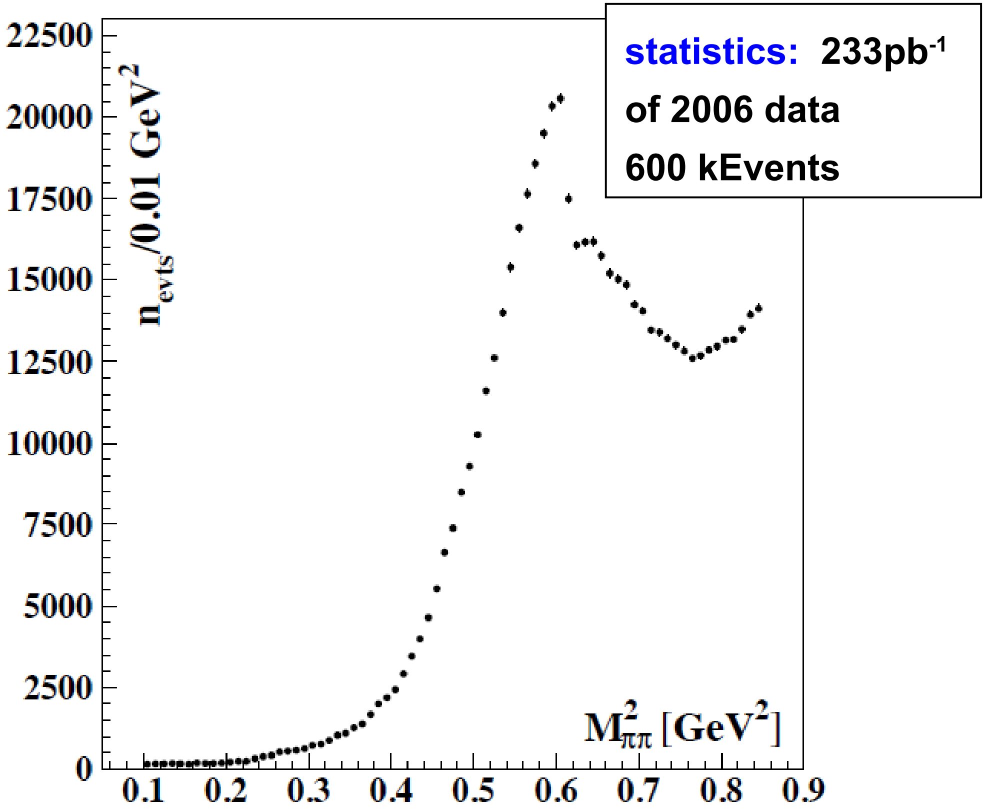
Photons at large angles

50° < θγ < 130°

- ✓ **independent complementary analysis**
- ✓ **threshold region $(2m_\pi)^2$ accessible**
- ✓ **γISR photon detected (4-momentum constraints)**

- ✓ **lower signal statistics**
- ✓ **larger contribution from FSR events**
- ✓ **larger $\phi \rightarrow \pi^+\pi^-\pi^0$ background contamination**
- ✓ **irreducible background from ϕ decays ($\phi \rightarrow f0\ \gamma \rightarrow \pi\pi\ \gamma$)**

At least 1 photon with 50°< θγ <130° **and Eγ > 20 MeV -> photon detected**



statistics: 233pb^{-1}
of 2006 data
600 kEvents

Use data sample taken at √**s≅1000 MeV**, 20 MeV below the ϕ-peak

KLOE10: Pion Form Factor

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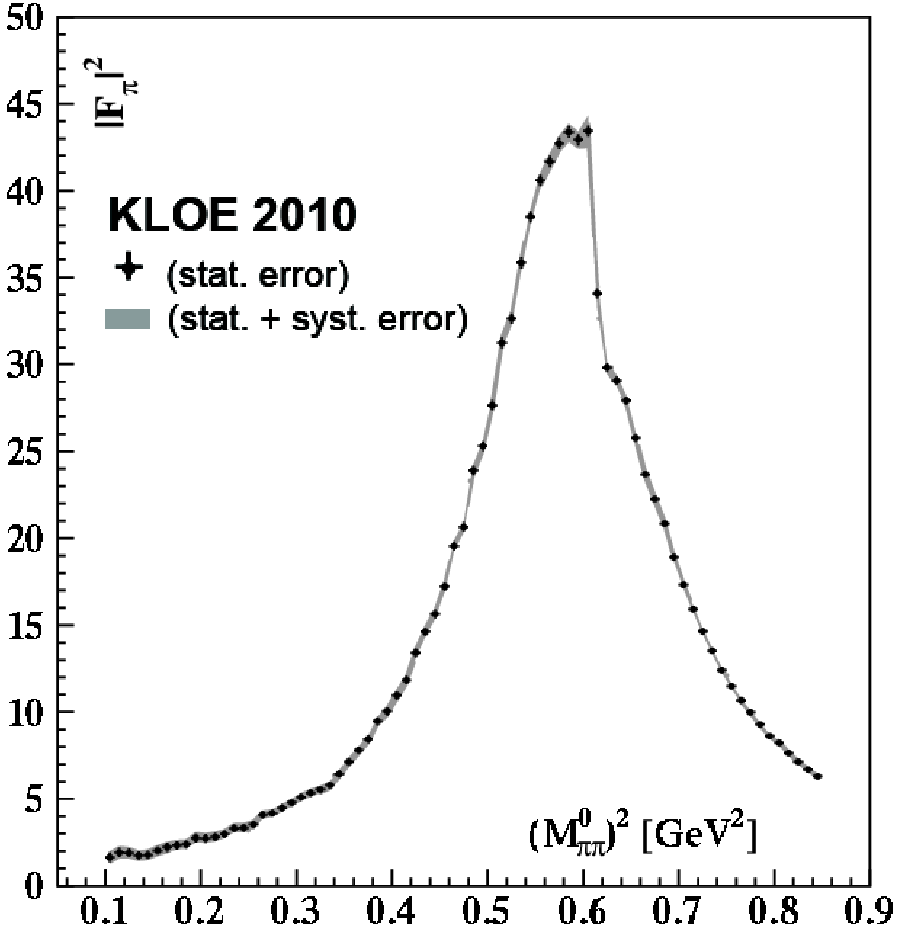


Disp. Integral:

$$a_\mu^{\pi\pi} = \int_{x1}^{x2} \sigma_{ee\to\pi\pi}(s)K(s)ds$$

Table of systematic errors on $a_\mu^{\pi\pi}(0.1\text{-}0.85\ \text{GeV}^2)$:

Reconstruction Filter	negligible
Background	0.5%
f0+ρπ	0.4%
Ω cut	0.2%
Trackmass	0.5%
p/e-ID and TCA	negligible
Tracking	0.3%
Trigger	0.2%
Acceptance	0.5%
Unfolding	negligible
Software Trigger	0.1%
Luminosity$(0.1_{th} \oplus 0.3_{exp})$%	0.3%

experimental fractional error on a_μ = 1.0 %

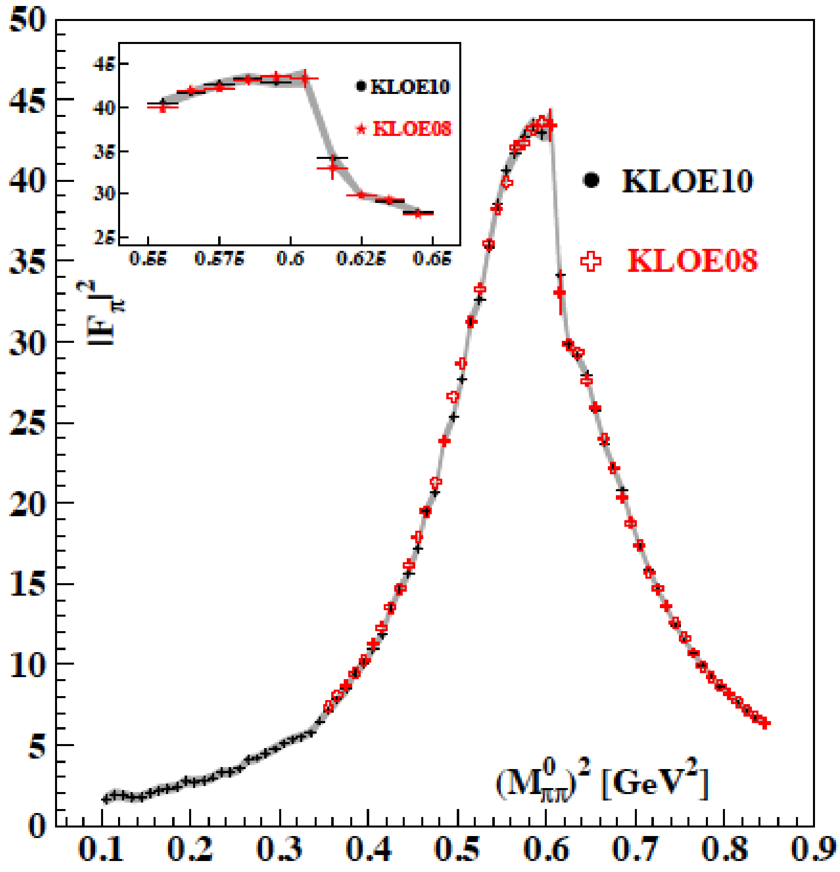
FSR treatment	0.8%
Radiator H	0.5%
Vacuum polarization	0.1%

theoretical fractional error on a_μ = 0.9 %

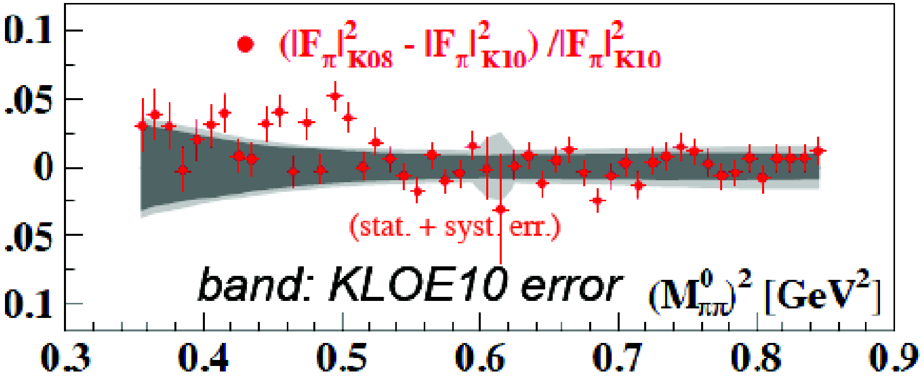
$a_\mu^{\pi\pi}(0.1\text{-}0.85\ \text{GeV}^2) = (478.5 \pm 2.0\ \text{stat} \pm 5.0\ \text{sys} \pm 4.5\ \text{theo}) \cdot 10^{-10}$

Comparison of results: KLOE10 vs KLOE08

KLOE08 result compared to KLOE10:



Fractional difference:



Excellent agreement with KLOE08, expecially above 0.5 GeV²

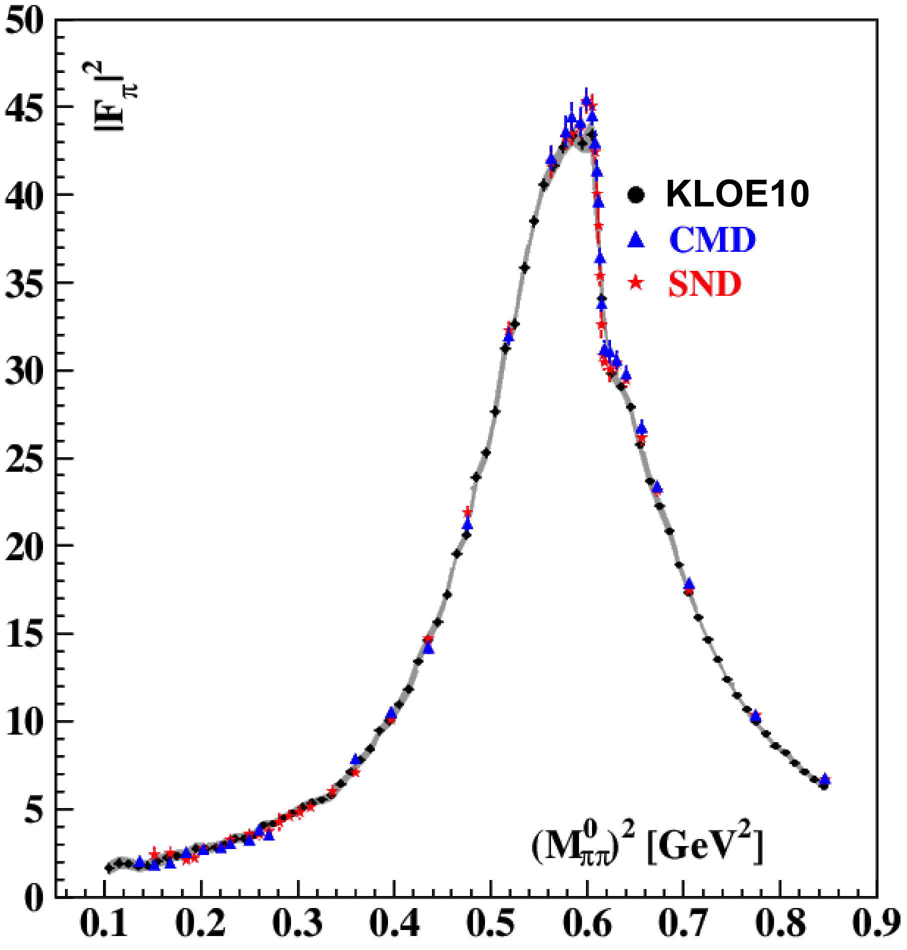
Combination of KLOE08 and KLOE10:

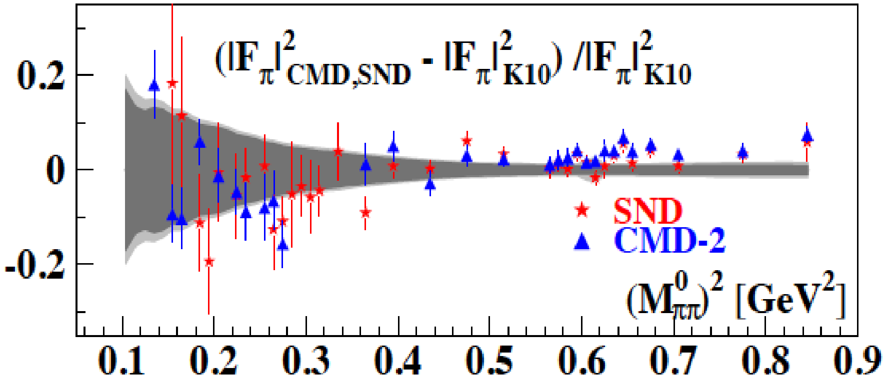
$a_\mu^{\pi\pi}$(0.1-0.95 GeV^2) = (488.6±6.0) · 10^{-10}

KLOE covers ~70% of total a_μ^{HLO} with a fractional total error of 1.2%

Comparison of results: KLOE10 vs CMD-2/SND

CMD and SND results compared to KLOE10: Fractional difference





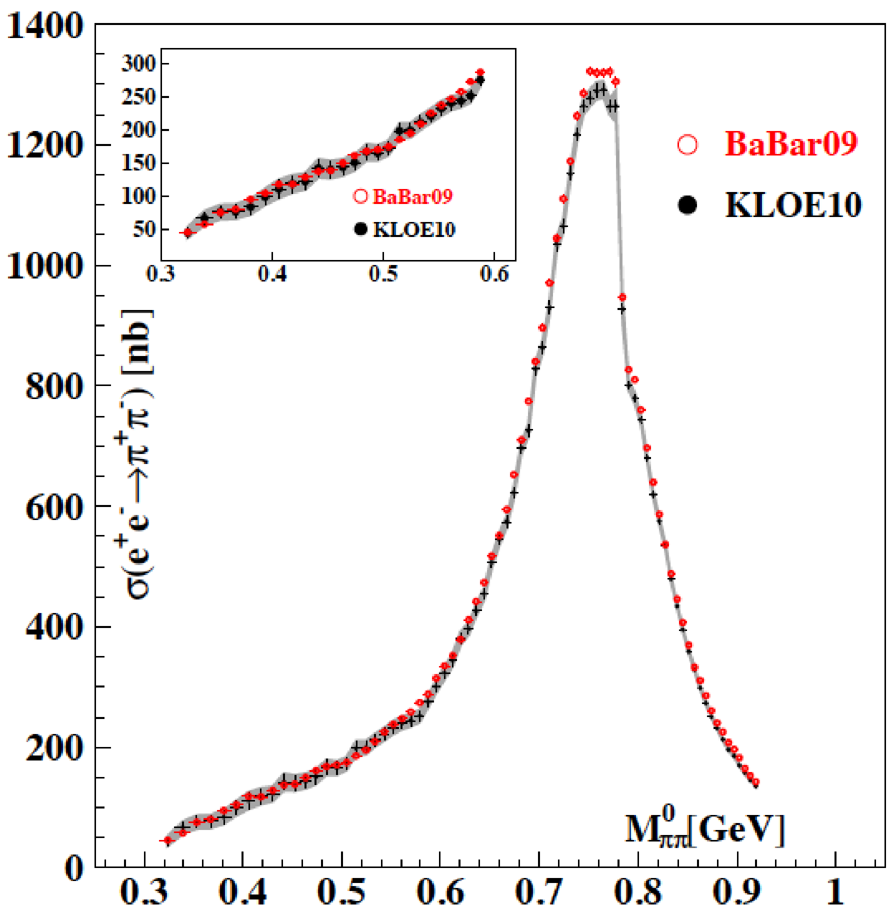
band: KLOE10 error

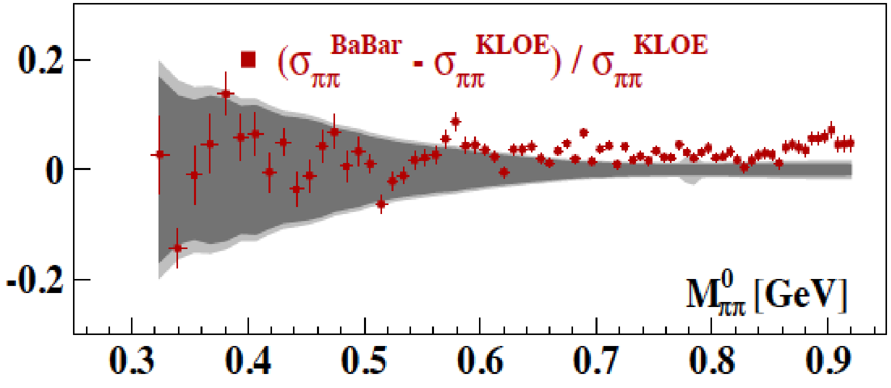
Below the ρ peak good agreement with CMD-2/SND.

Above the ρ peak KLOE10 slightly lower (as KLOE08)

Comparison of results: KLOE10 vs BaBar

BaBar results compared to KLOE10: Fractional difference





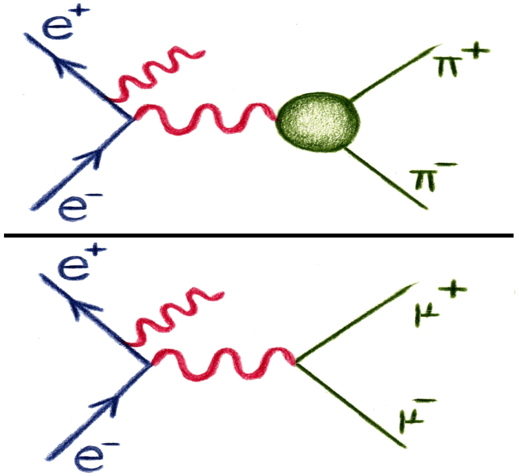
band: KLOE10 error

Agreement within errors below 0.6 GeV; BaBar higher by 2-3% above

New $\sigma_{\pi\pi}$ measurement from π/μ

An alternative way to obtain $|F_\pi|^2$ is the bin-by-bin ratio of pion over muon yields (instead of using absolute normalization with Bhabhas).

$$|F_\pi(s')|^2 \approx \frac{4\left(1+2m_\mu^2/s'\right)\beta_\mu}{\beta_\pi^3}\underbrace{\frac{d\sigma_{\pi\pi\gamma}/ds'}{d\sigma_{\mu\mu\gamma}/ds'}}_{\text{meas. quantities}}$$



Many radiative corrections drop out:

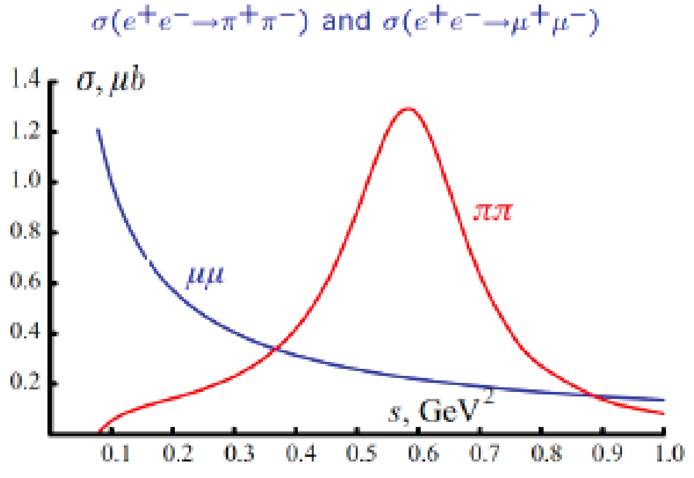
- **radiator function**
- **int. luminosity from Bhabhas**
- **Vacuum polarization**

$$\left(\sqrt{s}-\sqrt{|\mathrm{p}_+|^2+\mathrm{M}_{\mathrm{trk}}^2}-\sqrt{|\mathrm{p}_-|^2+\mathrm{M}_{\mathrm{trk}}^2}\right)^2-(\mathrm{p}_++\mathrm{p}_-)^2=0$$

Separation btw $\pi\pi\gamma$ and $\mu\mu\gamma$ using M_{TRK}

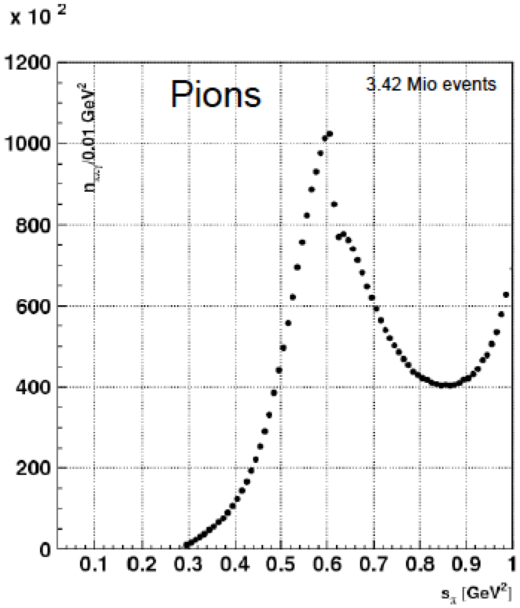
- *muons:* $M_{Trk} < 115$ *MeV*
- *pions :* $M_{Trk} > 130$ *MeV*

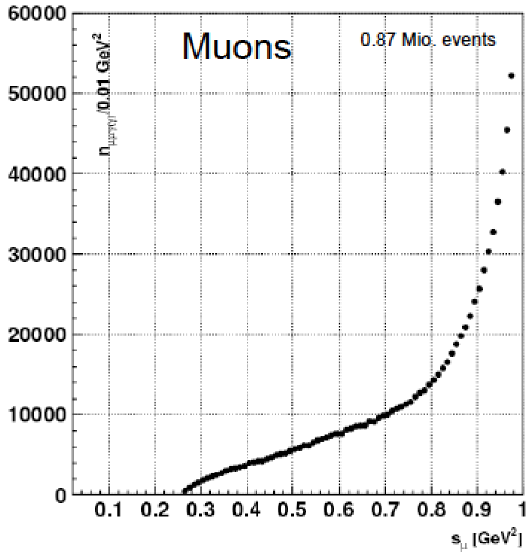
Very important control of π/μ *separation in the* ρ *region!* $(\sigma_{\pi\pi} >> \sigma_{\mu\mu})$

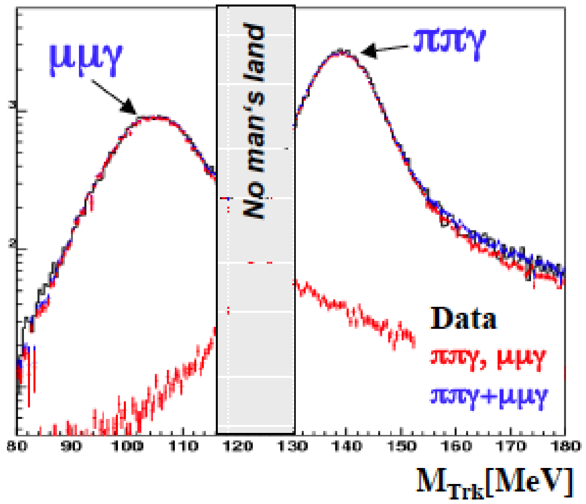


KLOE11: analysis of $\pi\pi\gamma/\mu\mu\gamma$

- 239.2 pb-1 of 2002 data sample (the same used in KLOE08 analysis), with photon at small angle : 0.87 Million $\mu\mu\gamma$ events (compared to 3.4 Million for $\pi\pi\gamma$)
- Careful work to achieve a control of ~1% in the muon selection, especially in the ρ region where $\pi\pi\gamma/\mu\mu\gamma \sim 10$. π/μ separation crosschecked in three different methods (MTRK fit, Kinematic fit, cut on σ_{MTRK})
- $\mu\mu\gamma$ (and $\pi\pi\gamma$) Efficiencies (Trk,Trg,PID) done on data
- Excellent data/MC agreement for many kinematic variables: M_{TRK}, track and γ polar angle, etc...







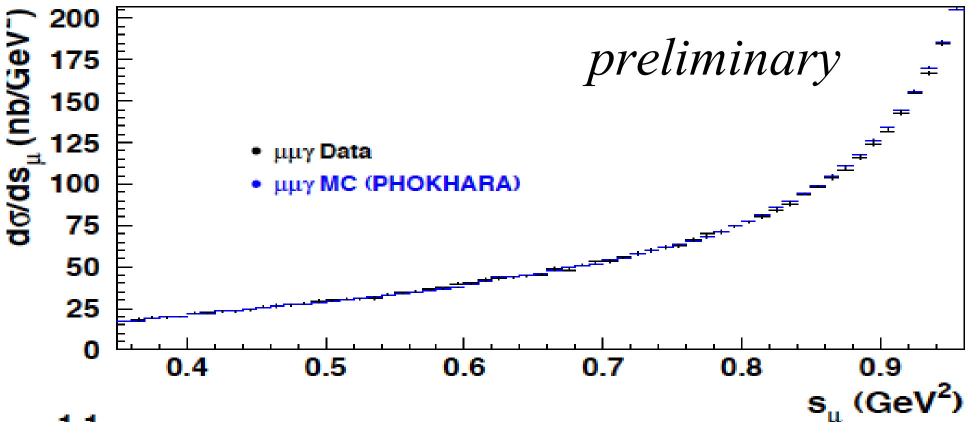
μμγ cross section: data/MC comparison

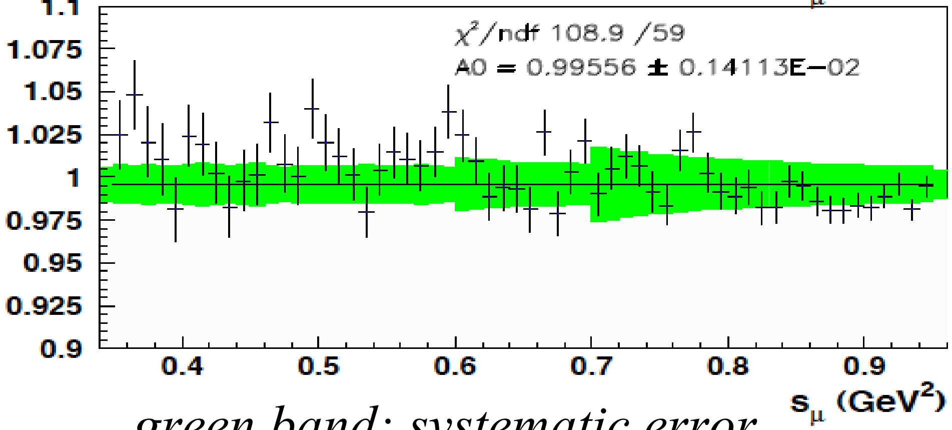
$$\frac{d\sigma^{obs}_{\mu\mu\gamma(\gamma)}}{dM^2_{\mu\mu}} = \frac{\Delta N_{\mathrm{Obs}} - \Delta N_{\mathrm{Bkg}}}{\Delta M^2_{\mu\mu}} \cdot \frac{1}{\varepsilon_{\mathrm{Sel}}} \cdot \frac{1}{\int L dt}$$

$$\frac{d\sigma^{DATA}_{\mu\mu\gamma(\gamma)}}{d\sigma^{MC}_{\mu\mu\gamma(\gamma)}} = 0.996 \pm 0.001_{stat} \pm 0.01_{syst}$$

The systematic error has been averaged on s_μ

Good agreement with PHOKHARA MC



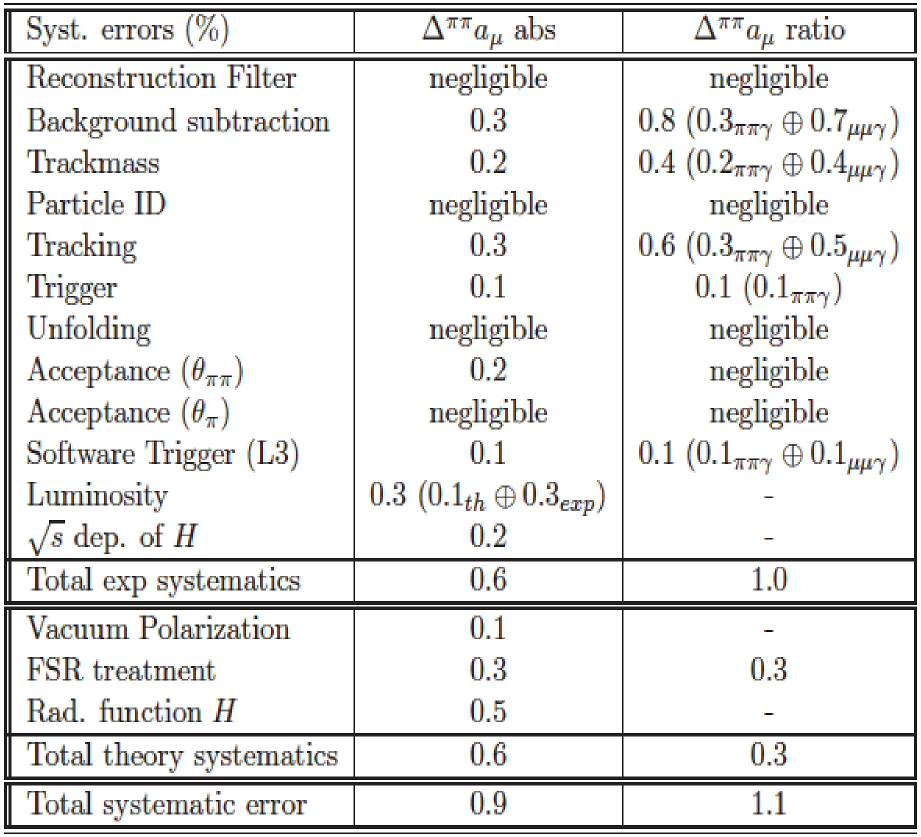


green band: systematic error

Consistency check of Radiator function, Luminosity, etc…

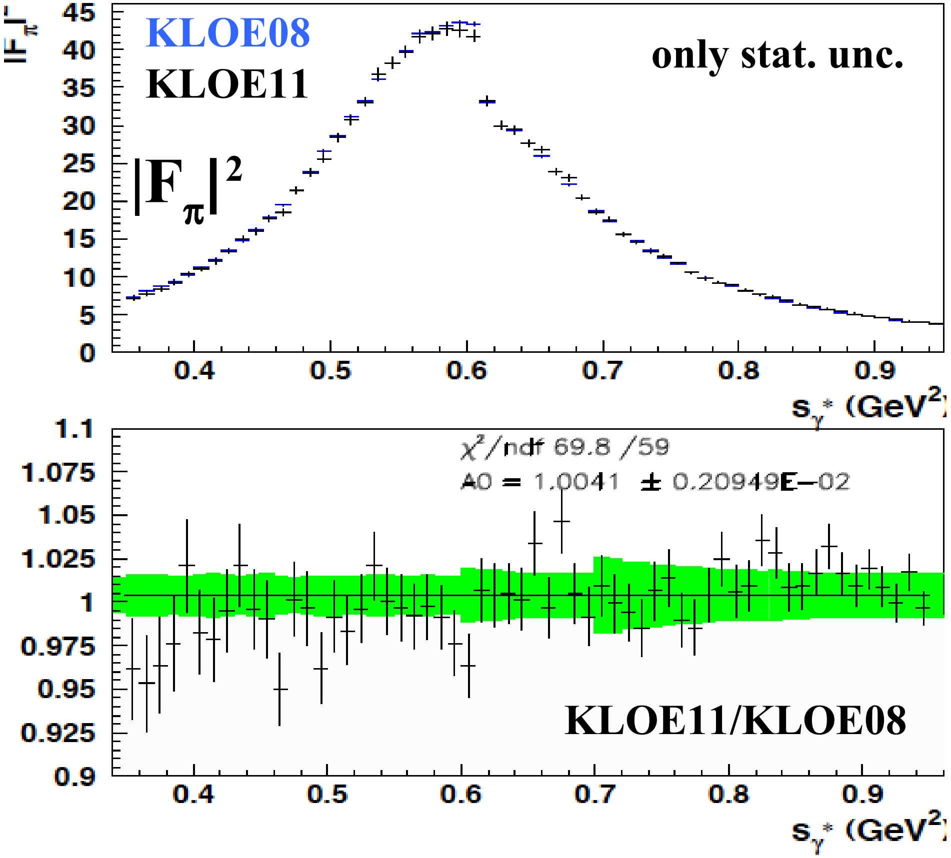
KLOE11 result on $|F_\pi|^2$ and comp. with KLOE08

KLOE08 **KLOE11**



Syst. errors (%)	$\Delta^{\pi\pi}a_\mu$ abs	$\Delta^{\pi\pi}a_\mu$ ratio
Reconstruction Filter	negligible	negligible
Background subtraction	0.3	$0.8\ (0.3_{\pi\pi\gamma} \oplus 0.7_{\mu\mu\gamma})$
Trackmass	0.2	$0.4\ (0.2_{\pi\pi\gamma} \oplus 0.4_{\mu\mu\gamma})$
Particle ID	negligible	negligible
Tracking	0.3	$0.6\ (0.3_{\pi\pi\gamma} \oplus 0.5_{\mu\mu\gamma})$
Trigger	0.1	$0.1\ (0.1_{\pi\pi\gamma})$
Unfolding	negligible	negligible
Acceptance ($\theta_{\pi\pi}$)	0.2	negligible
Acceptance (θ_π)	negligible	negligible
Software Trigger (L3)	0.1	$0.1\ (0.1_{\pi\pi\gamma} \oplus 0.1_{\mu\mu\gamma})$
Luminosity	$0.3\ (0.1_{th} \oplus 0.3_{exp})$	-
$\sqrt{s}$ dep. of H	0.2	-
Total exp systematics	0.6	1.0
Vacuum Polarization	0.1	-
FSR treatment	0.3	0.3
Rad. function H	0.5	-
Total theory systematics	0.6	0.3
Total systematic error	0.9	1.1

preliminary



Good agreement btw the two measurements, especially in the ρ region!!!

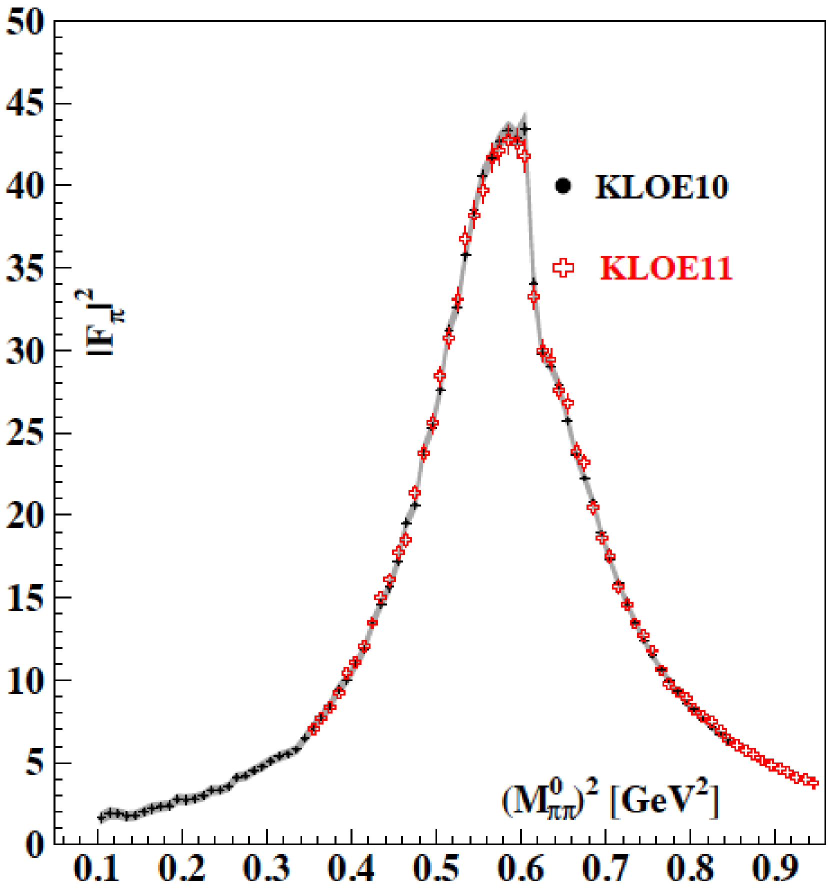
KLOE11: $a_\mu^{\pi\pi}$,(0.35-0.95 GeV^2) = (384.1 ± 1.2stat ± 4.0sys ± 1.2theo) · 10^{-10}

KLOE08: $a_\mu^{\pi\pi}$(0.35-0.95 GeV^2) = (387.2 ± 0.5stat ± 2.4sys ± 2.3theo) · 10^{-10}

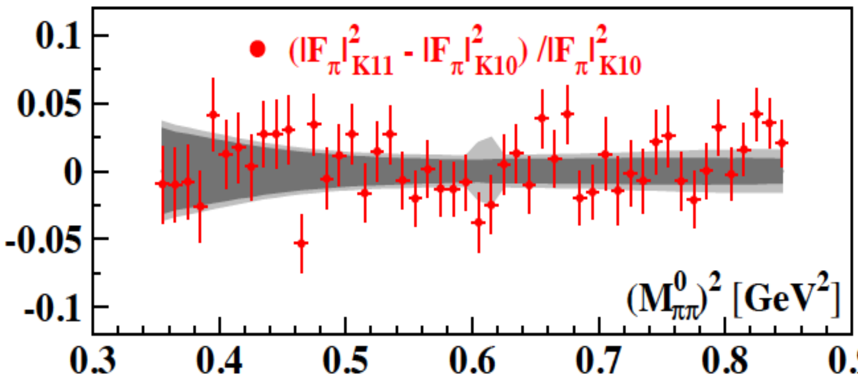
Comparison of results: KLOE11 vs KLOE10

KLOE11 result compared to KLOE10:

preliminary



Fractional difference:



band: KLOE10 error

Excellent agreement between the two measurements!

Comparison with other exp. in progress

KLOE11: $a_\mu^{\pi\pi}$,(0.35-0.85 GeV^2) = (376.4 ± 1.2stat ± 4.1sys tot) · 10^{-10}

KLOE10: $a_\mu^{\pi\pi}$,(0.35-0.95 GeV^2) = (376.6 ± 0.9stat ± 3.3sys tot) · 10^{-10}

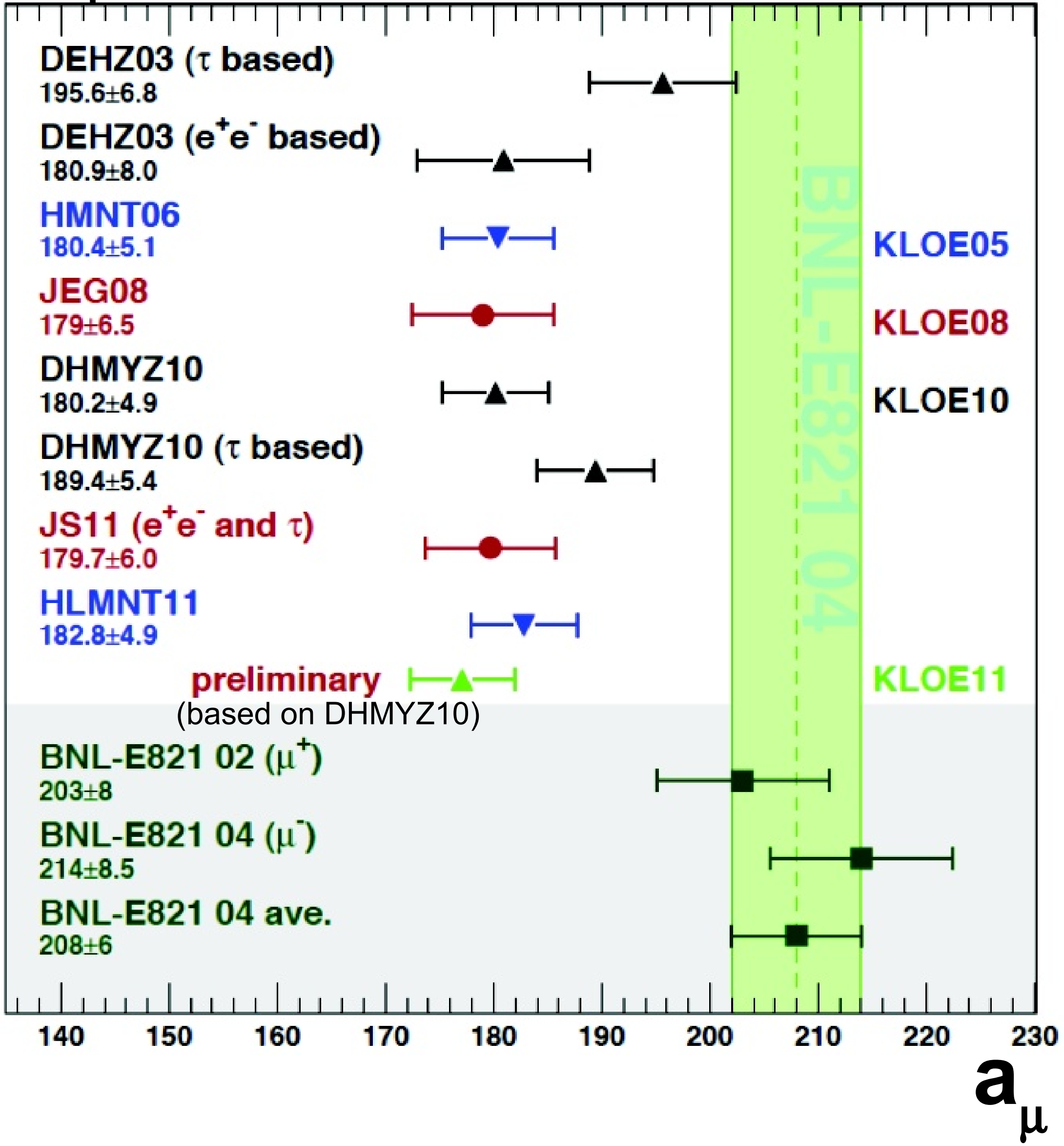
$a_\mu=(g_\mu-2)/2$:

Theoretical predictions compared to the BNL result (2009)

The latest inclusion of all e^+e^- data gives a discrepancy btw a_μ^{SM} and a_μ^{EXP} $\geq 3\,\sigma$

Preliminary KLOE11 in agreement with previous KLOE measurements and confirms this discrepancy!

Very important the new g-2 experiments (at FNAL and JPARC)!



Conclusion

During the last 10 years KLOE has performed a series of precision measurements with ISR which confirmed a 3σ discrepancy between a_μ^{SM} and the BNL measured value and allowed the measurement of $a_\mu^{\pi\pi}$ in the region 0.1-0.95 GeV^2(70% of a_μ^{HLO}) with 1.2% total error using KLOE data only.

A new (preliminary) measurement of $|F_\pi|^2$ from the $\pi\pi\gamma/\mu\mu\gamma$ ratio (based on 240 pb^{-1}) with 1.1% systematic error has been done. Preliminary results show good agreement for $\mu\mu\gamma$ cross section with PHOKHARA MC and for $|F_\pi|^2$ and $a_\mu^{\pi\pi}$ with previous KLOE published measurements.

Still more than1.5 fb^{-1} of KLOE data on tape. This is a ~4 improvement in statistics. We plan to analyse these data to improve $\sigma^{\pi\pi}$(and may be other channels) measurement.

In addition we expect about 25 fb^{-1} at KLOE-2, and that should allow to measure $\sigma^{\pi\pi}$ ($2m_\pi < s < 1\mathrm{GeV}$) with 0.4% error (now 0.7% averaging CMD-2/SND/KLOE).

Spares

Cross check of π/μ separation

- The π/μ separation has been crosschecked with two different (and independent) methods:

- A kinematic fit, in the hypothesis of 2 body+1γ (ISR) events.

- A cut on the quality of the fitted tracks, parametrized by σ_{MTRK}

$$\sigma^2_{M_{trk}} = \left(\frac{\partial M_{trk}}{\partial k_1} \quad \frac{\partial M_{trk}}{\partial \cot\theta_1} \quad \frac{\partial M_{trk}}{\partial \varphi_1} \quad \frac{\partial M_{trk}}{\partial k_2} \quad \frac{\partial M_{trk}}{\partial \cot\theta_2} \quad \frac{\partial M_{trk}}{\partial \varphi_2} \right) \cdot$$

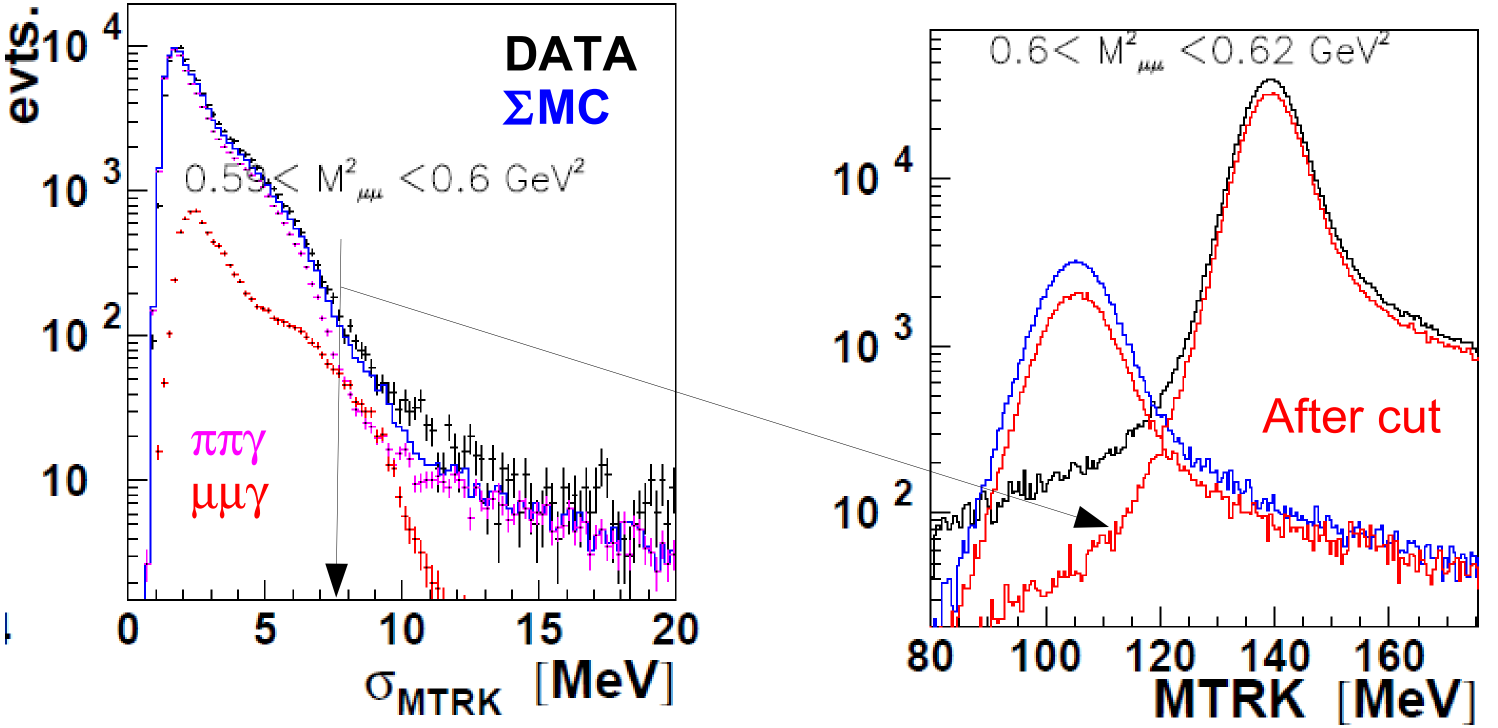
$$\begin{pmatrix} \sigma^2_{k_1} & \rho_{k_1 \cot\theta_1} & \rho_{k_1 \varphi_1} & 0 & 0 & 0 \\ \rho_{\cot\theta_1 k_1} & \sigma^2_{\cot\theta_1} & \rho_{\cot\theta_1 \varphi_1} & 0 & 0 & 0 \\ \rho_{\varphi_1 k_1} & \rho_{\varphi_1 \cot\theta_1} & \sigma^2_{\varphi_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{k_2} & \rho_{k_2 \cot\theta_2} & \rho_{k_2 \varphi_2} \\ 0 & 0 & 0 & \rho_{\cot\theta_2 k_2} & \sigma^2_{\cot\theta_2} & \rho_{\cot\theta_2 \varphi_2} \\ 0 & 0 & 0 & \rho_{\varphi_2 k_2} & \rho_{\varphi_2 \cot\theta_2} & \sigma^2_{\varphi_2} \end{pmatrix} \begin{pmatrix} \frac{\partial M_{trk}}{\partial k_1} \\ \frac{\partial M_{trk}}{\partial \cot\theta_1} \\ \frac{\partial M_{trk}}{\partial \varphi_1} \\ \frac{\partial M_{trk}}{\partial k_2} \\ \frac{\partial M_{trk}}{\partial \cot\theta_2} \\ \frac{\partial M_{trk}}{\partial \varphi_2} \end{pmatrix}$$

- π/μ separation obtained with these methods well in agreement with the standard one.

Cross check of π/μ separation σ_{MTRK}

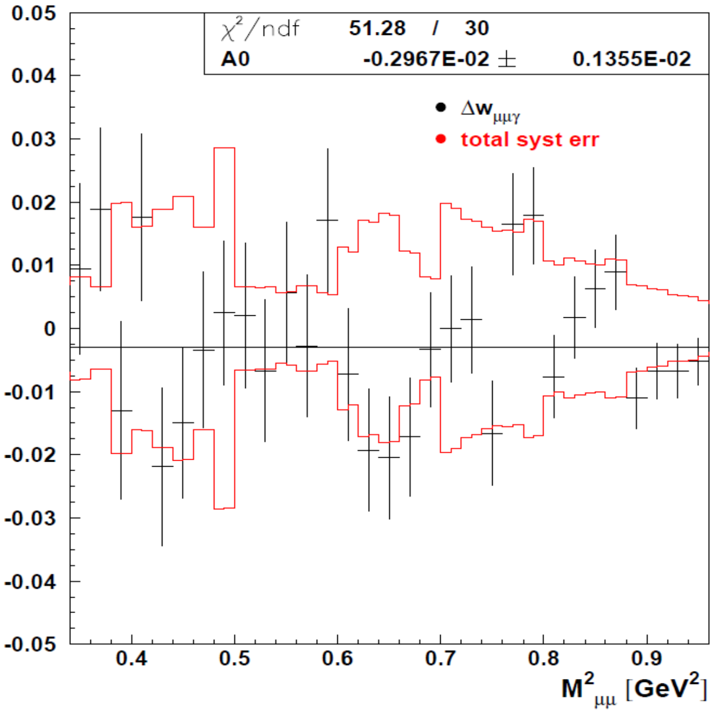
We have achieved an excellent Data/MC agreement for muons in many kinematic variables (as we did for pions)

BG reduction-Cut effect

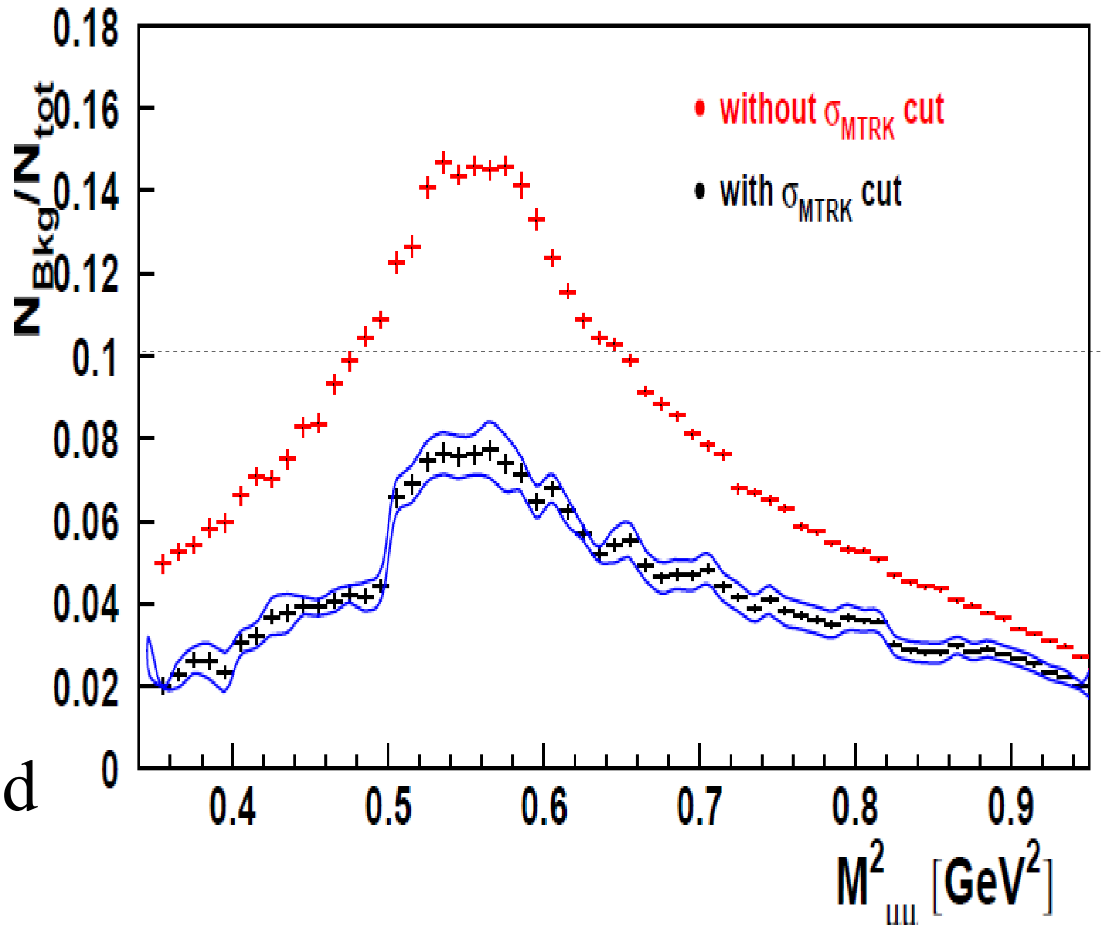


- π/μ separation obtained with these methods well in agreement with the standard one.

Cross check of π/μ separation σ_{MTRK}

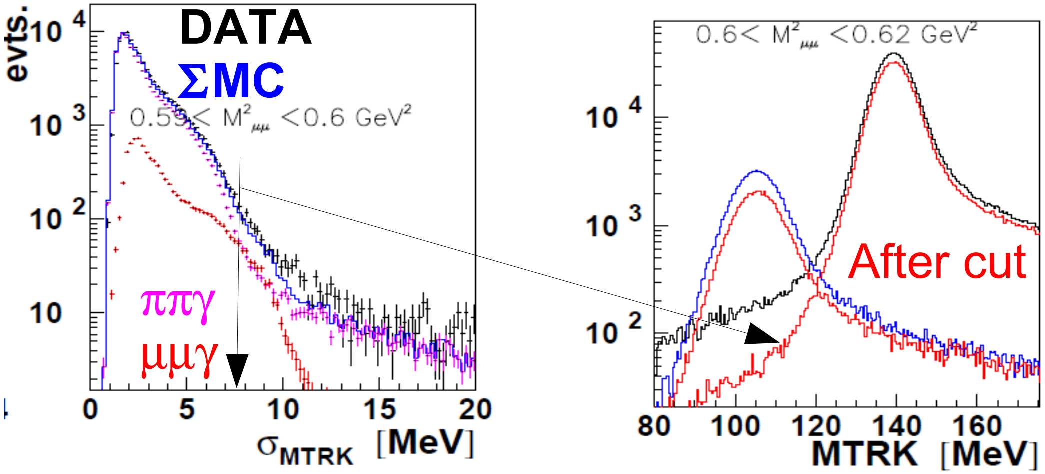


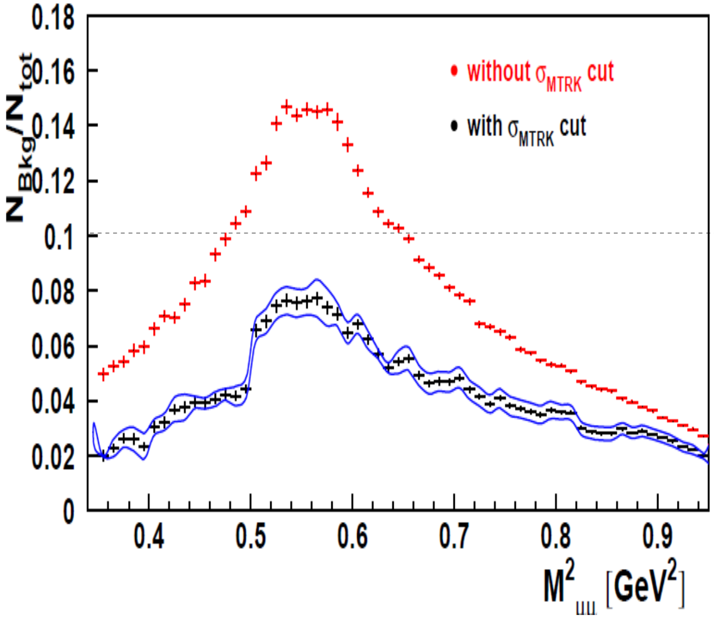
The $\mu^+\mu^-\gamma$ fraction obtained by the σ_{MTRK} cut are consistent with the standard procedure within the systematic error.



BG suppression under 10%

Cross check of π/μ separation σ_{MTRK}





- A cross check was realized by using a independent analysis based on a cut on the quality of the fitted tracks, parametrized by σ_{MTRK}
- The alternative analysis produced: a **BG contribution below 10%** (see figures); the $\mu^+\mu^-\gamma$ fraction determination consistent with the other procedure within the systematic error.

Extracting $\sigma_{\pi\pi}$ and $|F_\pi|^2$ from $\pi\pi\gamma$ events

a) Via absolute Normalisation to Bhabha events (KLOE05,08,10):

1) $$\frac{d\sigma^{obs}_{\pi\pi\gamma(\gamma)}}{dM^2_{\pi\pi}} = \frac{\Delta N_{\mathrm{Obs}} - \Delta N_{\mathrm{Bkg}}}{\Delta M^2_{\pi\pi}} \cdot \frac{1}{\varepsilon_{\mathrm{Sel}}} \cdot \frac{1}{\int L\,dt}$$

$d\sigma_{\pi\pi\gamma}(\gamma)/dM^2$ is obtained by subtracting background from observed event spectrum, divide by selection efficiencies, and ***int. luminosity***:

2) $$\sigma_{\pi\pi}(\mathrm{s}) \approx s\,\frac{d\sigma^{obs}_{\pi\pi\gamma(\gamma)}}{dM^2_{\pi\pi}} \cdot \frac{1}{H(\mathrm{s})}$$

Obtain $\sigma_{\pi\pi}$ from (ISR) - radiative cross section $d\sigma^{\pi\pi\gamma}(\gamma)/dM^2$ via theoretical radiator function H(s):

3) $$|\mathrm{F}_\pi|^2 = \frac{3s}{\pi\alpha^2\beta^3_\pi}\,\sigma_{\pi\pi}(\mathrm{s})$$

Relation between $|F_\pi|^2$ and the cross section $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$

b) Via bin-by-bin Normalisation to rad. Muon events (New measurement!)

Radiative Corrections

Radiator-Function $H(s,s_p)$ (ISR):

- ISR-Process calculated at NLO-level

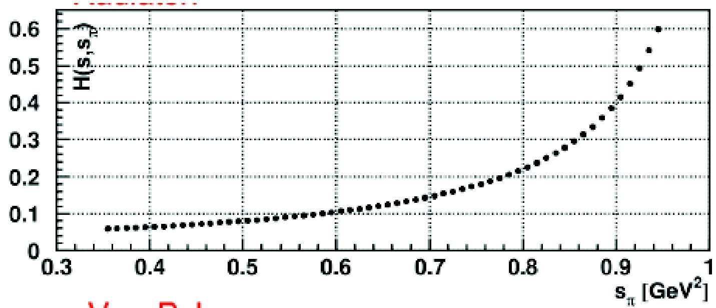
PHOKHARA generator

(H.Czyż, A.Grzelińska, J.H.Kühn, G.Rodrigo, EPJC27,2003)

Precision: 0.5%

$$\boxed{s \cdot \frac{d\sigma_{\pi\pi\gamma}}{ds_\pi} = \sigma_{\pi\pi}(s_\pi) \times H(s,s_\pi)}$$

Radiator:



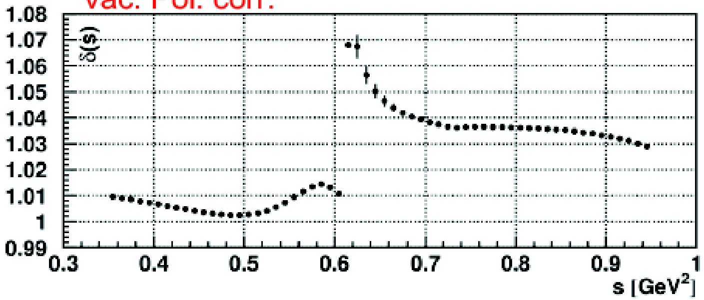
Radiative Corrections:

i) Bare Cross Section

divide by Vacuum Polarisation d(s)=(a(s)/a(0))²

➔ from F. Jegerlehner

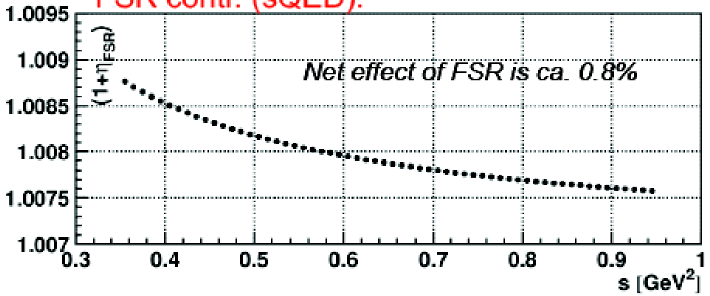
Vac. Pol. corr:



ii) FSR

Cross section s_{pp} must be incl. for FSR for use in the dispersion integral of a_m

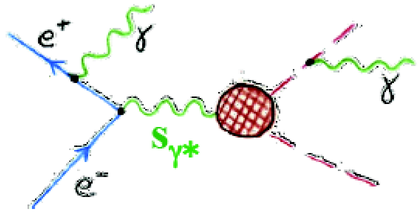
FSR contr. (sQED):



Net effect of FSR is ca. 0.8%

FSR corrections have to be taken into account in the efficiency eval. (Acceptance, M_{Trk}) and in the mapping $s_\pi \rightarrow s_{\gamma*}$

(H.Czyż, A.Grzelińska, J.H.Kühn, G.Rodrigo, EPJC33,2004)



$$\boxed{s_{\gamma*} > s_\pi}$$

$a_\mu^{\text{had,LO}}$:

Hadronic contribution to a_μ can be estimated by means of a dispersion integral:

$$a_\mu^{had} = \left(\frac{\alpha\ m\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds\ \frac{R(s)\,\hat{K}(s)}{s^2}$$

$$R(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to q\,\bar{q} \to hadrons)}{\sigma_{tot}(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

1 / s^2 makes low energy contributions especially important:

$$\boxed{e^+e^- \to \pi^+\pi^-}$$

in the range < 1 GeV contributes to 70% !

- K(s) = analytic kernel-function
- above sufficiently high energy value, typically 2…5 GeV, use *pQCD*

Input:

a) hadronic electron-positron cross section data

b) hadronic τ- decays, which can be used with the help of the CVC-theorem and an isospin rotation (plus isospin breaking corrections)