$g-2$

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1. The gyromagnetic ratio

By definition, the gyromagnetic ratio g of a state of angular momentum J and magnetic moment μ is:

$$g = \frac{\mu}{\mu_0} \bigg/ \frac{J}{\hbar}.$$

For a particle of charge e in a state of orbital angular momentum **L** we have:

$$\vec{\mu} = \mu_0 \mathbf{L}, \quad \mu_0 = \frac{e}{2m}, \quad g = 1.$$

For an electron $\mu_0 = \mu_{\rm B}$=5.788... $\times\, 10^{-11}$ MeV T^{-1} ($\pm$7 ppb).

The importance of g in particle physics is many-fold. A gross deviation from the expected value, 2 for charged spin 1/2 *Dirac* particles, is clear evidence for structure.

Thus the electron and the muon (g~2.002) are elementary particles while the proton, with g_p=5.6 is a composite object. For the neutron g should be zero, measurements give $g_n = -3.8$

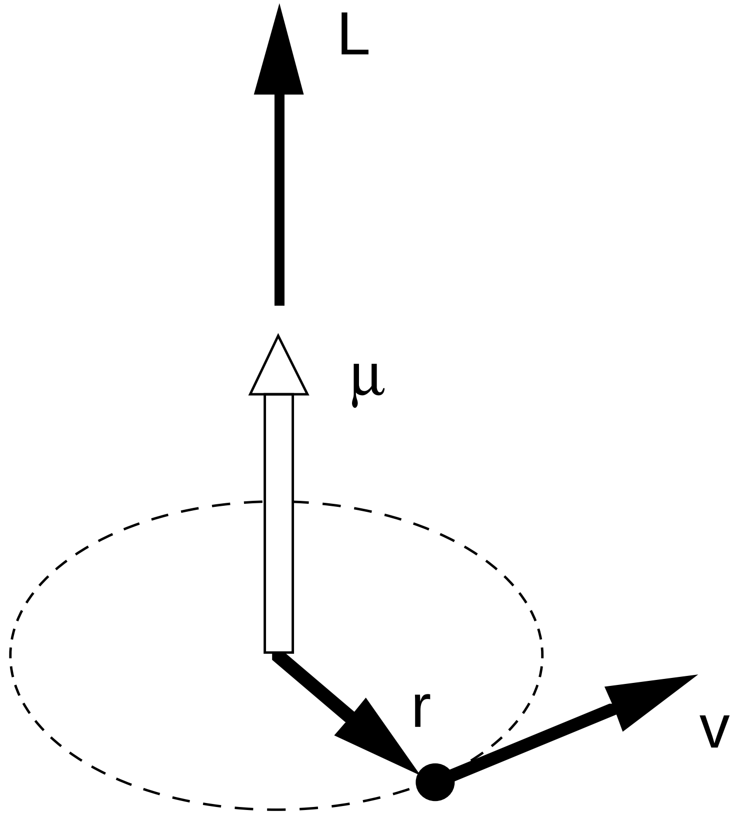
Small deviations from 2, ~0.1%, appear as consequence of the self interaction of the particles with their own field. Experimental verifications of the computed deviations are a triumph of QED.

We also define the anomaly, $a = (g-2)/2$, a measure of the so called anomalous magnetic moment, $(g-2)\mu_0$.

QED is not all there is in the physical world. The EW interaction contributes to a and new physics beyond the standard model might manifest itself as a deviation from calculations.

2. Magnetic moment

The classical physics picture of the magnetic moment of a particle in a plane orbit under a central force is illustrated on the side. $\vec{\mu}$ is along **L**, $\mu_0 = q/2m$ and g=1. This remains true in QM. For an electron in an atom, $\mu_\text{B} = e/2m_e$ is the Bohr magneton. $\mathbf{L} \parallel \vec{\mu}$ is required by rotational invariance.

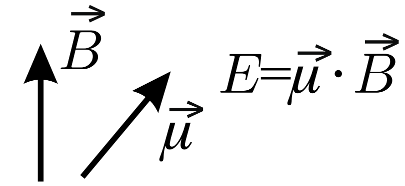


When we get to intrinsic angular momentum or spin the classic picture loses meanings and we retain only $\vec{\mu} \parallel \mathbf{L}$. We turn now to relativistic QM and the Dirac equation.

2.1 g for Dirac particles

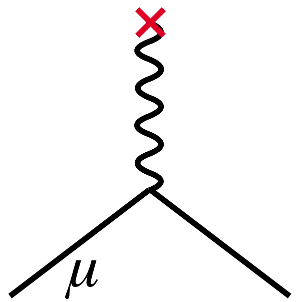
In the non-relativistic limit, the Dirac equation of an electron interacting with an electromagnetic field ($p_\mu \to p_\mu + eA_\mu$) acquires the term

$$\frac{e}{2m}\vec{\sigma}\cdot\mathbf{B} - eA^0$$



which implies that the electron's intrinsic magnetic moment is

$$\vec{\mu} = \frac{e}{2m}\vec{\sigma} \equiv g\frac{e}{2m}\mathbf{S} \equiv g\mu_B\mathbf{S},$$



where $\mathbf{S} = \vec{\sigma}/2$ is the spin operator and g=2.

The prediction g=2 for the intrinsic magnetic moment is one of the many triumphs of the Dirac equation.

3. Motion and precession in a B field

The motion of a particle of momentum p and charge e in a uniform magnetic field B is circular with $p = 300 \times B \times r$. For $p \ll m$ the angular frequency of the circular motion, called the cyclotron frequency, is:

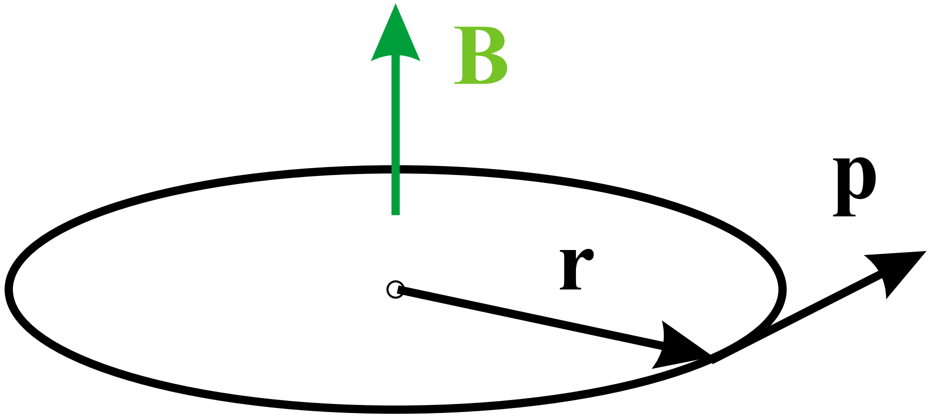
$$\omega_{\mathrm{C}} = \frac{eB}{m}.$$

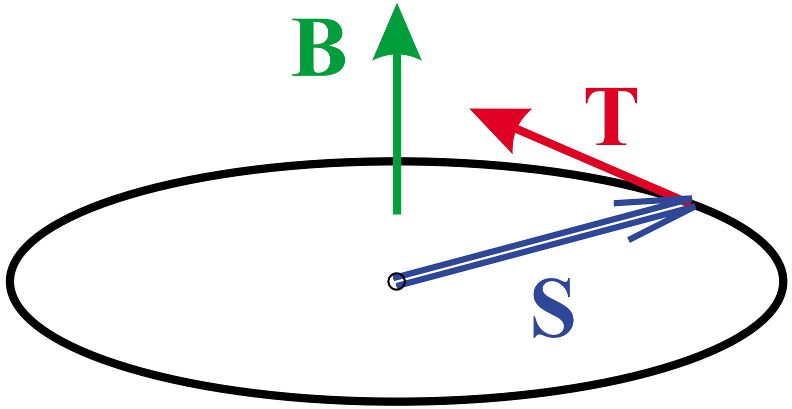
The spin precession frequency at rest is given by:

$$\omega_s = g\frac{eB}{2m}$$

which, for g=2, coincides with the cyclotron frequencies.

This suggests the possibility of directly measuring $g-2$.





'Cyclotron' orbit, ω_C Spin precession, ω_S

For higher momenta the frequencies become

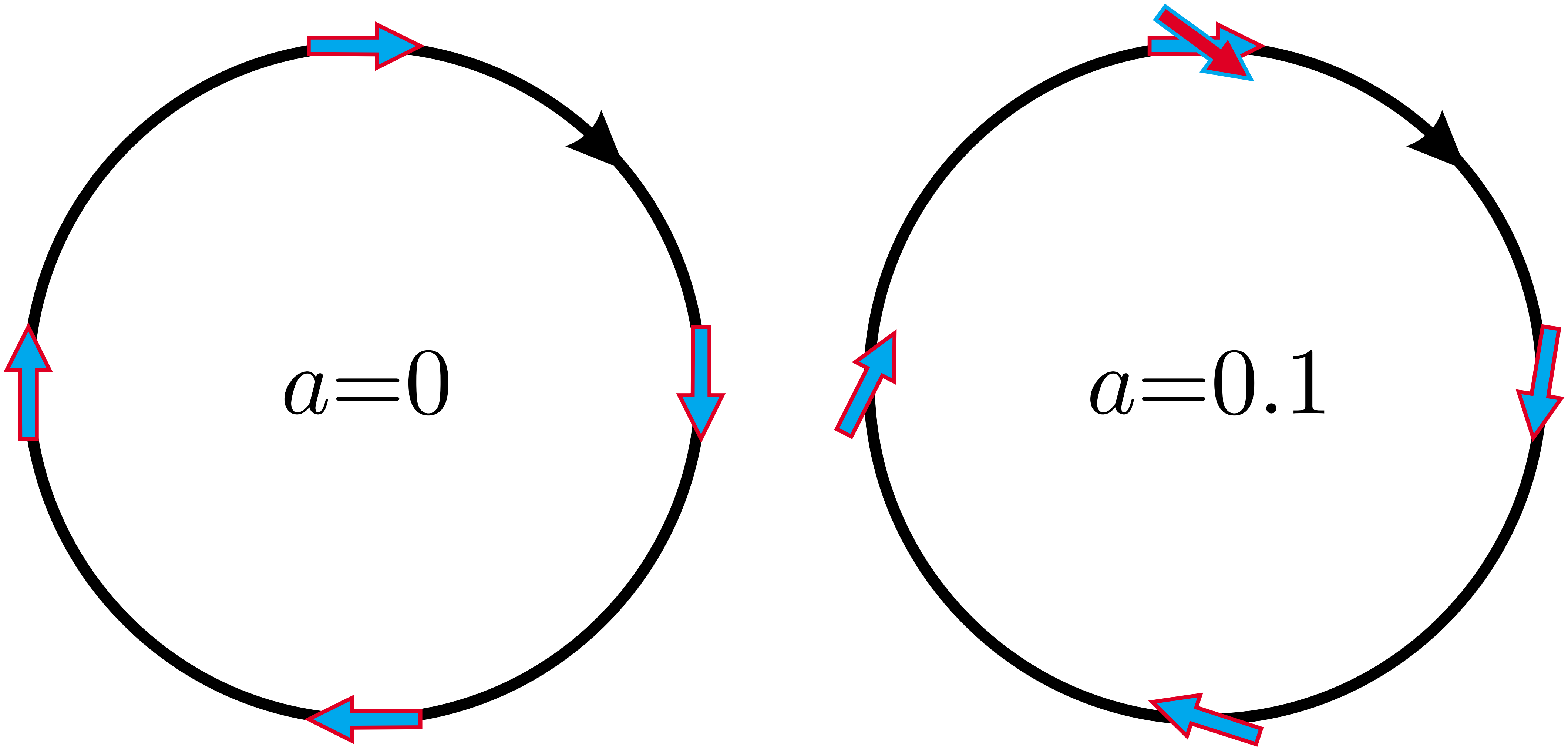
$$\omega_{\mathsf{C}} = \frac{eB}{m\gamma}$$

and

$$\omega_s = \frac{eB}{m\gamma} + a\frac{eB}{m}$$

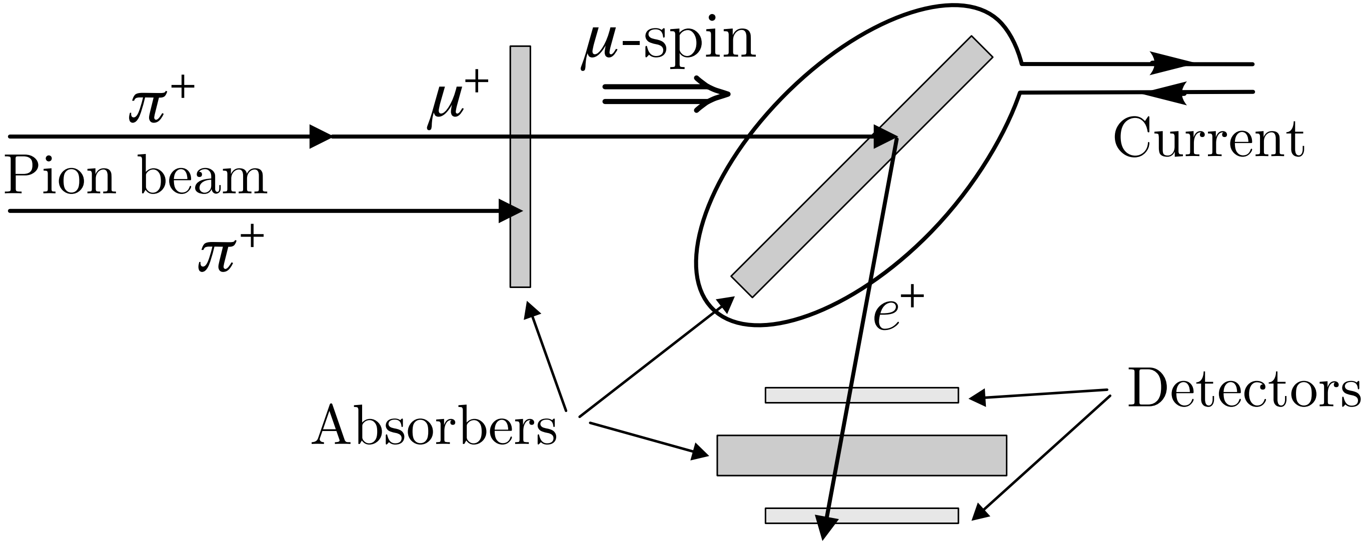
or

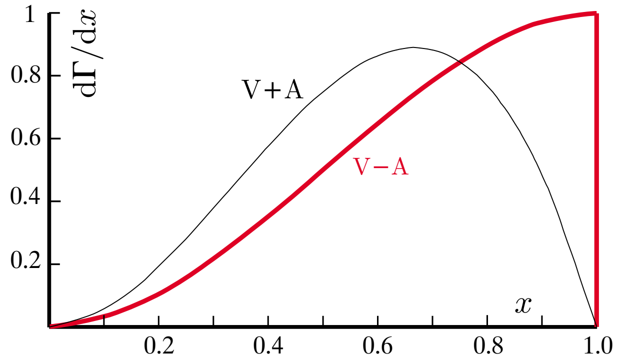
$$\omega_a = \omega_s - \omega_{\mathsf{C}} = a\frac{eB}{m} = a\gamma\omega_{\mathsf{C}}$$



For $a = 1$ (γ=1), spin rotates wrt momentum by 1/10 turn per turn.

4. $\pi \rightarrow \mu \rightarrow e$

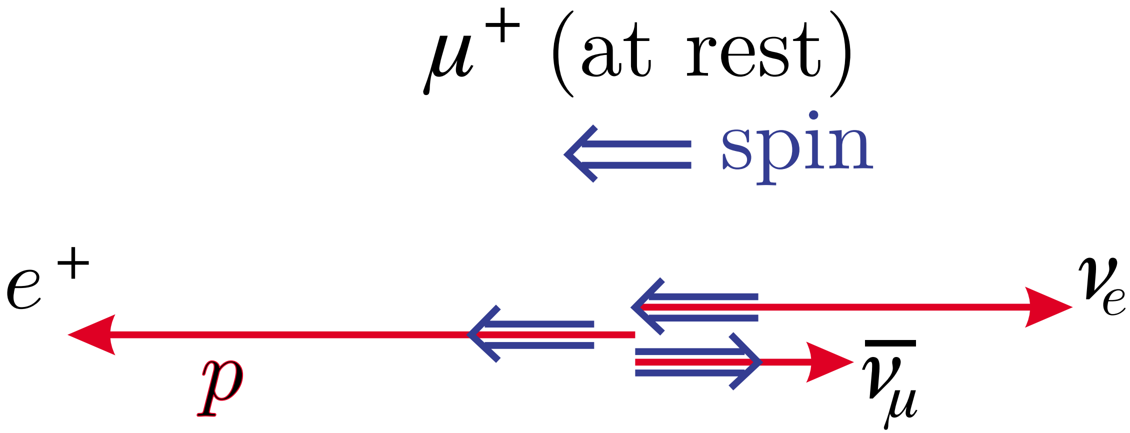




The rate of high energy decay electrons is time modulated with a frequency corresponding to the precession of a magnetic moment $e/m(\mu)$ or a muon with g=2. First measurement of $g(\mu)$!!

Also a proof that P and C are violated in both $\pi\mu\nu$ and $\mu \rightarrow e\nu\bar{\nu}$ decays.

S-**p** correlation fundamental to all muon anomaly experiments

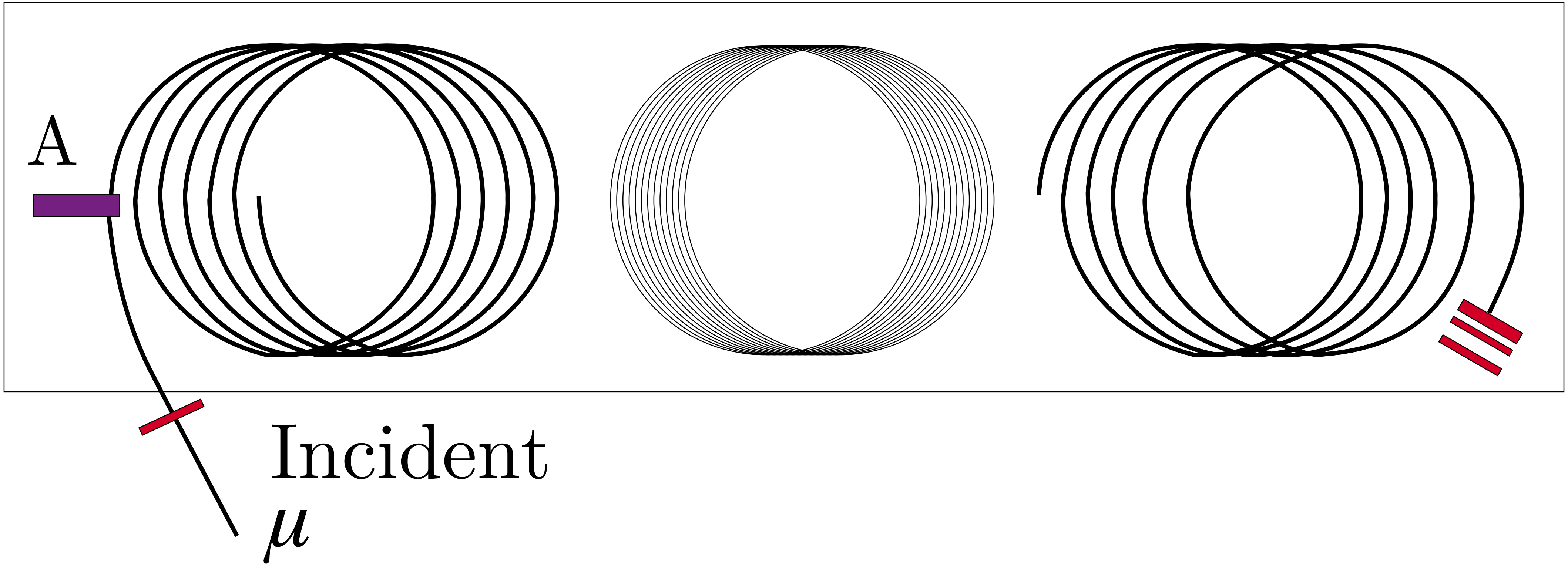


High energy positrons have momentum along the muon spin. The opposite is true for electrons from μ^-.

Detect high energy electrons. The time dependence of the signal tracks muon precession.

5. The first muon $g-2$ experiment

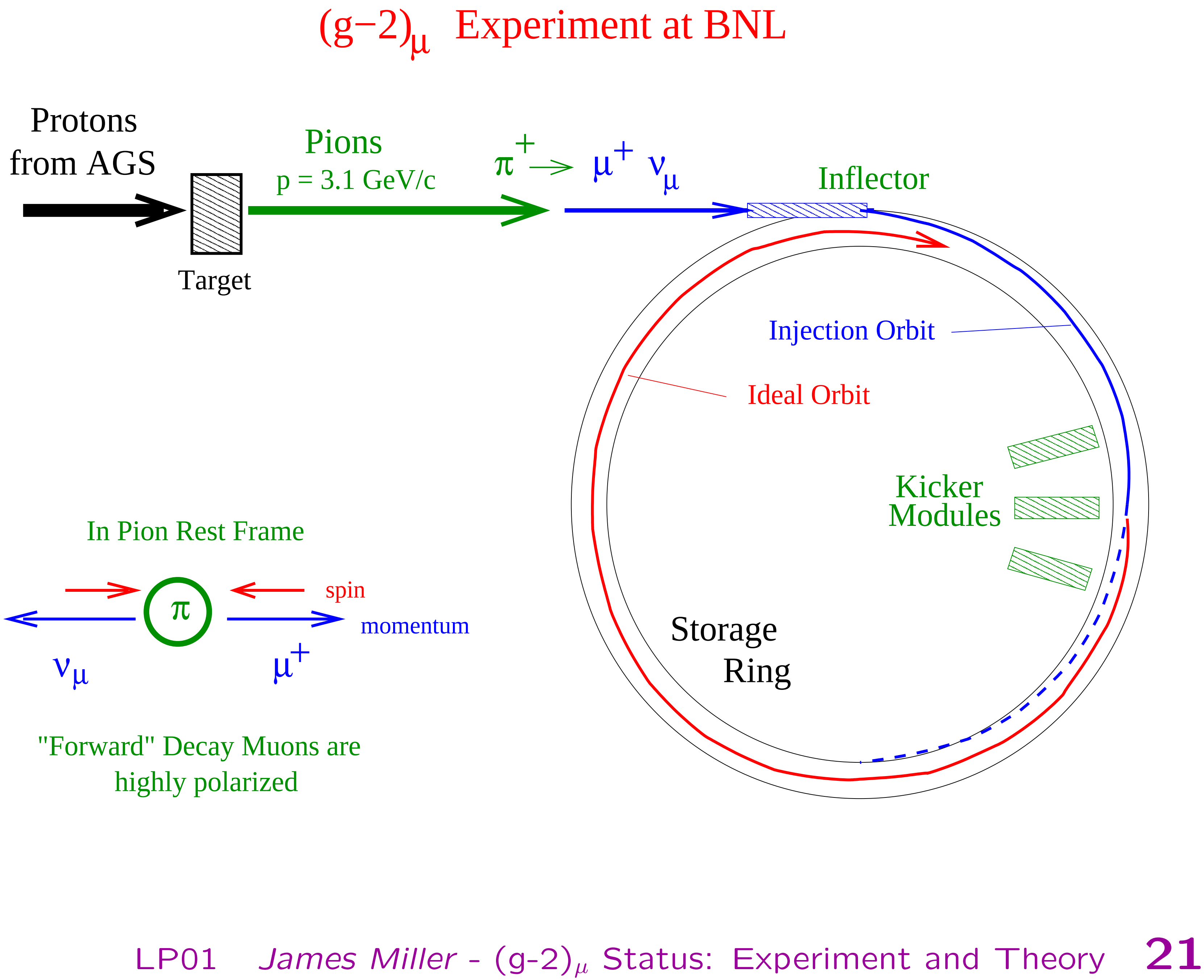
Shaped B field

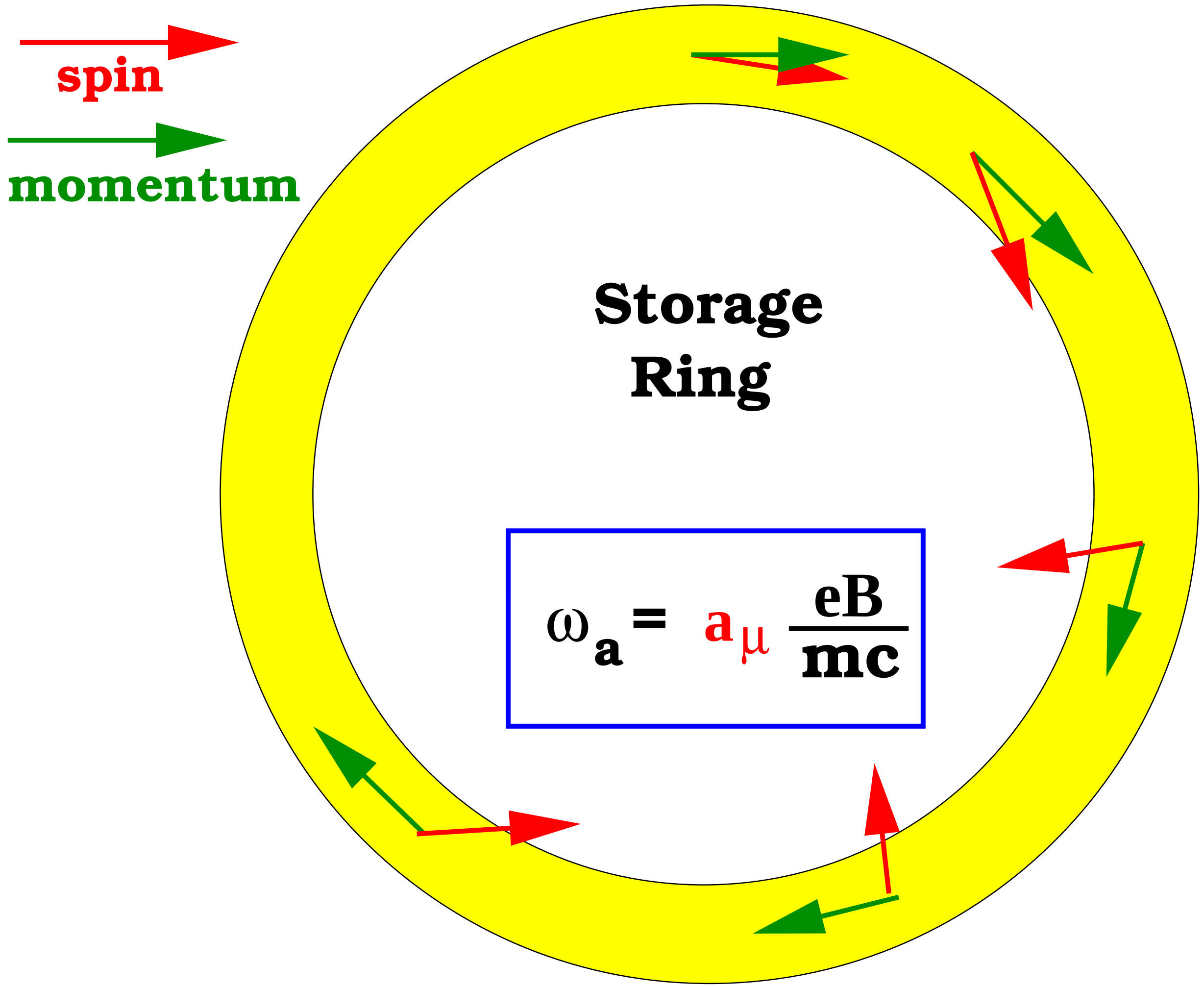


Performed in CERN, in the sixties. Need more turns, more γ.

Next step: a storage ring.

6. The BNL g-2 experiment





(exaggerated ~20x)

With homogeneous $\vec{B}$, all muons precess at same rate

With homogeneous $\vec{B}$, use quadrupole $\vec{E}$ to focus and store beam

Spin Precession with $\vec{B}$ and $\vec{E}$

$$\vec{\omega}_a = \frac{e}{mc}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta} \times \vec{E}]$$

Choose "Magic" $\gamma = \sqrt{\frac{1+a}{a}} \cong 29.3 \rightarrow$ Minimizes the $\vec{\beta} \times \vec{E}$ term

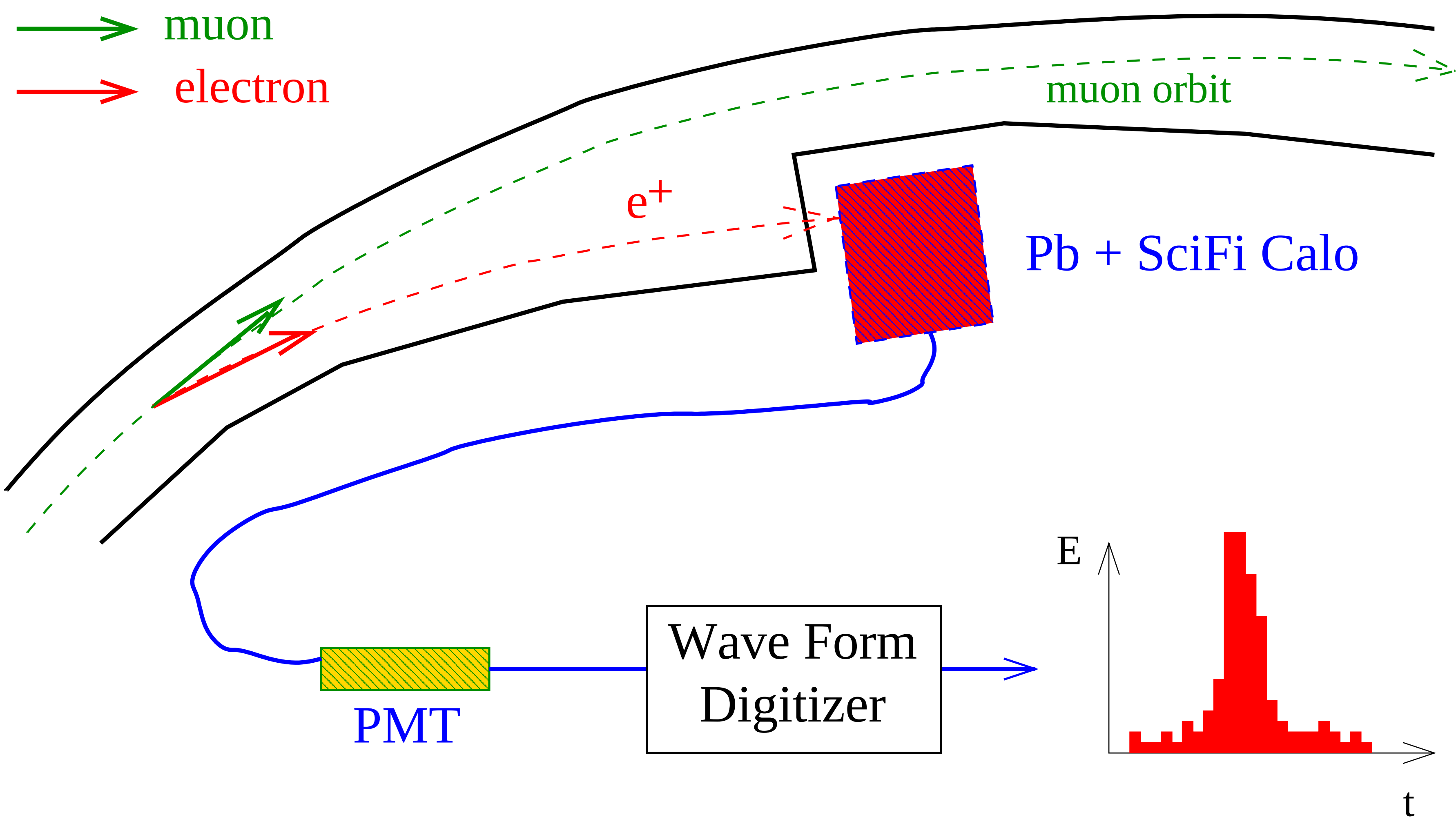
- $\gamma \cong 29.3 \rightarrow p_\mu \cong 3.094$
- $B \cong 1.4T \rightarrow$ Storage ring radius $\cong 7.112m$
- $T_c \cong 149.2ns \quad T_a \cong 4.365\mu s$
- $\gamma\tau \cong 64.38\mu s$

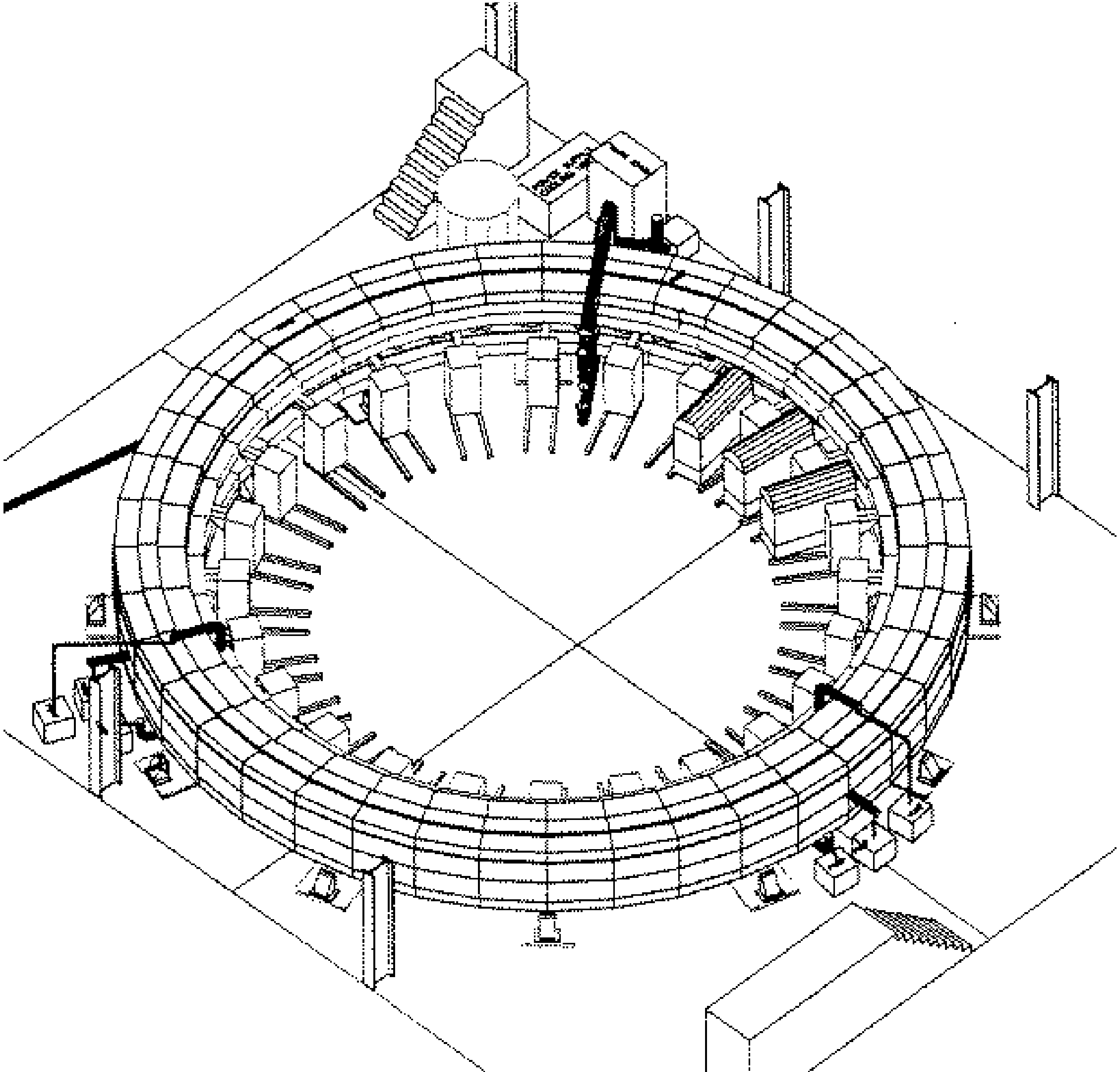
(Range of stored momenta: $\cong \pm 0.5\%$)

ω_a Measurement

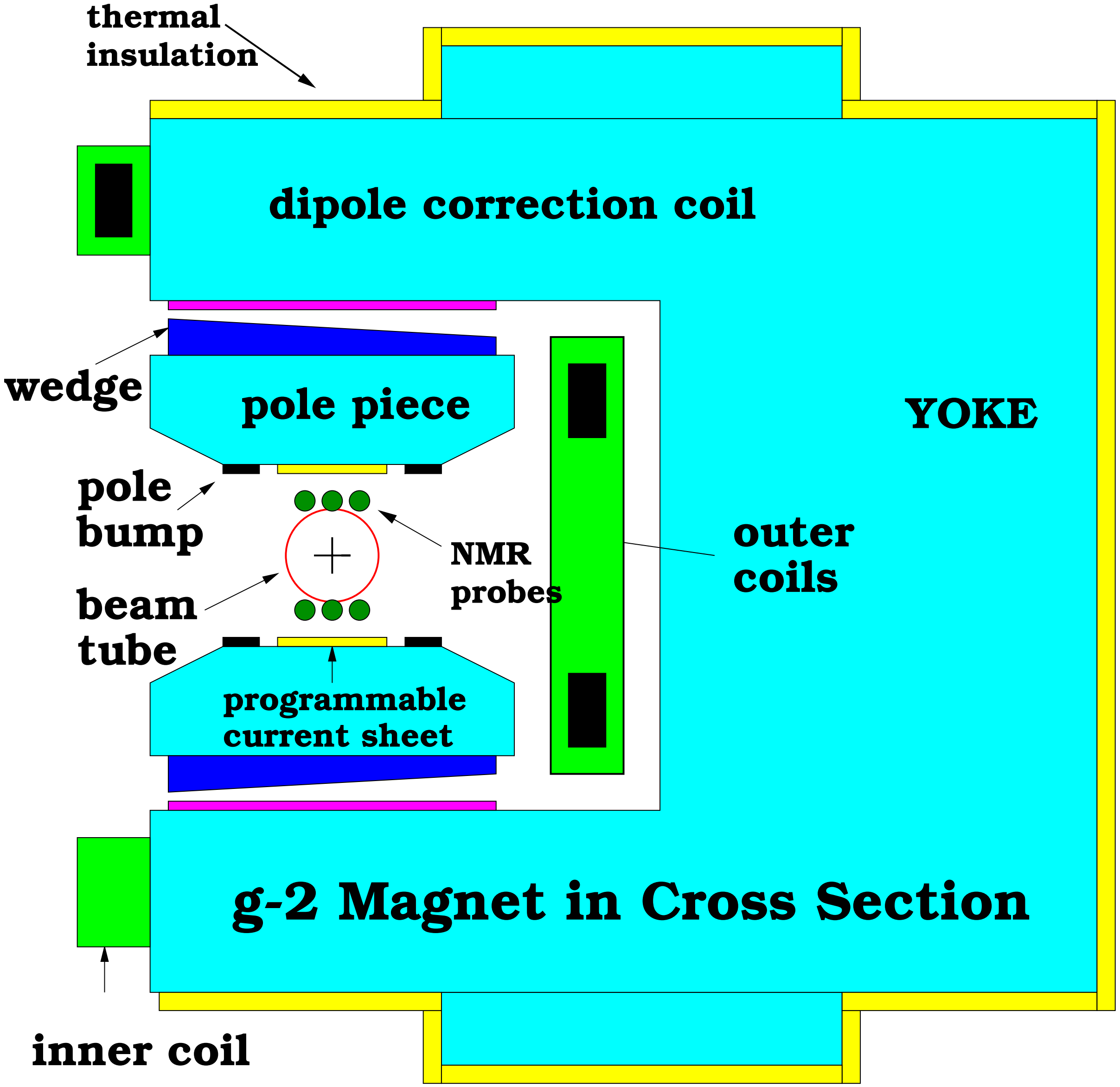
- $\mu^+ \rightarrow e^+\bar{\nu}_\mu \nu_e$, $0 < E_e < 3.1 GeV$
- Parity Violation $\rightarrow$ for given E_e, directions of $\vec{p}_{e^+}$ and $\vec{s}_\mu$ are correlated
  For high values of E_e, $\vec{p}_{e^+}$ is preferentially parallel to $\vec{s}_\mu$
- number of positrons with $E > E_{threshold}$
  $N(t) = N_0(1 + A(E)\cos(\omega_a t + \phi))$

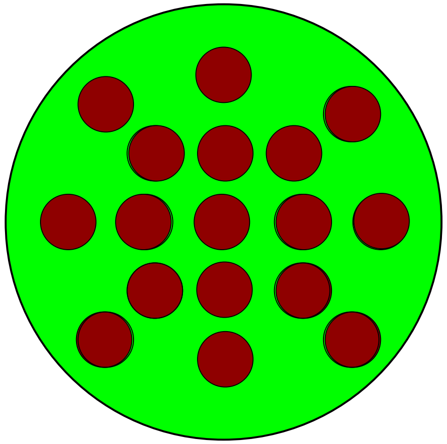
LP01 *James Miller* - (g-2)$_\mu$ Status: Experiment and Theory **24**


muon
electron
muon orbit
e+
Pb + SciFi Calo
E
Wave Form
Digitizer
PMT
t



LP01 *James Miller* - (g-2)$_\mu$ Status: Experiment and Theory **26**





Array of NMR probes moves through beam tube on cable car

Determination of Average B-field (ω_p) of Muon Ensemble

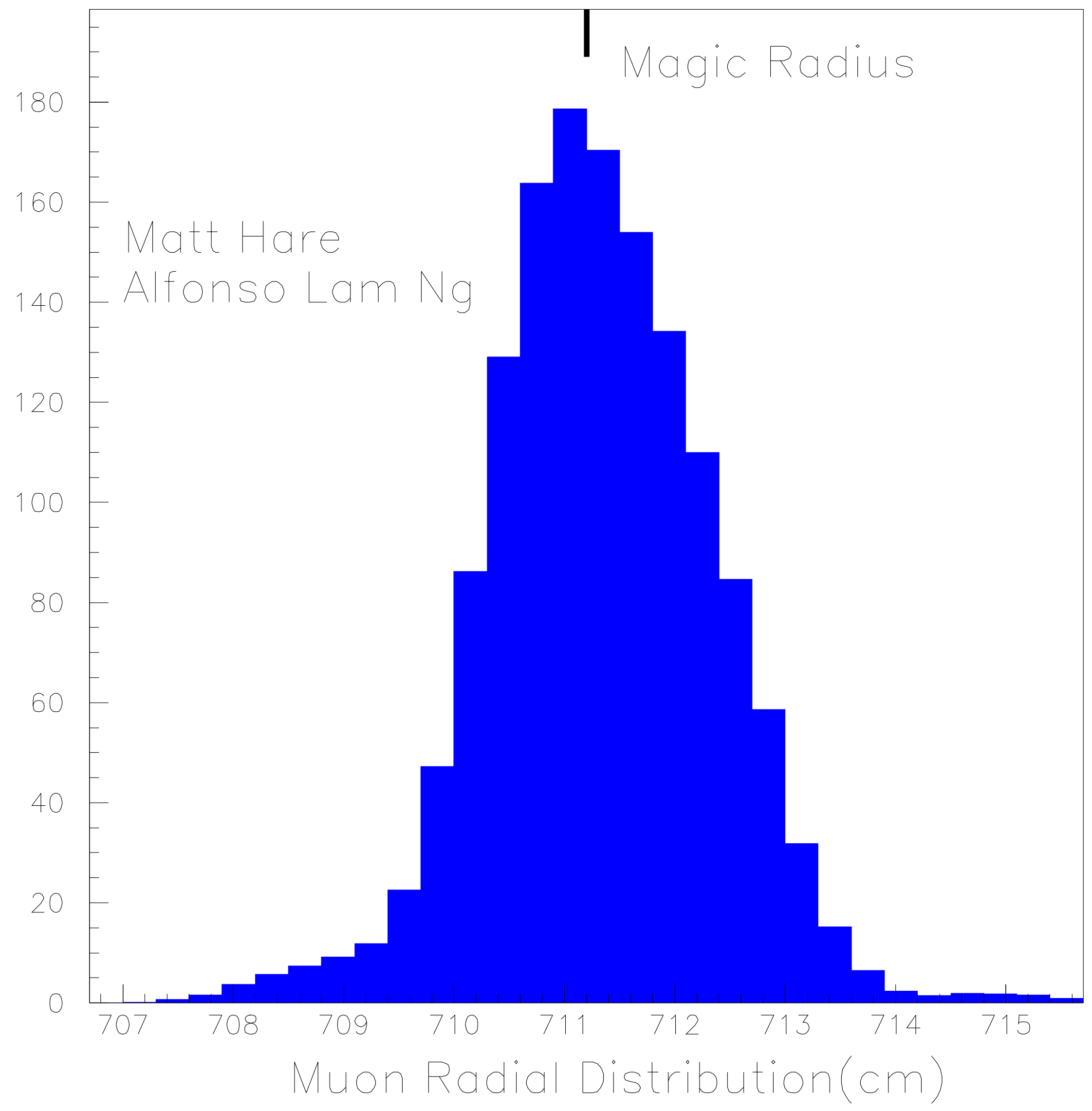
Mapping of B-field

- Complete B-Field map of storage region every 3-4 days
  Beam trolley with 17 NMR probes
- Continuous monitor of B-field with over 100 fixed probes

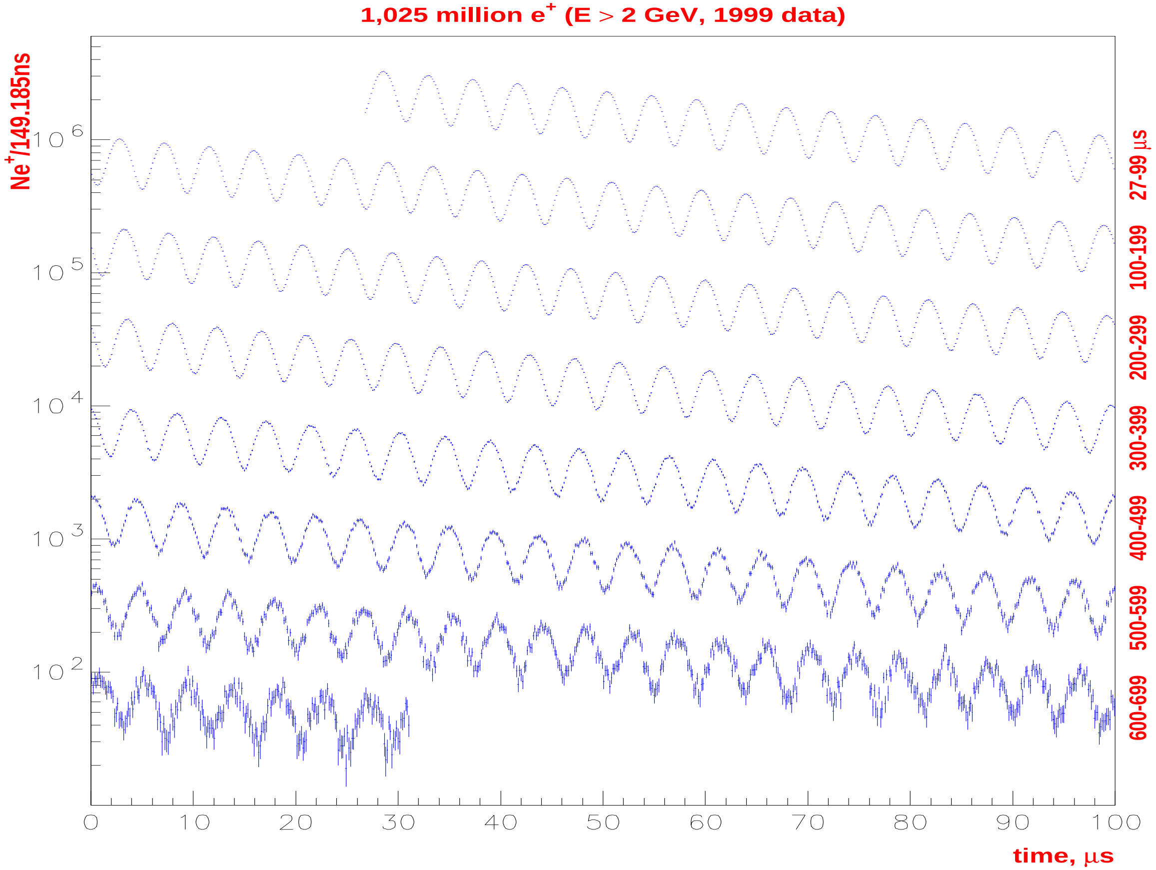
Determination of muon distribution

- Fit to bunch structure of stored beam vs. time

Determination of Muon Distribution



Log plot of 1999 data (10^9 e+)
149 ns bins
100 μs segments
Statistical error:
$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a \gamma \tau_\mu A \sqrt{N_e}}$$



5-parameter function (used to fit to 1998 data)

$$N(t) = N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

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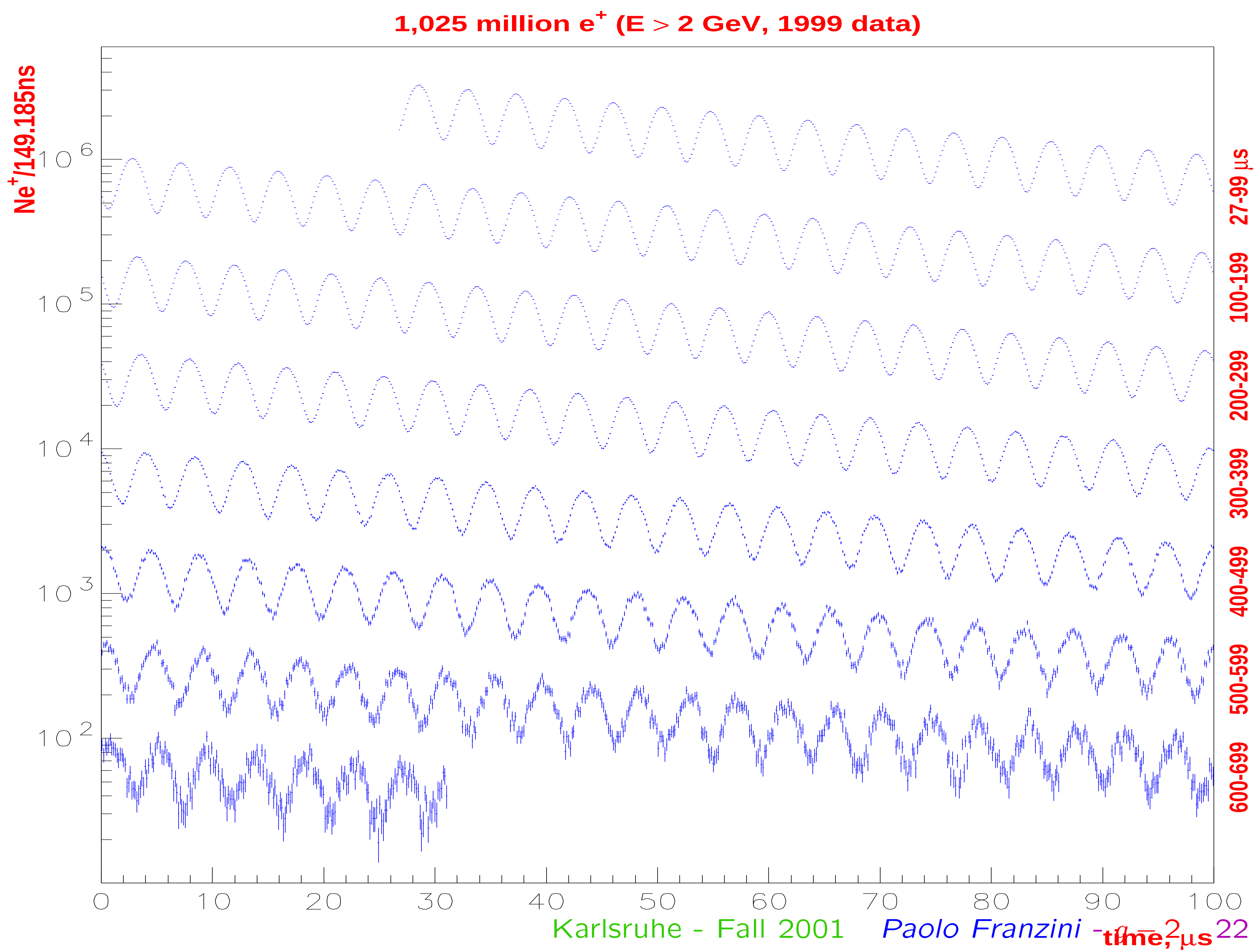

1,025 million e+ (E > 2 GeV, 1999 data)
Ne+/149.185ns
10 6
10 5
10 4
10 3
10 2
0
10
20
30
40
50
60
70
80
90
100
27-99 μs
100-199
200-299
300-399
400-499
500-599
600-699
time, μs
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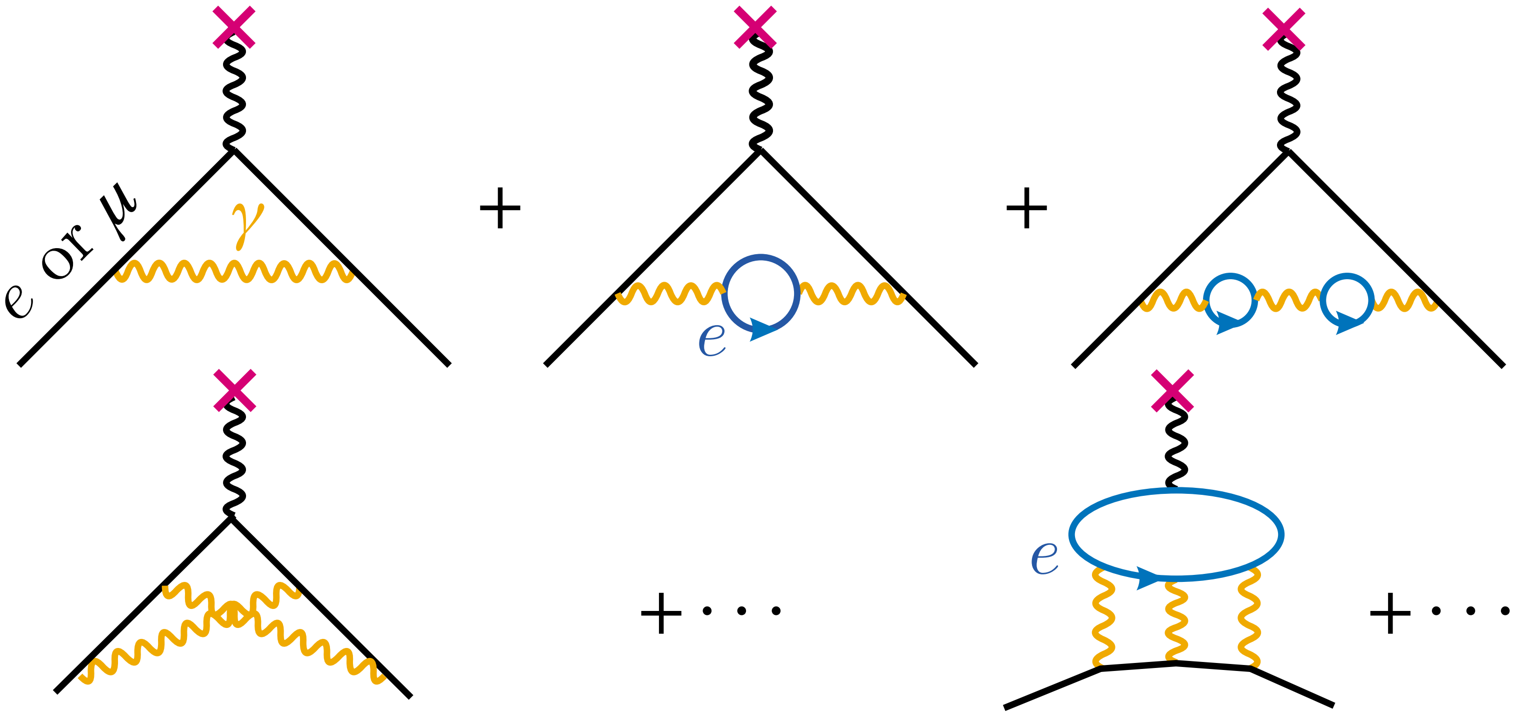
7. Computing $a = g/2 - 1$



$+ \ e \Rightarrow \mu, \tau; \ u, d, c, s, t, b; \ W^{\pm} \ldots$

$$a_e = \frac{\alpha}{2\pi} + \ldots c_4 (\frac{\alpha}{\pi})^4 = (115965215.4 \pm 2.4) \times 10^{-11}$$

Exp, e^+ and e^-: $\quad = (\quad \ldots 18.8 \pm 0.4) \times 10^{-11}$

Agreement to ∼30 ppb or 1.4 σ. What is α?

7.1 a_μ

Both experiment and calculation more difficult.

a_μ is m_μ^2/m_e^2~44,000 times more sensitive to high mass states in the diagrams above. Therefore:

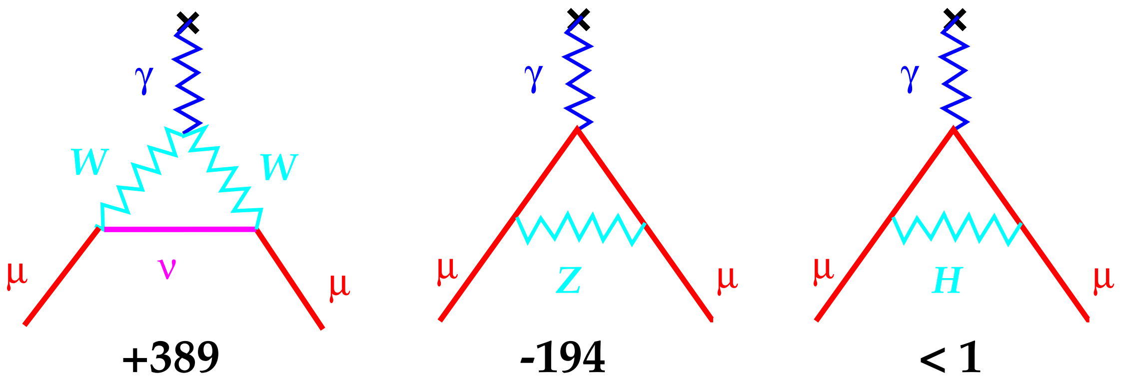
1. a_μ can reflect the existence of new particles - and interactions not observed so far.
2. hadrons - pion, etc - become important in calculating its value.

Point 1 is a strong motivation for accurate measurements of a_μ.

Point 2 is an obstacle to the interpretation of the measurement.

1. – New Physics

For calibration we take the E-W interaction

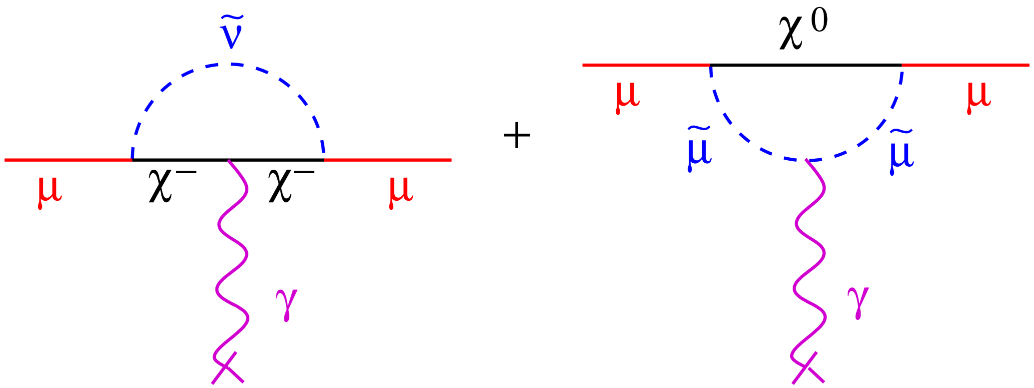


$\langle\phi\rangle = 236$ GeV

M$\sim$90 GeV

δa_μ(EW)$=150\times10^{-11}$

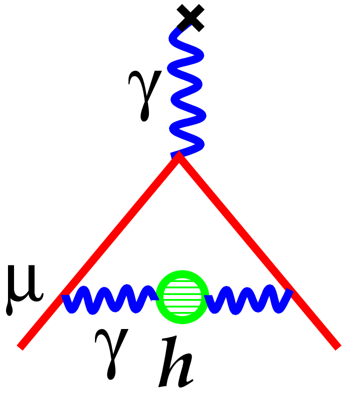
SUSY:



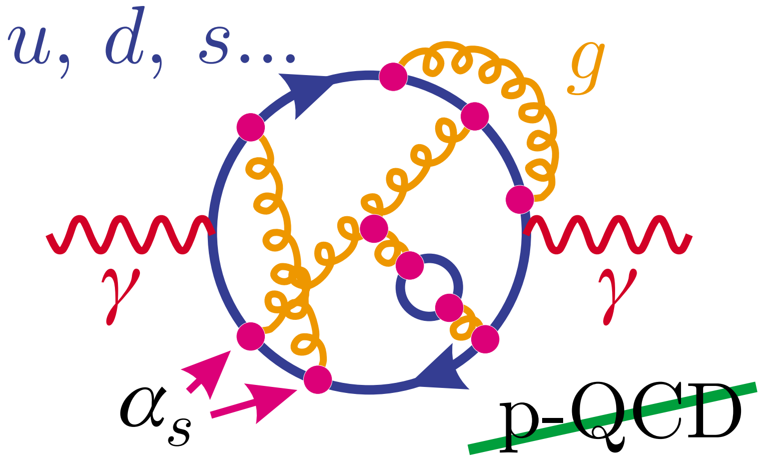
δa_μ(SUSY) $\sim150\times10^{-11}\times$ $(100\text{ Gev}/\tilde{M})^2\times\tan\beta$

2. – Hadrons

Need



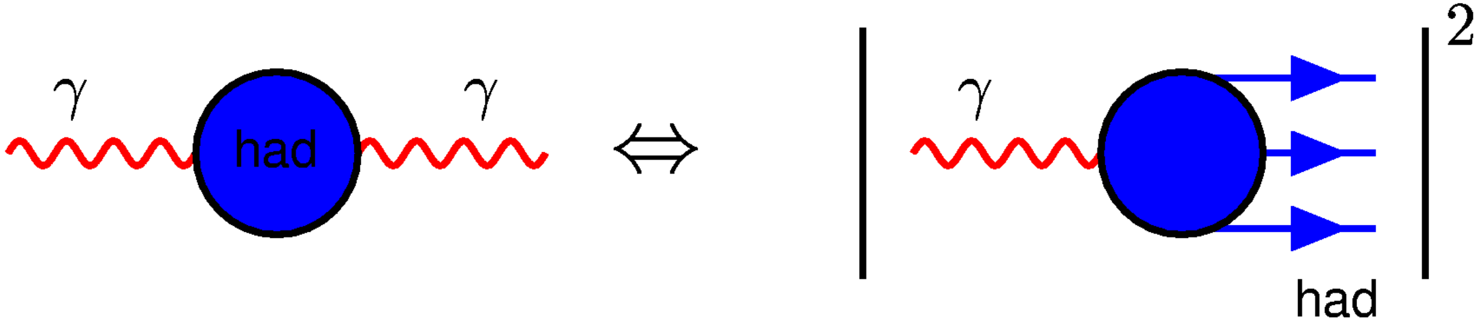
i.e.



which is not calculable at low q^2.

But. . .

Measure $\sigma(e^+e^- \to$ hadrons) and use dispersion relations:



$\Pi'_\gamma(q^2)$ $\sim \sigma^{\text{had}}_{\text{tot}}(q^2)$

δa_μ, (hadr - 1)$\sim 7000 \times 10^{-11}$

All these effects are irrelevant for a_e

$$a_\mu = \frac{\alpha}{2\pi} + \ldots c_4 (\frac{\alpha}{\pi})^4 = (116591596 \pm 67) \times 10^{-11}$$

$$\text{Exp}, \mu^+ : \qquad = (\qquad \ldots 2030 \pm 150) \times 10^{-11}$$

Measured-Computed=430±160 or 2.6 σ, ~3.7±1.4 ppm.

!!!!????

Standard Model Value for a_μ [1]

$$a_\mu(QED) = 116584706(3) \times 10^{-11}$$

$$a_\mu(HAD;1) = 6924(62) \times 10^{-11} \text{ (DH98)}$$

$$a_\mu(HAD;>1) = -100(6) \times 10^{-11} \text{ (Except LL)}$$

$$a_\mu(HAD;LL) = -85(25) \times 10^{-11}$$

$$a_\mu(EW) = 151(4) \times 10^{-11}$$

$$\text{TOTAL} = 116591596(67) \times 10^{-11}$$

[1] Czarnecki, Marciano, Nucl. Phys. B(Proc. Suppl.)76(1999)245

Used by the BNL g-2 experiment for comparison. Addition of above errors in quadrature is questionable.

8. $\sigma(e^+e^- \to \pi^+\pi^-)$

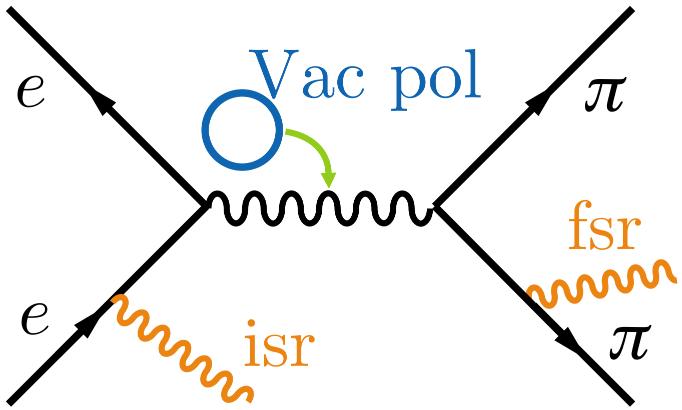
δa_μ(hadr - 1)$\sim 7000 \times 10^{-11}$

$\sigma(e^+e^- \to$ hadrons) is dominated below 1 GeV by $e^+e^- \to \pi^+\pi^-$. Low mass $\pi^+\pi^-$ (ρ, ω) contributes $\delta a_\mu \sim 5000 \pm 30$.

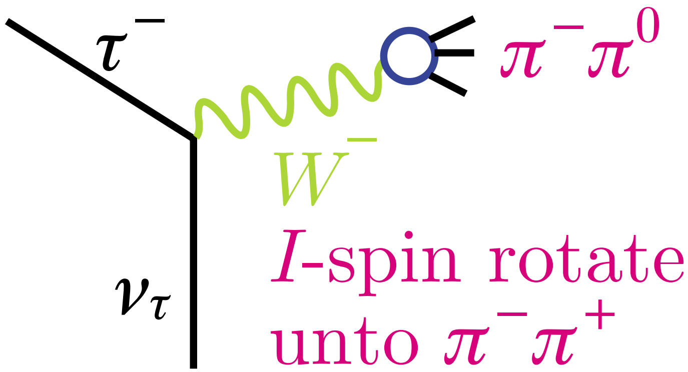
$\sigma(e^+e^- \to \pi^+\pi^-)$ or $(\gamma \to \pi^+\pi^-)$ is measured:

1. at e^+e^- colliders, varying the energy
2. in τ-lepton decays
3. at fixed energy colliders using radiative return

- 1. - Extensive measurements performed at Novosibirsk. Corrections for efficiency and scale plus absolute normalization (Bhabha, $e^+e^- \to e^+e^-$) are required for each energy setting. Data must also be corrected for radiation and vacuum polarization.



- 2. - τ data come mostly from LEP. To get σ(hadr) requires I-spin breaking, $M(\rho^\pm)$-$M(\rho^0)$, I=0 cntrib... corrections. Radiative corrections are also required.



- 3. - The radiative return method is being used by the KLOE collaboration, spear-headed by the Karlsruhe-Pisa groups.

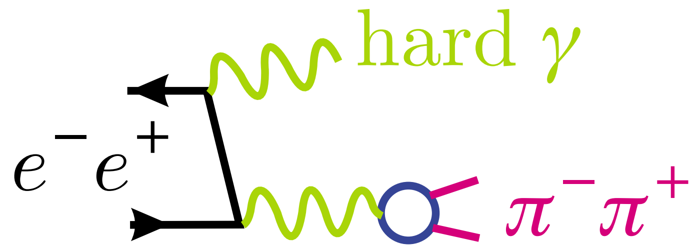
Can turn initial state radiation into an advantage.

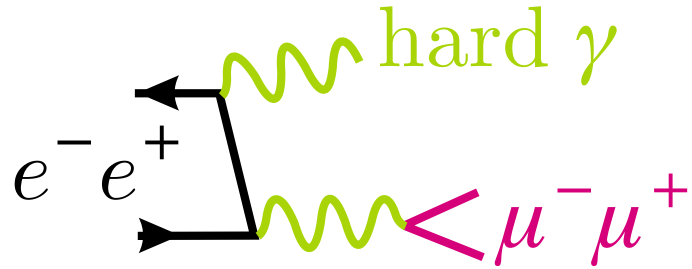
At fixed collider energy W, the $\pi^+\pi^-\gamma$ final state covers the di-pion mass range $280 < M_{\pi\pi} < W$ MeV.

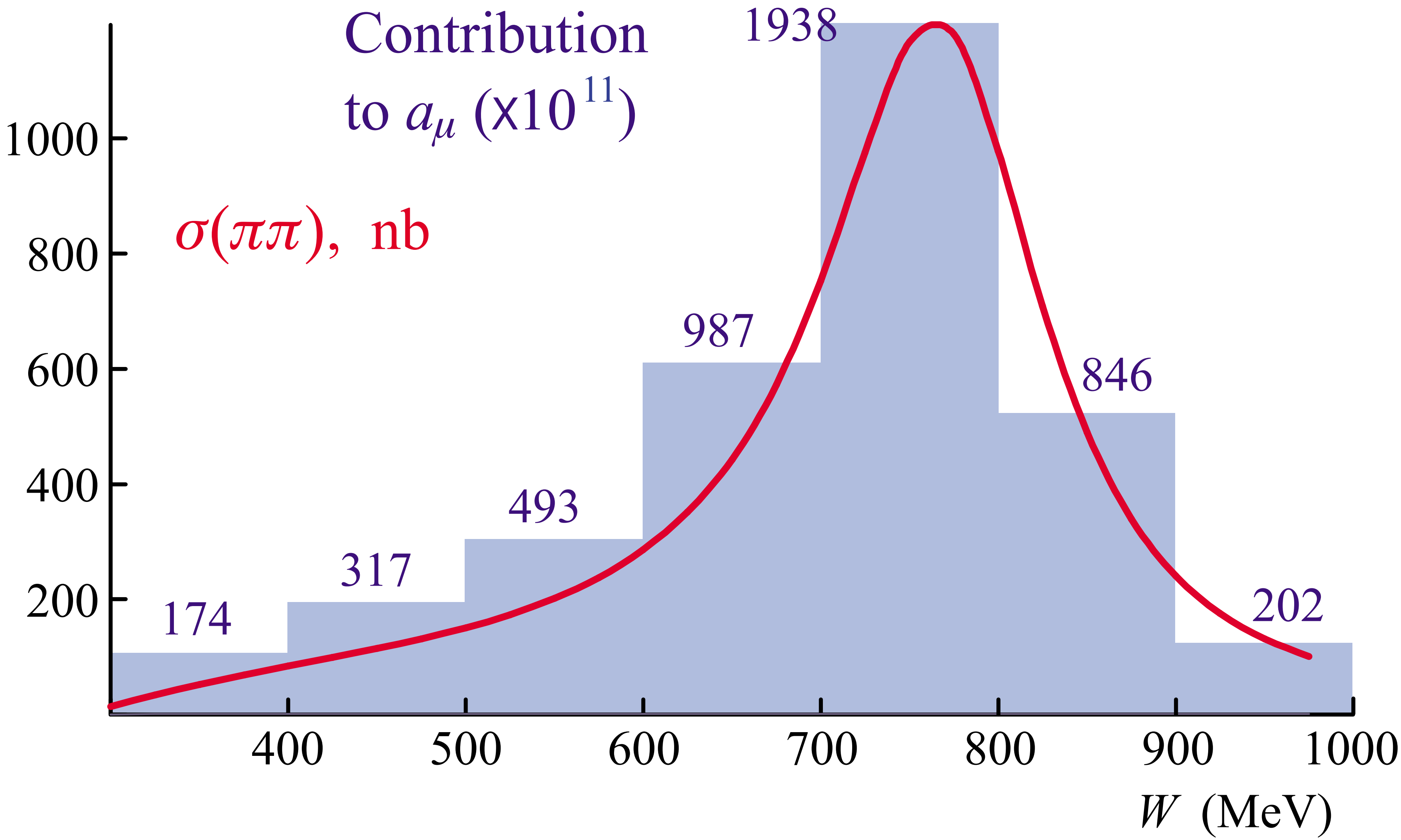
Correction for radiation and vacuum polarization are necessary.

All other factors need be obtained only once.

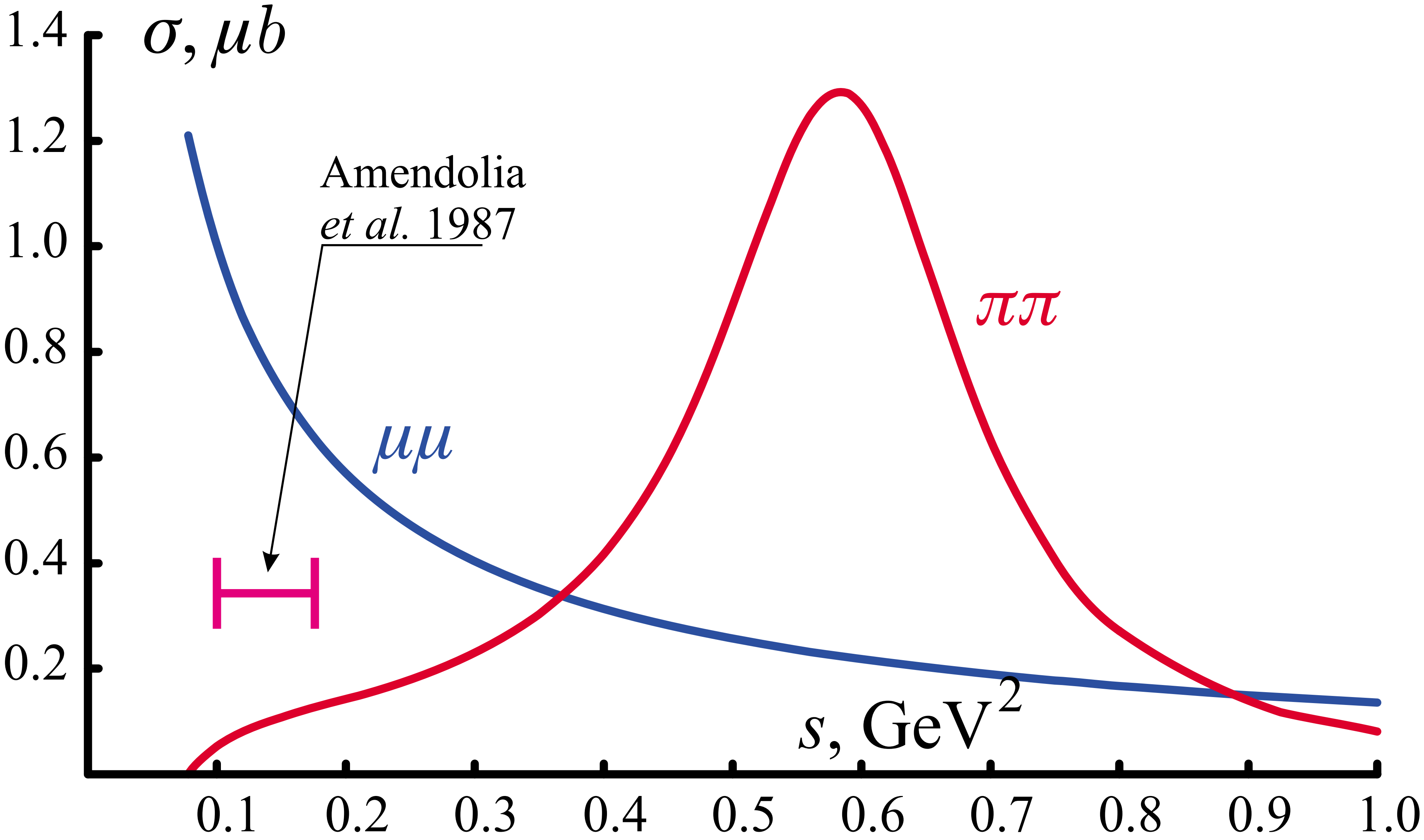
At low mass, di-muon production exceeds that of di-pion. ISR and vacuum polarization cancel.



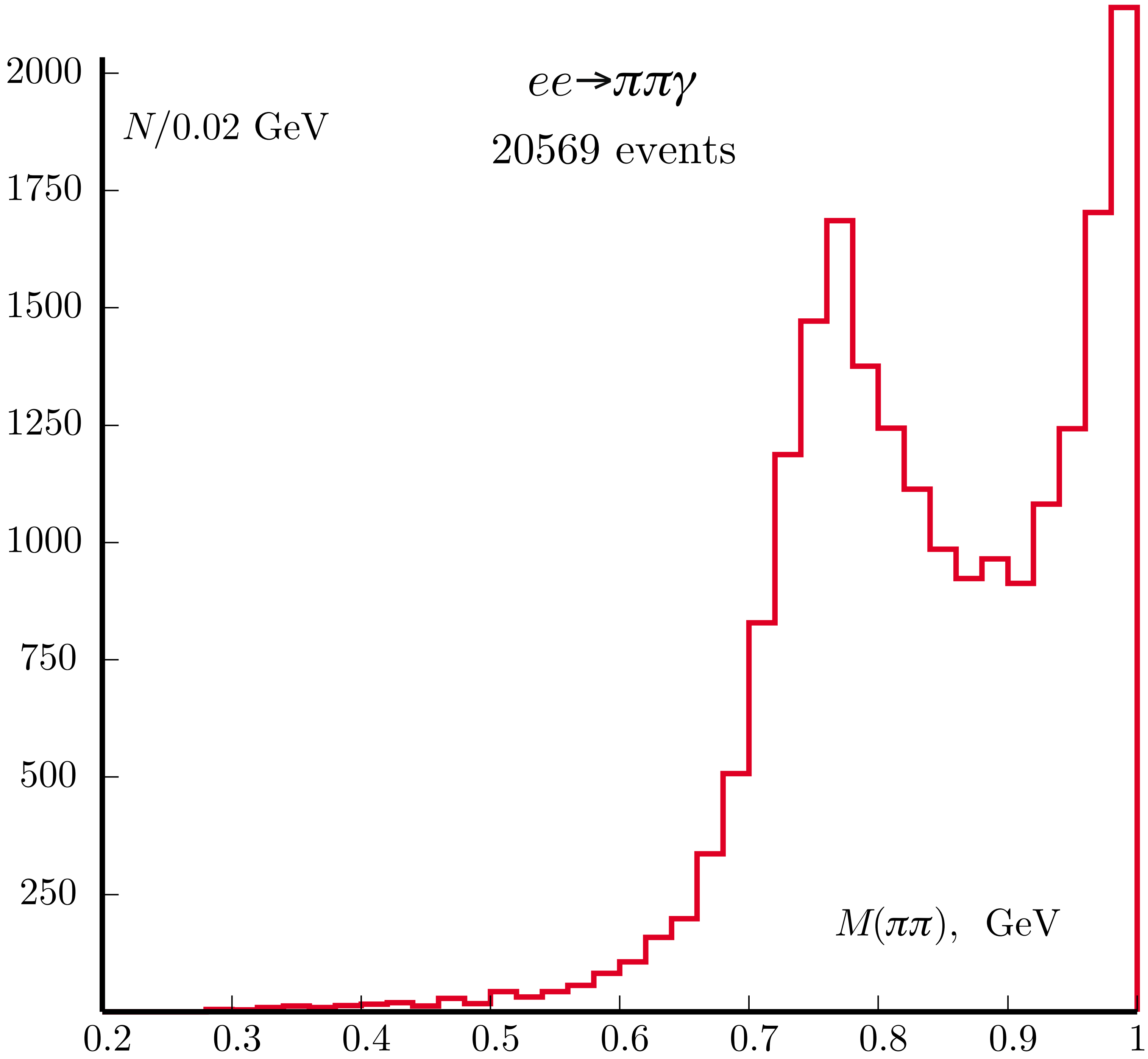


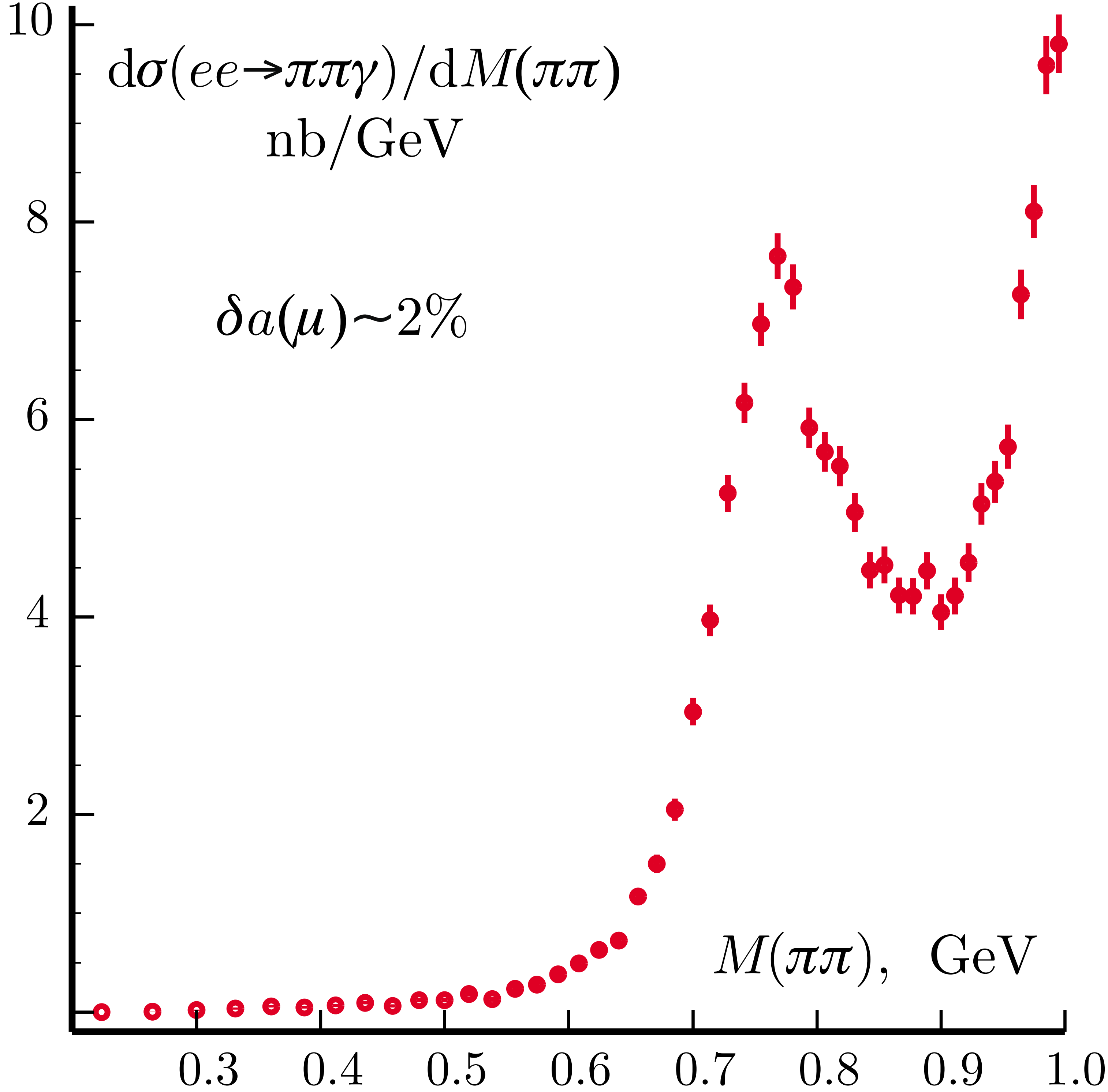


$$\sum(\ldots) = 5000$$

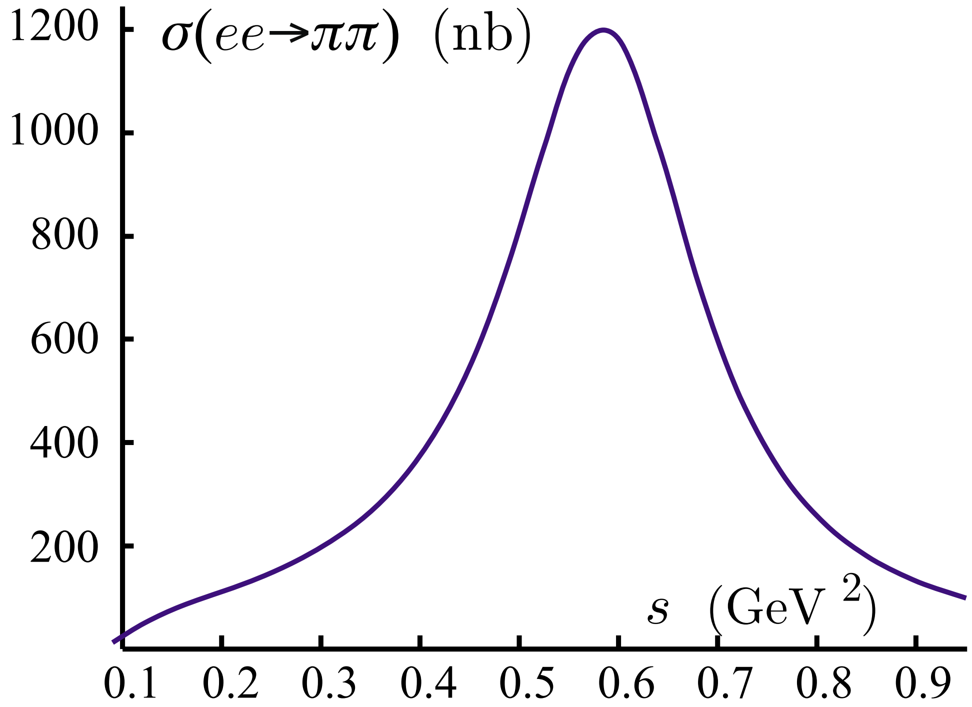


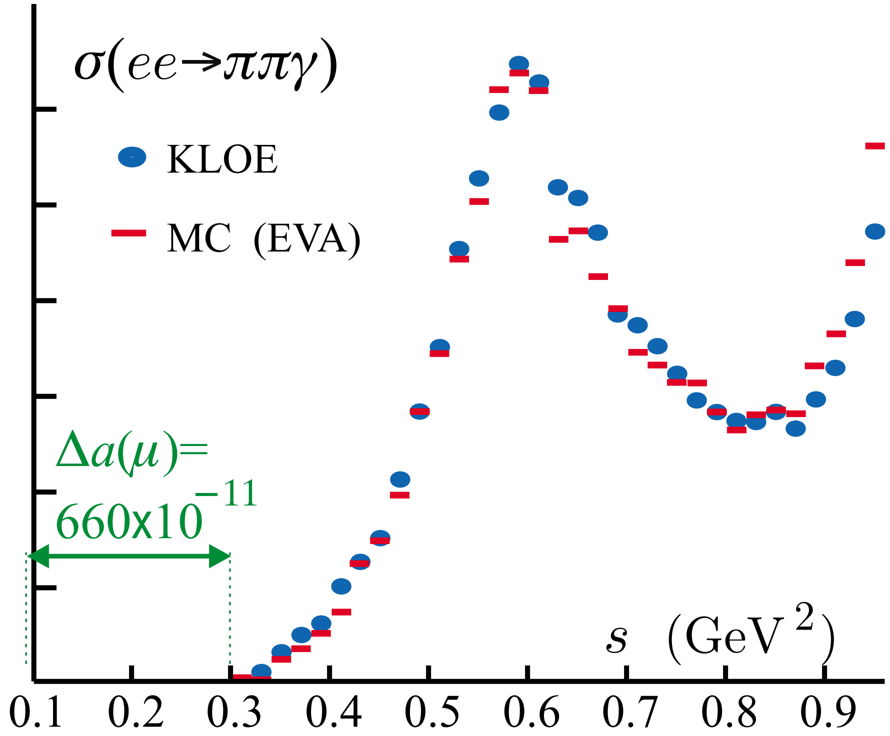
1.4
1.2
1.0
0.8
0.6
0.4
0.2
σ, μb
Amendolia
et al. 1987
μμ
ππ
s, GeV2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

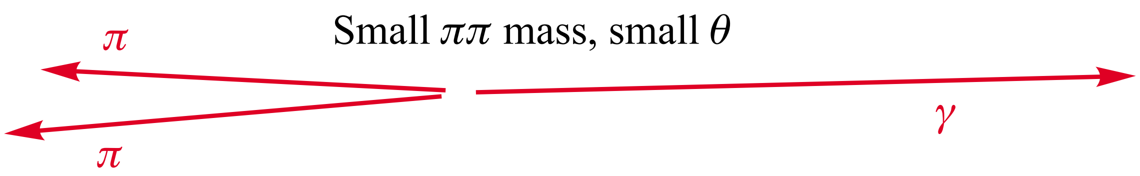

$ee \to \pi\pi\gamma$
20569 events
$N/0.02$ GeV
2000
1750
1500
1250
1000
750
500
250
$M(\pi\pi)$, GeV
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


$d\sigma(ee \to \pi\pi\gamma)/dM(\pi\pi)$
nb/GeV
$\delta a(\mu) \sim 2\%$
$M(\pi\pi)$, GeV
10
8
6
4
2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Use small angle radiation, higher x-section but miss low $M_{\pi^+\pi^-}$.







All lost in beam pipe.

Unsatisfactory points:

1. 2.6 σ is not very compelling and is also author dependent.
2. M-C is ~3×EW contribution. What about LEP, $b \to s\gamma$, M_W, $M_{\rm top}$, $\Re(\epsilon'/\epsilon)$, sin2β...
3. Hadronic corrections difficult, *e.g.* light-by-light
4. SUSY as a theory is not very precise at the moment. It has too many unknown, free parameters. There is no exp. evidence for it nor a prediction follows from the possible effect in the muon anomaly.

Soon better statistics and both signs muons.
Still very exciting at present.