From the Nucleus to the Muon

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THE BEGINNING

1. Rutherford Scattering
2. Ionization
3. Neutral Particles
4. The Pion Is the Muon

While I certainly had nothing to do with the discoveries of the first half of the 19$^{\text{th}}$ centuries, I want to go over some of them, for many reasons.

New ways of thinking and new instruments were developed in those years.

To a very large extent the instruments are the same today.

In a somewhat general sense so is the way we understand new results.

For example, often today we say "Rutherford scattering" when we talking about experiments probing the structure of some particles.

The understanding of the basic processes by which we "see" particles was developed very early in the last century and I wish to discuss some of the underlying principles.

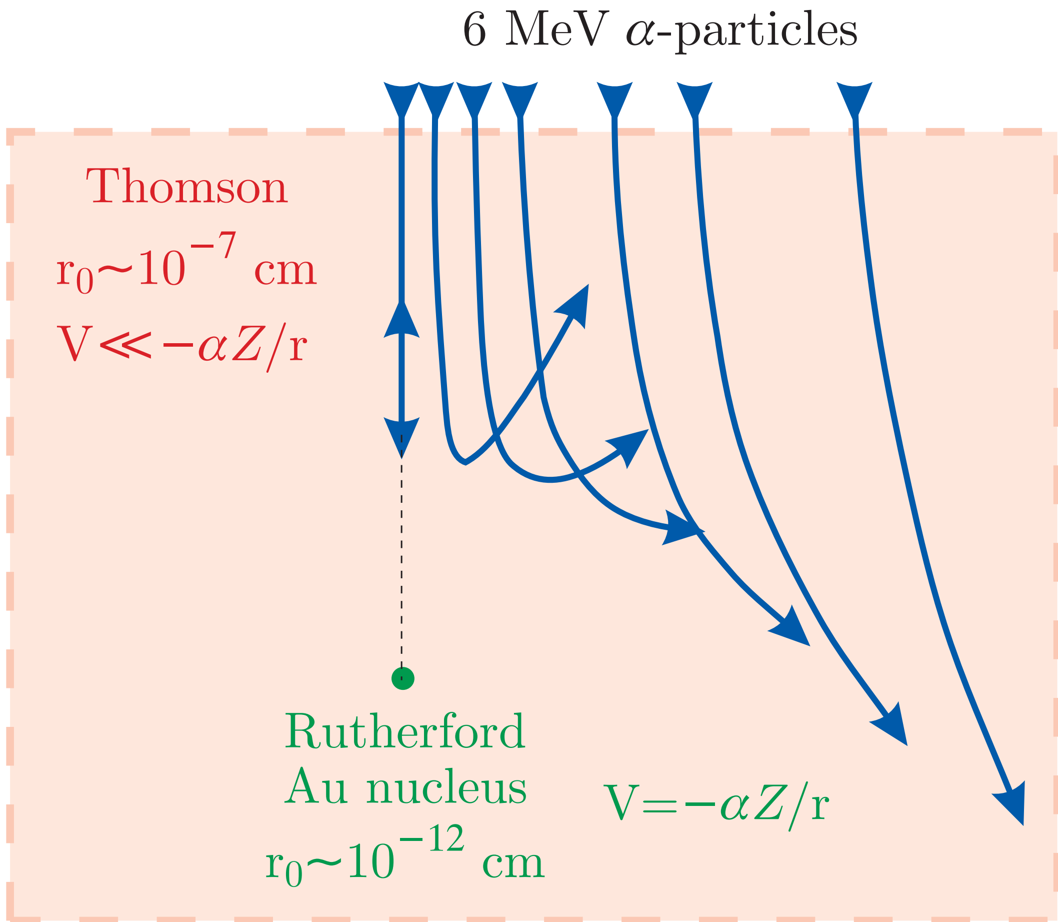
Radioactivity and cosmic rays were first observed as ionizing phenomena. Thanks to ionization we sense particles, and because of it we can even see their trajectories.

Together with ionization go scintillation, conversion of Ag salts to metallic silver, condensation of droplets in the Wilson or "cloud chamber", formation of bubbles in the bubble chamber and finally

the production of tiny electric signals in gas or semiconductors by which the microscopic and gigantic modern detectors can track charged particles.

The Geiger and Marsden experiment, often called the Rutherford experiment, used scintillation to detect the scattered α-particles.

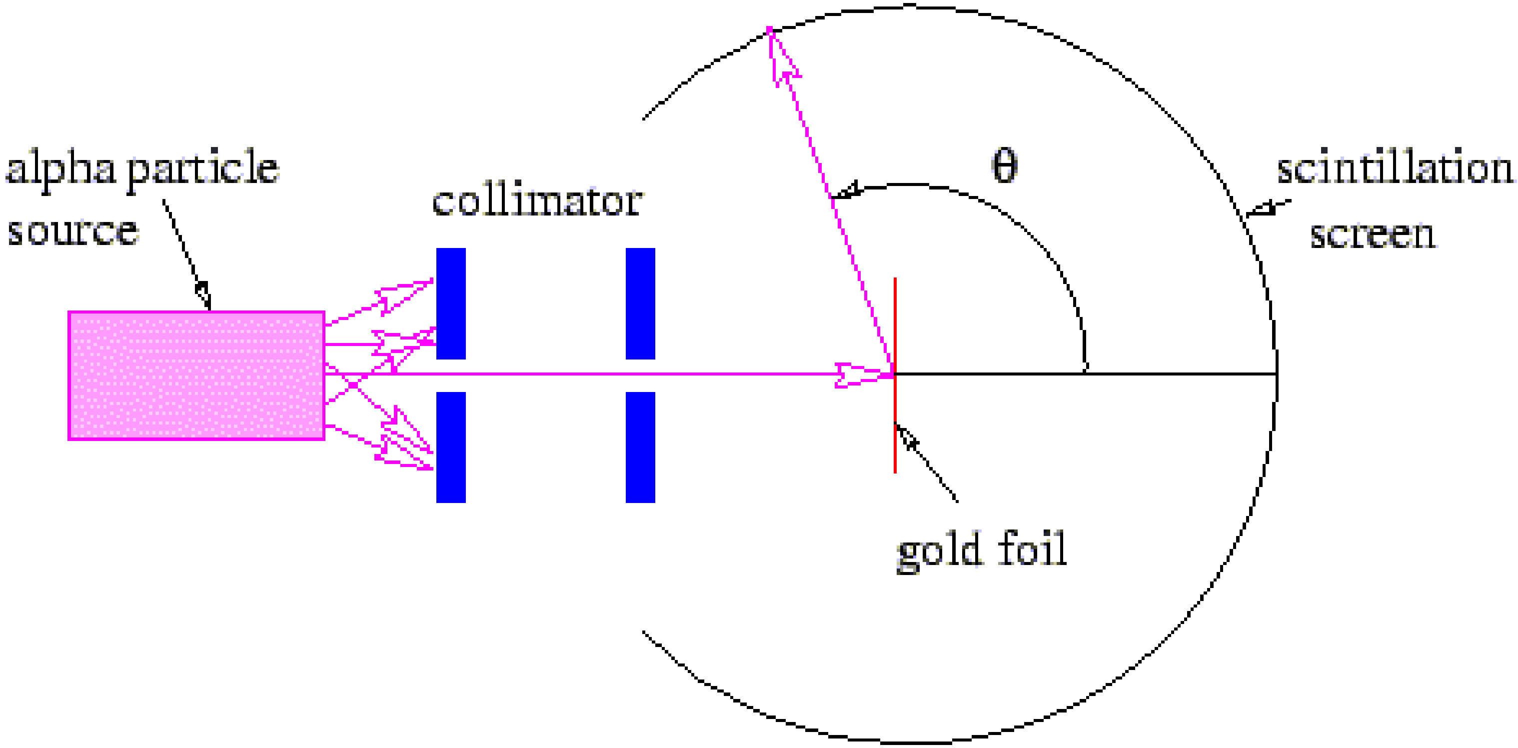
Rutherford scattering, classical



The scattering angle is a function of the impact parameter.

Geiger and Marsden, idealized

The experiment is often sketched as



without even mentioning that it all must be in vacuum.

Marsden was the student and so did all the observation, verifying the $1/\sin^4\theta/2$ dependence of the scattered α's.

Geiger and Marsden, in reality

The real set up.

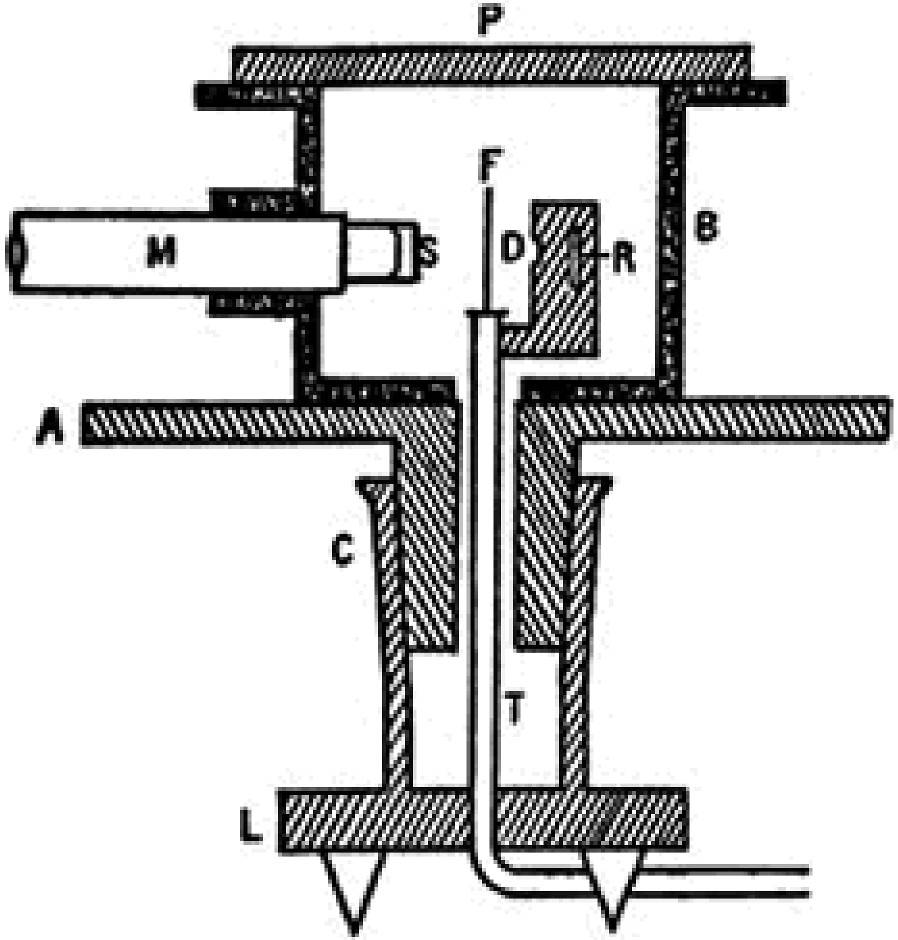
8 MeV α-particles from a radioactive source R in a piece of brass D with a hole

Gold foil F $\sim$400 atom thick

A ZnS coated screen S

A telescope M to observe light flashes

All in vacuum B

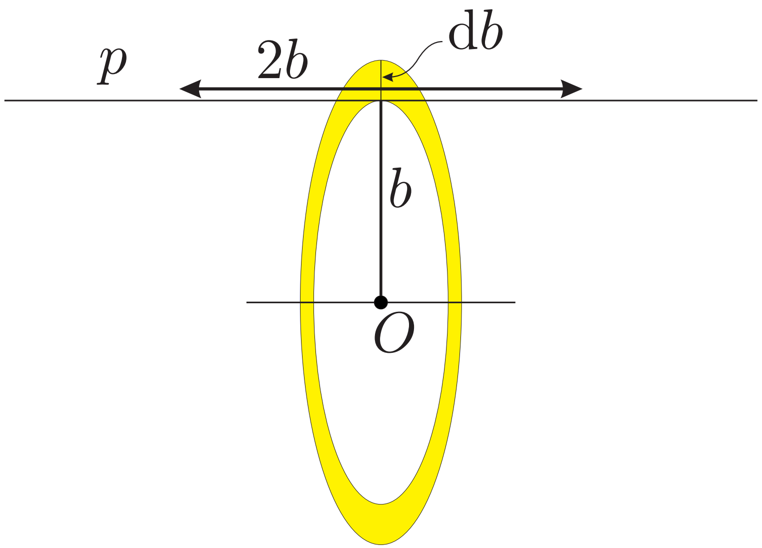


Q^2, t

The classical mechanics scattering.

The QED scattering.

2. Rutherford Scattering



The deflection angle of a particles travelling at a distance b from an infinite mass charge Ze nucleus is given by $\Delta p/p$ where the momentum change. equals the impulse of the force F acting for the time interval required to travel the distance $2b$ at a speed β

The relation between the scattering angle and the impact parameter b is:

$$\theta = \Delta p/p = F \times \Delta t/p = \alpha Z/b^2 \times \frac{2b}{\beta} \times \frac{1}{p} = \frac{2\alpha Z}{\beta b p} \text{ or } b = \frac{2\alpha Z}{\theta\beta p}.$$

The scattering cross section is then:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} = \frac{2\pi b\,\mathrm{d}b}{\mathrm{d}\theta} = \frac{8\pi\alpha^2 Z^2}{\beta^2 p^2 \theta} \times \left| \frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{\theta}\right) \right| = \frac{8\pi\alpha^2 Z^2}{\beta^2 p^2 \theta^3}$$

The Rutherford cross section is:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{\pi\alpha^2 Z^2}{2\beta^2 p^2 \theta} \frac{1}{\sin^4\theta/2}.$$

In the small angle limit, $\sin^4\theta/2 \sim \theta^4/16$, and with $\mathrm{d}\cos\theta \sim \theta\mathrm{d}\theta$ is:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} = \frac{8\pi\alpha^2 Z^2}{\beta^2 p^2 \theta^3}$$

The reason why we find the correct answer is because the crude approximation used for Δp is in fact exact as derived below.

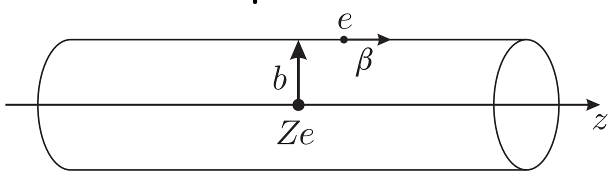
Rutherford was very proud that the scattering formula that he derived in classical mechanics was the correct answer also in QM.

The $\sin^4\theta/2$ term comes from $\mathrm{d}\sigma/\mathrm{d}\theta \propto 1/(\Delta p)^4$. Δp is the momentum transfer $\vec{p}_{\rm in} - \vec{p}_{\rm out}$ or $p_1 - p_2$. Then $(\Delta p)^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\theta$.

In the Rutherford limit, $p_1 = p_2 = p$, $(\Delta p)^2 = 2p^2\sin^2\theta/2$.

In the same limit, it is also $-t$, the Lorenz-invariant 4-momentum transfer squared.

Exact computation of Δp



Referring to fig., consider an infinite cylinder, of radius b around an axis z through the position of the heavy particle, parallel to the trajectory of the charge e moving withe velocity β. To compute $\Delta p = \int_{-\infty}^{\infty} F_\perp dt$ we use Gauss theorem:

$$\int_{S\ \mathsf{cyl}} \vec{E}\cdot \mathrm{d}\vec{\sigma} = Ze = \int_{-\infty}^{\infty} 2\pi b E_\perp dz \quad \text{or} \quad \int_{-\infty}^{\infty} E_\perp dz = \frac{eZ}{2\pi b}$$

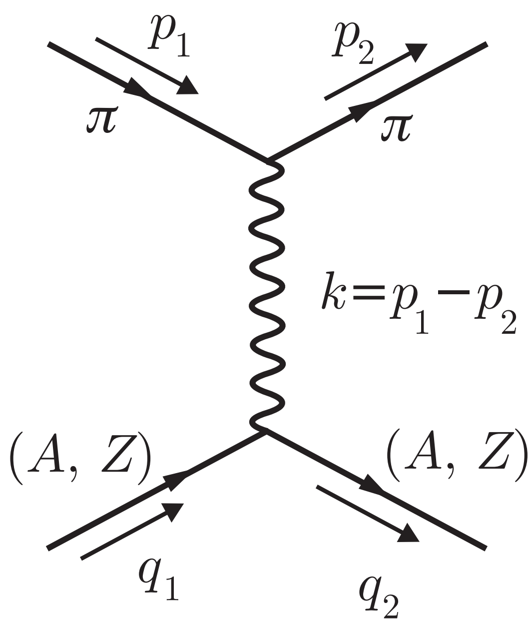
from which

$$\int_{-\infty}^{\infty} F_\perp dz = \frac{e^2 Z}{2\pi b} = \frac{2\alpha Z}{b}$$

and

$$\Delta p = \int_{-\infty}^{\infty} F_\perp dt = \int_{-\infty}^{\infty} F_\perp dz/\beta = \frac{2\alpha Z}{\beta b}.$$

In QED, Rutherford scattering is the scattering of a light, spin-zero particle (we call it a pion) from a heavy spin zero nucleus. The appropriate amplitude is given in the Feynman graph.



The upper vertex is given by $J_\mu(\pi)A^\mu$ and the lower by J_μ(nucleus)A$^\mu$. Both currents must be $\propto (p_{\text{in}} + p_{\text{out}})$ to satisfy gauge invariance and the proportionality constant is the particle charge. Including the photon propagator, $1/k^2$ and summing over its polarization we find

$$\mathfrak{M} = e^2 Z(p_1 + p_2)_\mu \frac{1}{k^2}(q_1 + q_2)^\mu$$

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Finally we compute:

$$\mathrm{d}^3\sigma = \frac{p_2 E_2 \mathrm{d}E_2 \mathrm{d}\Omega}{\beta} \frac{\delta(E_{in} - E_{out})}{(2\pi)^2\, 4E_1 E_3\, 4E_2 E_4} |\mathfrak{M}|^2$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = 8\pi\alpha^2 Z^2 E^2 \frac{1}{k^4}$$

From $p_1 - p_2 = \{0, \vec{p}_1 - \vec{p}_2\}$, $k^2 = -(p_1 - p_2)^2 = -4p^2 \sin^2\theta/2$ and

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right|_{\mathrm{Rutherford}} = \frac{\pi\alpha^2 Z^2}{2} \frac{1}{\beta^2 p^2} \frac{1}{\sin^4\theta/2}. \tag{1}$$

Eq. (1) acquires multiplicative factors when spin is taken into account (Mott, Rosenbluth) as well as from recoil.

The result is however valid strictly only for point-like particles.

Otherwise:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right|_{\mathrm{point}} \times \left|F(k^2 \text{ or } t)\right|^2$$

where the form factor $F(t)$ equals 1 for point like structures and $|F|^2 < 1$ signals the presence of structure.

3. Ionization

When a charged particle crosses a medium it suffers repeated collisions with electrons. It is easy, for $M > m_e$, to compute the average energy loss per unit path length, usually measured in g/cm^2.

By reasoning as in my derivation above of the Rutherford formula we get that the energy transferred to an electron at distance b from the trajectory of a charge z particle is:

$$\frac{p^2}{2m_e} = \frac{2z^2e^4}{m_e\beta^2b^2}$$

If $\mathcal{N}$ is the electron number density (N$_e$/cm^3), the energy lost per cm of trajectory to the electrons between b and $b + \mathrm{d}b$ is

$$-\mathrm{d}E(b) = \frac{4\pi z^2e^4\mathcal{N}}{m\beta^2}\frac{\mathrm{d}b}{b}$$

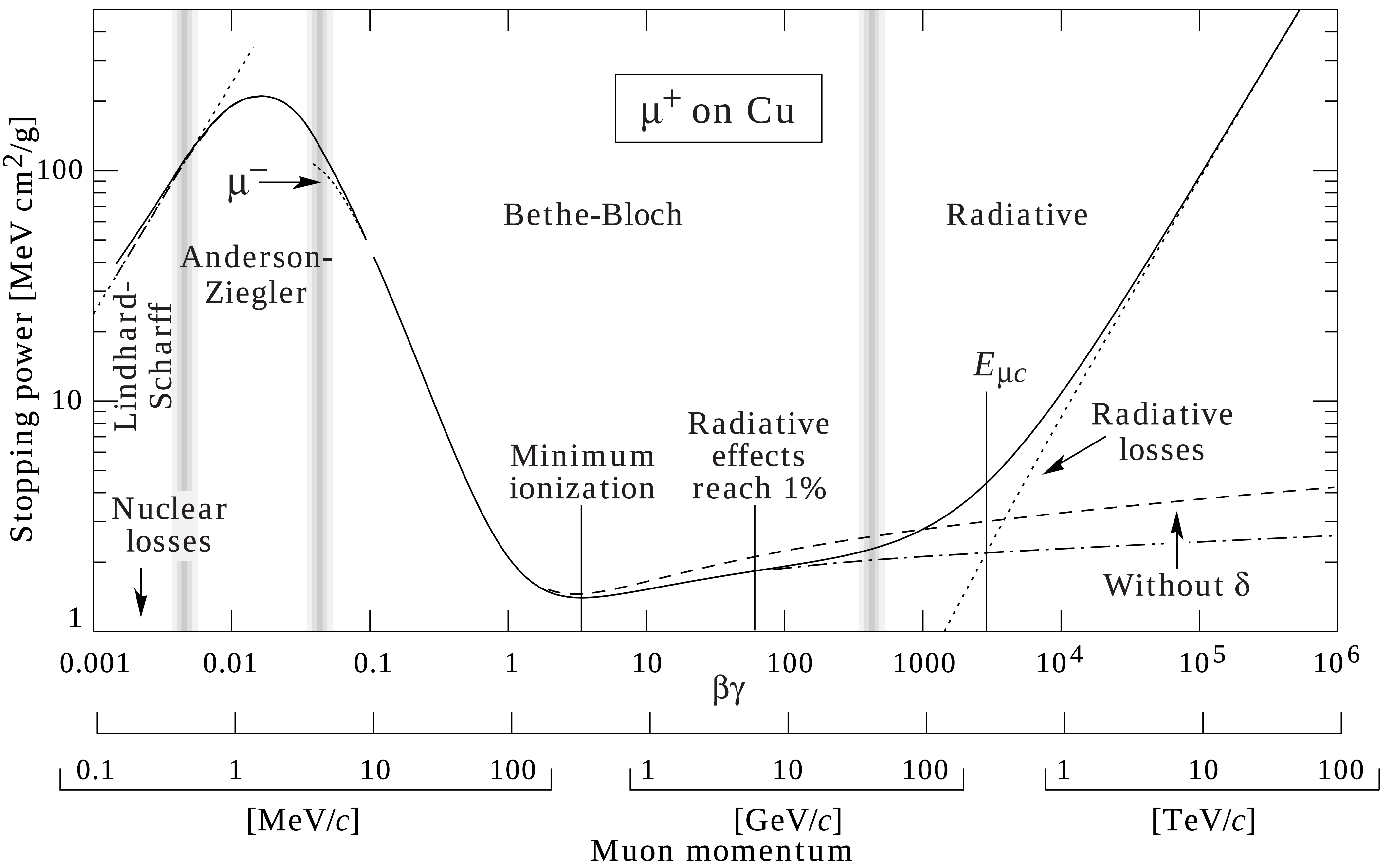
and the total energy loss per unit path lenght:

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi z^2 e^4 \mathcal{N}}{m\beta^2} \ \ln \frac{b_{\mathrm{max}}}{b_{\mathrm{min}}}$$

b_{max} is limited by the suppression of energy transfer if $1/\Delta t < \bar{f}$, where $\bar{f}$ is the average frequency of the bound electrons. b_{min} is obtained from the max KE Tmax that can be transferred to the electron in a collision. The result is not too different from the Bethe-Block formula:

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e \beta^2 \gamma^2 T_{\mathrm{max}}}{I^2} - \beta^2 - \frac{\delta}{2} \right]$$

The energy loss is in MeV/(g/cm^2), with $K = 4\pi N_{\mathrm{A}} r_e^2 m_e c^2$. There is a region where $-\mathrm{d}E/\mathrm{d}x \propto 1/\beta^2$. This has been very useful for particle identification. Deviation from Bethe-Bloch becomes large for $\beta < 0.03$ and $\beta\gamma, > 600$.


μ+ on Cu
Stopping power [MeV cm2/g]
100
10
1
μ−
Bethe-Bloch
Radiative
Anderson-
Ziegler
Lindhard-
Scharff
Eμc
Radiative
losses
Minimum
ionization
Radiative
effects
reach 1%
Nuclear
losses
Without δ
0.001
0.01
0.1
1
10
100
1000
10^4
10^5
10^6
βγ
0.1
1
10
100
1
10
100
1
10
100
[MeV/c]
[GeV/c]
[TeV/c]
Muon momentum

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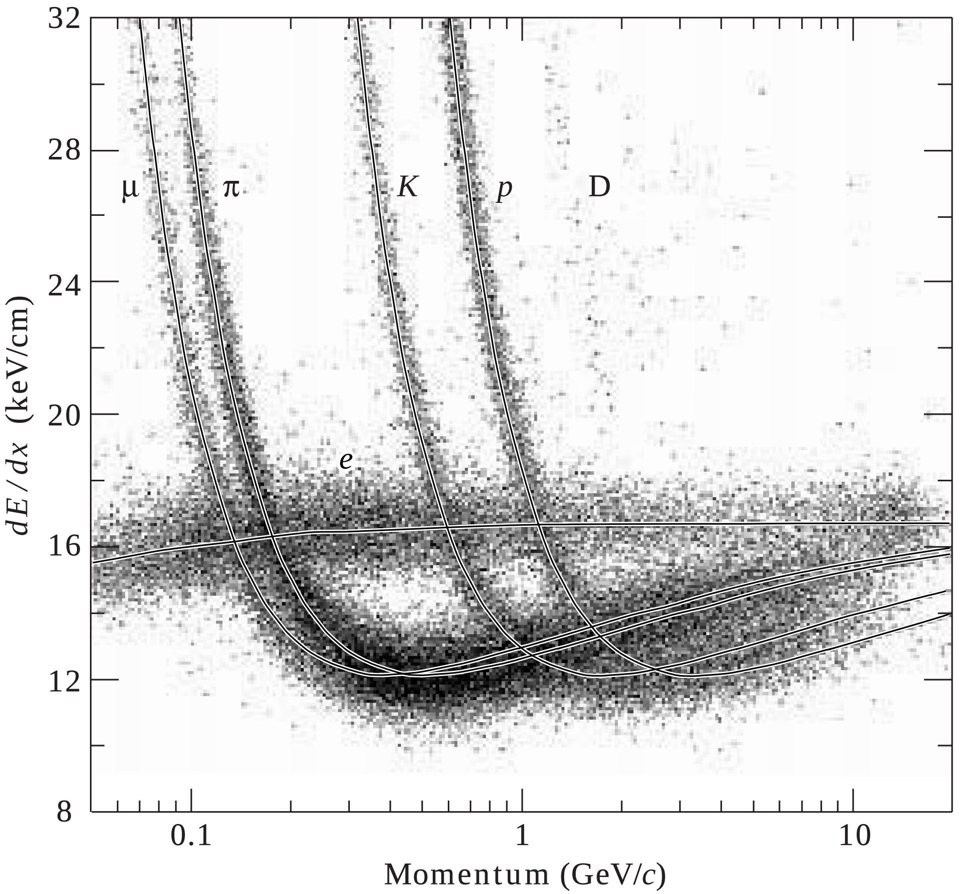
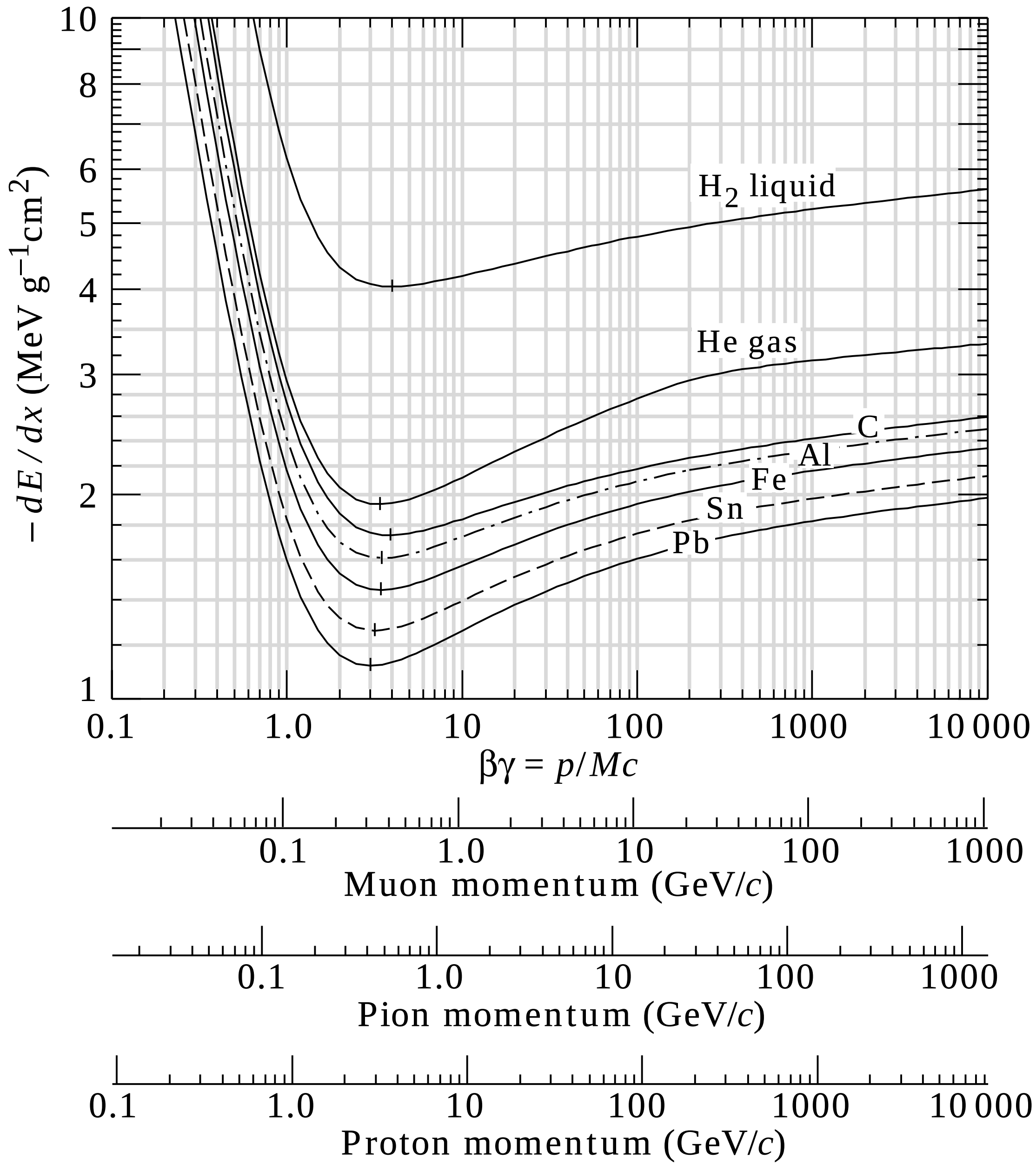




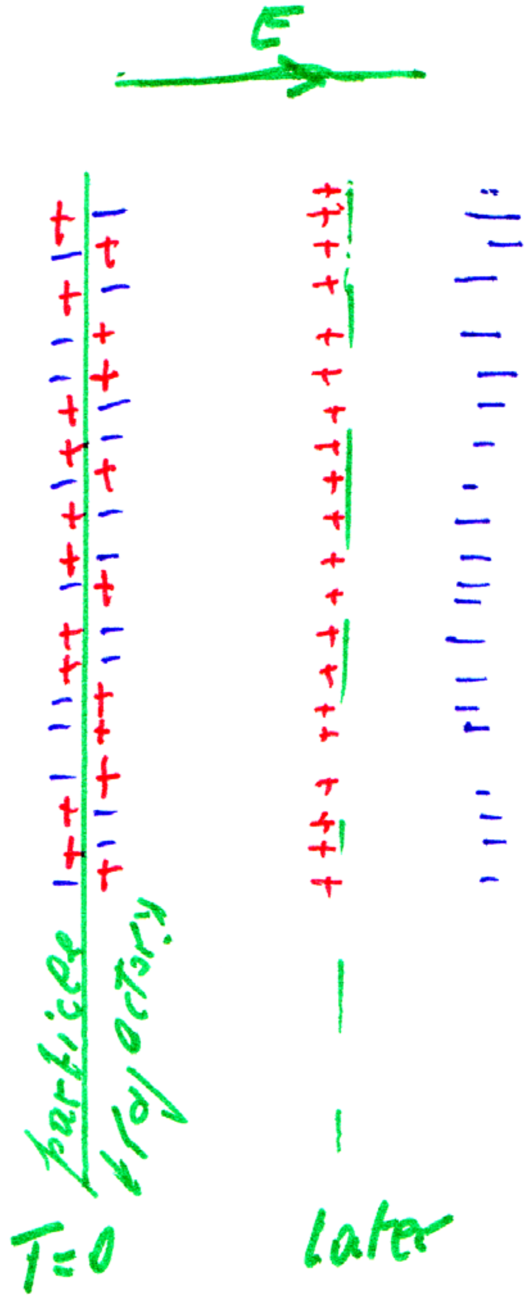
Specific ionization vs momentum as measured in the "Time Projection Chamber"

A convenient value to remember for the minimum of $-\mathrm{d}E/\mathrm{d}x$ is $\sim$2 MeV/g/cm^2, valid for He, D_2, n-(CH_2) (polyethylene, plastics). For H_2, $A = Z$, $-\mathrm{d}E/\mathrm{d}x|_{\mathrm{min}}$=4. For U it becomes about 1. The simple picture fails at low energy and for electrons.
At high energy radiation or bremsstrahlung dominates as we will discuss.

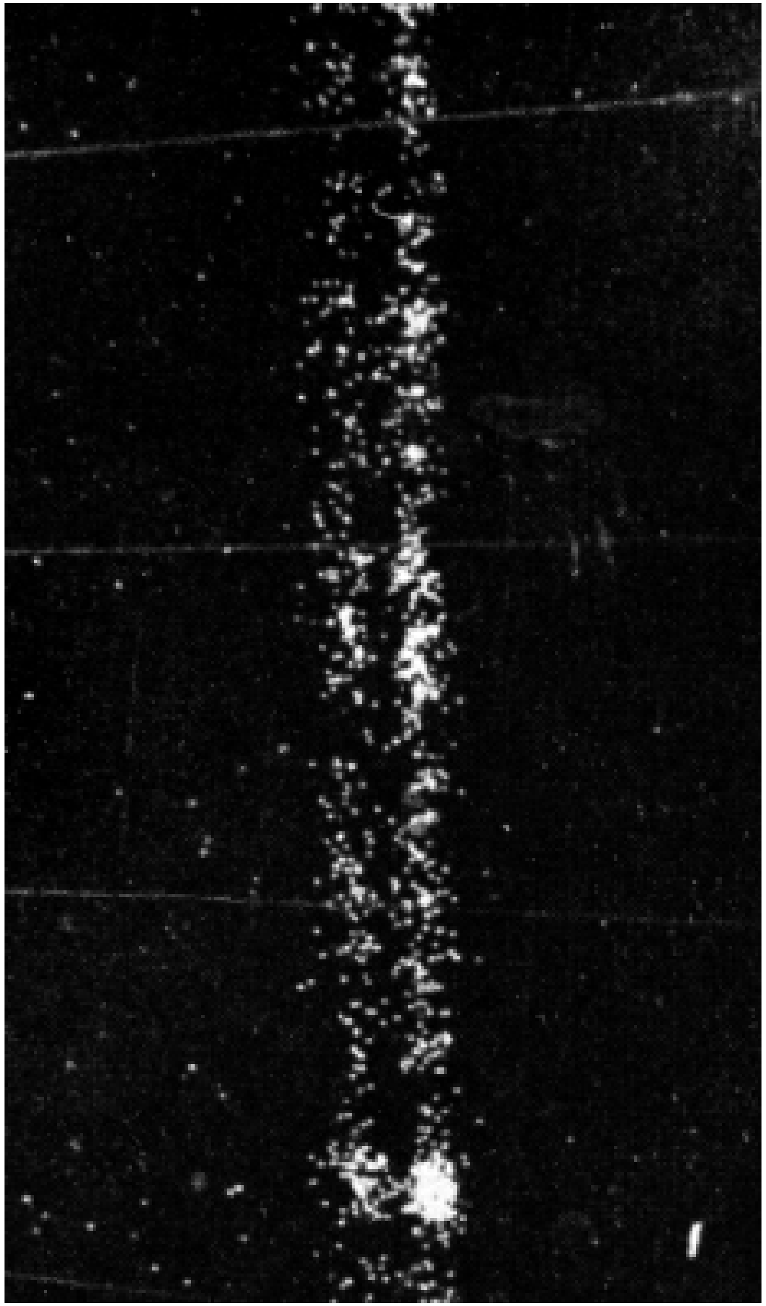
The energy lost in the medium by a charged particle goes initially into excitation and ionization, *i.e.* formation of electron-ion pairs. Ultimately the energy will thermalize.

In gases especially, where atomic and molecular collision are relatively rarer, the ion pairs can survive a long time and are the basis for seeing the trajectory of a charged particles and measuring some of its properties.

The initial ion-electron pairs can be drifted apart by an electric field. Drift velocities are vastly different for ions and electrons. Saturated drift velocities for electrons in gases are few cm/μs, $\sim$1000 time smaller for ions



Ion pairs are made dramatically evident in the Wilson chamber by liquid droplets condensing on the positive ions and the electrons, often a negative ion formed by the electron attached to electronegative atoms, oxygen foremost. An electric field has drifted apart positive and negative charges. Diffusion is also evident.



It is an experimental fact that the number of pairs is proportional to the energy deposited. (Until saturation at very high specific ionization.)

Gas	Pairs/cm	eV/pair
He	16	22
Ne	42	37
Ar	103	26
Xe	340	22

In noble gases and some materials, some fraction of the excitation energy is emitted as visible light. This process is called scintillation. It was discovered in the early days of discharge tubes, used by Thomson and many others.

It is still very important today in particle detection.

Drift velocities: 4 cm/μs, saturated, Ar, He...
1-8 cm/μs liq Ar.

There are some complications.

1. Drift velocity, larger for electrons than ions, field dependent
2. Recombination
3. Formation of negative ions in the presence of oxygen, halogens and so on
4. The signal, 100 $e{\sim}1.6 \times 10^{-17}$ C, is below the thermal noise of most, not all, amplification mechanism

Charged particles leave behind a trail of ions which allows us to track them. They are also deflected in magnetic fields

$$p = 300 \times B \times r,\ \text{B in Gauss, p in eV/c, r in cm}$$

.

Detection of photons

Neutral particles do not ionize. Neutron can be detected easily since they interact strongly with matter.

Photons interact with matter by several processes:
Photoelectric effect
Compton effect
Electron-positron pair production.

At high energy, $E \gg m$, or $\gamma \gg 1$, the last process is important.
The others are still relevant but we will not discuss them.

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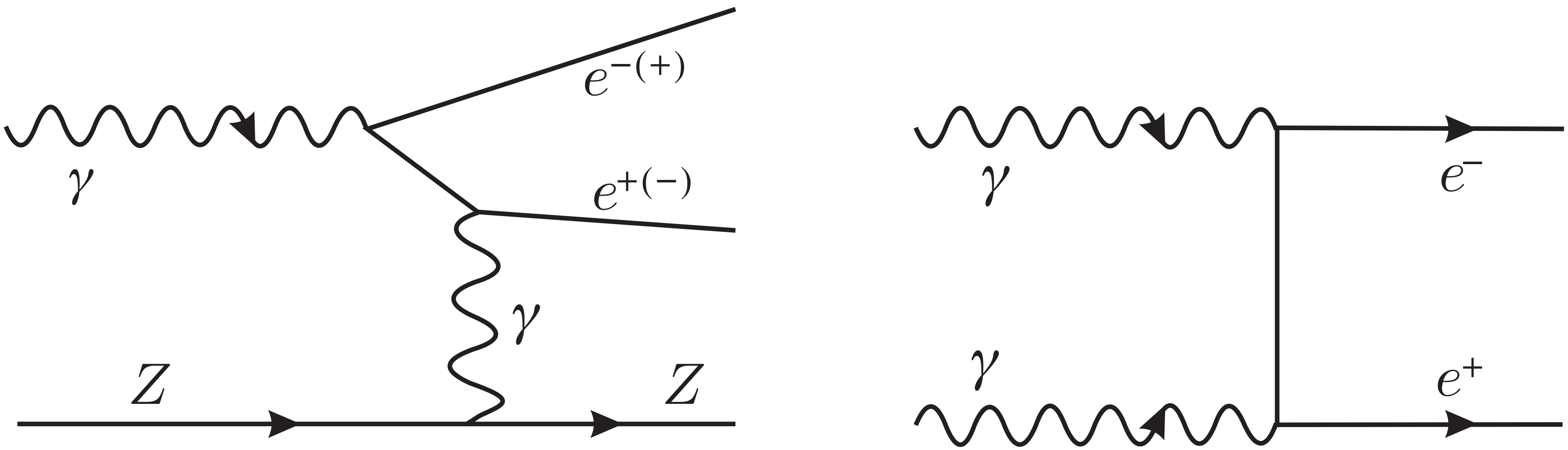
Pair Production

Pair production, $\gamma \to e^+ e^-$ can only take place in an external field, in the real world that provided by a nucleus of charge Ze. The nucleus absorbs the recoil momentum necessary not to violate Lorentz-invariance as illustrated.



Before dealing with pair production, we consider the two photon process $\gamma\gamma{\to}e^+e^-$. By counting powers of e and dimensional arguments we get a familiar result

$$\sigma(\gamma + \gamma \to e^+ + e^-) = \frac{\alpha^2}{s}$$

for high enough energies so that the electron mass can be neglected.

The connection between this last process and the one we are interested in, is clear: the Coulomb field of the nucleus, which we take to be of infinite mass, provides an almost real photon which, together with the initial photon, produces the e^+e^- pair.

The electrostatic Coulomb potential is Ze/r whose Fourier transform, $1/q^2$, is the spectrum of the virtual photons corresponding to the potential.

The 4-momentum of the virtual photon is $p = (0, \vec{p})$, *i.e.* the nucleus absorbs momentum but not energy because its mass is infinite. Because of this exchanged momentum the momenta of the two final electrons become independent in pair creation, contrary to the case of $\gamma\gamma \to e^+e^-$.

Therefore, the final state of the pair production process contains an extra factor d^3p/E for the two electrons. To get the total cross section, we must integrate.

Dimensionally this means that an extra factor $p^2 \sim s$ must appear in the cross section, cancelling the $1/s$ factor in the $\gamma\gamma$ result. Counting powers of e and Z, and taking into account the above, we finally write $\sigma = Z^2\alpha^3 f(m_e)$ and therefore:

$$\sigma_{\mathrm{Pair}} \sim \frac{Z^2\alpha^3}{m_e^2}.$$

The complete result is the Bethe-Heitler formula:

$$\sigma_{\mathrm{Pair}} = \frac{28}{9}\frac{Z^2\alpha^3}{m_e^2}\ln\frac{E_\gamma}{m_e}$$

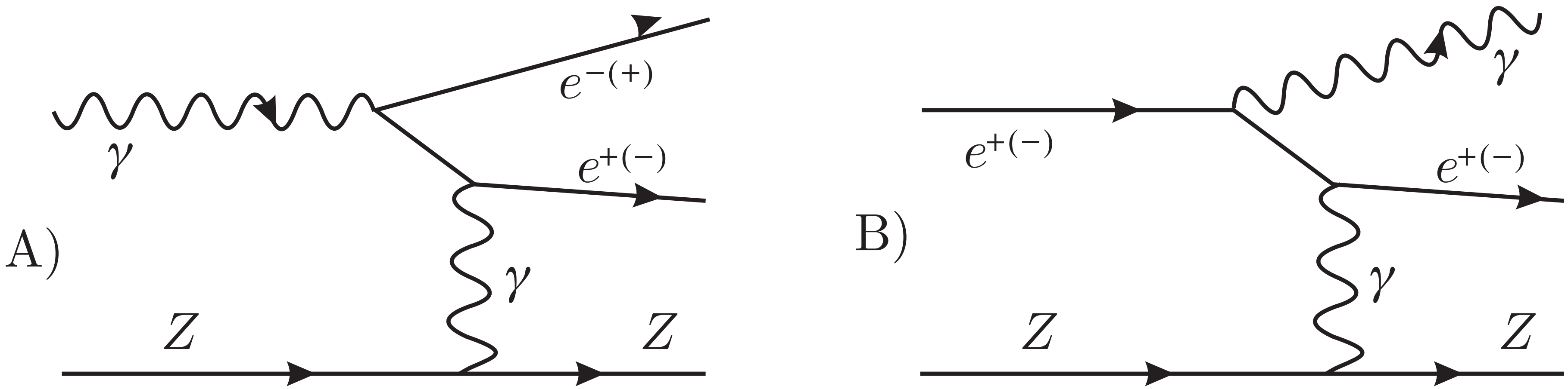
which is valid for a point nucleus, without screening.

The nucleus Coulomb field does not extend to infinity because of the atomic electron screening. Taking this into account, the log term becomes a constant, $\sim\ln(183 \times Z^{-1/3})$.

This is a very important result. At high energy, photons have a finite, non-zero, mean free path in matter, almost energy independent. A naive expectation could have been that absorption disappears at high energy.

Bremsstrahlung

The process by which a high energy electron, or positron, radiates crossing matter is identical to that of pair creation. The bremsstrahlung amplitude is the same as that of pair creation.



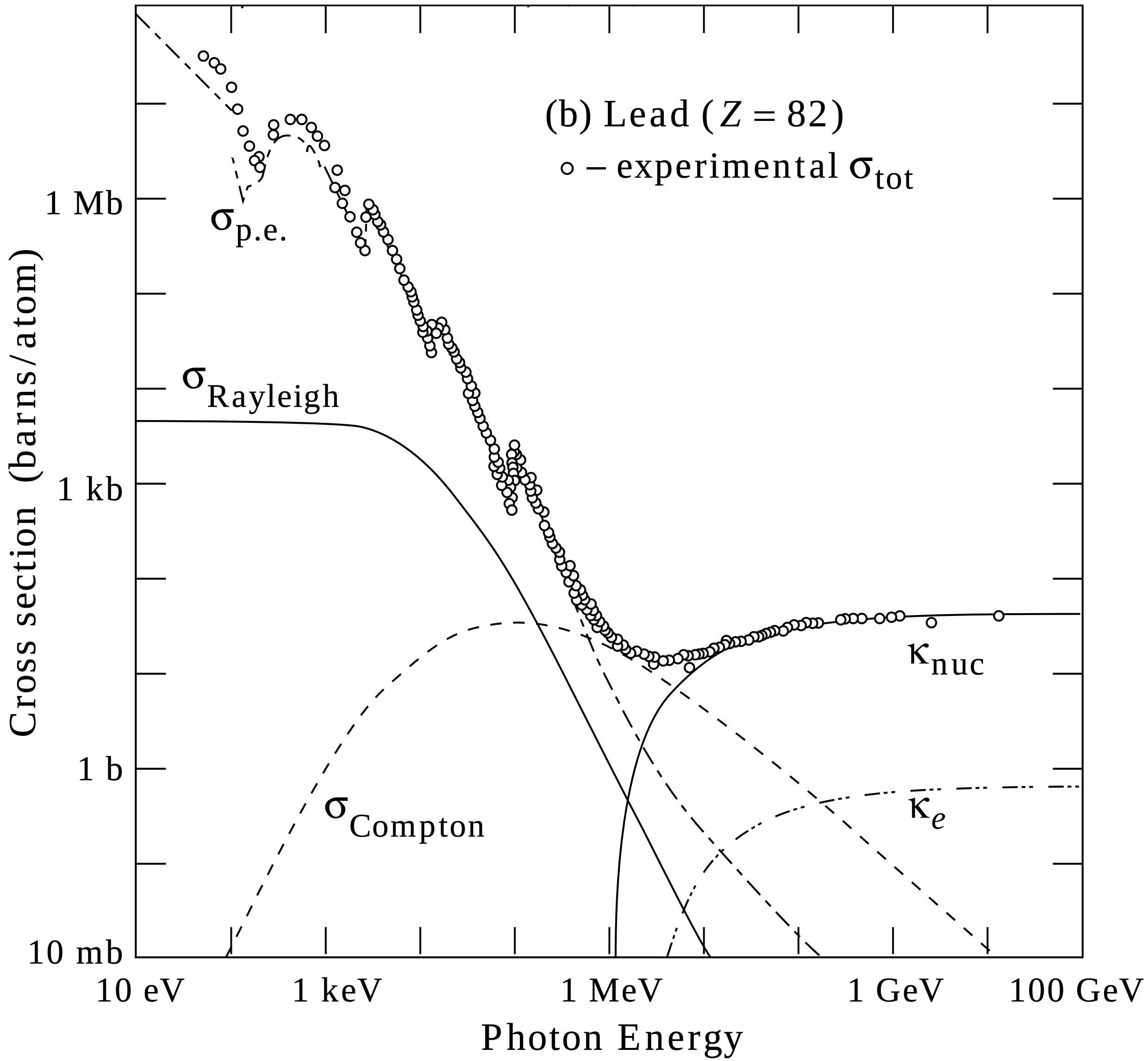
High energy electrons (positrons) also have a finite, non-zero, mean free path in matter. The radiated photon spectrum is $\propto 1/E$. Integration gives the electron energy loss which is proportional to the energy or

$$-\frac{1}{E}\frac{\mathrm{d}E}{\mathrm{d}x} \sim \text{const}$$

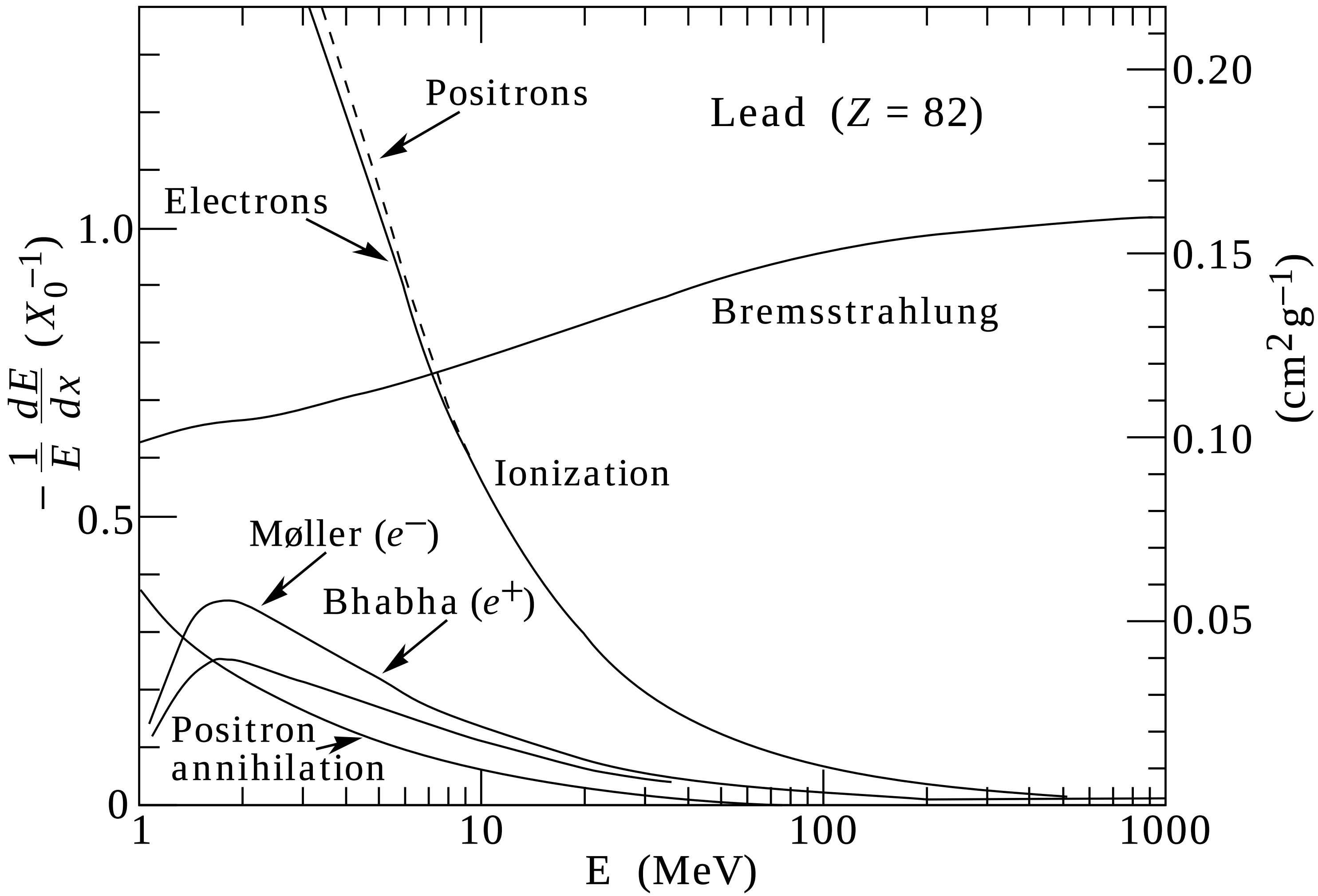
The radiation length X_0 is defined as the mean distance over which a high energy electron looses in average a fraction 1-e of its energy. $(7/9)X_0$ is also the mean free path for pair creation of high energy electrons.

X_0 for various material

Material	Z	X_0 g/cm^2	X_0 cm
H_2 liq.	1	61.3	866
He	2		4500 m
C	6	42.7	18.8
Fe	26	13.8	1.76
Pb	82	6.4	0.56
U	92	6.0	0.32
BGO		7.97	1.12
NaI		9.49	2.59



Total cross section for γ in lead



Fractional energy loss per radiation length, for electrons (positrons) in lead

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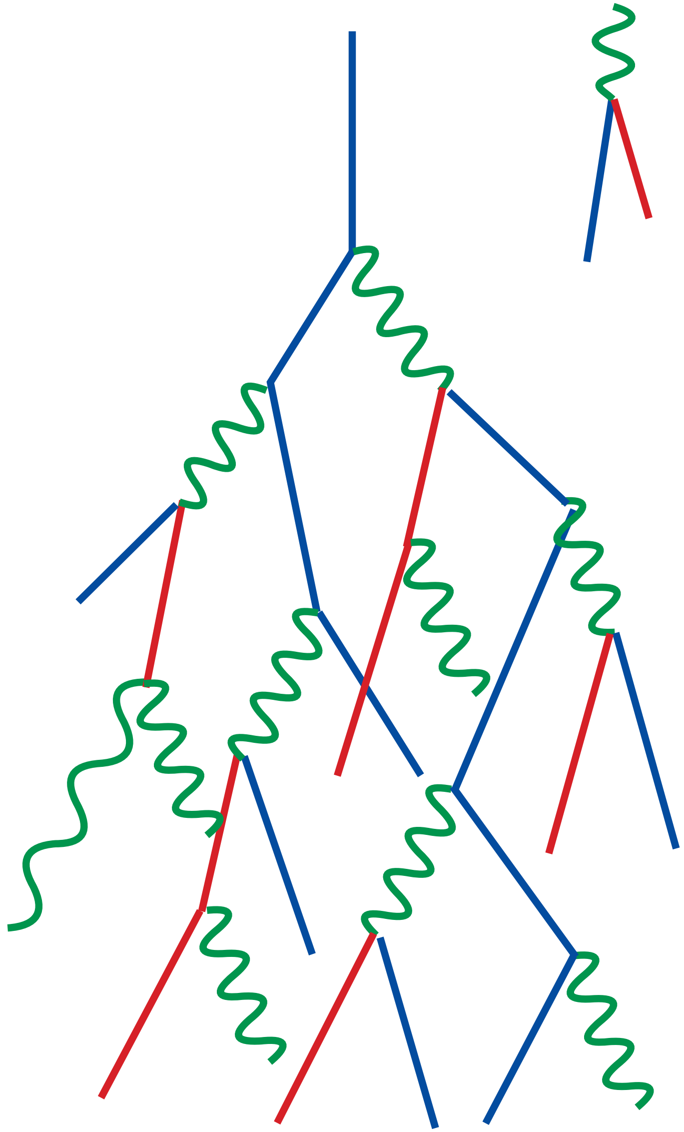
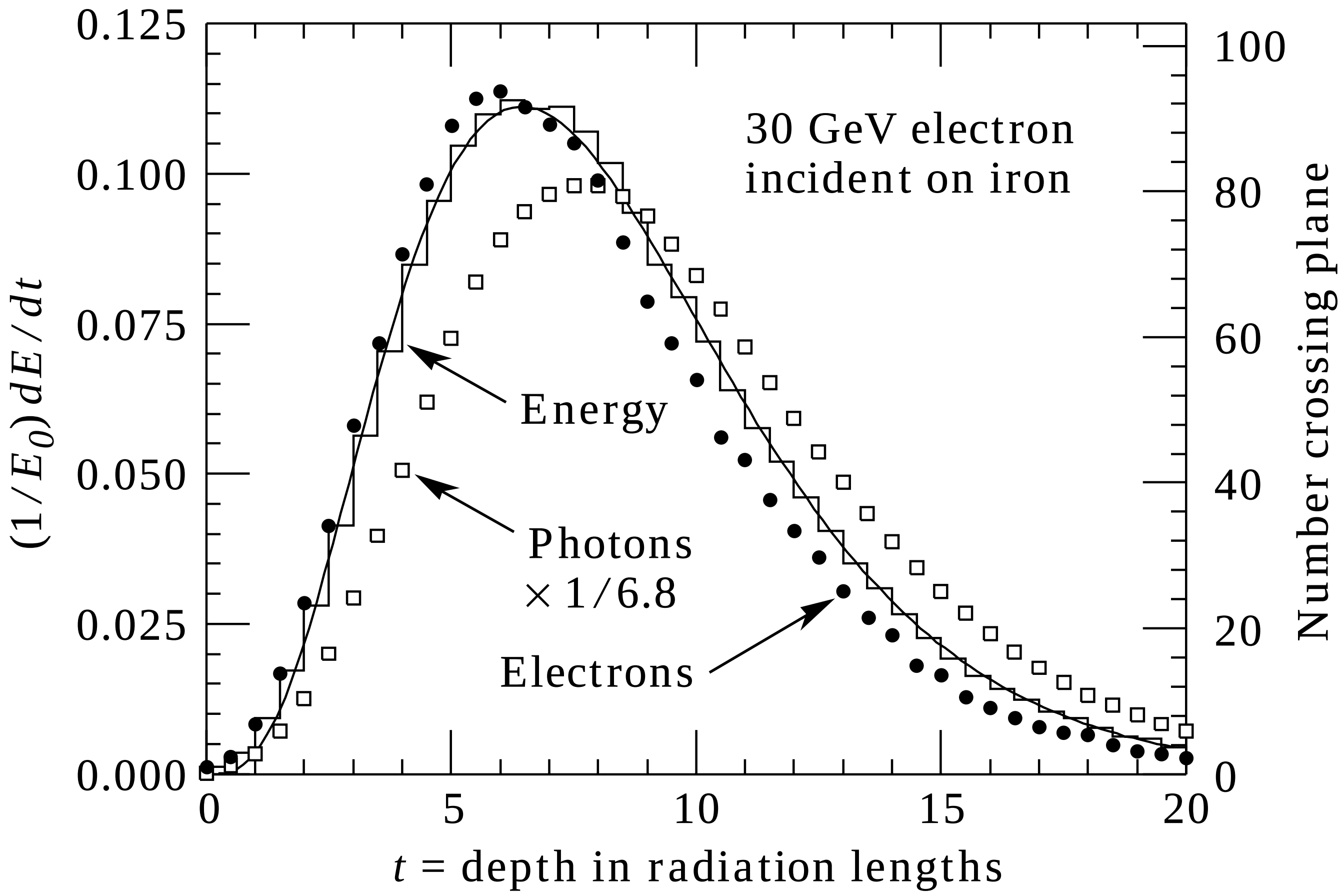
Electromagnetic cascade

At high energy both radiation and pair-production will happen repeatedly in cascade. It makes little difference whether it all started with an electron or a photon. As the cascade develops the number of particles and photons increases. Charged particles loose energy to the medium, the beginning a signal of the shower. At some point the energy of photon and electrons becomes low enough that radiation and pair production stop.



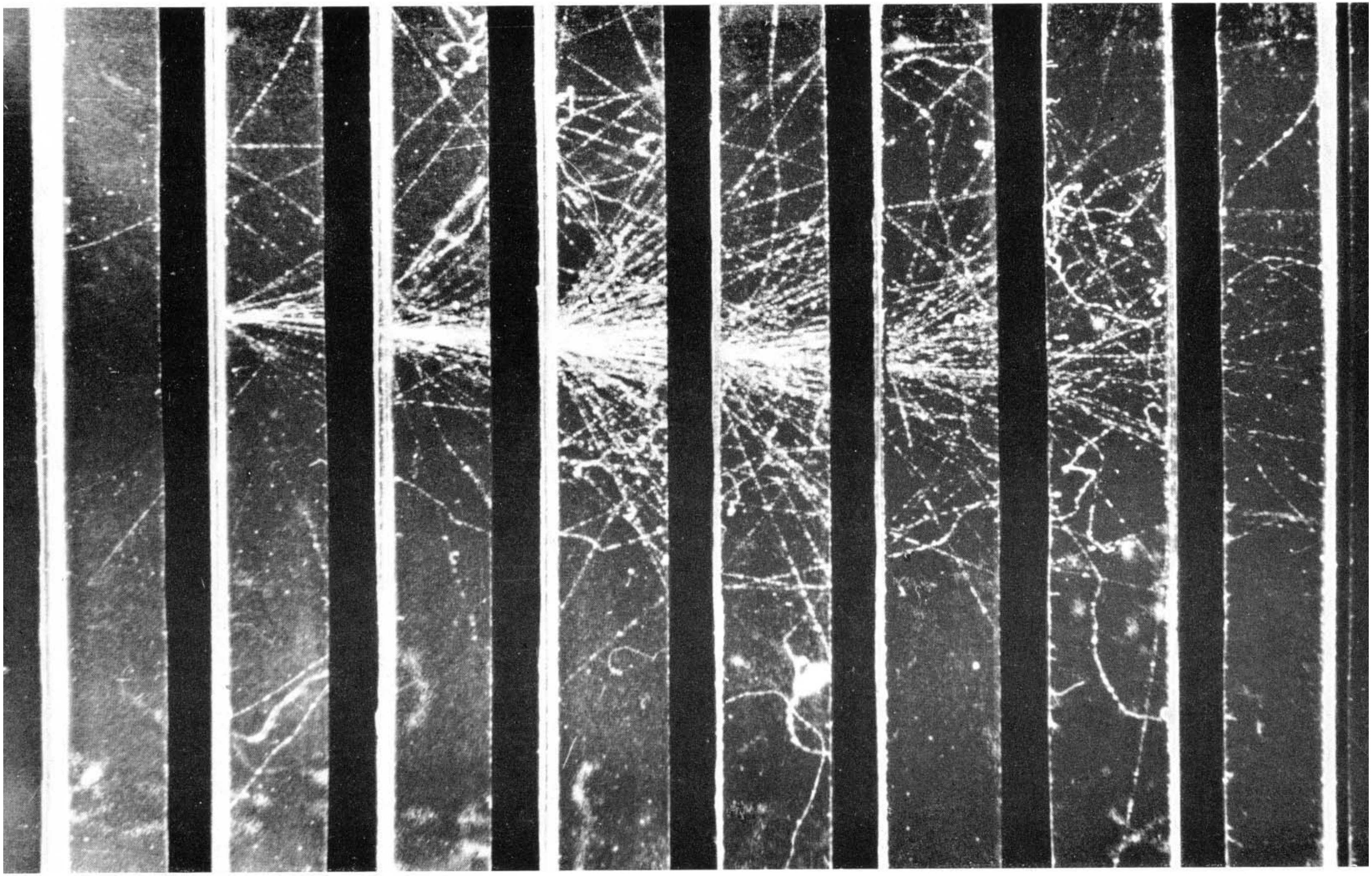
EM cascade



Shower profiles for fractional energy loss and number of particles.

Detection of the number of particles or the energy measures the photon (or electron or positron) energy.

EM shower developing in the plates in a Wilson Chamber.



4. The muon and the pion

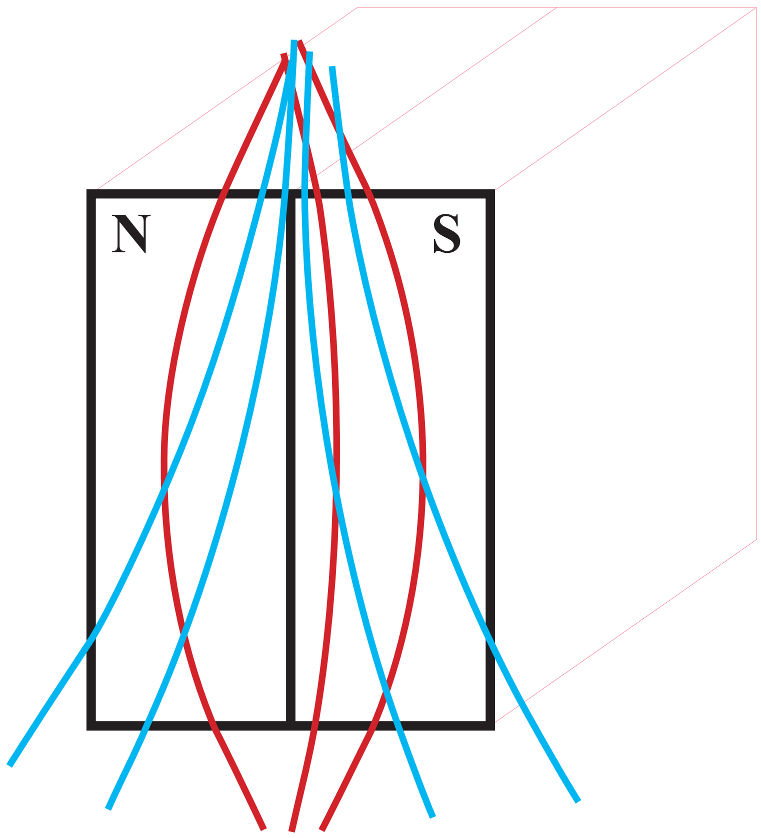
In 1937 Anderson and Neddermeyer proved the existence of a particle of mass $\gg m_e$, it did not radiate in a 1 cm platinum plate. Street and Stevenson measured $130 \times m_e$. It could have been Yukawa's mesotron, expected mass $\sim 1/r_p \sim 200$ MeV (m_μ=105 MeV). The mass is now 105.658357 MeV.

In 1947 Conversi, Pancini and Piccioni observed that the life-time of negative muons stopped in carbon equals that of positive muons. A definitive proof that the muon is not Yukawa's mesotron.

During the last war years the Rossi group had developed a method to select positive and negative muons in the cosmic radiation at sea level. This was done with the so-called magnetic lenses.

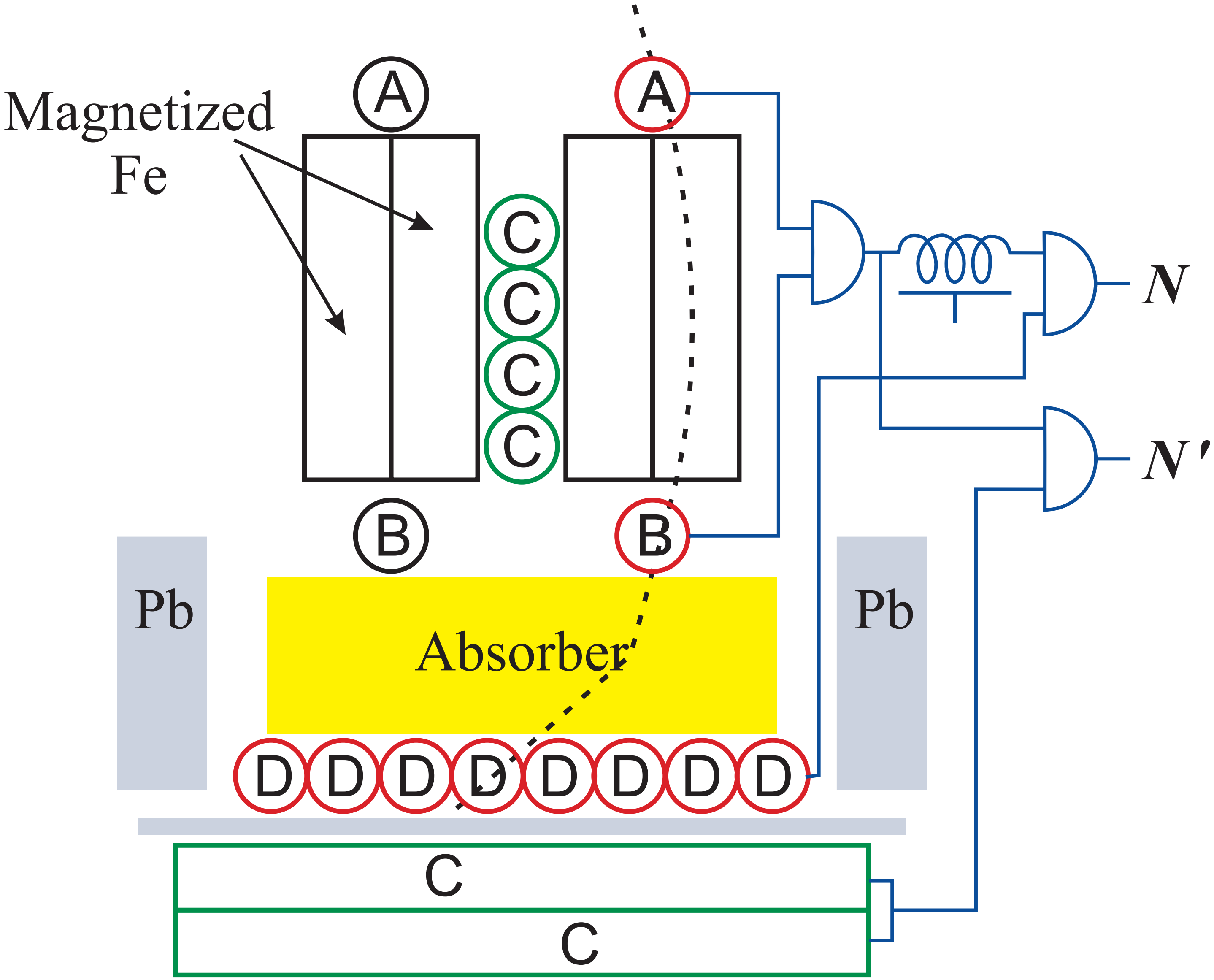
The magnetic lens

Two Fe blocks, magnetized with a current in opposite directions, have a focussing effect on particles of one sign and defocussing on the opposite sign.



The Conversi Pancini Piccioni experiment

A late signal from the D counters in coincidence with the A and B counters delayed, signals the arrival of a positive (negative) muon which stops in the absorber and then decays into an electron.



So what?

The muon lifetime in vacuum is 2.2 μs, measured since the early days. This immediately tells us that the decay is due to the weak interaction.

The Yukawa mesotrons have however strong interaction with nucleons. This means that a mesotron interacts with a nucleon with a reaction rate of $\sim 3 \times 10^{23}$ per second (s^{-1}).

When muons come at rest in matter, ($<$10 ps), positive muons can only decay as in vacuum while negative muons will bind in hydrogen-like structures. In condensed matter, thermal muons bind into S-wave orbits also in very short times. Mostly by Stark effect.

For muons in a 1S states we estimate the overlap integral as just the ratio of the nucleus radius to the Bohr radius, cubed.

If you do not remember the Bohr radius, begin writing the energy:

$$E = V + T = -\frac{\alpha}{r} + p^2/2m_e.$$

Quantize the system by the uncertainty principle, $p = 1/r$ ($\hbar$=c=1) and find the minimum energy:

$$\frac{\mathrm{d}}{\mathrm{d}p}(-\alpha p + p^2/2m_e) = -\alpha + p/m_e = 0$$

giving $p = m_e\alpha$ from which the Bohr radius $a_\infty = 1/(m_e\alpha)$, in natural units.

Reintroducing $\hbar$ and c we find a (maybe??) more familiar expression $a_\infty = 4\pi\epsilon_0\hbar^2/m_e e^2$ in SI units, $a_\infty = \hbar^2/m_e e^2$ in cgs units. $a_\infty \sim 5 \times 10^{-8}$ cm.

In natural units the Rydberg constant is $\alpha^2 m_e/2$ or $R_\infty = 510\ 999$ eV$/(2 \times 137.036^2) = 13.605\ldots$eV.

The Bohr radius is $R_{\mathrm{Bhor}}{=}1/(m_r Z\alpha){=}2.8\times10^{-11}/Z$ cm and the overlap integral

$$\frac{A(1.4\times10^{-13})^3 Z^3}{(1.9\times10^{-11})^3}=1.2\times10^{-7}Z^3A.$$

Even for hydrogen the absorption rate for Yukawa mesons is

$$\Gamma_{\text{abs., Yukawa}}=3\times10^{23}\times1.2\times10^{-7}\sim3\times10^{16}\ \mathrm{s}^{-1}$$

or 10^8 times larger then the decay rate.

Very grossly, taking the reaction rate for $\mu^-+N\to N'+\nu_e$ as $1/\tau_\mu{\sim}10^6$/s, the weak μ^- absorbtion rate is

$$\Gamma_{\text{weak}}\sim0.12\times Z^3A\sim0.24\times Z^4\ \mathrm{s}^{-1}.$$

For carbon, $Z=6$ and $\Gamma_{\text{weak}}{\sim}300$, which is much smaller than the decay rate of the muon. Positive and negative muons equally decay when stopped in carbon.

However, for $Z = (10^6/.24)^{1/4}{\sim}45$, the weak capture rate becomes significant. Conversi et al. did observe that for negative muons stopping in Fe, Z=26, the free decay is almost not present.

The estimates above are of course very crude. The Z^3A dependence of the capture rate has been however verified (later) by measuring $\tau_\mu = 1/(\Gamma(dec) + \Gamma(cap))$ in different materials.

From the observation of Conversi *et al.* the muon was born. The names π, μ are due to the Bristol nuclear emulsion group of Lattes, Occhialini and Powell. The muon is a basic constituent of the so called standard model and as best as we can tell an elementary particle. The π-meson is the Yukawa mesotron.

The pion

Yukawa's mesotrons was found soon after, observing the full decay chain $\pi \to \mu \to e$. In fact the cosmic ray muons that reach the earth surface all come from the reaction steps:

$$p + (A, Z) \to \pi^{\pm} + \ldots, \quad \pi \to \mu\nu.$$

Pions and muons are not present in the primary radiation because of their short lifetimes.

Pions hardly reach sea level because of their short lifetime and large absorption. They are found at high altitude or in interactions.

Negative pions stopped in H_2 "NEVER" decay, as expected.

Reflections

Establishing the existence of the muon, as was done in the '47 experiment, is one of the milestones of today's physics. Leptons, the electron discovered in 1897, the muon and a few (too many) other particles are what we like today to call elementary particles.

The pion today has lost its status. It is just another of the few hundred states made up of a quark and an antiquark. It is not an elementary particle.

Conversi was my teacher in the early fifties and I have always admired his passion for physics.