



LISA - PennState, July 22, 2002

# Bars in the short- and medium-term future

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and INFN Roma 2*

# Principle of operation of resonant-mass detectors

GWs excite those vibrational modes of a resonant body that have a mass quadrupole moment, such as the fundamental longitudinal mode of a cylindrical antenna.

$$u(x, t) = \sum_n A_n(t) \psi_n(x)$$

$$\ddot{A}_n(t) + \tau^{-1} \dot{A}_n(t) + \omega_0^2 A_n(t) = F_n(t)$$

$$F_n(t) = M^{-1} R_{iojo} \int \psi_n^{i*} x^j \rho d^3x$$

Force acting on the n-mode, described by the eigenfunction  $\psi_n$

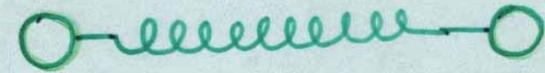
$$F_{1L} \neq 0; \quad F_{2L} = 0$$

## Cylindrical Bar

Mass **M**, length **L**, speed of sound **v<sub>s</sub>**, resonant frequency **f=v<sub>s</sub>/2L**

$$f_o = \frac{\omega_o}{2\pi} = 1kHz \left( \frac{v_s}{5.4 \times 10^3 \text{ m/s}} \right) \left( \frac{3m}{L} \right)$$

## Equivalent oscillator



M/2

$$l = 4L/\pi^2$$

M/2

Equation governing the response:

$$\ddot{x}(t) + \tau^{-1} \dot{x}(t) + \omega_o^2 x(t) = \frac{l}{2} \ddot{h}(t)$$

**Experimental domain: the measurement of weak forces acting on a mechanical oscillator**

## Bars with respect to IFOs

- Technology  
different principles and instrumentation
- Frequency Band  
Complementary; HF signals allow to study unique features of compact objects
- Symmetry properties  
discriminating the signal quadrupole character

## Detectors main features

	ALLEGRO	AURIGA	EXPLORER	NAUTILUS	NIOBE
Bar Material	Al5056	Al5056	Al5056	Al5056	Nb
Bar Mass [kg]	2296	2230	2270	2260	1500
Bar Length [m]	3.0	2.9	3.0	3.0	2.75
Freq. - [Hz]	895	912	905	908	694
Freq. + [Hz]	920	930	921	924	713
Q ± [1E6]	2	3	1.5	0.5	20
Bar Temp. [K]	4.2	0.25	2.6	0.1	5
Misalignment *	6°	5°	3°	2°	16°

\* Angle between bar axis and the perpendicular to the Earth great circle closer to the five detectors.

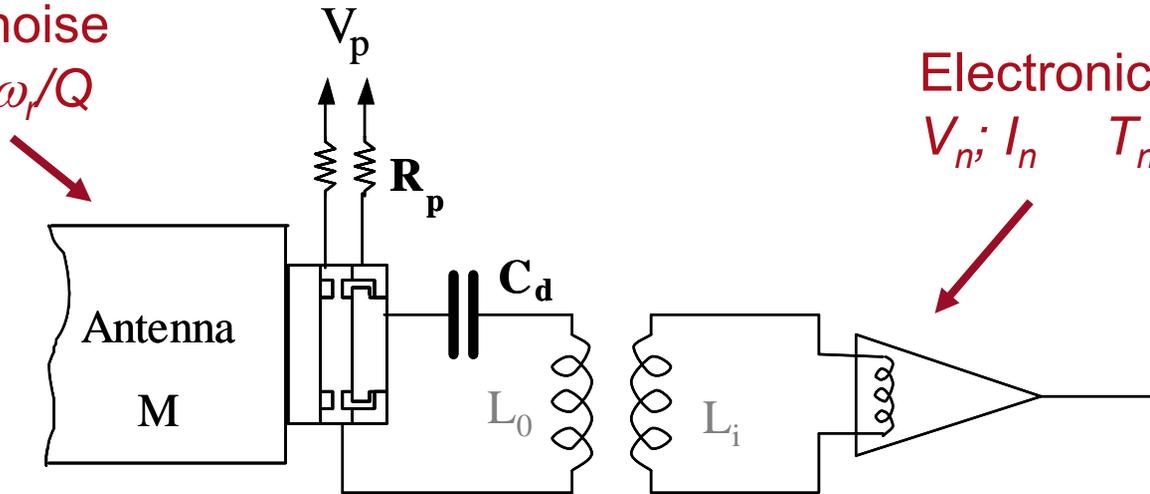
- **almost parallel** detectors
- resonant **frequencies** span from **694 to 930 Hz**
- typical **frequency bandwidths** per each resonance **~ 1 Hz**
- typical **amplitude thresholds** for bursts search in 1997–1998 at resonances:

$$H_{th} \sim 1.5 - 4 \times 10^{-21} \text{ /Hz} \quad \text{Fourier component of the g.w. burst amplitude}$$

$$h_{th} \sim 1.5 - 4 \times 10^{-18} \quad \text{strain g.w. amplitude for a conventional } \sim 1\text{ms burs}$$

# MAIN FEATURES

Thermal noise  
 $S_F = MkT\omega_r/Q$



Electronic noise  
 $V_n, I_n \quad T_n = \sqrt{V_n^2 I_n^2} / k$

**The mechanical oscillator**

Mass **M**  
 Speed of sound  $v_s$   
 Temperature **T**  
 Quality factor **Q**  
 Res. frequency  $f_r$

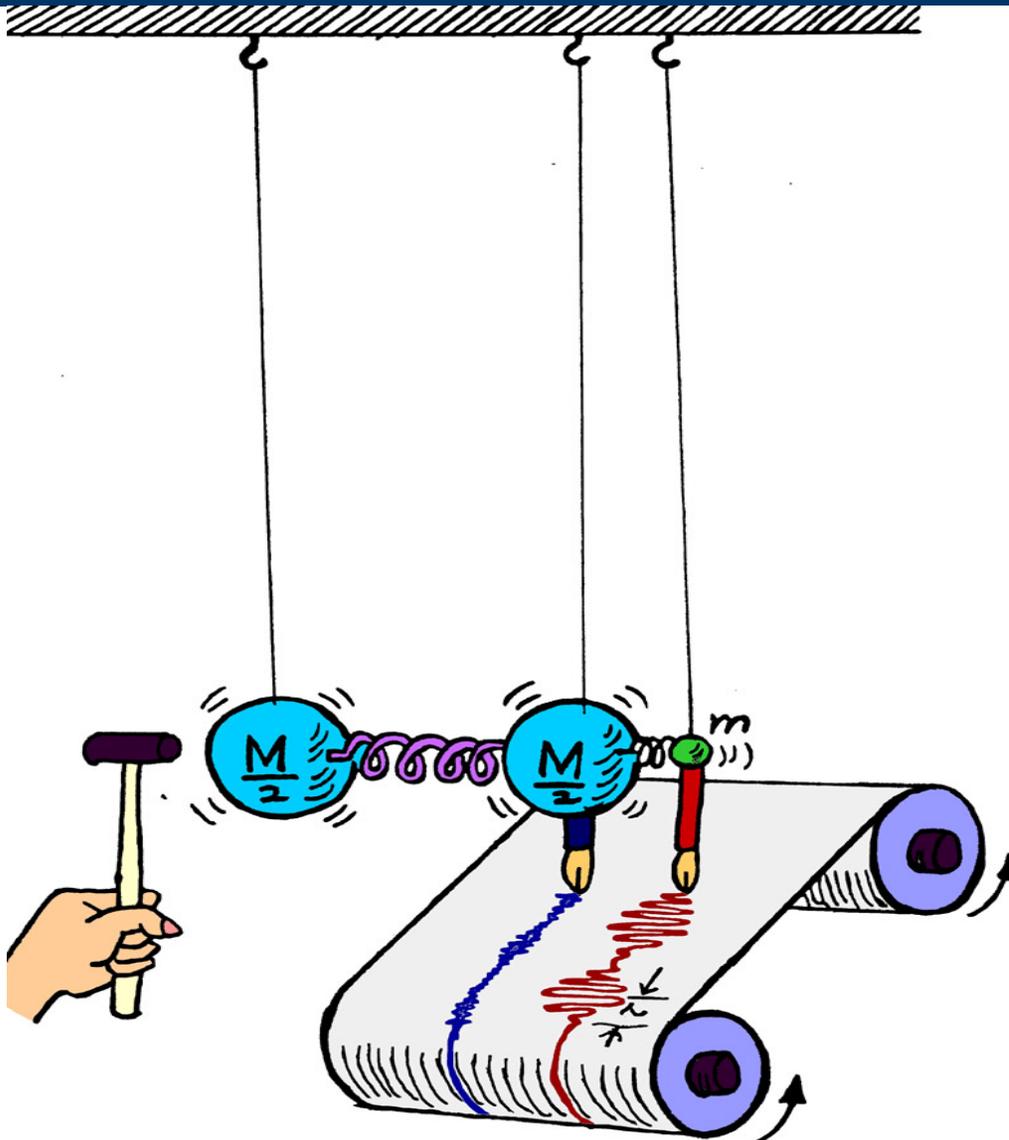
**The transducer**

Efficiency  $\beta$

**The amplifier**

Noise temperature  $T_n$

# Principle of a Resonant Transducer



The displacement of the secondary oscillator modulates a dc electric or magnetic field or the frequency of a s.c. cavity

$$x_m = \sqrt{\frac{M}{m}} x_M$$

## NAUTILUS sensitivity

Strain sensitivity, i.e. minimum impulsive signal detectable with SNR = 1,

$$\bar{h}_{\min} \approx \left( \frac{T}{MQ} \right)^{1/2} \quad [1/\sqrt{\text{Hz}}]$$

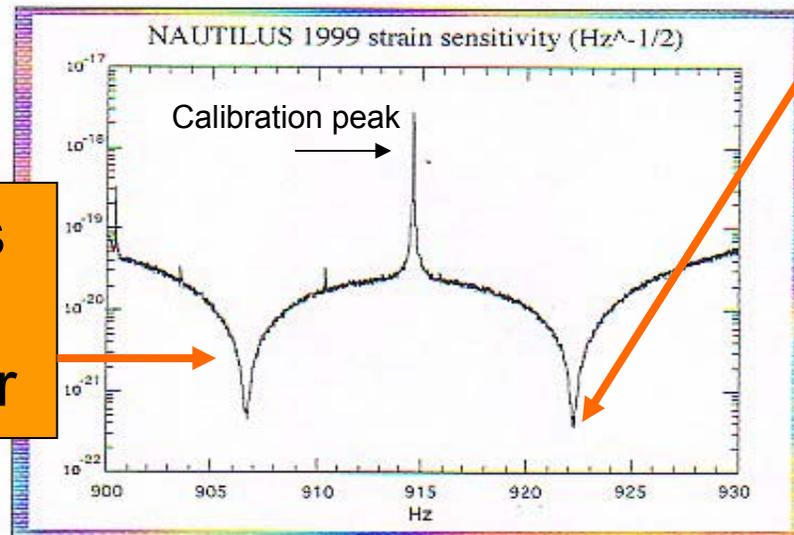
T Thermodynamic Temperature  
M Mass  
Q Quality Factor

$$\Delta f \approx \left( \frac{\beta}{T_n} \right)^{1/2} \quad [\text{Hz}]$$

$\beta$  Capacitive transducer  
 $T_n$  Electronic temperature

The peak sensitivity depends on T/MQ

The bandwidth depends mainly on the transducer and amplifier

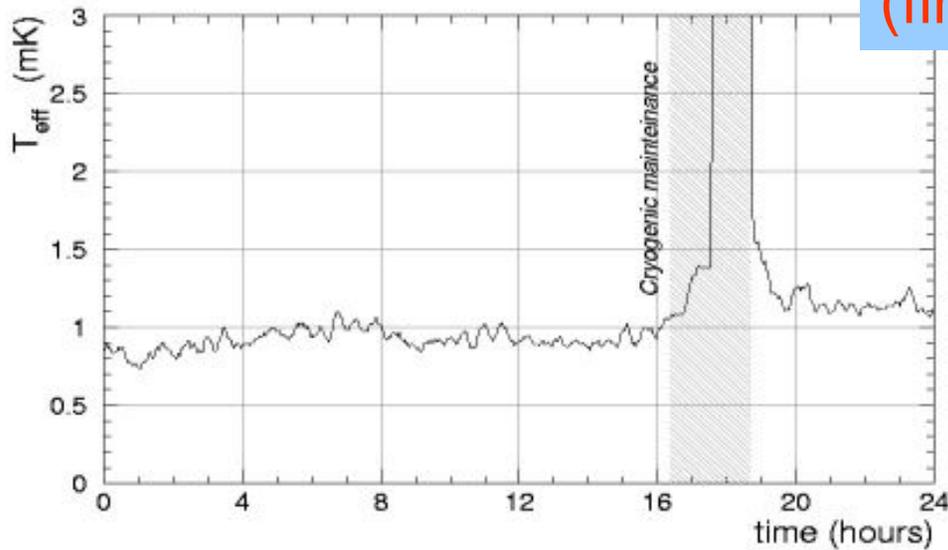


The bandwidth of the antenna can be increased acting on the transducer-amplifier of the signals, by increasing  $\beta$  and/or decreasing  $T_n$

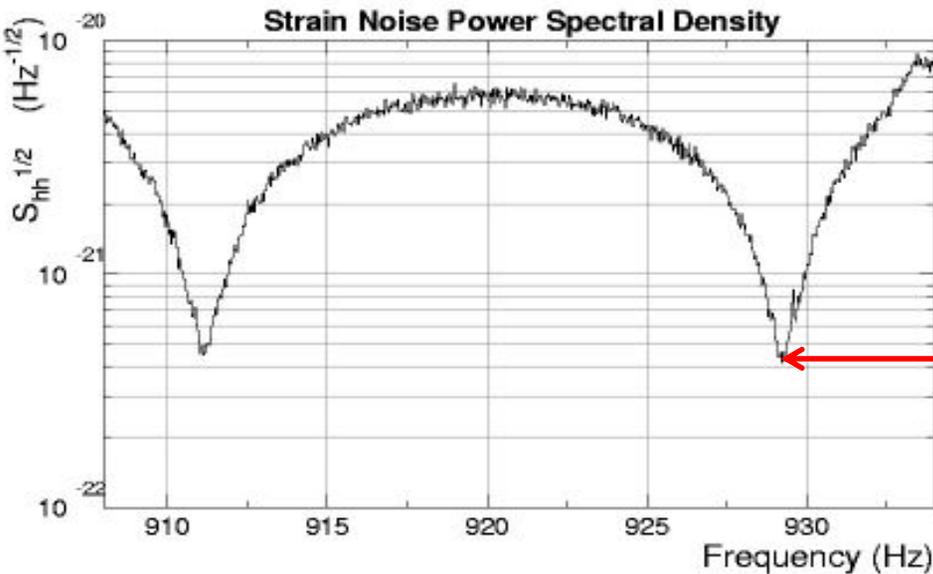
# The AURIGA sensitivity (first run 97-99)

Bar at 0.2 K

DAY 185, SUNDAY Jul 4, 1999

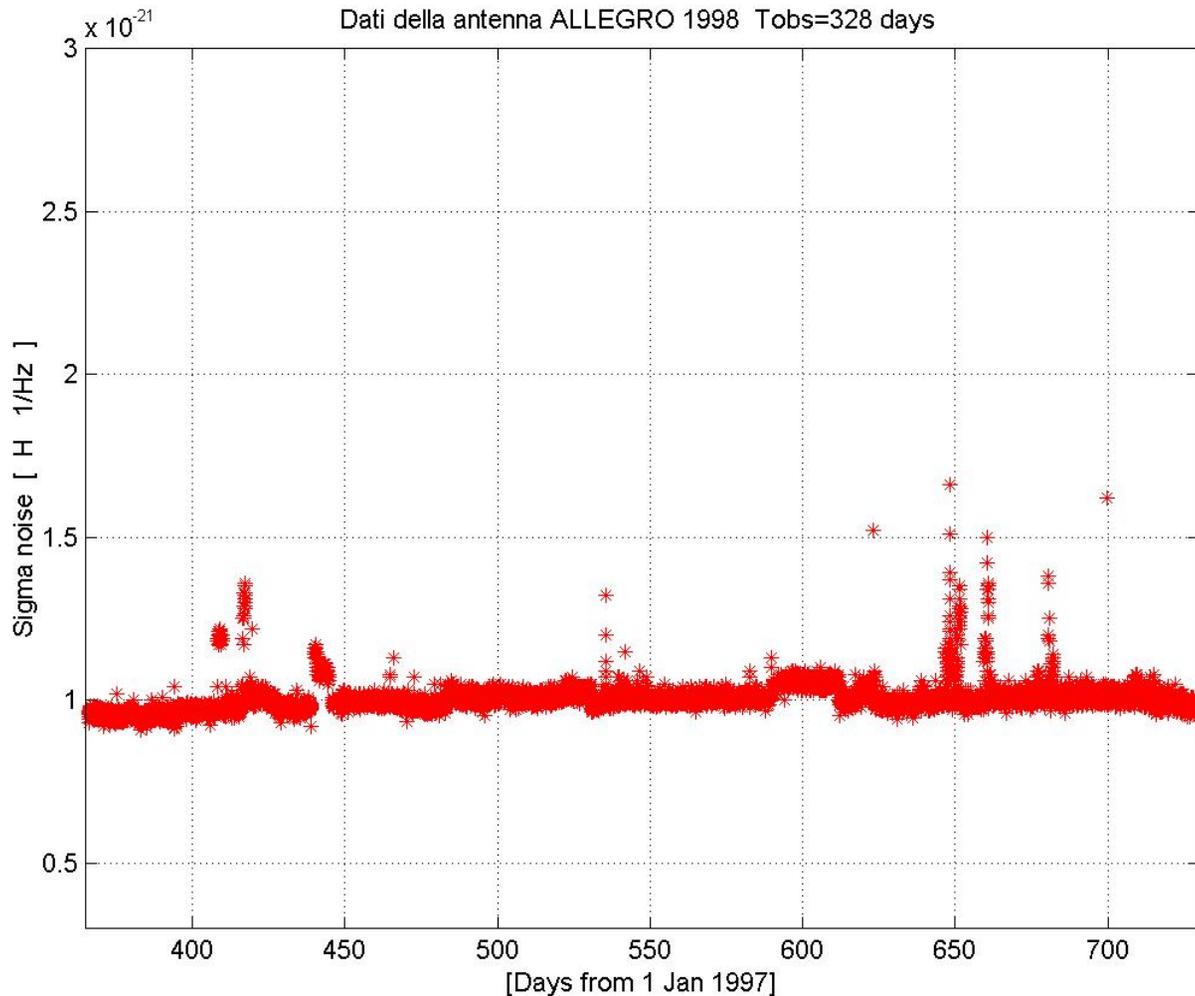


$h=2.5 \cdot 10^{-19}$



$4 \cdot 10^{-22} \text{ Hz}^{-1/2}$   
 $B=6 \text{ Hz at } 10^{-21} \text{ Hz}^{-1/2}$

# A STATIONARY AND VERY HIGH DUTY CYCLE DETECTOR: ALLEGRO



ALLEGRO  
1998

Tobs=  
328 days !

$h=1 \times 10^{-18}$

# Detectable signals today

**BURSTS:** Black Hole ( $M \sim 10M_{\odot}$ ) formation,  $10^{-4}M_{\odot}$  into GW

$$SNR = 6 \times 10^3 \left( \frac{10 \text{ kpc}}{r} \right)^2 \left( \frac{10^{-44} \text{ Hz}^{-1}}{\tilde{h}^2} \right) \left( \frac{\Delta f}{1 \text{ Hz}} \right)$$

**SPINNING NEUTRON STARS:** Non axisymmetric ( $\epsilon \sim 10^{-6}$ ) pulsar,  $M \sim 1.4M_{\odot}$

$$SNR \approx 30 \left( \frac{10 \text{ kpc}}{r} \right)^2 \left( \frac{10^{-44} \text{ Hz}^{-1}}{\tilde{h}^2} \right) \left( \frac{\epsilon}{10^{-6}} \right) \left( \frac{T_{obs}}{1 \text{ y}} \right)$$

**COALESCING BINARIES:** Inspiring NS-NS system,  $M \sim 1.4M_{\odot}$

$$SNR \approx 10^3 \left( \frac{10 \text{ kpc}}{r} \right)^2 \left( \frac{10^{-44} \text{ Hz}^{-1}}{\tilde{h}^2} \right) \left( \frac{\Delta f}{1 \text{ Hz}} \right)$$

**STOCHASTIC BACKGROUND:** 2 detectors, at distance  $d \ll \lambda_{GW}$

$$\Omega_{GW} \approx 2 \times 10^{-3} \left( \frac{f}{900 \text{ Hz}} \right)^3 \left( \frac{\tilde{h}_{1,2}}{10^{-22} \text{ Hz}^{-1/2}} \right)^2 \left( \frac{1 \text{ Hz}}{\Delta f} \right)^{1/2} \left( \frac{1 \text{ y}}{T_{obs}} \right)^{1/2}$$

## Bursts

*Phys. Rev. Lett.* **85**, 5046 (2000)  
*Class. Quant. Grav.* **18**, 43 (2001)

## Continuous signals

*Phys. Rev. D* **65**, 022001(2002)  
*Phys. Rev. D*, **65**,042003 (2002)

## Stochastic Background

*Astron. Astrophys.* **351**, 811 (1999)

## more

*Search for correlation with GRB's*  
*Astron. Astrophys.* **138**, 603 (1999)  
*Phys. Rev. D* **63**, 082002 (2001)  
*astro-ph/0206431*, submitted to PRD.

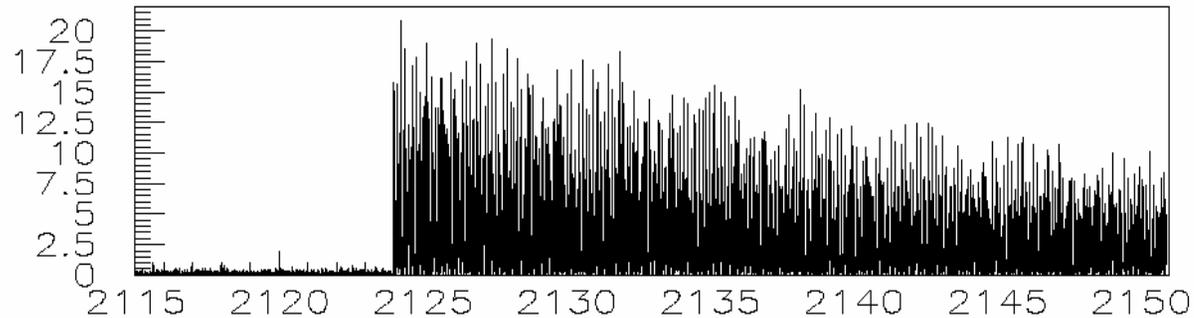
*Gravitational near field*  
*Eur. J. Phys. C* **5**, 651 (1998)

*Effect of cosmic rays*  
*Phys. Rev. Lett.* **84**, 14 (2000)  
*Phys. Lett. B* **499**, 16 (2001),  
*Phys. Lett. B* (2002), in press.

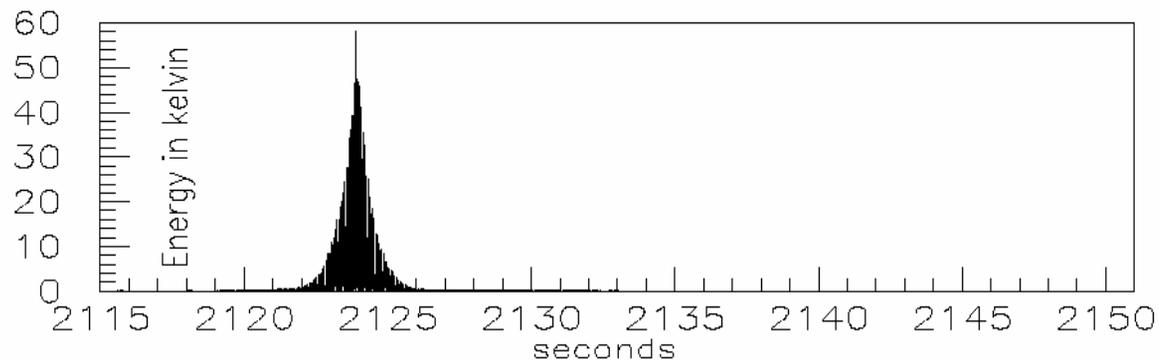
**Burst event for a present bar:** a millisecond pulse, a signal made by a few millisecond cycles, or a signal sweeping in frequency through the detector resonances. The burst search with bars is therefore sensitive to different kinds of gw sources such as a stellar gravitational collapse, the last stable orbits of an inspiraling NS or BH binary, its merging, and its final ringdown.

## Real data: the arrival of a cosmic ray shower on NAUTILUS

Unfiltered  
signal ( $V^2$ )



The signal  
after  
filtering  
(kelvin)



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**Number 514 (Story 1)**, 29 November 2000 by Phillip F. Schewe and Ben Stein

## New Upper Limit on Gravity Wave Events in Our Galaxy

The International Gravitational Event Collaboration (IGEC) is the first ever network of cryogenic resonant-cylinder gravity wave detectors. It consists of five widely spaced detectors: one in the US (Baton Rouge), two in Italy (Legnaro and Frascati), one in Switzerland (at CERN), and one in Australia (Perth).

Searching for passing gravity waves is a delicate art since it involves sensing deformations much smaller than the size of an atomic nucleus in huge detectors meters or kilometers in size. In the resonant detector approach this means watching for longitudinal vibrations in chilled automobile-sized metal cylinders. In the interferometer approach (used at LIGO; see, for example, [Update 442](#)) the deformation is the change in the separation of distant mirrors attached to test masses. Gravity waves strong enough to be detected will most likely come from events such as the coalescence of black holes or neutron stars, and these are rare. IGEC reports now that in its first operational period it has observed no gravity waves. From this they calculate an upper limit of the order of one per year in the rate at which such gravity wave events occur in our galaxy.

GEC is not only striving to have the sensitivity to record gravity waves from events out to distances of 100 million light years but is also hoping to be able to locate the source of the waves in the sky.

**No detection of g.w. bursts above  $h = 4 \cdot 10^{-18}$**

**That is, 0.07 solar masses in the GC**

**IGEC is currently analysing the observations from 1997 to 2000 (up to 4 det. operating simultaneously)**

•1detector	•1322 days
•2 detectors	•713 days
•3 detectors	•178 days
•4 detectors	•29 days
•5 detectors	•0 days

Total observation  
time = 1460 days

**Three detectors in accidental coincidences  $1 / 10^4$ y**



# Explorer and Nautilus 2001

- **EXPLORER (CERN)**

- ON from March to December
- Bandwidth = 9 Hz
- T = 2.6 K
- Duty Cycle=267/294=91%
- Average sensitivity

**$h=4.5 \cdot 10^{-19} \rightarrow$**

**$1.2 \cdot 10^{-4} M_0$  in GC**

- **NAUTILUS (LNF)**

- ON from January to December
- Bandwidth = 0.4 Hz
- T = 1.5 K
- Duty Cycle=291/365 =80%
- Average sensitivity

**$h=5.7 \cdot 10^{-19} \rightarrow$**

**$2 \cdot 10^{-4} M_0$  in GC**

**Coincident operation for 213.5 days**

# Stochastic Background

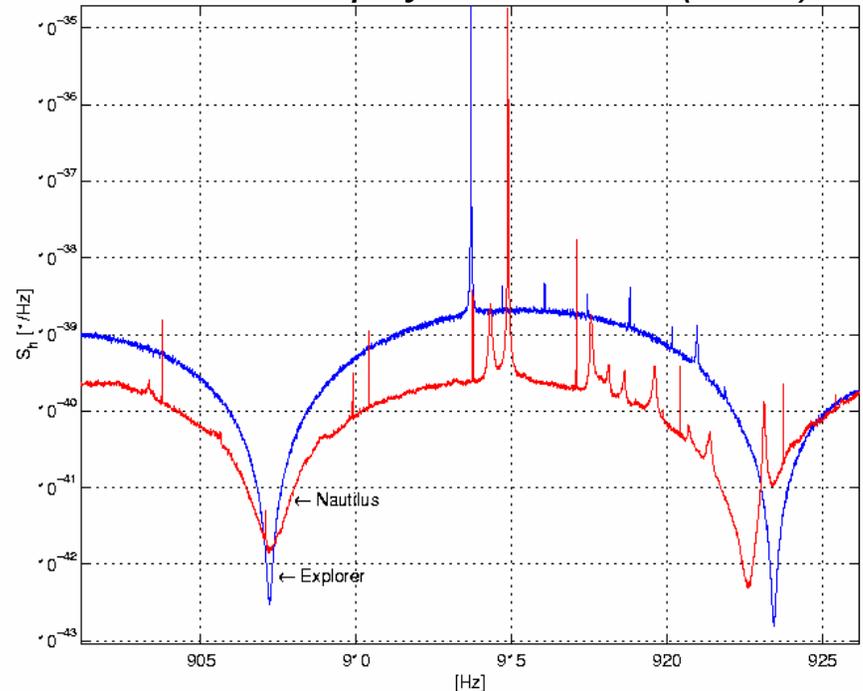
12 hours of data

$B = 0.1$  Hz

$S_{12} < 1 \times 10^{-22} \text{ Hz}^{-1/2}$

Crosscorrelation measurement of stochastic background of GW with two resonant detectors

*Astron. Astrophys.* 351, 811 (1999)



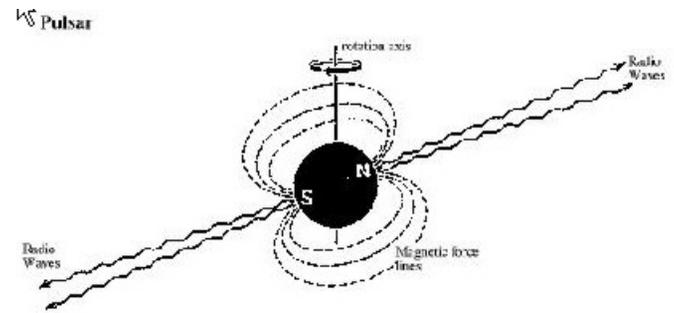
$$\Omega_{\text{GW}} < 60$$

# Stochastic background

Limits of  $\Omega_{\text{gw}} < 1$  achievable cross-correlating data from a bar-bar couple or from a bar-interferometer couple:

- The cross-correlation of 4 months of NAUTILUS and AURIGA data, at the sensitivity expected in the next run, would put the limit at  $\Omega_{\text{gw}} \leq 0.1$
- Joint analyses with VIRGO, NAUTILUS and AURIGA may put limits at the level  $\Omega_{\text{gw}} \leq 3-5 \cdot 10^{-3}$  (1y integration time, NAUTILUS upgraded and AURIGA phase 2, and VIRGO at  $10^{-22} \text{ Hz}^{-1/2}$  @900 Hz)

# Continuous waves



- ALLEGRO put upper limits ( $4 \cdot 10^{-23}$  over 1 Hz band) on signals from the GC and 47Tucanae using one month of data
- Limit for signals in the GC, using 95 days of EXPLORER data  $h_c = 3 \cdot 10^{-24}$  (*Astone et al. PRD 65, 022201, 2002*)
- Overall sky search over 2 days of data is now running: limit at the level of  $h_c = 3 \cdot 10^{-23}$  (1 million points, by choosing spin-down parameter and position randomly) (*Astone, Borkowsky, Jaranowsky, Krolak, PRD, 65, 042003, 2002*)
- Collaboration with VIRGO-Rome group. Application of the strategy for the pulsar search to the EXPLORER and NAUTILUS data

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## Data analysis for the search of continuous waves

Continuous signals are emitted by various astrophysical sources (rotating neutron star, neutron star in a binary system). They carry important information on their sources and on fundamental physics.

The signals are very weak but, due to the fact they are continuous, it is possible to implement search procedures that build up the SNR with the time.

The sensitivity of the detectors to continuous signals is a function of the noise spectral amplitude ( $1/\sqrt{\text{Hz}}$ ) and of the observation time.

Read [this note](#) for an introduction to the analysis of continuous waves with resonant detectors.

We have developed a procedure for the search of signals from periodic sources in the data of gravitational wave detectors and we have used it to analyze one year of data from the resonant detector Explorer, searching for sources located in the Galactic Center (GC).

No signals with amplitude greater than  $2.9 \cdot 10^{-24}$ , in the range 921.32-921.38 Hz, were observed using data collected over a time period of 95.7 days, (for a source located at right ascension =  $17.70 \pm 0.01$  hours and declination =  $-29.00 \pm 0.05$  degrees).

The procedure and the results have been published on PRD 65,022201 (2002): [click here for the PDF file](#)

We are now running, in collaboration with A. Krolak, K. Borkovsky and P. Jaranovsky (Poland), an All-Sky search using two days of the Explorer 1991 data.

The procedure is described [here \(PDF file\)](#) and has been published on PRD 65,042003 (2002)

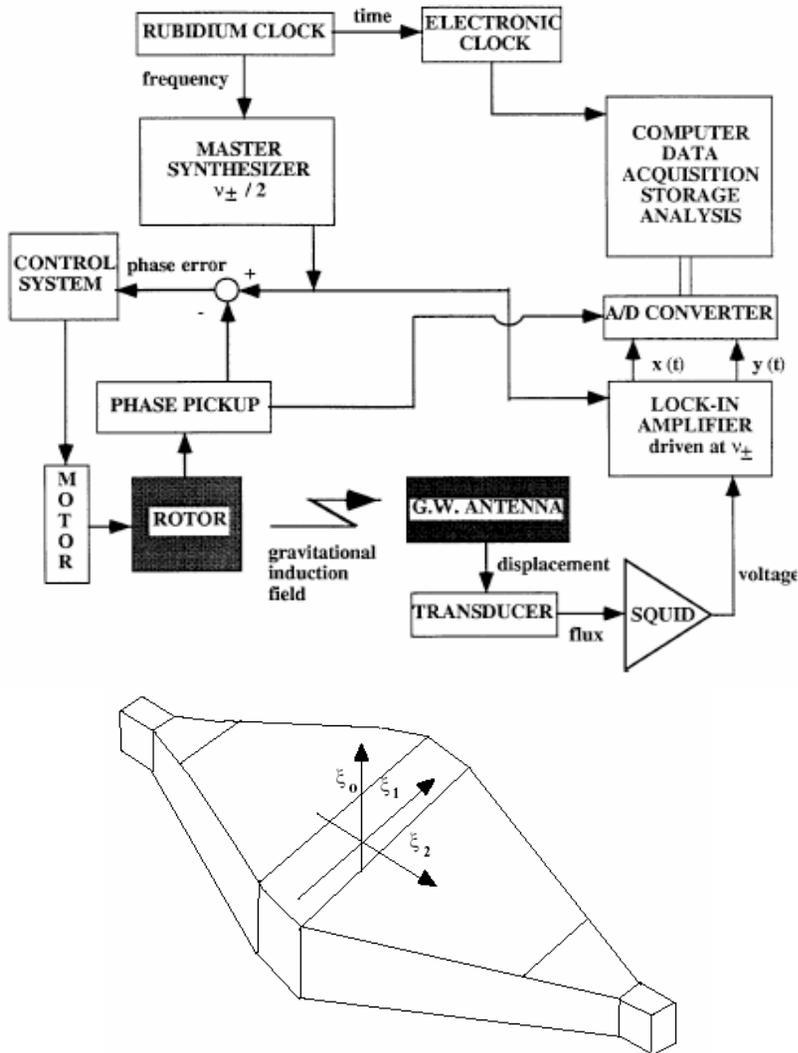
The web site for the ALL-SKY search is [HERE!](#)



# The search of continuous signals

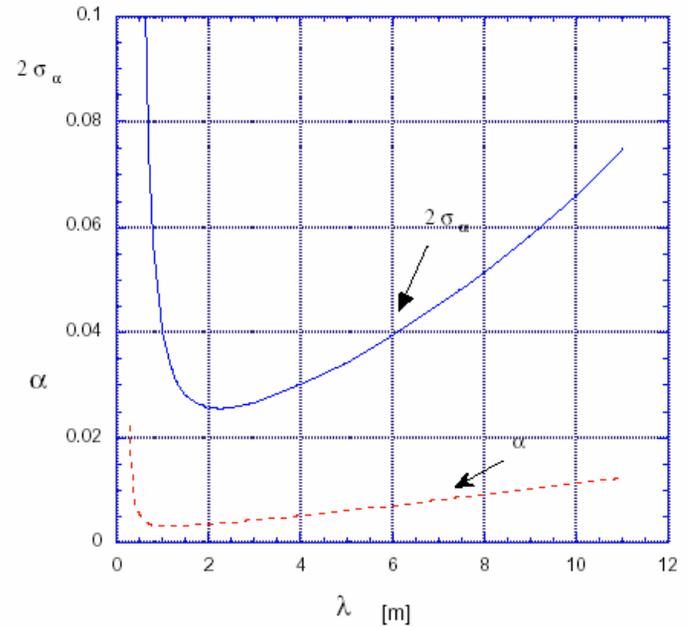
- The search method is based on a hierarchical method.
  - Filling a Short FFT data base ( the data are divided in different band of frequencies)
  - Construction of Time Frequency maps
  - Hough Transform
  - Candidate Selection
  - Coherent search in the selected frequency ranges (Zooming, Doppler correction , FFT.....)
  - New iteration
- The search is applied on the reduced data transferred in Rome and stored in the SFFT data base

# Detector calibration - Deviation from Newton law



$$V = -\frac{Gm}{r} \left(1 - \alpha e^{-r/\lambda}\right)$$

EXPLORER, J.of Phys. C (1999)



# Nautilus Side View

Al 5056;

$L = 3\text{ m}; D = 0.6\text{ m}$

$M = 2300\text{ kg}$

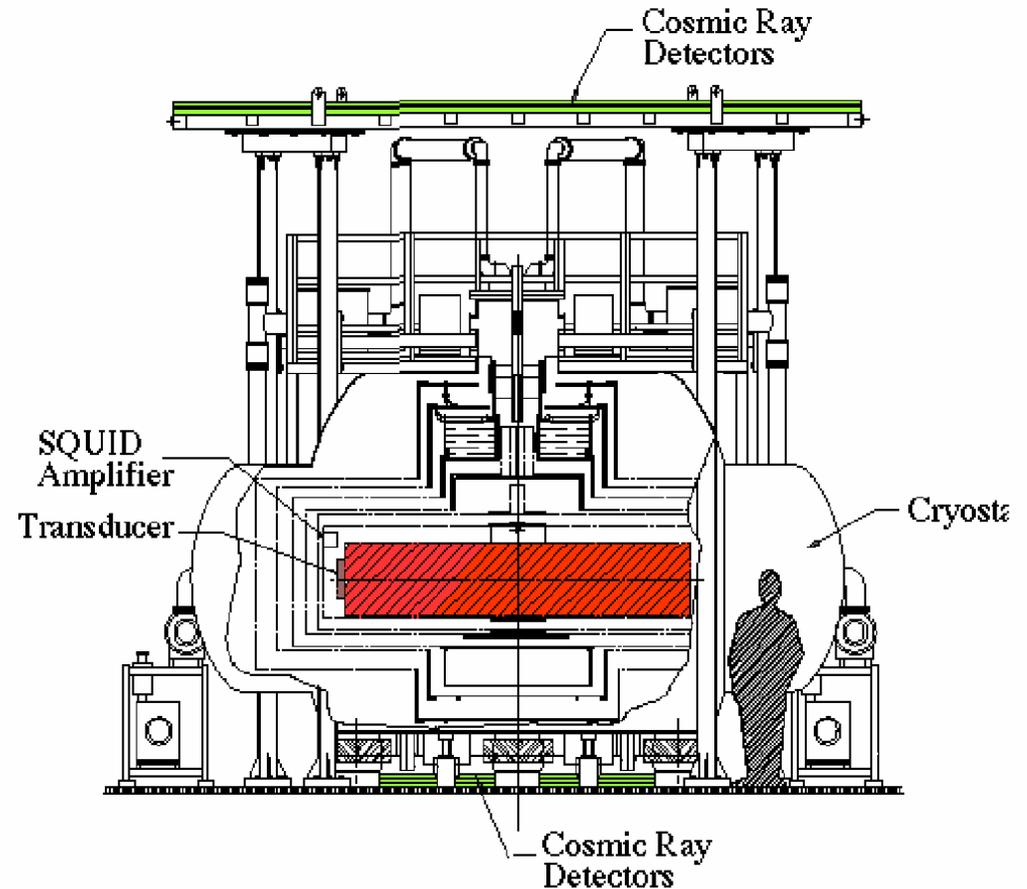
$T = 0.1\text{ K}$

$f_{\pm} = 907\text{ Hz}, 922\text{ Hz}$

$h = 3 \times 10^{-22}\text{ Hz}^{-1/2}$

$h_{\text{pulse}} = \Delta L/L = 4 \times 10^{-19}$

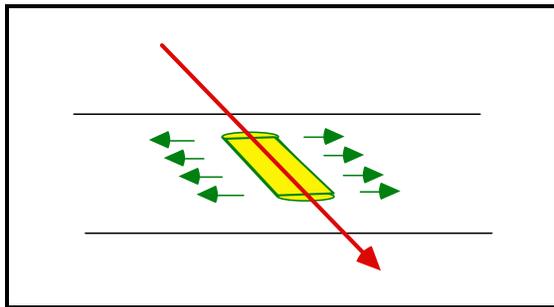
$\Delta E = 2\text{ mK} = 0.3\ \mu\text{eV}$



# Cosmic ray interaction in the bar

## Thermo-Acoustic Model:

the **energy deposited** by the particle is converted in a **local heating** of the medium:



$$\delta T = \frac{\delta E}{\rho C V_0}$$

$$\delta p = \gamma \frac{\delta E}{V_0} \quad \gamma = \frac{\alpha Y}{\rho C}$$

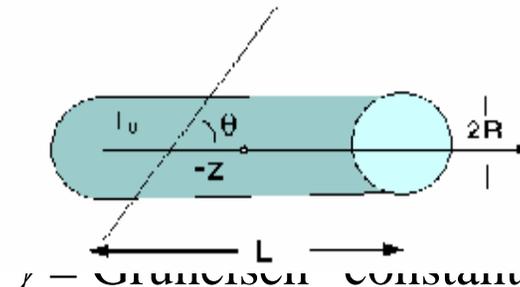
Excitation of the longitudinal modes of a cylindrical bar

$$E_n = \frac{1}{2} \frac{l^2}{V} \frac{G_n^2}{\rho v^2} \gamma^2 \left( \frac{dE}{dX} \right)^2$$

Allega A.M. & Cabibbo N. Lett Nuovo Cim 38 (1983) 263-  
A. De Rujula & B. Lautrup, Nucl Phys. B242 (1984) 93-144

$G_n$  form factor

A resonant gw detector used as a particle detector is different from any other particle detector





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## Physics News Update

### The American Institute of Physics Bulletin of Physics News

[Number 465](#) (Story #3), January 7, 2000 by Phillip F. Schewe and Ben Stein

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COSMIC RAYS OBSERVED BY GRAVITY-WAVE DETECTOR at the Frascati Laboratory in Italy consists of a 2300-kg aluminum cylinder cooled to a temperature of 0.1 K. The plan is that a passing gravitational wave (broadcast, say, by the collision of two neutron stars) would excite a noticeable vibration in the cylinder. NAUTILUS has not yet recorded any gravitational waves, but scientists have now witnessed the cylinder vibrated by energetic particle showers initiated when cosmic rays strike the atmosphere. The signal generated by the rays is believable because conventional cosmic-ray detectors surrounding the bar also lit up when they were struck by the particles. In effect the detector is able to discern a mechanical vibration as small as  $10^{-18}$  meters, corresponding to an energy deposit as small as  $10^{-6}$  eV. ([Astone et al.](#), *Physical Review Letters*, 3 January 2000; [Select Article](#). Contact Giuseppina Modestino, [modestino@lnf.infn.it](mailto:modestino@lnf.infn.it), 011-39-694-032-756.)



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# Summary of results

1999-2001

- We have found **for the first time** the cosmic rays induced signal in a resonant cryogenic detector. Very nice technical result ( $\Delta x \approx 10^{-18}$  meters!)
- Several checks show that we have indeed a **mechanical excitation of the bar**.
- The **thermo-acustical model** is  $\approx$  correct for most of the events. But for a fraction  $\approx 10\%$  of the showers we have signals much larger (2 order of magnitude) than expected.
- Interesting problem involving **gravitational waves, cosmic rays, particle detection and low temperature physics**

This unexpected result was obtained with a NAUTILUS bar temperature of 0.14 K (below  $T_c = 0.92$  K)

There are no anomalous large signals if the bar temperature is 1.5 K (normal material)

## The cosmic ray possibility to explain data

As was pointed by Barish and Liu (PRD **61**, 271, 1988) acoustic detectors are different from normal particle detectors based on ionization.

In a gas detector for example you need to excite some atomic level==>> threshold in velocity (around  $\beta \gg 10^{-3}$ )

**In acoustic detectors no threshold**

Several kind of massive slow particles proposed in the past (monopoles, nuclearites..etc) would give rise to large events in a resonant-mass detector Cabibbo Lett. Nuovo Cim. **83**, 263 (1983), De Rujula Nucl. Phys. B **242**, 93 (1984)

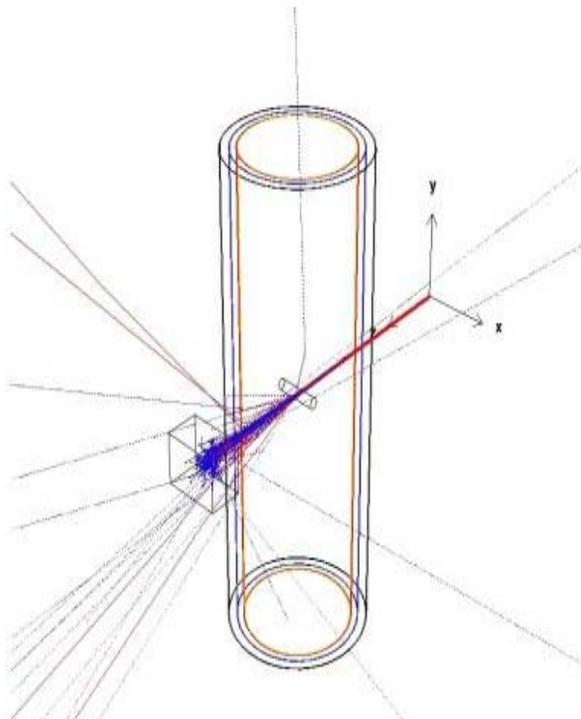
Very good limits have been set to these particles (for example with the MACRO) with underground experiments but not with experiments at sea level

The energy of the NAUTILUS anomalous events is in the region of the cosmic ray knee where we know that *something* should happen



## Rivelazione Acustica di Particelle

Search for thermo-acoustic effects on a cryogenic target by a particle beam at the [DAPHNE beam-test facility \(BTF\)](#)



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## Study of this effect in a controlled environment: the RAP experiment

we plan to excite vibrations of a small aluminium cylindrical bar at temperatures below and above 1 Kelvin, using the 500 MeV electron beam of the DAFNE BTF in Frascati

- useful for the future generations of gw detectors, including interferometers
- of general interest to understand the effect of particles on a superconductive medium

ALLEGRO:

AURIGA:

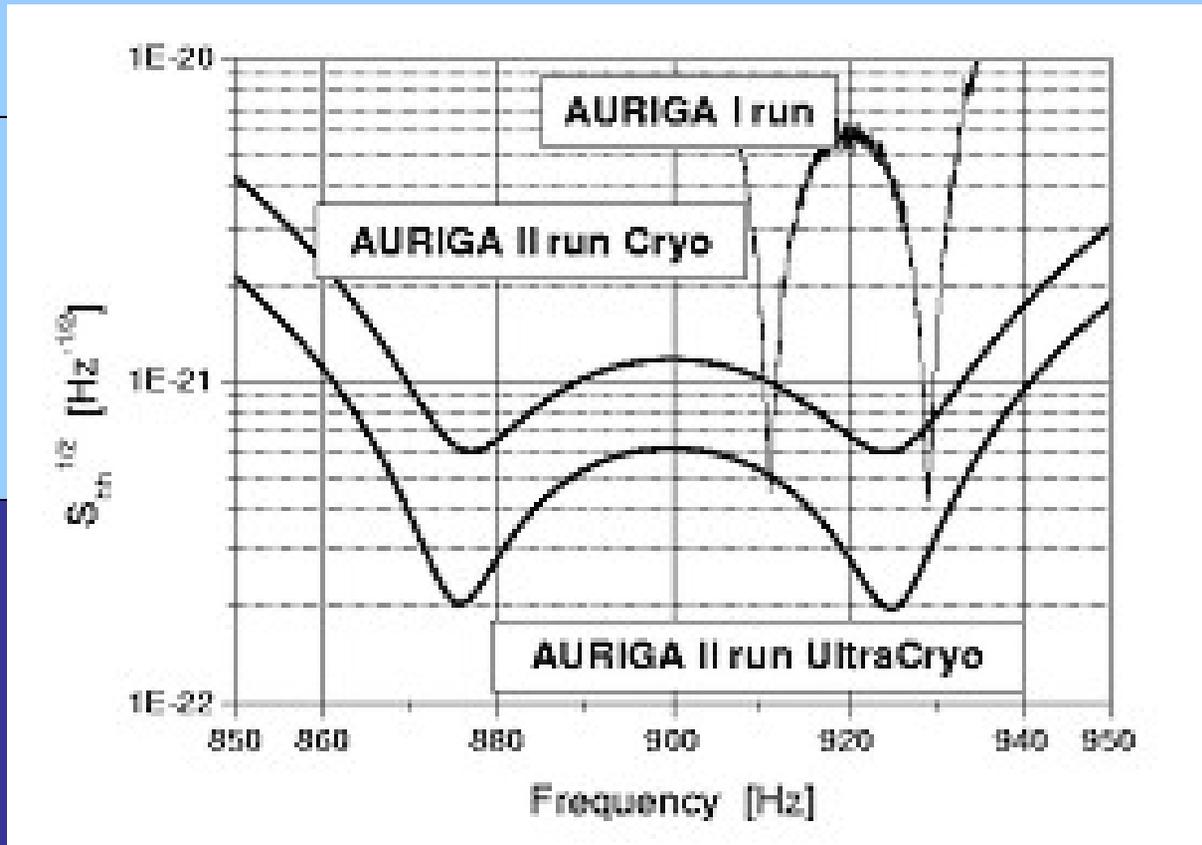
EXPLORER:

NAUTILUS:

NIOBE:

Actually in stand-by

The expected AURIGA strain sensitivity:  
*Phase I (2002)*



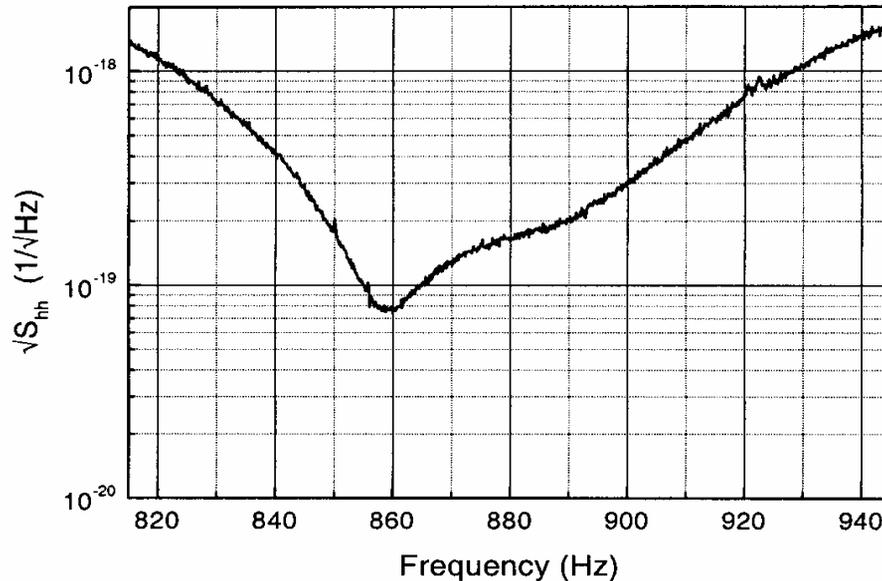
$$2 \cdot 10^{-22} \text{ Hz}^{-1/2}$$

$$B=80 \text{ Hz @ } 1 \cdot 10^{-21} \text{ Hz}^{-1/2}$$

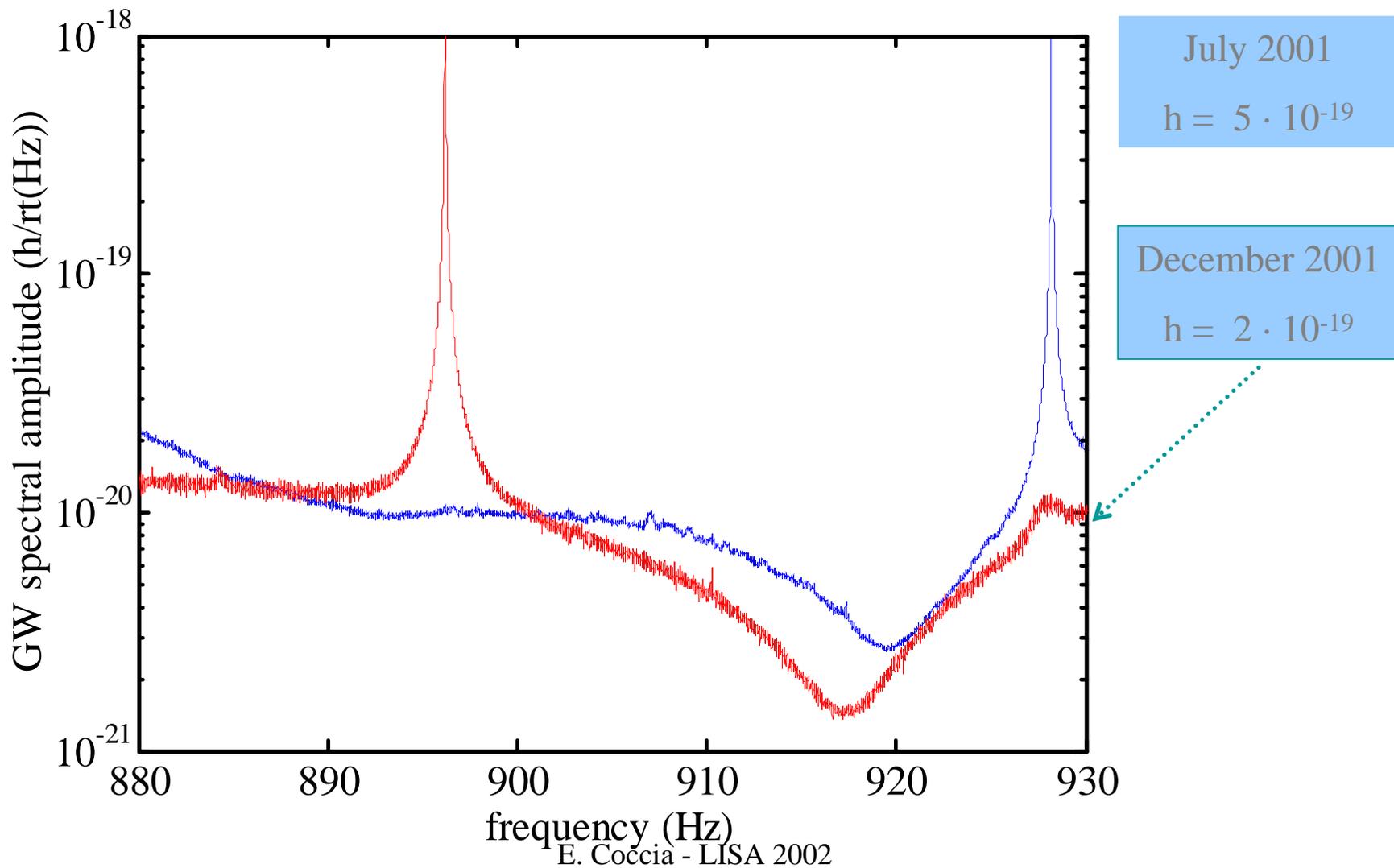
$$h=5 \cdot 10^{-20} (\text{burst})$$

# AURIGA Optomechanical readout

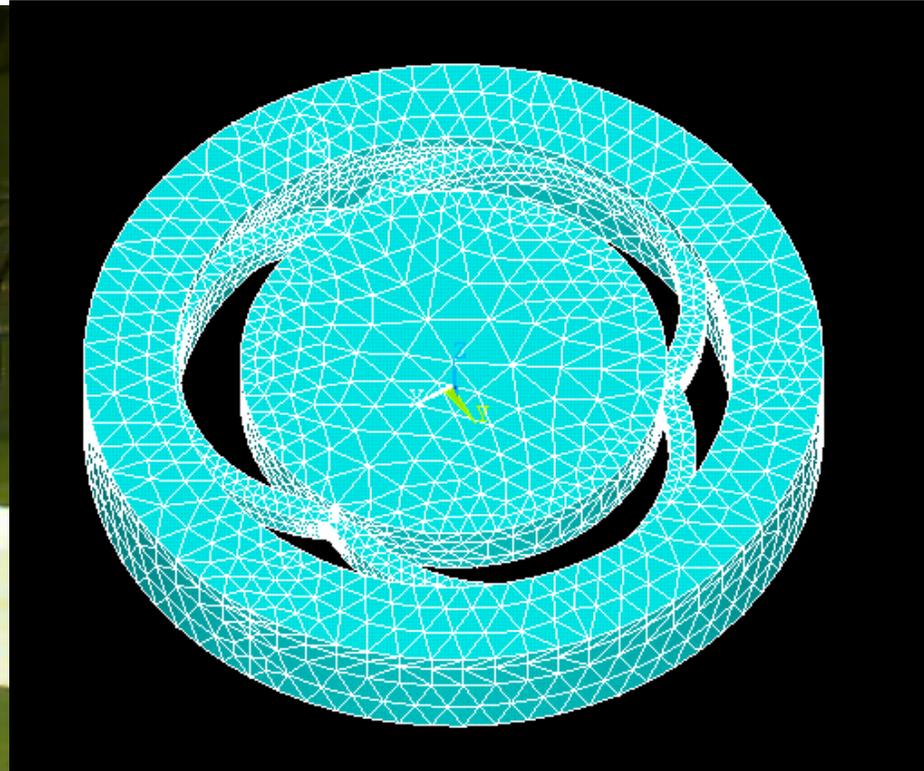
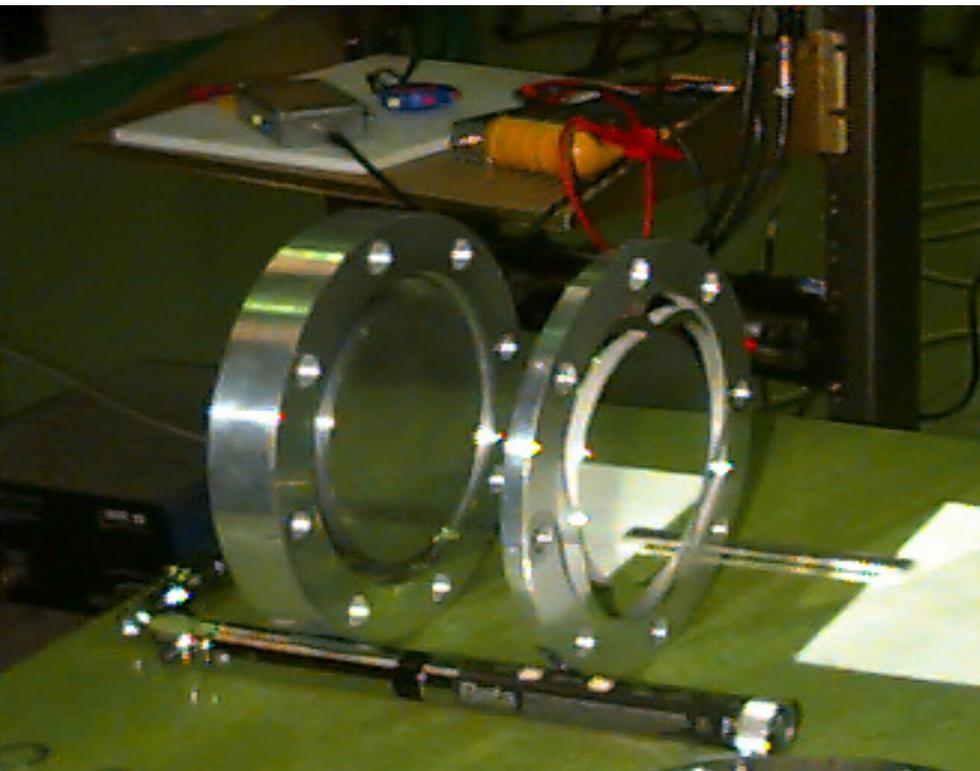
Bar motion read by **Fabry-Perot cavity** mounted between bar and resonant transducer. Length reference given by external Fabry-Perot cavity. **T Room bar** operated for 30 days.



# EXPLORER PERFORMANCES



# MICROMECHANICS

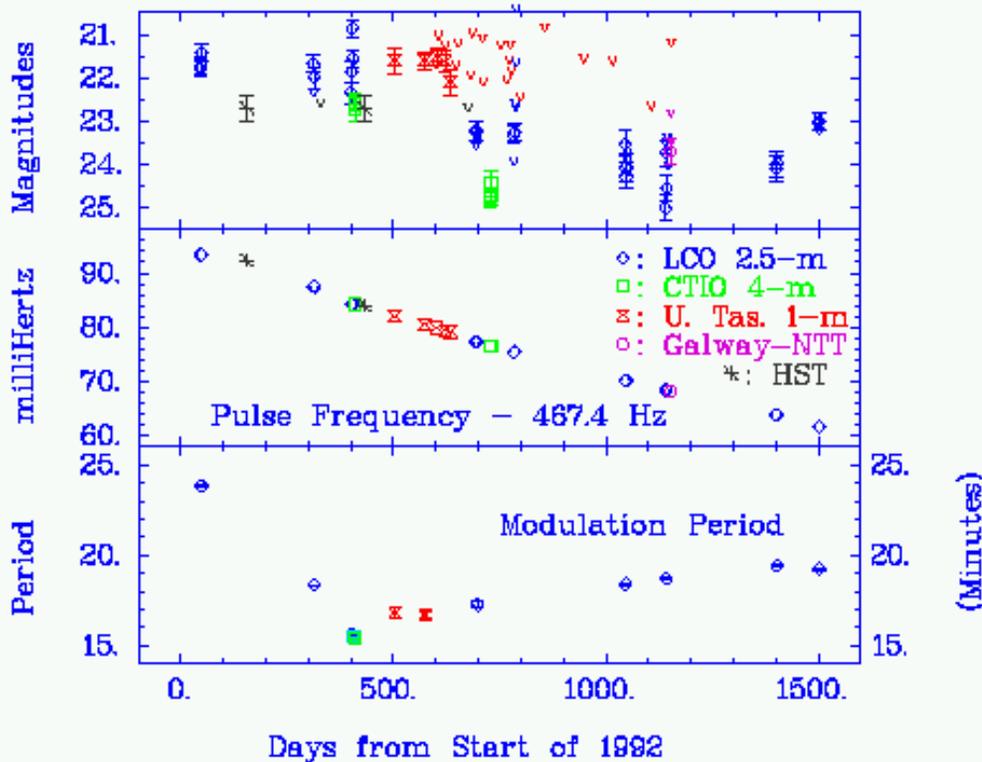


The rosette capacitive transducer; gap=9 $\mu$ m



Supernova 1987a

Evidence for a triaxial faint pulsar associated with SN1987a has been found in data taken from 5 detectors in the optical/near-infrared bands in the years 1992-1996 (Middleditch *et al.*, *New Astronomy*, 5, 243, **2000**).



$$P = 2.14ms;$$

$$\dot{P} \approx 2 \cdot 10^{-10} \text{ Hz/s};$$

$$P_{\text{mod}} \approx 10^3 s$$

$$\Rightarrow f_{GW} = 935.0Hz$$

$$\Rightarrow h_{\text{max}} \approx 4.7 \times 10^{-26}$$

# NAUTILUS

INFN Frascati Nat. Labs

2002: Tuning of the Nautilus antenna at 935 Hz for a possible detection of GW from the pulsar associated with SN1987A



E. Coccia - LIS.

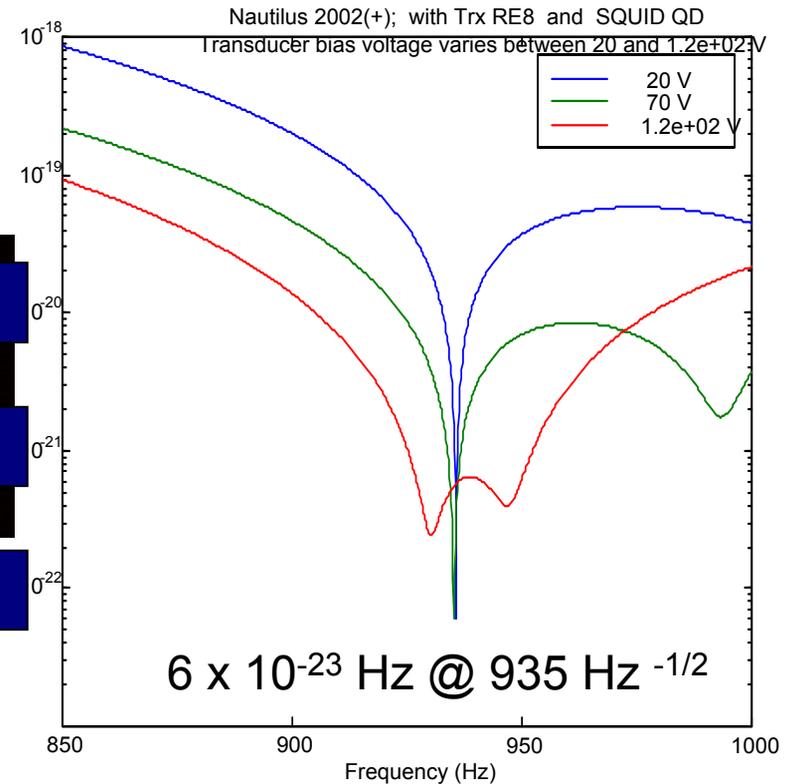
If the observed pulsar spindown is due to GW emission, we expect  $h=4.7 \times 10^{-26}$  on Earth.

NAUTILUS can reach this sensitivity (SNR=1) with 1 month integration time if its spectral sensitivity at 935 Hz is  $h=6 \times 10^{-23} \text{ Hz}^{-1/2}$

$10^{-20}$

$10^{-21}$

$10^{-22}$



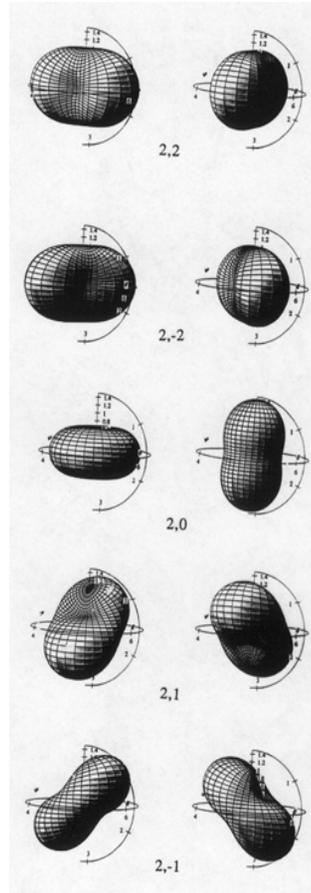
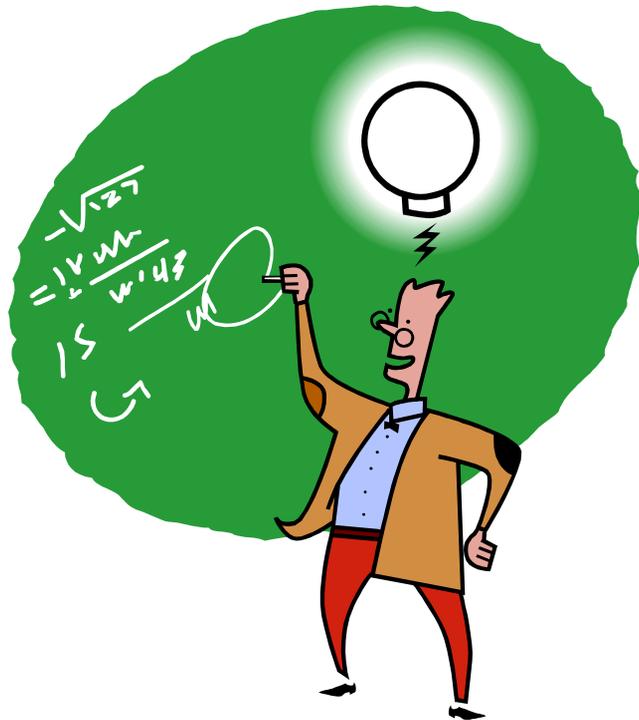
## Bars

- Operating bars are capable of monitoring the strongest galactic sources with a very low false alarm probability when at least three detectors are in simultaneous operation.
- R&D to improve the readout and enlarge the bandwidth are giving results
- Confidence of detection can be increased by the possible measurement of the signal tensor character

## Bars and Interferometers

- FRAMES to exchange data in a collaborative data analysis with GEO, LIGO1, TAMA and VIRGO
- Interferometers-Bars correlation perspectives: GW transient signals, Stochastic Background, ms pulsars.

# Exploiting the resonant-mass detector technique: the spherical detector



- 5 quadrupole modes
- Source direction
- Wave polarization

**M = 1-100 tons**

**Sensitivity:**

**$10^{-23} - 10^{-24} \text{ Hz}^{-1/2}$ ;**

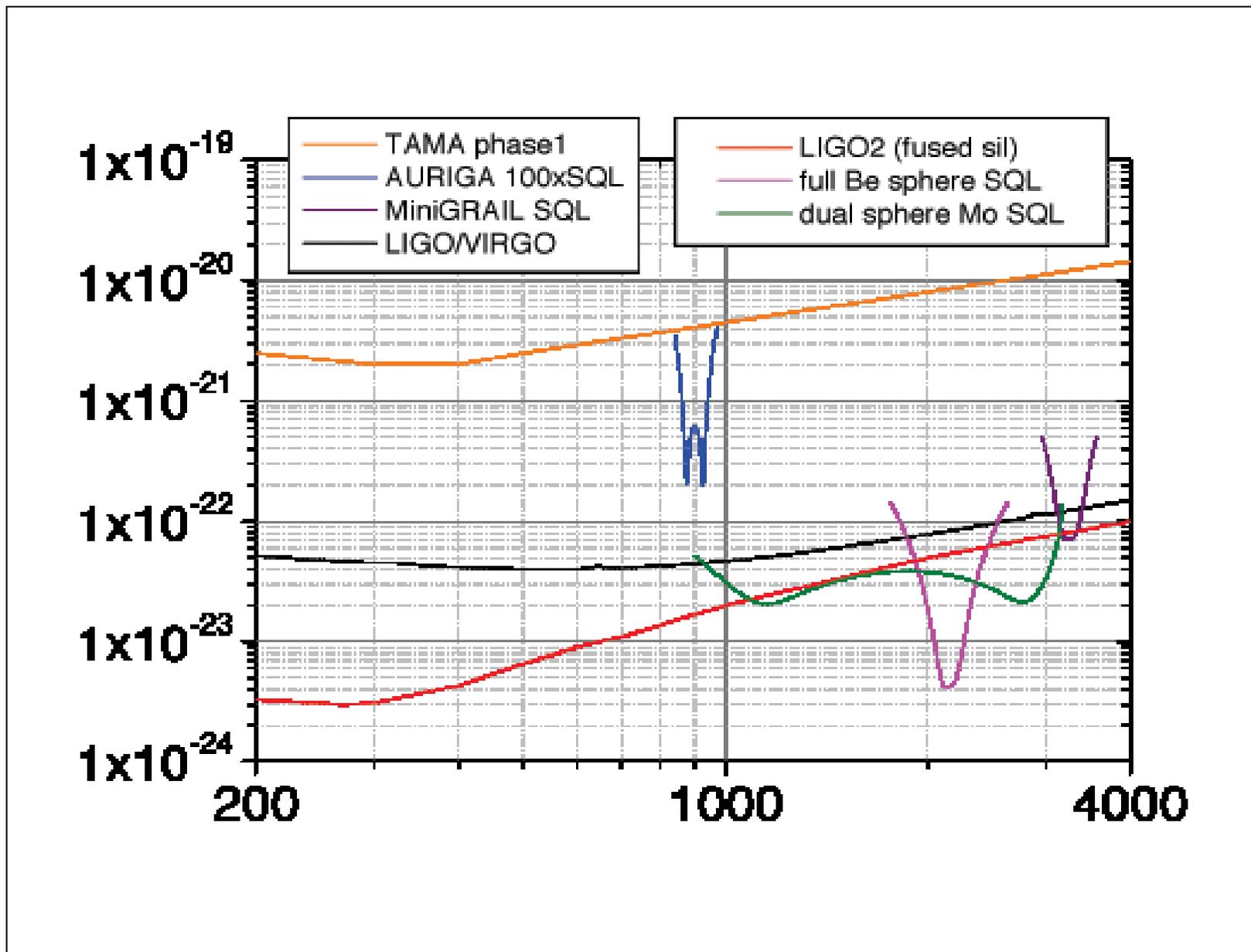
**$h \sim 10^{-21} - 10^{-22}$**

***TIGA, PRL 1993***

***Hollow sphere, PRD 1998***

***Dual sphere, PRL 2001***

# Strain sensitivity ( $\text{Hz}^{-1/2}$ )



$$\text{Sensitivity to short bursts: } h_c \sim h_{peak} \Delta f^{-1/2} \tau^{-1}$$

Detectors having the same burst sensitivity  $h_c$

<i>detector</i>	<i>strain sens.</i>	$\Delta f$	$h_c$
EXPLORER	$10^{-21} \text{ Hz}^{-1/2}$	10 Hz	$5 \cdot 10^{-19}$
Equivalent	$5 \cdot 10^{-21} \text{ Hz}^{-1/2}$	250 Hz	$5 \cdot 10^{-19}$

NAUTILUS, AURIGA	$2 \cdot 10^{-22} \text{ Hz}^{-1/2}$	1 Hz	$4 \cdot 10^{-19}$
Equivalent	$5 \cdot 10^{-21} \text{ Hz}^{-1/2}$	600 Hz	$4 \cdot 10^{-19}$

Advanced bar	$10^{-22} \text{ Hz}^{-1/2}$	50 Hz	$3 \cdot 10^{-20}$
Equivalent	$5 \cdot 10^{-22} \text{ Hz}^{-1/2}$	1000 Hz	$3 \cdot 10^{-20}$

*Here is the test to find whether  
your mission on earth is finished:*

*If you are alive, it is not.*

*Richard Bach*