Spectral techniques for gw stochastic background detection

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Abstract. I will present and discuss some aspects of the analysis to search for a gw stochastic background (SB).

1 Introduction to the SB search

The search of a SB of gw is very interesting, as it might give information on the very early stages of the Universe and its formation. Several sources of stochastic background have been considered in the past years [17] and recently a source based on the string theory of matter has been proposed [8, 12], which predicts relict gw whose density increases with the frequency f to the third power. The sensitivity of a gw antenna to the SB is given in terms of its noise spectral density $S_h(f)$, in unit $1/Hz$. The relation between the SB density, expressed as a function of the frequency, $\Omega(f) = \frac{d\Omega}{d(\ln f)}$ and the detector spectral density $S_h(f)$ is [4]:

$$\Omega(f) = \frac{4\pi^2}{3}\frac{f^3}{H^2}S_h(f) = 1.25 \cdot 10^{45} \left(\frac{f}{1\ kHz}\right)^3 \left(\frac{100\ km\ s^{-1}Mpc^{-1}}{H}\right)^2 S_h(f) \tag{1}$$

where H is the Hubble constant and Ω the ratio of the gw energy density to the critical density needed for a closed universe. $S_h(f)$ is the quantity we measure. Using one detector it is only possible to put upper limits [5], as we cannot model in a trustful way the detector noise [1]. The measure is then based on the use of two (or more) detectors. To discriminate between different models we have to measure in different frequency ranges.

Let us consider two " near" and "aligned" antennae [2] with spectral densities $S_{1h}(f)$ and $S_{2h}(f)$. Their crosscorrelation function $R_{12}(t)$ only depends on the

[1] In a pulsars search the spectral characteristics of the noise and of the signal are different. The noise is roughly white while the signal is expected to be a delta in the frequency domain. Hence it is possible to smooth the spectra and to subtract the noise to estimate the amplitude of a spectral line. This is not possible in all those cases in which the spectral characteristics of the noise and of the signal are expected to be the same, as for the SB.

[2] "near" here means reasonably close together, such that the correlation may contain the information on the SB, but not too close, such that the local noises are uncorrelated.

"aligned" means that their relative orientation is not too far from the optimal value common excitation of the detectors, as due to the gw stochastic background spectrum $S_{gw}(f)$, and is not affected by the noises acting independently on the two detectors. The Fourier transform of $R_{12}(t)$ is the cross spectrum. This is a complex quantity $S_{12}(f) = C_{12}(f) - jQ_{12}(f)$. The estimate, obtained over a finite observation time t_m, has a statistical error. It can be shown [7] that the standard deviation of each sample is

$$\delta C_{12}(f) \leq \frac{\sqrt{S_{1h}(f) \cdot S_{2h}(f)}}{\sqrt{t_m \delta f}} \quad ; \quad \delta Q_{12}(f) \leq \frac{\sqrt{S_{1h}(f) \cdot S_{2h}(f)}}{\sqrt{t_m \delta f}} \tag{2}$$

where t_m is the measuring time and δf the frequency step in the spectra. According to eq.2 there is no improvement by using two detectors instead of one when the frequency step δf of the spectrum is equal to $1/t_m$. In this case the statistical improvement factor $\sqrt{\delta f t_m}$ reduces to unity and the sensitivity, for two identical detectors, coincides with that of a single detector. The cross spectrum is always less or equal to the square root of the product of the two spectra $S_{12} <= \sqrt{S_{1h} S_{2h}}$, the equal sign being valid if the two detectors are totally correlated. If the background spectrum is expected to be approximately constant the estimation of its intensity over different spectral intervals Δf, larger than the spectral step δf, is very simple. The intervals should be properly chosen, for example such that over Δf the two spectral densities are quite flat. The uncertainty of this estimate is obtained from eq.2:

$$\delta S_{gw}(f) \leq \frac{\sqrt{\frac{1}{\Delta f} \int_{\Delta f} S_{1h}(f) \cdot S_{2h}(f) df}}{\sqrt{t_m \Delta f}} \tag{3}$$

We expect at first just to put upper limits. In this case the estimated spectrum $S_{gw}(f)$ will be zero with a standard error given by eq.3, which thus gives the overall sensitivity of the experiment.

1.1 Effect of the location and relative orientation of the detectors

The previous formulae apply to "near" detectors (at a distance smaller than $d_{max} = \lambda_{gw}/(2\pi)$) but not "too near", such to be not excited by the same local noises. They should be parallel, that is sensitive to the same polarization plane of the wave, at each time. If the distance is $d > d_{max}$ there is a decrease in the efficiency of the detection [13, 18]. The distance d_{max} is a function of the frequency: it is roughly 50 Km at 1 kHz, where we have operating resonant bars, and 250 km at 200 Hz, where we expect the interferometers to have the best sensitivity. We have $\Omega(f) = \left(\frac{\Omega_{opt}(f)}{\gamma(f)}\right)$. Here $\gamma(f)$ is the overlapping reduction function, as explained by Flanagan [11], and gives the degradation with respect to the optimal situation, in which we can measure the smallest value of $\Omega(f)$,

$\Omega_{opt}(f)$. The quantity $\gamma(f)$ is a function of the frequency of the wave, the location of the detectors and their relative orientation. In case of two detectors the search can be performed by computing their cross correlated spectrum:

$$S_{12}(\Delta f) = \frac{1}{\Delta f}\int\limits_{\Delta f} H_1(f) \cdot H_2^*(f)df \tag{4}$$

where $H_i(f)$ $(i = 1, 2)$ are the Fourier Transforms of the data of the two detectors, and Δf is the bandwidth. This is the optimal choice if the bandwidth is so small that we can neglect the frequency variations of $\gamma(f)$, of the detector spectral densities and of the signal. When we consider larger bandwidths the optimal choice is to filter the data using a weighting function, $Q(f) = \frac{\gamma(f)\Omega_{gw}(f)}{f^3 S_{1h}(f) S_{2h}(f)}$. Thus different filters are needed, for different models of the signal. The optimal estimation [11, 18] is then

$$\tilde{S}_{12}(\Delta f) = \frac{1}{\Delta f}\int\limits_{\Delta f} Q(f) \cdot H_1(f) \cdot H_2^*(f)df \tag{5}$$

It is shown in [11] how the effect of correlated noise affects the SB detection. In case of "very near" detectors the correlation due to the local noises and the correlation due the SB are not distinguishable. In principle they could be distinguishable if we know the characteristics of the local noises, but this is not the case. In addition it will always be that this effect is order of magnitude greater than the SB, thus blinding the search. This is what we do when searching for bursts: we apply vetoes on the data, but we know that in this way we are vetoing only a small percentage of the events due to noise and, for better vetoing, we need a coincidence experiment with other detectors.

2 The frequency domain data base

Here we propose an analysis procedure for the SB based on a frequency domain data base (**FD DB**) and on the software procedures operating on it. The data base is the same we use for the pulsars search. In particular, for the pulsar search, the software procedures are designed in order to reduce the computing time problems that arise in the case of a "blind" search (source position and/ or frequency unknown) [6, 14]. The **FD DB** is composed by the first half part of the FFT of 2N data. Each basic FFT, $b-fft$, in the data base is completely characterized. This is important, because it allows to take into account the data taking interruptions and the non-stationarity in the noise of the detector, which represent difficult real problems. The information to register in the header of the $b-fft$ should be such to help during the analysis and can regard both enviromental information (for example flags of the experimenter concerning the apparatus, i.e. normal operation, working around the system etc.) or the data quality (i.e. sensitivities to bursts at that time in different frequency bandwidths, integrals over sub-bandwidths etc.) It is thus possible to choose thresholds for vetoing the data or criteria to weight them.

For the choice of the length of the basic element of the data base one can use different criteria, but it is important to have a time period during which the system is stationary.

To do a cross-correlation experiment the length of the b–fft is not important, as we need to integrate over the overlapping bandwidth of the experiments we want to correlate. Hence we can use the elements optimized for the pulsar search, where a reasonable choice is to consider the maximum expected Doppler shift. For interferometers, planned to reach good sensitivities over a large bandwidth, the FD DB can be organized with FFTs of different lengths for searches in the different frequency bands [6].

2.1 Spectral data base applied to the stochastic search

We show here how to use the FD DB to perform an efficient cross-correlation analysis with two experiments. In particular, we stress again it is important the philosofy of the FD DB (data organized in short time pieces, completely characterized and thus easily handled or vetoed) as the quality of the data is a crucial point in the analysis.

The only agreement that should be reached among the experiments that will do the analysis are the lengths of the $b-ffts$ and the rule to choose the inizial time of the first spectrum of each new run. As a consequence (same length for the time duration of each basic FFT) all the data bases will have the basic elements with the same time covering.

Suppose we have M $b-fft$ in the FD DB, each one of duration t_0 and be Δf the overlapping bandwidth.

$$S_{12_i}(f) = (b-fft)_{1i} \cdot (b-fft)^*_{2i} \tag{6}$$

as a function of the frequency

$$R_{12_i}(t) = FFT^{-1}[S_{12_i}] \tag{7}$$

as a function of time.

Here $i = 1, M$ is the number of the considered FFT and the product in eq.6 is done after bandpass filtering and extraction of the bandwidth Δf. The time domain sequence is thus subsampled a factor that depends on the ratio between Δf and the total band of the detector. This saves computational time and lead to a result that, even if regarded in the time domain, contains only the information on that particular chosen bandwidth. In case it is necessary, eq.6 can be weighted using $Q(f)$, as explained in section 1.1.

Then

$$R_{12}(t) = 1/M \sum_{i=1}^{M} [R_{12_i}(t)] \tag{8}$$

is the final result in the time domain, and

$$S_{12}(f) = FFT^{-1}[R_{12}(t)] \tag{9}$$

is the result in the frequency domain.

Obviously, $R_{12}(0) = \int_{\Delta f} S_{12}(f)df$

The M spectra should not be consecutive in time, and in practice they will not, because of the vetoes or data taking interruption. Eq.3 shows that it may be convenient to reduce the observation time t_m and to consider only the times when the two spectra S_{1h} and S_{2h} are very good. The choice depends on the relative improvement or loss in the two quantities, spectra and times of observation.

3 Correlation of the Explorer and Nautilus detectors

The detectors we have used for the cross correlation experiment are Explorer at Cern and Nautilus in Frascati. They are parallel, but due to the distance the cross correlation should be corrected by a factor 6 [13, 18]. We tuned the two detectors to have the same resonance frequency at one of the modes, $\nu_- =$ 907.20 Hz. The overlapping bandwidth was ± 0.05 Hz. We run the experiment for few days and we obtained $t_m = 12.57$ hours of "good" data, on which the cross correlation was done. The overlapped data cover a period of 12.57 hours from 1997 february, 7^{th},22 h to the 8^{th},12 h. The details of the analysis are described in [3].

The result of the experiment [10] is: $S_{12} = 6 \cdot (2.1 \pm 0.9)10^{-44}$ 1/Hz

We believe that the small excess, 2 when expecting 1, is due to a systematic error in the applied mathematical algorithm.

Reasoning in terms of $\Omega(f)$ we get $\Omega(907.2)$ smaller than $6 \cdot 20$. With the present sensivities and bandwidths, running the experiment for 1 year, we could reach $\Omega(f)$ less than the unity.

We note that [4] for resonant bars or spheres, the spectral density at the resonances increases as $\frac{T}{MQ}$ where T the temperature, M the mass, Q the merit factor. Then increasing the mass M by a factor K the sensitivity in $\Omega(f)$ improves by a factor K. The sensitivity decreases with the detectors distance, hence it could be convenient, also in terms of expenses and feasibility, to consider cross-correlation experiments between an interferometer and a near bar or a big sphere [15, 4].

The sensitivity for the Nautilus detector has been measured to be 5 10^{-43}/Hz [10], and its target value for the year 2000 is 7 10^{-45}/Hz, over a bandwidth of $(5-6)$ Hz at the two resonances [4]. Two near antennae like Nautilus 2000, in operation for one year, can reach the limit $\Omega(f) \simeq 5 \cdot 10^{-4}$. With one year of operation and $\Delta f \simeq 100$ Hz we get $\Omega(f) \simeq 1 \cdot 10^{-4}$. An additional increase in the mass of the resonant detector by a factor of 100 and further cooling of the detector will improve the sensitivity such as to make it possible to measure values as small as $\Omega(f) \simeq (4-5) \cdot 10^{-7}$.

With $S_{920Hz} = 7\ 10^{-45}$/Hz for Nautilus and Auriga ($d \simeq 400$ Km) we can reach $\Omega(920\ \text{Hz}) \simeq 5\ 10^{-4} \cdot 10$ in a bandwidth of 5 Hz (t_m=1 year).

Using a simple characterization for Virgo and Geo600, as in [15], with Virgo (at the knee frequency): $S_{200\ Hz} = 1\ 10^{-46}$/Hz, Geo600: $S_{200\ Hz} = 3\ 10^{-45}$/Hz we obtain:

Nautilus-Virgo ($d \simeq 260$ Km) $\Omega(920\ \text{Hz}) \simeq 2\ 10^{-4} \cdot 10$, in a bandwitdh of 5 Hz.
Auriga-Virgo ($d \simeq 220$ Km) $\Omega(920\ \text{Hz}) \simeq 2\ 10^{-4} \cdot 5$, in a bandwidth of 5 Hz.
Nautilus-Geo600 $\Omega(920\ \text{Hz}) \simeq 1\ 10^{-3} \cdot 20$, in a bandwidth of 5 Hz.
A similar result can be obtained for Auriga-Geo600, or even better due to the smaller distance.

These all are very interesting limits to the SB.

References

[1] B. Allen, R. Brustein, 1997 Phys Rev D,55,3260-3264

[2] B. Allen, 1997, Proc. of the 2 Amaldi conf, CERN.

[3] P. Astone, 1997, Proc. of the 2 Amaldi conf, CERN.

[4] P. Astone, G.V. Pallottino, G. Pizzella, 1997 CQG, 14, 2019.

[5] P. Astone et al., 1996, Physics Letters B, 385, 421

[6] P. Astone, S. Frasca, M.A. Papa, 1997 Aspen Winter conference on g.w. detection

[7] J.S. Bendat and A.G. Piersol, 1966 eds. John Wiley & Sons, New York, *Measurement and analysis of random data*

[8] R.Brustein,M.Gasperini,M.Giovannini,G.Veneziano, 1995, Phys Let B,361,45

[9] K. Compton, PCD thesis,march 1996

[10] V. Fafone et al.,1997, in Proc. of MGR8, Jerusalem.

[11] E. Flanagan, 1993, Phys. Rev D 48, 6, 2389

[12] M. Gasperini, 1997, Proc. of the 2 Amaldi conf, CERN.

[13] P.F.Michelson, 1987, Mon. Not. R astr. Soc. 227, 933

[14] M. A. Papa, 1997, these Proceedings

[15] B. F. Schutz, 1996 Proc. of the VIRGO Int. Conf. on g.w., ed. I. Ciufolini, F. Fidecaro. World scientific.

[16] B. F. Schutz, 1996, AEI-003, Max Plank Institut.

[17] K.S.Thorne,1987 *300 years of Gravitation* Hawking and Israel,Univ. Press

[18] S. Vitale, M. Cerdonio, E. Coccia, A. Ortolan, 1997, CQG 14, 1487