

$g - 2$

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Karlsruhe - Fall 2001

1. The gyromagnetic ratio

By definition, the gyromagnetic ratio g of a state of angular momentum J and magnetic moment μ is:

$$g = \frac{\mu}{\mu_0} / \frac{J}{\hbar}.$$

For a particle of charge e in a state of orbital angular momentum \mathbf{L} we have:

$$\vec{\mu} = \mu_0 \mathbf{L}, \quad \mu_0 = \frac{e}{2m}, \quad g = 1.$$

For an electron $\mu_0 = \mu_B = 5.788 \dots \times 10^{-11} \text{ MeV T}^{-1}$ (± 7 ppb).

The importance of g in particle physics is many-fold. A gross deviation from the expected value, 2 for charged spin 1/2 *Dirac* particles, is clear evidence for structure.

Thus the electron and the muon ($g \sim 2.002$) are elementary particles while the proton, with $g_p = 5.6$ is a composite object. For the neutron g should be zero, measurements give $g_n = -3.8$

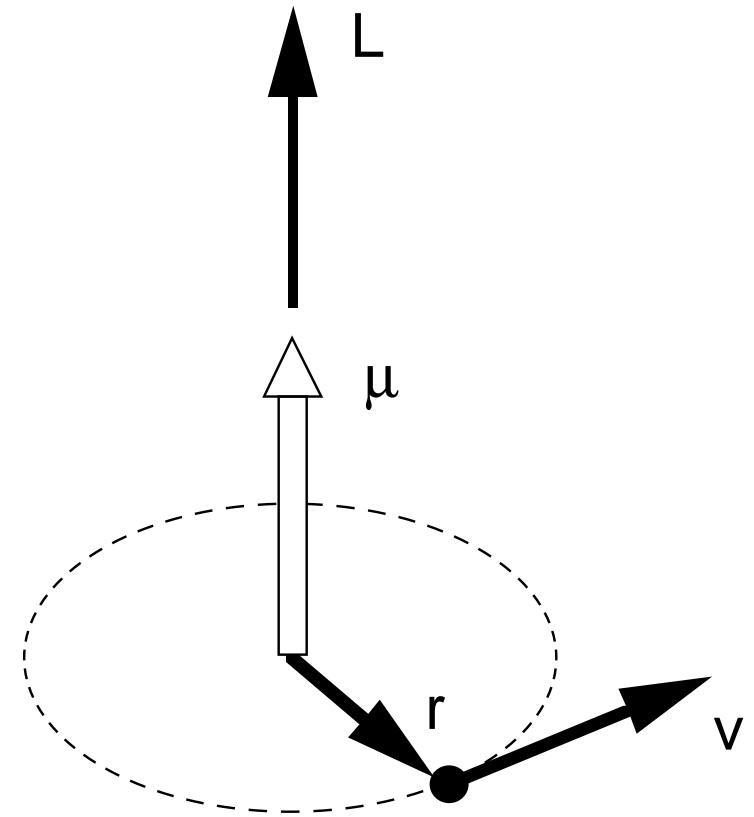
Small deviations from 2, $\sim 0.1\%$, appear as consequence of the self interaction of the particles with their own field. Experimental verifications of the computed deviations are a triumph of QED.

We also define the anomaly, $a = (g - 2)/2$, a measure of the so called anomalous magnetic moment, $(g - 2)\mu_0$.

QED is not all there is in the physical world. The EW interaction contributes to a and new physics beyond the standard model might manifest itself as a deviation from calculations.

2. Magnetic moment

The classical physics picture of the magnetic moment of a particle in a plane orbit under a central force is illustrated on the side. $\vec{\mu}$ is along \mathbf{L} , $\mu_0 = q/2m$ and $g=1$. This remains true in QM. For an electron in an atom, $\mu_B = e/2m_e$ is the Bohr magneton. $\mathbf{L} \parallel \vec{\mu}$ is required by rotational invariance.



When we get to intrinsic angular momentum or spin the classic picture loses meanings and we retain only $\vec{\mu} \parallel \mathbf{L}$. We turn now to relativistic QM and the Dirac equation.

2.1 g for Dirac particles

In the non-relativistic limit, the Dirac equation of an electron interacting with an electromagnetic field ($p_\mu \rightarrow p_\mu + eA_\mu$) acquires the term

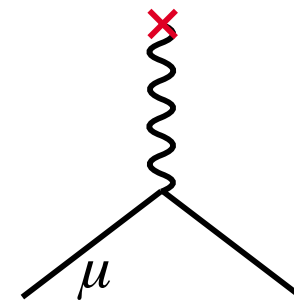
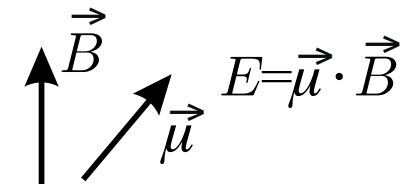
$$\frac{e}{2m} \vec{\sigma} \cdot \mathbf{B} - eA^0$$

which implies that the electron's intrinsic magnetic moment is

$$\vec{\mu} = \frac{e}{2m} \vec{\sigma} \equiv g \frac{e}{2m} \mathbf{S} \equiv g\mu_B \mathbf{S},$$

where $\mathbf{S} = \vec{\sigma}/2$ is the spin operator and $g=2$.

The prediction $g=2$ for the intrinsic magnetic moment is one of the many triumphs of the Dirac equation.



3. Motion and precession in a B field

The motion of a particle of momentum p and charge e in a uniform magnetic field B is circular with $p = 300 \times B \times r$. For $p \ll m$ the angular frequency of the circular motion, called the cyclotron frequency, is:

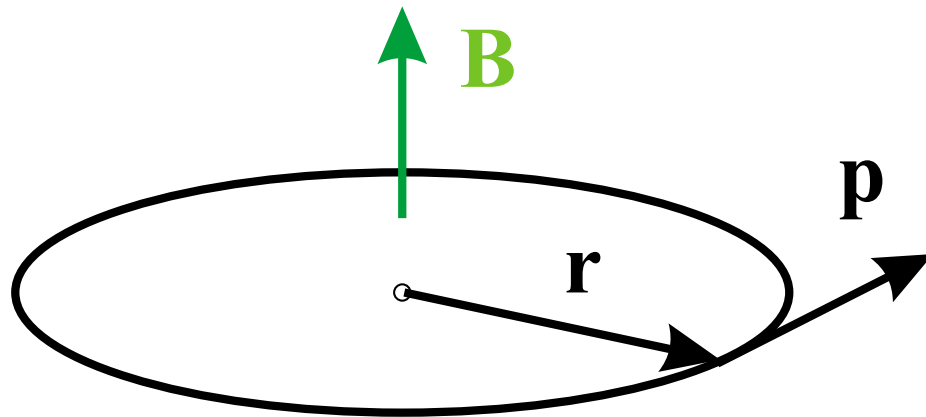
$$\omega_c = \frac{eB}{m}.$$

The spin precession frequency at rest is given by:

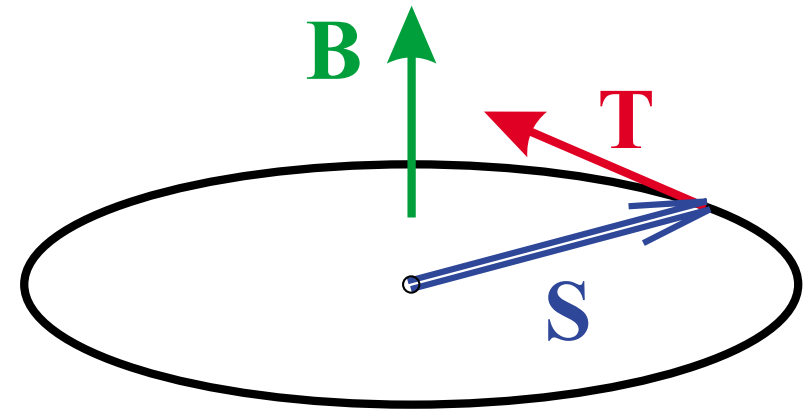
$$\omega_s = g \frac{eB}{2m}$$

which, for $g=2$, coincides with the cyclotron frequencies.

This suggests the possibility of directly measuring $g - 2$.



'Cyclotron' orbit, ω_C



Spin precession, ω_S

For higher momenta the frequencies become

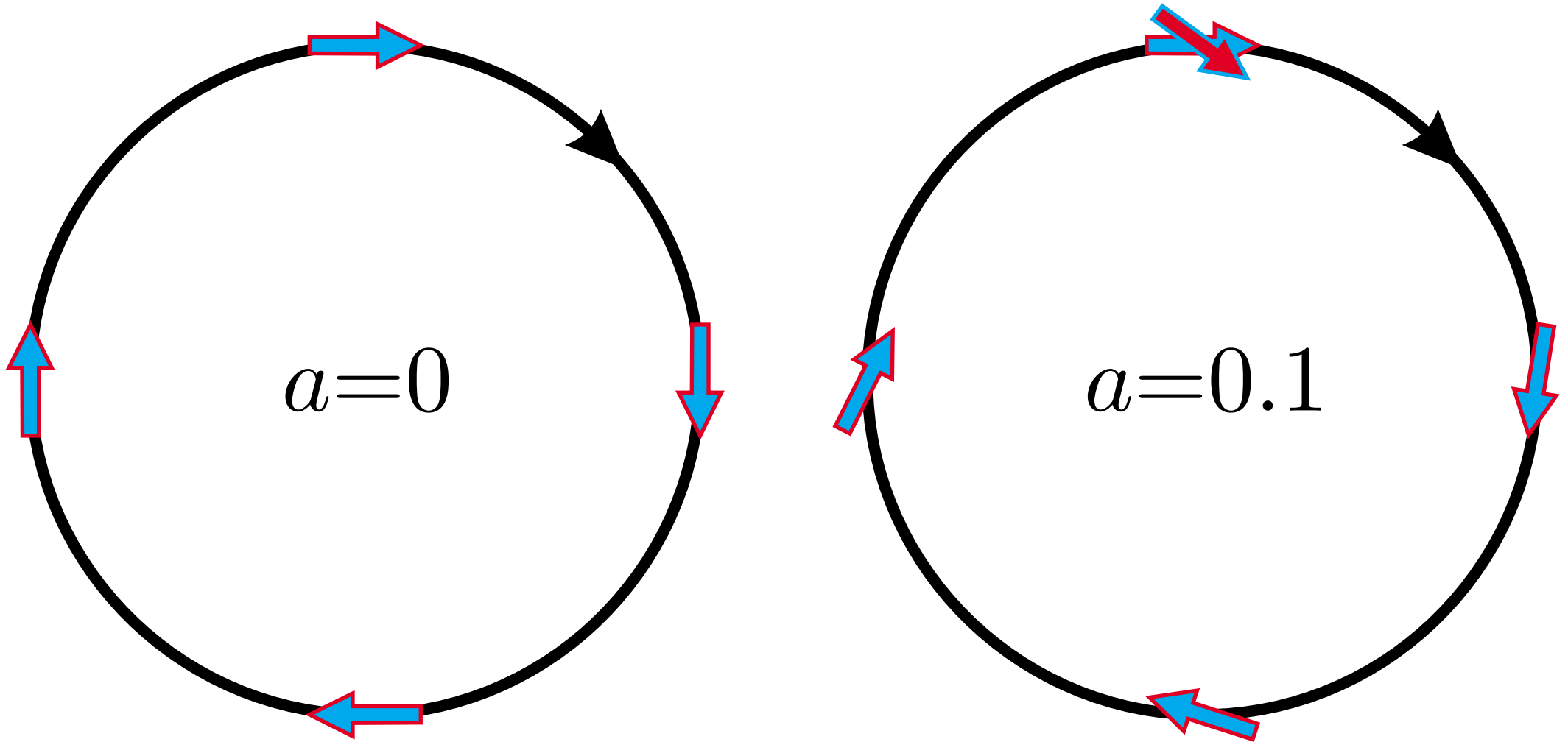
$$\omega_C = \frac{eB}{m\gamma}$$

and

$$\omega_S = \frac{eB}{m\gamma} + a \frac{eB}{m}$$

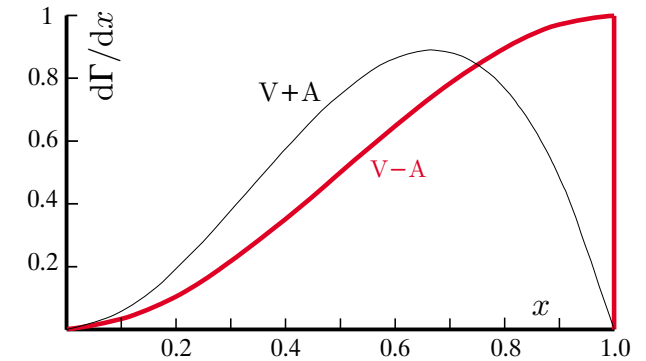
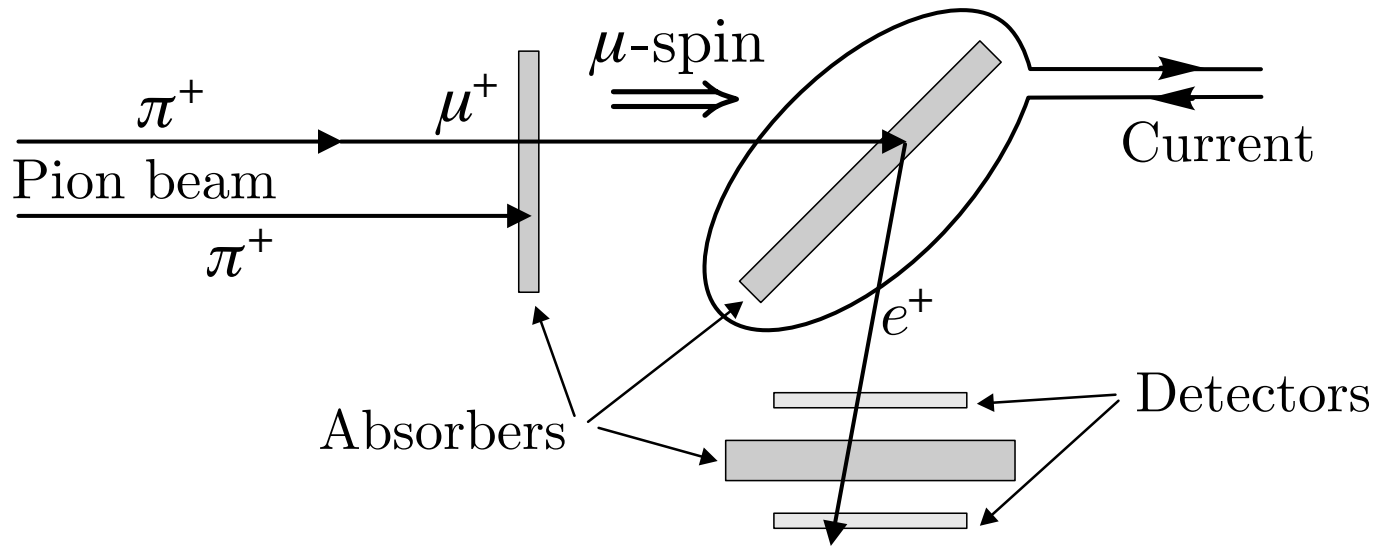
or

$$\omega_a = \omega_S - \omega_C = a \frac{eB}{m} = a\gamma\omega_C$$



For $a = 1$ ($\gamma=1$), spin rotates wrt momentum by $1/10$ turn per turn.

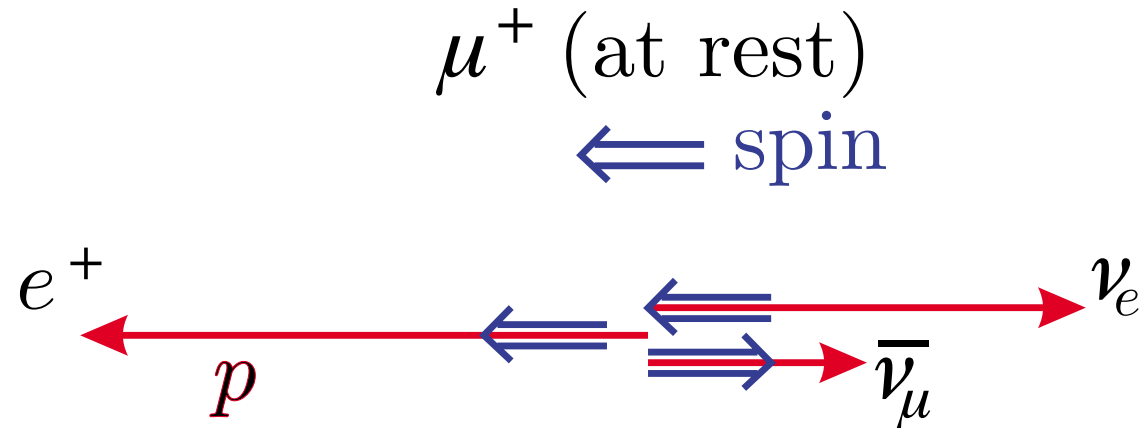
4. $\pi \rightarrow \mu \rightarrow e$



The rate of high energy decay electrons is time modulated with a frequency corresponding to the precession of a magnetic moment $e/m(\mu)$ or a muon with $g=2$. **First measurement of $g(\mu)$!!**

Also a proof that P and C are violated in both $\pi\mu\nu$ and $\mu \rightarrow e\nu\bar{\nu}$ decays.

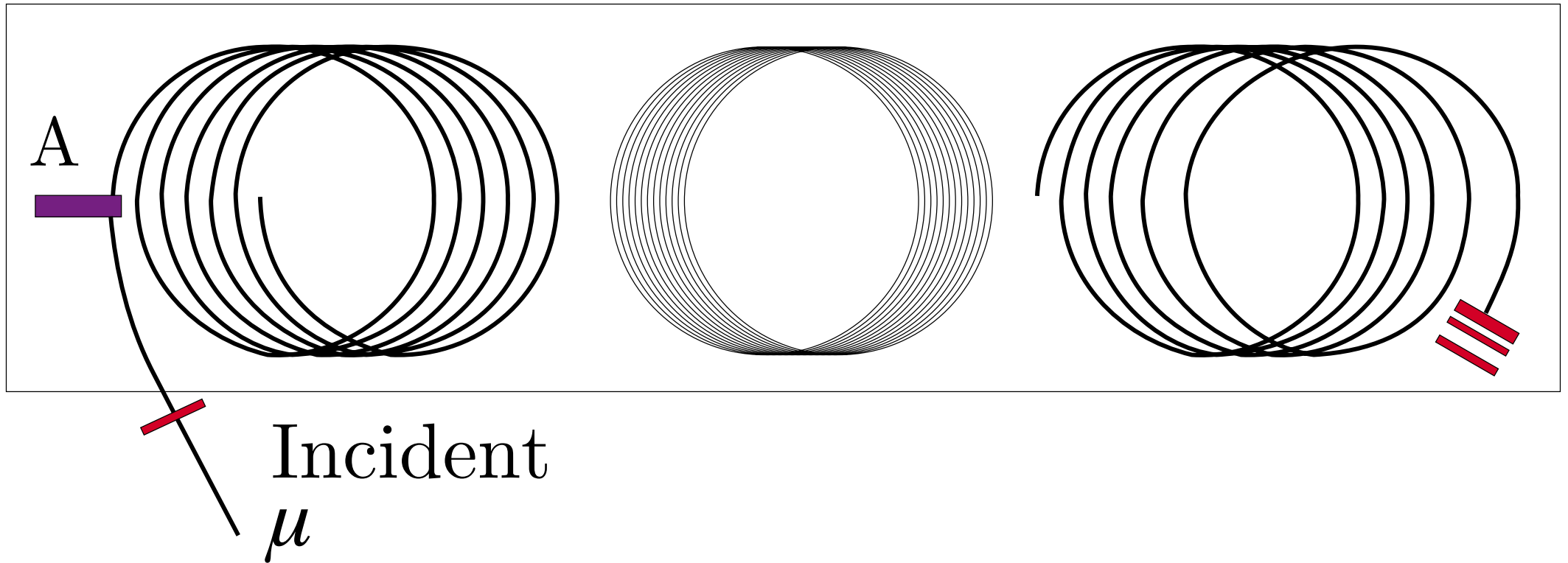
S-p correlation fundamental to all muon anomaly experiments



High energy positrons have momentum along the muon spin. The opposite is true for electrons from μ^- .

Detect high energy electrons. The time dependence of the signal tracks muon precession.

5. The first muon $g - 2$ experiment Shaped B field

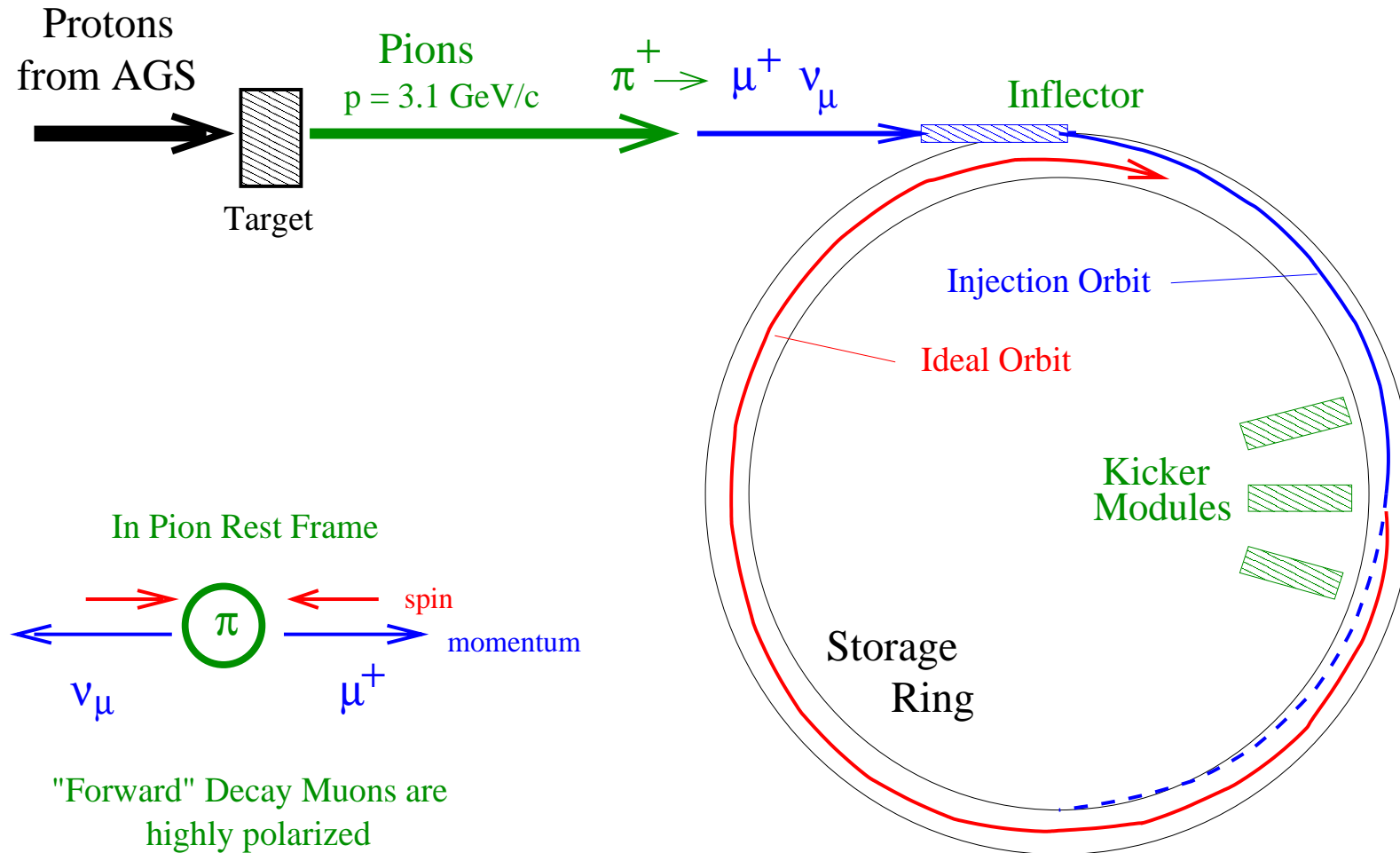


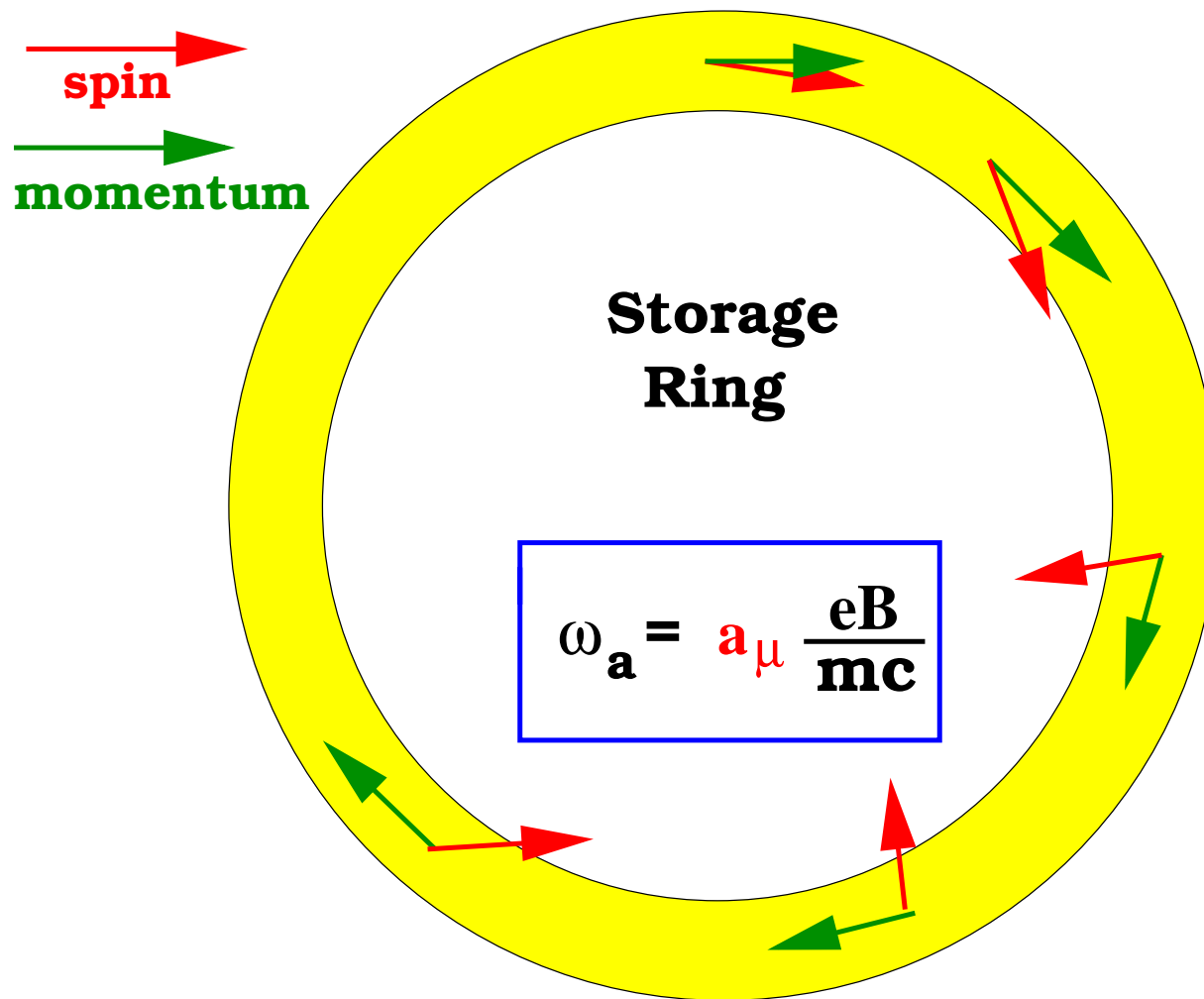
Performed in CERN, in the sixties. Need more turns, more γ .

Next step: a storage ring.

6. The BNL g-2 experiment

$(g-2)_\mu$ Experiment at BNL





(exaggerated ~20x)

With homogeneous \vec{B} , all muons precess at same rate

With homogeneous \vec{B} , use quadrupole \vec{E} to focus and store beam

Spin Precession with \vec{B} and \vec{E}

$$\vec{\omega}_a = \frac{e}{mc} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right]$$

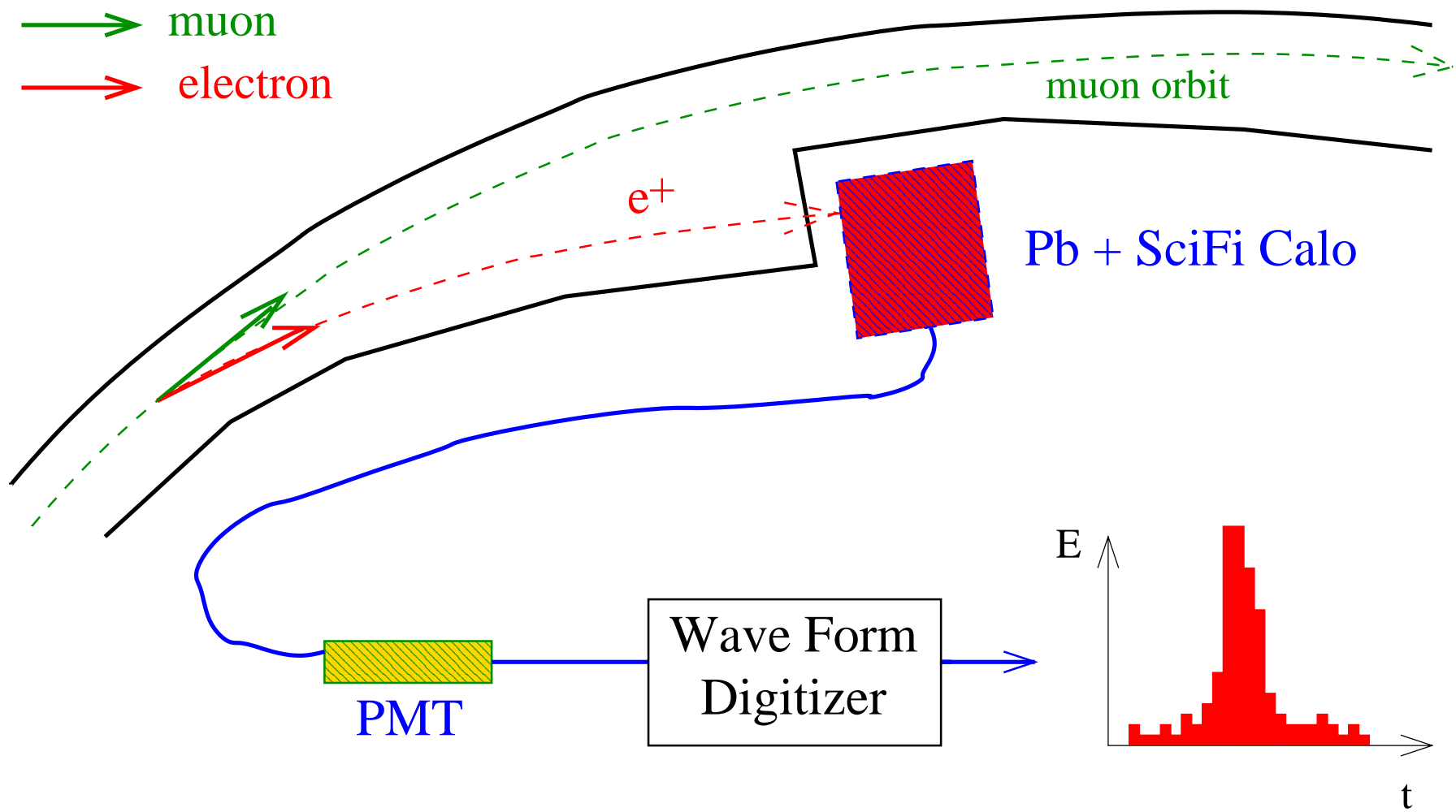
Choose “Magic” $\gamma = \sqrt{\frac{1+a}{a}} \cong 29.3 \rightarrow$ Minimizes the $\vec{\beta} \times \vec{E}$ term

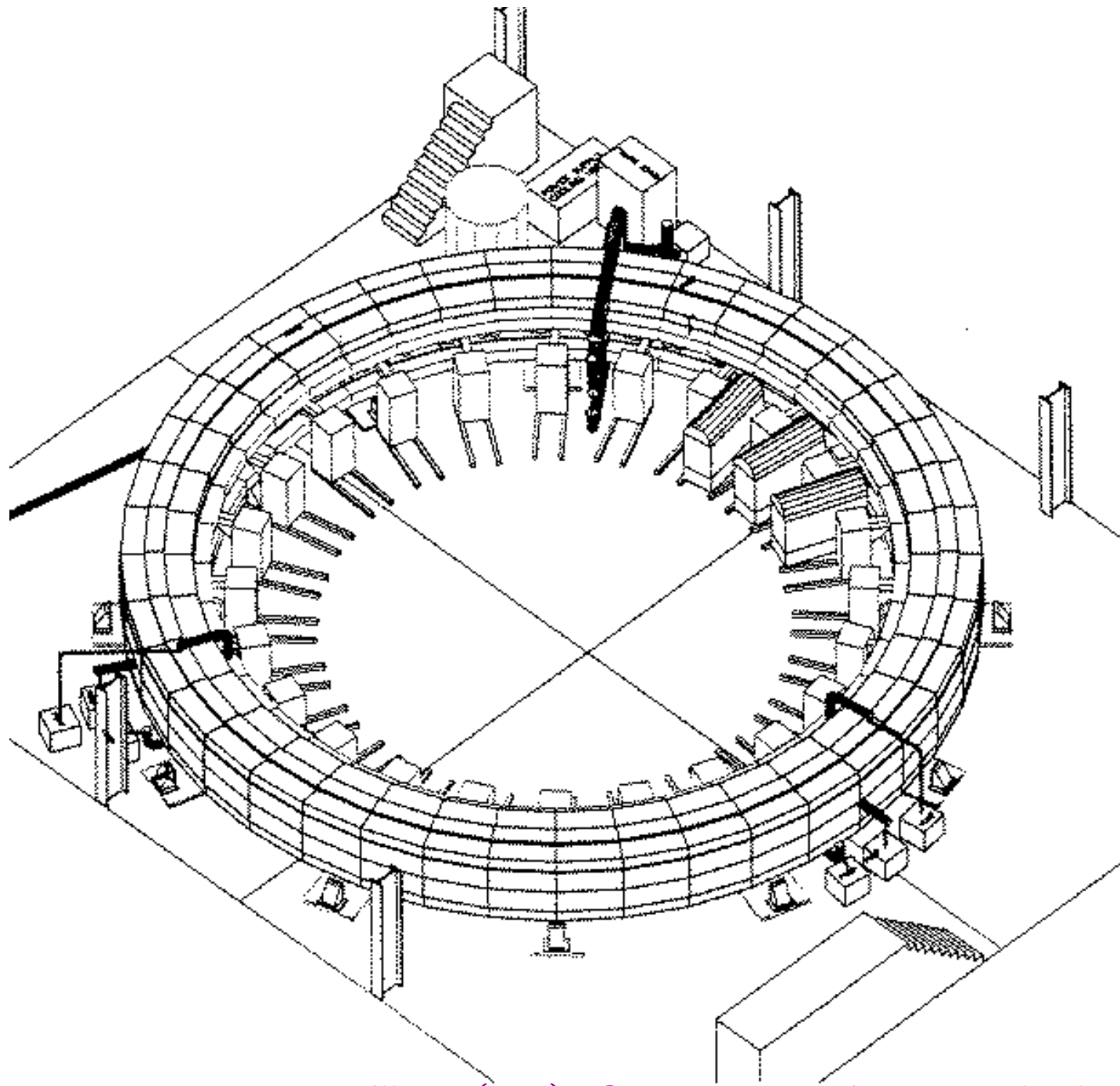
- $\gamma \cong 29.3 \rightarrow p_\mu \cong 3.094$
- $B \cong 1.4T \rightarrow$ Storage ring radius $\cong 7.112m$
- $T_c \cong 149.2ns \quad T_a \cong 4.365\mu s$
- $\gamma\tau \cong 64.38\mu s$

(Range of stored momenta: $\cong \pm 0.5\%$)

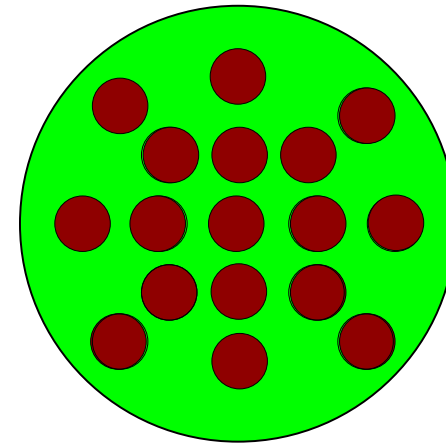
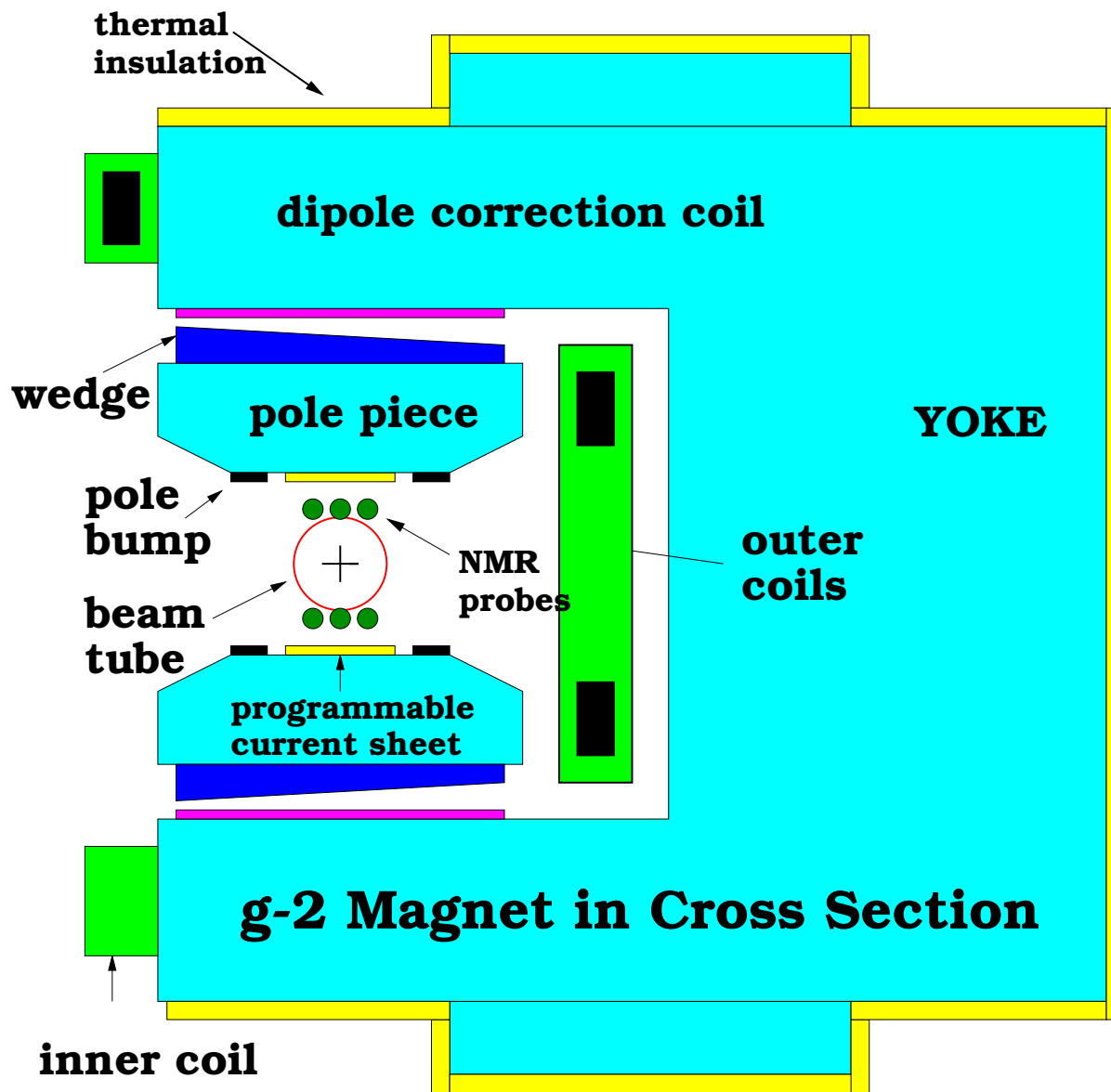
ω_a Measurement

- $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$, $0 < E_e < 3.1 \text{ GeV}$
- Parity Violation \rightarrow for given E_e , directions of \vec{p}_{e^+} and \vec{s}_μ are correlated
For high values of E_e , \vec{p}_{e^+} is preferentially parallel to \vec{s}_μ
- number of positrons with $E > E_{\text{threshold}}$
$$N(t) = N_0(1 + A(E) \cos(\omega_a t + \phi))$$





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Array of NMR probes moves through beam tube on cable car

Determination of Average B-field (ω_p) of Muon Ensemble

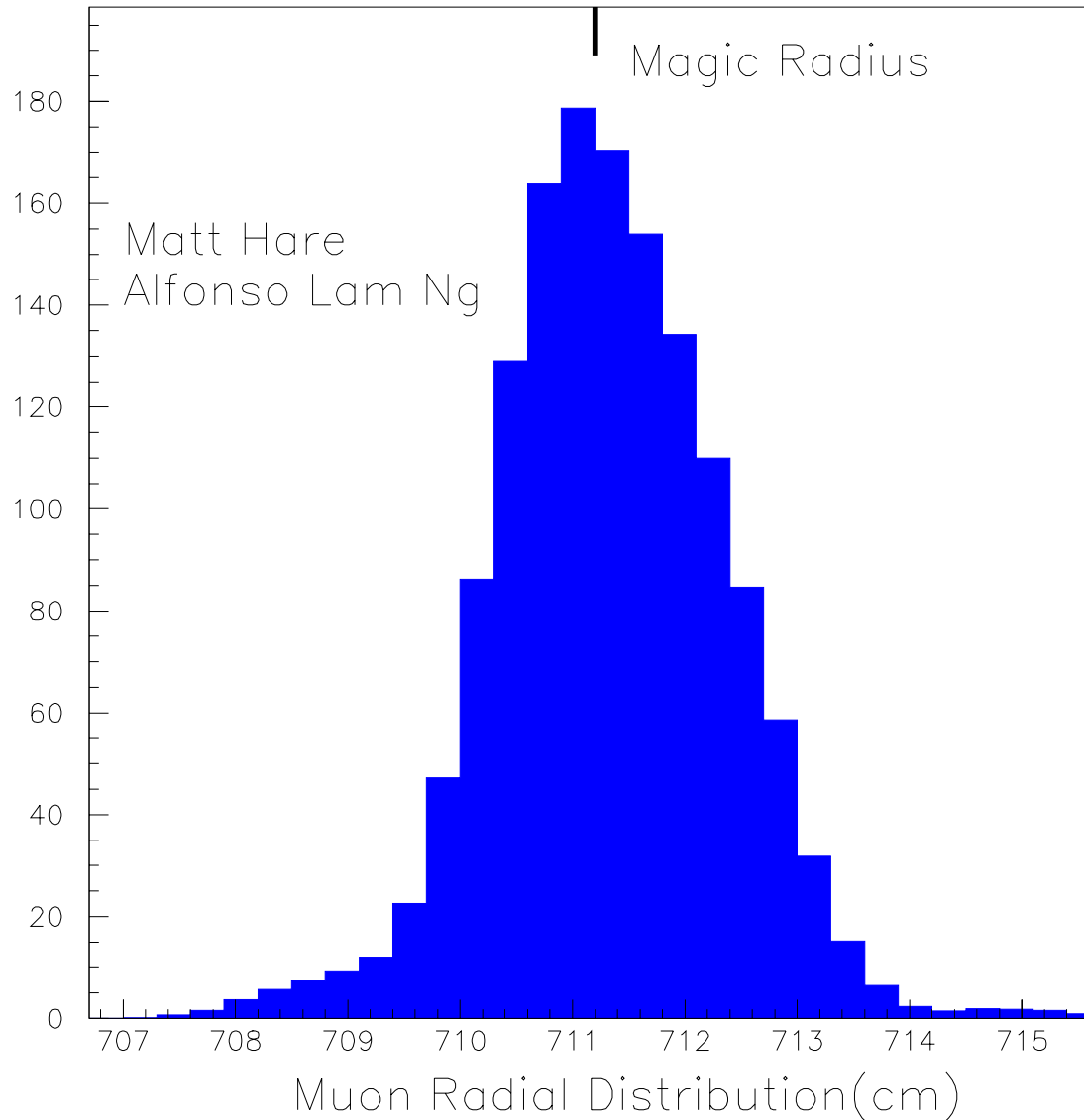
Mapping of B-field

- Complete B-Field map of storage region every 3-4 days
Beam trolley with 17 NMR probes
- Continuous monitor of B-field with over 100 fixed probes

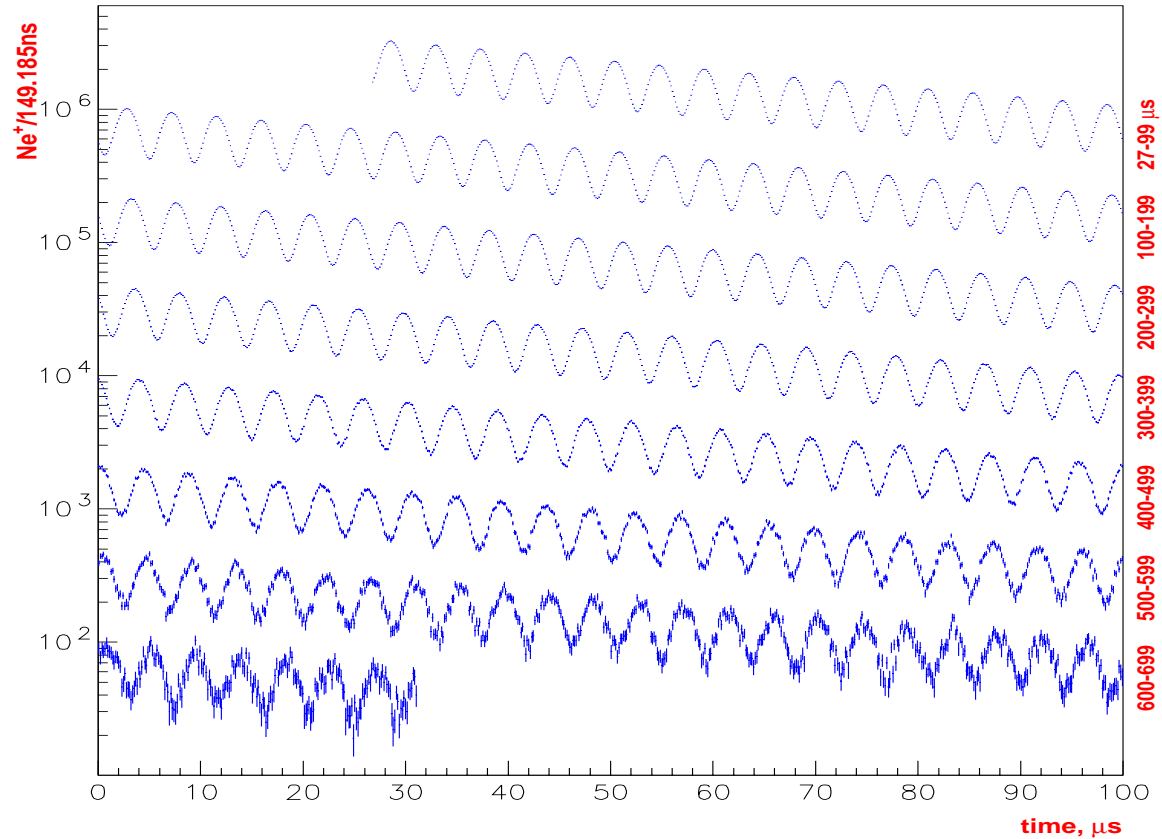
Determination of muon distribution

- Fit to bunch structure of stored beam vs. time

Determination of Muon Distribution



1,025 million e⁺ (E > 2 GeV, 1999 data)



Log plot of 1999
data (10⁹ e⁺)

149 ns bins

100 μs segments

Statistical error:

$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a\gamma\tau_\mu A\sqrt{N_e}}$$

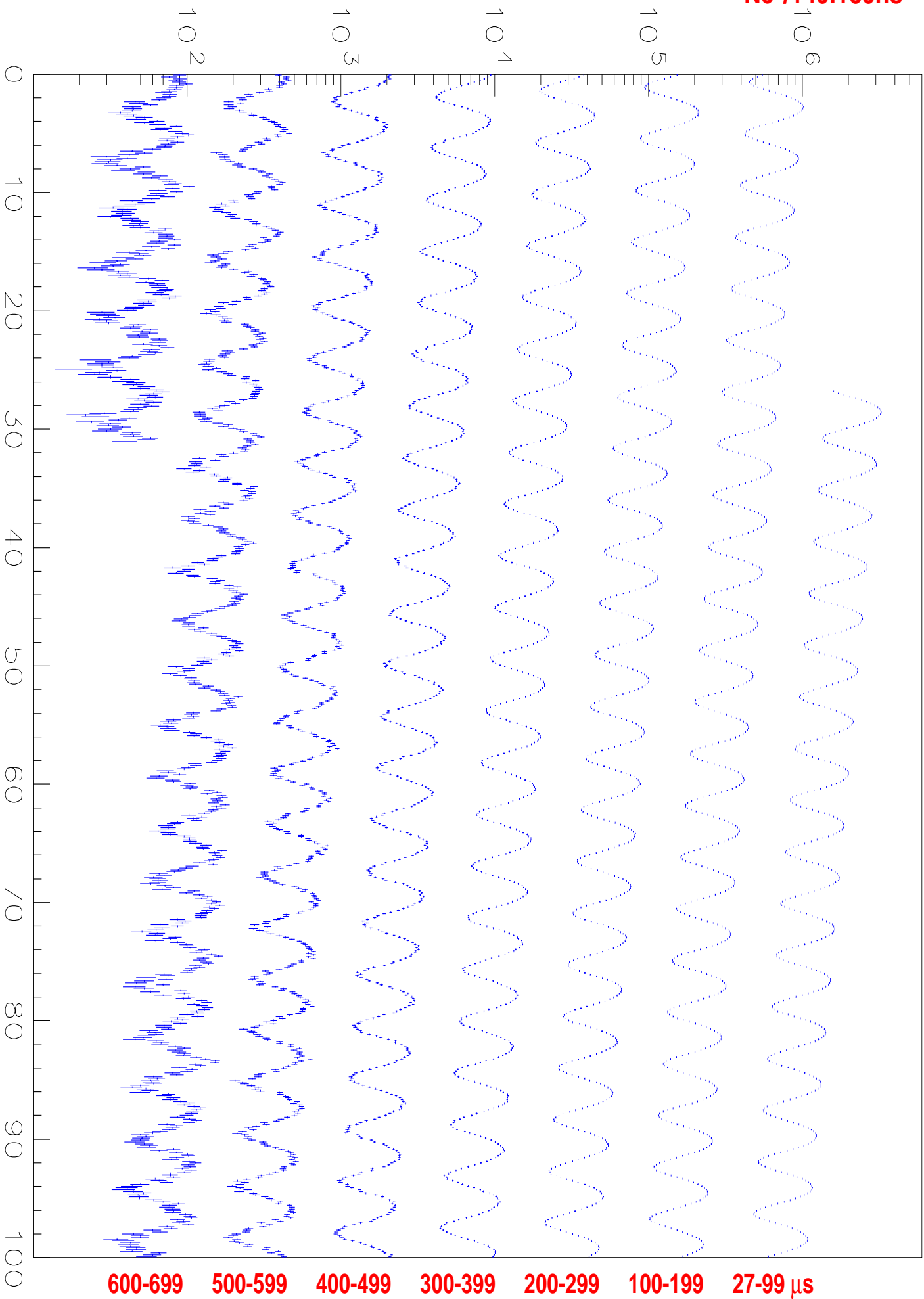
5-parameter function (used to fit to 1998 data)

$$N(t) = N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)]$$

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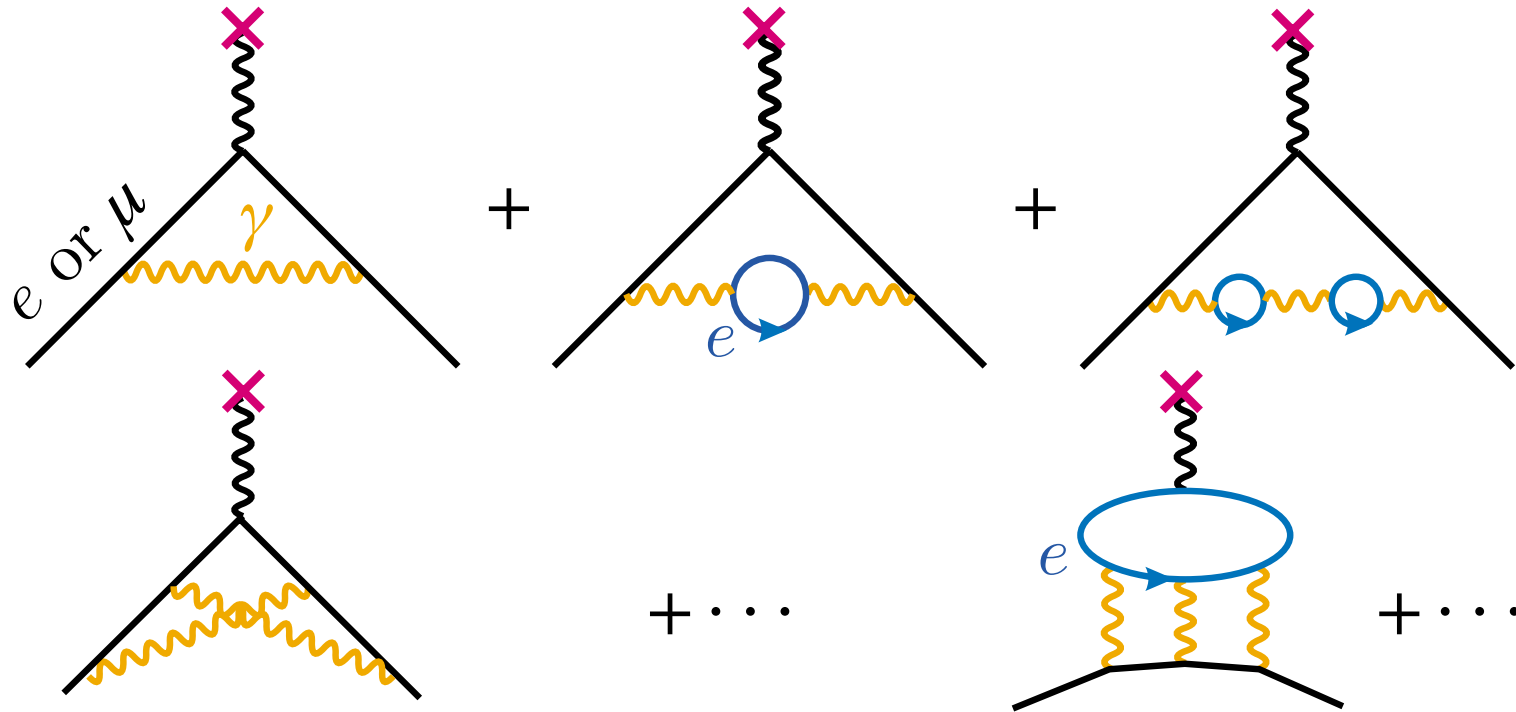
1,025 million e⁺ (E > 2 GeV, 1999 data)

Ne⁺/149.185ns



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7. Computing $a = g/2 - 1$



+ $e \Rightarrow \mu, \tau; u, d, c, s, t, b; W^\pm \dots$

$$a_e = \frac{\alpha}{2\pi} + \dots c_4 \left(\frac{\alpha}{\pi}\right)^4 = (115965215.4 \pm 2.4) \times 10^{-11}$$

$$\text{Exp, } e^+ \text{ and } e^-: \quad = (\quad \dots 18.8 \pm 0.4) \times 10^{-11}$$

Agreement to ~ 30 ppb or 1.4σ . What is α ?

7.1 a_μ

Both **experiment and calculation** more difficult.

a_μ is $m_\mu^2/m_e^2 \sim 44,000$ times more sensitive to high mass states in the diagrams above. Therefore:

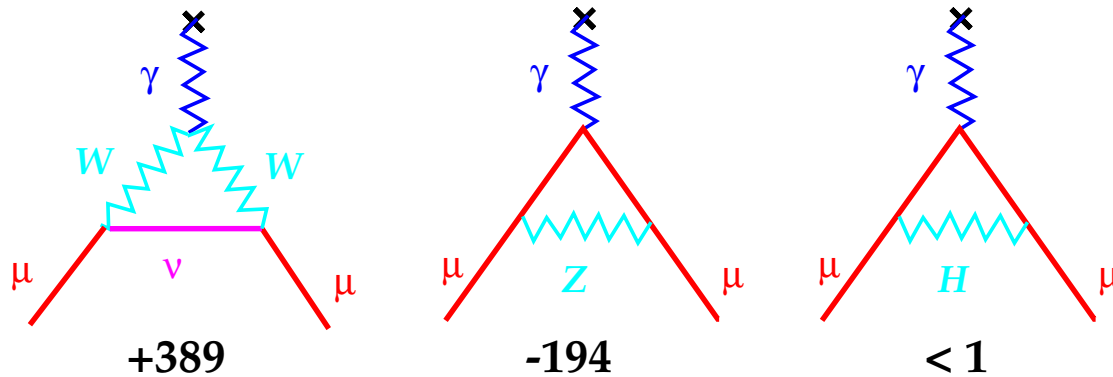
1. a_μ can reflect the existence of new particles - and interactions not observed so far.
2. hadrons - pion, etc - become important in calculating its value.

Point 1 is a strong motivation for accurate measurements of a_μ .

Point 2 is an obstacle to the interpretation of the measurement.

1. – New Physics

For calibration we take the E-W interaction

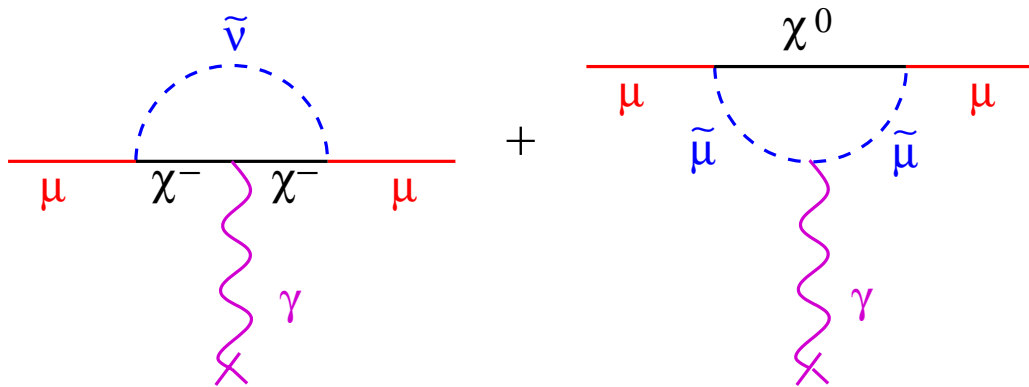


$$\langle \phi \rangle = 236 \text{ GeV}$$

$$M \sim 90 \text{ GeV}$$

$$\delta a_\mu(\text{EW}) = 150 \times 10^{-11}$$

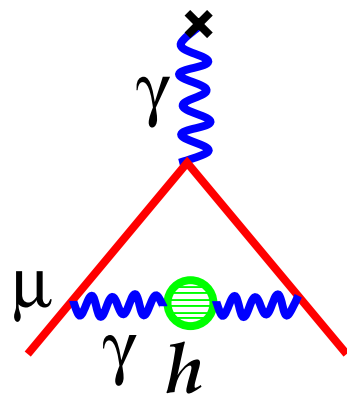
SUSY:



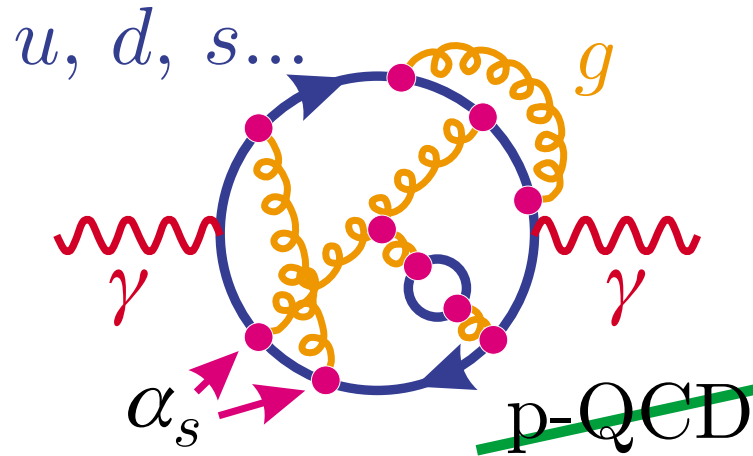
$$\delta a_\mu(\text{SUSY}) \sim 150 \times 10^{-11} \times (100 \text{ GeV} / \tilde{M})^2 \times \tan \beta$$

2. – Hadrons

Need



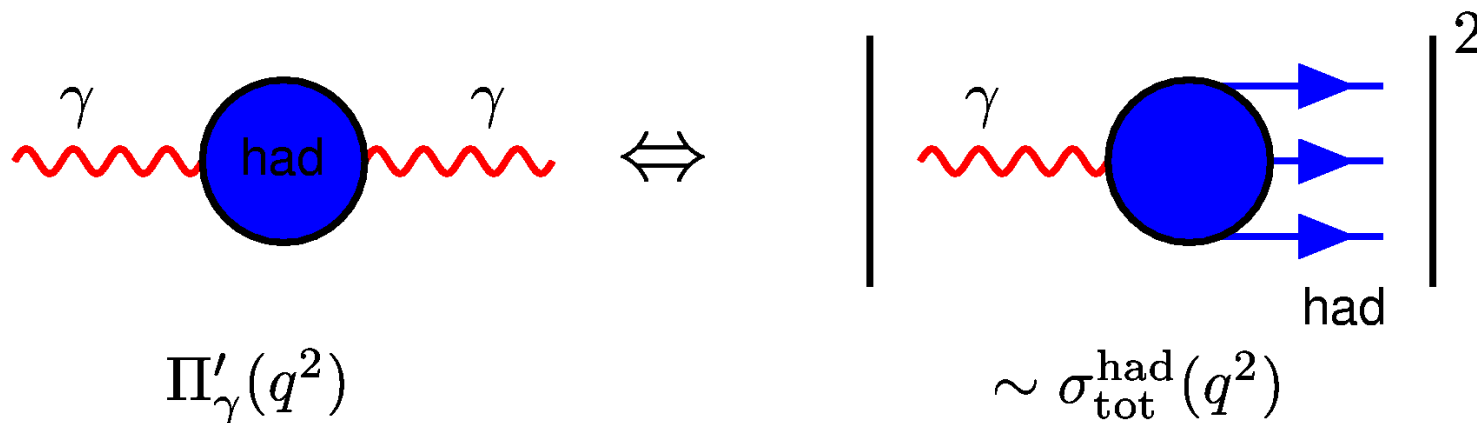
i.e.



which is not
calculable at
low q^2 .

But...

Measure $\sigma(e^+e^- \rightarrow \text{hadrons})$ and use dispersion relations:



$$\delta a_\mu, (\text{hadr} - 1) \sim 7000 \times 10^{-11}$$

All these effects are irrelevant for a_e

$$a_{\mu} = \frac{\alpha}{2\pi} + \dots c_4 \left(\frac{\alpha}{\pi}\right)^4 = (116591596 \pm 67) \times 10^{-11}$$

$$\text{Exp, } \mu^+ : \quad = (\quad \dots 2030 \pm 150) \times 10^{-11}$$

Measured-Computed= 430 ± 160 or 2.6σ , $\sim 3.7 \pm 1.4$ ppm.

!!!!?????

Standard Model Value for a_μ [1]

$$\begin{aligned}a_\mu(QED) &= 116584706(3) \times 10^{-11} \\a_\mu(HAD; 1) &= 6924(62) \times 10^{-11} \text{ (DH98)} \\a_\mu(HAD; > 1) &= -100(6) \times 10^{-11} \text{ (Except LL)} \\a_\mu(HAD; LL) &= -85(25) \times 10^{-11} \\a_\mu(EW) &= 151(4) \times 10^{-11} \\TOTAL &= 116591596(67) \times 10^{-11}\end{aligned}$$

[1] Czarnecki, Marciano, Nucl. Phys. B(Proc. Suppl.)76(1999)245

Used by the BNL $g-2$ experiment for comparison. Addition of above errors in quadrature is questionable.

$$8. \sigma(e^+e^- \rightarrow \pi^+\pi^-)$$

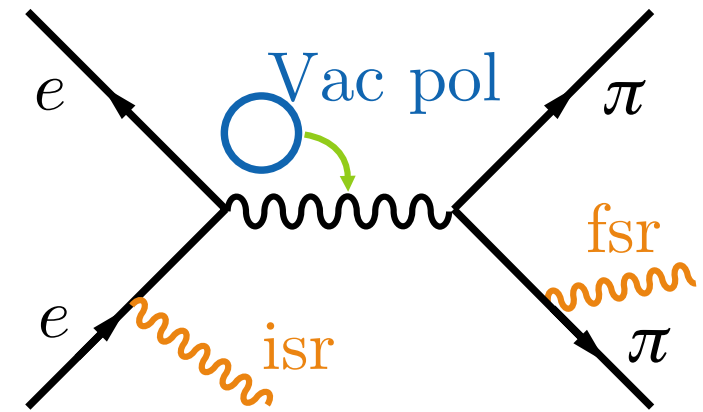
$$\delta a_\mu(\text{hadr} - 1) \sim 7000 \times 10^{-11}$$

$\sigma(e^+e^- \rightarrow \text{hadrons})$ is dominated below 1 GeV by $e^+e^- \rightarrow \pi^+\pi^-$.
Low mass $\pi^+\pi^-$ (ρ, ω) contributes $\delta a_\mu \sim 5000 \pm 30$.

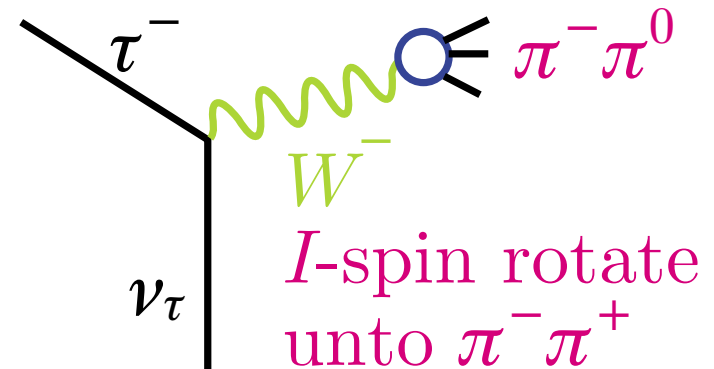
$\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ or ($\gamma \rightarrow \pi^+\pi^-$) is measured:

1. at e^+e^- colliders, varying the energy
2. in τ -lepton decays
3. at fixed energy colliders using radiative return

- 1. - Extensive measurements performed at Novosibirsk. Corrections for efficiency and scale plus absolute normalization (Bhabha, $e^+e^- \rightarrow e^+e^-$) are required for each energy setting. Data must also be corrected for radiation and vacuum polarization.



- 2. - τ data come mostly from LEP. To get $\sigma(\text{hadr})$ requires I -spin breaking, $M(\rho^\pm) - M(\rho^0)$, $I=0$ contrib... corrections. Radiative corrections are also required.



- 3. - The radiative return method is being used by the KLOE collaboration, spear-headed by the Karlsruhe-Pisa groups.

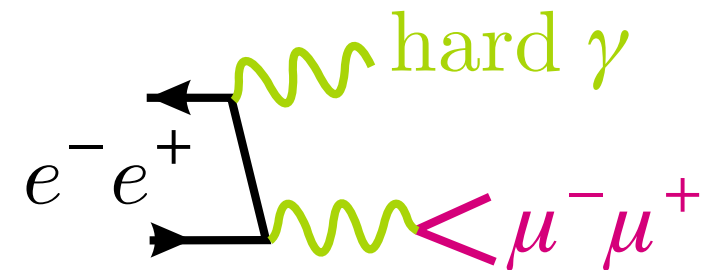
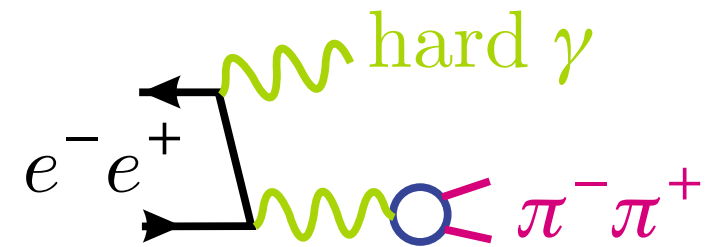
Can turn initial state radiation into an advantage.

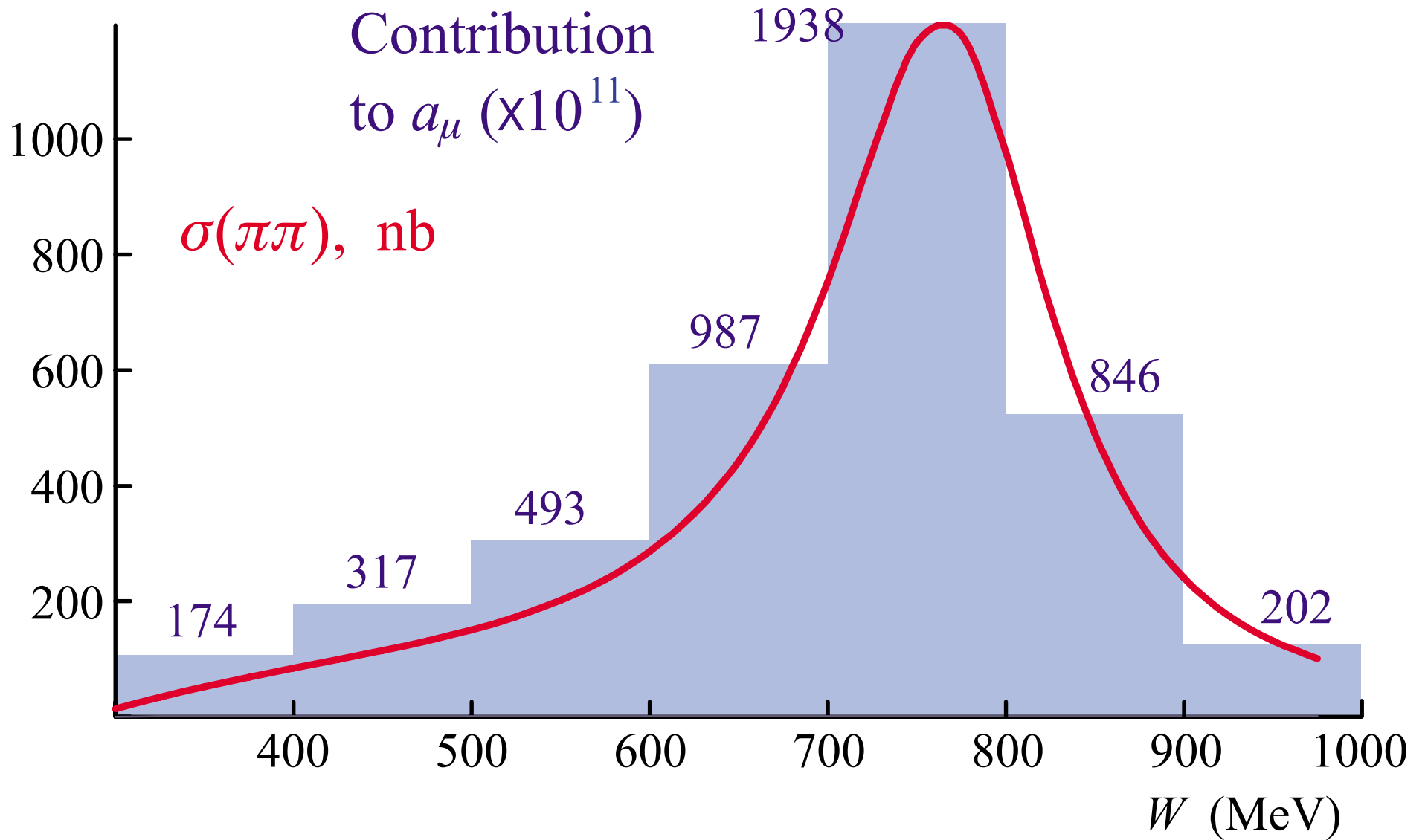
At fixed collider energy W , the $\pi^+\pi^-\gamma$ final state covers the di-pion mass range $280 < M_{\pi\pi} < W$ MeV.

Correction for radiation and vacuum polarization are necessary.

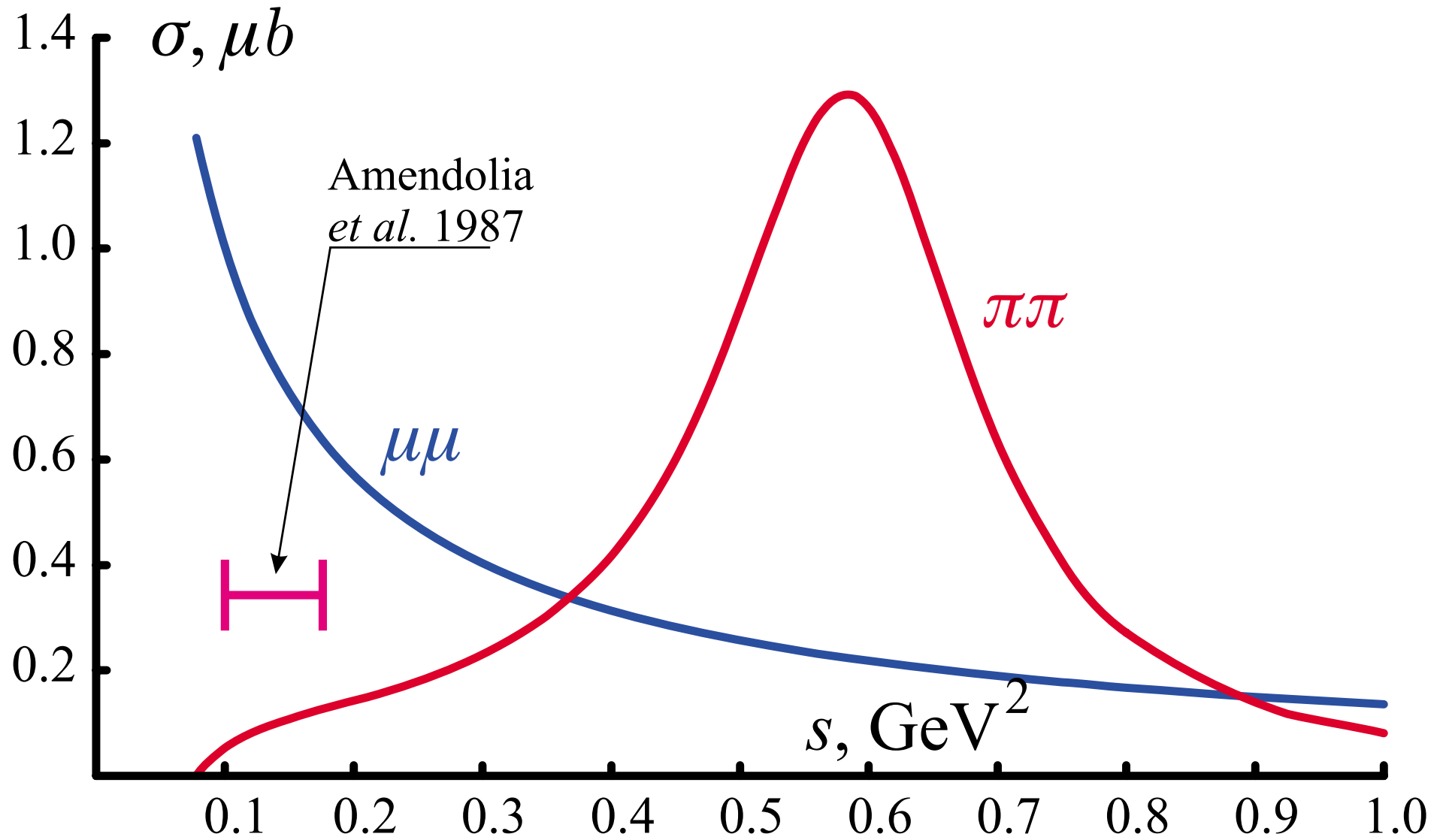
All other factors need be obtained only once.

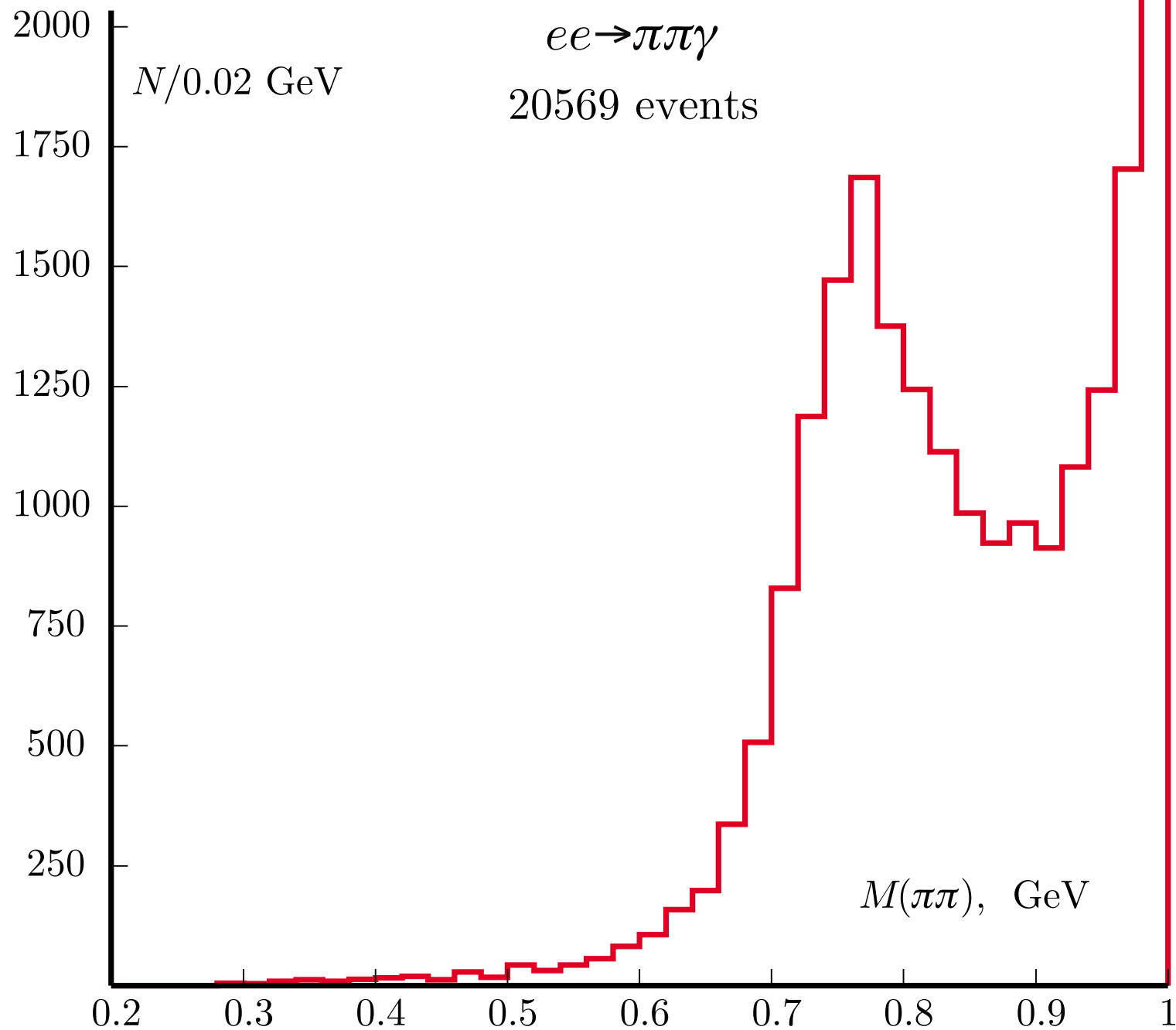
At low mass, di-muon production exceeds that of di-pion. ISR and vacuum polarization cancel.

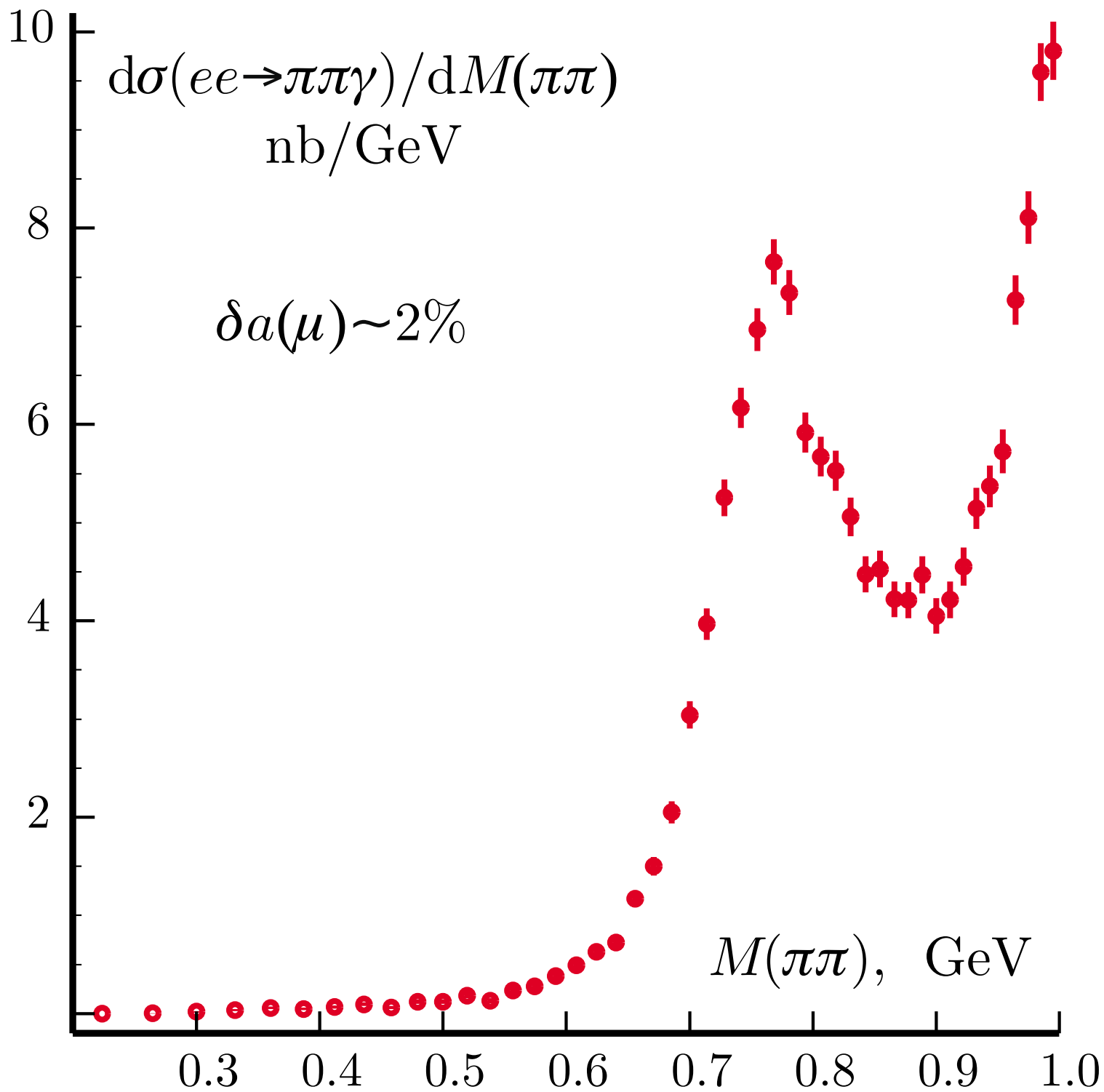




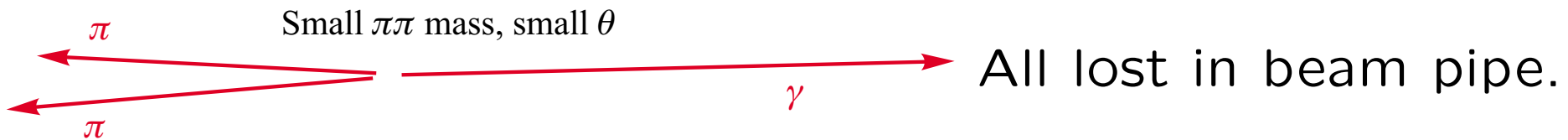
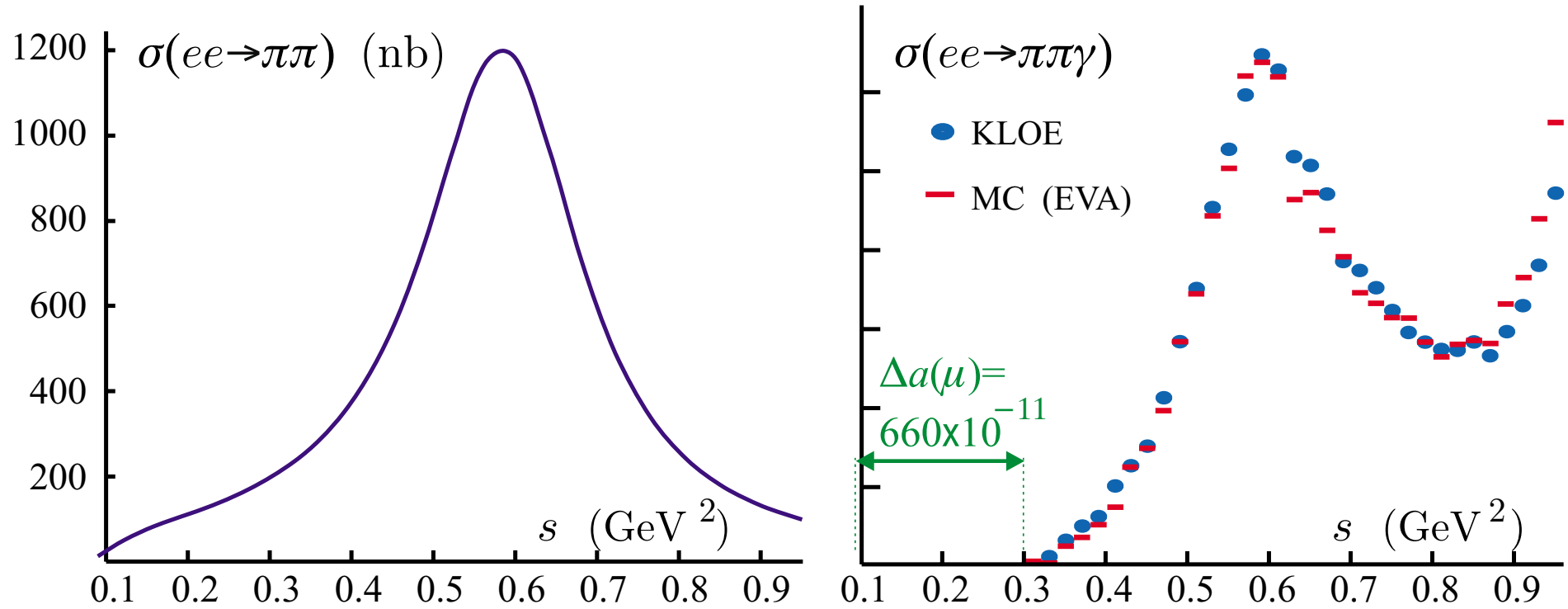
$$\Sigma(\dots) = 5000$$







Use small angle radiation, higher x-section but miss low $M_{\pi^+\pi^-}$.



Unsatisfactory points:

1. 2.6σ is not very compelling and is also author dependent.
2. M-C is $\sim 3 \times$ EW contribution. What about LEP, $b \rightarrow s\gamma$, M_W , M_{top} , $\Re(\epsilon'/\epsilon)$, $\sin 2\beta$...
3. Hadronic corrections difficult, e.g. light-by-light
4. SUSY as a theory is not very precise at the moment. It has too many unknown, free parameters. There is no exp. evidence for it nor a prediction follows from the possible effect in the muon anomaly.

Soon better statistics and both signs muons.

Still very exciting at present.