Introduction to

Chiral Perturbation Theory

Gino Isidori

INFN −Laboratori Nazionali di Frascati

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- Motivations and basic principles
- Lowest−order Lagrangians
- Non−leptonic weak interactions
- Beyond lowest order
- A detailed example: K_{l3} decays and the Cabibbo angle
- Other interesting issues in kaon physics
- Conclusions

Motivations and basic principles

At low energies ($E \ll 1$ GeV) QCD is in a highly non−perturbative regime

- very difficult to describe the (low−energy) hadronic world in terms of partonic degrees of freedom.

However...

- the hadronic spectrum is very simple at low energies: only 8 (3) pseudoscalar fields separated by a mass gap from the heavier states

- the interactions among the pseudoscalar mesons become weak in the limit $E \to 0$

$\Downarrow$

Reasonable to expect that QCD can be treated in a perturbative way even at low energies with a <u>suitable choice of degrees of freedom</u>:

(q, G) pQCD	$\longleftrightarrow$	(π, K, η) CHPT
[*perturbative @ high E*]		[*perturbative @ low E*]

CHPT is a typical example of *Effective Quantum Field Theory*, or of a QFT which has an intrinsic limitation in the (energy) range of validity ($E < \Lambda$).

All QFT we know (including the Standard Model) can be considered as EQFT, or as low energy approximations of "more fundamental" theories, valid up to higher energy scales.

basic principle:	Only few degrees of freedoms are relevant in a given energy range: the heavy (lighter) ones can be integrated out.

Do we need renormalizability (in the "classical" sense) within EQFT ?

Renormalizable theories are a particular subset of EQFT with

- the virtue of being very predictive (only a finite set of couplings need independently of the energy range)
- the disadvantage of non containing explicit indications about their validity range

General consistency conditions for EQFT:

[replacing the "classical" requirement of renormalizability]

The operators needed to regularize the theory must be organized in a power series (defining the perturbative expansion) such that

- for any $n \geq 0$ there is only a finite set of operators contributing at $O(E^{n})$ to the physical amplitudes
- the coefficients of these operators, $C_i^{(n)}$, scale according to $C_i^{(n+k)}/C_j^{(n)} \sim O(1/\Lambda^k)$

$$\Downarrow$$

Only a finite set of couplings is needed to describe the physics at $E < \Lambda$ with arbitrary precision [the theory is "renormalizable order by order" in the energy expansion of the physical amplitudes]

$$A_{phys}(E) = \sum_i a_n \left(\frac{E}{\Lambda}\right)^n$$

Chiral Symmetry:

N_f massless quarks $\Rightarrow$ $$L_{QCD}^{[0]} = \sum_{i=0}^{N_f} \bar{q}^i iD_\mu q^i - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} + L_{heavy\,quarks}$$

Global invariance: $\underbrace{SU(N_f)_L \times SU(N_f)_R}_{\text{[chiral group G]}} \times U(1)_V \times U(1)_A$

not manifest in the hadronic spectrum
(only an approximate $SU(N_f)_V$ is realized)

realized
in nature
[CVC]

broken at the quantum level
by the axial anomaly

Basic assumptions of CHPT (N_f = 2 or 3):

- G is spontaneously broken into $H = SU(N_f)_{L+R}$

  (π) or (π,K, η) → Goldstone bosons of G/H [correct quantum numbers, small masses, vanishing interactions for $E \to 0$]

- The $O(m_q)$ explicit breaking terms can be treated as small perturbations around the chiral limit

consistency check $\Rightarrow$ $\boxed{\begin{array}{l} M_\pi^2 / M_\rho^2 \sim 0.03 \\ M_K^2 / M_\rho^2 \sim 0.4 \end{array}}$

Chiral realization of the QCD functional:

$$e^{Z[J]} = \int Dq\, D\bar{q}\, DG \;\; e^{\int L_{QCD}[q,\bar{q},G;\, J]} \;\Leftrightarrow\; \int D\phi \;\; e^{\int L_{\chi}[\phi;\, J]}$$

ϕ = Goldstone boson fields, parameterizing *G/H*

[transforming non linearly under *G*]

J = external sources

[transforming linearly under *G* (or *H*)]

$$L_{\chi} = L_{\chi}^{(2)} + L_{\chi}^{(4)} + L_{\chi}^{(6)} + \ldots$$

= most general series of operators transforming linearly under *G*,
($\Rightarrow$ same symmetry properties of L_{QCD}), written in terms of ϕ and *J*,
organized in a power series according to the number of derivatives
($\Rightarrow$ powers of the external momenta in physical amplitudes)

S. Weinberg, *Phys. Rev. Lett. 18 (67) 188*

N.B: CHPT is not a model of QCD !

The general procedure to construct non−linear realizations of a spontaneously broken group, in terms of its Goldstone boson fields, has been analyzed in the classical papers by Callan, Coleman and Zumino

S. Coleman, J. Wess, B. Zumino, *Phys. Rev. D177 (69) 2239*
C.G. Callan, S. Coleman, J. Wess, B. Zumino, *Phys. Rev. D177 (69) 2239*

The key element is the so−called *connection* of G/H:

$$u(\phi) \xrightarrow{G} g_L\, u(\phi)\, h^{-1} = h\, u(\phi)\, g_R^{-1} \qquad [\, g_{L,R} \in G, \quad h \in H \,]$$

starting from u and ext. sources we get all possible ops. transforming linearly under G:

$$U = u \,\mathrm{I}\, u = u^2 \rightarrow g_L U g_R^{-1}$$
$$U^+ = u^+ \,\mathrm{I}\, u^+ \rightarrow g_R U^+ g_L^{-1}$$
$$\vdots$$

There are several ways to express u in terms of ϕ, but they are all equivalent and leads to the same physical results.

Simplest choice:

$$\boxed{u(\phi) = e^{i\phi/\sqrt{2}F}}$$

F = free parameter [dimension ~ E] $\rightarrow F_\pi$

Lowest−order Lagrangians

$$\phi = \begin{bmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -\eta\sqrt{2/3} \end{bmatrix} \qquad U(\phi) = e^{i\sqrt{2}\phi/F}$$

If we do not include external fields (and symmetry breaking terms), the first non-trivial operator invariant under G appears at $O(p^2)$ and has a unique structure:

$$L_\chi^{(2)} = \frac{F^2}{4} tr\left(\partial_\mu U^+ \partial^\mu U\right) = \frac{1}{2} tr\left(\partial_\mu \phi \partial^\mu \phi\right) + \frac{1}{6F^2} tr\left([\partial_\mu \phi, \phi][\partial^\mu \phi, \phi]\right) + \ldots$$

Coupling fixed to get the standard kinetic term

Infinite series of interaction terms:

all of $O(p^2)$

all ruled by a single coupling: F

To get in contact with the real (non−chiral...) world, we must introduce also the explicit symmetry−breaking terms $\Rightarrow$ this can be done very efficiently by means of suitable external fields:

J. Gasser, H. Leutwyler, *Ann. Phys, 158 (84) 142*

$$L_{QCD}(l^\mu, r^\mu, s, p) = L_{QCD}^{[0]} + \bar{q}_L \gamma^\mu l_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R - \bar{q}_R (s+ip) q_L - \bar{q}_L (s-ip) q_R$$

- We can preserve a (formal) invariance under G choosing appropriate transformation properties for the ext. fields [E.g.: $(s-ip) \rightarrow g_L (s-ip) g_R^{-1}$]
- But we recover the breaking terms *freezing* the ext. fields to appropriate vevs [E.g.: $\langle s \rangle = \mathrm{diag}\,(m_u, m_d, m_s)$]

The other big advantage of this procedure is that, by means of vector and axial vector sources, we systematically describe also the interactions of the pseudo−Goldstone boson fields with the external SM electroweak fields [γ, W & Z]:

$$Q = \mathrm{diag}\,(2/3, -1/3, -1/3)$$

$$r_\mu = -e Q A_\mu + O(Z_\mu)$$

$$l_\mu = -e Q A_\mu - \frac{e}{\sqrt{2}\sin(\vartheta_W)} (T_+ W_\mu^+ + h.c.) + O(Z_\mu)$$

$$T_+ = \begin{bmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The lowest−order chiral realization of $L_{QCD}(l^{\mu}, r^{\mu}, s, p)$ is simply given by:

$$L_{\chi}^{(2)}(l^{\mu}, r^{\mu}, s, p) = \frac{F^2}{4} \, tr\left(D_{\mu} U^{+} D^{\mu} U + \chi^{+} U + U^{+} \chi\right)$$

where $\quad D_{\mu} U = \partial_{\mu} U + i U l_{\mu} - i r_{\mu} U \,, \qquad \chi = 2B(s+ip) \sim O(p^2)$

and it's completely determined in terms of two couplings: F & B

Looking at the derivatives of the generating functional with respect to the external fields, it is easy to realize that F & B can be determined by the two following (non−perturbative) expectation values:

$$\langle 0|\bar{u}_L \gamma^{\mu} d_L|\pi^{-}(p)\rangle \stackrel{def}{=} \frac{-i}{\sqrt{2}} F_{\pi} p^{\mu} = \frac{-i}{\sqrt{2}} F\,[1 + O(m_q)]\, p^{\mu}$$

$$\langle 0|\bar{q} q|0\rangle = -B F^2 \;\Rightarrow\; \begin{aligned} (M_{\pi})^2 &= B\,(m_u + m_d) \\ (M_{K^+})^2 &= B\,(m_u + m_s) \\ (M_{K^0})^2 &= B\,(m_d + m_s) \end{aligned}$$

1 parameter−free prediction:

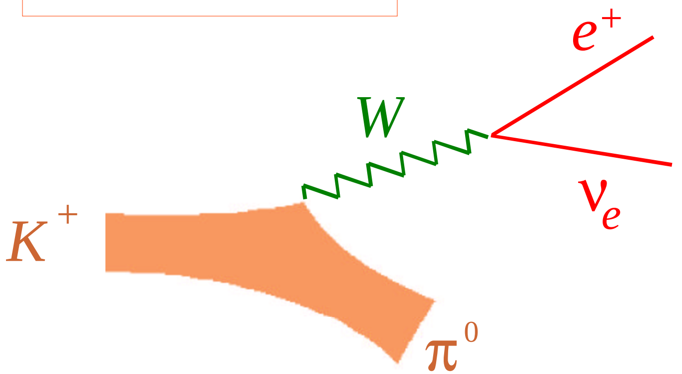
$$3 M_{\eta^8}^2 = 4 M_K^2 - M_{\pi}^2$$

[Gell−Mann−Okubo mass formula]

Computing physical processes by means of $L_\chi^{(2)}(l^\mu, r^\mu, s, p)$ we recover all known results of current algebra in a very simple way

Explicit example of a tree−level process:

$$K^+ \to \pi^0 e^+ \nu_e$$



To a good approximation, within this process the *W* boson can be considered as an external source from the point of view of QCD (or CHPT)

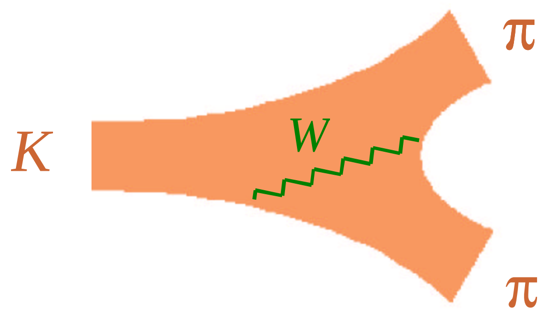
$$l_\mu \to -\frac{e\,V_{us}^*}{\sqrt{2}\sin\vartheta_W} W_\mu^- \delta_{31}$$

$$L_{1-l_\mu}^{(2)} = \frac{i F^2}{2}\, tr\left(l^\mu \partial_\mu U^+ U\right) = -\frac{i e F^2 V_{us}^*}{2\sqrt{2}\sin\vartheta_W} W^\mu \,(\partial_\mu U^+ U)_{13} + \ldots$$

$$= -\frac{i e V_{us}^*}{4\sin\vartheta_W} W^\mu \,(\partial_\mu \pi^0 K^+ - \partial_\mu K^+ \pi^0) + \ldots$$

$$\Rightarrow \quad A(K^+ \to \pi^0 e^+ \nu_e)^{(2)} = \frac{i G_F V_{us}^*}{2}(p_K + p_\pi)_\mu \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5) v(p_e)$$

• Non−leptonic weak interactions



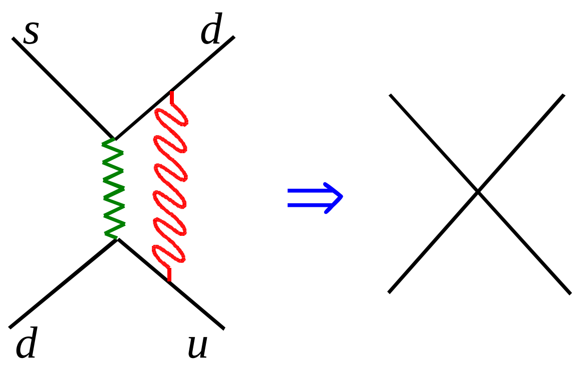
Within these processes the W cannot be considered anymore as an external field

two very different scales involved: M_K & M_W

"double" EQFT approach to simplify the problem

1) Construction of the *partonic* effective $|\Delta S|=1$ Hamiltonian

Purpose: resummation of the *large logs* generated in pQCD



G. Altarelli, L. Maiani, *'74*

M.K. Gaillard, B.W. Lee, *'74*

⋮

A.J. Buras *et al.*, *'93−'94*

M. Ciuchini *et al.*, *'93−'94*

large *logs* of the type $\alpha_S \log\left[p^2/M_W^2\right]$ p ~ external momenta

$$H_{eff}^{|\Delta S|=1} = \sum_i C_i(\mu) O_i \quad + \quad O(\mu^2/M_W^2)$$

Wilson coefficients

$\alpha_S^n(\mu) \cdot \log(\mu/M_W)^n$,

$\alpha_S^{n+1}(\mu) \cdot \log(\mu/M_W)^n, \ldots$

resummed by means of RGE

4−quark operators

$\sim \quad \bar{s}^\alpha \Gamma d^\beta \quad \bar{u}^\beta \Gamma u^\alpha$

μ = renormalization scale

$\Lambda_{QCD} \ll \mu \ll M_W$

$$A(K \to \pi\pi) = \sum_i C_i(\mu) \; \langle \pi\pi | Q_i | K \rangle(\mu)$$

pQCD effects

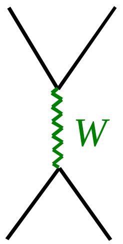
non−perturbative dynamics

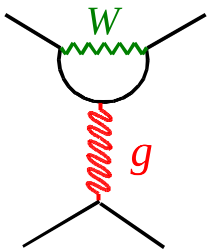
Original problem reduced to the evaluation of hadronic matrix elements of a finite set of four−quark operators (11 for $m_c < \mu < m_b$)

2) *Chiral realization* of the $|\Delta S|=1$ non−leptonic Lagrangian

$$H_{eff}^{|\Delta S|=1} = \sum_i C_i(\mu) Q_i$$ transforms linearly under $SU(3)_L \times SU(3)_R$

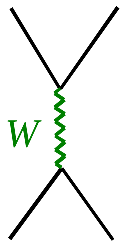
$(8_L, 1_R)$





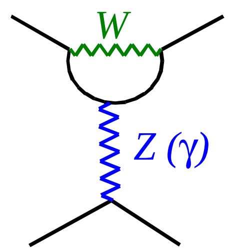
first non−vanishing chiral realization at $O(p^2)$ (unique)

$(27_L, 1_R)$



first non−vanishing chiral realization at $O(p^2)$ (unique)

$(8_L, 8_R)$



non−vanishing chiral real. at $O(p^0)$

$\Rightarrow$ coeff. suppressed by $O(e^2)$

relevant to *CPV* only

N.B.: Chiral symmetry alone does not help to evaluate hadronic matrix elements of 4−quark operators (new unknown couplings), it only helps to relate each other the matrix elements of a given operator in different processes

Lowest−order non−leptonic weak Lagrangian:

$$
\begin{aligned}
L_W^{(2)} \;&=\; G_8 F^4\, tr\left(\hat{\lambda}\, L_\mu L^\mu\right) \\
&+ G_{27} F^4 \left[(L_\mu)_{23} (L^\mu)_{11} + \frac{2}{3} (L_\mu)_{21} (L^\mu)_{13} \right] \\
&+ G_{\underline{8}} F^6\, tr\left(\hat{\lambda}\, U^+ Q\, U\right) \;+\; h.c.
\end{aligned}
$$

$$L_\mu = (L_\mu)^+ = i\, U^+ D_\mu U = u^+ u_\mu u$$

$$\hat{\lambda} \;=\; \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Most general structure of $O(e^0 p^2)$ & $O(e^2 p^0)$

G_i = new unknown couplings to be determined by data (or Lattice−QCD)

Using the exp. results on $A(K \to 2\pi)$:

$$|G_8| \;\simeq\; 9.1 \times 10^{-6} \;\; \text{GeV}^{-2}$$

$$|G_{27}/G_8| \;\; 5.7 \times 10^{-2}$$

Parameter−free predictions for the leading CP−conserving amplitudes of $K\to 3\pi$, $K\to 2\pi\gamma$ & $K\to 3\pi\gamma$, which typically differ by factors of 20−30% from experimental data

Beyond lowest order

The present accuracy of experimental data on K decays is certainly a good motivation to look for NLO terms in the chiral expansion

$$e^{Z[J]} = \int Du(\phi)\ e^{\int L_\chi^{(2)} + L_\chi^{(4)} + \ldots.} \quad \Rightarrow \quad Z[J] = Z^{(2)}[J] + Z^{(4)}[J] + \ldots$$

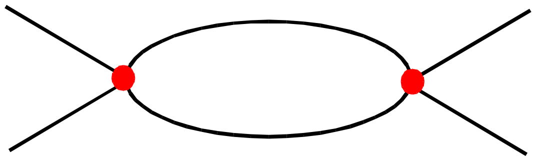
$Z^{(2)}[J]$ = tree−level amplitudes from $L_\chi^{(2)}$

$Z^{(4)}[J] = L_\chi^{(2)} \otimes L_\chi^{(2)}$ loops

+ tree–level from $L_\chi^{(4)}$ [$\supset$ counterterms needed to regularize one−loop div.]

+ chiral realization of the axial anomaly [Wess−Zumino−Witten term]

$\vdots$



$$\frac{p^2}{F_\pi^2} \times \frac{1}{16\pi^2} \times \frac{p^2}{F_\pi^2}$$

- The loop expansion provides an independent indication for the effective scale controlling the chiral expansion:

$$\Lambda_\chi = 4\pi F_\pi \sim 1.2 \text{ GeV} \quad [\Rightarrow \text{good convergence}]$$

- Unfortunately, the predictivity of the theory decreases (*but does not vanish!*) because of the increase of free parameters

Present status of NLO chiral Lagrangians in the mesonic sector:

$O(p^4)$
Gasser−Leutwyler
’84−’85
$[L_i : \underline{10} + 2]$

$\xrightarrow{\text{2 meson loops}}$

$O(p^6)$
Fearing−Scherer ’96
Bijnens *et al.* ’99
$[C_i : \underline{90} + 4]$

$\xrightarrow{\text{virtual photons}}$

$O(e^2p^2)$
Urech ’95
Neufeld−Rupertsberger ’96
$[K_i : \underline{11} + 3]$

$\xrightarrow{\text{virtual leptons}}$

$O(e^2p^2)$
Knecht *et al.* ’99
$[X_i : \underline{5} + 2]$

$\xrightarrow{\text{internal W's}}$

$O(G_8\, p^4)$
Kambor *et al.* ’91
Ecker *et al.* ’94
$[N_i : \underline{22}\ (\underline{11}+11)+15]$

$\oplus$

$O(G_{27}\, p^4)$ $[D_i : \underline{23}+5]$
Kambor *et al.* ’91, E.−Farese ’91

$\xrightarrow{\text{virtual photons}}$

$O(G_8\, e^2p^2)$
Cirigliano *et al.* ’99
Ecker *et al.* ’00
$[Z_i : \underline{15} + 17]$

$\underline{\text{N}}$ = n. of relevant physical couplings

$\underline{\text{N}}$ = n. of independent combinations in allowed processes

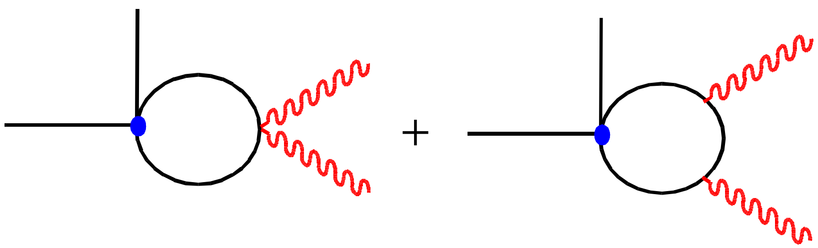
Despite the large number of new parameters, the theory still have a significant predictive power, since:

- In several subsets of processes we can identify more observables that CT combinations [e.g.: $K \to 3\pi$ Dalitz plots]

$K \to 3\pi$ amplitudes:

	$O(p^2)$	$O(p^4)$	Exp.
α_1	$+74.0$	input	$+92.12 \pm 0.32$
β_1	-16.5	input	-26.73 ± 0.27
ζ_1	$-$	-0.51 ± 0.18	-0.38 ± 0.19
ξ_1	$-$	-1.66 ± 0.19	-1.80 ± 0.29
α_3	-4.1	input	-6.41 ± 0.44
β_3	-1.0	input	-2.26 ± 0.44
γ_3	$+1.8$	input	$+2.89 \pm 0.28$
ζ_3	$-$	-0.01 ± 0.01	-0.09 ± 0.10
ξ_3	$-$	$+0.07 \pm 0.03$	$+0.17 \pm 0.15$
ξ_3'	$-$	-0.15 ± 0.08	-0.57 ± 0.41

- In some cases symmetry arguments forbids any CT contribution [e.g.: $K_S \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$, $\eta \to \pi^0\gamma\gamma$, ...]

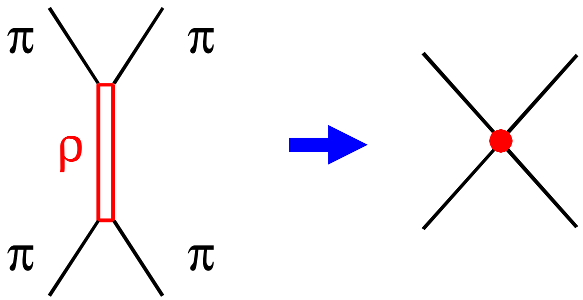


$+ \ldots \quad \Rightarrow$ finite $O(p^4)$ amplitude

- In some cases the (loop−induced) chiral logs dominate over local CT [e.g.: $K_L \to e^+e^- \quad \Rightarrow \quad \log(m_K^2/m_e^2)$]

Moreover, we can try to estimates the value of the CT using non−perturbative information about QCD (i.e. going beyond the pure CHPT approach):

- Resonance saturation (and large N_c)



good agreement with exp. determinations in the strong sector.

i	$10^3 L_i^r(M_\rho)$	V	A	S	S_1	η_1	Total
1	0.53 ± 0.25	0.6	0	−0.2	$0.2^{b)}$	0	0.6
2	0.71 ± 0.27	1.2	0	0	0	0	1.2
3	-2.72 ± 1.12	−3.6	0	0.6	0	0	−3.0
4	-0.3 ± 0.5	0	0	−0.5	$0.5^{b)}$	0	0.0
5	0.91 ± 0.15	0	0	$1.4^{a)}$	0	0	1.4
6	-0.2 ± 0.3	0	0	−0.3	$0.3^{b)}$	0	0.0
7	-0.32 ± 0.15	0	0	0	0	−0.3	−0.3
8	0.62 ± 0.2	0	0	$0.9^{a)}$	0	0	0.9
9	6.9 ± 0.7	$6.9^{a)}$	0	0	0	0	6.9
10	-5.5 ± 0.7	−10.0	4.0	0	0	0	−6.0

a) Input. b) Estimate based on the limit $N_c \to \infty$.

- Matching between CHPT & dispersion relations

  excellent tool in (strong) processes with large amount of data, e.g.: ππ→ππ

- Matching between CHPT & Lattice−QCD

  most promising tool in the case of non−leptonic weak interactions however, present simulations are still far from precision estimates

A detailed example: K_{l3} decays and the Cabibbo angle

The rates of the four K_{l3} decays [$K=K^+,K_L$ $l=e,\mu$] can be written as

$$\Gamma = G_F^2 M_K^5 \times |V_{us}|^2 \times |f_+(0)|^2 \times I(df/dt)$$

kinematical integral: mild sensitivity to df_+/dt (and f_-/f_+ for $l=\mu$) and e.m. corrections

vector form factor at zero momentum transfer [$t=(p'-p)^2=0$]

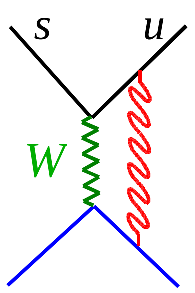
$$\langle K(p')|\bar{u}\gamma_\mu s|\pi(p)\rangle = C\left[(p'+p)_\mu f_+(t) + (p'-p)_\mu f_-(t)\right]$$

CVC $\Rightarrow$ $f_+(0) = 1$ in the $SU(3)$ limit $m_s = m_u = m_d$

Three main issues to address in order to extract $|V_{us}|$:

- estimates of the $SU(3)$ breaking term $f_+(0)-1$
- e.m. corrections
- kinematical dependence of the form factors [mainly an exp. issue]

E.m. corrections:



I. short−distance corrections to the $s \to u\, l\, \nu_l$ eff. Hamiltonian

sizable [$\sim \alpha \log(\mu_{\mathrm{had}}/M_W)$ $\Rightarrow$ $\delta\Gamma \sim 1\%$]

well known

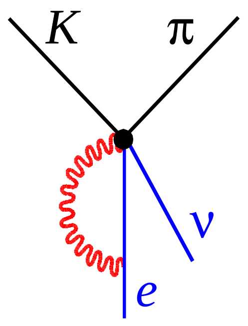
Marciano & Sirlin, '70− '80

II. pure long−distance corrections (IR div. & bremss.)

sizable [$\sim \alpha \log(M_K/m_e)$ $\Rightarrow$ $\delta\Gamma \sim 1\%$]

only partially known till last year

Ginsberg, '66− '69
(Coulomb corrections)



III. structure−dependent (intermediate−scale) terms

small [no large logs $\Rightarrow$ $\delta\Gamma \sim 0.1\%$]

model dependent

Coherent analysis of the 3 effects (particularly II. + III.) possible in the framework of CHPT [non−trivial results at $O(e^2p^2)$]

Cirigliano *et al.* '01

Systematic estimate of the model−dependent terms obtained by varying the $O(e^2p^2)$ counterterms within conservative ranges

A crucial point in the analysis of e.m. corrections is the identification of I.R. safe observables.

most convenient choice: $\Gamma(K_{l3})_{\text{incl.}} = \sum \Gamma(K \to \pi\, l\, \nu_l + n\gamma)$

The recent work by Cirigliano et al. provides a clear prescription to separate, in this observable, model−independent non−local terms [which modify the decay distrbution] from the local counterterms of $O(e^2p^2)$ [which can be re−absorbed in $f(0)$]

e.g.: $\Gamma(K^+_{e3})_{\text{incl.}} \propto |f_+(0)|^2 \times I(df/dt)$

$\delta_{CT} = (+0.36 \pm 0.16)\%$

$\delta_{QED} = -1.27\%$

N.B.: are we sure that the (old) PDG data on $\Gamma(K_{l3})$ are completely inclusive ?

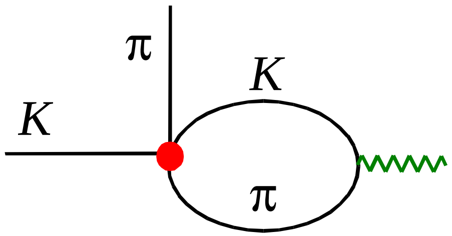
$\Rightarrow$ important exp. issue to extract V_{us} (together with the kinem. dependence of the form factor) in view of new precise measurements

Th. estimates of $f_+(0) - 1$

- No linear corrections in $(m_s - m_u)$ [Ademollo−Gatto theorem, '64]

$$\Downarrow$$

- At $O(p^2)$ [LO in CHPT]: $\delta = f_+^{K^0\pi^+}(0) - 1 = 0$
- At $O(p^4)$: finite (unambiguous) non−polynomial corr. induced by meson loops
  [$\sim m_P \log m_P \Rightarrow \sim (m_s - m_u)^2 / m_s$] numerically small: $\delta^{(4)} = -2.2\%$



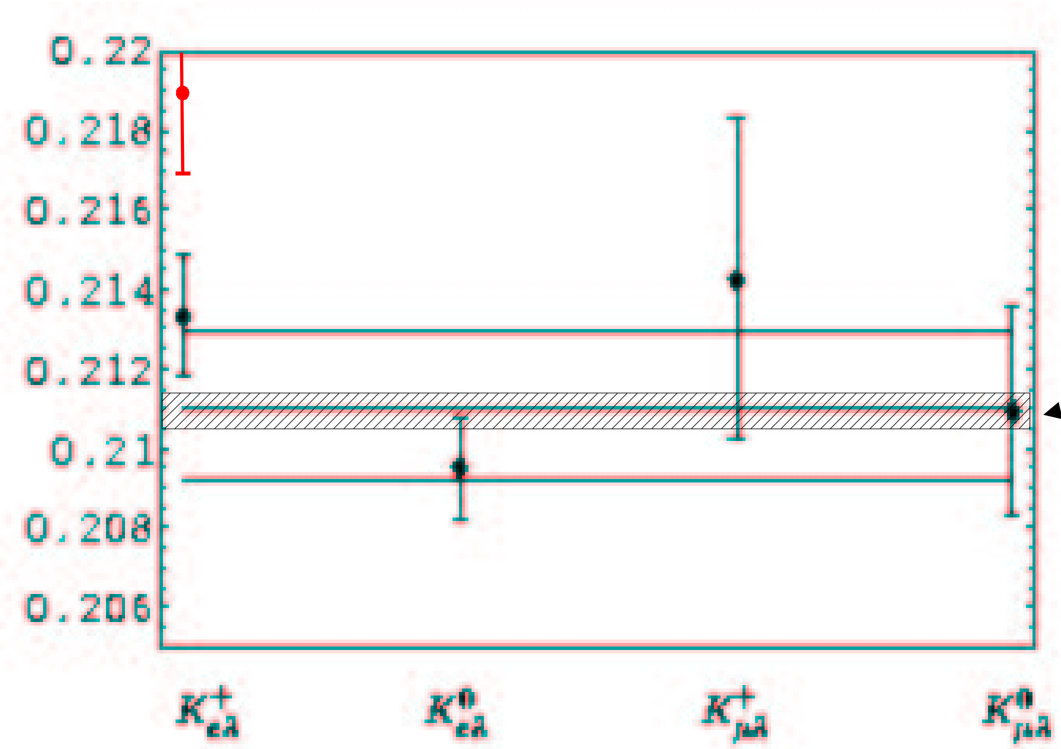
$\Rightarrow$ precise determination of the isospin−breaking ratio $f_+^{K^0\pi^+}(0) / f_+^{K^+\pi^0}(0)$

- At $O(p^6)$: appearance of $B^2 (m_s - m_u)^2 / \Lambda_\chi^4$ local terms
  reasonable estimate: $\delta^{(6)} = -1.6 \pm 0.8\ \%$ [Leutwyler−Roos, '84]

Is it really conservative? Can we improve the determination of CT by means of other processes and/or Lattice−QCD ? $\Rightarrow$ interesting on−going activity

The extraction of $|V_{us}|$

$f(0) \times V_{us}$



data after the inclusion of $SU(2)$ corrections

th. error [on $SU(2)$ corr.]

Cirigliano, Colangelo, Lopez−Castro & G.I. ’02 [CERN CKM workshop]

● = 2002 PDG values [old data] ● = prelim. BNL−E865

$(V_{us})_{today} = 0.2196 \pm 0.0026$ $(\Delta V_{us})_{today} = 0.86\%_{exp} + 0.82\%_{th.-f(0)}$

$(V_{us})_{unit.} = [1-|V_{ud}|^2-|V_{ub}|^2]^{1/2} = 0.2269 \pm 0.0021$ 2.2σ (3%) difference!

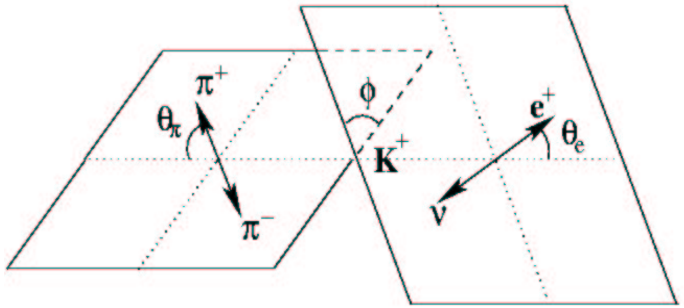
A rather intriguing situation which KLOE could substantially help to clarify, already with the available data

Other interesting issues in kaon physics

K_{l4} and $\pi\pi$ phase shifts

$K^+ \to (\pi^+\pi^-) l^+ \nu$ form factors: $F_i(s) = f_i^o(s)\, e^{i\delta_0^0(s)} + \ldots$ strong $\pi\pi$ phases

possible to isolate the contribution of the δ's by looking at the *asymmetry* in the distribution of the angle between $\pi\pi$ and $l\nu$ planes



$\pi\pi$ phase shifts near thresholds [$\Leftrightarrow a_J^I$ scattering lengths] are among the most precise observables we can compute in CHPT, and also among the most interesting ones [a_0^0 strongly depends from the beahaviour of $\langle 0|\bar{q}q|0\rangle$ in the chiral limit]:

$$\delta_0^0(s) \Leftrightarrow a_0^0 = \left[\begin{array}{lll} 0.16 & O(p^2) & \text{Weinberg '79} \\ 0.20 \pm 0.01 & O(p^4) & \text{Gasser \& Leutwyler '83} \\ 0.220 \pm 0.005 & O(p^6) & \text{Bijens, Colangelo, Ecker, Gasser \& Leutw. '99} \\ & & \text{Ananthanarayan } et\ al. \text{ '01} \end{array}\right.$$

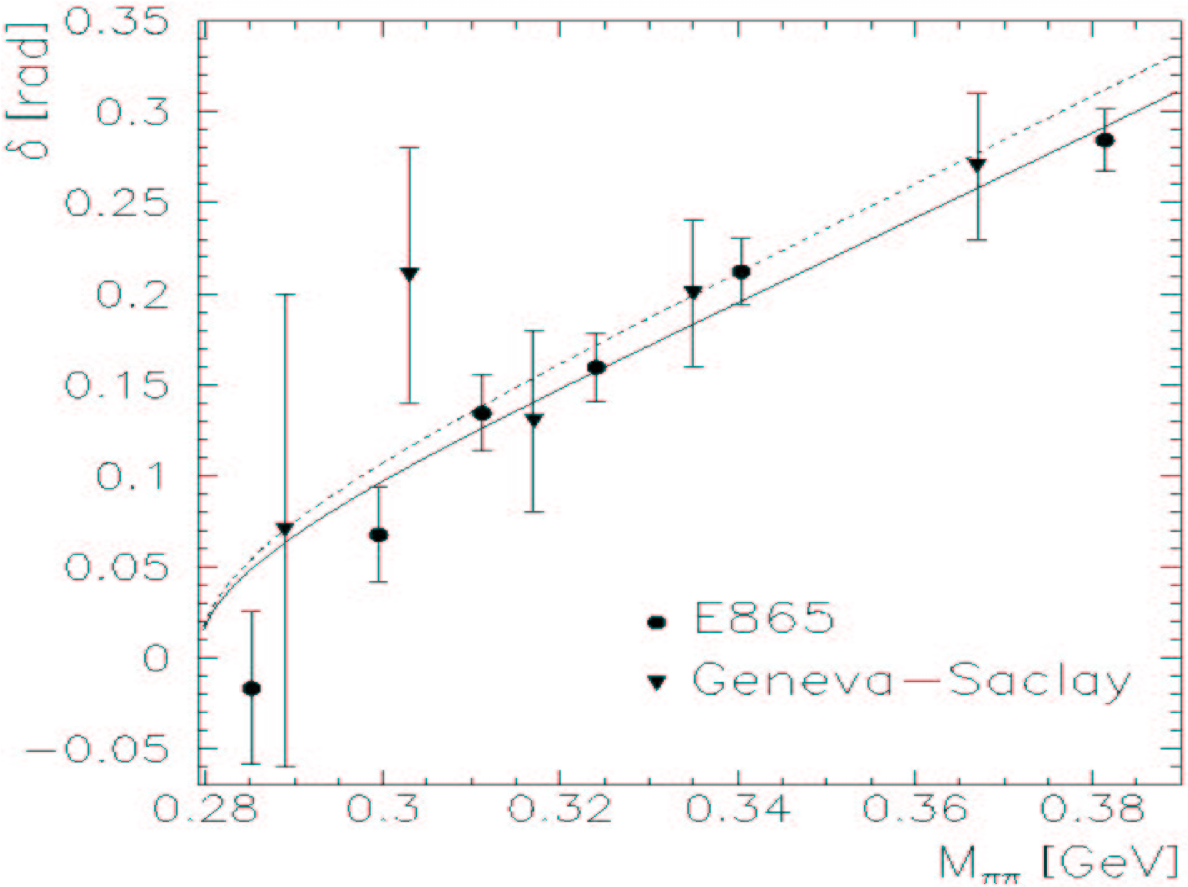
A recent measurement by BNL−E865 [hep−hp/0301040] as provided an important check of CHPT expectations:

$$\boxed{a_0^0 = 0.216 \pm 0.013}$$

BNL−E865 [+ th. contsr. on a_2]

$$\boxed{a_0^0 = 0.220 \pm 0.005}$$

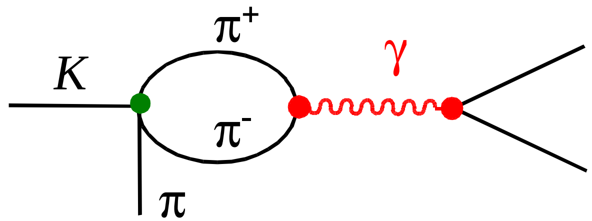
CHPT [+ disp. relations]



Probably we will learn even more from future data by KLOE & NA48b

$K \to \pi l^+ l^-$ decays & short–distance dynamics

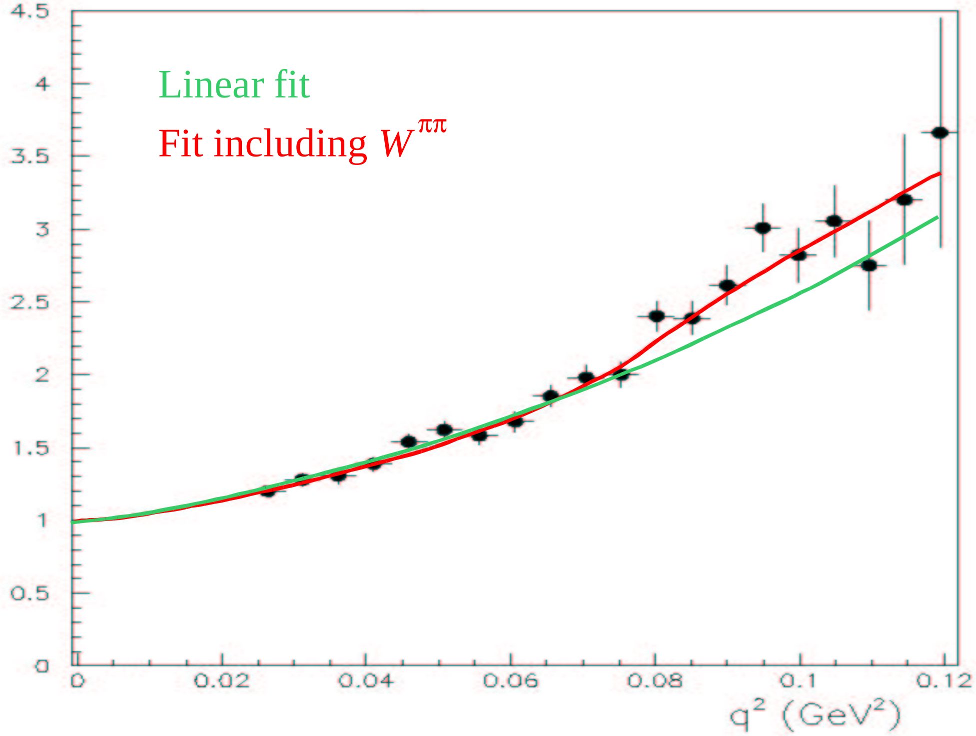
The rare FCNC modes $K^{\pm} \to \pi^{\pm} l^+ l^-$ & $K_S \to \pi^0 l^+ l^-$ are dominated by long–distance amplitudes $\Rightarrow$ calculable in CHPT up to local terms:



$\Rightarrow$ weak $K \to \pi\gamma^*$ form factor: $F(z) = W^{\pi\pi}(z) + W^{\rm pol}(z)$

model–independent

2 independent CT for charged & neutral channels: we cannot predict $B(K_S \to \pi^0 e^+ e^-)$ using the measured $B(K^+ \to \pi^+ e^+ e^-) = (2.94 \pm 0.14) \times 10^{-7}$



Recent data on $K^+ \to \pi^+ e^+ e^-$ show a significant evidence for $W^{\pi\pi}(z)$ [good consistency of the theory]

however, the value of the CT is larger than expected in naïve power counting

⬇

Important clue of VMD in weak decays

To fully understand the VMD mechanism of these transition we would need to observe also the neutral mode: $K_S \to \pi^0 e^+ e^-$ [VMD expectation: BR ~ $10^{-8} - 10^{-9}$]

Then we would have all the ingredients to extract the interesting FCNC short−distance amplitude from a (future) measurement of $B(K_L \to \pi^0 e^+ e^-)$

Components of the $K_L \to \pi^0 e^+ e^-$ amplitude [single γ exchange forbidden by CP]:

1. direct CPV amplitude

   short−distance dominated, proportional to $\mathrm{Im}(V_{ts}^* V_{td})$

   $B(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV-dir}} \sim 4 \times 10^{-12}$

2. indirect CPV: $B_{\mathrm{CPV-ind}} = 3 \times 10^{-3} \times \mathrm{B}(K_S \to \pi^0 e^+ e^-)$

   if the VMD description works, we will be able to determine also the sign of the interference

3. CPC amplitude: $B_{\mathrm{CPC}} < 10^{-12}$

   strong constraints derived again from CHPT, be means of the recent data on $K_\mathrm{L} \to \pi^0 \gamma\gamma$ [NA48 '00]

Conclusions

CHPT is a rather mature EQFT [NNLO analyses of several observables have already been performed] and there is no doubt that it provides a very powerful tool to for precise/systematic studies of low−energy physics.

The fact that CHPT is a mature subject does not mean is not anymore an interesting field:

- We don't need anymore to *tests* CHPT [there is no doubt the the theory works...] but we still need to investigate it's validity range

- A lot of activity is presently focused on how to merge CHPT with other non−perturbative methods in oder to enhance the predictive power [two main directions: lattice QCD, large N_c]