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All-sky search of EXPLORER data: search for coincidences

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Abstract

We present the result of a search for coincidences among the triggers found in the analysis of three 2-day-long stretches of data from the EXPLORER bar detector. The data were searched for nearly periodic gravitational waves from rapidly rotating neutron stars in our Galaxy. In this paper we propose a numerical procedure to search for coincidences and we present a theoretical formula for the expected number of coincidences. Our numerical analysis revealed *no double* and consequently *no triple* coincidences among the triggers from the three data sets. The results are in agreement with the expected number of coincidences that we estimate by our theoretical formula.

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1. Introduction

We have performed an analysis of three data sets, each 2 days long, from the EXPLORER resonant bar detector [1]. We have searched for continuous gravitational-wave signals from spinning neutron stars. Our data analysis technique was based on the maximum likelihood detection method. Under the assumption of normality of the noise maximum likelihood detection is equivalent to matched filtering [5]. In our search we have set a low threshold corresponding to a signal-to-noise ratio of 6.7. We have obtained a number of candidate signals with the signal-to-noise ratio exceeding the threshold value. We call these signals triggers. We have subjected the triggers to a verification procedure consisting of searching the events in a different data set and for a longer observation time. The search and the subsequent verification procedure did not produce any viable candidates for gravitational-wave signals. The main outcome of our analysis [6, 7] is an upper limit of 1×10^{-22} for the dimensionless amplitude of a continuous gravitational-wave signal. The upper limit is for any source location in the sky, any polarization of the wave and for signals of frequency from 921.00 Hz to 921.76 Hz and with spin down from -2.36×10^{-8} Hz s^{-1} to $+2.36 \times 10^{-8}$ Hz s^{-1}.

In this paper we perform yet another verification of the triggers. We search for coincidences among the triggers.

This paper is organized as follows. In section 2 we discuss a formula for the coincidences and a method to extract the coincidences among the searches in a multidimensional parameter space. In section 3 we present results of search for coincidences among events in the three data sets from the EXPLORER detector that we have analysed and we discuss the results.

The data analysis was performed by a team consisting of Pia Astone, Kaz Borkowski, Piotr Jaranowski, Andrzej Królak and Maciej Pietka and was carried out on the basis of a memorandum of understanding between the ROG collaboration and Institute of Mathematics of Polish Academy of Sciences. More details about the search can be found in [8].

2. A general formula for the number of accidental coincidences

In this section we calculate the expected number of coincident triggers among the searches that occur by pure chance, i.e., when the data are only noise and there are no signals.

The parameters of the signal that we search for can be divided into two classes: *extrinsic parameters* and *intrinsic parameters* [5]. For extrinsic parameters we can find the maximum likelihood estimators in a close analytic form and we can eliminate them from the likelihood function. The reduced likelihood function that depends only on intrinsic parameters is called the $\mathcal{F}$-statistic. The search for signals consists of evaluating the $\mathcal{F}$-statistic on a grid in the parameter space and comparing the values with a fixed threshold $\mathcal{F}_o$. Parameters of the grid points for which there is a threshold crossing are registered as triggers [2, 5].

Let the response s of the detector to a graviational-wave signal, as a function of time t, be of the form

$$s(t; A_o, \phi, \xi_k) = A_o \cos\left(\sum_{k=1}^{K} \xi_k l_k(t) + \phi\right), \tag{1}$$

where $l_k(t)$ are known functions of time. The amplitude A_o of the signal (1) is constant and its phase is a linear function of the K parameters ξ_k $(k = 1, \ldots, K)$ and the parameter ϕ. One can show that ξ_k are the intrinsic parameters of the signal and that the $\mathcal{F}$-statistic is a homogeneous random field [3–5]. Consequently the autocovariance function $C(\xi, \xi')$ of $\mathcal{F}$ depends only on the difference $\xi - \xi'$ of the parameter values and not on the values of the parameters themselves.

The region of the parameter space where $C \leqslant 1/2$ is called an *elementary cell*. Using Taylor expansion of the autocovariance function (see [3, 5] for details) we can approximate the hypervolume V_{cell} of a cell by the following formula

$$V_{\text{cell}} = \frac{(\pi/2)^{K/2}}{\Gamma(K/2+1)\sqrt{\det G}}, \tag{2}$$

where Γ denotes the Gamma function and the matrix G is the reduced Fisher matrix for the intrinsic parameters. For the linear model given by equation (1) the components of G are constant, independent of the values of the parameters. We estimate the number N_c of elementary cells by dividing the total Euclidean volume V of the intrinsic K-dimensional parameter space by the volume V_{cell} of the elementary cell, i.e., we have

$$N_c = \frac{V}{V_{\text{cell}}}. \tag{3}$$

Let us assume that we analyse the same parameter space M times and that each analysis is statistically independent. For example, this may be a search of a certain time interval for impulses using stretches of data in that interval from M different detectors or it may be a search of a certain frequency band for monochromatic signals using M non-overlapping time stretches of data from the same detector. Suppose that the Ith analysis produced N_I triggers. Consequently, as the cells are independent the probability of occurrence of a trigger in the Ith search, in each cell is N_I/N_c. The probability of occurrence of a trigger in a given cell in M independent searches is just the product $\prod_{I=1}^{M}(N_I/N_c)$ of probabilities for all M searches. Consequently the number of expected coincidences N_{coi} in the whole parameter space consisting of N_c cells is given by

$$N_{\text{coi}} = N_c \prod_{I=1}^{M} \frac{N_I}{N_c}. \tag{4}$$

Formula (4) is similar to the formula for number of coincidences used in the search for burst signals by a network of bar detectors [9]. In the search for bursts one is looking for coincidences in the time-of-arrival of impulses within a certain time window $\Delta\tau$. If we define the number of cells as $N_c = T_o/\Delta\tau$ where T_o is the total observation time, the formulae are equivalent.

We perform the coincidence search in the following way. For each parameter ξ_k, $k = 1, \ldots, K$, we calculate all the differences $\Delta_{IJ}\xi_k = \xi_k^I - \xi_k^J$ between all the ξ_k^I parameter values of the triggers for the Ith search and the parameter values ξ_k^J for the Jth search and for each set of K differences we calculate the distance

$$d_{IJ} = \sum_{k=1}^{K}\sum_{l=1}^{K} G_{kl}\Delta_{IJ}\xi_k\Delta_{IJ}\xi_l. \tag{5}$$

If the distance d_{IJ} is $\leqslant 1/2$ the two triggers are within one cell and we report a coincidence.

It is very common that the data from the detectors contain various interferences that may generate an excess of triggers. For example, the quasi-monochromatic interferences that originate from the electronics of the instrument can mimic nearly periodic gravitational-wave signals that are emitted by rotating neutron stars. Moreover this excess of triggers can generate an excess of coincidences among triggers in various searches that are not related to gravitational radiation. As usual with the search for bursts, to estimate this excess background level of coincidences one can perform shifts of the values of the parameters of triggers obtained from one data set and then perform the coincidences. To obtain the number of background events one can average the number of coincidences obtained for each shift. This estimate of the background is a generalization of the estimate of the background of coincidences used in the search of bar data for bursts. In the burst search one applies time shifts to time-of-arrival of events to estimate the background rate of coincidences [9].

Table 1. The expected number of coincidences among the triggers for the three searches of EXPLORER data.

Coincidence	Number of coincidences
n_{12}	2.5×10^{-3}
n_{23}	5.6×10^{-3}
n_{13}	6.4×10^{-3}
n_{123}	2.3×10^{-10}

3. Results of coincidence search of EXPLORER data

In our search we have analysed three data sets taken in November 1991 from the resonant bar detector EXPLORER. The three data sets had non-overlapping observation times of 2 days. Consequently we can treat these three data sets as independent. The modified Julian dates of the first samples of the three sets are 48 580.7909, 48 590.3221 and 48 582.7854, respectively. In order to insure that all the three searches refer to the same parameter space we need to bring the parameters of all the searches to the same epoch. We choose the epoch as the time of the first sample of the first data stretch that we analysed. Thus the frequencies f_i of the triggers obtained in the search of the second data set need to be shifted by $\dot{f}_i \Delta T_{21}$, where $\dot{f}_i$ is the spin-down parameter of the ith trigger and ΔT_{21} is the difference between the times of the first samples of the second and the first search.

In the three searches we have obtained 66 996, 58 090 and 151 861 triggers, respectively. Note that these numbers are higher than the numbers of triggers reported in table 1 of [6] which are 37 264, 21 549 and 42 626, respectively. The numbers reported in table 1 of [6] are independent numbers of triggers, i.e., in any cell in the parameter space we choose only one trigger: the one which has the highest signal-to-noise ratio. The number of cells N_c in our parameter space is estimated from the formula (3) and it is approximately equal to 1.6×10^{12}. The expected number of double coincidences n_{IJ} among the three searches and the expected number of triple coincidences n_{123} obtained using equation (4) are given in table 1.

In our search we have four intrinsic parameters: angular frequency ω_0 of the gravitational-wave signal, its spin-down parameter ω_1, and the two angles giving position of the source in the sky. The response of the EXPLORER detector to a nearly periodic gravitational wave is given by equations (1)–(6) of [6] and it does not have the form of equation (1). However, for the purpose of calculation of the number of cells and the construction of the grid of templates, we have found (see [4]) a suitable approximation of the response that has the linear form of equation (1) given by

$$s(t; A_o, p, p_0, p_1, A, B) = A_o \cos(p + p_0 t + p_1 t^2 + A \cos(\Omega_{\mathrm{r}} t) + B \sin(\Omega_{\mathrm{r}} t)), \tag{6}$$

where Ω_{r} is the rotational angular velocity of the Earth. The parameters A and B can be related to the right ascension α and the declination δ of the gravitational-wave source through the equations

$$A = \frac{\omega_0 r}{c} \cos\delta \cos(\alpha - \phi_{\mathrm{r}}), \tag{7a}$$

$$B = \frac{\omega_0 r}{c} \cos\delta \sin(\alpha - \phi_{\mathrm{r}}), \tag{7b}$$

where c is the speed of light, ω_0 is the angular frequency of the gravitational-wave signal, r is the equatorial component of the detector's radius vector with respect to the centre of the

Earth, and the angle ϕ_r determines the position of the Earth in its diurnal motion at $t = 0$. The parameters p, p_0 and p_1 contain contributions both from the intrinsic evolution of the gravitational-wave source and the modulation of the signal due to the motion of the Earth around the Sun.

Once a trigger is found we perform a transformation of its parameters (p_0, p_1, A, B) to the astrophysical parameters $(\omega_0, \omega_1, \delta, \alpha)$. To search for true gravitational waves we need to search for coincidences among the astrophysical parameters of the triggers. However in the astrophysical parametrization the response of the detector does not have the linear form (1) that warrants the application of equation (5) to determine the coincidences because the phase of the signal is a nonlinear function of the position angles δ and α (see equations (1)–(6) of [6]). To overcome this difficulty we use yet another linear model of the response introduced in [3] (section 4, equations (48) and (49)). As in the model given by equation (1), in this new model we neglect the amplitude modulation of the signal. In addition we neglect a small term in the signal's phase corresponding to the motion of the detector in the direction perpendicular to the ecliptic plane. Then the linear model is achieved by introducing the two new parameters α_1 and α_2 defined by

$$\alpha_1 := \frac{\omega_0 + \omega_s}{2\pi}(\cos\varepsilon \sin\alpha \cos\delta + \sin\varepsilon \sin\delta), \tag{8a}$$

$$\alpha_2 := \frac{\omega_0 + \omega_s}{2\pi}\cos\alpha \cos\delta, \tag{8b}$$

where ε is the angle between ecliptic and the Earth's equator and the frequency $f_s = \omega_s/(2\pi) \simeq 921$ Hz is an offset frequency by which we heterodyne the original time series to reduce the sampling rate of the data. We thus transform the astrophysical parameters $(\omega_0, \omega_1, \delta, \alpha)$ of the triggers to parameters $(\omega_0, \omega_1, \alpha_1, \alpha_2)$ and for this linear parametrization we perform the coincidence search using the criterion given by equation (5), where the matrix G is the reduced Fisher matrix for the signal

$$\begin{aligned} s(t; A_o, \phi, \omega_0, \omega_1, \alpha_1, \alpha_2) = A_o \cos\Big\{&\phi + \omega_0 t + \omega_1 t^2 \\ &+ \frac{2\pi}{c}[\alpha_1(1\mathrm{AU}\sin(\phi_\mathrm{o} + \Omega_\mathrm{o} t) + r\cos\varepsilon \sin(\phi_\mathrm{r} + \Omega_\mathrm{r} t)) \\ &+ \alpha_2(1\mathrm{AU}\cos(\phi_\mathrm{o} + \Omega_\mathrm{o} t) + r\cos(\phi_\mathrm{r} + \Omega_\mathrm{r} t))]\Big\}, \end{aligned} \tag{9}$$

where Ω_o is the mean orbital angular velocity of the Earth, ϕ_o is a deterministic phase which defines the initial position of the Earth in its orbital motion, and AU stands for the astronomical unit.

We have applied this procedure to triggers obtained from all three data sets that we had searched. The procedure revealed *zero number of double coincidences* n_{12}, n_{23}, n_{13} and consequently *no triple coincidences* n_{123}. This result is in accordance with the expectations quantified in table 1.

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