Υ Spectroscopy and lattice QCD

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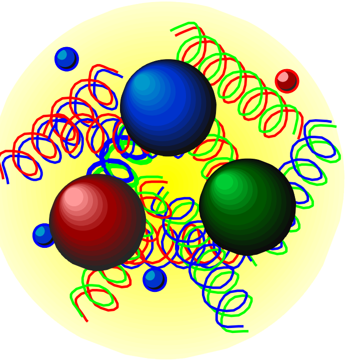
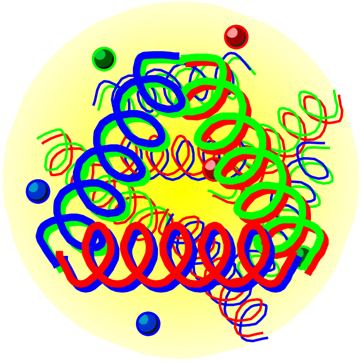
HPQCD (Glasgow, Cornell, OSU, SFU, FNAL) $+$ MILC collaborations.

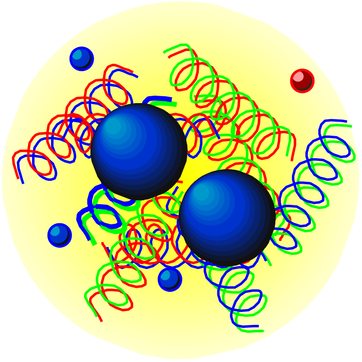
QCD describes strong interactions of quarks and gluons.



Proton



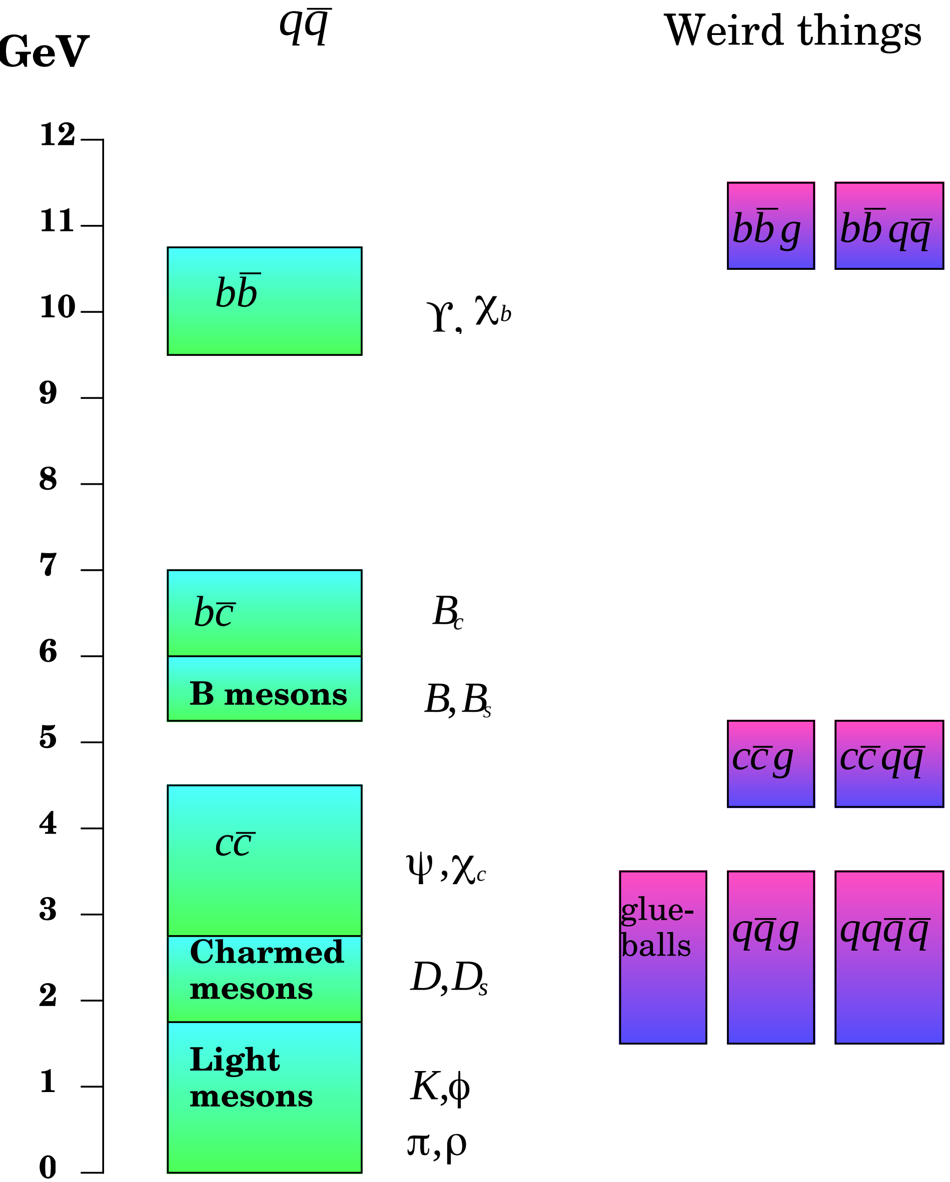
Glueball



Pion

QCD is confining

Quarks and gluons are bound into hadrons. Need numerical methods to solve QCD in this regime.

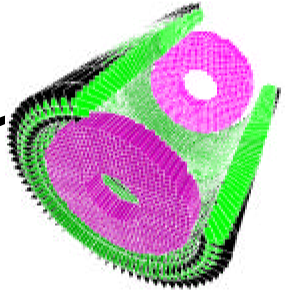


The Meson Spectrum

Spectrum of hadronic states made from u, d, s, c, b quarks is very rich.

Spectrum is predicted by QCD and we would like to calculate it from the theory.

Hadrons made from the heavy c, b quarks turn out to be particularly good ones to look at, both heavy-heavy and heavy-light.



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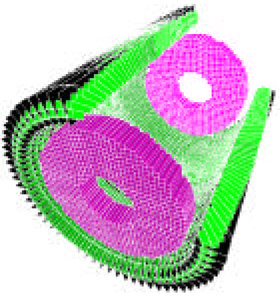
FORCES		Sys-tem	$(v/c)^2$	Ground triplet state 1^3S_1		Number of states below dissociation energy	
binding	decay			Name	Γ (MeV)	n^3S_1	all
POSITRONIUM							
EM	EM	e^+e^-	~0.0	Ortho-	$5\ 10^{-15}$	2	8
QUARKONIUM							
STRONG	STRONG	$u\bar{u},d\bar{d}$	~1.0	ρ	150.00	0	0
		$s\bar{s}$	~0.8	ϕ	4.40	“1”	“2”
	EM	$c\bar{c}$	~0.25	ψ	0.09	2	8
		$b\bar{b}$	~0.08	Υ	0.05	3	30
	weak	$t\bar{t}$	<0.01		3000.00	0	0

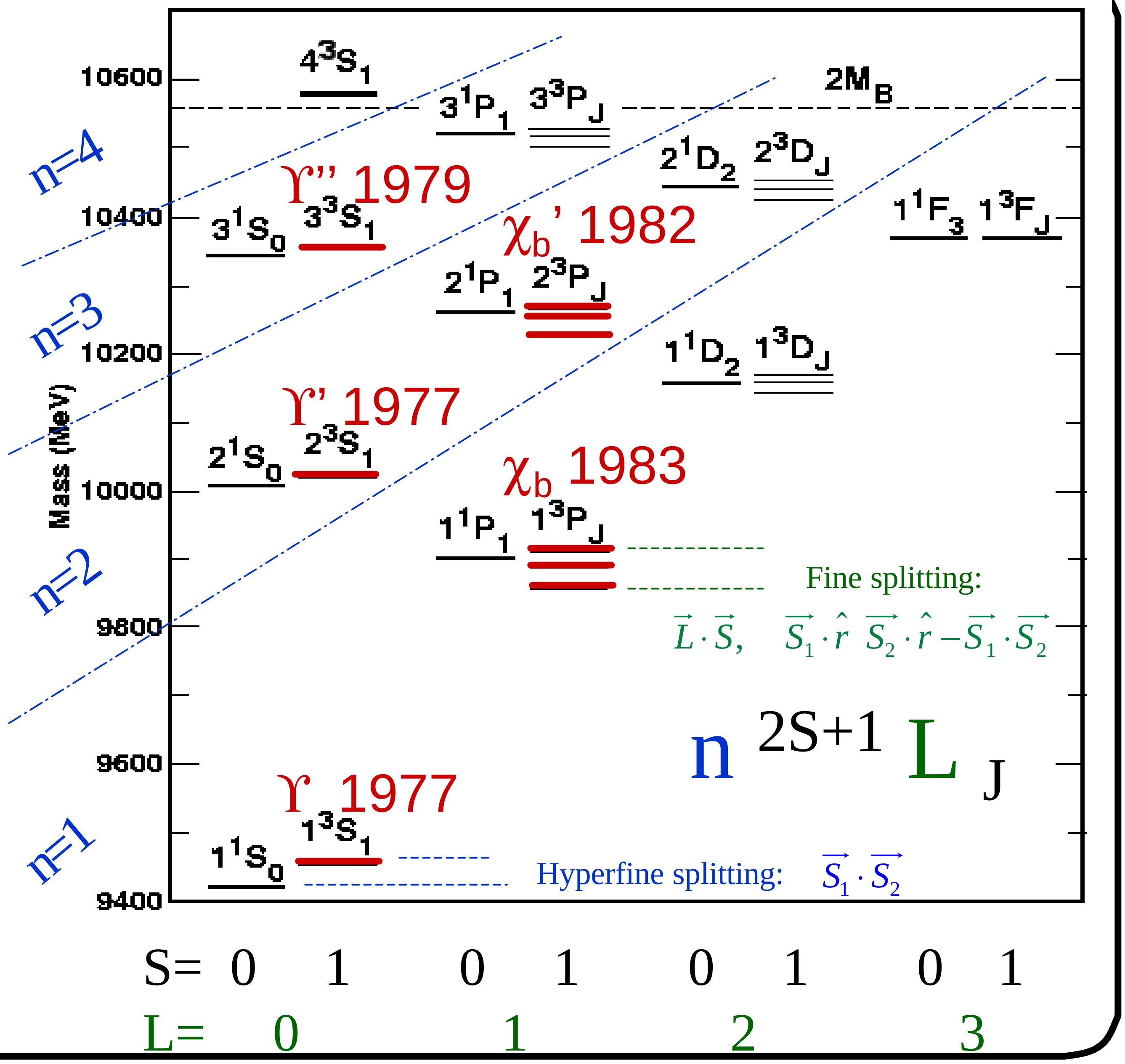
- Heavy quarkonia hold a promise of playing a similar role for QCD as positronium did for QED
- Upsilons are the most non-relativistic (i.e. simplest) states among all long-lived quarkonia states
- The Upsilon system also has the largest number of stable states

↦ Upsilons play a special role in probing the strong interactions (tests of lattice QCD, potential models)



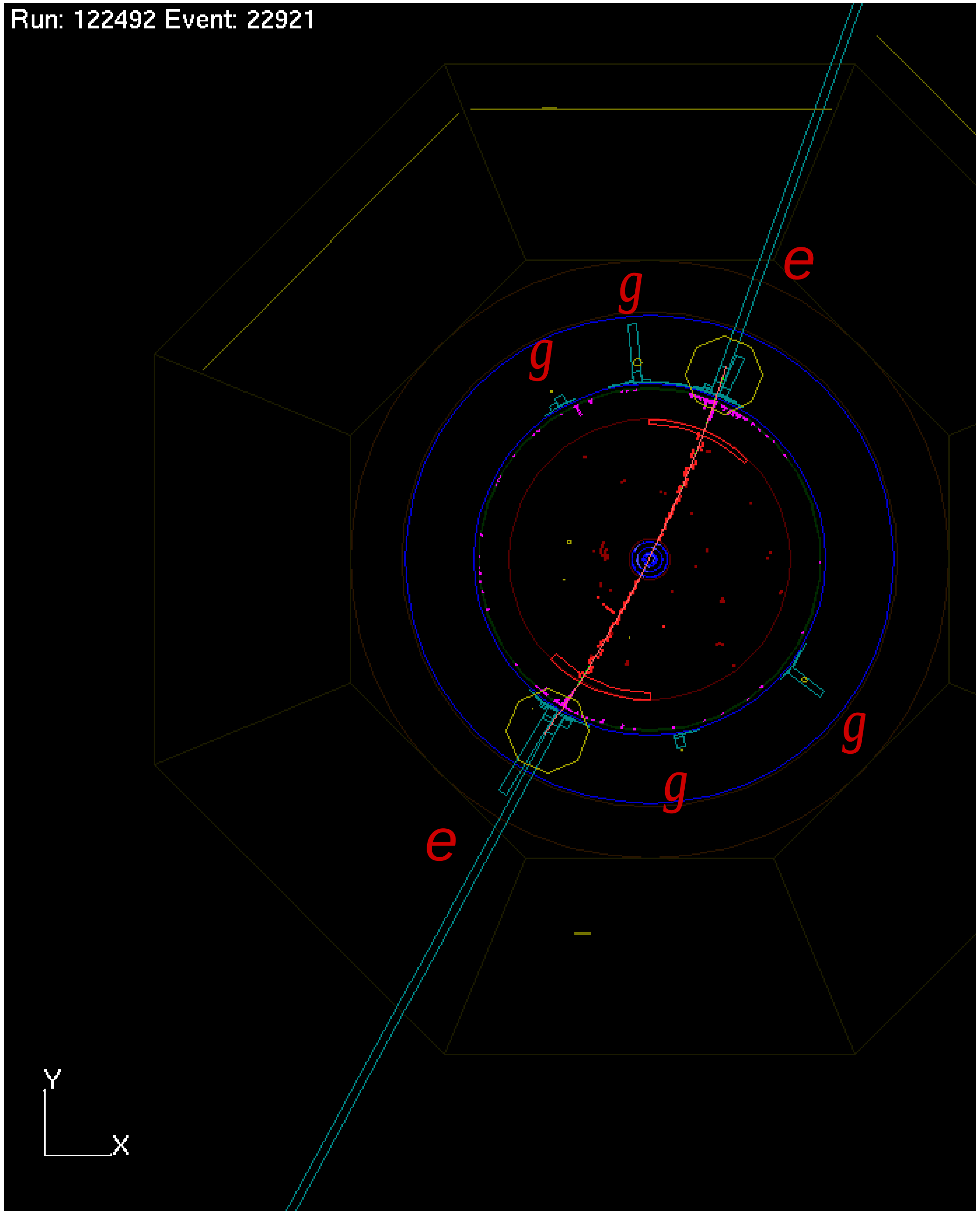
Upsilon States

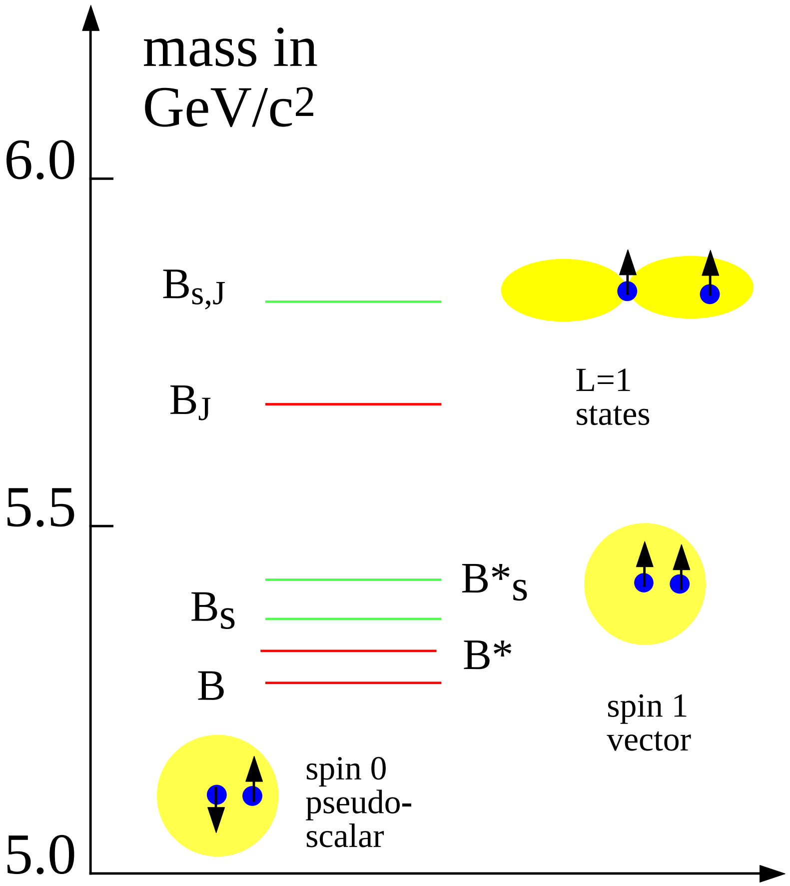
- Only 9 out of 30 narrow states observed so far
- No spin-singlet states observed
- No new states observed in 19 years



(Extra Slide)

γγγγee 1D candidate

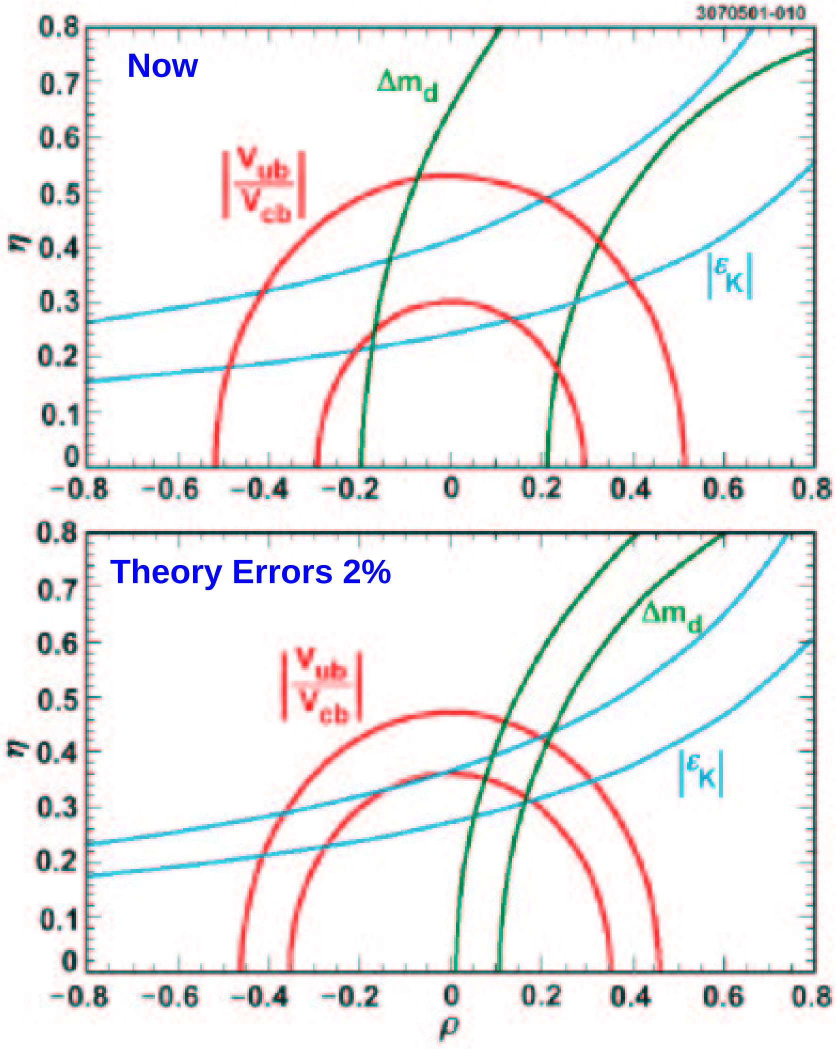




Heavy-light spectrum is not so well mapped out.

Can make mesons from b quarks and u, d or s quarks. Think of these as 'hydrogen atom' type states with heavy quark at centre and light quark cloud around.

CP violation



B mesons are being studied extensively at B-factories to extract parameters of the Standard Model (CKM matrix elements) relevant to CP violation.

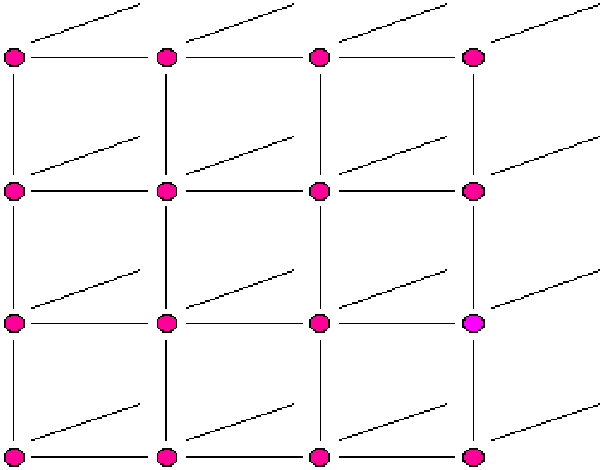
Need theory for this - lattice QCD.

Need to test the theory on other systems.

Lattice QCD calculations =

Euclidean space-time lattice + QCD Lagrangian (discretised)

$$\begin{aligned}\mathcal{L}_{QCD} &= \mathcal{L}_g + \mathcal{L}_q \\ &= \frac{1}{2g^2} Tr F_{\mu\nu}^2 + \overline{\psi}(\gamma \cdot D + ma)\psi\end{aligned}$$

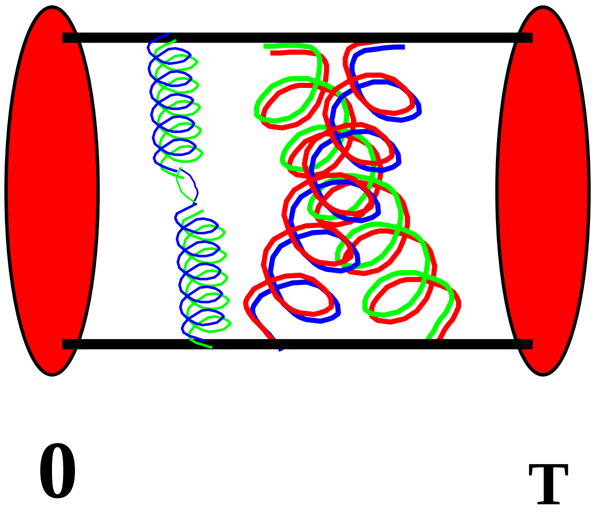


Lattice spacing (a) is implicit u.v. cutoff

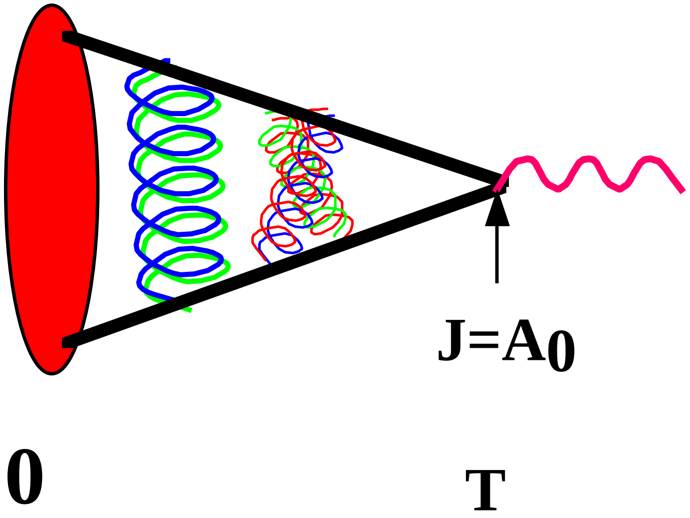
Parameters are : Gauge coupling g^2, Quark masses, $m_i a$

Calculation done by Monte Carlo integration of the Feynman Path ntegral. Generate gluonic ‘vacuum snapshots’ called configurations on which subsequent calcs are done.

Calculate quark propagators on gluon fields. Put together into hadron correlators of appropriate J^{PC}.



2pt function for spectrum

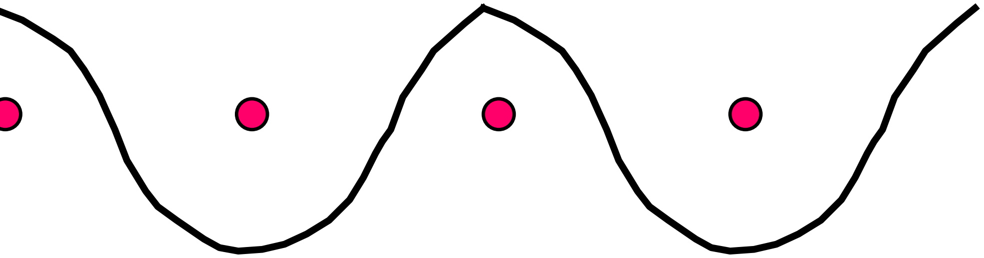


2pt function for decay constant

Fit to $A_0e^{-E_0T} + A_1e^{-E_1T}\ldots.$ E_0, E_1 given in units of a.
Set one mass equal expt to fix a, all others then in GeV.
n principle must then take $a \longrightarrow 0$ and $V \longrightarrow \infty$. Discretisation errors arise from mismatch of lattice and continuum actions. Remove with mproved actions.

Handling b quarks on the lattice

Problem: at $a \approx 0.1$ fm $M_b a \approx 2.5$ $(M_c a \approx 0.75)$
→ huge discretisation errors for methods based on the Dirac action.
Errors come from $\vec{p} \approx M$, i.e. relativistic momenta, distorted on the attice.



BUT, b and c quarks are non-relativistic in both HH and HL bound states. (radial excitation energy $<<$ mass)
→ M is not an important dynamical scale.
nstead focus on simulating scales like Mv and $\frac{1}{2}Mv^2$ accurately.

NRQCD is non-relativistic version of QCD

$$\mathcal{L}_Q = \overline{\psi}(D_t - \frac{\vec{D}^2}{2M_Q a} - c_4 \frac{\vec{\sigma}\cdot\vec{B}}{2M_Q a} + \ldots)\psi$$

ψ a 2-component spinor.

$M_Q a$ determined by getting heavy hadron mass correct.

NRQCD correct for important low-momentum physics ($p < M, \pi/a$).

ncorrect for irrelevant high-momentum physics.

Effect of missing high-momentum modes is short distance - correct by adjusting coeffs c_i in action to match QCD e.g perturbatively.

Cannot take a to 0 but improve until a-dependence small enough.

Power counting and error estimation

1. HH spectrum

Action is expansion in powers of v^2/c^2 ($\approx$ 0.1 for $b\bar{b}$, 0.3 for $c\bar{c}$) Current action inc. v^2, v^4, a^2v^4. $\rightarrow$ radial, orbital splittings in $b\bar{b}$ spectrum accurate to $\approx$ 1%. Spin splittings to $\approx$ 10%.

2. HL spectrum

Action is expansion in Λ/M. ($\approx$ 0.1 for $b\bar{l}$, 0.3 for $c\bar{l}$). Current action nc. $1/M$, $1/M^2$, a^2/M, but errors from light quark action.

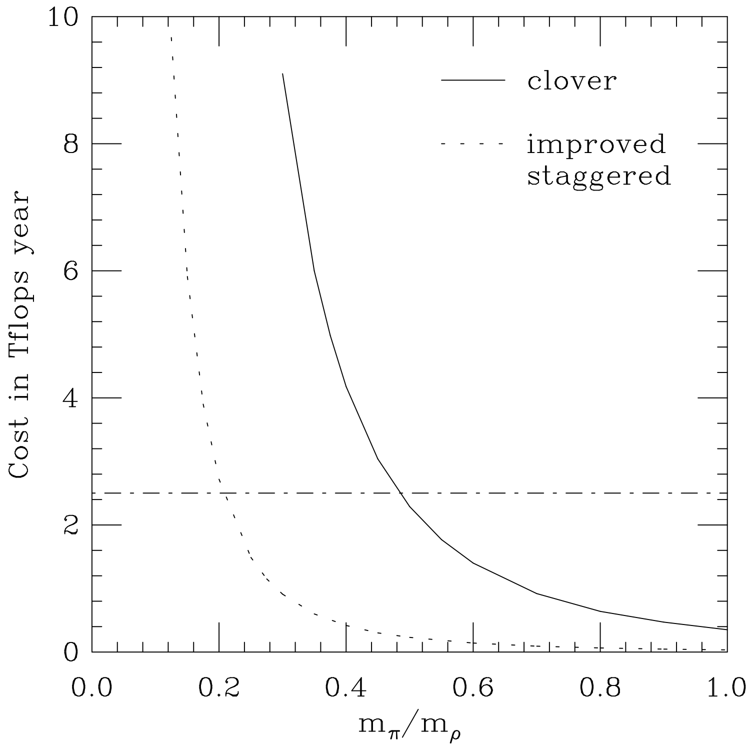
Calculations provide good test of QCD.

Major problem for lattice calculations : DYNAMICAL QUARKS

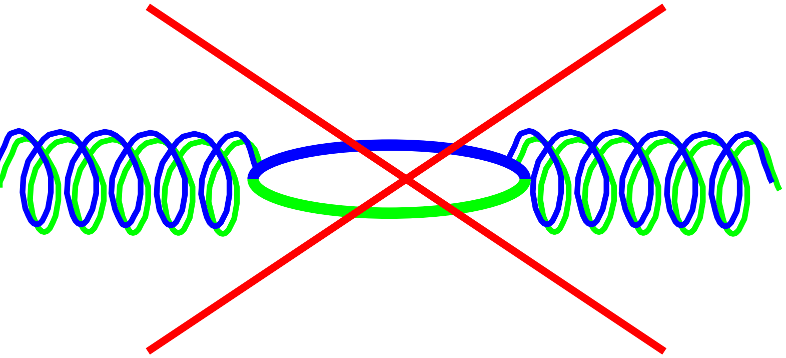
Quarks are fermions so computers cannot handle them explicitly.

ntegrate out of the Feynman P.I. ⇒ dynamical quarks give contrib. to S_{QCD} of $\ln(\det(M))$ where M is a huge $(2 \times 10^6$ on a side) sparse matrix.

Cost of inc. dynamical quarks is enormous. Most important and hardest are light u, d ones. Cost grows as $m_q \to 0$.



Solution until recently was to miss out dynamical quarks.
Quenched Approximation’.



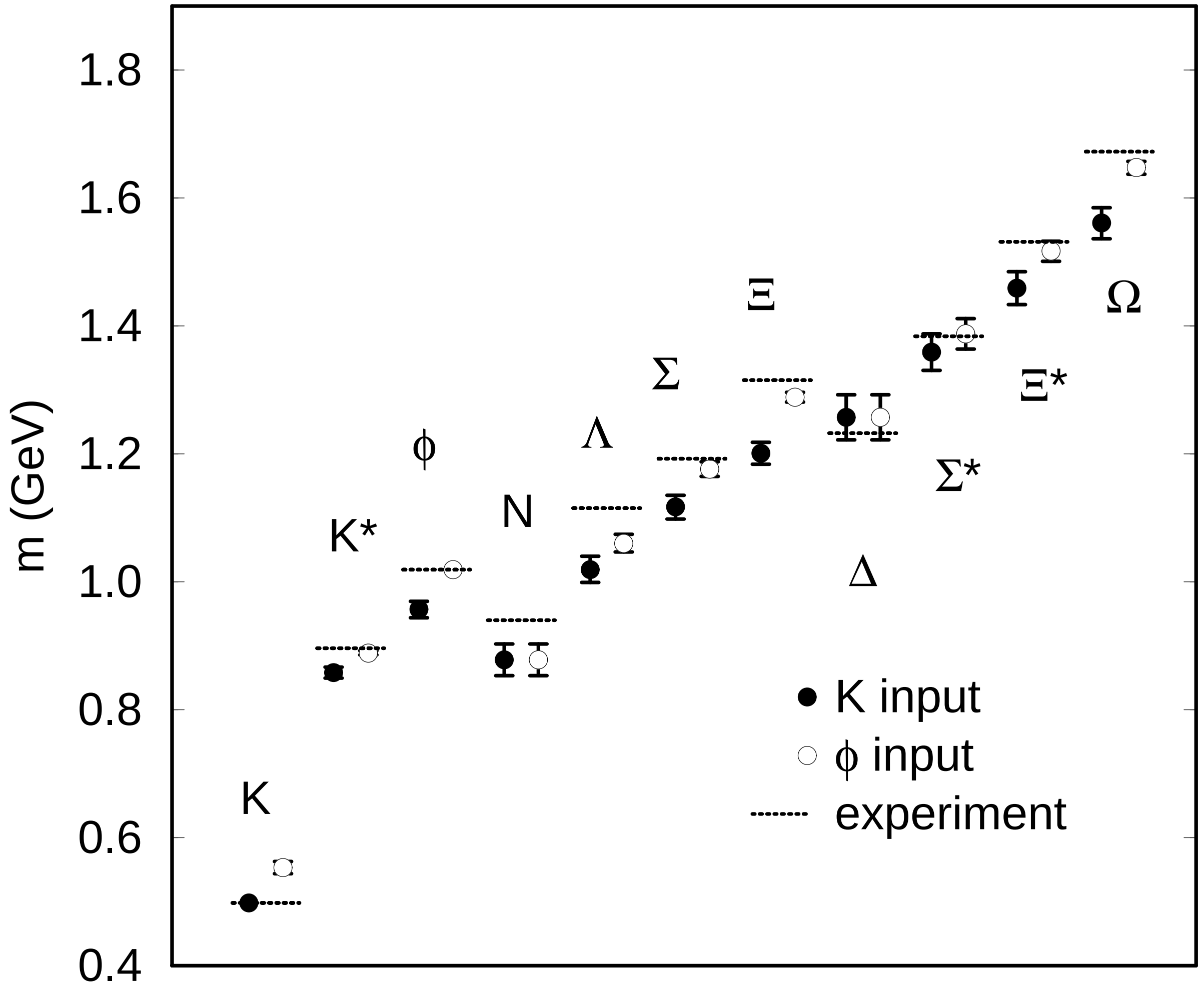
≡ cutting out feedback between quark and gluon sectors.
How wrong is this?

One effect is no ‘screening’ of charge, only anti-screening by gluons ⇒ charge doesn’t vary correctly with distance. If results sensitive to different distance/energy scales are compared, answer will be wrong.

Errors from quenching obscured for many years by discretisation errors. mproved actions now control these.

Quenched results

The light hadron spectrum is about 10% wrong, and is mpossible to consistently match to heavy hadron spectrum. Quark masses cannot be fixed consistently either.



(CP-PACS collaboration)

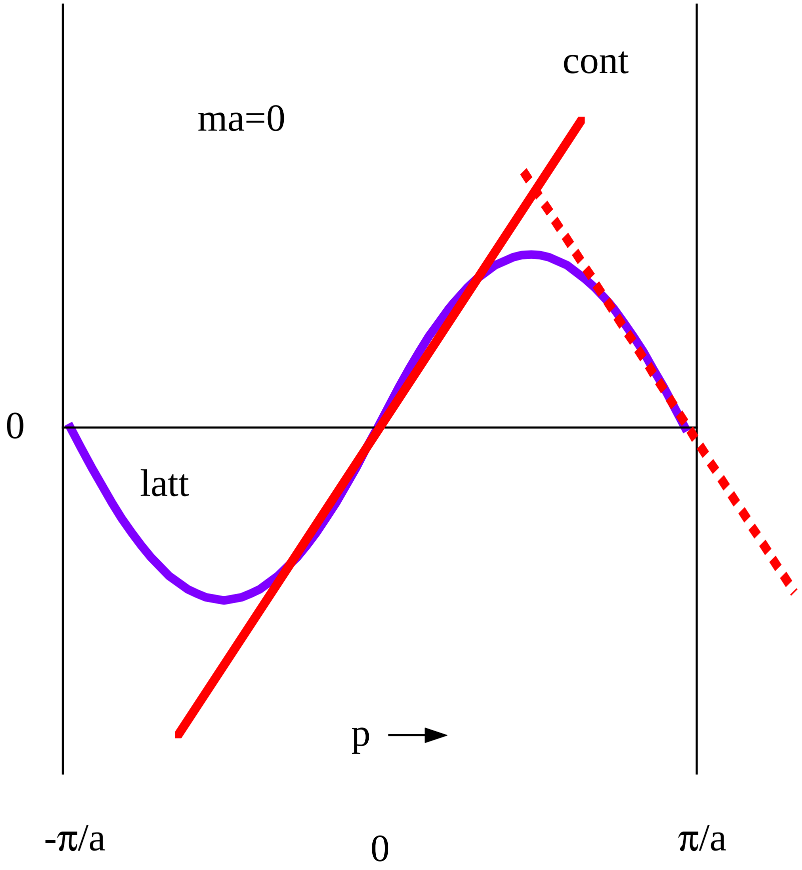
ncluding dynamical quarks

Fermion doubling' problem is additional headache.

Naive discretisation of the Dirac equation gives 2^d quarks instead of 1 i.e. 16 in 4-d. Comes from discretisation of simple deriv.

$D\psi \rightarrow ip\tilde{\psi}(p)$ in continuum

$D\psi \equiv 0.5*(\psi(x+1)-\psi(x-1)) \rightarrow isin(pa)\tilde{\psi}(p)$ on lattice



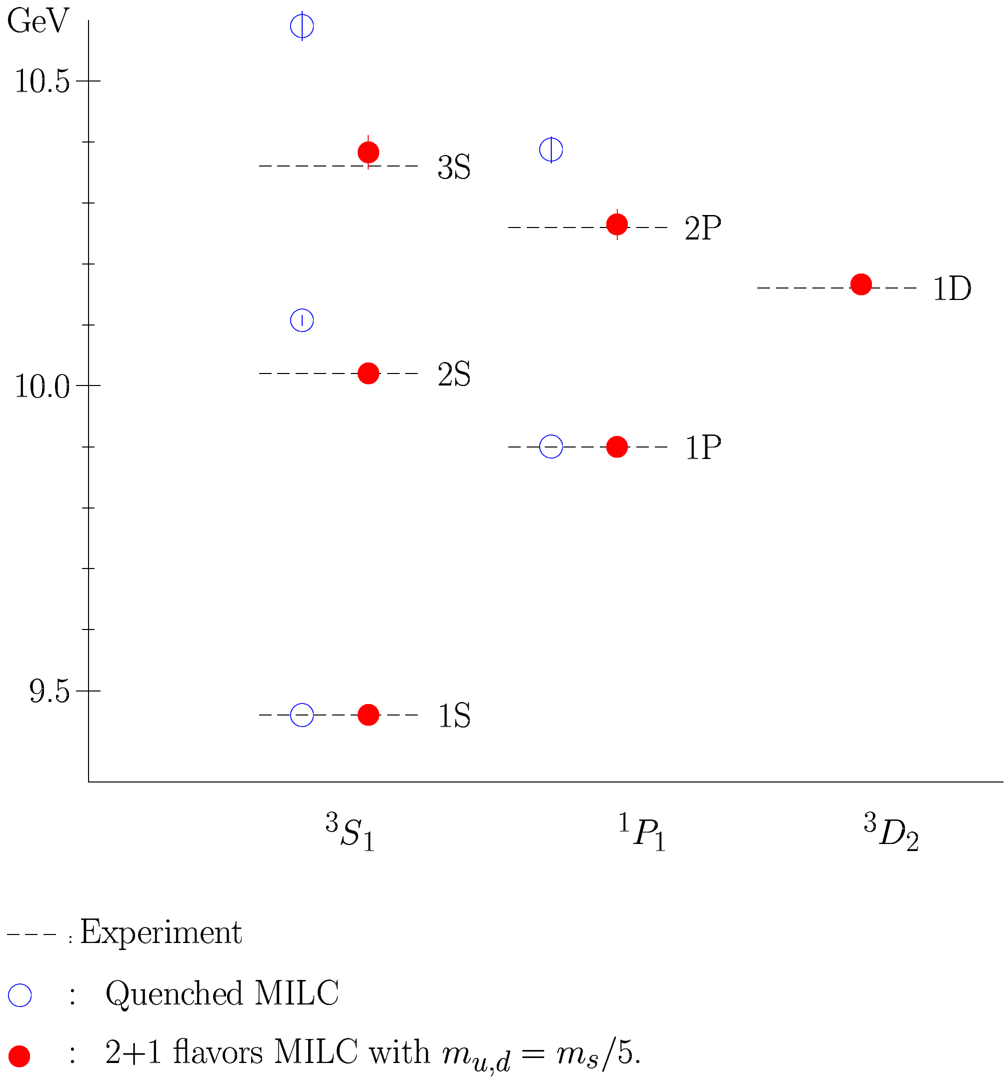
Different formulations address this in different ways at different costs. Traditional method used by SESAM is Wilson quarks; UKQCD/JLQCD use improved ‘clover’ quarks. Configs made with 2 flavours of dynamical quarks, $m \geq m_s$, small vols.

New‘improved staggered’ formalism much faster. Keeps 4 doublers and divides effects by 4 ($\sqrt[4]{det}$).

MILC have used improved staggered action to generate ensembles of configurations including the effect of 2+1 flavours of dynamical quarks for the first time. Sustained computing power = 0.25 Tflops.

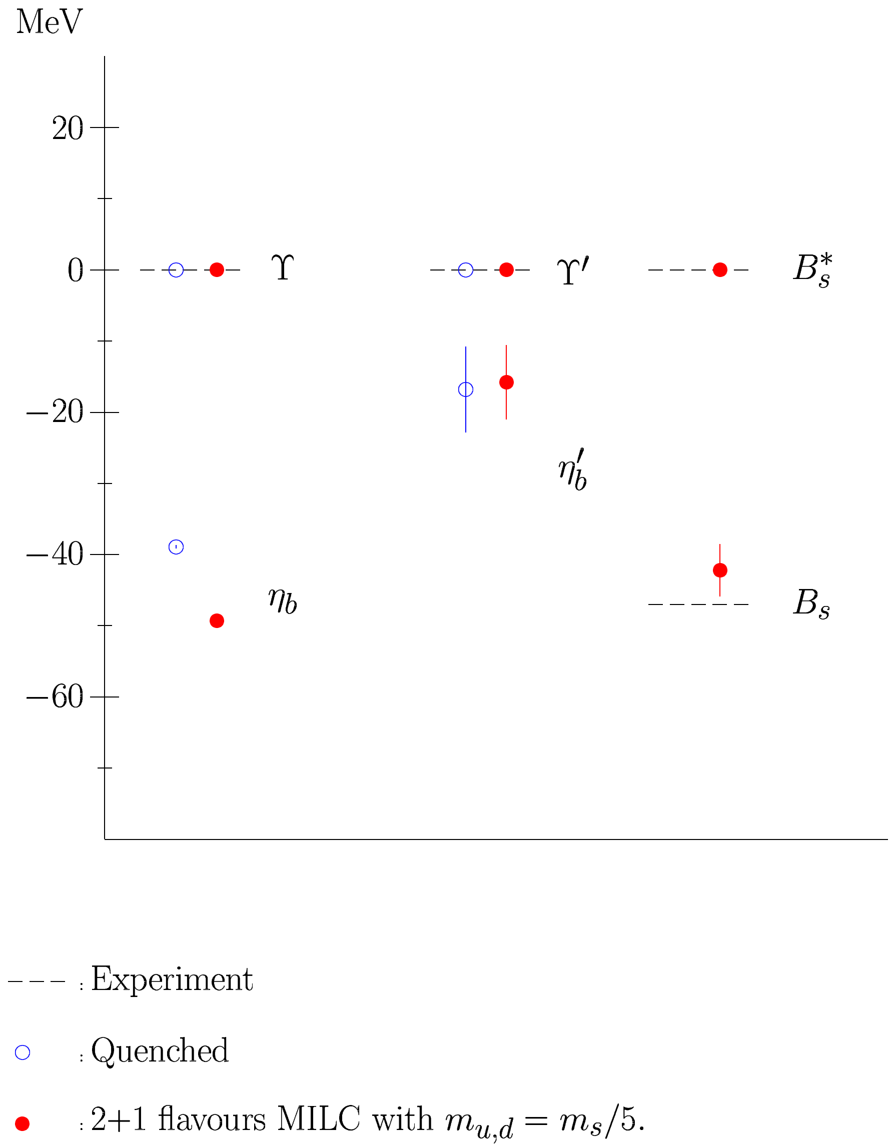
$2 = u, d$ with masses down to $m_s/5$
$1 = s$

2 sets, $a \approx$ 0.12 fm and $a \approx$ 0.08 fm. Coarse set light hadron results, hep-lat/0104002.



Υ spectrum from lattice QCD, Alan Gray, CD et al

Study $b\bar{b}$ mesons on these configurations to see if quenching errors have been removed. Good thing to study since no valence light quarks. Focus on radial and orbital excitations since these are most precise. Things look good.

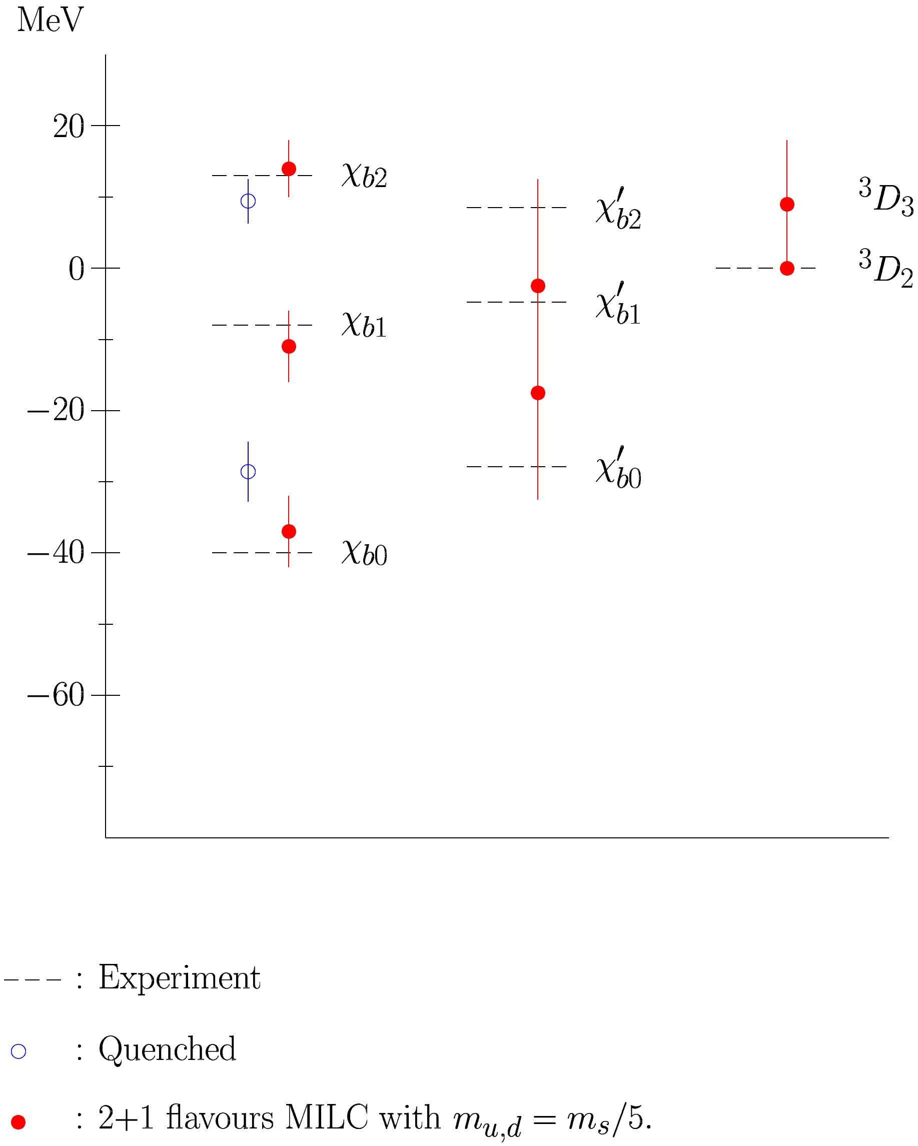


Calculation of the fine structure in the spectrum currently has systematic errors from relativistic corrections not included of order 10%.

Comparison to $B^* - B$ splitting shows these are not severe.

Aim to predict η_b mass for experiment.

Υ spectrum from lattice QCD, Alan Gray, CD, Matt Wingate, Junko Shigemitsu et al, HPQCD



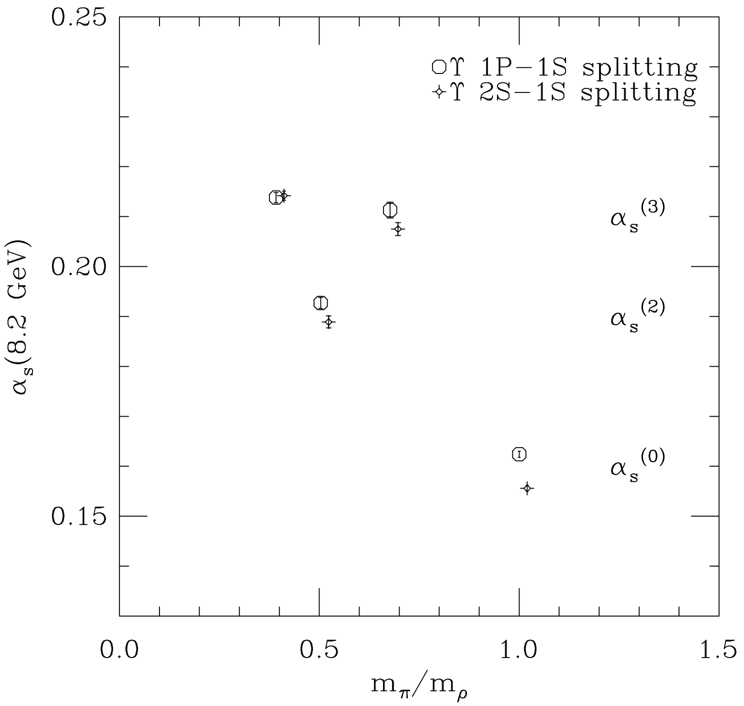
The P fine structure likewise has systematic errors from relativistic corrections not included of order 10%.

Also need improved statistical precision.

Aim to predict mass of $^3D_1$ inc. mixing with Υ'.

Υ spectrum from lattice QCD, Alan Gray, CD et al

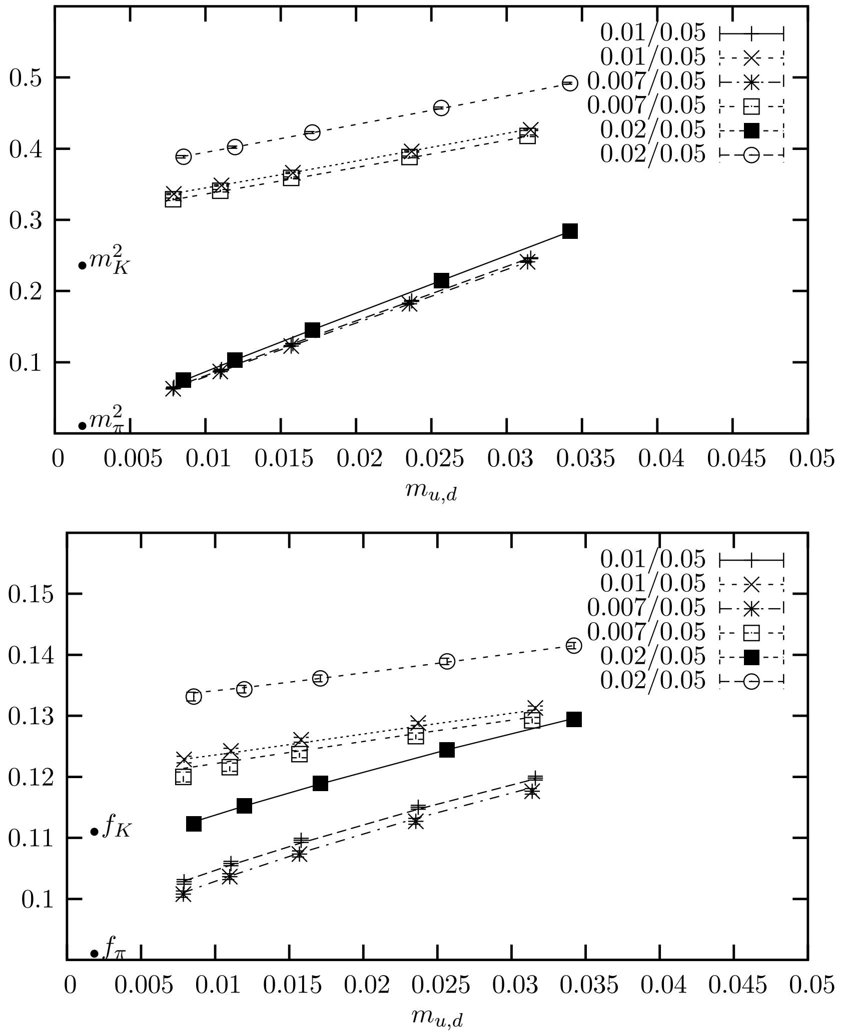
Determination of α_s



α_s determination from lattice QCD, Howard Trottier, Quentin Mason, Peter Lepage, CD et al

The strong coupling constant, α_s, can be determined precisely. Use a gluonic matrix element measured on lattice + lattice perturbation theory to get α_s. Fix scale from $1P - 1S$ splitting of Υ. Convert to $\overline{MS}$, run : $\alpha^{(5)}_{\overline{MS}}(M_Z) = 0.121(3)$

Light hadron results

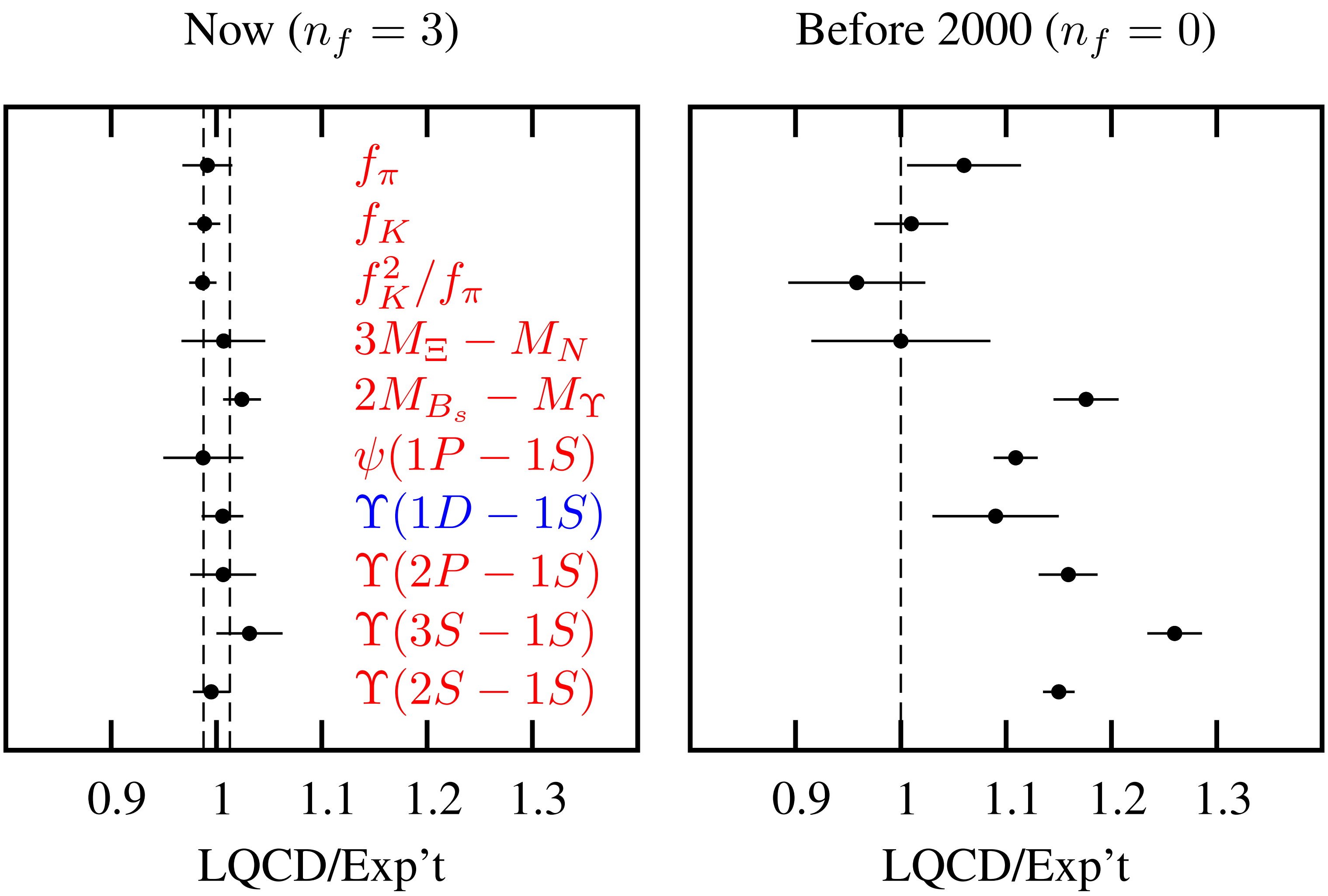


Light hadron masses must be extrapolated to the real world, 'chiral limit' using chiral pert. th.

Use to fix $m_{u,d}, m_s$, predict decay constants, f_π, f_K, amplitude for leptonic decay.

Light hadron decay constants, Peter Lepage et al + MILC

Putting it all together



nconsistencies of quenched approximation disappear!

Future lattice calculations

With 5 Tflops machine (UKQCD 2003) can make 500 config ensembles with 2 ($m < m_s/4$) + 1 (m_s) dynamical quarks and $a \leq$ 0.1fm using mproved staggered formulation. Others (APENEXT 2003) will explore more expensive Ginsparg-Wilson formalism.

Focus on staggered configurations will move to more complicated matrix elements and hadron masses (glueballs, hybrids), determination of quark masses.

Matrix elements to focus on are:

- Υ radiative decays and leptonic widths
- B, D, leptonic and semi-leptonic decays and mixing
- Nucleon structure function moments
- $K \rightarrow \pi\pi$ decays