

Particle Physics - V

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5. CKM

5.1 The Mixing Matrix

The Standard Model has a *natural* place for CP violation (Cabibbo, Kobayashi and Maskawa).

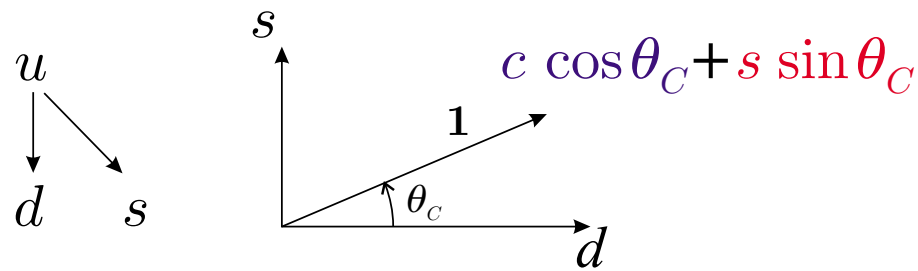
In fact, it is the discovery of CP violation which inspired KM⁽¹⁾ to expand the original Cabibbo⁽²⁾ -GIM⁽³⁾ 2×2 quark mixing matrix, to a 3×3 one, which allows for a phase and therefore for CP violation. This also implied an additional generation of quarks, now known as the b and t , matching the τ in the SM.

According to KM the six quarks charged current is:

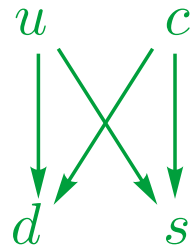
$$J_{\mu}^{\dagger} = (\bar{u} \ \bar{c} \ \bar{t}) \gamma_{\mu} (1 - \gamma_5) \mathbf{M} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

where \mathbf{M} is a 3×3 unitary matrix: $\mathbf{M}^{\dagger} \mathbf{M} = 1$.

Since the relative phases of the 6 quarks are arbitrary, \mathbf{M} contains 3 real parameter, the Euler angles, plus a phase factor, allowing for \mathcal{CP} .



$\Gamma(s \rightarrow d) \propto \sin^2 \theta_C$ - Strange part. decays



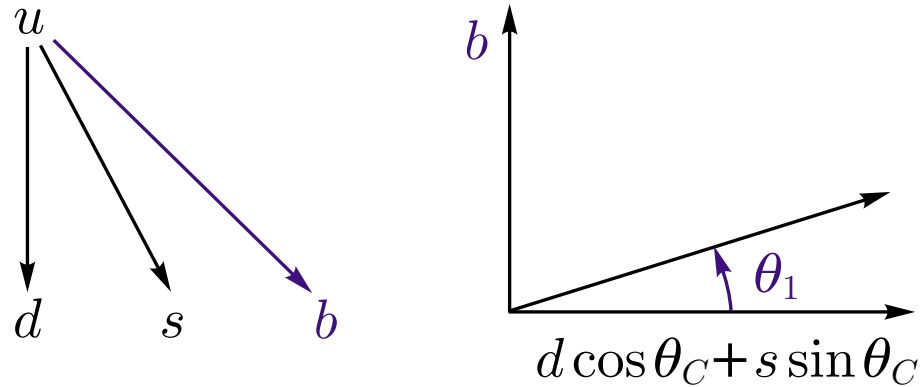
GIM, neutral currents,
2 by 2 unitary matrix,
calculable loops

$$\mathbf{V}_{\text{C/GIM}} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix}$$

$$J_\mu^+(udcs) = \bar{u}(\cos \theta_C d + \sin \theta_C s) + \bar{c}(-\sin \theta_C d + \cos \theta_C s)$$

$$J_\mu^0 = \bar{d}d + \bar{s}s \quad - \quad \text{No FCNC: } K^0 \rightarrow \mu\mu \text{ suppression.}$$

Three down like quarks require one more mixing angle:



and the u, dsb current is now given by

$$J_{\mu}^{\dagger}(u, dsb) = \bar{u}[\dots]_{\mu}(\cos \theta_C \cos \theta_1 d + \sin \theta_C \cos \theta_1 s + \sin \theta_1 b)$$

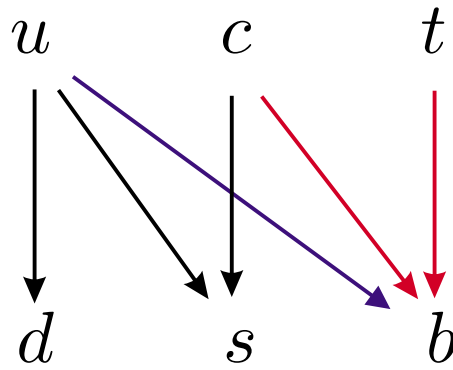
If the c quarks and the t quarks are included, one more angle is necessary to account for $c \rightarrow b$ transitions.

These geometric illustrations are justified by counting parameters in an $n \times n$ unitary matrix. $2n^2$ real numbers define a complex matrix, of which n^2 are removed requiring unitarity. $2n - 1$ phases are unobservable and can be reassorbed in the definition of $2n - 1$ quark fields. In total we are left with $(n - 1)^2$ parameters. In n dimensions there are $n(n - 1)/2$ orthogonal rotation angles since there are

$$n - 1 + n - 2 + \dots + 1 = n(n - 1)/2$$

planes.

Thus the $n \times n$ unitary matrix contains $n(n - 1)/2$ rotations and $(n - 1)(n - 2)/2$ phases. For $n = 3$ we have three angles and one phase.



$$\Gamma(b \rightarrow c) \propto \sin^2 \theta_2$$

$$B \rightarrow D, D^* \dots + \dots$$

$$V_{\text{CKM}} = \begin{pmatrix} \cos \theta_c \cos \theta_1 & \sin \theta_c \cos \theta_1 & \sin \theta_1 \\ & \sin \theta_2 \cos \theta_1 & \\ & \cos \theta_2 \cos \theta_1 & \end{pmatrix}$$

The complete form of the matrix, in the Maiani notation, is:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & c_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

with $c_{12} = \cos \theta_{12} = \cos \theta_C$, etc.

While a phase can be introduced in the unitary matrix \mathbf{V} which mixes the quarks

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$

the theory does not predict the magnitude of the effect.

The constraint that the mixing matrix be unitary corresponds to the desire of having a universal weak interaction.

Our present knowledge of the magnitude of the V_{ij} elements is

given below.

$$\begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.047 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - .9993 \end{pmatrix}$$

The diagonal elements are close to, but definitely are not equal to unity. If such were the case there could be no CP violation.

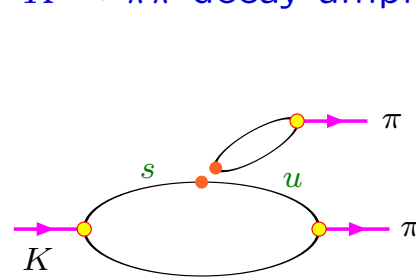
However, if the violation of CP which results in $\epsilon \neq 0$ is explained in this way then, in general, we expect $\epsilon' \neq 0$.

For *technical* reasons, it is difficult to compute the value of ϵ' . Predictions are $\epsilon'/\epsilon \leq 10^{-3}$, but cancellations can occur, depending on the value of the top mass and the values of appropriate matrix elements, mostly connected with understanding the light

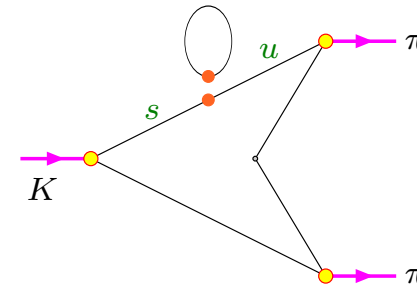
hadron structure. I quote from LP01 on lattice QCD's progress.

Prospects for Lattice Calculations $K \rightarrow \pi\pi$ Decays are exciting!

- A number of different mechanisms contribute to $K \rightarrow \pi\pi$ decay amplitudes, e.g.



Disconnected Emission



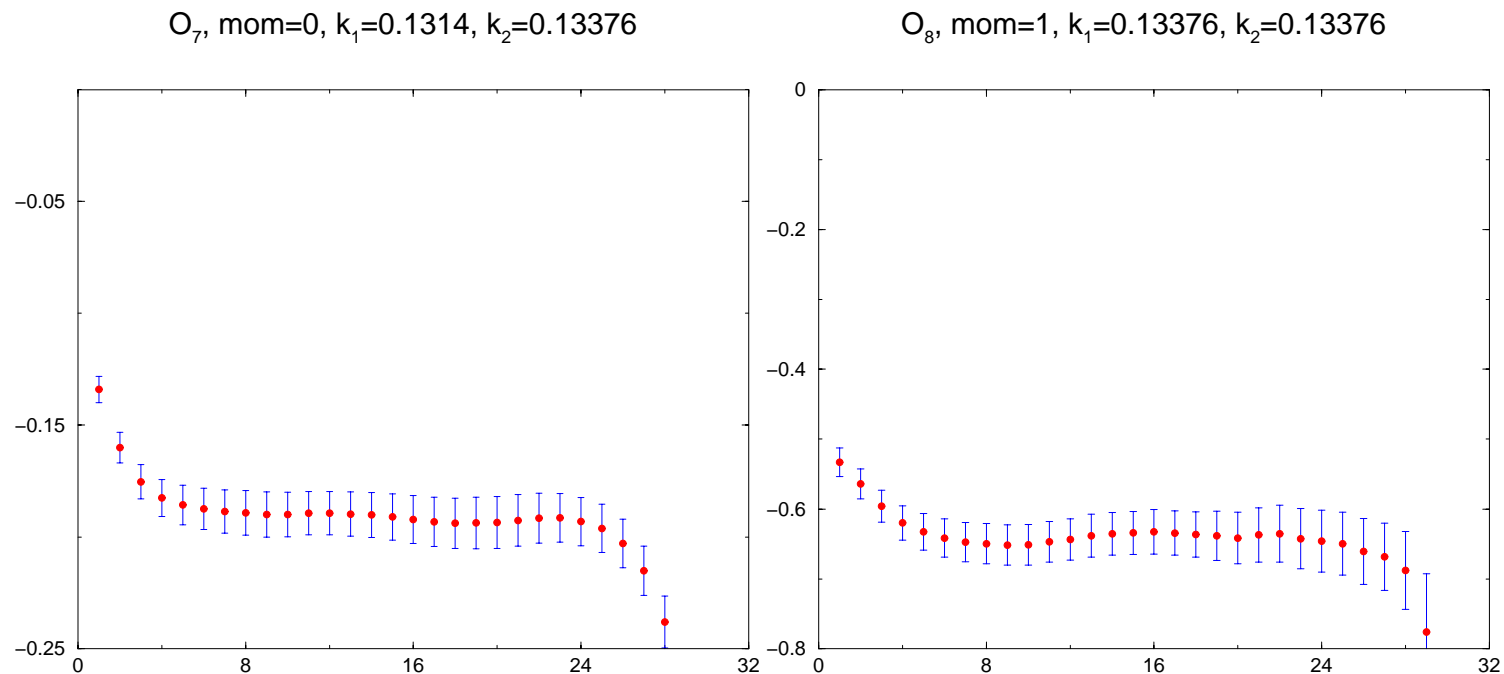
Disconnected Penguin

- Lattice calculations have shown that it is not possible to explain the $\Delta I=1/2$ rule with emission diagrams only.
- In order to obtain the physical contribution from the penguin diagrams in general we have to subtract large unphysical terms (power divergences). This is the reason for the absence up to now, of sufficiently precise results for $\Delta I = 1/2$ decays.
- There is a signal for $\Delta I = 1/2$ matrix elements
⇒ data which we can analyse
⇒ studies of $\Delta I = 1/2$ rule and evaluation of ϵ'/ϵ .

fig

$K \rightarrow \pi\pi$ Decays - Cont:

- Calculations of amplitudes for $\Delta I = 3/2$ transitions are relatively straightforward compared to $\Delta I = 1/2$ ones and the signals are very strong. We are currently undertaking a detailed study, up to NLO in the chiral expansion, for these matrix elements (Q_4 , EW-penguins $Q_{7,8}$).



$\Delta I=3/2$ matrix elements of the electroweak penguin operators O_7 (left) and O_8 (right).

PRELIMINARY!

SPQ_{CD}R Collaboration

$K \rightarrow \pi\pi$ Decays - Cont:

- The matrix element of Q_8 is important in the evaluation of ε'/ε . Lattice results (from $K \rightarrow \pi$ matrix elements + soft-pion theorems and χ PT at leading order) are significantly smaller than other determinations, e.g. a lattice simulation gives

$$|_{I=2}\langle \pi\pi | Q_8 | K^0 \rangle| = (0.5 \pm 0.01) \text{ GeV}^3$$

in the NDR renormalization scheme at $\mu = 2 \text{ GeV}$.

A.Donini, V.Giménez, L.Giusti and G.Martinelli 1999

This can be compared to:

$1.3 \pm 0.3 \text{ GeV}^3$ (Donoghue and Golowich) and

$3.5 \pm 1.1 \text{ GeV}^3$ (Knecht, Peris and de Rafael).

Large matrix element of $O_8 \Rightarrow$ very large matrix element of Q_6 in order to explain the measured value of ε'/ε .

- Interesting studies using Domain Wall Fermions also underway (RBC, CP-PACS). $K \rightarrow \pi\pi$ amplitudes can be determined from $K \rightarrow \pi$ matrix elements if the chiral symmetry is sufficiently precise.
- Preliminary results indicate that it is possible to obtain a $\Delta I = 3/2$ $K \rightarrow \pi\pi$ decay amplitude which is consistent with experiment and yet with a large B_K (as indicated above).

Much exciting work do be done during the coming year or two!!!

. A fundamental task of experimental physics today is the determination of the four parameters of the CKM mixing matrix, including the phase which results in \mathcal{CP} . A knowledge of all parameters is required to confront experiments. Rather, many experiments are necessary to complete our knowledge of the parameters and prove the uniqueness of the model or maybe finally break beyond it.

5.2 Cabibbo's Angle

$$\Gamma(K \rightarrow \pi l \nu) \propto |V_{us}|^2$$

From PDG

	m MeV	Δ MeV	Γ 10^7 s^{-1}	BR($e3$)	$\Gamma(e3)$ 10^6 s^{-1}
K^\pm	493.677	358.190	8.07	0.0482	3.89
error	-	-	0.19%	1.24%	1.26%
K_L	497.672	357.592	1.93	0.3878	7.50
error	-	-	0.77%	0.72%	1.06%

The above rates for K_{e3} determine, in principle, $|V_{us}|^2$ to 0.8% and $|V_{us}|$ to 0.4%. Yet in PDG

$$|V_{us}| = 0.2196 \pm 1.05\%.$$

The problem is estimating-guessing matrix element corrections due to isospin and $SU(3)_{\text{flavor}}$ symmetry breaking.

Decay rates for $|i\rangle \rightarrow |f\rangle$ are obtained from the transition probability density $\bar{w}_{fi} = |T_{fi}|^2$ ($S = 1 + iT$):

$$\bar{w}_{fi} = (2\pi)^4 \delta^4(p_i - p_f) (2\pi)^4 \delta^4(0) |\mathfrak{M}|^2$$

where

$$\mathfrak{M} = \langle f | \mathcal{H} | i \rangle$$

from which

$$d\Gamma = \frac{1}{8M(2\pi)^3} |\mathfrak{M}|^2 dE_1 dE_2.$$

$\Gamma(\ell 3) \propto G_F^2 \times |V_{us}|^2$ but we must deal with a few details.

1. Numerical factors equivalent to an overlap integral between final and initial state. Symmetry breaking corrections, both isospin and $SU(3)_F$.

2. An integral over phase space of $|\mathfrak{M}|^2$.
3. Experiment dependent radiative corrections. Or, bad practice, correct the data.

$$\langle \pi^0 | J_\alpha^H | K^+ \rangle = \langle (u\bar{u} - d\bar{d})/\sqrt{2} | u\bar{u} \rangle = 1/\sqrt{2}$$

$$\langle \pi^- | J_\alpha^H | K^0 \rangle = \langle d\bar{u} | d\bar{u} \rangle = 1$$

$$\langle \pi^+ | J_\alpha^H | \bar{K}^0 \rangle = \langle \bar{d}u | \bar{d}u \rangle = 1$$

$$\langle \pi^+ | J_\alpha^H | K_L \rangle = -\langle \bar{d}u | \bar{d}u \rangle/\sqrt{2} = -1/\sqrt{2}$$

$$\langle \pi^- | J_\alpha^H | K_L \rangle = \langle \bar{d}u | \bar{d}u \rangle/\sqrt{2} = 1/\sqrt{2}$$

$$\langle \pi^+ | J_\alpha^H | K_S \rangle = \langle \bar{d}u | \bar{d}u \rangle/\sqrt{2} = 1/\sqrt{2}$$

$$\langle \pi^- | J_\alpha^H | K_S \rangle = \langle \bar{d}u | \bar{d}u \rangle/\sqrt{2} = 1/\sqrt{2} \quad (\times f_+(q^2)q_\alpha \langle J^L \rangle^\alpha \dots)$$

Ignoring phase space and form factor differences:

$$\begin{aligned}\Gamma(K_L \rightarrow \pi^\pm e^\mp \bar{\nu}(\nu)) &= \Gamma(K_S \rightarrow \pi^\pm e^\mp \bar{\nu}(\nu)) \\ &= 2\Gamma(K^\pm \rightarrow \pi^0 e^\pm \nu(\bar{\nu}))\end{aligned}$$

An approximate integration gives

$$\Gamma = \frac{G^2 |V_{us}|^2}{768\pi^3} |f_+(0)|^2 M_K^5 (0.57 + 0.004 + 0.14\delta\lambda_+)$$

with $\delta\lambda = \lambda - 0.0288$. Integration over phase space gives a leading term $\propto \Delta^5$, where $\Delta = M_K - \Sigma_f(m)$.

$$(\Delta_+^5 - \Delta_0^5) / \Delta^5 = 0.008.$$

From data, $\Gamma_0 = (7.5 \pm 0.08) \times 10^6$, $2\Gamma_+ = (7.78 \pm 0.1) \times 10^6$ and $(2\Gamma_+ - \Gamma_0) / \Sigma = (3.7 \pm 1.5)\%$.

This is quite a big difference, **though only 2σ** , but typical of **violation of I-spin invariance**.

The slope difference is ~ 0.001 , quite irrelevant. The big problem remains the $s - u, d$ mass difference. For K^0 the symmetry

breaking is $\propto (m_s - \langle m_{u,d} \rangle)^2$ in accordance with A-G. But then $(m_s - \langle m_{u,d} \rangle)^2$ acquires dangerous divergences, from a small mass in the denominator. It is argued that it is not a real problem.

Leutwyler and Roos (1985) deal with all these points and radiative corrections. They are quoted by PDG (Gilman *et al.*, 2000), for the value of $|V_{us}|$. After isospin violation corrections, K^0 and K^+ values agree to 1%, experimental errors being 0.5%, 0.6%.

Reducing the errors on Γ_+ and Γ_0 , coming soon, will help understand whether we can properly compute the corrections.

5.3 Wolfenstein parametrization

Nature seems to have chosen a special set of values for the elements of the mixing matrix: $|V_{ud}| \sim 1$, $|V_{us}| = \lambda$, $|V_{cb}| \sim \lambda^2$ and $|V_{ub}| \sim \lambda^3$.

On this basis Wolfenstein found it convenient to parameterize the mixing matrix in a way which reflects more immediately our present knowledge of the value of some of the elements and has the CP violating phase appearing in only two off-diagonal elements, to lowest order in $\lambda = \sin \theta_{\text{Cabibb}}$ a real number describing mixing of s and d quarks.

The Wolfenstein⁽⁴⁾ *approximate* parameterization up λ^3 terms is:

$$\mathbf{V} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

A , also real, is close to one, $A \sim 0.84 \pm 0.06$ and $|\rho - i\eta| \sim 0.3$.

CP violation requires $\eta \neq 0$. η and ρ are not very well known. Likewise there is no CP if the diagonal elements are unity.

The Wolfenstein matrix is not exactly unitary: $V^\dagger V = 1 + \mathcal{O}(\lambda^4)$. The phases of the elements of \mathbf{V} to $\mathcal{O}(\lambda^2)$ are:

$$\begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix}$$

which defines the angles β and γ .

Several constraints on η and ρ can be obtained from measurements. ϵ can be calculated from the $\Delta S=2$ amplitude of fig. 19, the so called box diagram.

At the quark level the calculations is straightforward, but complications arise in estimating the matrix element between K^0 and \bar{K}^0 .

Apart from this uncertainties ϵ depends on η and ρ as $|\epsilon| = a\eta + b\eta\rho$ a hyperbola in the η, ρ plane as shown later in figure 23.

The calculation of ϵ' is more complicated. There are three $\Delta S=1$ amplitudes that contribute to $K \rightarrow \pi\pi$ decays, given below to lowest order in λ for both the real and imaginary parts.

They correpondig to a u , c and t quark in the loop and are illus-

trated in fig. 19, just look above the dashed line.

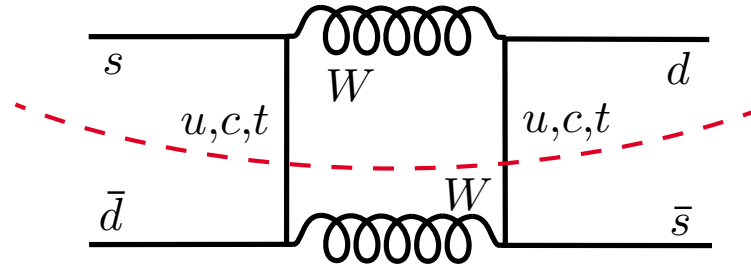


Fig. 19. Box diagram for $K^0 \rightarrow \bar{K}^0$

$$A(s \rightarrow u\bar{u}d) \propto V_{us}V_{ud}^* \sim \lambda \quad (1)$$

$$A(s \rightarrow c\bar{c}d) \propto V_{cs}V_{cd}^* \sim -\lambda + i\eta A^2 \lambda^5 \quad (2)$$

$$A(s \rightarrow t\bar{t}d) \propto V_{ts}V_{td}^* \sim -A^2 \lambda^5 (1 - \rho + i\eta) \quad (3)$$

where the amplitude (1) correspond to the simplest way for computing $K \rightarrow \pi\pi \rightarrow \bar{K}^0$ (it diverges by itself but cancels with(2)) in

the standard model and the amplitudes (2), (3) account for direct CP .

If the latter amplitudes were zero there would be no direct CP violation in the standard model. The flavor changing neutral current (FCNC) diagram of fig. 20 called the penguin diagram, contributes to the amplitudes (2), (3).

Estimates of $\Re(\epsilon'/\epsilon)$ range from $\text{few} \times 10^{-3}$ to 10^{-4} .

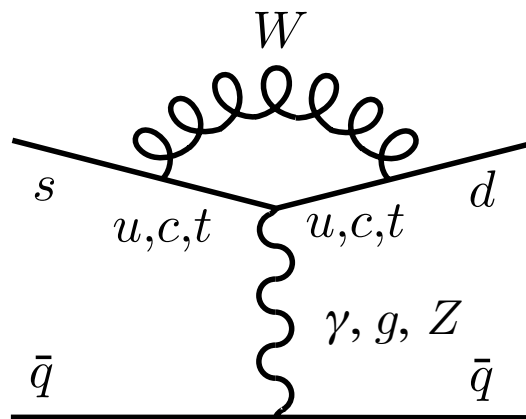


Fig. 20. Penguin diagram, a flavor changing neutral current effective operator

5.4 Unitary triangles

We have been practically inundated lately by very graphical presentations of the fact that the CKM matrix is unitary, ensuring the renormalizability of the $SU(2) \otimes U(1)$ electroweak theory.

The unitarity condition

$$V^\dagger V = 1$$

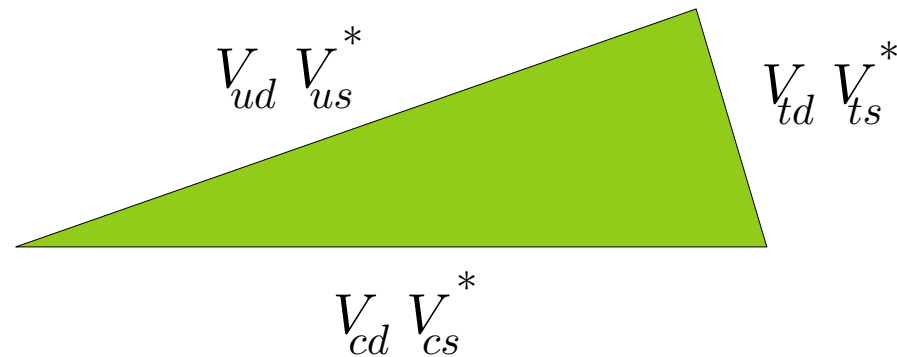
contains the relations

$$\sum_i V_{ij}^* V_{ik} = \sum_i V_{ji}^* V_{ki} = \delta_{jk}$$

which means that if we take the products, term by term of any one column (row) element with the complex conjugate of another (different) column (row) element their sum is equal to 0.

Geometrically it means the three terms are sides of a triangle.
Two examples are shown below.

1, 2 triangle



1, 3 triangle

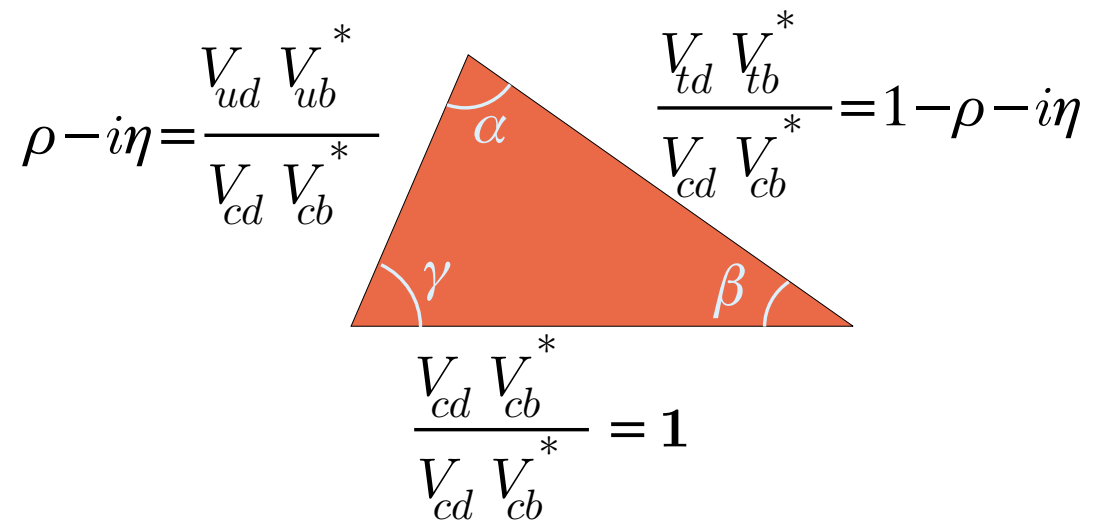


Fig. 21. The (1,2) and (1,3) Unitarity triangles

The second one has the term $V_{cd}V_{cb}^*$ pulled out, and many of you will recognize it as a common figure used when discussing measuring CP violation in the B system.

Cecilia Jarlskog in 1984 observed that any direct CP violation is proportional to twice the area which she named J (for Jarlskog ?) of these unitary triangles, whose areas are of course equal, independently of which rows/columns one used to form them.

In terms of the Wolfenstein parameters,

$$J \simeq A^2 \lambda^6 \eta$$

which according to present knowledge is $(2.7 \pm 1.1) \times 10^{-5}$, very small indeed! This number has been called the price of CP . Its smallness explains why the ϵ' experiments are so hard to do, and also why B factories have to be built in order to study CP violation

in the B system, despite the large value of the angles in the B unitary triangle.

An illustration of why CP effects are so small in kaon decays is given in fig. 22. The smallness of the height of the kaon triangle wrt two of its sides is the reason for CP there being a 10^{-6} effect. The B triangle has all its sides small and the CP effects are relatively large.

Measuring the various J 's to high precision, to check for deviations amongst them, is a very sensitive way to probe for new physics! Small perturbations due to phenomena beyond presently understood physics stand a better chance to disturb strongly suppressed effects in the standard model.

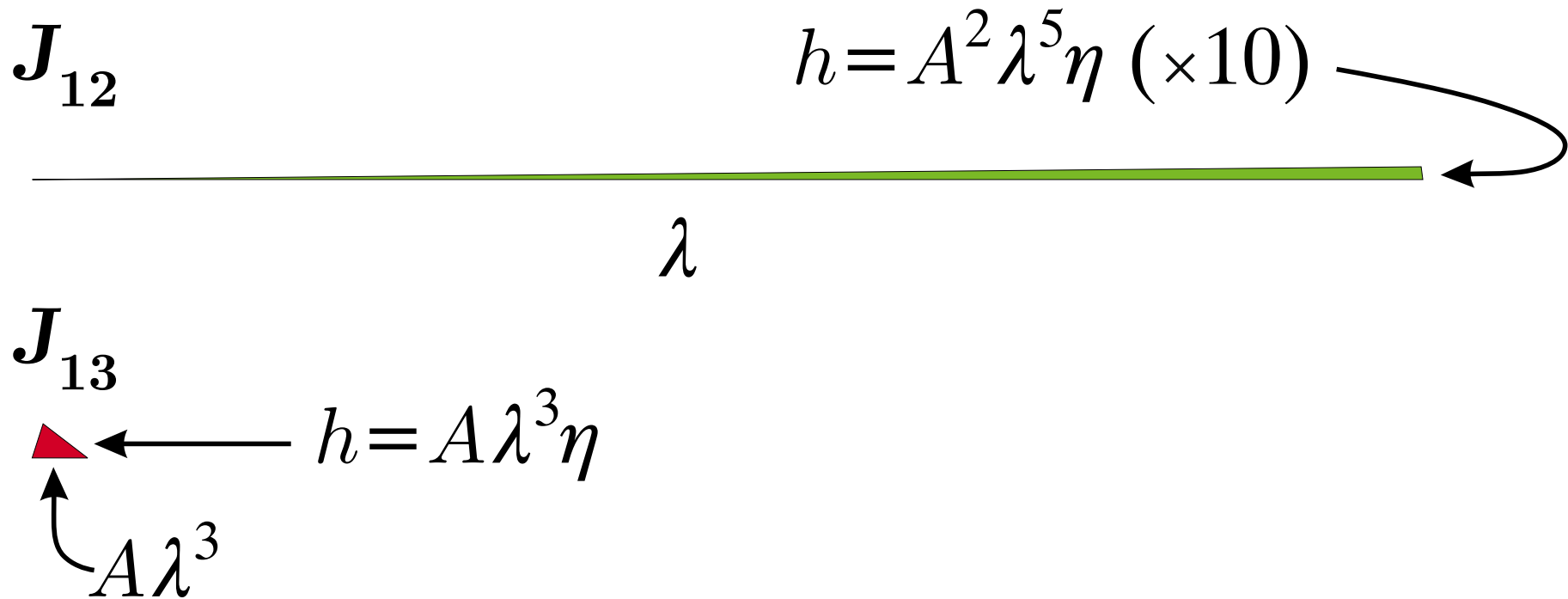


Fig. 22. The B and K Unitarity triangles

5.5 Rare K Decays

Rare K decays offer several interesting possibilities, which could ultimately open a window beyond the standard model.

The connection with ρ and η is shown in fig. 23.

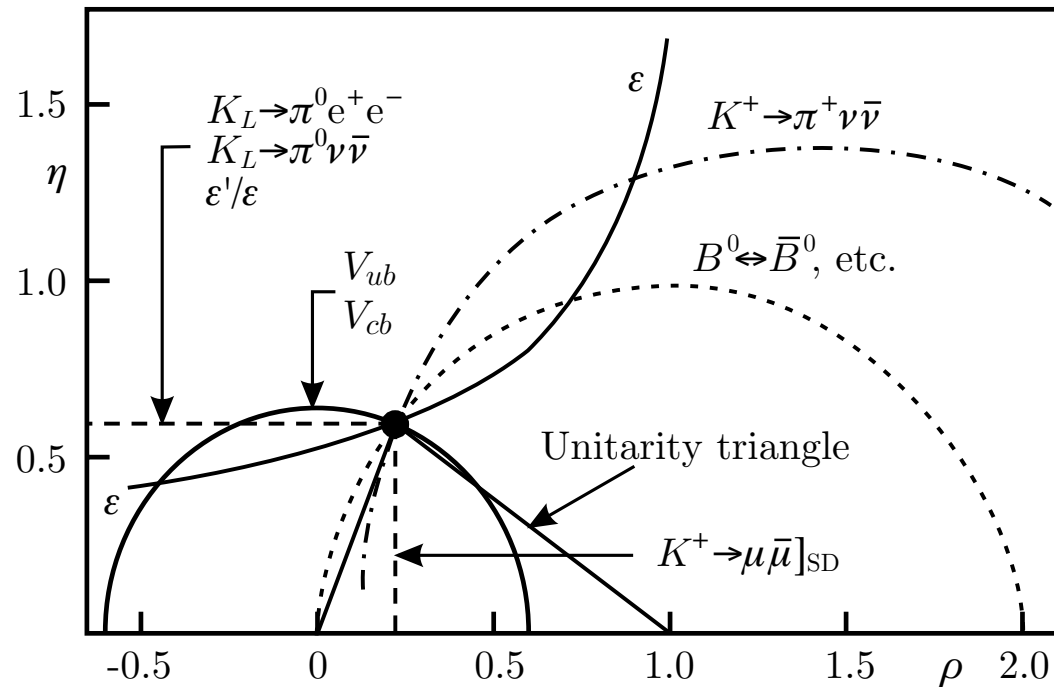


Fig. 23. Constraints on η and ρ from measurements of ϵ , ϵ' , rare decays and B meson properties.

Rare decays also permit the verification of conservation laws which are not strictly required in the standard model, for instance by searching for $K^0 \rightarrow \mu e$ decays.

The connection between ϵ' and η is particularly unsatisfactory because of the uncertainties in the calculation of the hadronic matrix elements. This is not the case for some rare decays.

A classification of measurable quantities according to increasing uncertainties in the calculation of the hadronic matrix elements has been given by Buras.⁽⁵⁾

1. $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$,
2. $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$,
3. $\text{BR}(K_L \rightarrow \pi^0 e^+ e^-)$, ϵ_K ,
4. ϵ'_K , $\text{BR}(K_L \rightarrow \mu \bar{\mu}]_{\text{SD}})$, where SD = *short distance* contributions.

The observation $\epsilon' \neq 0$ remains a unique proof of direct CP. Measurements of 1 through 3, plus present knowledge, over determine

the CKM matrix.

Rare K decay experiments are not easy. Typical expectations for some BR's are:

$$BR(K_L \rightarrow \pi^0 e^+ e^-, \text{dir}) \sim (5 \pm 2) \times 10^{-12}$$

$$BR(K_L \rightarrow \pi^0 \nu \bar{\nu}) \sim (3 \pm 1.2) \times 10^{-11}$$

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \sim (1 \pm .4) \times 10^{-10}$$

Note that the uncertainties above reflect our ignorance of the mixing matrix parameters, not uncertainties on the hadronic matrix element which essentially can be “taken” from $K_{\ell 3}$ decays.

The most extensive program in this field has been ongoing for a long time at BNL and large statistics have been collected recently and are under analysis.

Sensitivities of $\sim 10^{-10}$ are attainable now, however $10^{-(12 \text{ or } 13)}$ is

really necessary. Experiments with high energy kaon beams have been making excellent progress toward observing rare decays.

5.6 Search for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

This decay, CP allowed, is best for determining V_{td} . At present after analyzing half of their data, E781-BNL obtains BR is about 2.4×10^{-10} . This estimate is based on ONE event which surfaced in 1995 from about 2.55×10^{12} stopped kaons. The SM expectation is about half that value. Some 100 such decays are enough for a first V_{td} measurements. **Another event was found!**

5.7 $K_L \rightarrow \pi^0 \nu \bar{\nu}$

This process is a “pure”, direct CP signal. The $\nu \bar{\nu}$ pair is an eigenstate of CP with eigenvalue $+1$. Thus CP is manifestly

violated.

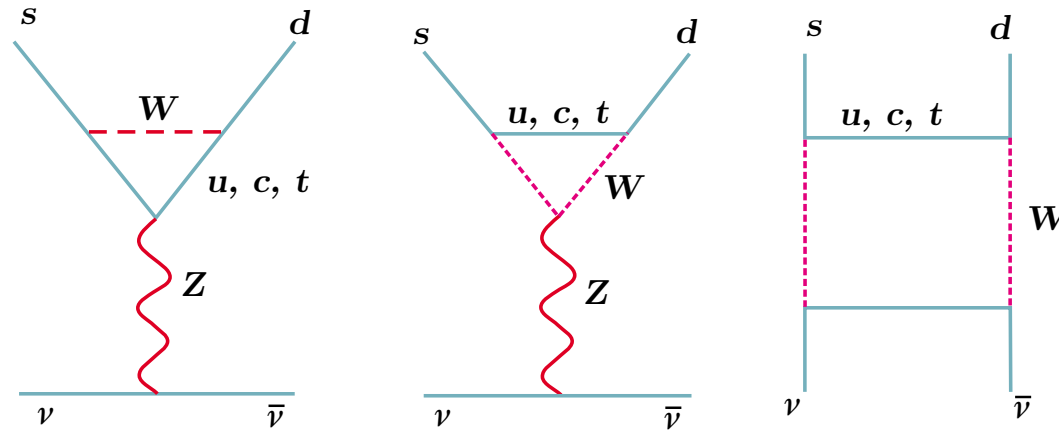


Fig. 24. Feynman Diagrams for $K_L \rightarrow \pi^0 \nu \bar{\nu}$

It is theoretically particularly “pristine”, with only about 1-2% uncertainty, since the *hadronic* matrix element need not be calculated, but is directly obtained from the measured $K_{\ell 3}$ decays.

Geometrically we see it as being the altitude of the J_{12} triangle.

$$J_{12} = \lambda(1 - \lambda^2/2)\Im(V_{td}V_{ts}^*) \approx 5.6[B(K_L \rightarrow \pi^0 \nu \bar{\nu})]^{1/2}$$

The experimental signature is just a single unbalanced π^0 in a hermetic detector. The difficulty of the experiment is seen in the present experimental limit from E799-I, $BR < 5.8 \cdot 10^{-5}$.

The sensitivities claimed for E799-II and at KEK are around 10^{-9} , thus another factor of 100 improvement is necessary.

The new FNAL and BNL proposals at the main injector are very ambitious. There is “hope” to make this measurement a reality early in the third millennium.

CP violation (as of 2001)

$$\mathcal{L}^{CP} = \mathcal{L}^{\Delta F=0} + \mathcal{L}^{\Delta F=1} + \mathcal{L}^{\Delta F=2}$$

$$\Delta F = 0 \quad d_e < 1.5 \cdot 10^{-27} \text{ e cm}, \quad d_N < 6.3 \cdot 10^{-26} \text{ e cm}$$

Cummings, Regan; Rutherford-Sussex-ILL

$$\Delta F = 1 \quad \text{YES,} \quad \frac{\epsilon'}{\epsilon}$$

$$\Delta F = 2 \quad \text{YES,} \quad \epsilon, a_{\psi K_S}$$

(E.g.: $\delta(CKM) = 0$ rather strongly disfavoured)

Concentrate on $\Delta F = 2$

In the SM

$$\mathcal{L}^{\Delta F=2} = \sum_{i,j=d,s,b} (V_{td_i} V_{td_j}^*)^2 \mathcal{C} (\bar{d}_i \gamma_\mu (1 - \gamma_5) d_j)^2$$

In general

$$\mathcal{L}^{\Delta F=2} = \sum_\alpha \sum_{i,j=d,s,b} (V_{td_i} V_{td_j}^*)^2 \mathcal{C}_{ij}^\alpha O_{ij}^\alpha$$

where

α = different Lorentz structures for 4F-operators

\mathcal{C}_{ij}^α = complex num. coeff.s (properly normalized)

from which $\langle K | \mathcal{L}^{\Delta F} | \bar{K} \rangle$, $\langle B_d | \mathcal{L}^{\Delta F} | \bar{B}_d \rangle$, $\langle B_s | \mathcal{L}^{\Delta F} | \bar{B}_s \rangle$ (6 par.s)