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STOCHASTIC BACKGROUND OF GRAVITATIONAL WAVES

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The search for a stochastic background of gravitational waves is very interesting, both from a cosmological and astrophysical point of view. We give here a brief summary of the status of theoretical predictions, experimental results and data analysis activities.

1. GW Experiments and Early Universe Cosmology

Gravitational waves (g.w.) are a unique source of information on the very early Universe. The detection of a background of g.w. would in fact have a big impact on early Universe cosmology and on high energy physics, as relic g.w. decoupled from the primordial plasma below the Plank scale, when the Universe had a temperature $T_{dec} \simeq 10^{19}$ Gev. The reason why they are so interesting is the same reason why they are so difficult to detect: due to their small cross section, g.w. produced in the very early Universe still retain in their spectrum the information of the Universe at the time they have been decoupled.

The cosmological stochastic g.w. background is expected to be isotropic, stationary and unpolarized. It is characterized by its frequency spectrum, both in terms of energy density per unit logarithmic interval of frequency, $h_0^2\,\Omega_{gw}(f)$ (where $H_0 = h_0 \cdot 100$ Km/(s Mpc) is the Hubble constant) or in terms of the spectral density of the signal $S_h(f)$ in units of 1/Hz.

The dimensionless function of the frequency $\Omega_{gw}(f)$ and the spectral density of the signal $S_h(f)$ are related by the formula:

$$\Omega_{gw}(f) = \frac{S_h(f) f^3 4\pi^2}{3H_0^2} \tag{1}$$

Then:

$$\Omega_{gw}(f) = 1.25 \cdot 10^{45} \left(\frac{f}{1\ kHz}\right)^3 \left(\frac{100\ km\ s^{-1} Mpc^{-1}}{H_0}\right)^2 S_h(f) \tag{2}$$

Gravitational wave experiments with interferometers and with resonant mass detectors can do searches for stochastic gravitational waves. The detector's sensitivity to the stochastic background is obtained using Eq. 2, where $S_h(f)$ is the detector's strain noise spectral density (expressed in units of 1/Hz).

Using one detector the measurement of the noise spectrum only provides an upper limit for the g.w. stochastic background spectrum. This limit can be considerably improved, or even an estimation of the spectrum can be attempted by cross-correlating the output signals of two (or more) antennas.

M. Maggiore[1] has done a very interesting and complete review of both theoretical and experimental aspects of the search for stochastic GW backgrounds of cosmological origins. In his report an extended bibliography on the subject may also be found.

2. Upper limits on the Stochastic g.w. background

We recall here that upper limits for the stochastic background, in the frequency range of $\simeq$ 1 kHz, have been set using bar detectors at room temperature in Glasgow[2,3] ($\Omega_{gw} \leq 10^4$), interferometers[4] ($\Omega_{gw} \leq 3\ 10^5$) (Garching-Glasgow), the small antenna Altair[5], and the two resonant g.w. detectors Explorer and Nautilus[6].

The result obtained with Explorer and Nautilus is very interesting as it is the first time that two cryogenic resonant detectors have been used to do this measurement. As this is presently the best upper limit in the frequency range of 1 kHz, we recall here briefly the result. The detector noise spectral densities have minima at the two resonances, $\simeq$ 904 and $\simeq$ 921 Hz, and are shown in Fig.1. The analysis has been done over a bandwidth of $\simeq \pm 0.05$ Hz, from 907.1508 to 907.2574 Hz. In this bandwidth the averaged Nautilus spectrum is constant at the level $1.5 \cdot 10^{-42}$ /Hz, and the Explorer spectrum varies by a factor 2, from $3 \cdot 10^{-43}$ /Hz to $6 \cdot 10^{-43}$ /Hz

The overlapped data cover a period of 12.57 hours from February, 7^{th}, 1997, 22 h, 18 m (day=35466.9298) to the 8^{th}, 12 h, 11 m (day=35467.5916).

The result of the cross-correlation analysis of the data of the two detectors is shown in Fig.2: the lower curve shows the modulus of the cross spectrum $S_{12}(f)$, compared to the square root of the product of the two spectra $\sqrt{S_{1h}(f) S_{2h}(f)}$. In the case of total correlation the two curves should coincide .[a]

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[a] We recall that $|S_{12}| \leq \sqrt{S_{1h} S_{2h}}$, the equal sign being valid only if the spectra of the two detectors are totally correlated.

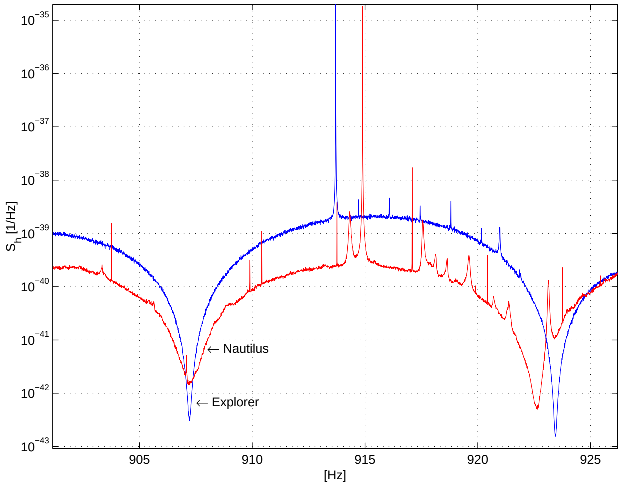

Fig. 1. Averaged noise spectra of Explorer and Nautilus used for the cross-correlation experiment. The minima correspond to the resonances of the detectors.

In case of null correlation we expect the standard deviation to be smaller than that obtained with only one detector by a factor $(t_m \, \Delta f)^{1/2} \simeq 70$, when integrating over the bandwidth $\Delta f = 0.1$ Hz. This factor represents the sensitivity improvement of the cross-correlation experiment with respect to the use of only one detector, if they were "near" and had the same sensitivity.

The result obtained by averaging over a 0.1 Hz bandwidth is:

$$S_{12}(907.20; 0.1) = (1.0 \pm 0.6) 10^{-44} \; Hz^{-1} \tag{3}$$

By expressing the above in terms of $\Omega_{gw}(f)$, that is using Eq.1 (with $S_{12}(f)$ in place of $S_h(f)$) and taking into account the factor 6, that is the worsening due to the distance between Explorer and Nautilus, we get

$$\Omega_{gw}(907.20; 0.1) \leq 6 \cdot 10 \tag{4}$$

We note that by extending the period of correlation to one year, we can obtain, with the detectors in operation now, an upper limit less than unity. This would be already very interesting for the various theoretical scenarios of the gravitational wave background.

3. Stochastic Background of Gravitational Waves by Cosmological Populations of Astrophysical Sources

A stochastic background of g.w. may also be of astrophysical origin. In fact the emission of g.w. from a large number of unresolved astrophysical sources can

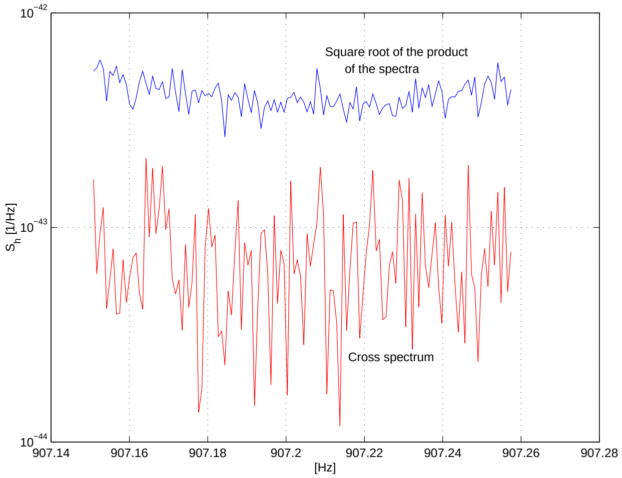

Fig. 2. The lower curve shows the result of the cross-correlation experiment (modulus of the cross spectrum). The upper curve shows the square root of the product of the spectra obtained with the two detectors. In case of total correlation of the two detectors the two curves should coincide. In the figure the factor $(t_m \ \delta f)^{1/2}$ is $\simeq 6$ at each frequency.

create a stochastic background of g.w. The work done by the Rome group (prof. V. Ferrari and collaborators) on the study of the stochastic background of gravitational radiation is focused on the contributions given by cosmological populations of astrophysical sources, i.e. sources distributed throughout the universe from the onset of the galaxy formation up to the present time. In collaboration with Raffaella Schneider and Sabino Matarrese they have elaborated a general method to evaluate the spectral properties of the stochastic background generated by astrophysical sources, whose rate of formation can be deduced from the rate of formation of the corresponding stellar progenitor. The star formation rate they use is based on observations collected by the Hubble space Telescope, Keck and other large telescopes on the UV emission of stars forming galaxies up to redshifts of about $z \sim 5$.

A global picture of the contributions they have calculated by a number of sources is shown in Fig.3 (see caption and references therein) and the corresponding picture in terms of the closure density Ω_{gw} is given in Fig.4.

For reviews on the subject refers to Refs.[7,8,9].

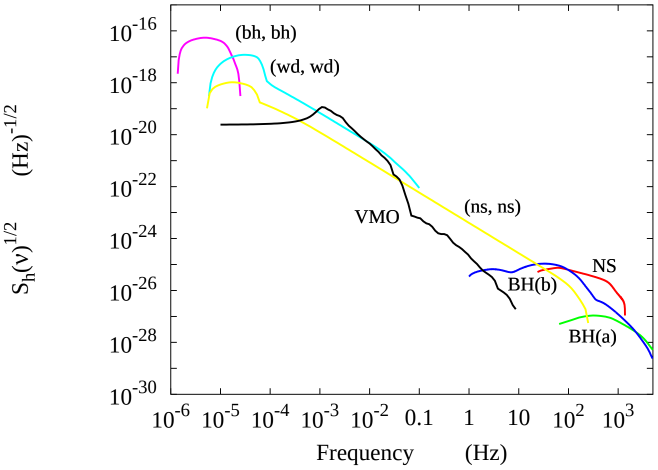

Fig. 3. The predicted strain amplitude of the stochastic backgrounds produced by the following extragalactic populations of gravitational sources: r-mode instability of rotating neutron stars (*V. Ferrari, S. Matarrese, R. Schneider, MNRAS 303, 259, 1999*), two possible signals emitted by populations of massive stars collapsing to black holes (BH(a)-*V. Ferrari, S. Matarrese, R. Schneider, MNRAS 303, 247, 1999*, BH(b)-*C.L. Freyer, ApJ 522, 413, 1999*), three different populations of coalescing binary systems (*R. Schneider, V. Ferrari, S. Matarrese, S. Portegies Zwart, submitted to MNRAS*) and pre-galactic contribution of population III stars which are assumed to be Very Massive Objects (VMO) whose individual fate is to collapse to very massive remnants (*R. Schneider, A. Ferrara, B. Ciardi, V. Ferrari, S. Matarrese, submitted to MNRAS, astro-ph/9909419*).

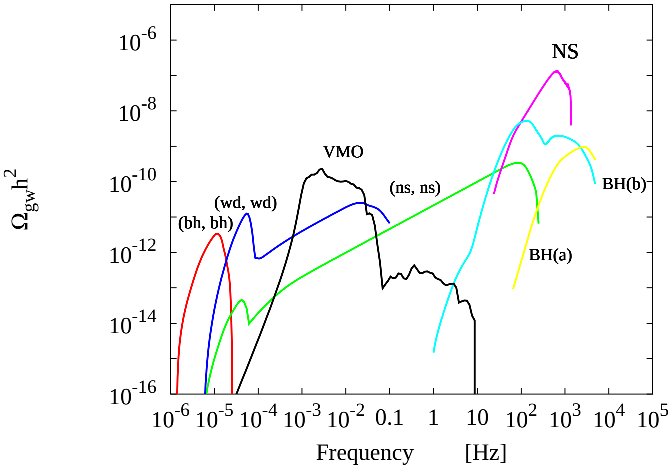

Fig. 4. The function $h^2\Omega_{gw}$ of the stochastic backgrounds produced by the extragalactic populations of gravitational sources of the previous Fig.

4. Data analysis strategies

The problem of the optimal detection strategy to look for a stochastic g.w. background has been analyzed in many papers. For reviews on the subject refers to Refs.[10,11,12,13,14] and to the bibliography therein. The detection is based on the use of two or more detectors that should be properly oriented and "near", that is such that the correlation may contain the information on the stochastic background, but not too close, such that the local noises are uncorrelated. The analysis strategy is usually based on the cross-correlation function, that only depends on the common excitation of the detectors, as due to the gw stochastic background spectrum, and is not affected by the noises acting independently on the two detectors. The sensitivity to the stochastic signal improves with the observation time and with the bandwidth, as shown in all the cited papers.

The analysis of the Explorer and Nautilus detectors has been done, as described in [15], in the frequency domain using the Fourier Transforms of a Data Base.

Anyway, the problem of the optimal signal detection strategy is still open as many problems have to be studied and solved.

For example, the resonant detectors presently in operation show non stationary noise and also future detectors are expected to have non stationary noise. Another important point to be solved is the possibility of correlated noise in the outputs of the detectors. Joe Romano, in collaboration with B. Allen, J. Creighton, L. S. Finn, E. Flanagan and others, is studying the different statistics for this search and work is in progress in trying to solve the problems, as briefly described in the next session.

4.1. *Detection algorithms and unresolved problems for stochastic gravity-wave searches*

Three different statistics for stochastic background searches are currently under investigation: (i) the standard cross-correlation statistic, (ii) the maximum-likelihood statistic, and (iii) a robust version of the cross-correlation statistic for non-gaussian detector noise. Monte Carlo simulations have shown that for simple models the robust statistic performs better than the standard cross-correlation statistic; while the maximum-likelihood statistic performs either better or worse than the standard cross-correlation statistic, depending on how well one can estimate the auto-correlated detector noise in advance. There are a number of important unresolved problems facing stochastic background searches. These include: (i) how to best estimate the auto-correlated detector noise, (ii) how to set a threshold for signal detection, (iii) how to deal with non-gaussian and/or non-stationary detector noise, (iv) how to distinguish cross-correlated environmental noise from a cross-correlated stochastic gravity-wave signal, and (v) whether or not one can claim detection (or only set an upper limit) given a two detector cross-correlation.

Fig.5 compares the performance of the standard cross-correlation and maximum-likelihood statistics for a simple model problem consisting of a white isotropic and unpolarized stochastic gravitational wave signal incident on two coincident and coaligned detectors having uncorrelated white noise. Both the signal and the noise are gaussian-stationary. The different curves show the performance of the two statistics with and without an error in the estimate of the auto-correlated detector noise equal to the stochastic background signal strength.

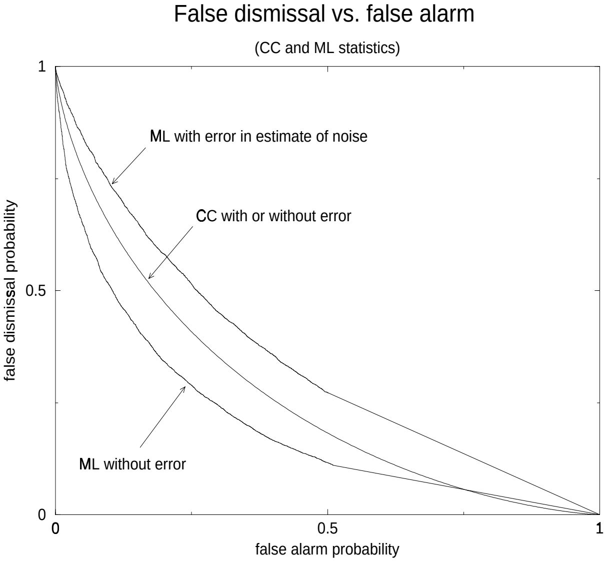

Fig. 5. Comparison of the performances of the standard cross-correlation and maximum-likelihood statistics. The different curves show the performance of the two statistics. See the text for explanations.

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