



Resonant Bars: Time dispersion and efficiency detection for impulsive signals of known shape using the "ROG Delta Filter"

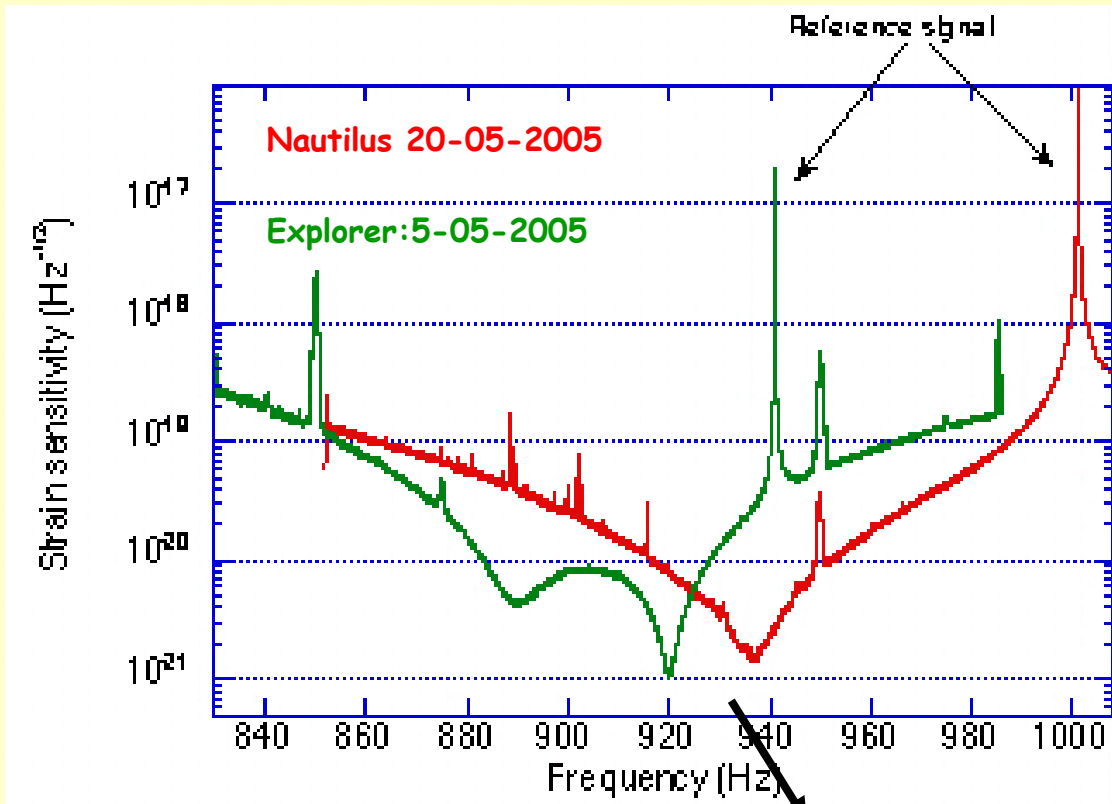


Sabrina D'Antonio for the ROG Collaboration
INFN sez. University of Rome "Tor Vergata "

Archana Pai, Pia Astone

INFN Sez. University of Roma "La Sapienza"

Status of the EXPLORER- NAUTILUS resonant bars



EXPLORER 2005:

$$\sqrt{S_n(f)} \approx 10^{-21} / \sqrt{\text{Hz}}$$

BW ~ 50 Hz at $10^{-20} / \sqrt{\text{Hz}}$

T=3 K

Teff ~ 2mK

NAUTILUS 2005:

$$\sqrt{S_n(f)} \approx 1.5 * 10^{-21} / \sqrt{\text{Hz}}$$

BW ~ 40 Hz at $10^{-20} / \sqrt{\text{Hz}}$

T=3.3K

Teff ~ 2mK

Investigating the Data Analysis Issues for Improved Bandwidth (BW)

- Presently the resonant bars Explorer and Nautilus are working with broader BW.
- ➔ It is worth investigating different kind of signals in addition to the delta signals as done in the past.
- It is necessary to reassess the assumption of delta-like signals for matched filtering as the filtered output crucially depends on the shape of the waveform.
- ➔ In this study we discuss this problem with an example of GW signals: quasi-normal-modes of proto-NS during the cooling phase after the supernova core collapses: Damped sinusoid evolving in frequency and damping time.

Investigating the Data Analysis Issues for Improved Bandwidth (BW)

□ Topic of this talk:

Validate ROG delta filters :

- a). Loss in SNR using ROG delta filters compared to SNR for Matched filter (in case of known signal)
- b). Accuracy in the signal arrival time measured by ROG delta filter compared to that obtained with the Matched filter.

Important while searching coincidences between resonant bars

- c) Amplitude signal estimation using the ROG delta filter (A_d) compared to that obtained with the matched filter (A_m).

□ signal parameters: [?] where :

□ → we can use the Delta matched filter

□ we have to use a "bank of filters"

Investigating the Data Analysis Issues for Improved Bandwidth (BW)

First Step (discussed in this presentation):

In this preliminary work we consider damped sinusoid signals of known shape with constant frequency F_0 and damping time τ .

This study is performed by analytical approach

&

Simulations: injection of signals of known shape at the input of adaptive delta/signals matched filters

We have used Explorer data "Dec 2004":

Resonant frequencies at the minus/plus mode= $F_m=904.7\text{Hz}/F_p=927.45\text{Hz}$.

Second Step: (Future Work)

Study for signals of unknown shape

Quasi-normal Modes of a newly born hot NS : Signal

[V. Ferrari *et al* , MNRAS 338 (2003) 389]

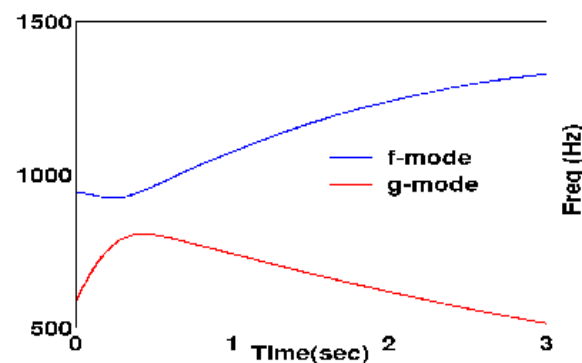
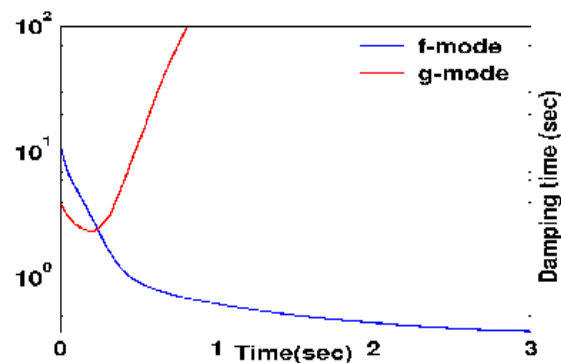
Scenario: Stellar Core Collapse → Neutrino Diffusion & Thermalisation
→ Cooling & Contraction → Energy Dissipation *via* Neutrino emission

Various QNM of a star are excited in above physical processes ⇒ GW Emission

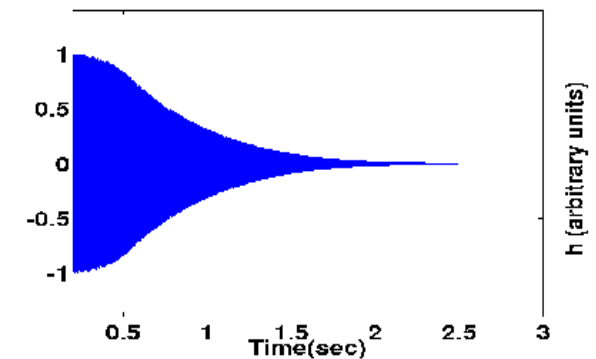
Signal: Damped Sinusoid with varying damping time (τ) and frequency (f)
(Variation depends on the equation of state)

$$h(t) = h_0 \exp^{-(t-t_0)/\tau(t)} \sin[2\pi f(t)(t - t_0)]$$

Example:



$h(t)$ of f-mode



f-mode : Fundamental Mode, Frequency in kHz region

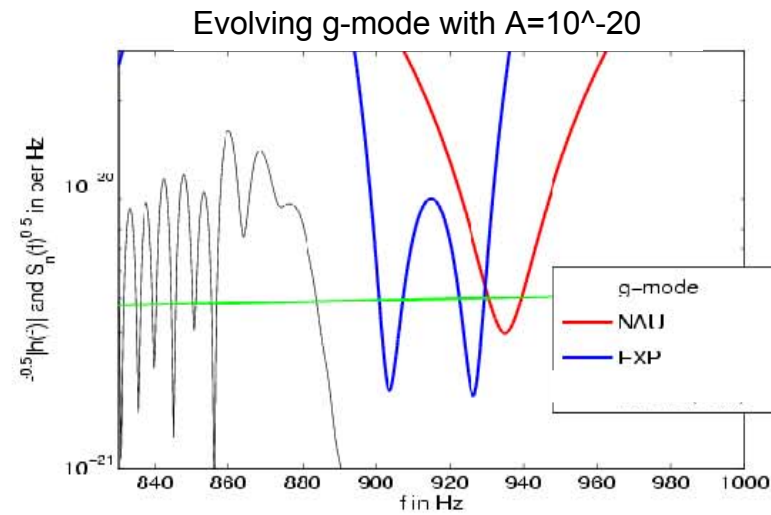
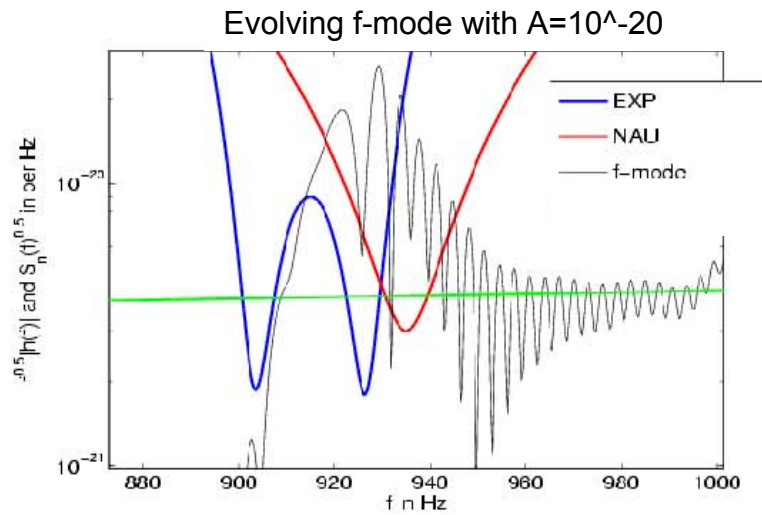
g-mode : Due to entropy and composition gradients in proto-NS

Typically energy: Core collapse $\Delta E \sim 10^{-6} - 10^{-8} M_{sun} c^2$ QNM: $\Delta E \sim 10^{-9} M_{sun} c^2$.

QNM of a newly born hot NS: Detectability

Matched Filter SNR:

$$SNR^2 = 4 \int_0^\infty \frac{|h(f)|^2}{S_n} df$$



To obtain $SNR=5$ for galactic source ($r = 10$ kpc), minimum energy ΔE required in QNM (in $M_{sun}c^2$)

	f-mode	g-mode
Explorer	10^{-4}	1

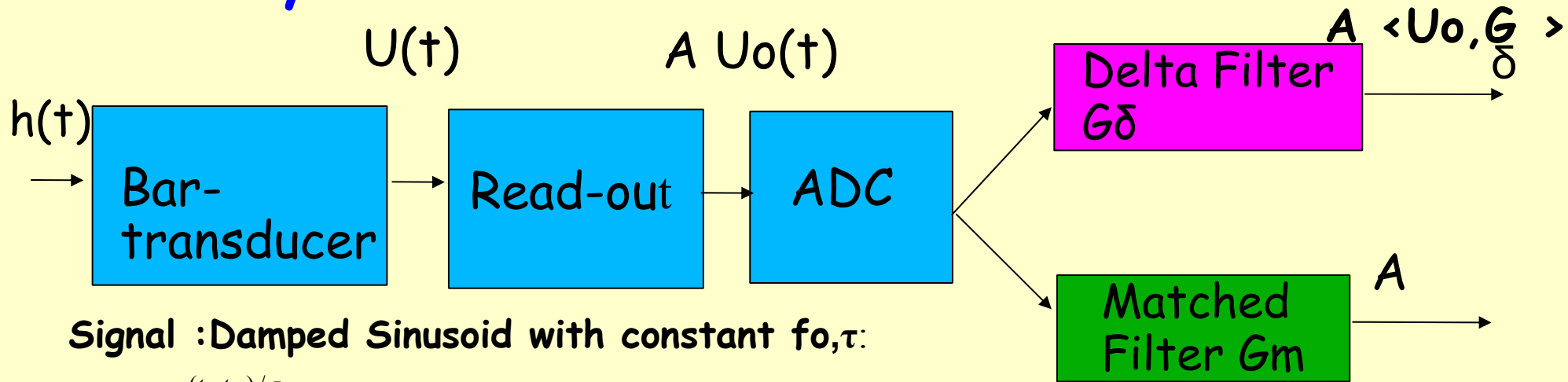
Typical energy: SN Core collapse $\Delta E \sim 10^{-6} - 10^{-8} M_{sun}c^2$ QNM: $\Delta E \sim 10^{-9} M_{sun}c^2$

Recent simulations [Shibata&Uryu,2002 Simulation] have shown that a merger of two NS with equal mass and low compactness can give rise to short lived supra massive NS.

Typical energy during merger $\Delta E \approx 10^{-2} M_{sun} c^2$

The QNM in such a merger may have more energy than those during the cooling phase after the core collapse $\Delta E \approx 10^{-4} M_{sun} c^2$

Layout-ROG Detector + Filters



Signal : Damped Sinusoid with constant f_0, τ :

$$h(t) = e^{-(t-t_0)/\tau} \sin(\omega_0(t-t_0)) * \theta(t-t_0)$$

Force :

$$F(t) = m_x \frac{L}{2} \frac{\partial^2 h(t)}{\partial^2 t}$$

L is the effective length $L = l / 4\pi^2$
 m_x is the reduced mass $m_x = M / 2$

**Amplitude, $A \sim A(\tau, F_0, h_0, S)$
S=System parameters**

Response of Bar-Transducer to $h(t)$:

**SNRd (delta matched filter) = $A \langle U_0, G\delta \rangle / \sigma_d$
**SNRm (matched filter) = A / σ_m
 σ_d, σ_m :std dev of the filtered noise****

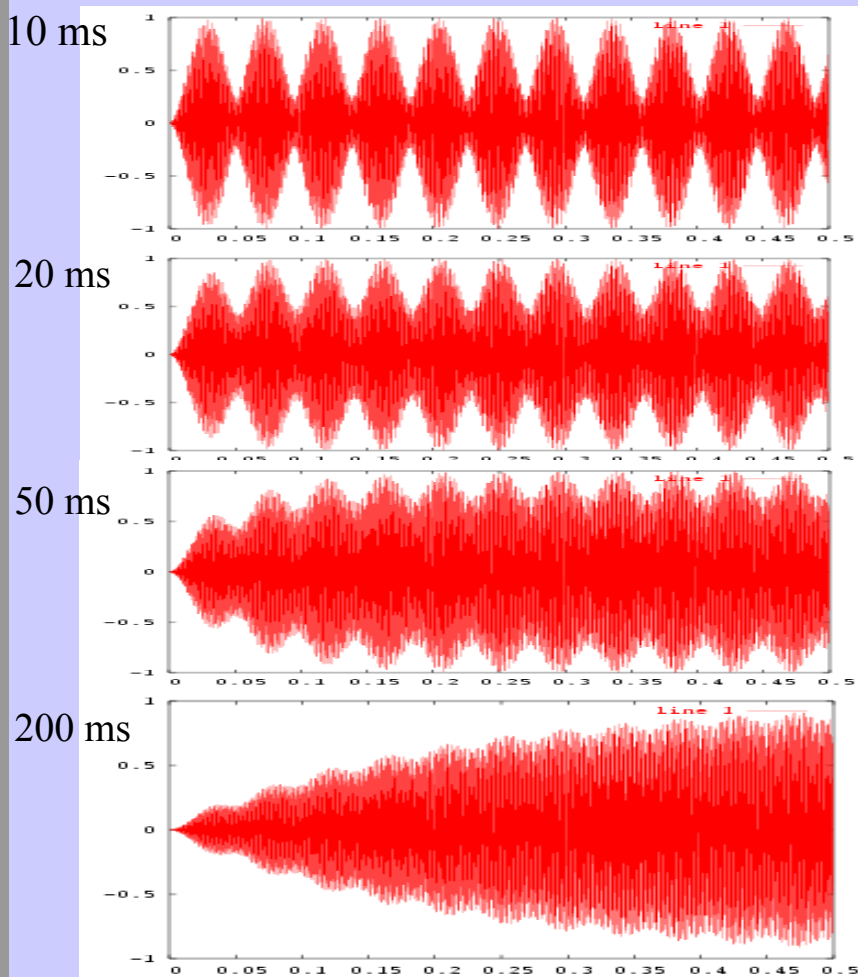
$$U(j\omega) = W_{ux}(j\omega) * F(j\omega) = -\omega^2 m_x \frac{L}{2} * \underset{\uparrow}{W_{ux}(j\omega)} * H(j\omega)$$

System transfer function

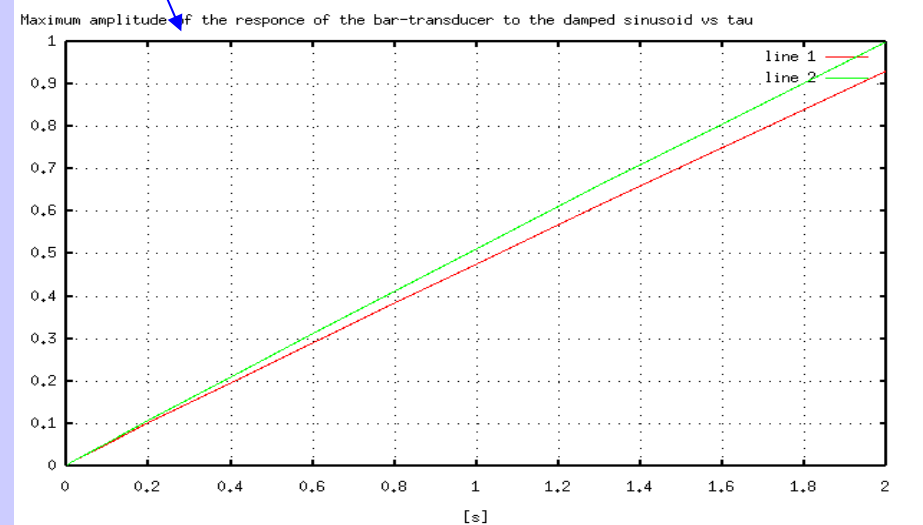
Response of the bar-transducer to the damped sinusoid (summary)

$$u(t) = A_+(t, \tau, F_o, S) * \sin(\omega_+ t + \phi_+) - A_-(t, \tau, F_o, S) * \sin(\omega_- t + \phi_-) - A_o(t, \tau, F_o) * \sin(\omega_o t + \phi_o)$$

↓
Case I: $F_o = F_m, \tau : [10 \text{ ms} - 200 \text{ ms}]$



Maximum amplitude $A = u(T_o)$ vs τ
 T_o increases with τ

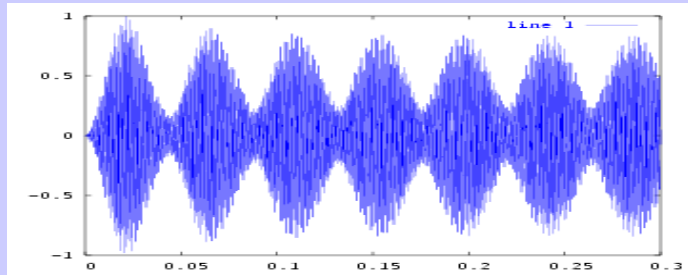


For $F_o = F_m$:
 For $\tau \geq 50 \text{ ms}$
 $A_+ \ll A_-$
 A_+ and $A_o \propto \tau$

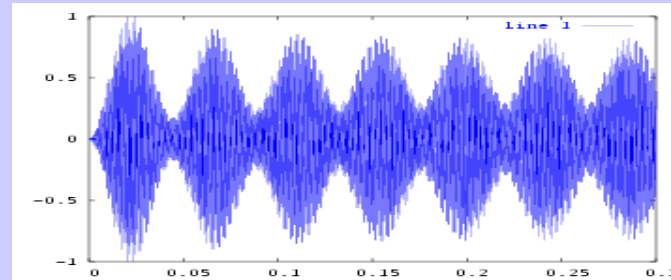
Response of the bar-transducer to the damped sinusoid (summary)

Case III : $F_m < F_o < F_p$, let $\tau = 50$ ms

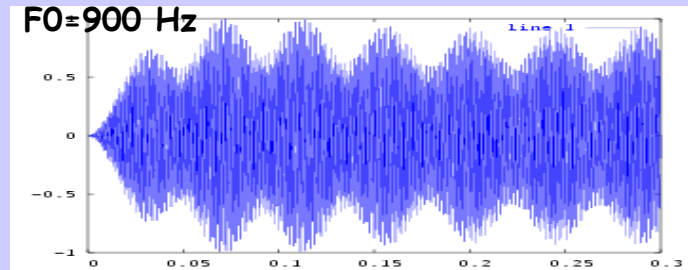
$F_0 = 880$ Hz



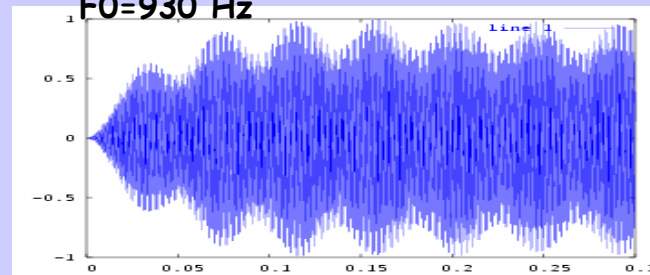
$F_0 = 950$ Hz



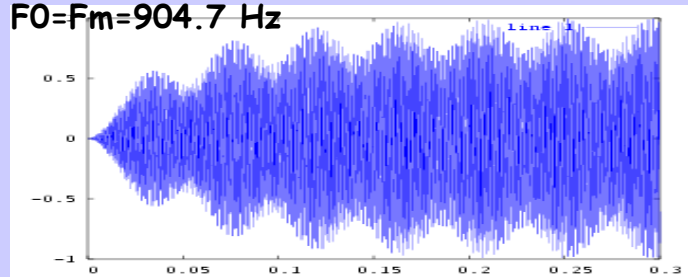
$F_0 = 900$ Hz



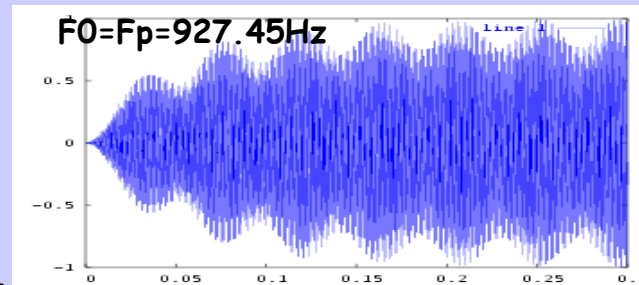
$F_0 = 930$ Hz



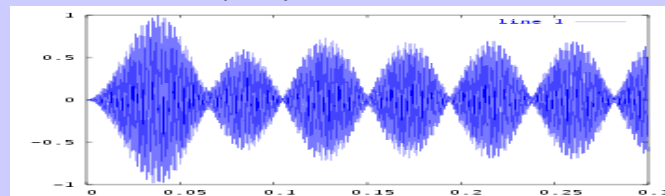
$F_0 = F_m = 904.7$ Hz



$F_0 = F_p = 927.45$ Hz



$F_0 = 915$ Hz



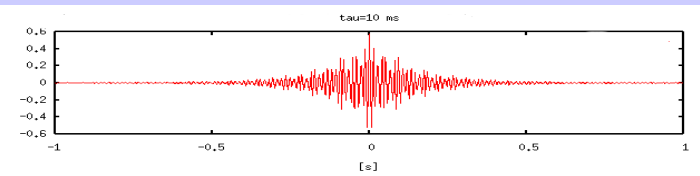
MATCHED FILTER G_m

Filtered signal $g(t)$

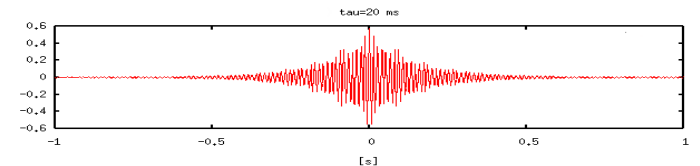
▶ Example: $F_o = F_m$, τ : [few ms till 1 sec]

τ [ms]

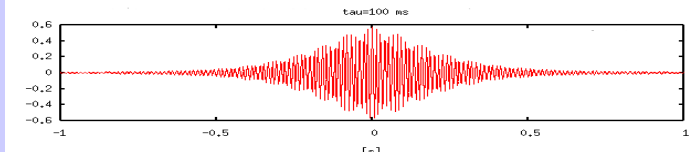
10 ms



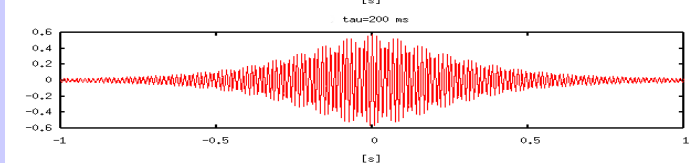
20 ms



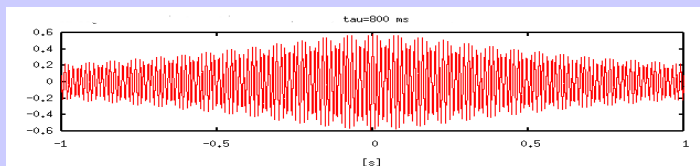
100 ms



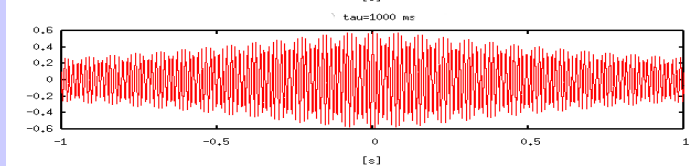
200 ms



0.8 s



1 s



The decay time of $g(t)$, (τ_g) depends on BW and τ .

$$\tau_{BW} = 1 / (\pi * BW) \sim 140 \text{ ms}$$

For $\tau < 0.1 * \tau_{BW}$, $\tau_g \sim 1/BW$.

For $\tau > 1/(\pi * BW)$, τ_g increases with τ

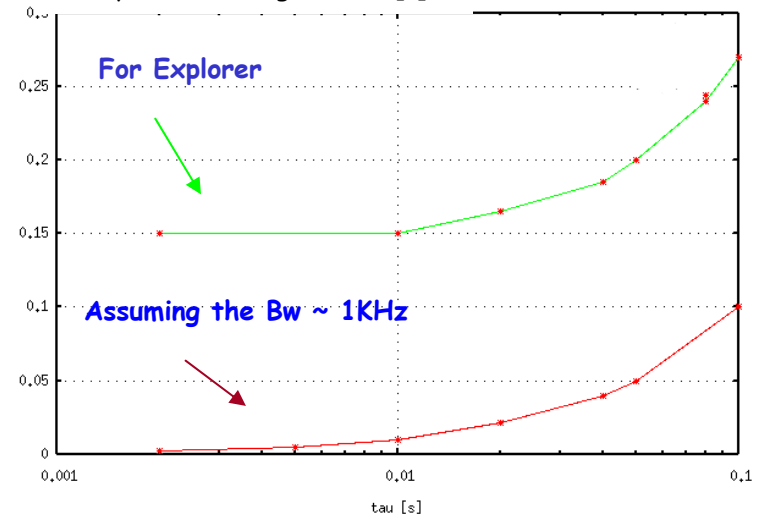
! (Independent of detectors' bandwidth)

Max($g(t) = g(T_o)$) is not well-peaked.

With noise, the error in T_o can be large !

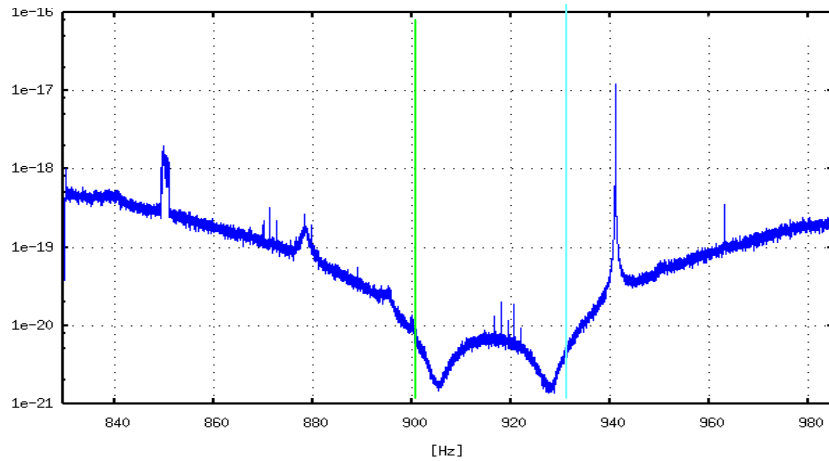
τ_g [s]

Decay time [s] of $g(t)$ vs τ [s]

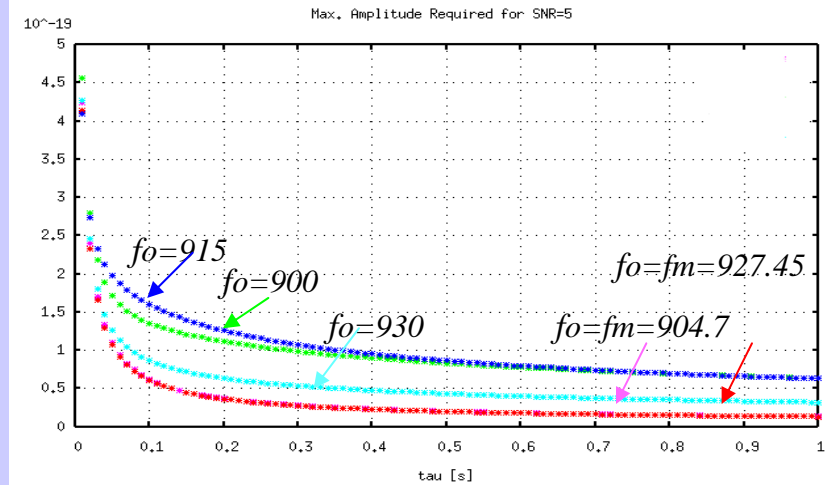


MATCHED FILTER SNR

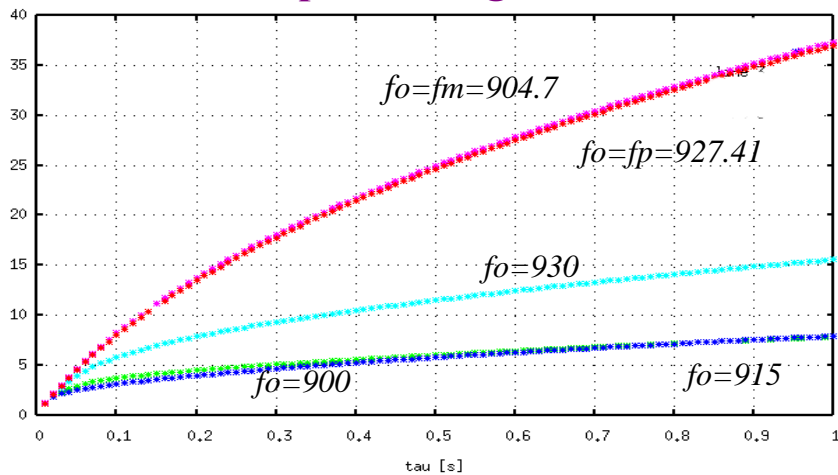
Explorer:Dec 2004 (Data used for simulations)



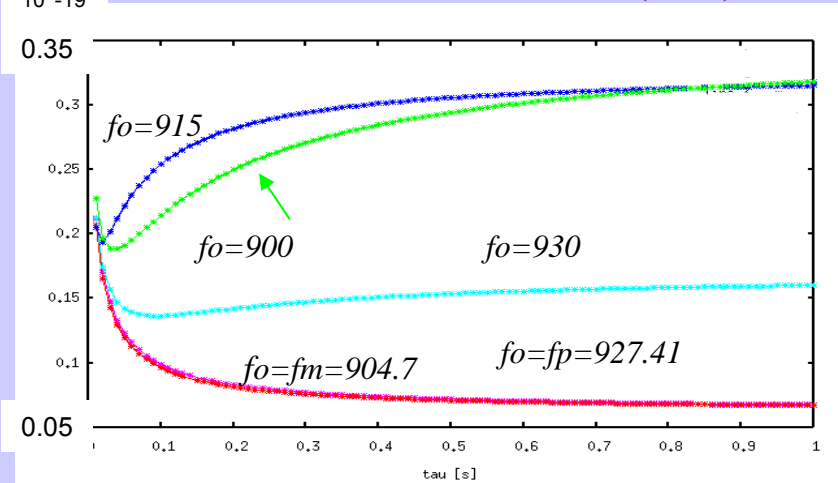
For a detectable SNR=5, min(ho) vs τ



SNR vs τ for damped sine signal with $h_o = 10^{-19}$

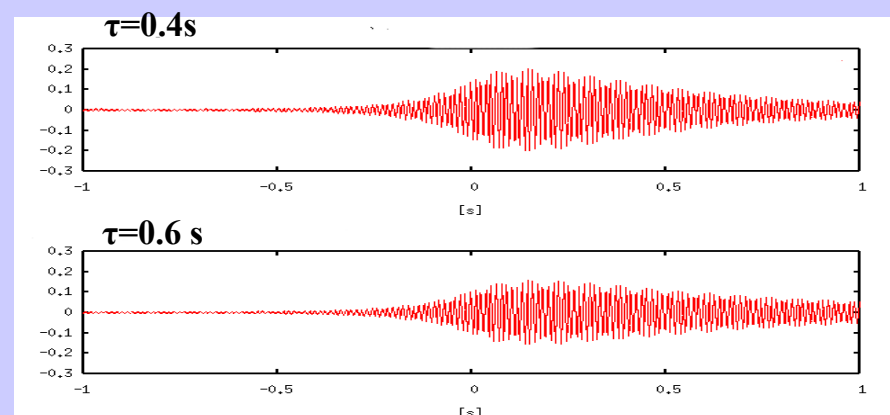
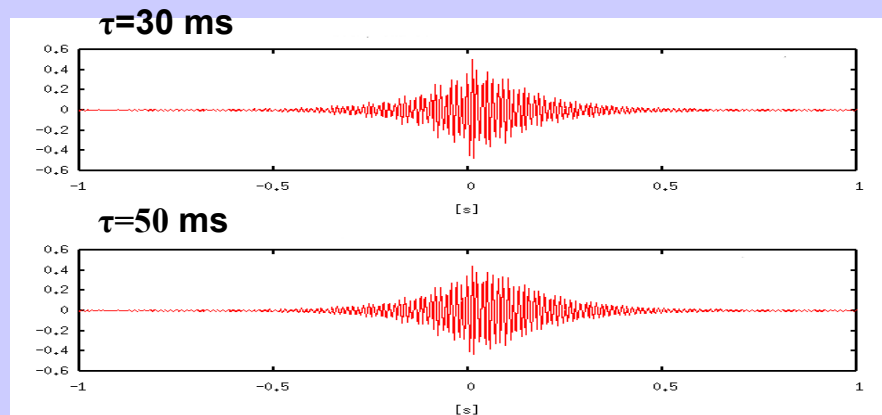
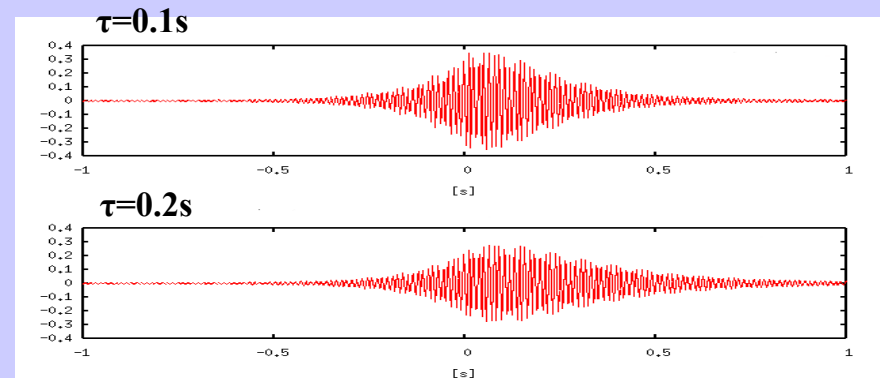
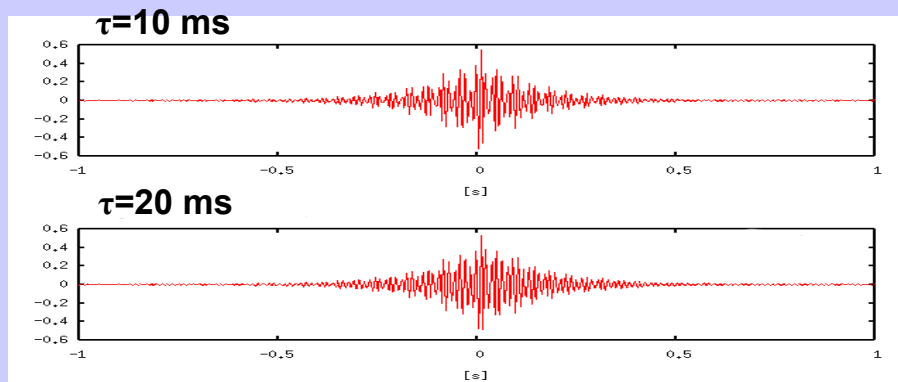


For a detectable SNR=5, min(hrss) vs τ



Damped sinusoid filtered with Delta Filter

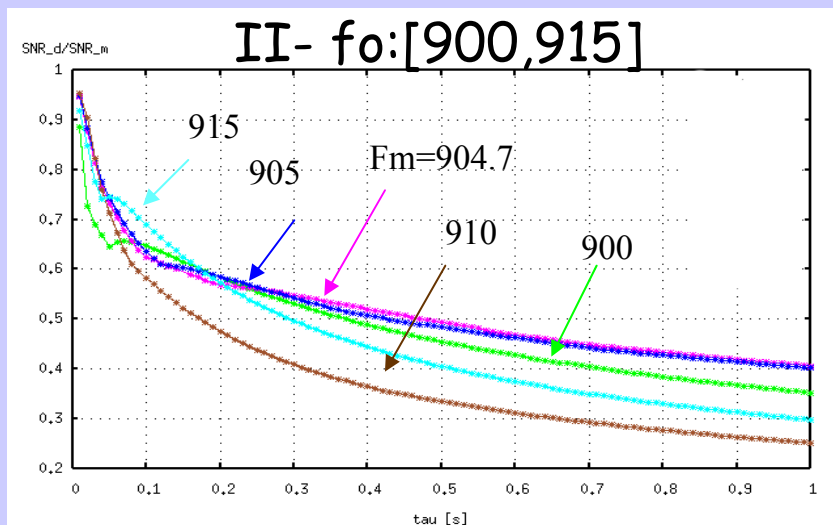
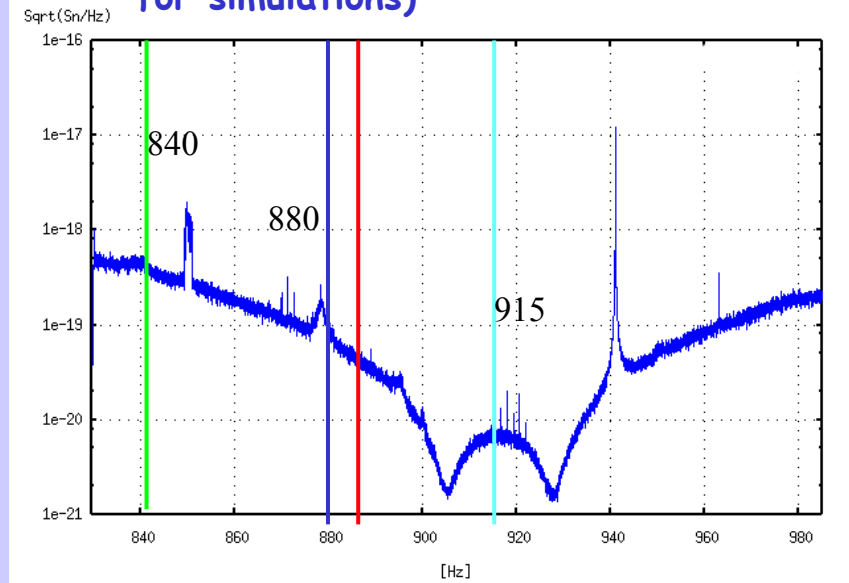
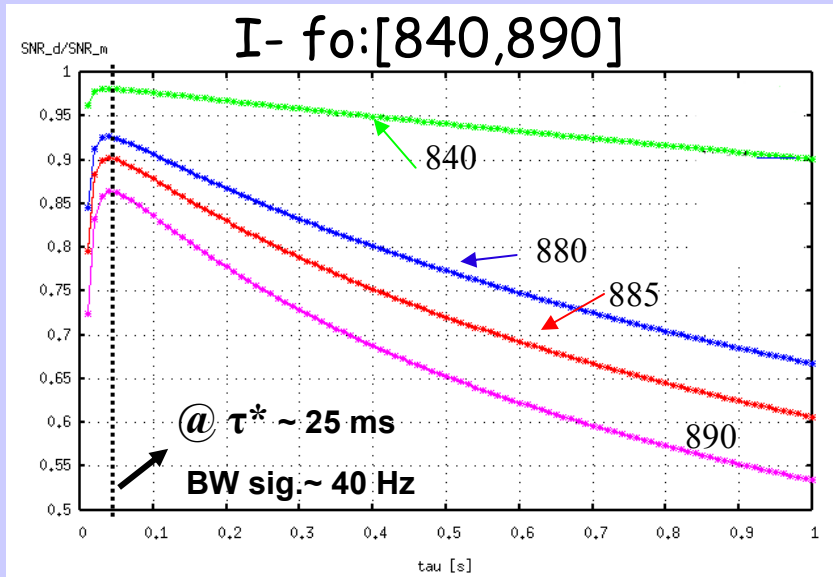
Case $f_o=f_m$, τ [10 ms-0.6 s]



Delta Filter: SNR COMPARISON

SNR-Delta Filter to SNR-Matched Filter

Explorer: Dec 2004 (Data used for simulations)



I- Fo far from the resonant frequencies, for $\tau < \tau^*$ the loss in the SNR decreases with τ because the frequency band of the signal falls in the more sensitive band of the detector till $\tau = \tau^*$ -> important to apply the matched filter for $\tau < \tau^*$.

For $\tau > \tau^*$ the loss in the SNR increases with τ : the signal stays in the detector for more time => important to apply the matched filter

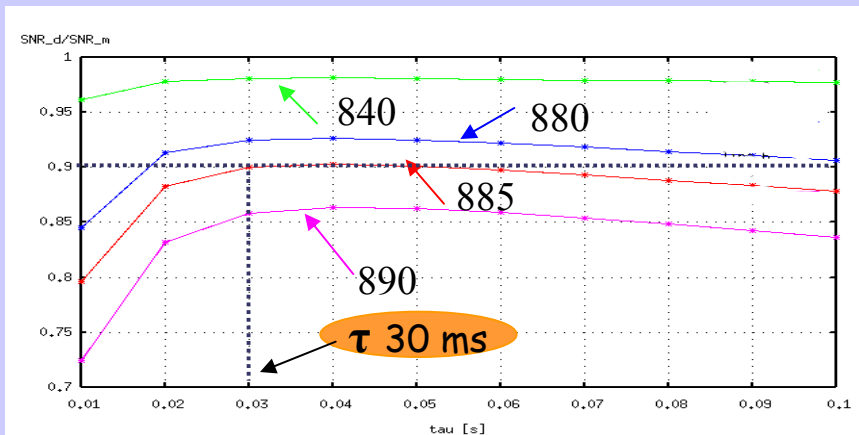
II- Fo near the resonant frequencies

The loss in the SNR rapidly increases with τ

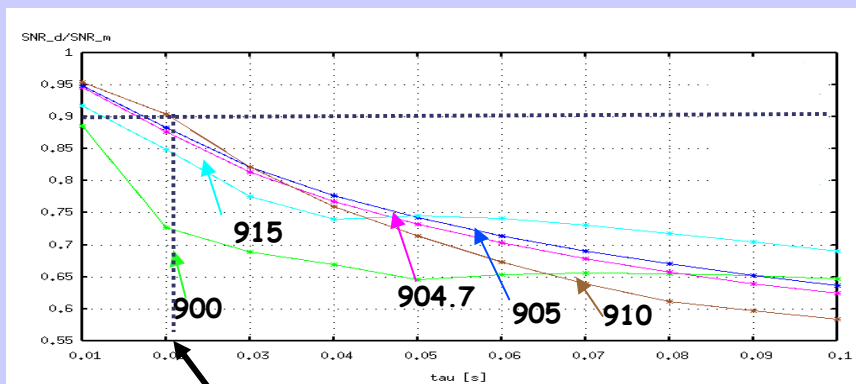
Delta Filter: SNR COMPARISON

SNR-Delta Filter to SNR-Matched Filter

Fo:[840,890] Hz < Fm = 904.7 Hz



Fo:[900,915] Hz around Fm=904.7 Hz



Accepted loss in SNR with Delta filter ~ 10%

1. Fo < Fm (Fo > Fp) and away from the freq -band

$$\text{SNR}_D \sim \text{SNR}_M,$$

2. For [900, 930], Max(τ) to be searched

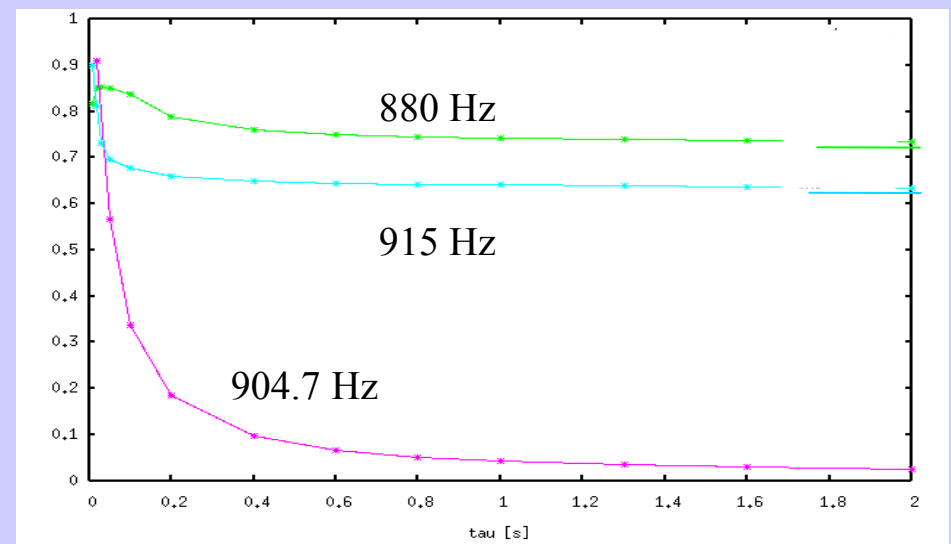
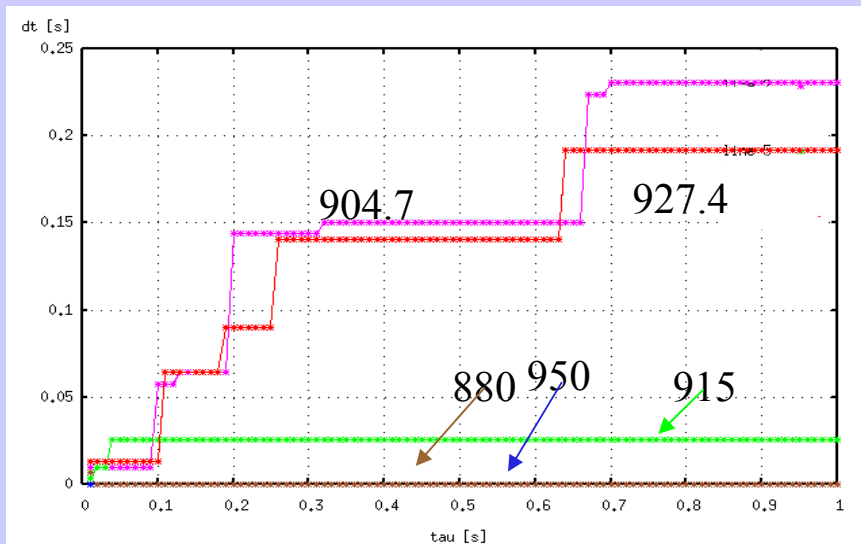
Fo	Max(τ)	Q
Fm=904.7	18ms	~51
910	20ms	~57
915	12ms	~35

τ 20 ms

DELTA FILTER : Time Dispersion and Amplitude Estimation

Time dispersion : Δt Vs τ

Ad/Am: Vs τ



1. Time dispersion:

- For F_0 near the two resonant frequencies Δt increases with τ .

Constant after $\tau > 1$ sec is misleading

Important while fixing the coincidence window

- For F_s away from the freq-band, $\Delta t \rightarrow 0$ msec

2. The error in the amplitude estimation is

- For F_0 near the two resonant frequencies the error increases faster with τ
- For F_s away from the freq-band. The error is $\sim 20\%$

Conclusions on the use of the Delta Filter

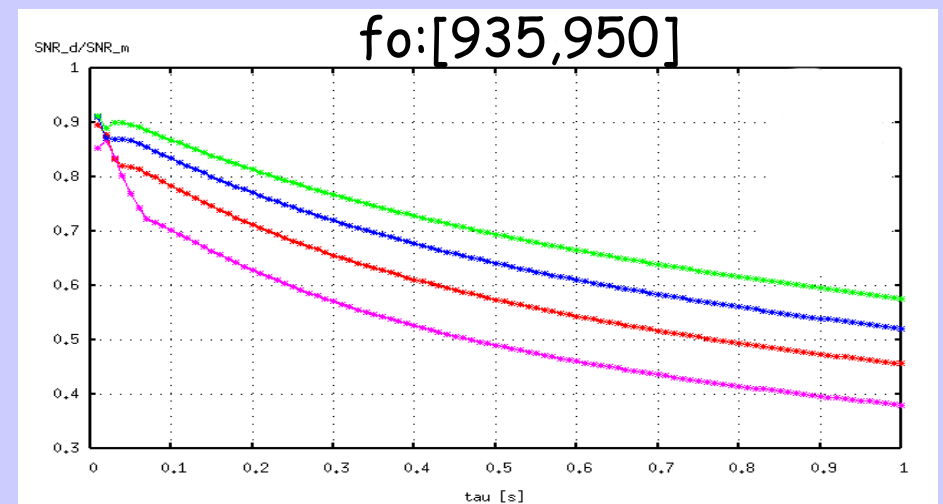
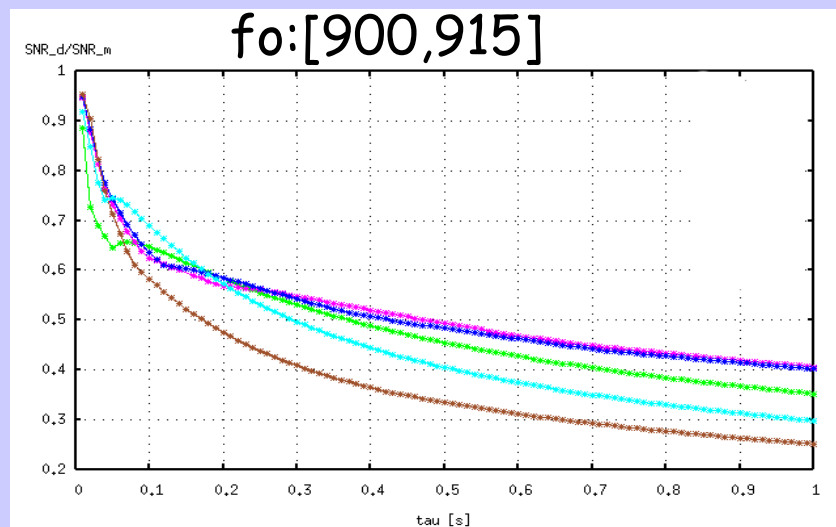
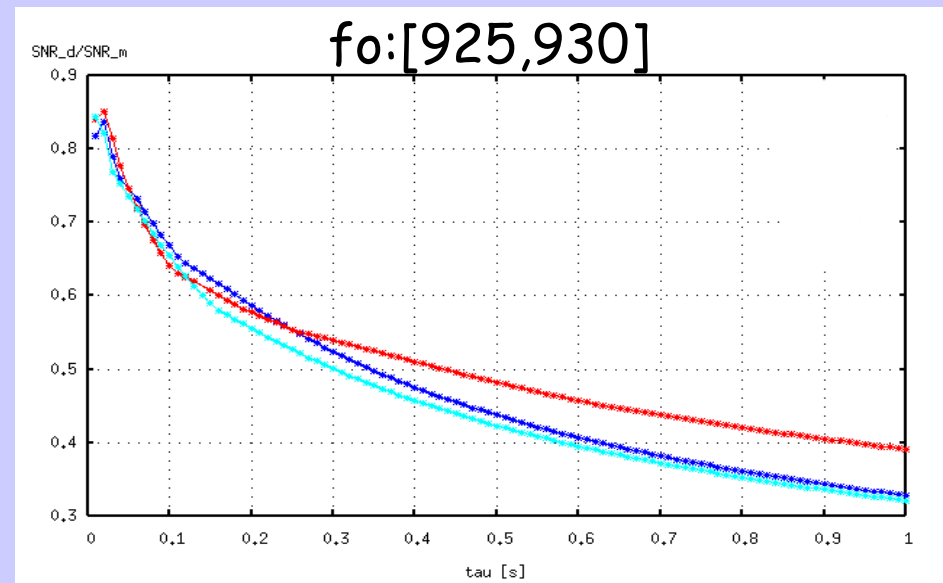
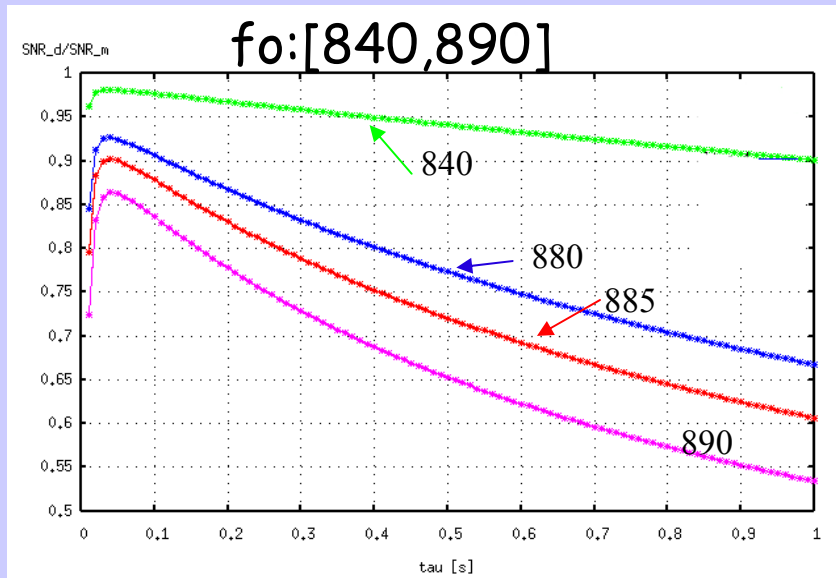
Signal parameters	Dt	Error in the amplitude estimation	Loss in SNR
$F_o < 880 \text{ Hz}$ $F_o > 935 \text{ Hz}$ $T < 100 \text{ ms}$	0	15%	10%
Delta filter is ok			
$F_o \text{ near } F_m, F_p$ $T < 20 \text{ ms}$	Increases with T < Few ms	Increases with T < 30%	Increases with T < 10%
Delta filter is ok			

Bank of Matched filters should be used for the other cases !

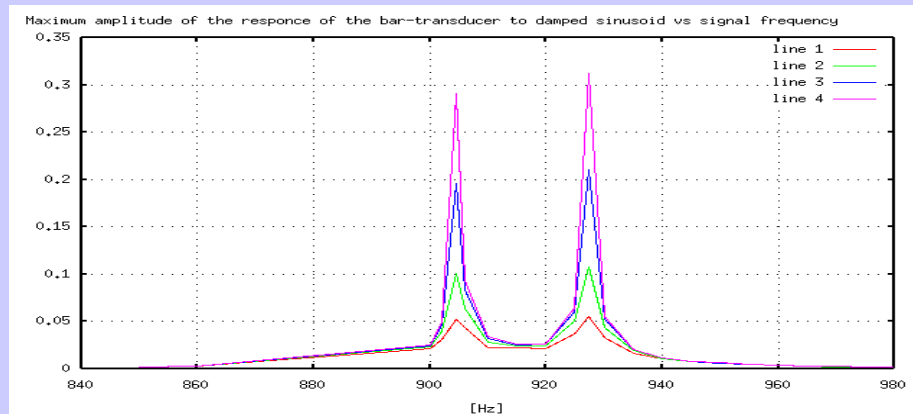
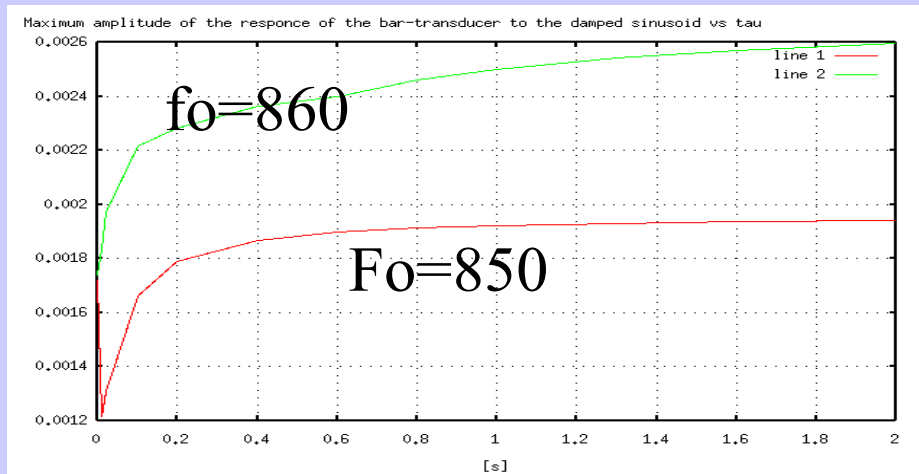


Delta Filter: SNR COMPARISON

SNR-Delta Filter to SNR-Matched Filter



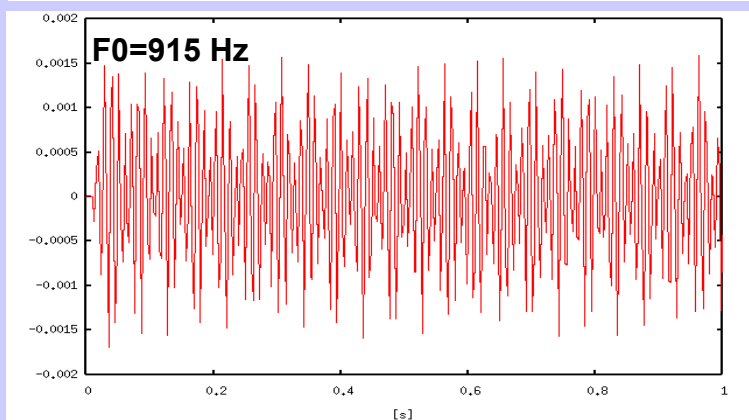
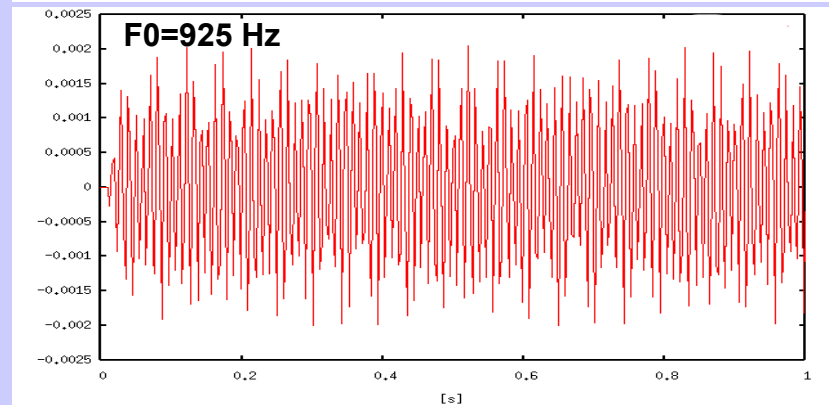
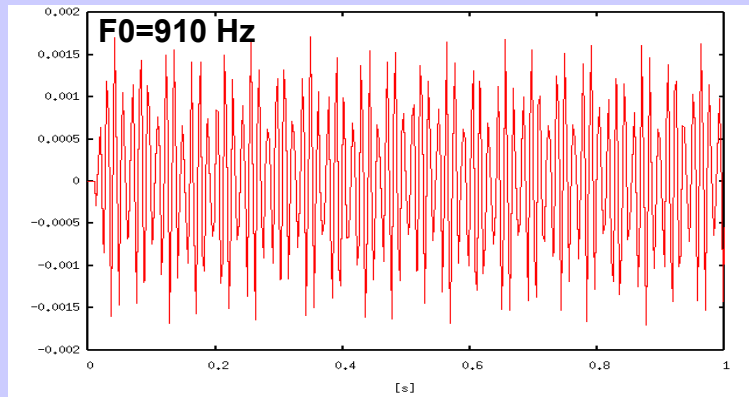
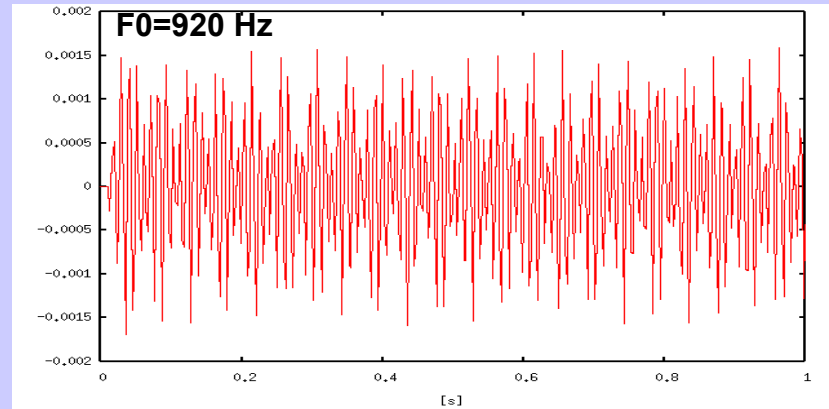
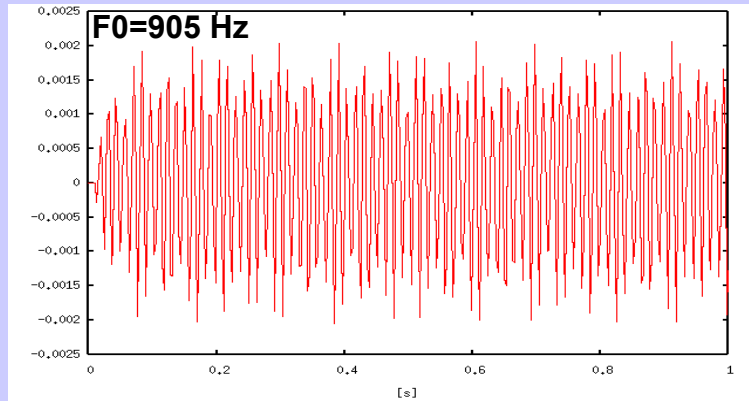
Response of the bar-transducer ...



$\tau = 0.1, 0.2, 0.4, 0.6$
ms

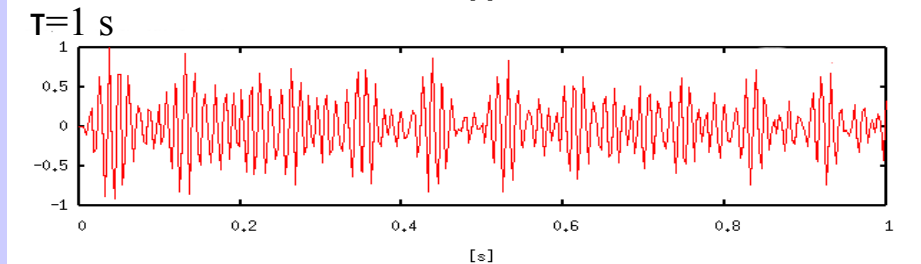
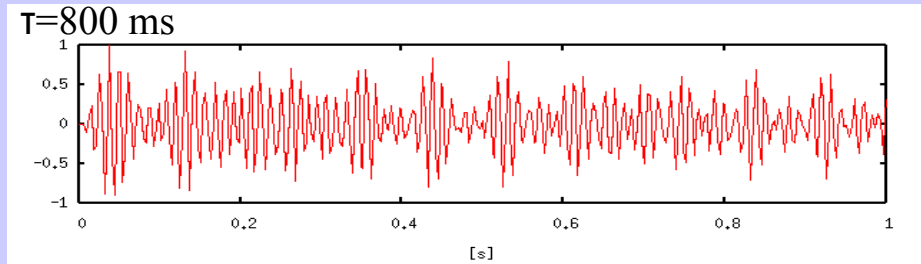
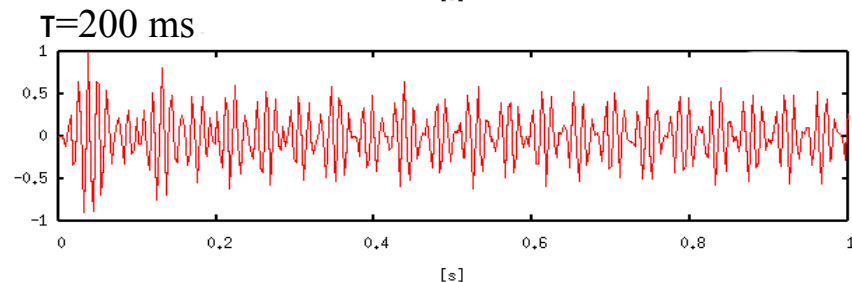
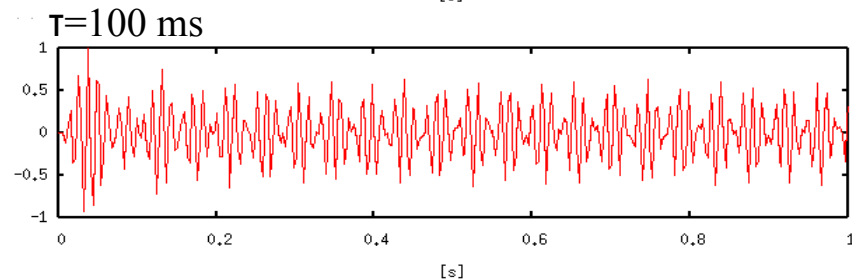
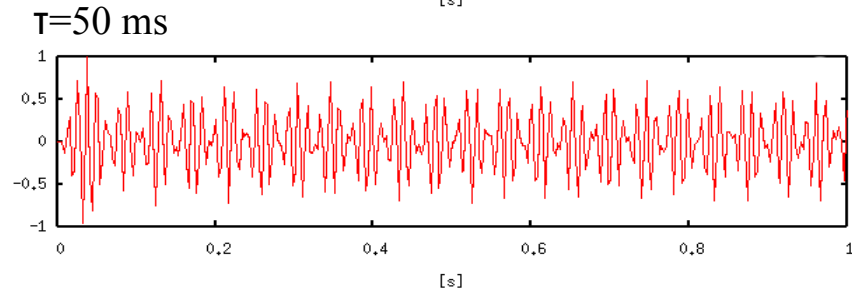
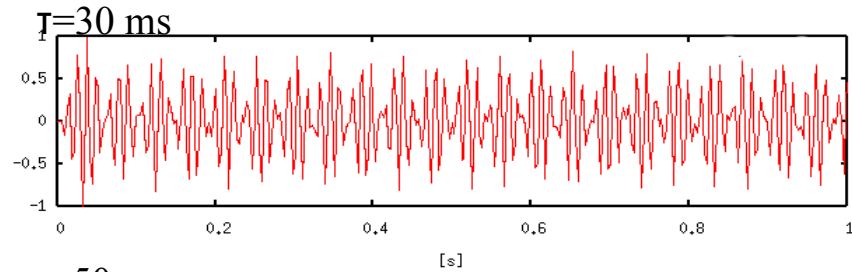
Response of the bar-transducer to the damped sinusoid

Case III : $F_m < F_0 < F_p$, let $\tau = 30$ ms



Response of the bar-transducer to the damped sinusoid (summary)

Case II : $F_m < F_o < F_p$, τ increasing from 30 ms till 1 s



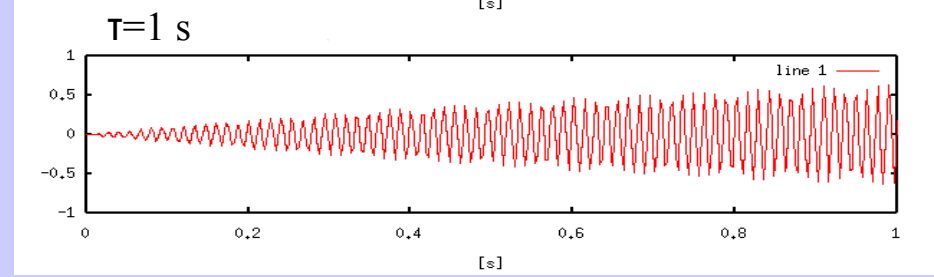
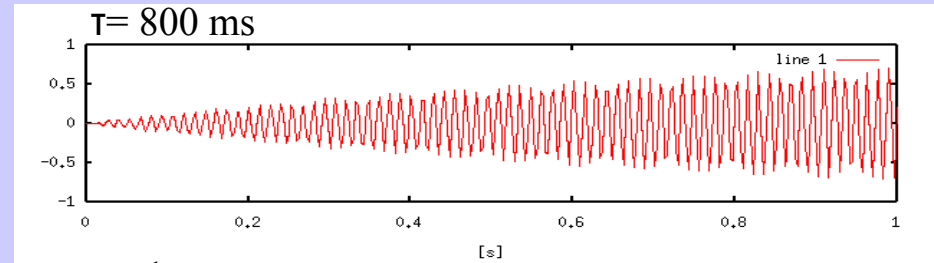
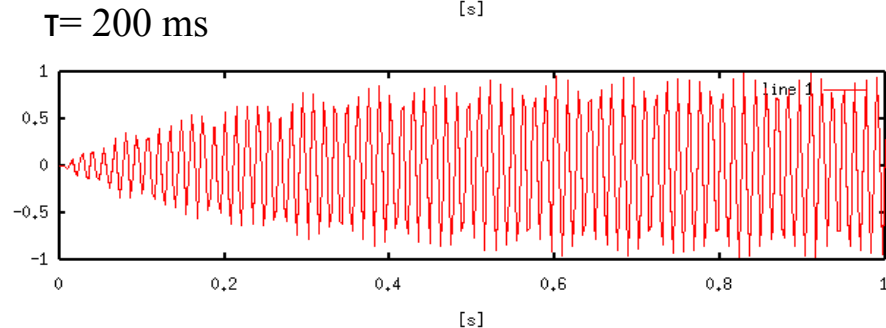
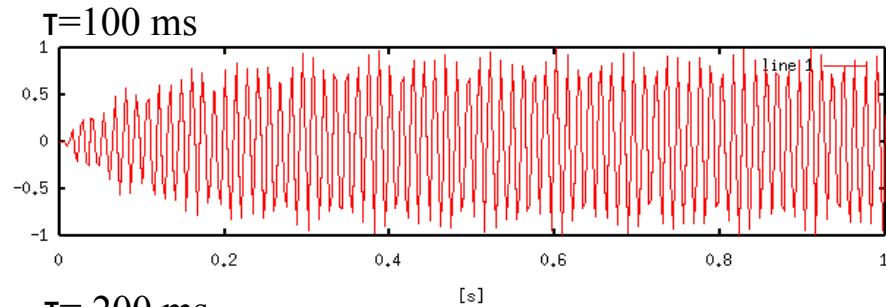
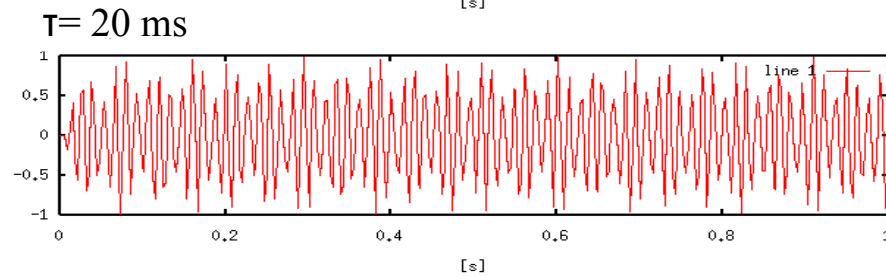
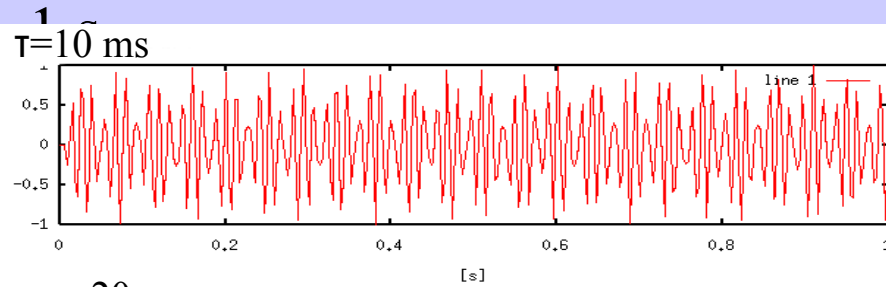
The maximum amplitude $\mathbf{A}=u(\mathbf{t}_o)$ is a function of τ

\mathbf{t}_o is constant in this particular case:

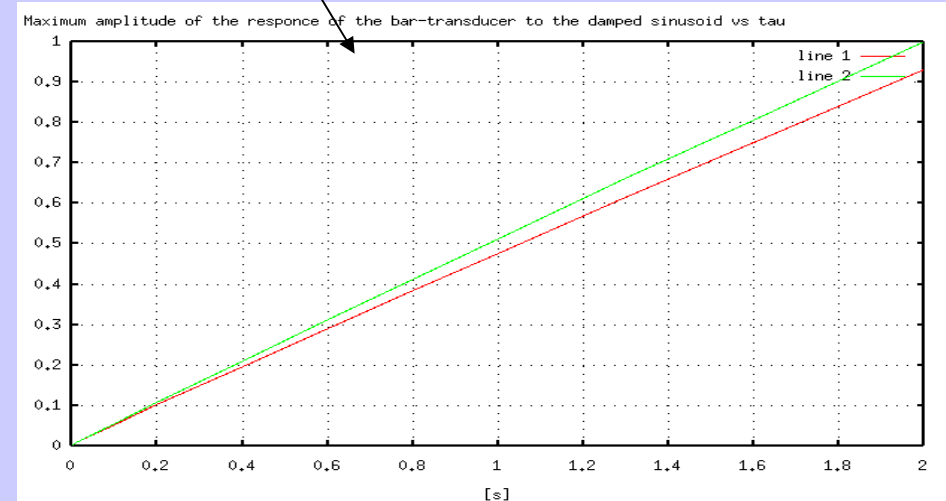
$$F_o=(F_m+F_p)/2$$

Response of the bar-transducer to the damped sinusoid

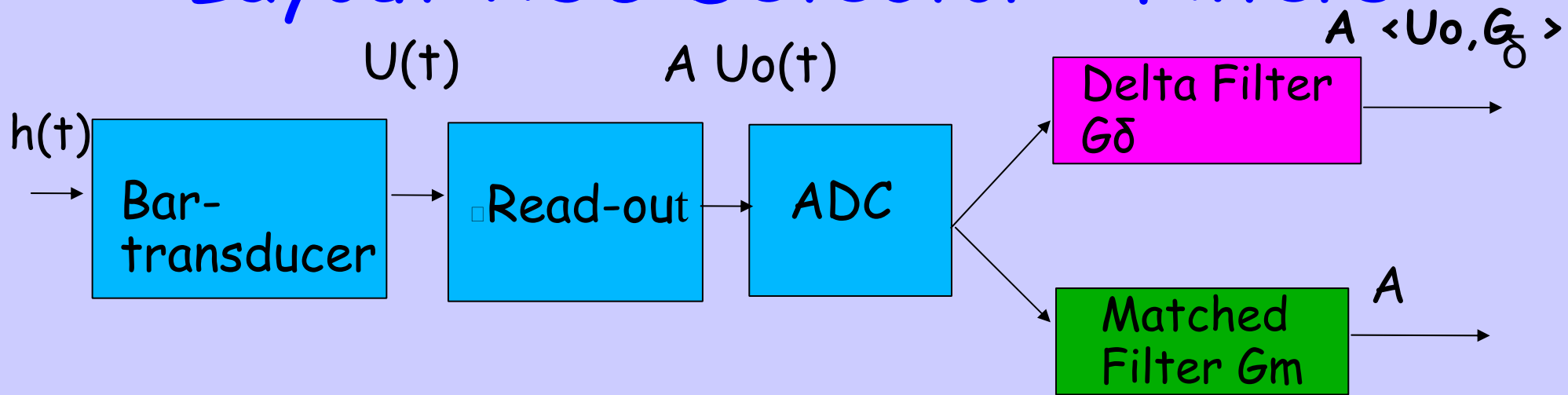
Case I: $F_o = F_m$, τ increasing from 10 ms till



The maximum amplitude $A = u(t_0)$ is a linear function of τ .



Layout-ROG Detector + Filters



Amplitude, $A \sim A(\tau, F_o, h_o, S)$

S = System parameters

ROG Delta Filter: $G\delta = N_d W_{ux}(f) / (S(f) * M_d)$

$SNR = A \langle U_o, G\delta \rangle / N_d$

Matched filter: $G_m = N_m U_o^*(f) / (S(f))$

$SNR = A / N_m$

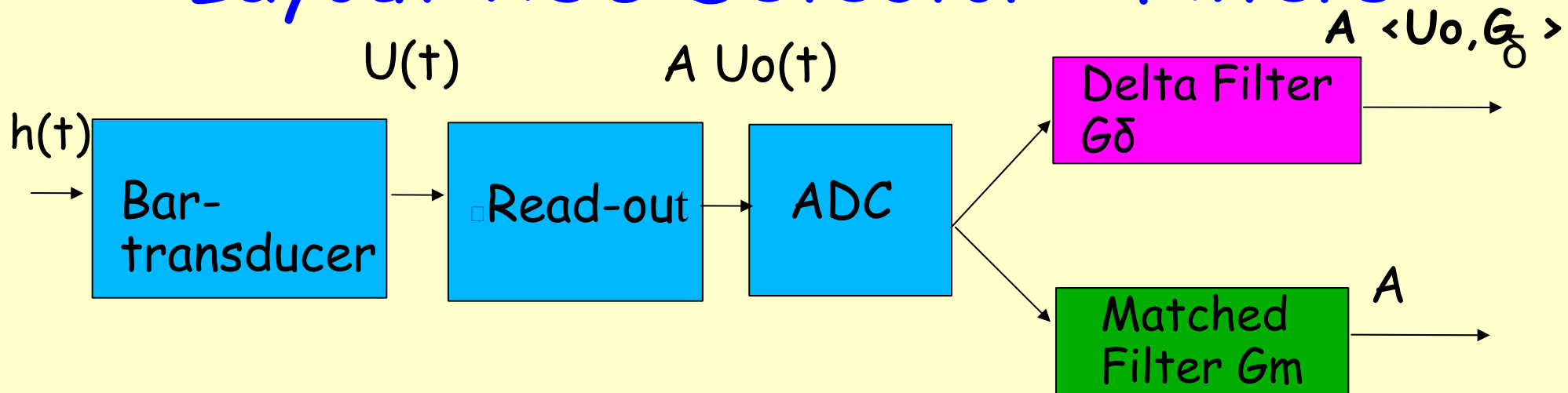
$S(f)$: Noise power spectra at the output

N_m, N_d : std dev of the filtered noise

$$N_m^2 = \left[\frac{1}{2\pi} \int \frac{|U_o(j\omega)|^2}{S(\omega)} d\omega \right]^{-1}$$

$$N_d^2 = \left[\frac{1}{2\pi} \int \frac{|W_{ux}(j\omega)|^2}{S(\omega)(M_d)^2} d\omega \right]^{-1}$$

Layout-ROG Detector + Filters



Amplitude, $A \sim A(\tau, F_0, h_0, S)$

S = System parameters

ROG Delta Filter: $G\delta = N_d W \cdot u_x(f) / (S(f) \cdot M_d)$

$SNR = A \langle U_0, G_\delta \rangle / N_d$

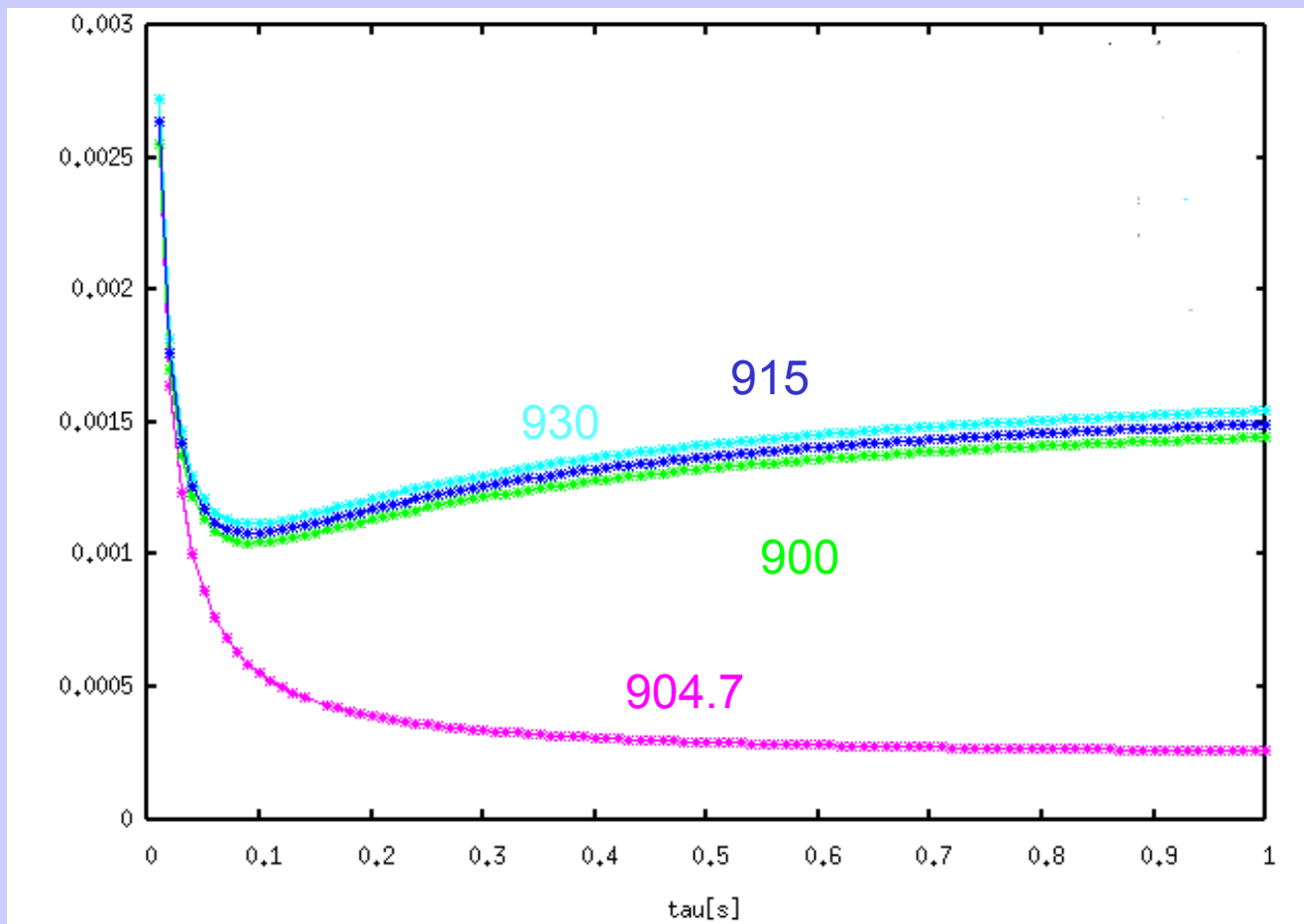
Matched filter: $G_m = N_m U_0^*(f) / (S(f))$

$SNR = A / N_m$

$S(f)$: Noise power spectra at the output

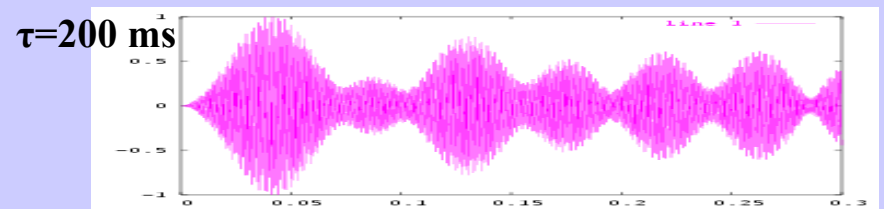
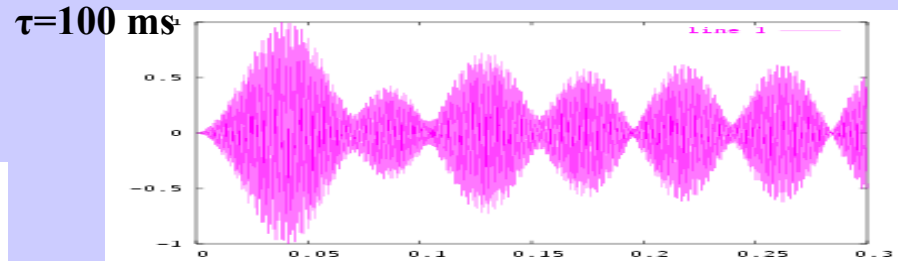
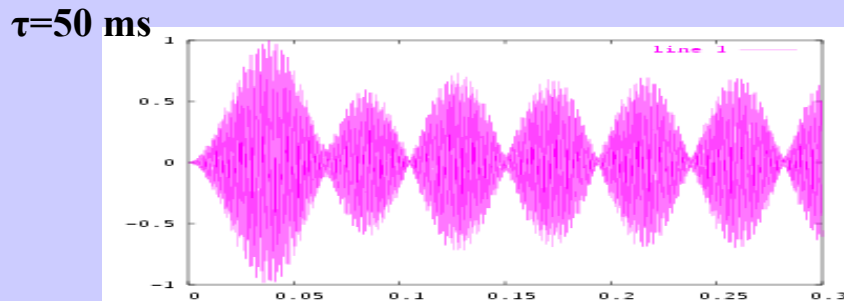
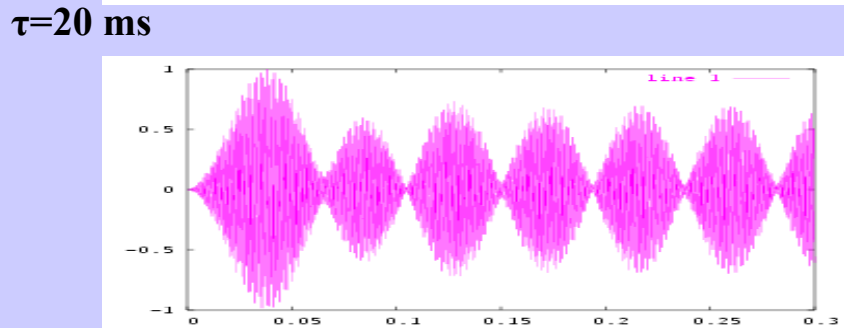
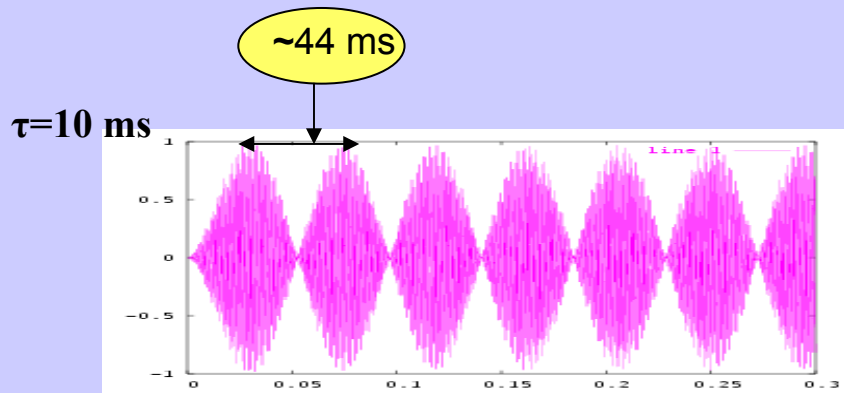
N_m, N_d : std dev of the filtered noise

Minimum energy (Mc^2) to be emitted in QNM to obtain SNR=5



Response of the bar-transducer to the damped sinusoid (summary)

Case II : $F_o = (F_p + F_m)/2 = 915$, τ : [10 ms till 200ms]

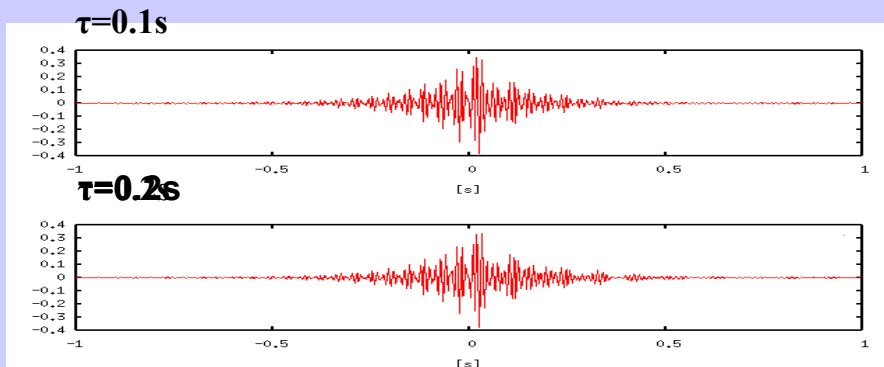
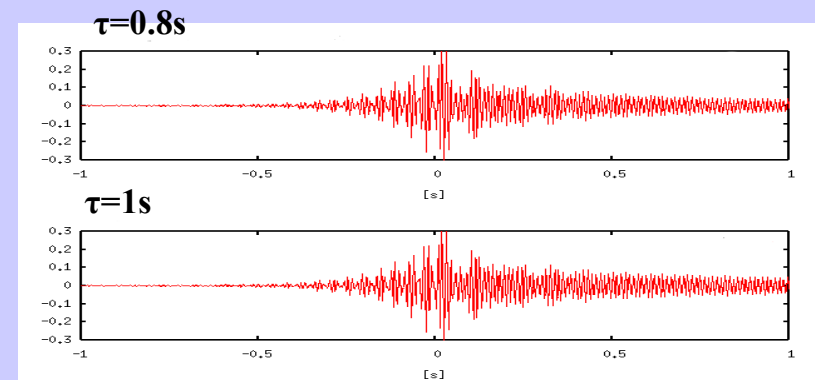
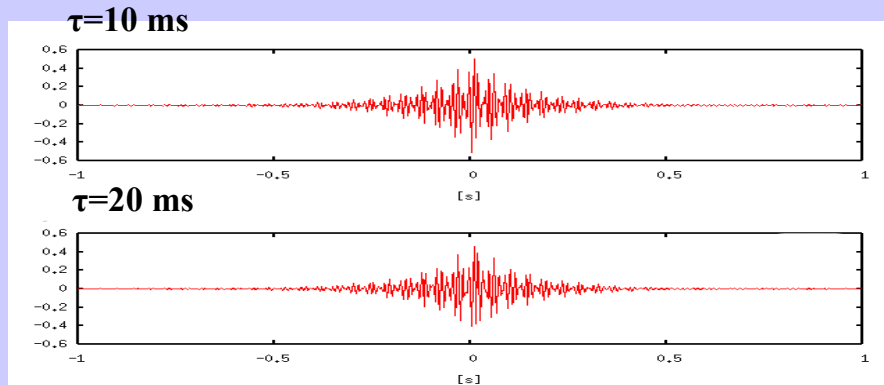


1. Maximum amplitude $A=u(T_o)$ increases with τ
2. T_o is constant.

Depends on the modulation freq of the envelope i.e. $(F_p - F_m) \sim 22.7\text{Hz}$

Damped sinusoid filtered with Delta Filter

CASE $f_o = (f_m + f_p)/2$, τ [10 ms - 1s]



The temporal position of $\text{Max}(A \langle U_o, G \rangle)$ is independent of τ .

Because the time T_o corre. to $\text{max}(u(t))$ is independent of τ