Preliminary Measurements of the Dominant \(K_L\) Absolute Branching Ratios, the \(K_L\) Lifetime, and \(V_{us}\) with the KLOE Detector

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Abstract

From a sample of about $10^9 \phi$-mesons produced at DAΦNE, we select $K_L$ mesons tagged by observing $K_S \rightarrow \pi^+\pi^-$ decays. We present preliminary results on the major $K_L$ branching ratios, including the semileptonic decays relevant for the $|V_{us}|$ determination. A preliminary measurement of the $K_L$ lifetime, using $K_L \rightarrow \pi^0\pi^0\pi^0$ decays, is also given.

1 Introduction

We describe in the following the measurement of the $\pi^\pm e^\mp \nu$, $\pi^\pm \mu^\mp \nu$, $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$ $K_L$ decay absolute branching ratios. These branching fractions are fundamental experimental parameters used to determine the CKM element $|V_{us}|$. There are no good measurements of the absolute value of these branching fractions; at present, the PDG “fit” values for this branching ratio have 0.5–2% relative errors, and are obtained from various $K_L$ branching ratio and rate measurements[1]. A recent analysis from KTeV[2] of the relative $K_L$ main branching fractions has shown discrepancies with respect to the PDG values up to 8% depending on the channel.
2 Measurement of absolute branching ratios

The measurement of the $K_L$ absolute branching fractions is a unique possibility of the $\phi$-factory. A pure sample of almost monochromatic $^1$ $K_L$ can be selected by the identification of the $K_S$ decay. The $K_L$ absolute branching fractions can be determined on a tagged $K_L$ events sample by counting the fraction of $K_L$ decays in each channel, correcting for acceptances, reconstruction efficiencies and background. However, the tagging procedure is not perfect, because the tagging efficiency depends slightly on the $K_L$ evolution. We include events with the $K_L$ interacting in the calorimeter, escaping the detector and all $K_L$ decays. The difference in tagging efficiency depends on the tagging algorithm and its minimization is used to optimize the tagging criteria. To correct for this effect we define a tag bias defined as the ratio of the tagging efficiency of each channel and the overall tagging efficiency.

Knowledge of the $K_L$ lifetime $\tau(K_L)$ is very important not only for the determination of $|V_{us}|$ but it is also necessary for the absolute branching fraction measurement. $\tau(K_L)$ can be obtained from the $K_L$ proper time distribution and also by measuring all $K_L$ decay branching ratios in a given fiducial volume, FV, by requiring they add up to 1. Results from both methods will be given.

The analysis presented uses all data collected by KLOE at DAΦNE during the years 2001 and 2002, corresponding to an integrated luminosity of about 410 pb$^{-1}$. The data sample has been divided in eighteen subsamples to account for changes in detector operation and DAΦNE running conditions. Each subsample is divided in two parts, one used for the measurement and the other to evaluate corrections.

3 The $K_L$ tag

Candidate events are identified by the presence of a $K_S \rightarrow \pi^+ \pi^-$ decay. We require a vertex with two opposite curvature tracks within a cylindrical FV of radius $r < 10$ cm and height $h < 20$ cm, around the center of the collision region, obtained from Bhabha events for each run. The two-track, assumed pions, invariant mass must be within 5 MeV of $m_{K_S}$. The magnitude of the total momentum of the two tracks must be within 10 MeV of the value expected from the value of $p_\phi$. The $K_S$ momentum is obtained from the decay kinematics of $\phi \rightarrow K_S K_L$ using the $K_S$ direction reconstructed from the measured momenta of the $\pi^+ \pi^-$ tracks and the known value of $p_\phi$.

The position of the $\phi$ production point, $x_\phi$, is determined as the point of closest

$^1$ $\phi$ mesons at DAΦNE are produced almost at rest with a small horizontal momentum of about 12 MeV.
approach of the $K_S$ momentum, propagated backwards from the $K_S$ vertex, to the beam line. The $K_L$ line-of-flight is then given by the $K_L$ momentum $\vec{p}_{K_L} = \vec{p}_{\phi} - \vec{p}_{K_S}$ and the production vertex at $\vec{x}_{\phi}$.

The main source of tag bias is due to the dependence of the trigger efficiency on the $K_L$ behavior. The hardware calorimeter trigger, which requires two local energy deposits above some threshold (50 MeV on the barrel and 150 MeV on the end caps), is used for the present analysis.

The trigger efficiency is essentially 100% for $\pi^0 \pi^0 \pi^0$, between 95-85% for charged decays and lower for $K_L$ interacting in the calorimeter or escaping. It is fundamental to know whether the $K_S$ alone can satisfy the calorimeter trigger in order to reduce the tag bias. The minimal requirement, two clusters from the $K_S \rightarrow \pi^+ \pi^-$ decay associated with fired trigger sectors, has been reinforced, using the reconstructed event information by verifying the trigger with a lower cut on the clusters energy $E_{\text{cluster}} > E_{\text{threshold}}$, where $E_{\text{threshold}}$ is the energy of the trigger sector efficiency plateau. This condition is sufficient to ensure at 99.8% the trigger. For this analysis only events with barrel-barrel trigger configuration are used and a value of 80 MeV has been used for $E_{\text{threshold}}$. The overall efficiency of this tagging is about 20%. A very small fraction of events tagged by the $K_S$ decay is rejected by the cosmic-ray veto being reached by the $K_L$ decay products. The value of the tag bias is corrected using an events sub-sample for which the cosmic-ray veto presence was recorded but not enforced.

Another contribution to the tag is due to the dependence of the tracking efficiency for the $K_S \rightarrow \pi^+ \pi^-$ pions on others tracks in the drift chamber. We will call this effect tracking interference in the following. This effect produces a difference in the tagging efficiency among charged and neutral $K_L$ modes which depends on the $K_L$ decay point position. The tag bias has been evaluated with the KLOE Monte Carlo simulation (MC), Geant3 [3]. A data-MC comparison of the vertex distribution for charged and neutral modes is used to validate or correct the Monte Carlo results.

The FV used for the analysis is defined inside the drift chamber by $35 < \sqrt{x^2 + y^2} < 150$ cm and $|z| < 120$ cm, where $(x, y, z)$ are the $K_L$ decay vertex position coordinates. Since the $K_L$ mean decay length in our experiments is 340 cm, the FV contains ~26.1% of the $K_L$ decays. This choice minimizes the difference in tag bias among decay modes. The average tag bias is .985, 0.99 and 1.02 for $\pi^\pm e^\mp \nu$ or $\pi^\pm \mu^\mp \nu$, $\pi^+ \pi^- \pi^0$ and $\pi^0 \pi^0 \pi^0$ decays, respectively.

4 $K_L$ charged decay modes

All tracks in the chamber, after removal of those from the $K_S$ decay and their descendants, are extrapolated to their points of closest approach to the $K_L$ line-
of-flight. For each candidate track, we obtain the point of closest approach to the
$K_L$ line-of-flight $\vec{x}_c = (x, y, z)$. We then compute the distance of closest approach,
$d_c$, the momentum $\vec{p}_c$ of the track at $\vec{x}_c$ and the extrapolation length, $l_c$. Tracks
satisfying $d_c < a/\sqrt{x^2 + y^2 + b}$, with $a=0.03$ and $b=5$ cm are accepted as $K_L$
decay products.

The tracking efficiency can be determined by counting the number of events with
at least one found $K_L$ decay track and the number of events in which there are
two opposite sign decay tracks. The difference between data and MC for this
efficiency is used to determine the tracking efficiency correction. The correction
is evaluated as function of the track momentum using $K_L \rightarrow \pi^+\pi^-\pi^0$ and $\pi^\pm e^\mp \nu$ events. In $K_L \rightarrow \pi^+\pi^-\pi^0$ events, the momentum of one decay track is calculated
with a resolution of about 10 MeV using the two photons from the $\pi^0$ decay
and the momentum of the other decay track. In order to obtain the correction
at higher momenta $\pi^\pm e^\mp \nu$ decays are used, identifying the electrons by time of
flight with a purity of $\sim$95%. The momentum of the second $K_L$ track is evaluated
from the missing energy with a resolution of about 30 MeV.

In the following we require two good $K_L$ decay tracks forming a vertex to improve
the momentum resolution of the $K_L$ decay products. The average tracking efficiency is 60.5% for $K_{e3}$ 58.5% for $K_{\mu3}$ and 43.0% for $\pi^+\pi^-\pi^0$ as evaluated from
Monte Carlo simulation. The correction, evaluated as described above, ranges
between 1.03 and 0.99 and depends on the channel and on the subsample.

From Monte Carlo studies we found that the best discriminant amongst the $K_L$
charged decay modes is the smallest of the two values of $\Delta_{\mu\pi}=|\vec{p}_{\text{mis}}|-E_{\text{mis}}$,
where $\vec{p}_{\text{mis}}$ is the missing momentum and $E_{\text{mis}}$ is the missing energy evaluated
assigning to the two particles the pion and muon masses. The background due to
$K_L \rightarrow \pi^+\pi^-$ and $K_L \rightarrow \pi^0\pi^0\pi^0$ with Dalitz conversion amounts to about 0.3%. The
shape $\Delta_{\mu\pi}$ spectrum is very sensitive to the radiative corrections to $K_L$ decay
processes. This is particularly evident for $K_{e3}$ decays. The KLOE Monte Carlo
simulation includes very accurately the radiative corrections as described in [4].

5 Neutral $K_L$ decays

The position of the $K_L$ vertex for decays to neutrals, $\pi^0$, is obtained from the
photon time of arrival, TOA, at the EMC. Each photon determines the $K_L$ decay
length $L_K$ up to a two-fold ambiguity, one easily discarded. The best value of $L_K$ is
the energy weighted average of each measurement. The accuracy of this method
is checked with $K_L \rightarrow \pi^+\pi^-\pi^0$ decays, comparing the position of the $K_L$
decay vertex from tracking with the $\pi^+\pi^-$ pair and TOA for the $\pi^0$ photons. Decays to
$\pi^+\pi^-\pi^0$ are also used to measure the single photon efficiency in order to correct
the Monte Carlo simulation. At least three photons with energy greater than
20 MeV originating from the $K_L$ decay are required for the $K_L \rightarrow \pi^0\pi^0\pi^0$ event
Table 1
Summary of systematic uncertainties on the absolute branching fraction measurement.

<table>
<thead>
<tr>
<th></th>
<th>$\pi^+e^+\nu$</th>
<th>$\pi^+\mu^+\nu$</th>
<th>$\pi^+\pi^0$</th>
<th>$\pi^0\pi^0\pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0002</td>
<td>0.0015</td>
</tr>
<tr>
<td>Kinematic Shape</td>
<td>0.0010</td>
<td>0.0008</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Tag bias</td>
<td>0.0022</td>
<td>0.0016</td>
<td>0.0006</td>
<td>0.0011</td>
</tr>
<tr>
<td>$K_L$ lifetime</td>
<td>0.0023</td>
<td>0.0017</td>
<td>0.0007</td>
<td>0.0012</td>
</tr>
<tr>
<td>Total</td>
<td>0.0034</td>
<td>0.0025</td>
<td>0.0010</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

The same selection used for the measurement of the ratio $\Gamma(K_L \to \gamma\gamma)/\Gamma(K_L \to \pi^0\pi^0\pi^0)$ has been used for the lifetime measurement\cite{5}. The selection has been slightly modified for the branching fraction measurement. The background contamination, concentrated in three and four photons event sample, is dominated by $\pi^+\pi^-\pi^0$ events with accidental clusters due machine background. They are strongly suppressed by requiring that the less energetic photon has an energy greater than 50 MeV and polar angle satisfying $|\cos \theta| < .88$. The selection efficiency is about 99% with a residual contamination of 1.3%.

6 Systematic uncertainties

Systematic errors due to the still limited statistics in the evaluation of corrections, data Monte Carlo consistency, signal extraction stability, and limited knowledge of the $K_L$ lifetime have been studied. They are summarized in Table 1 for each $K_L$ decay mode. A check on tracking efficiency correction has been performed comparing data and Monte Carlo efficiency as a function of the decay track momentum transformed to the $K_L$ center of mass (CM), $p^*$, which can be calculated for each decay track. No deviation from the Monte Carlo estimates has been observed. The statistical accuracy of the check is included as a systematic uncertainty on the tracking efficiency in addition to the statistical error on the correction.

To assess the uncertainty on $K_{e3}$ and $K_{\mu3}$ measurement due to the limited knowledge of the momentum resolution and the simulation of the radiation, we checked the data Monte Carlo agreement in the $|\vec{p}_{\text{mis}}| - E_{\text{mis}}$ distribution. The check has been performed independently both on a $K_{e3}$ and on a $K_{\mu3}$ enriched samples selected via electron identification with the calorimeter. The check on $K_{e3}$ and on $K_{\mu3}$ are shown in figure 1 and figure 2. In addition the measurement has been repeated in different mass hypothesis. The largest observed deviation, amounting to 0.25% is assigned as the systematic error.

The uncertainties on the $K_L \to \pi^0\pi^0\pi^0$ signal extraction has been preliminarily
Fig. 1. $\Delta_{e\pi}$ distribution with electron identification by calorimeter time of arrival and energy deposition for all data sample and Monte Carlo. The contribution of $K_{e3}\gamma$ with $E_\gamma > 1$ MeV is also shown.

Fig. 2. $\Delta_{\mu\pi}$ distribution with muon/pion identification by calorimeter time of arrival and energy deposition pattern for all data sample and Monte Carlo. The contributions from different channels is also given.
evaluated by checking the stability of the result with modified selection criteria. The effects of photon efficiency corrections and the cluster splitting recovery are below 0.1%. The largest deviation occurs if the lower cut on the number of $K_L$ photons is reinforced from three to five. This cut strongly reduce the background contamination to about 0.1% maintaining a signal efficiency of 88%. A change in the result of 0.7% has been observed. It has been conservatively considered as systematic.

The studies on uncertainties due to the tagging method are ongoing. Preliminary results show that a conservative estimate of this uncertainty is about 0.5%. The uncertainty on the tag bias corrections amount to 0.2% and is dominated by the method used for the tracking interference correction.

The FV acceptance depends on the value of the $K_L$ lifetime. A variation in lifetime of 1% translates in a change of 0.8% in the acceptance. The 0.8% current accuracy on the $K_L$ lifetime leads to a 0.64% uncertainty on the absolute branching fraction measurements. The systematic uncertainties on the $K_L$ lifetime due to the neutral vertex efficiency and calibration are determined from data using $K_L \to \pi^+\pi^-\pi^0$ events. The preliminary estimate amounts to 0.3% and 0.1% respectively. The change of the result obtained by varying the fit interval has also been considered as systematic uncertainty. The systematic error is at present dominated by the Monte Carlo statistics contributing with a 0.6%.

7 Results and $|V_{us}|$ determination

7.1 Absolute $K_L$ branching fractions

A total of about $13 \times 10^6$ tagged $K_L$ events are used for the measurement of the branching fractions. Almost twice as many additional events provide calibration. The results of the measurement of the absolute branching fractions obtained using for the lifetime our result $\tau(K_L) = 51.15 \pm 0.20_{\text{stat}} \pm 0.40_{\text{syst}}$ ns, reported later in section 7.2, are:

$$BR(K_L \to \pi^\pm e^\mp \nu) = 0.3994 \pm 0.0006_{\text{stat}} \pm 0.0034_{\text{syst}}$$
$$BR(K_L \to \pi^\pm \mu^\mp \nu) = 0.2708 \pm 0.0005_{\text{stat}} \pm 0.0025_{\text{syst}}$$
$$BR(K_L \to \pi^0\pi^0\pi^0) = 0.2014 \pm 0.0003_{\text{stat}} \pm 0.0022_{\text{syst}}$$
$$BR(K_L \to \pi^+\pi^-\pi^0) = 0.1271 \pm 0.0004_{\text{stat}} \pm 0.0010_{\text{syst}}$$

The fit to the $|\vec{p}_{\text{mis}}| - E_{\text{mis}}$ distribution from data to a linear combination of Monte Carlo distributions for $K_L \to \pi^\pm e^\mp \nu$, $K_L \to \pi^\pm \mu^\mp \nu$ and $K_L \to \pi^+\pi^-\pi^0$ is shown in figure 3. The three large charged decay modes are well separated. The systematic error due to the selection efficiency uncertainties is uncorrelated among the
different modes.

Fig. 3. $\Delta_{\mu\pi}$ distribution for one data sample and Monte Carlo. The contributions from different channels is also shown.

The shape errors result however in a $O(15\%)$ anticorrelation between the $K_{e3}$ and $K_{\mu3}$ modes. The errors due to tag bias uncertainties and lifetime uncertainty affect equally all channels.

The sum of all measured branching fraction above, plus the PDG value for rare decays, 0.0036, is $\sum BR_i = 1.0023 \pm 0.0009 \pm 0.0077$, where the result, as remarked earlier depends on $K_L$ lifetime. Turning the argument around, by renormalizing the sum to 1.0 we obtain the $K_L$ lifetime:

$$\tau(K_L) = 51.35 \pm 0.05_{\text{stat}} \pm 0.26_{\text{syst}} \text{ ns}$$

### 7.2 $K_L$ lifetime measurement from $K_L \rightarrow \pi^0\pi^0\pi^0$

In figure 4 we show the $K_L$ proper time distribution obtained with $\sim 15 \times 10^6$ $K_L \rightarrow \pi^0\pi^0\pi^0$ decays. The distribution is fitted with an exponential inside the FV, ranging from 50 to 160 cm. In this interval the decay reconstruction efficiency is flat to $\sim 0.3 \%$. The two peaks visible at short values of $t^*$ are due to $K_L \rightarrow K_S$ regeneration in the beam pipe and the inner wall of the drift chamber. The regeneration region is removed by the choice of FV. Variations of the fit result vs the FV choice are well within the statistical accuracy of the fit itself. Our preliminary result is:

$$\tau(K_L) = 51.15 \pm 0.20_{\text{stat}} \pm 0.40_{\text{syst}} \text{ ns}$$
The systematic error is at present dominated by the Monte Carlo statistics and should decrease with the increasing of the generated Monte Carlo sample. Combining the two lifetime values above with those of [6,7], we obtain the preliminary new lifetime value:

$$\tau(K_L) = 51.35 \pm 0.25 \text{ ns}$$

or $$\Gamma(K_L) = 1.947 \pm 0.0076 \times 10^7 \text{ s}.$$  

Finally, the BR given above add up to 0.9987 while the sum of the BR for all other modes is between 0.0035 and 0.0036. Renormalizing to $$\sum = 1 - 0.0035 = 0.9965$$ gives our preliminary branching ratio values:

$$BR(K_L \rightarrow \pi^\pm e^\mp \nu) = 0.3985 \pm 0.0035$$
$$BR(K_L \rightarrow \pi^\pm \mu^\mp \nu) = 0.2702 \pm 0.0025$$
$$BR(K_L \rightarrow \pi^0\pi^0\pi^0) = 0.2010 \pm 0.0022$$
$$BR(K_L \rightarrow \pi^+\pi^-\pi^0) = 0.1268 \pm 0.0011$$

The systematic errors are in part due to common scale uncertainties and should not appear in the errors above. We prefer at the moment not to reduce the errors.

7.3 $|V_{us}|$ determination

The most precise check on the unitarity of the CKM mixing matrix is provided by measurements of $|V_{us}|$ and $|V_{ud}|$, the contribution of $V_{ub}$ being at the level of $10^{-5}$. $|V_{us}|$ is proportional to the square root of the kaon semileptonic partial width. Many factors are necessary for reaching the desired result [8]. In general
we can write

\[ |V_{us}| \times f_+^{K^0}(0) = \left[ \frac{192 \pi^3 \Gamma}{G^2 M^2 S_{ew} I_i(\lambda'_+, \lambda''_+, \lambda_0, 0)} \right]^{1/2} \times \frac{1}{1 + \delta_{em}^i + \Delta I_i/2} \]

where \( f_+^{K^0} \) is the normalization of the form factors at zero momentum transfer and \( I_i(\lambda', \lambda'', \lambda_0) \) is the integral of the phase space density, factoring out \( f_+^{K^0} \) and without radiative corrections. Short distance radiative corrections are in the universal term \( S_{ew} \) [9]. In addition long distance radiative corrections [10,11] for form factor and phase space density are included as \( \delta_{em}^i \) and \( \Delta I_i(\lambda) \). \( \lambda' \) and \( \lambda'' \) are the slope and curvature of the vector form factor \( f_+ \). \( \lambda_0 \) is the slope of the scalar form factor. Using \( \lambda'_+ = 0.0206 \pm 0.0018 \), \( \lambda''_+ = 0.00032 \pm 0.0007 \) and \( \lambda_0 = 0.0137 \pm 0.0013 \) from KTeV [12] we obtain:

\[
\begin{align*}
    f_+^{K^0} \times |V_{us}| &= 0.2147 \pm 0.0014 \text{ from K}\ell_3 \\
    f_+^{K^0} \times |V_{us}| &= 0.2167 \pm 0.0015 \text{ from K}\mu_3
\end{align*}
\]

A precise estimate \( f_+^{K^0}(0) = 0.961 \pm 0.008 \) was first given in 1984 [13]. More recent estimates are subject to large uncertainties [14,11]. Very recently however, lattice calculations [15] have given \( f_+^{K^0}(0) = 0.960 \pm 0.009 \), in excellent agreement with [13]. Using the above value we find:

\[
\begin{align*}
    |V_{us}| &= 0.2237 \pm 0.0025 \text{ from K}\ell_3 \\
    |V_{us}| &= 0.2258 \pm 0.0026 \text{ from K}\mu_3
\end{align*}
\]

The average of the above values is 0.2248\pm0.0022, taking into account the fact that the full error on \( f_+^{K^0} \) must be used for the mean. This result can be compared with 0.2265\pm0.0021 obtained from unitarity, with \( V_{ud} = 0.9740 \pm 0.005 \).

8 Conclusions

Figure 5 shows the value of the product \( V_{us} \times f_+^{K^0}(0) \) as obtained from all KLOE results, see also [16], as well as the recent results reported by the KTeV collaboration. The possible violation of unitarity in the first row of the CKM matrix which followed from the value for BR(\( K_\ell_3 \)) given in the PDG particle listings is clearly no longer present. Our results are preliminary at the moment.
We expect to be able to reduce the errors and eagerly await for improved estimates of the value of the form factor at \( t = 0 \). There is still some question about the form factor parameters, \( \lambda \). We have used the very recent results of KTeV, which are somewhat different from previous values. However, the more recent measurements by ISTRA+ [17] are not accurate enough to prove the existence of a \( (\lambda''/2)(t/m_\pi^2) \) term.

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References


