Astron. Astrophys. 351, 811–814 (1999)

ASTRONOMY AND ASTROPHYSICS

Crosscorrelation measurement of stochastic gravitational waves with two resonant gravitational wave detectors

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Received 2 October 1998 / Accepted 21 September 1999

Abstract. We report on the crosscorrelation analysis of the data recorded by the gravitational wave (g.w.) resonant detectors Explorer and Nautilus, performed to obtain information on the g.w. stochastic background. We found that the quantity $\Omega_{gw}(f)$, that measures the closure of the Universe, is $\Omega_{gw}(f) \leq 6 \cdot 10$, at 907.20 ± 0.05 Hz, where the factor 10 is obtained by the crosscorrelation analysis and the factor 6 takes into account the distance ($\simeq 600\,$km) between Explorer and Nautilus. This is the first experiment where the data of two g.w. cryogenic resonant detectors are crosscorrelated.

Key words: gravitation – methods: data analysis – cosmology: early Universe

1. Introduction

The measurement of a stochastic background of g.w. is very interesting as it might give information on the very early stages of the Universe. Various theories (Thorne 1987, Brustein et al. 1995) describe different scenarios for the generation of a stochastic background of g.w., where the intensities of the predicted phenomena are given in terms of Ω_{gw}, the ratio of the g.w. energy density to the critical density needed for a closed Universe.

To discriminate between the various models we need measurements over different frequency ranges, as provided by the different families of detectors that are now in operation or will start operating in the next future (Schutz 1997).

The Rome group at present has two detectors, Explorer (Astone et al. 1993) and Nautilus (Astone et al. 1997a), operating around 1 kHz. Their data have been used, separately, to put limits on Ω_{gw} in this frequency range (Astone et al. 1996). The limit was $\Omega_{gw} \leq 100$.

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The problem when using one detector only is that only an upper limit can be obtained. Instead, by crosscorrelating the data of two or more experiments a measurement of the stochastic background can be obtained, or an upper limit if the measurement turns out to be zero within the statistical error.

In the following we shall report on the result obtained when crosscorrelating the data obtained with Explorer and Nautilus.

We recall here that upper limits for the stochastic background, in the same frequency range, have been previously set also using bar detectors at room temperature in Glasgow, $\Omega_{gw} \leq 10^4$ (Hough et al. 1975, Zimmerman & Hellings 1980), interferometers $\Omega_{gw} \leq 3\ 10^5$ (Garching-Glasgow) (Compton et al. 1994), and recently the antenna Altair, operating at 1752 Hz, $\Omega_{gw} \leq 10^3$ (Astone et al. 1999).

2. Stochastic g.w. search

The sensitivity of a g.w. antenna is usually given in terms of its strain noise spectral density $S_h(f)$ or spectral amplitude $\tilde{h}(f) = \sqrt{S_h(f)}$ (unit of $1/\sqrt{Hz}$). Using $\tilde{h}(f)$ it is easy to infer the detector sensitivity for various classes of signals, as bursts, periodic signals and stochastic g.w. (Astone et al. 1997b).

As regards stochastic g.w., the dimensionless function of the frequency (Brustein et al. 1995) $\Omega_{gw}(f)$

$$\Omega_{gw}(f) = \frac{d\Omega_{gw}}{d(lnf)} \tag{1}$$

is related to the detector sensitivity, $S_h(f)$ by the formula (Astone et al. 1996):

$$\Omega_{gw}(f) = \frac{S_h(f) f^3 4\pi^2}{3H_0^2} \tag{2}$$

where H_0 is the Hubble constant. Then we have

$$\Omega_{gw}(f) = 1.25 \cdot 10^{45} \left(\frac{f}{1\ kHz}\right)^3 \left(\frac{100\ km\ s^{-1} Mpc^{-1}}{H_0}\right)^2 S_h(f) \tag{3}$$

Using one detector the measurement of its noise spectrum only provides an upper limit for the g.w. stochastic background spectrum. This limit can be considerably improved, or even an estimation of the spectrum can be attempted by crosscorrelating the output signals of two (or more) antennas (Michelson 1987, Astone et al. 1997b). Let us consider two " near" and "aligned" antennas[1] with spectral densities $S_{1h}(f)$ and $S_{2h}(f)$. The crosscorrelation function $R_{12}(\tau)$ only depends on the common excitation of the detectors, as due to the g.w. stochastic background spectrum acting on both of them, and is not affected by the noises acting independently on the two detectors.

As the analysis of the data is usually performed in the frequency domain, we consider the cross spectrum that is the Fourier transform of $R_{12}(\tau)$, where all the signals are properly normalized to represent the input strain of the detectors. The cross spectrum is a complex quantity $S_{12}(f) = C_{12}(f) - jQ_{12}(f)$, which is identically zero, for each frequency, in the case of no correlation between the two detectors, therefore ideally providing unlimited sensitivity for any common excitation. The actual sensitivity, however, is limited because the estimate obtained over a finite observation time has a statistical error. It can be shown (Bendat & Piersol 1966) that the standard deviation of each sample of the spectrum is

$$\delta C_{12}(f) \leq \frac{\sqrt{S_{1h}(f) \cdot S_{2h}(f)}}{\sqrt{t_m \, \delta f}} \tag{4}$$

$$\delta Q_{12}(f) \leq \frac{\sqrt{S_{1h}(f) \cdot S_{2h}(f)}}{\sqrt{t_m \, \delta f}} \tag{5}$$

where t_m is the total measuring time and δf is the frequency step in the spectrum.

If the g.w. background spectrum is expected (Brustein et al. 1995) to be approximately constant over a few hertz or a few tens of hertz, the statistical error can be reduced by estimating its intensity over spectral intervals Δf larger than the spectral step δf. In this case the uncertainty of the estimate, obtained from Eq. (5),

$$\delta S_{12}(f; \Delta f) \leq \frac{\sqrt{\frac{1}{\Delta f} \int\limits_{\Delta f} S_{1h}(f) S_{2h}(f) df}}{\sqrt{t_m \, \Delta f}} \tag{6}$$

represents the overall sensitivity of the experiment.

The above expression shows that the spectral interval Δf has to be carefully chosen: as large as possible to increase the statistics, but small enough to avoid including spectral samples of larger value, outside the flat regions of the two noise spectra.

The analysis is therefore performed by computing the cross spectrum of the data of the two detectors, averaged over Δf:

$$S_{12}(f; \Delta f) = \frac{1}{\Delta f} \int_{f-\frac{\Delta f}{2}}^{f+\frac{\Delta f}{2}} H_{1h}(f') H^*_{2h}(f') df' \tag{7}$$

[1] "near" here means reasonably close together, such that the correlation may contain the information on the stochastic background, but not too close, such that the local noises are uncorrelated.
"aligned" means that the axes of the two detectors are parallel one to each other, so that the two detectors have equal sensitivity for g.w. with any direction and polarization.

where $H_{ih}(f)$ with $(i = 1, 2)$ are the Fourier transforms of the data of each detector, properly normalized, and * indicates the complex coniugate.

The optimal sensitivity $\Omega^{opt}_{gw}(f)$ is obtained (Michelson 1987, Flanagan 1993, Vitale et al. 1997) when the detectors, besides being aligned, are at a distance $d \leq d_{max}$, where d_{max} is a function of the frequency of the wave, $d_{max} = \lambda_{gw}/2\pi$, roughly 50 km at 1 kHz.

If the distance is $d > d_{max}$ there is a decrease in the efficiency of the detection, as the crosscorrelation falls down due to the phase shift between the waves acting on the two detectors. We have, in general:

$$\Omega_{gw}(f) = \frac{\Omega^{opt}_{gw}(f)}{\gamma(f)} \tag{8}$$

where $\gamma(f)$ is the overlapping reduction function, discussed by Flanagan (1993). The quantity $\gamma(f)$ is a function of the frequency of the wave, of the location of the detectors and of their relative orientation, which is equal to the unity for "near" and "aligned" detectors.

We note, for sake of completeness, that Eq. (7) is the optimal detection strategy only if the integration bandwidth Δf is so small that we can neglect the frequency variations of $\gamma(f)$, of the detector noise spectrum and of the signal $\Omega_{gw}(f)$. A larger bandwidth requires (Michelson 1987, Flanagan 1993, Vitale et al. 1997) to apply to the data a weight function $Q(f)$ which takes into account all the frequency dependences. This is not the case in the present analysis.

2.1. Use of the frequency domain data base

The analysis of the data is done in practice using a frequency domain data base, where each basic FFT is completely characterized by recording information on the status of the experimental apparatus and on the quality of the data. This allows to take into account the data taking interruptions and the non-stationarity of the noise.

For a crosscorrelation experiment the length of the basic FFT is not crucial, as we need to integrate over the overlapping bandwidths of the experiments we want to correlate. We have used the length optimized for the pulsar search, that is $t_0 = 0.6617$ hours.

The analysis procedure is based on the use of Eq. (7) and it is described in (Astone 1997). We note here that in the organization of the data base for crosscorrelation analysis common criteria should be used for the different detectors (the lengths of the basic FFTs, the rule for choosing the inizial time of the first spectrum of each new run, and possibly the sampling times)

3. Correlation of the Explorer and Nautilus detectors

The detectors used for the crosscorrelation experiment are Explorer (Astone et al. 1993) at CERN and Nautilus (Astone et al. 1997a) in Frascati: two aluminum cylinders with mass of 2200 kg, equipped with a capacitive resonant transducer. These detectors are parallel but, due to the distance ($\simeq 600$ km), the crosscorrelation should be corrected by a factor that is 6 (Michelson 1987, Vitale et al. 1997).

Their noise spectral densities have minima at the two resonances, $\simeq 907$ and $\simeq 923$ Hz, as shown in Fig. 1.

We recall (Astone et al. 1997b) that the spectral density of a gravitational wave background which can be measured with signal to noise ratio equal to unity with a resonant detector is related to the detector characteristics by the formula

$$S_h(f) = \frac{\pi}{2}\frac{kT_e}{MQv^2}\frac{1}{f_0}U(f) \quad (9)$$

Here Q is the merit factor of the detector, M the mass, T_e the temperature, v the sound velocity in the material, f_0 the resonance frequency, and $U(f) \geq 1$ a frequency dependent term that reduces to unity for $f = f_0$.

The bandwidth is usually smaller than 1 Hz, thus we can neglect the frequency dependences concerning both the g.w. background and the function $\gamma(f)$, and use in the analysis Eq. (7), only considering the detector spectra frequency dependence. For this experiment we have tuned[2] the two detectors in order to have the same resonance frequency at one of the two resonant modes, $f_- = 907.20$ Hz.

We choose an overlapping bandwidth of $\simeq \pm 0.05$ Hz, from 907.1508 to 907.2574 Hz: in this bandwidth the averaged Nautilus spectrum is constant at the level $1.5 \cdot 10^{-42}$ /Hz, and the Explorer spectrum varies a factor 2, from $3 \cdot 10^{-43}$ to $6 \cdot 10^{-43}$ /Hz

The Explorer data are sampled in a bandwidth of the order of 27.5 Hz from 900 to 927.5 Hz, with a sampling time of 18.18 ms; the Nautilus data in a bandwidth from 900 to 955.0, with a sampling time of 9.09 ms.

We performed the experiment, by tuning the detectors, only for a relatively short period of time, obtaining $t_m = 12.57$ hours of "good" data, on which the crosscorrelation was applied. The overlapped data cover a period of 12.57 hours from February, 7^{th}, 1997, 22 h, 18 m (day=35466.9298) to 8^{th}, 12 h, 11 m (day=35467.5916). The noise spectra of Explorer and Nautilus during this period are shown in Fig. 1.

We considered "good" the Explorer basic FFT's where the noise spectral density, averaged in the bandwidth of 0.1 Hz around the resonance frequency, was smaller than $0.05 \cdot 10^{-40}$ /Hz, thus vetoing 20% of the spectra. From these data alone, using Eq. (2), we obtained the limit

$$\Omega_{gw}(907.20; 0.1) \leq 300 \quad (10)$$

As regards Nautilus, we considered "good" the FFT's with noise below $0.1 \cdot 10^{-40}$ /Hz, thus vetoing less than the 20% of the spectra. From the Nautilus data alone, we obtained

$$\Omega_{gw}(907.20; 0.1) \leq 2000 \quad (11)$$

We remark here that at the time we performed the experiment the Nautilus sensitivity was worse than usual (Astone et al. 1997a), roughly by a factor 10 (as this detector usually works at the same sensitivity of Explorer).

[2] This is done by changing the transducer voltage.

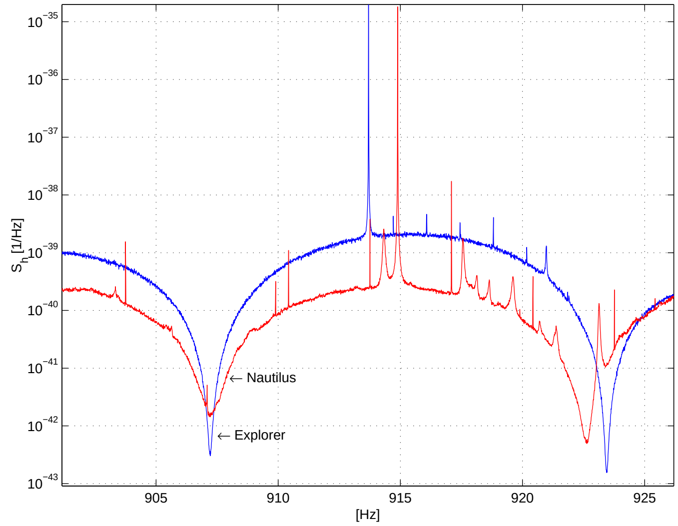

Fig. 1. Averaged noise spectra of Explorer and Nautilus used for the cross-correlation experiment. The minima correspond to the resonances of the detectors.

The above thresholds has been chosen to optimize the contribution of S_{1h}, S_{2h}, t_m in Eq. (6). The Explorer detector was the one which limited the overall bandwidth of the experiment, as clearly shown in Fig. 1.

The result of the crosscorrelation analysis of the data of the two detectors is shown in Fig. 2: the lower curve shows the modulus of the cross spectrum $S_{12}(f)$, compared to the square root of the product of the two spectra $\sqrt{S_{1h}(f)S_{2h}(f)}$[3]. In the case of total correlation the two curves should coincide. In case of null correlation we expect the standard deviation be smaller than that obtained with only one detector by a factor $(t_m\ \Delta f)^{1/2} \simeq 70$, when integrating over the overlapping bandwidth $\Delta f = 0.1$ Hz. This factor represents the sensitivity improvement of the crosscorrelation experiment with respect to the use of only one detector, if they were "near" and had the same sensitivity.

The numerical results obtained by averaging over a 0.1 Hz bandwidth are:

$$Re[S_{12}(907.20; 0.1)] = (7 \pm 6)10^{-45} \text{ 1/Hz} \quad (12)$$

$$Im[S_{12}(907.20; 0.1)] = (7 \pm 5)10^{-45} \text{ 1/Hz} \quad (13)$$

$$|S_{12}|(907.20; 0.1) = (1.0 \pm 0.6)10^{-44} \text{ 1/Hz} \quad (14)$$

By expressing the above in terms of $\Omega_{gw}(f)$, that is using Eq. (2) (with $S_{12}(f)$ in place of $S_h(f)$) and taking into account the factor 6 due to the distance, we get

$$\Omega_{gw}(907.20; 0.1) \leq 6 \cdot 10 \quad (15)$$

Comparing this result with those given by Eqs. (10) and (11), we notice that the gain obtained by crosscorrelating the two data sets is a factor $\simeq 5$ for Explorer and a factor $\simeq 30$ for Nautilus.

[3] We recall that $|S_{12}| \leq \sqrt{S_{1h}S_{2h}}$, the equal sign being valid only if the spectra of the two detectors are totally correlated.

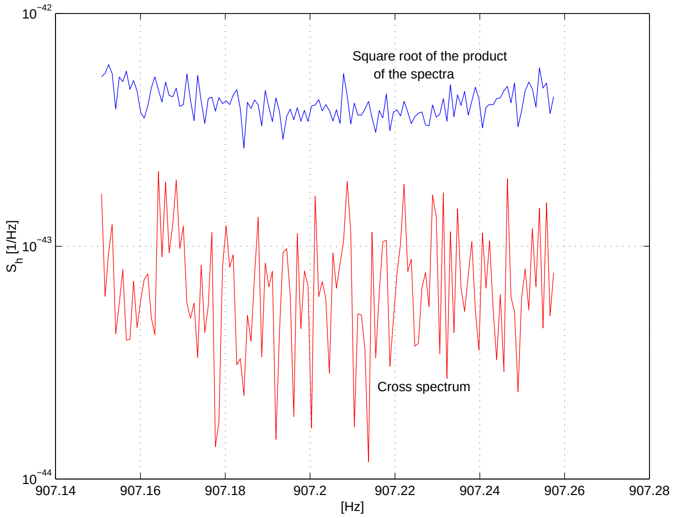

Fig. 2. The lower curve shows the result of the cross-correlation experiment (modulus of the cross spectrum). The upper curve shows the square root of the product of the spectra obtained with the two detectors. In case of total correlation of the two detectors the two curves should coincide. In the figure the factor $(t_m\ \delta f)^{1/2}$ is $\simeq 6$ at each frequency.

4. Conclusion

The limit $\Omega_{gw}(907.20; 0.1) \leq 60$ improves by a factor of five our previosuly published result obtained with just one detector.

But let us remark again that the basic difference between the two cases, of one detector alone and of the correlation between two detectors, is not just that of improving the upper limit. In the first case, one detector alone, only an upper limit can be estimated, because our knowledge of the detector background would be never sufficiently good to subtract it from the signal. In the second case instead, crosscorrelation of the data of two detectors, a real measurement of the background is possible, which reduces to an upper limit estimation if the signal is null within the statistical error.

This is, in fact, the new result presented in this paper. Null value of the gravitational wave background, giving the above upper limit.

By extending the period of correlation to one year, we can obtain, with the detectors in operation now, an upper limit of less than unity. This would be already very interesting for the various theoretical scenarios of the gravitational wave background.

We finally remark the very interesting perspective of performing correlations between a large interferometric detector, one of those now being built in Europe and in the USA, and an advanced resonant detector located a few tens of km apart.

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