Particle Physics - IV

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Karlsruhe - Fall 2001

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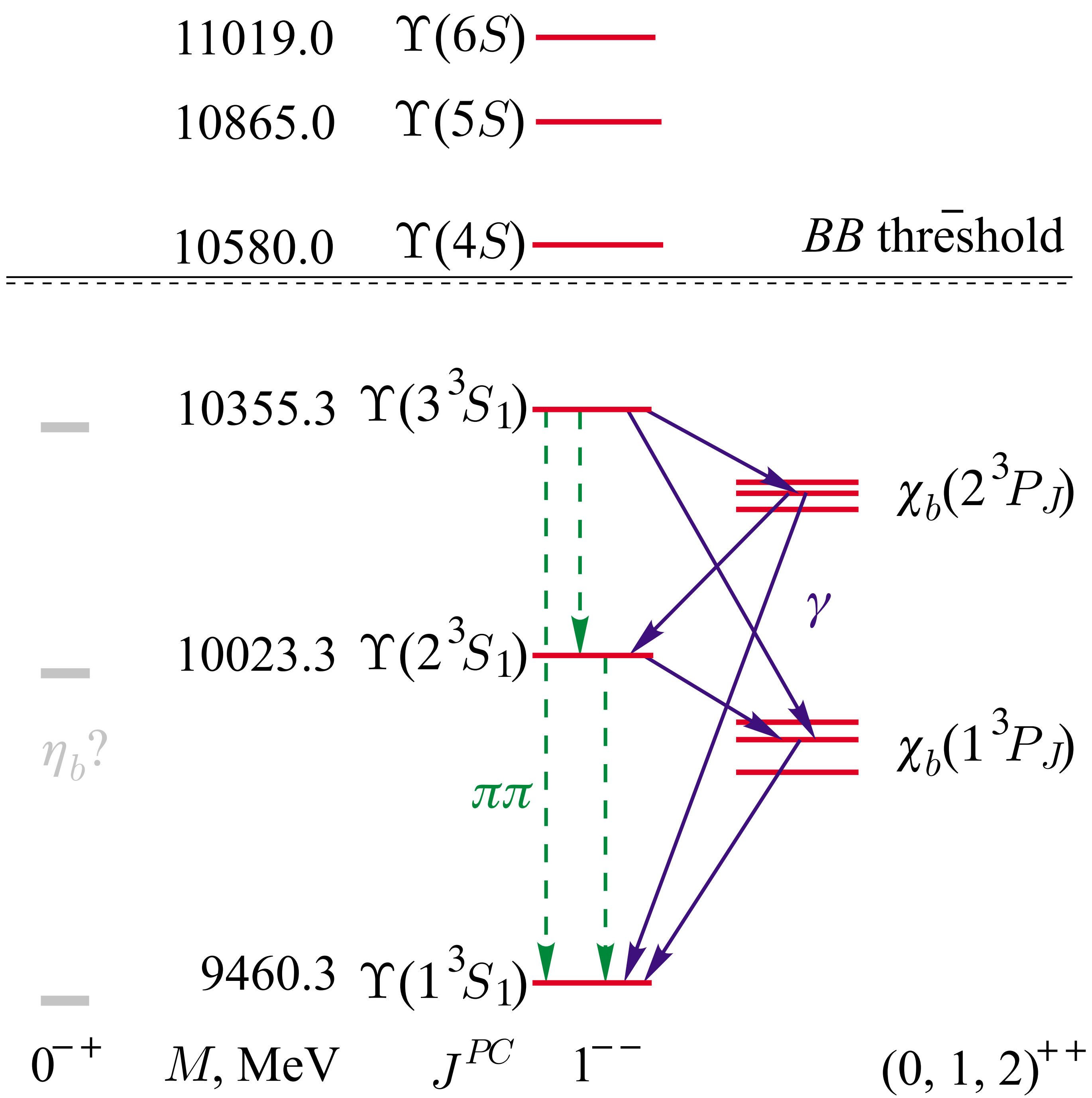
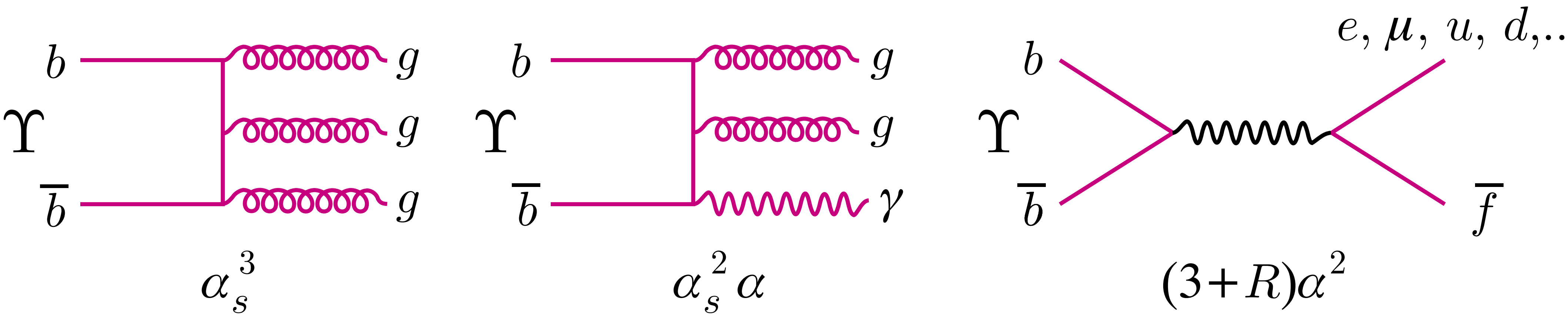
4. The Beauty Quark

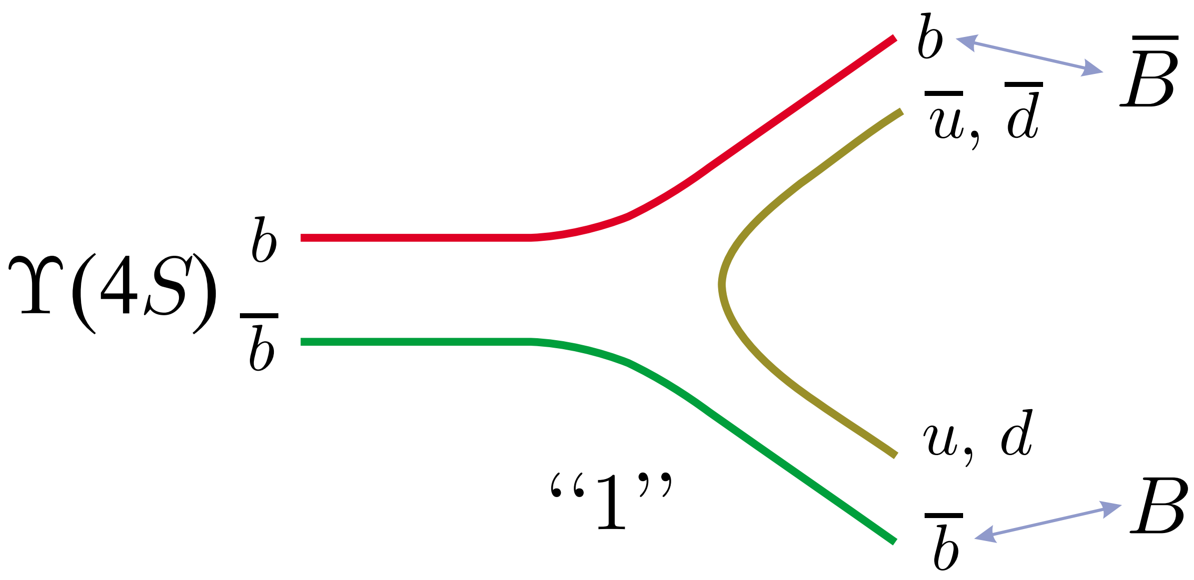
The discovery at Fermilab, in 1977, of the Υ, with mass of $\sim$10 GeV, was immediately taken as proof of the existence of the b quark. Actually another set of quarks had been proposed by Kobayashi and Maskawa in 1973 to enlarge the quark sector to six quarks. This allows the introduction of a phase in the quark mixing matrix which could explain CP violation, as we'll see later. The new quark had already been christened: b for beauty or bottom. The b quark has B flavor B=-1.

The *Upsilons* are bound states of a $\bar{b}b$ pair, and have a complex level diagram. There are 9 (+3) states below the $\bar{B}B$ threshold. A B meson is a bound state of a b quark and a light u, d, s, c quark. We usually denote by a subscript the light quark content.


11019.0 Υ(6S)
10865.0 Υ(5S)
10580.0 Υ(4S)
BB̄ threshold
10355.3 Υ(3 ³S₁)
χ_b(2 ³P_J)
γ
10023.3 Υ(2 ³S₁)
η_b?
χ_b(1 ³P_J)
ππ
9460.3 Υ(1 ³S₁)
0⁻⁺
M, MeV
J^PC
1⁻⁻
(0, 1, 2)⁺⁺



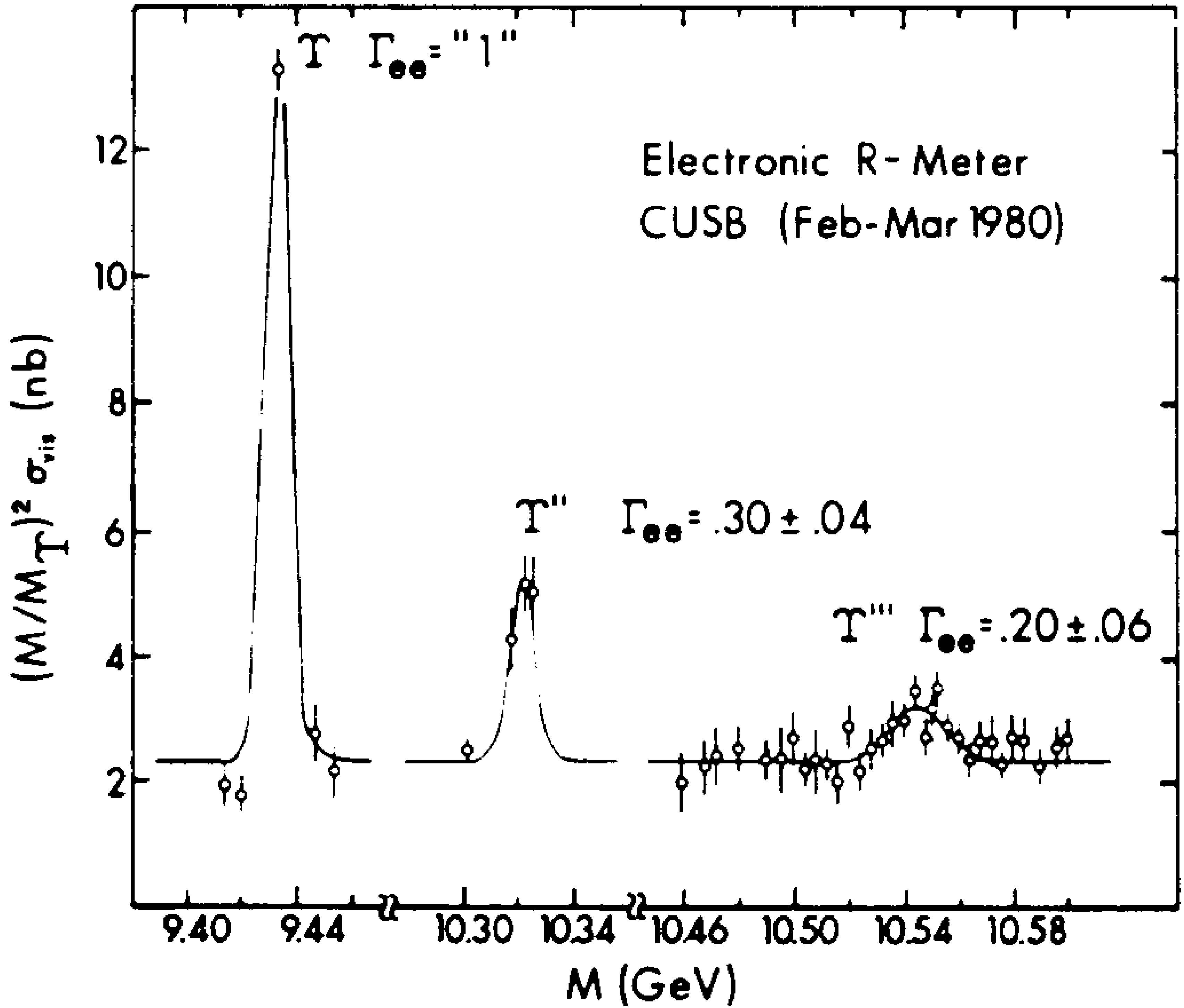
b
Υ
$\bar{b}$
g
g
g
α_s^3
b
Υ
$\bar{b}$
g
g
γ
$\alpha_s^2\,\alpha$
b
Υ
$\bar{b}$
$e,\ \mu,\ u,\ d,..$
$\bar{f}$
$(3+R)\alpha^2$



$\Upsilon(4S)$
b
$\bar{b}$
b
$\bar{u},\ \bar{d}$
$\bar{B}$
$u,\ d$
$\bar{b}$
B
"1"

The vector bound states decay by annihilation into three gluons, $2g+\gamma$, a photon and by dipole transitions. Just like the J/ψ they have very narrow widths. The visible width of the first three Υ peaks, which are below threshold (the first and third are shown in an antique picture of CUSB), are in fact due to experimental resolution, and, in the original Lederman experiment which discovered them, they were merged into one peak.

The fourth Υ, however, is just above threshold for it to decay into a $B\bar{B}$ pair, where the B mesons are $b\bar{u}$, $b\bar{d}$ and their charge conjugate states.

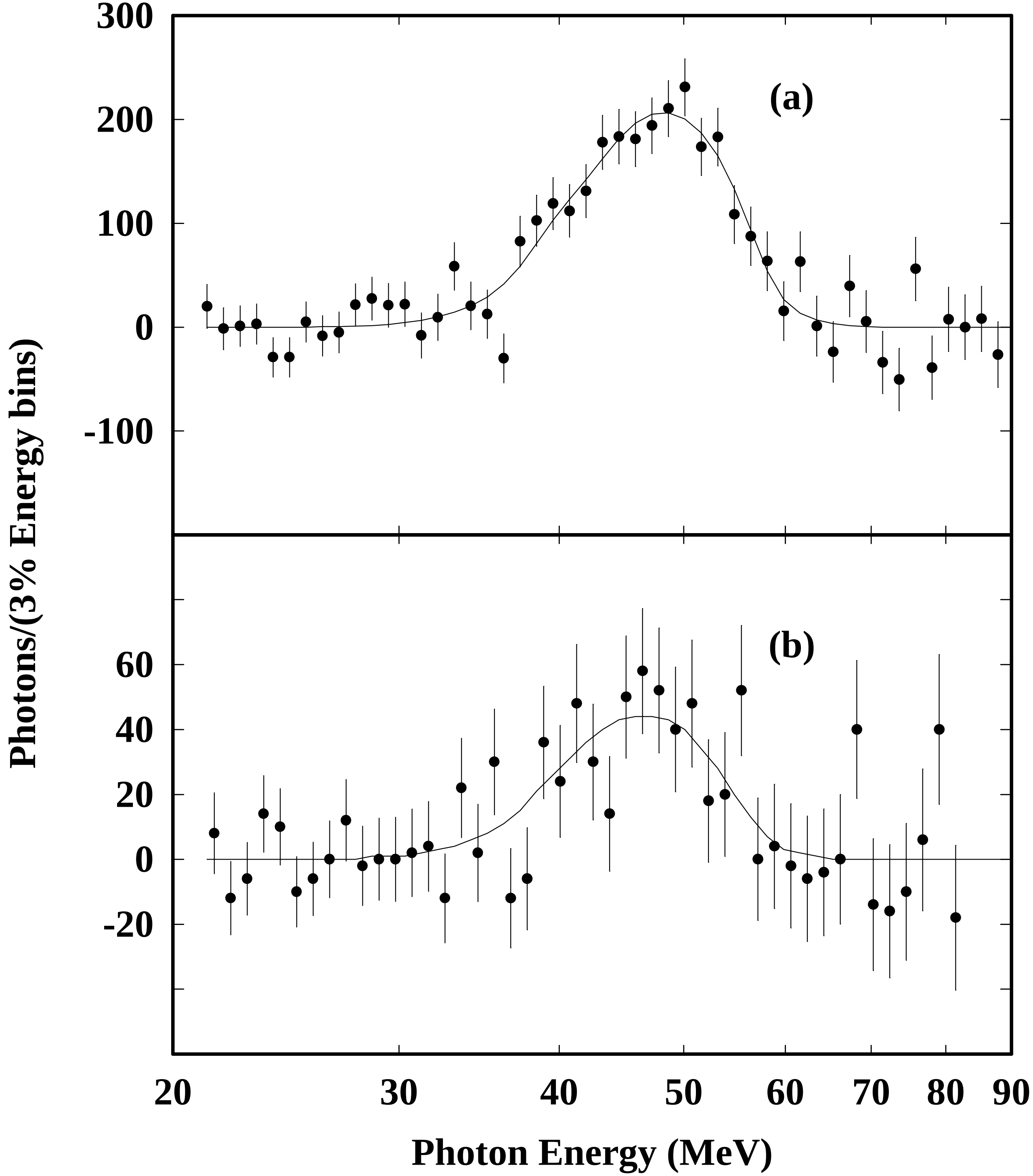


ϒ Γee="1"
Electronic R-Meter
CUSB (Feb-Mar 1980)
ϒ'' Γee=.30 ± .04
ϒ''' Γee=.20 ± .06
(M/Mϒ)² σvis (nb)
12
10
8
6
4
2
9.40
9.44
10.30
10.34
10.46
10.50
10.54
10.58
M (GeV)

In complete analogy to charged and neutral kaons, B^0 and $\overline{B}^0$ are not self conjugate states.

That the $\Upsilon(4S)$ decays only to $B^0\overline{B}^0$ pairs was demonstrated by CUSB searching (and not finding) low energy photons from B^* decays.

The photons from $B^* \to B + \gamma$ where observed at the next resonance, $\Upsilon(5S)$. The signal is shown in the next figure.



(a)
(b)
300
200
100
0
-100
60
40
20
0
-20
Photons/(3% Energy bins)
20
30
40
50
60
70
80
90
Photon Energy (MeV)

By disentangling the complex structure above the $\bar{B}B$ threshold, using coupled channel formalism, CUSB also determined the thresholds for B^*B, B^*B^*, $B_s\bar{B}_s$ etc. production.[1] This suggests that one can chose $B^0\overline{B}^0$ or B^* mesons by just changing the energy of a B factory.

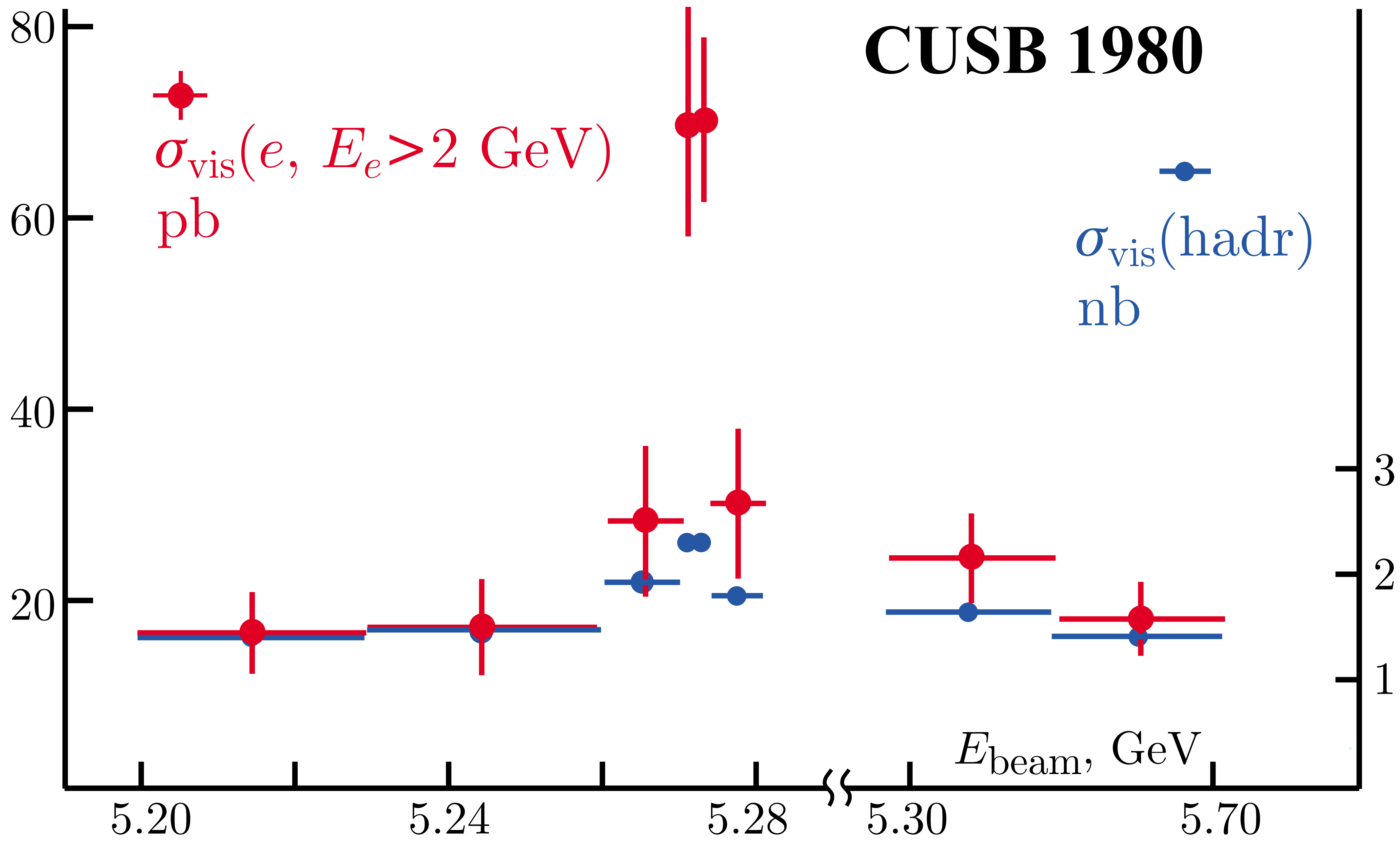
$B^0\overline{B}^0$'s at the $\Upsilon(4s)$ are produced in a C-odd state while at the $\Upsilon(5S)$ the $B^0\overline{B}^0$ pair is in an even state. At slightly higher energies, only about 1 GeV higher, B_S are produced. However, because the $\bar{B}_S \leftrightarrow B_S$ oscillations (see later) for B_S's are much faster than that of B_d's, one needs to go to the TeVatron and LHC to measure it.
Strictly speaking, one does not have to go to TeV-1 and LHC energies to study CP violation in the B_s system. But you better do it for reasons of statistics!

4.1 B semileptonic decays

Because of their massiveness, B's can decay - weakly - into many more channels than the K's. We might recall that we owe the long lifetime of the K_L to the smallness of the phase space for 3 body decays. The average particle multiplicity in the decay of B and $\bar{B}$ is about six. This also implies that we cannot have two vastly different lifetimes for neutral B's.

The leptonic modes have a branching ratio of about 25%, with a unique signature, namely a lepton with energy up to half that of the parent Υ.

The sudden appearance of high energy electrons in e^+e^- collisions at ∼10.6 GeV gave unambiguous prove of the existence of the b quark in 1980. Υ's have B=0 and beauty is hidden in them.


CUSB 1980
$\sigma_{\rm vis}(e,\ E_e\!>\!2\ {\rm GeV})$
pb
$\sigma_{\rm vis}({\rm hadr})$
nb
80
60
40
20
3
2
1
$E_{\rm beam}$, GeV
5.20
5.24
5.28
5.30
5.70

At quark level beauty decays are: $b \to c\ell^-\bar{\nu}$ and $b \to u\ell^-\bar{\nu}$ via a W exchange. With the selection rule $\Delta B{=}\Delta Q$ operating in a similar way as for $\Delta S{=}\Delta Q$, $\bar{b}$ decays to positive leptons, b decays to negative leptons.

The endpoint of the lepton spectrum and its shape depend on the flavor of the hadronic system appearing in the final state. We define as X_c a hadronic system with charm $C = \pm 1$ and $U{=}0$ where U is the *uppityness*. Likewise X_u has $U = \pm 1$ and $C{=}0$.

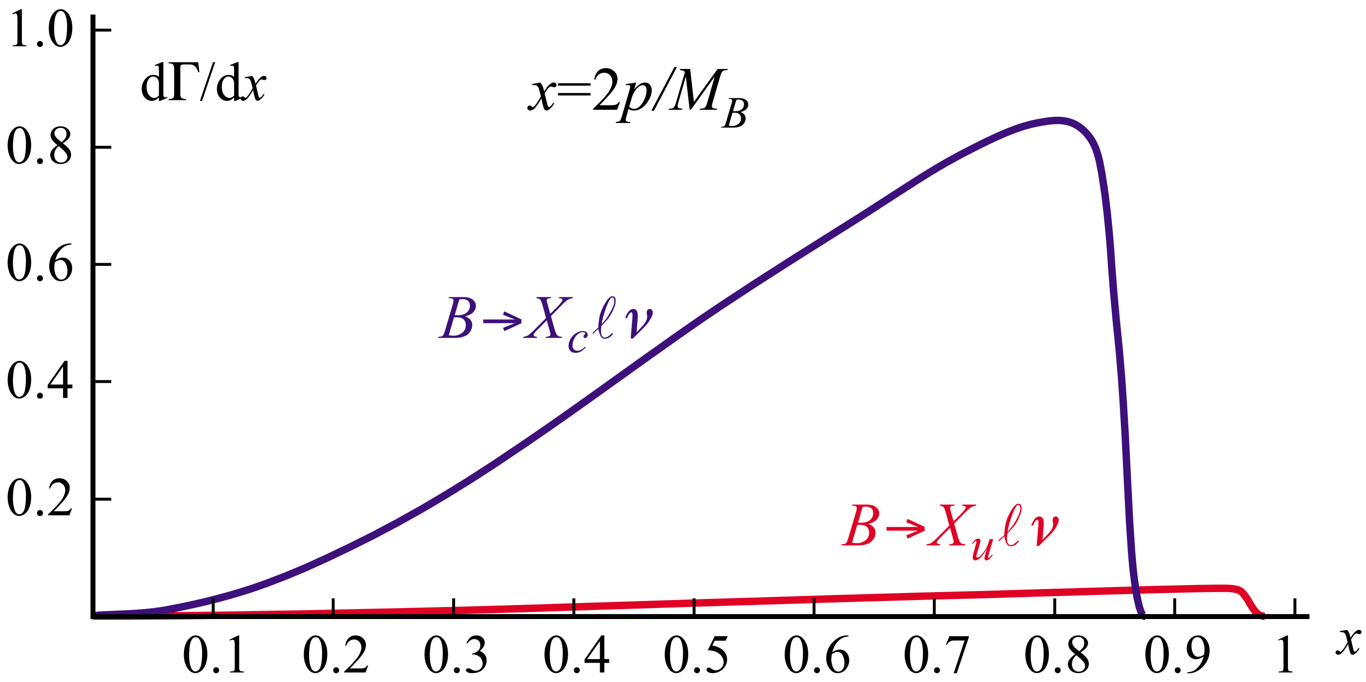
The leptonic decays are:

$$B \to \ell^{\pm} + \nu(\bar{\nu}) + X_c$$

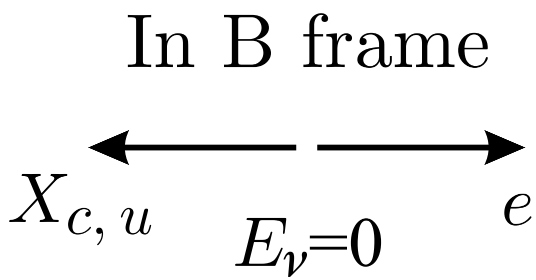
$$B \to \ell^{\pm} + \nu(\bar{\nu}) + X_u$$

where $X_c{=}D$, $D^* \ldots$ with $\overline{M}(X_c){\sim}2$ GeV and $X_u{=}\pi$, ρ.. with $\overline{M}(X_u){\sim}0.7$ GeV.

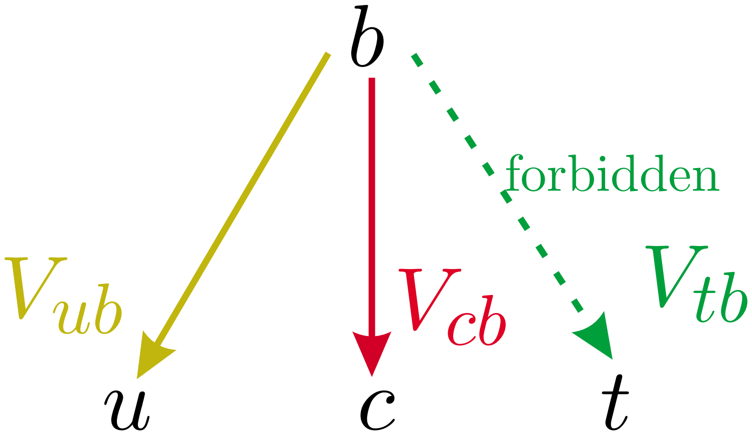
The expected lepton spectra are shown in fig.



End point kinematics

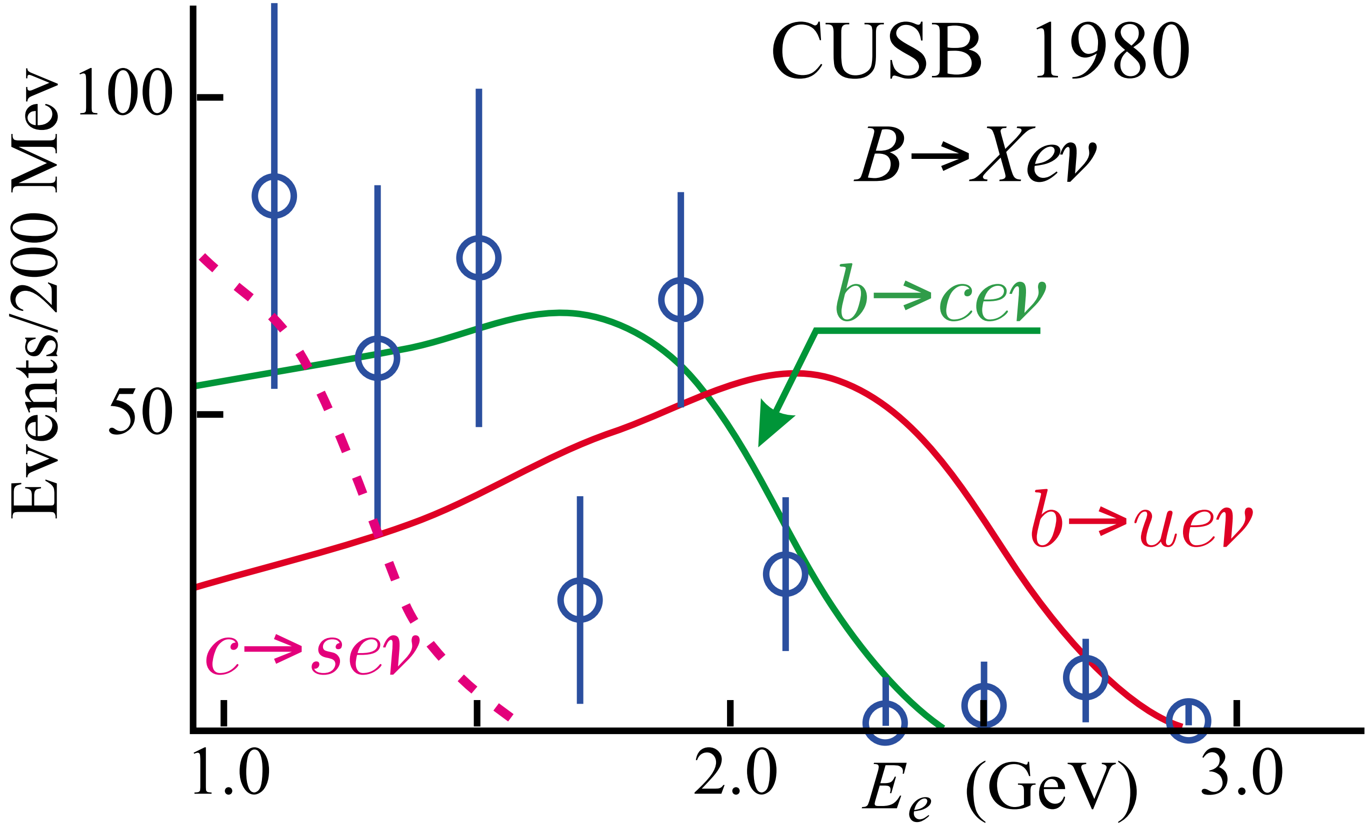


Total decay rates, *i.e.* the inverse of the lifetimes, and branching ratios of B mesons provide the determination of $|V_{cb}|$ and $|V_{ub}|$. The correct way is to measure the semileptonic branching ratio by integrating over the whole spectrum.

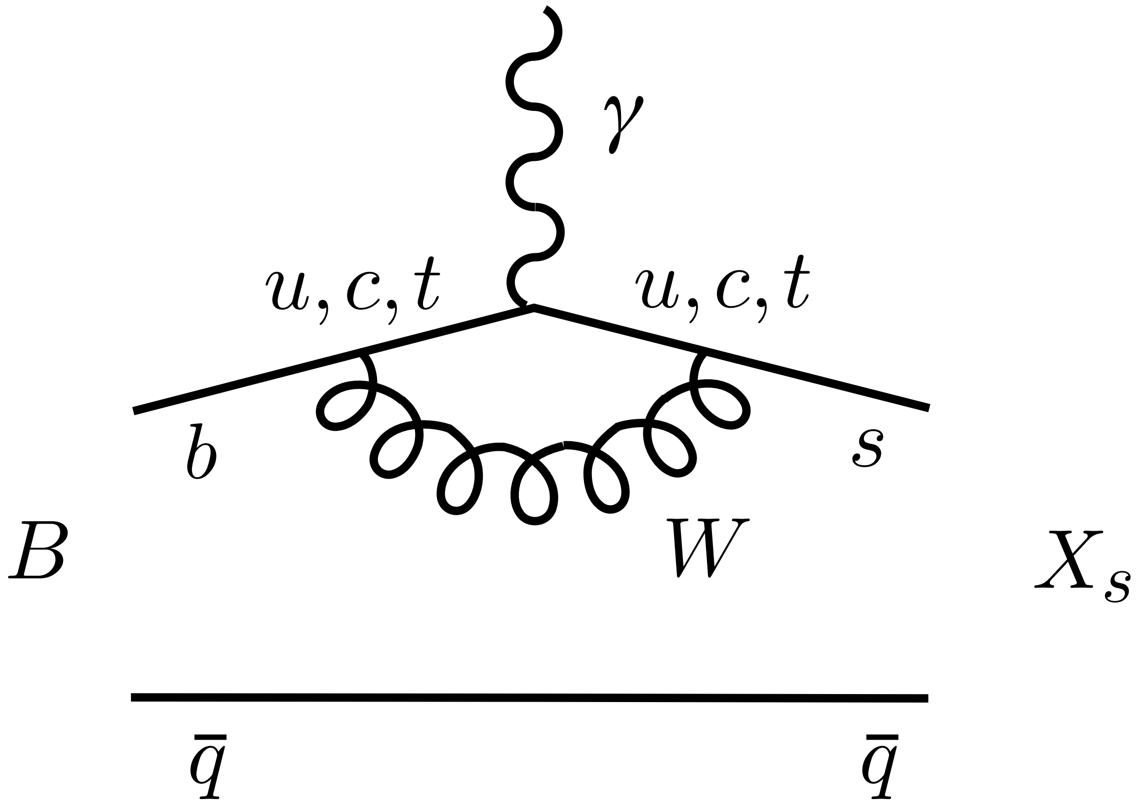


Definition of the b quark couplings. $|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2 = 1$. $V_{tb} \sim 1$ but $b \to t\ell\nu$ is energetically forbidden.

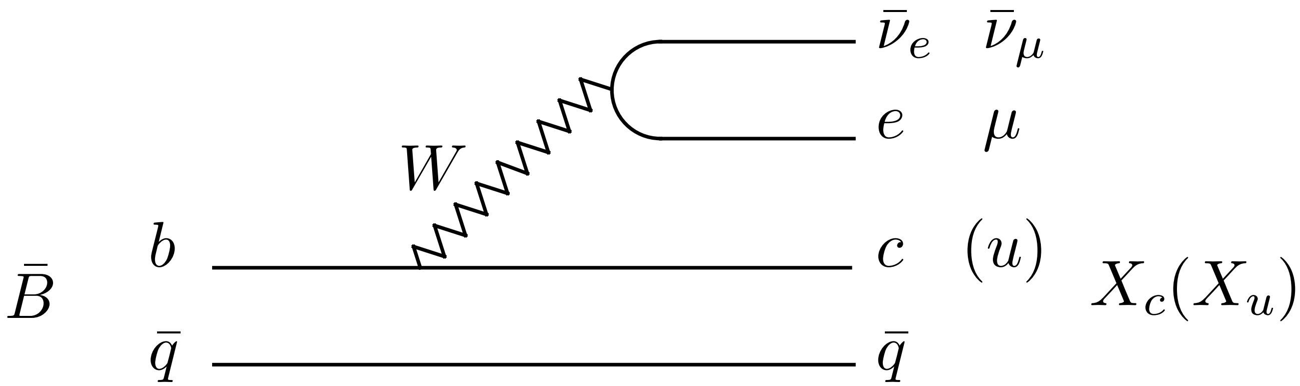
An early determination by CUSB already indicated that $|V_{ub}/V_{cb}|$ is less than 1/10.



Uncertainties in the calculation of the hadronic matrix elements and the shape of the spectrum near the end point introduce errors in the extraction of $|V_{ub}/V_{cb}|$. The most recent attempts to circumvent the first problem is to use the $b \to s\gamma$ decay, as reported by Cassel at LP01.



Measuring $|V_{cb}|$ and $|V_{ub}|$



$|V_{cb}|$ and $|V_{ub}|$ can be determined from semileptonic decays

$$\Gamma^c_{SL} \equiv \Gamma(\bar{B} \to X_c \ell \bar{\nu}) = \frac{\mathcal{B}(\bar{B} \to X_c \ell \bar{\nu})}{\tau_B} = \gamma_c |V_{cb}|^2$$

$$\Gamma^u_{SL} \equiv \Gamma(\bar{B} \to X_u \ell \bar{\nu}) = \frac{\mathcal{B}(\bar{B} \to X_u \ell \bar{\nu})}{\tau_B} = \gamma_u |V_{ub}|^2$$

The theoretical parameters γ_c and γ_u are a serious problem

- $b \to s\gamma$ decays can substantially reduce γ_c model dependence

$|V_{cb}|$ from Hadronic Mass Moments and $b \to s\gamma$

$\Gamma(\bar{B} \to X_c \ell \bar{\nu})$ can be written in the form

$$\Gamma_{SL}^{c} = \frac{G_F^2 |V_{cb}|^2 M_B^5}{192\pi^3} \Big[\mathcal{G}_0 + \frac{1}{M_B} \mathcal{G}_1(\bar{\Lambda}) + \frac{1}{M_B^2} \mathcal{G}_2(\bar{\Lambda}, \lambda_1, \lambda_2)$$
$$+ \frac{1}{M_B^3} \mathcal{G}_3(\bar{\Lambda}, \lambda_1, \lambda_2 | \rho_1, \rho_2, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4) + \mathcal{O}\left(\frac{1}{M_B^4}\right) \Big]$$

- $\bar{\Lambda}, \lambda_1, \lambda_2, \rho_1, \rho_2, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4$ are nonperturbative parameters,
- We use theoretical estimates of $\rho_1, \rho_2, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4$.

There are similar expressions for the moments

- $\langle (M_X^2 - \bar{M}_D^2) \rangle$ of the $\bar{B} \to X_c \ell \bar{\nu}$ mass spectrum ($\mathcal{M}_n$) and
- $\langle E_\gamma \rangle$ of the $b \to s\gamma$ energy spectrum ($\mathcal{E}_n$).

$|V_{cb}|$ from Hadronic Mass Moments and $b \to s\gamma$

To extract $|V_{cb}|$ we

- determine λ_2 from $M_{B^*} - M_B$, and
- determine $\bar{\Lambda}$ and λ_1 from $\langle(M_X^2 - \bar{M}_D^2)\rangle$ and $\langle E_\gamma \rangle$.

Other experimental parameters:

- $\mathcal{B}(\bar{B} \to X_c \ell \bar{\nu}) = (10.39 \pm 0.4)\%$ from CLEO,
- τ_{B^-} and τ_{B^0} from PDG 2000, and
- $(f_{+-}\tau_{B^-})/(f_{00}\tau_{B^0}) = 1.11 \pm 0.08$ from CLEO.
  - $f_{+-} \equiv \mathcal{B}(\Upsilon(4S) \to B^+ B^-)$
  - $f_{00} \equiv \mathcal{B}(\Upsilon(4S) \to B^0 \bar{B}^0)$

Theoretical functions:

- $\langle(M_X^2 - \bar{M}_D^2)\rangle$ are measured for $E_\ell > 1.5$ GeV,
- $\langle E_\gamma \rangle$ are measured for $E_\gamma > 2.0$ GeV, and
- $\mathcal{G}_n$, $\mathcal{M}_n$, and $\mathcal{E}_n$ calculated with same cuts using Falk-Luke, Phys. Rev. D **57**, 1 (1998)

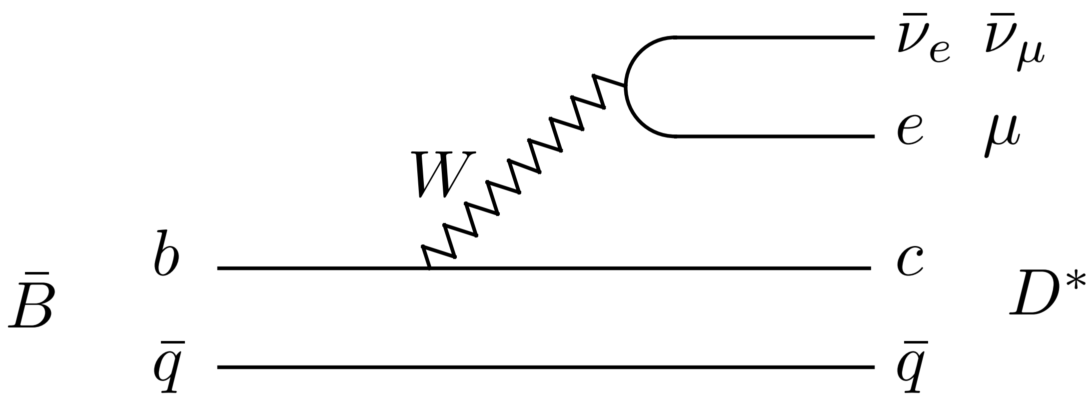
$$|V_{cb}| = (40.4 \pm 0.8 \pm 0.5 \pm 0.8) \times 10^{-3}$$
$$|V_{ub}| = (4.09 \pm 0.14 \pm 0.66) \times 10^{-3}$$

Methods (HQET) have been developed to make use of exclusive channels. A good twenty years have been spent in refining:

1. the theory
2. the exclusive channel measurements
3. performing additional measurements to provide parameters necessary for the application of the method.

As shown in five talks at LP01, both theory and experiments still have room for improvements.

$|V_{cb}|$ from $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ Decay



The differential decay width for $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ decay is

$$\frac{d\Gamma(w)}{dw} = \frac{G_F^2}{48\pi^3} \mathcal{G}(w) \, |V_{cb}|^2 \, \mathcal{F}_{D^*}^2(w)$$

with $w \equiv v_B \cdot v_{D^*} = \dfrac{\mathcal{E}_{D^*}}{M_{D^*}} = \dfrac{M_B^2 + M_{D^*}^2 - q^2}{2 M_B M_{D^*}}$ (Isgur – Wise)

- v_B and v_{D^*} are the 4-velocities of the B and D^*,
- $\mathcal{E}_{D^*}$ is the energy of the D^* in the B rest frame.

w range is $(1.00 < w < 1.51)$ for $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ decay.

$|V_{cb}|$ from $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ Decay

The form factor $\mathcal{F}_{D^*}(w)$

- parameterizes the w dependence of the hadronic current and
- is constrained by Heavy Quark Effective Theory (HQET)
  - $\mathcal{F}_{D^*}(1) \approx \eta_A[1 + \mathcal{O}(1/m_Q^2)]$ for large heavy quark masses

However $\mathcal{G}(1) = 0$ (phase space) so

- measure $d\Gamma(w)/dw$ over the full w range,
- extract $|V_{cb}|\mathcal{F}_{D^*}(w)$ and fit it over the full w range,
- extrapolate $|V_{cb}|\mathcal{F}_{D^*}(w)$ to $w = 1$ to get $|V_{cb}|\mathcal{F}_{D^*}(1)$.

Reduce w dependence to form factor ratios $R_1(1)$ and $R_2(1)$, previously measured by CLEO, and a slope parameter ρ^2 in the fit (Caprini-Lellouch-Neubert)

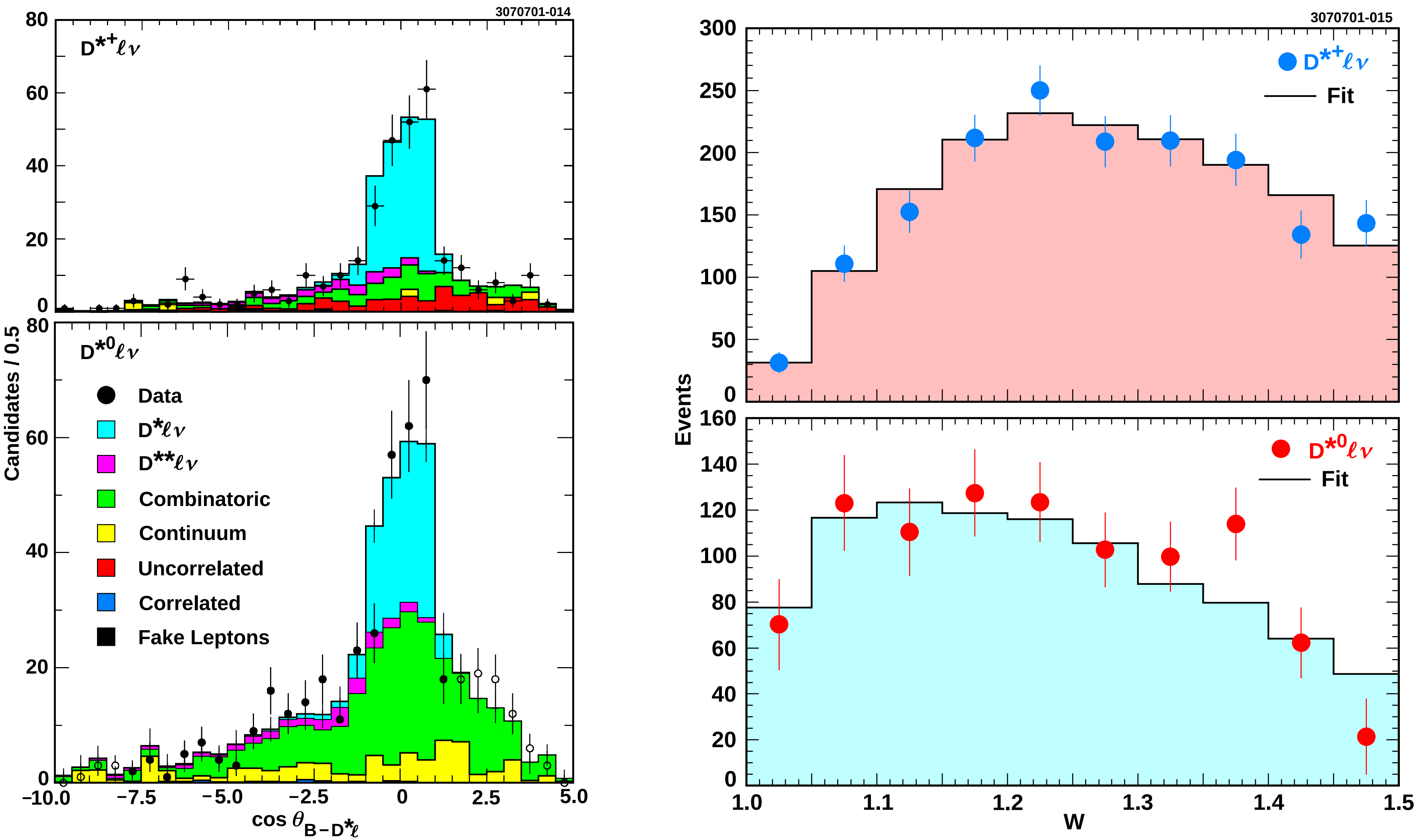
$|V_{cb}|$ from $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ Decay

After reconstructing the D^* and the ℓ, separate the signal from background using the angle $\theta_{B-D^*\ell}$ between the momenta of the B and the $D^*\ell$ combination

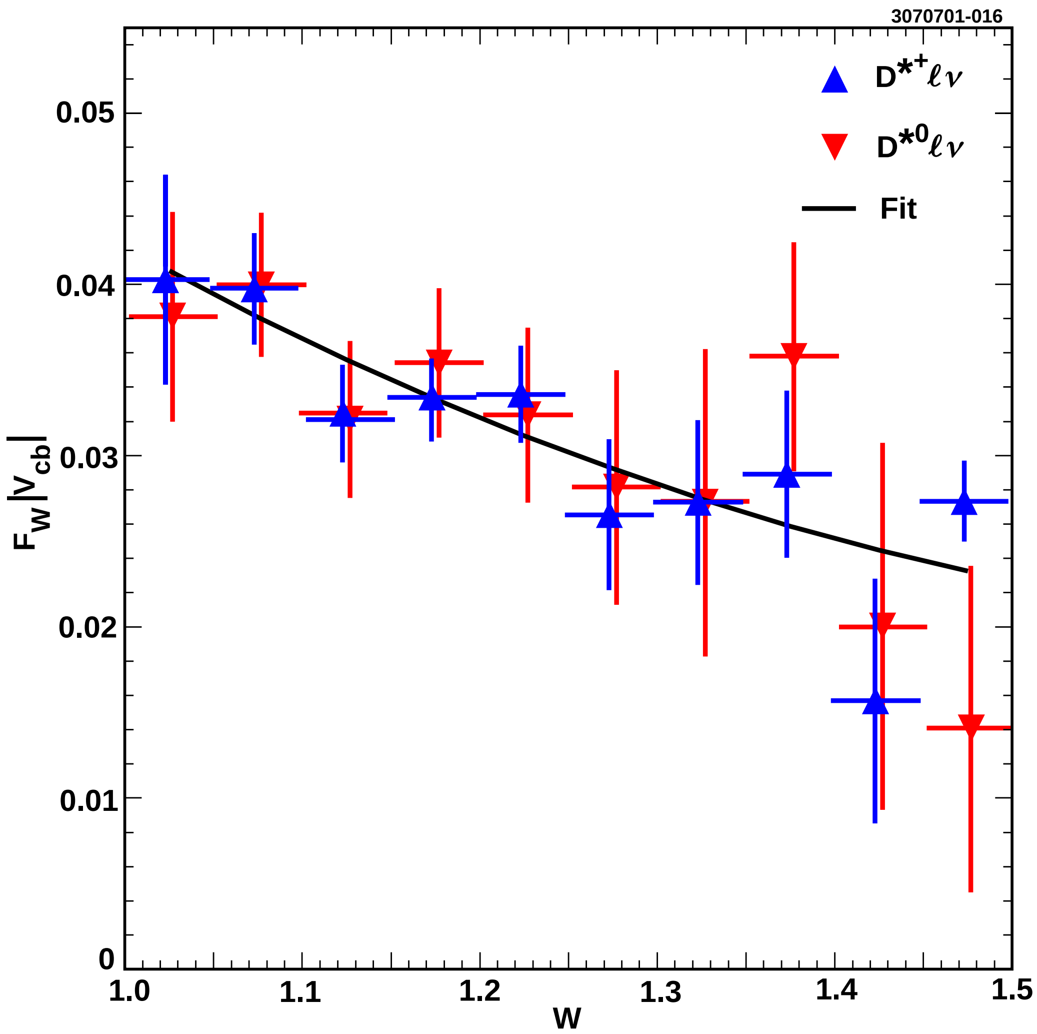
$$\cos\theta_{B-D^*\ell} = \frac{2E_B E_{D^*\ell} - M_B^2 - M_{D^*\ell}^2}{2\,P_B\,P_{D^*\ell}}$$

We now have results for $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$ and $B^- \to D^{*0}\ell^-\bar{\nu}_\ell$ (previously, *e.g.*, ICHEP 2000 Osaka, $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$ only)

$|V_{cb}|$ from $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ Decay



$|V_{cb}|$ from $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ Decay



The fit uses:

$\mathcal{F}_{D^*}(w)$ from Caprini-Lellouch-Neubert

$\mathcal{F}_{D^{*+}}(w) = \mathcal{F}_{D^{*0}}(w)$

$\Gamma(D^{*+}\ell\bar{\nu}) = \Gamma(D^{*0}\ell\bar{\nu})$

CLEO measurement of

$(f_{+-}\tau_{B^-})/(f_{00}\tau_{B^0})$

$|V_{cb}|$ from $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ Decay

From the $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ fit and systematic error estimates:

$$\begin{aligned} |V_{cb}|\mathcal{F}_{D^*}(1) &= (42.2 \pm 1.3 \pm 1.8) \times 10^{-3} \\ \rho^2 &= 1.61 \pm 0.09 \pm 0.21 \\ \Gamma(\bar{B} \to D^* \ell^- \bar{\nu}_\ell) &= (0.0376 \pm 0.0012 \pm 0.0024)\ \mathrm{ps}^{-1} \end{aligned}$$

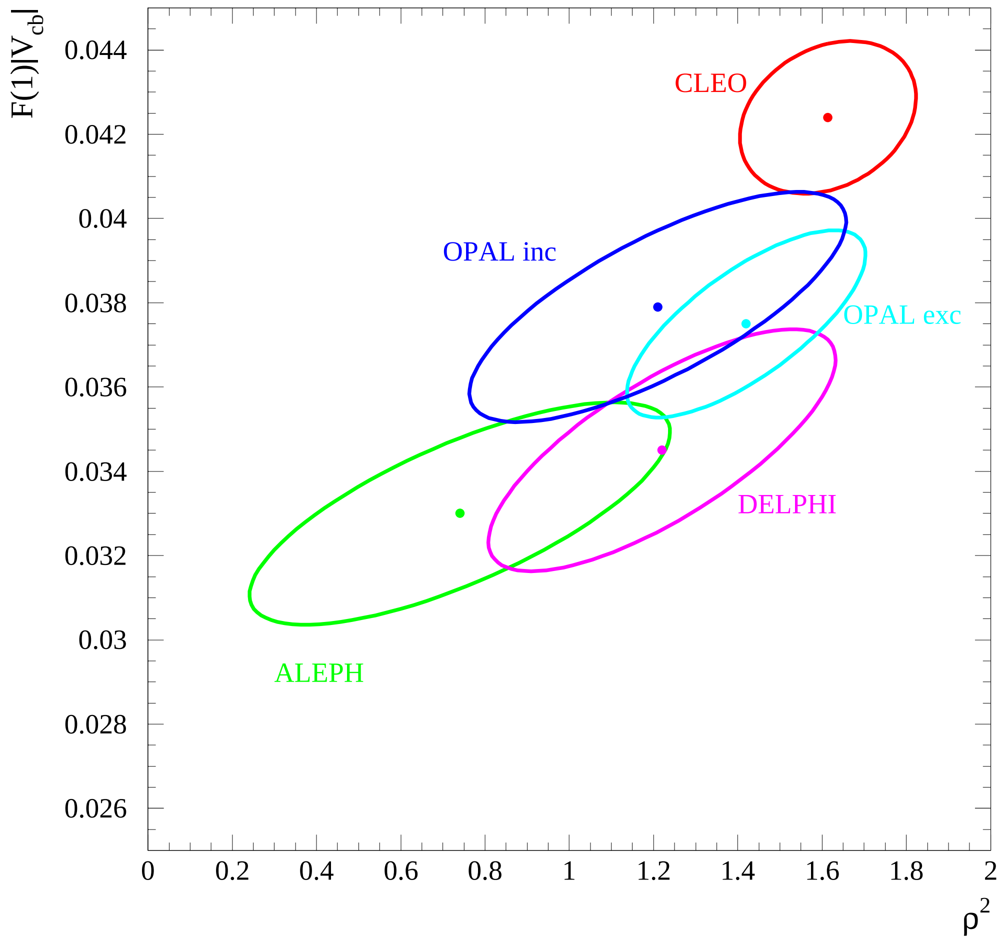
Using PDG 2000 lifetimes and branching fractions

$$\begin{aligned} \mathcal{B}(B^- \to D^{*0} \ell^- \bar{\nu}_\ell) &= (6.21 \pm 0.20 \pm 0.040)\% \\ \mathcal{B}(\bar{B}^0 \to D^{*+} \ell^- \bar{\nu}_\ell) &= (5.82 \pm 0.19 \pm 0.037)\% \end{aligned}$$

Using $\mathcal{F}_{D^*}(1) = 0.913 \pm 0.042$ (BaBar Physics Book)

$$|V_{cb}| = (46.2 \pm 1.4_{\mathrm{stat}} \pm 2.0_{\mathrm{syst}} \pm 2.1_{\mathrm{thry}}) \times 10^{-3}$$

$|V_{cb}|$ from $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ Decay



Possible sources of apparent difference between CLEO and the LEP experiments

- $D^* X \ell \bar{\nu}$ component
  - CLEO fits
  - LEP uses model
- Large $\rho^2 - \mathcal{F}_{D^*}|V_{cb}|$ correlation at LEP

4.2 Mixing and its discovery

Just as with neutral K-mesons, neutral B mesons are not C eigenstates and can mix, i.e, transitions $B^0 \leftrightarrow \overline{B}^0$ are possible.

The first observation of mixing was reported by Argus at the DESY DORIS collider running on the $\Upsilon(4S)$.

Mixing results in $B\,\overline{B} \to B\,B$ or $\overline{B}\,\overline{B}$ resulting in decays to same sign lepton pairs $\ell^+\ell^+$ and $\ell^-\ell^-$. Defining the ratio

$$r = \frac{\ell^+\ell^+ + \ell^-\ell^-}{\ell^+\ell^- + \ell^-\ell^+ + \ell^+\ell^+ + \ell^-\ell^-}$$

$r \neq 0$ is proof of mixing, not however of $\not{C}\not{P}$.

Today, instead of r, the χ_d parameter, which is a measure of the time-integrated B^0-$\overline{B}^0$ mixing probability that a produced $B^0(\overline{B}^0)$ decays as $\overline{B}^0(B^0)$, is used.

They are related simply by $r = \chi_d/(1-\chi_d)$. The present value of χ_d is 0.172 ± 0.01.

4.3 Formalism

We define, analogously to the $K^0\overline{K}^0$ system,

$$B_\mathsf{L} = p\,|\,B^0\,\rangle + q\,|\,\overline{B}^0\,\rangle$$
$$B_\mathsf{H} = p\,|\,B^0\,\rangle - q\,|\,\overline{B}^0\,\rangle$$

with $p^2 + q^2 = 1$

Here L, H stand for light and heavy. The B_d's also have different masses but very similar decay widths.

Mixing is calculated in the SM by evaluating the standard "box" diagrams with intermediate u, c, t and W states. We define:

$$\Delta M = M_H - M_L, \quad \Delta\Gamma = \Gamma_H - \Gamma_L$$

note that ΔM is positive by definition.

The ratio q/p is given by:

$$q/p = (\Delta M - i/2\Delta\Gamma)/2(M_{12} - i/2\Gamma_{12}) =$$
$$2(M_{12}^* - i/2\Gamma_{12}^*)/(\Delta M - i/2\Delta\Gamma)$$

where

$$\Gamma_{12} \propto [V_{ub}V_{ud}^* + V_{cb}V_{cd}^*]^2 m_b^2 = (V_{tb}V_{td}^*)^2 m_b^2$$

and $M_{12} \propto (V_{tb}V_{td}^*)^2 m_t^2$, so they have almost the same phase.

x and y, for B_d and B_s mesons are:

$$x_{d,s} = \Delta M_{d,s}/\Gamma_{d,s}, \qquad y_{d,s} = \Delta\Gamma_{d,s}/\Gamma_{d,s}$$

y_d is less than 10^{-2}, and x_d is about 0.7, and if we ignore the width difference between the two B_d states,

$$\frac{q}{p}\Big|_{B_d} \approx \frac{(V_{tb}^* V_{td})}{(V_{tb}V_{td}^*)} = e^{-2i\beta}$$

Therefore $|q/p|_d$ is very close to 1 and since $2\Re\epsilon_{B_d} \approx 1 - |q/p|_d$, ϵ_{B_d} is imaginary. y_s is about 0.2, and x_s theoretically could be as large as 20, so far only lower bounds are quoted.

χ_d as defined before, in terms of q, p, x, y is

$$\chi_d = \frac{1}{2}\left|\frac{q}{p}\right|^2 \frac{x_d^2 - y_d^2/4}{(1+x_d^2)(1-y_d^2/4)}$$

which reduces to a good approximation:

$$\chi_d = \frac{x_d^2}{2(1+x_d^2)},$$

from which one obtains that $x_d = 0.723 \pm 0.032$.

In summary, from evaluating the box diagrams, one finds:

$$x_l \propto \ m_t^2 \tau_{B_l} m_{B_l} |V_{tl} V_{tb}^*|^2.$$

where the subscript l refers to the light meson partner which makes up the B meson, i.e. $l =s$ or d.[(2)]

An amusing historical note. The surprisingly large amount of mixing seen required that the top mass be larger than the then acceptable value of about 20 GeV. Stretching beyond reason the limits for $|V_{ub}|$ and the value of r than known, a lower limit $M_{\text{top}} \geq$ 40 GeV was obtained.

The first CUSB limit on $|V_{ub}|$ already implied M_{top} >120 GeV. Theorist were at that time still misled by prejudice and intimidated by very wrong experimental findings and only had the courage to claim a top mass of ∼40 GeV.

Using the top mass today known, and ΔM measured from the $B^0\overline{B}^0$ oscillation frequency from experiments at FNAL and LEP, one obtains the estimates $|V_{td}| = (8.4\pm1.4)\times10^{-3}$ and $\Im(V_{td}V_{tb}^*) = (1.33 \pm 0.30) \times 10^{-4}$.

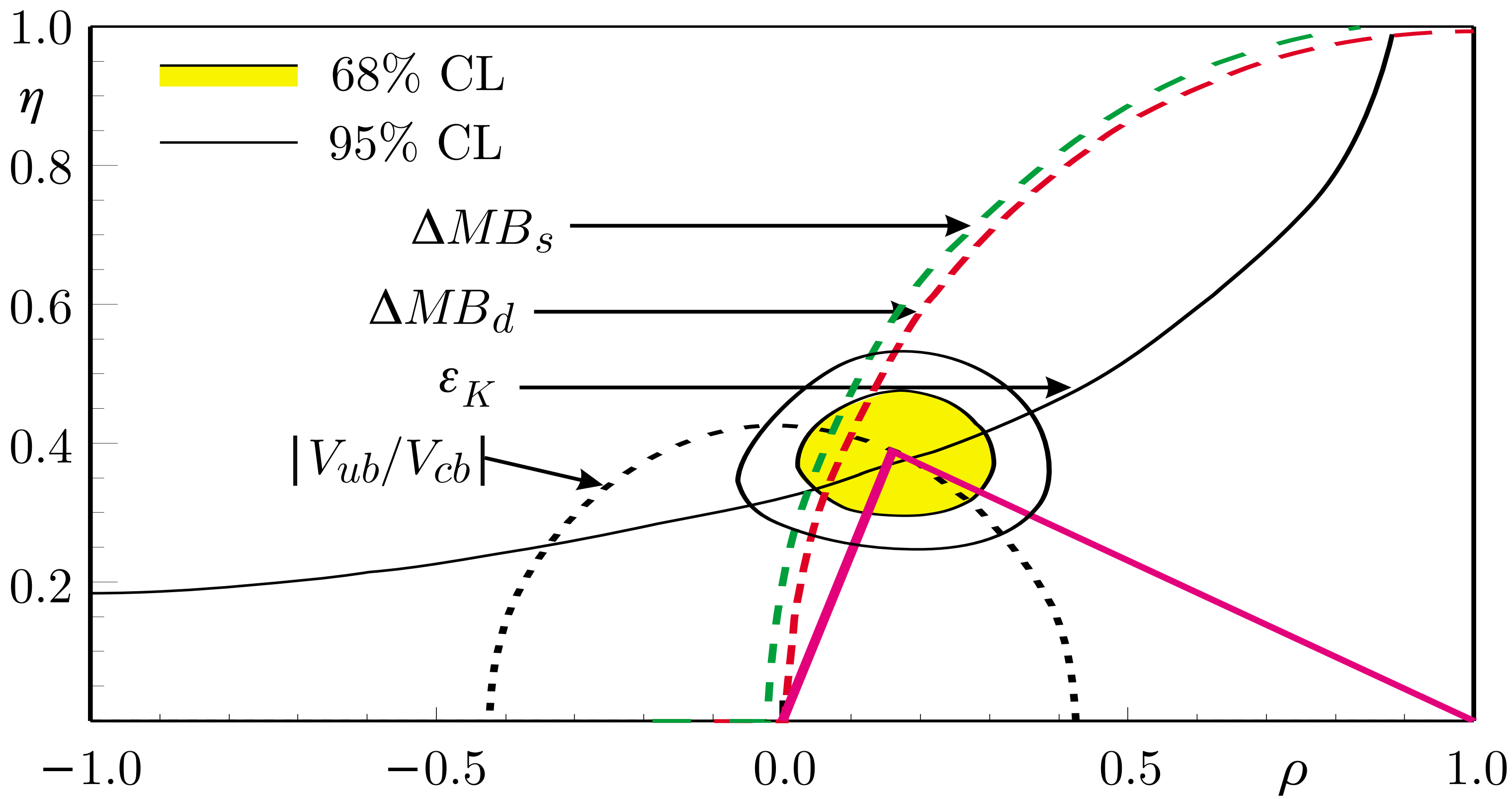


Fig. 26. Fit to data in the η-ρ plane.

From a fit, shown in fig. 26, Parodi *et al.*,[3] obtain

$$\sin 2\beta = 0.71 \pm 0.13\,, \qquad \sin 2\gamma = 0.85 \pm 0.15.$$

Of course the whole point of the exercise is to measure directly η and ρ and then verify the uniqueness of the mixing matrix.

Especially one should not use, implicitly or otherwise the unitarity of the mixing matrix in order to verify whether the matrix is indeed unitary.