On the crosscorrelation between Gravitational Wave Detectors for detecting association with Gamma Ray Bursts

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Crosscorrelation of the outputs of two Gravitational Wave (GW) detectors has recently been proposed [1] as a method for detecting statistical association between GWs and Gamma Ray Bursts (GRBs). Unfortunately, the method can be effectively used only in the case of stationary noise. In this work a different crosscorrelation algorithm is presented, which may effectively be applied also in non-stationary conditions for the cumulative analysis of a large number of GRBs. The value of the crosscorrelation at zero delay, which is the only one expected to be correlated to any astrophysical signal, is compared with the distribution of crosscorrelation of the same data for all non-zero delays within the integration time interval. This background distribution is gaussian, so the statistical significance of an experimentally observed excess would be well-defined. Computer simulations using real noise data of the cryogenic GW detectors Explorer and Nautilus with superimposed delta-like signals were performed, to test the effectiveness of the method, and theoretical estimates of its sensitivity compared to the results of the simulation. The effectiveness of the proposed algorithm is compared to that of other cumulative techniques, finding that the algorithm is particularly effective in the case of non-gaussian noise and of a large (100-1000s) and unpredictable delay between GWs and GRBs.

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I. INTRODUCTION

Over the last decade, Gamma Ray Bursts have been successfully investigated with the satellite experiments BATSE [2] and Beppo-SAX [3]. The large database now available includes information, for more than 2,000 GRBs, on the GRB arrival time, duration, intensity in some frequency bands, sky position of the source, and (for a small GRB subset) redshift. On the basis of all this information it is possible to design cumulative algorithms to detect a statistical association between the Gravitational Wave (GW) detector signals and the GRBs. It is well known that the present sensitivity of GW detectors is not sufficient to detect single events unambiguously, with the exception of very nearby GW sources, which are expected to be very rare. For this reason, much effort has been devoted to the development of data analysis techniques for the detection of coincidences between the events recorded by different detectors. The main problem facing this kind of analysis is that the event lists of both detectors are dominated by the contribution of non-gaussian and non-stationary noise [4]. Observation of a large number of GRBs, probably associated with explosive events capable of producing a large GW signal, has afforded the possibility to analyze the GW detector data around the GRB arrival times. Cumulative techniques have been proposed to detect a statistically significant association between GW signals and GRBs [5–9]. A difficulty arises from the theoretical uncertainty about the time delay between the GRB and GW arrival times. This delay depends on the models used to describe GRB dynamics, and, however, it is not expected to be constant. Theoretical predictions [10–12], and the interpretation of experimental results [13] based on a fireball model [14] suggest that delays up to 1hr should be expected. Thus, any cumulative analysis technique of GW detector data, synchronized to the GRB arrival time, would dramatically lose effectiveness due to the uncertainty about the GW-GRB time delay. Recently, a crosscorrelation method has been proposed by Finn et al. [1], to detect statistical association between GWs and GRBs. The method is based on measurement of the average of the crosscorrelation of two detectors (the two LIGO interferometers were considered), on a set (named on-GRB set) of time windows centered at the arrival times of the GRB events. An off-GRB set is used as a reference, and the statistical significance of the difference between the average crosscorrelation of the two sets is evaluated. As correctly claimed by the authors, the non-gaussian nature of the noise does not affect the method, but the hypothesis of stationarity of the GW detector noise is necessary to obtain a meaningful result. Gravitational wave detectors are typically affected by noise that is not only non-gaussian, but also non-stationary. The non-stationarity of the noise and uncertainty about the GW-GRB time delay make it difficult to choose a suitable off-GRB data set providing an unbiased background. In fact, it would be necessary to choose the off-GRB data samples near those of the on-GRB set in order to decrease the uncertainty introduced by the noise non-stationarity. On the other hand, due to GW-GRB

delay uncertainty, unbiased reference samples should be chosen far from the GRB trigger times. Thus it is very important to design a different crosscorrelation technique, capable of overcoming this difficulty with optimal choice of the reference quantity. In this work it will be shown that the reasonably obvious assumption of simultaneity of any real astrophysical GW signal on the two detectors may be used to design a simple technique for detecting association of GW signals with GRBs, which is affected neither by the non-stationarity nor the non-gaussianity of the detectors' noise. Simultaneity is assumed to hold within a Wiener filter characteristic time, which, for the narrow-band resonant GW detectors considered here, is always much longer than the physical delay associated with the distance between the GW detectors. In Section 2 the proposed method will be described. The results of numerical simulations and analytical computations will be shown and discussed in Section 3.

II. METHOD

Let $x_i(t)$ and $y_i(t)$ be the Wiener filtered outputs of the GW detectors X and Y, during a time interval T, centered at $t_{\gamma i}$, the time of arrival of the i-th GRB. The data are placed on a circular buffer, to compute on the same data set the crosscorrelation between the two GW detectors as a function of delay τ:

$$\chi_i(\tau) = \frac{1}{T} \int_{t_{\gamma i}-T/2}^{t_{\gamma i}+T/2} dt x_i(t) y_i(t+\tau) \tag{2.1}$$

The average $< \chi_i >$ and the standard deviation $\sigma_{\chi i}$ of the i-th $\chi_i(\tau)$ distribution are then computed, and the normalized, zero-mean quantity is obtained:

$$c_i(\tau) = \frac{\chi_i(\tau) - < \chi_i >}{\sigma_{\chi i}} \tag{2.2}$$

The functions $c_i(\tau)$ associated with each GRB are then averaged, yielding a cumulative zero-mean crosscorrelation, which is a function of delay τ:

$$C(\tau) = \frac{1}{N} \sum_i c_i(\tau) \tag{2.3}$$

For sufficiently long integration times, the quantity $C(\tau)$ is a random variable with gaussian distribution. The crosscorrelation signal to noise ratio SNRC is then obtained by dividing the zero-delay crosscorrelation C(0) by the standard deviation σ_C of the $C(\tau)$ distribution:

$$SNRC = \frac{C(0)}{\sigma_C} \tag{2.4}$$

We note here that this procedure, truncated at the step of Eq.2.2, may also be applied to single GRB events. The behavior of a $c_i(\tau)$, and the crosscorrelation snr, $SNRc_i$ may be evaluated for any single GRB. Analysis of an individual GRB could be interesting in the case of very peculiar GRBs (high power and/or low redshift). However, the method becomes particularly effective for the cumulative analysis of a large number of GRBs, because the cumulative SNR advantage, as the square root of the number of samples, is fully obtained also if the delay between the GRB and the GW is unknown and variable. The only requirement of the GRB-GW delay is that it must be smaller than the integration time, for all GRBs. In this case the crosscorrelation contributions due to differently delayed GW signals add coherently, while, in the case of other cumulative algorithms using one detector only, GRB-GW simultaneity is needed to obtain the full cumulative SNR advantage. It should be added that the integration interval doesn't need to be centered around the GRB arrival time, as it is in Eq.2.1. It may be arbitrarily shifted to test the predictions of different theoretical models.

III. RESULTS AND DISCUSSION

The sensitivity of the above-described procedure has been evaluated, both analytically and with computer simulations, using real Wiener filtered [15] data of the GW detectors Nautilus [16] and Explorer [17]. A period of one year of GW data was considered. The data sampling time is 0.296 s. The adaptive Wiener filter smooths the noise with the time constant $\tau_3 \approx 1$ s for both detectors, corresponding to an effective bandwidth $\beta_3 \approx 1$ Hz. The noise level of the

filtered data is expressed by the effective temperature T_{eff}, which is the minimum energy variation detectable by the antenna with signal to noise ratio equal to unity. One-hour-long common data stretches were used, centered at the times given by a dummy GRB time list with the same experimental BATSE GRB rate (1/day) [2]. The data stretch associated with each GRB was selected for the analysis if the average effective temperature proved less than 20 mK on both detectors. This choice was suggested by the noise distribution of the detectors in the considered year. The noise distributions of both detectors were peaked in the 10-15 mK range, so a higher threshold would have not significantly improved the statistics. The above constraints yielded N = 27 selected one-hour common data stretches of 12000 samples each, with an average effective temperature T_{eff} = 13 mK. These figures give the important information about the size of the statistical sample that can reasonably be obtained for a crosscorrelation analysis of two real GW detectors. It is clear that the requirement of having both detectors simultaneously in operation with low noise performance severely reduces the size of the available statistical sample. The same constraint applied to one detector only would give a much larger sample (N=150). This is clearly a drawback of the proposed method, which could be limited by using pairs of data coming from more than two GW detectors. In Fig.1 the energy histogram of the filtered data of both detectors is shown. The energy is normalized to T_{eff} of the selected data. Looking at the distribution of Fig.1, it is clear that the large non-gaussian tail of the energy event distribution makes it impossible to detect unambiguously individual events with E/T_{eff} = 5-10.
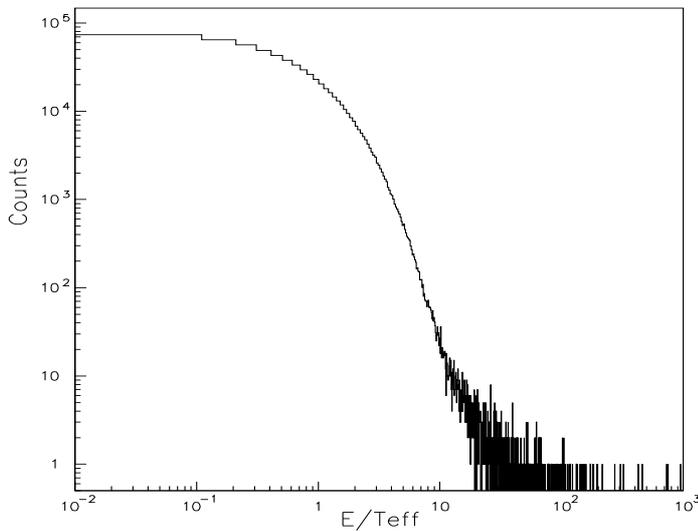
FIG. 1. Energy histogram of the Wiener filtered data of the detectors in the 27 selected common hours. The energy is normalized to T_{eff} of the selected data.

This is a well-known problem in GW data analysis, which has generally led the GW data analysis community to define as GW events, to be used for a coincidence analysis, those with a very large value of E/T_{eff}. The procedure was applied to the N one-hour periods. The choice of using one hour integration time is proposed to include the contribution of GW signals associated to GRBs, according to the delay predicted by most theoretical models. This choice could be optimized, as will be discussed later. In Fig.2 the normalized average crosscorrelation $C(\tau)/\sigma_C$ and its histogram are shown for the 27 selected hours. The distribution of the crosscorrelation variable $C(\tau)/\sigma_C$ shows a gaussian shape, without any significant tail, while the original distributions of the two detectors' noise were both markedly non-gaussian.

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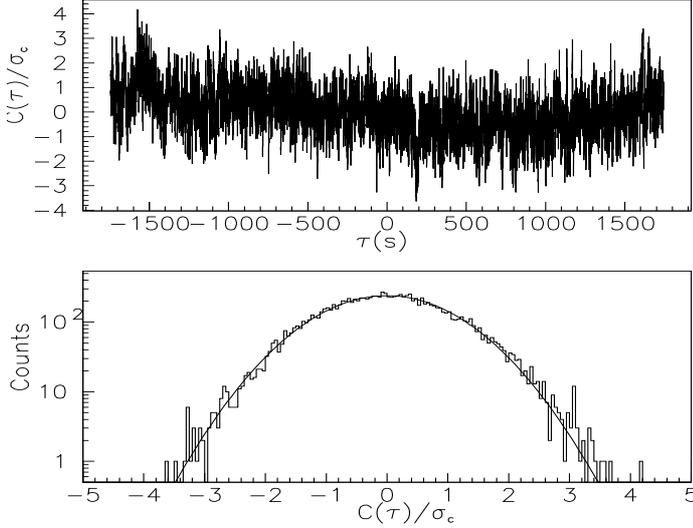
FIG. 2. Cumulative crosscorrelation of the noise of the two GW detectors, and its statistical distribution, which is clearly well approximated by a gaussian function.

This result, in the light of the central limit theorem, is a quite obvious consequence of the averaging processes (time integration and sample average) applied to the original data to give the variable $C(\tau)$. The gaussianity of the crosscorrelation variable is very important, as it gives a well-defined statistical meaning to the experimental result. The probability that a given high value of the experimental crosscorrelation quantity be due to chance may reliably be computed, as well as the upper limit on the source average power implied by a low experimental value.

An analytical estimate of the sensitivity of the method has been obtained, assuming gaussian noise. The expected normalized zero-mean crosscorrelation SNRC is given by:

$$SNRC = (SNR_x SNR_y + SNR_x + SNR_y)\sqrt{\frac{N}{N'}} \qquad (3.1)$$

where SNR_x and SNR_y are the snr's of the two detectors, in terms of E/T_{eff}, and N' = T β_3, is the number of independent samples in the integration interval. A numerical simulation was performed to test the sensitivity of the algorithm using the non gaussian noise data of two real GW detectors, Nautilus and Explorer. Impulsive signals of energy E were added to the N stretches of noise data, randomly delayed with respect to $t_{\gamma i}$ within the integration interval, but simultaneous on the two detectors X and Y. In Fig.3 the normalized cumulative crosscorrelation $C(\tau)/\sigma_C$ is plotted as a function of τ, for three values of the added signal amplitude, corresponding to increasing SNR on the single detector: $E/T_{eff} = 2.2, 4.5, 9.1$. The data of Fig.3 show the corresponding crosscorrelation signal C(0), emerging from the gaussian background. Low E/T_{eff} (e.g. between 3 and 10) events, which would be totally immersed in the non-gaussian tails of Fig.1, distinctly emerge from the gaussian tails of the distribution of Fig.2.
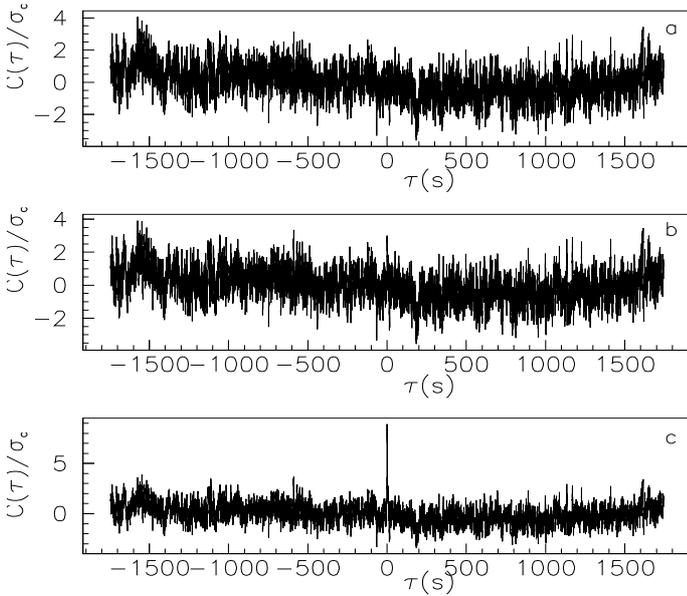
FIG. 3. Cumulative crosscorrelation of the noise of the two GW detectors, Nautilus and Explorer, with superimposed signals of increasing energy: $2.2T_{eff}$, $4.5T_{eff}$. $9.1T_{eff}$. The superimposed GW signals are simultaneous on the two GW detectors, but variably delayed with respect to the GRB arrival time.
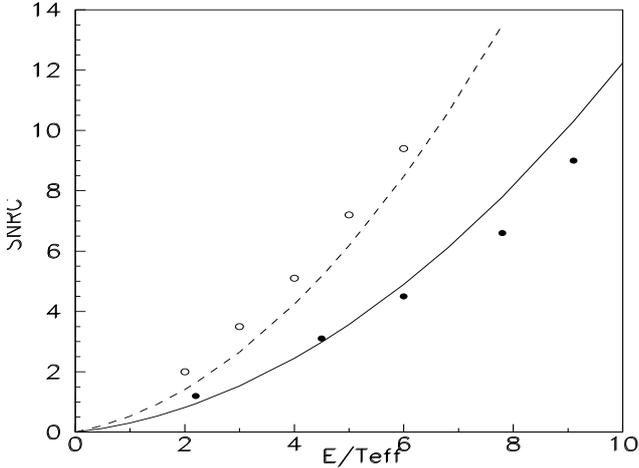
FIG. 4. Crosscorrelation snr, SNRC, plotted as a function of the added signal snr, E/T_{eff} ($\bullet$ for T = 1 hr, $\circ$ for T = 20 min). The numerical results are compared to the analytical estimation Eq.3.1.

The sensitivity of the algorithm, as computed by the numerical simulations, is shown in Fig.4, where the adimensional crosscorrelation signal to noise ratio SNRC, defined by Eq.2.4, is plotted as a function of the signal SNR, E/T_{eff}, for two values of the integration time T. The simulation shows that SNRC decreases as the square root of the integration time, provided that T is larger than the maximum delay between GRBs and GW signals, as in this simulation, and that the relation between SNRC and E/T_{eff} is in agreement with Eq.3.1. As discussed above, a given value of the SNR is much more significant for the crosscorrelation variable, due to the gaussianity of the crosscorrelation background. The sensitivity of the algorithm may be compared to that of a low-threshold coincidence search, which is a possible alternative two-detector method. Setting an event threshold at SNR_t one can compute [18] the probability of getting an event larger than the threshold in the presence of gaussian noise, as a function of the signal SNR, $SNR_s = E/T_{eff}$:

$$P_x(SNR_s, SNR_t) = \int_{SNR_t}^{\infty} dx \frac{\exp \frac{-(SNR_s+x)}{2} \cosh \sqrt{xSNR_s}}{\sqrt{2\pi x}} \qquad (3.2)$$

The expected number of coincidences due to the N added signals would be:

$$N_C = P_x P_y N \tag{3.3}$$

For $SNR_s = SNR_t$ we obtain $P_x = P_y = 0.5$ and the coincidence excess is $N_C \approx 7$. The coincidence background during the whole time interval NT may also be computed for gaussian noise:

$$N_{acc} = P(0, SNR_t) P(0, SNR_t) N N' \tag{3.4}$$

The coincidence excess with respect to the Poisson distribution of the accidental coincidences has a probability of being due to chance that may be compared to the corresponding probability of getting by chance the crosscorrelation excess with respect to the gaussian crosscorrelation background found with the simulation. This comparison is shown in Table I for three values of the coincidence SNR threshold, in the case $SNR_s = SNR_t$ and $T = 1h$. P_{SNRC} is the probability of finding by chance the corresponding value of SNRC, assuming gaussian statistics. $P_{c,g}$ is the probability of finding by chance the coincidence excess N_C, assuming the accidental coincidences background $N_{acc,g}$, computed in the case of gaussian noise on both detectors. $P_{c,r}$ is the probability of finding by chance the same coincidence excess N_C, assuming the accidental coincidences background $N_{acc,r}$, computed from the real event distribution of the detectors, shown in Fig.1. The evaluation of the accidental coincidences background based on gaussian statistics is not adequate to describe real GW detectors, whose noise is not gaussian. A more realistic evaluation of the accidental coincidences background is found using the real event rate of the two GW detectors as a function of the threshold level SNR_t. From Table I it is clear that the coincidence technique would be more effective for a hypothetical detector with gaussian noise, while the crosscorrelation method proposed here proves preferable in the more realistic case of non gaussian noise. It should also be added that the coincidence search method is also affected, for non-delta-like signals, by an uncertainty about the event maximum time that leads either to a decrease of the coincidence detection efficiency, or to the choice of a longer coincidence window, with a correspondingly higher rate of accidental coincidences. Of course, other cumulative single-detector techniques could also be effective, but only in the case of close simultaneity between the GW event and the GRB trigger time. This model-dependent assumption is not needed for the effectiveness of the proposed crosscorrelation technique. The advantage of the proposed method is the absence of hypotheses, excluding the obvious assumptions that the signals of the two GW detectors be simultaneous. No hypothesis is made on both the gaussianity and the stationarity of the noise. A similar method, proposed by Finn et al. [1], is critically based on the hypothesis of stationarity of the GW detector noise, which is not generally true for present GW detectors. The intrinsic fluctuation of the noise contribution to the crosscorrelation of the on-GRB sample would require a very large number of low-noise data samples. The advantage of the method proposed here has nothing to do with the extraction of a crosscorrelation quantity with the maximum signal contribution. Rather, it lies in the choice of the reference quantity to which the measured crosscorrelation is compared to evaluate its statistical significance. In the case of Finn et al., the reference quantity is found by choosing reference data uncorrelated to the GRB arrival times, according to the noise stationarity hypothesis. In the present work, only the obvious physical hypothesis that the signals be simultaneous on the two GW detectors is used to define the reference quantity, which makes it possible to avoid the problem of non-stationarity of the GW detector noise. Model-dependent assumptions on the GRB physics could be considered to increase the sensitivity of the method. For example, the uncertainty about the time delay between GRBs and GWs would suggest computing the crosscorrelation on a time window wide enough to include this uncertainty. As the sensitivity of the method is decreasing with the width of the window, the model-dependent hypothesis that the delay uncertainty is correlated to the GRB duration could be used, for example, to optimize the method by choosing each time window width according to the GRB duration. Unfortunately, as pointed out above, the obvious drawback of any crosscorrelation technique is the reduced size of the data sample, due to the requirement of simultaneous low-noise operation of two GW detectors. Multiple crosscorrelation with n detectors is also a natural extension of the proposed method.

TABLE I. Comparison between the crosscorrelation and the low-threshold coincidence technique.

E/T_{eff}	$SNRC(T = 1h)$	P_{SNRC}	$N_{acc,g}$	$P_{c,g}$	$N_{acc,r}$	$P_{c,r}$
2	0.8	0.21	200	0.3	2000	0.4
3	1.5	$7 \cdot 10^{-2}$	1	10^{-5}	250	0.3
5	3.5	$2 \cdot 10^{-4}$	0		22	$9 \cdot 10^{-2}$

IV. CONCLUSIONS

A new method is proposed to detect statistical association between GW detector signals and GRBs, by using a cumulative crosscorrelation technique. The originality of the proposed method lies in the choice of the reference background quantity to which the crosscorrelation should be compared, thanks to which unbiased meaningful results can also be obtained in the case of non-stationary noise. The obvious physical constraint, i.e. that the signals must be simultaneous on the two GW detectors, is used to select as physically relevant the crosscorrelation at zero delay only, while the crosscorrelation integrals computed on the same circularly permuted data for other delays are used as a background distribution, providing a well-defined estimate of the statistical significance of the zero-delay result, in the eventuality of an excess, or of an absence of excess. Computer simulations using the real noise data of the cryogenic GW detectors Nautilus and Explorer have been performed and compared to analytical estimates of the algorithm sensitivity. The sensitivity of the proposed algorithm has also been compared to that of a low-threshold coincidence search method, finding that the crosscorrelation method proves more effective in the case of non-gaussian noise.

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