# The sound of **R**ecoiling ex-**A**l **P**articles in Nautilus?

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**Remarks in advance**

- **Energy-amplitude relation:**

  ·thermo acoustic (TA)

$$\epsilon_{TA} = E_{absorbed}/(YV)$$

bar volume $V$, Young modulus $Y$, amplitude $s = L * \epsilon_{TA}$, bar length $L$,

  ·mechanical (ME)

$$\epsilon_{ME} = \sqrt{E_{ME}/(YV)}$$

- **Production rate:**

  ·0.6 m Nautilus diameter:
  20% of 1 GeV hadrons interact
  with Al-nucleus

  ·cosmic ray hadrons $E > 1$ GeV:
  8/s hit Nautilus.

**Essentials of a mechanical model**

- **5 step mechanical sound excitation model:**
  - ·1: cosmic ray hadron + Al nucleus
    $\longrightarrow$ multiple hadronic interactions
  - ·2: nuclei recoil
  - ·3: ions forms clusters of holes
  - ·4: holes $\equiv$ deformation, strain
  - ·5: sudden strain
    $\longrightarrow$ sound mode excited
- **presentation**
  - ·first global walk through
  - ·then repeat each step with some detail

## First round

- **Recoil nucleus:**

  ·hadron throws $\geq$ 1 nucleon out of Al-nucleus

  ·nucleus acquires recoil energy $\geq$ 115 MeV/c, 250 keV in bar's frame of reference.

- **Multiplication:**

  ·repeated hadronic interactions multiply recoiling nuclei

  ·I ***assume*** $N_{rec} \approx 50$.

- **Lattice holes:**

  ·recoiling nucleus of 250 keV

  ·energy transferring cascade

  ·producing cluster with $N_h \approx 2100$ lattice vacancies in volume of $\approx 0.2\ \mu$m radius.

- **Mechanical energy:**

  ·I ***assume*** mechanical deformation energy of a hole = lattice binding energy of Al atom

  ·$E_{ME}/hole = 3$ eV.

- displacement,

  unlike thermo-acoustic heating,

  more like momentum transfer,

  hammer's effect:

  "mechanical".

- **Sound signal:**

  ·effective temperature

  of the mechanically excited sound mode

  for Nautilus

  $\cdot T^{ME}(in\ K) = 2 * 10^{22} * E_{ME}(in\ J) *$

  $* \left[\alpha \sin^2(\pi \frac{x}{l}) + (1-\alpha)\cos^2(\pi \frac{x}{l})\right] *$

  $* \ \Sigma_{clusters}(2\pi * \frac{\overline{d^{eff}}}{L})^2,$

  $\alpha = 0$ or 1, at random for any event.

- **Mechanically excited mode temperature:**

  ·for the measured mean Nautilus value

  $\cdot T^{ME}_{hadron} = T^{TA} \approx 10^{-2}$ K,

  $\rightarrow \overline{d^{eff}} = 2 * 10^{-6}$ m.

- **Signal spread:** combined statistical distributions of

  ·1) the impinging spectrum and number of nuclear interactions per hadron,

  ·2) the number of impinging hadrons piling up in one triggered filter integration time,

  ·3) the number of shower particles producing a recoil nucleus,

  ·4) the energy of the recoiling nucleus,

  ·5) the number of cascade atoms produced by the recoiling nucleus,

  ·6) the spatial distribution of holes and interstitial atoms in the cascade cluster, and

  ·7) their possible recombination.

- **Cosmic ray counter coincidence signals**

  ·correlated ME

  from hadrons coming along with TA

  ·continuous uncorrelated background

  impinging cosmic ray nucleons: 8/s

  piling up within filter time

- **Muons, electrons**

  ·produce recoiling nuclei too

  ·cross section too small

  for the sensitivity at hand.

- **Tests on existing Nautilus data:**

  ·1) If indeed pile up
  on the TA triggered signals
  ·then ME rate should vary with filter time
  ·2) If an x-position can be deduced
  from the cosmic ray counters,
  ·the x-asymmetry might be exploited
  ·inner half of the detector
  contains a mixture
  ·$\longrightarrow$ in outer half:
  mean $T^{TA}$-value reduced by a factor 5,
  some, $(1-\alpha)$-part, $T^{ME}$-values also
  some, $\alpha$-part, enhanced by a factor 5
  if $\overline{\alpha} \neq 0$

- **Other tests:**
  - ·The *NIKHEF 1998 electron* beam
    producing no recoil atoms,
    should show no ME signals,
    none were found.
  - ·*Milano 1980 proton* beam
    producing recoils,
    rough estimate $T^{ME} \approx T^{TA}$,
    no such signals were reported:
    $\rightarrow$ estimate too rough.

- **New tests:**
  - ·1) RAP in e-beam very important
    test applicability of TA model
    at super conducting condition.
  - ·2) RAP instrument in proton beam?
    test applicability of the ME model
  - ·3) 5 m concrete around Nautilus,
    ME noise drop by factor $\approx 10$?

# Second round

## recoil nucleus

- **For** $\mathbf{E}_{hadron} \gg 1$ **GeV**
- nucleon momenta in nucleus
- residual nucleus recoils: $E_{Al} = \frac{(pc)^2}{M_{Al}c^2}$

  ·Al, or Mg, or ...
- nucleon density formula from geant4 manual

  $\cdot\rho = \rho_0/(1 + e^{(r-R)/a})$,

  $\cdot R = r_0 A_{Al}^{1/3} = 3$ fm, $r_0 = 1$ fm,

  $A_{Al} = 27, a = 0.54$ fm, $\rho_0 = 0.006$

  ·take nucleon at Fermi value $p_F$,

  $\cdot\rho = (p_F/\hbar)^3/(3\pi^2)$, $E_{rec} = (p_F c)^2/(2Mc^2)$.

·Then

$$dN/dE_{rec} = dN/dp_f * dp_F/dE_{rec},$$

$$dp_F/dE_{rec} = Mc^2/p_F,$$

$$dN/dp_F = d\rho/dp_F * dr/d\rho * dN/dr,$$

$$dN/dr = 4\pi\rho r^2,$$

$$d\rho/dr = \rho^2 a\rho_0 exp((r-R)/a),$$

$$dp_F/d\rho = p_F/3\rho_0.$$

- From this $\rightarrow 0 < p_N < 115$ MeV/c

  ·cut off $\frac{1}{2}M_{Al}v_s^2 = 3$ eV, $v_s = 5000$ m/s

  ·23% recoils 3 eV< $E_{rec}$ <100 keV,

  ·14% recoils 100 keV< $E_{rec}$ <150 keV,

  ·23% recoils 150 keV< $E_{rec}$ <200 keV,

  ·40% recoils 200 keV< $E_{rec}$ <250 keV.

- independent of hitting hadron energy

**For $E_{hadron}$ around 1 GeV**

- more than a single nucleon launched from the Al-nucleus

  ·remaining nucleus recoils with larger momentum than 115 MeV/c,

  ·now depending on $E_{hadron}$

- the nucleons react further,

  ·for instance n+Al$\rightarrow x_1 N + X_1$

  $\rightarrow X_1 + x_1(x_2 N + X_2) \rightarrow ...$

  ·producing more recoiling nuclei

# Ion displaces atoms

- recoiling ion displaces Al-atoms

  ·mostly in a cluster around end of path,

  ·Ziegler's SRIM-2000:

| ion energy | number of holes/ion | cluster diameter |
|---|---|---|
| keV | | $\mu$m |
| 1 | 20 | 0.002 |
| 10 | 160 | 0.02 |
| 100 | 1000 | 0.07 |
| 250 | 2100 | 0.1 |
| 1000 | 3700 | 0.3 |
| 10000 | 5100 | 0.4 |

- mean spacing $\overline{a_{cluster}(250)} =$

  $0.1 * 10^{-6}/(2100)^{1/3} = 10^{-8}$ m

- interstitial excites phonons

  high frequency lattice vibrations

  ·BUT some might compensate

  with pressure, the hole pulling

  ·net effect

  depending on spatial distribution

# thermo-acoustic model

- energy absorption $E_{abs}$
- heats material along particle track
- produces excess pressure $p$
- excites sound mode
- $\tau_p = d/c \ll \tau_{mode} \approx L/c_s \ll \tau_{diff}$
  - $\cdot d$ diameter, $L$ length of bar
  - $\cdot c = 3 * 10^8$ m/s, velocity of light,
  - $\cdot c_s = 5000$ m/s velocity of sound,
  - $\cdot \tau_{diff}$ thermal diffusion time
- extra heated by temperature $\Delta T = E_{abs}^{TA}/(c_v \rho V)$
  - $\cdot c_v$ specific heat, $\rho$ density, $V$ volume
- expands $\Delta V = \alpha \Delta T V = \alpha E_{abs}^{TA}/(\rho c_v)$
  - $\cdot \alpha$ expansion coefficient
- relative expansion $\epsilon = \Delta V/V = \alpha E_{abs}^{TA}/(\rho c_v V)$
  - ·dimensionless Grueneisen constant
    $\gamma = \alpha Y/(\rho c_v) \simeq 1.6$
  - ·Young modulus $Y = 7 * 10^{10}$ N/m$^2$

- maximum mode amplitude from:

$$\epsilon_{TA} = (\frac{\Delta x}{L})_{TA} = \frac{\gamma}{Y}\frac{E^{TA}_{abs}}{V} \simeq 2 * 10^{-11}\frac{E^{TA}_{abs}}{V}.$$

  ·Nautilus $V = 0.85$ m$^3$

  ·super hit:

  $E^{TA}_{abs} = 1.3 * 10^{-5}$ J $= 87$ TeV

  $T_{mode} = 58$ K

  $\rightarrow \epsilon_{TA} \simeq 3 * 10^{-16}$

# TA mode formula check

- strain → sound mode excitation
- $\cdot T_{mode} = (M * f^2 * L^2)/k) * \epsilon^2$
  $=1,22 * 10^{33} * \epsilon^2$ K
  ·bar mass $M$, length $L$, frequency $f$

  Boltzmann $k = 1,4 * 10^{-23}$ J/K
  $\cdot \epsilon = 2,2 * 10^{-16}$: $T_{mode} = 58$ K
- Thermo-acoustical $\epsilon^{TA} = \frac{E_{abs}(TA)}{YV}$
  $\cdot T^{TA}_{mode} =$
  $= (M*f^2*L^2)/(k*V^2*Y^2)*E_{abs}(TA)^2$
  $=3.5 * 10^{11} E_{abs}(TA)^2$ K
  ·bar volume $V$, Young modulus $Y$
  $\cdot E_{abs}(TA) = 1,3 * 10^{-5}$ Joule:
  $T^{TA}_{mode} = 58$ K

## mechanical model

- ion deforms material along track
- $\rightarrow$ strain $\rightarrow$ excites sound mode
- Deformation energy

  $\cdot W = \int_0^{\Delta x_0} F(\Delta x) d(\Delta x)$

  force F, displacement $\Delta x$

  $\cdot F(\Delta x) = O' Y \Delta x / L'$ Young modulus $Y$

  $\cdot W = \frac{O'Y}{2L'} \Delta x_0^2$

  $\cdot \Delta x_0 = L' \epsilon_0$

  $\cdot W = O' Y L' \epsilon_0^2 / 2 = \frac{V'Y}{2} \epsilon_0^2$

- $\epsilon_{ME} = (\frac{\Delta x}{L})_{ME} = \sqrt{\frac{2}{Y} \frac{E_{ME}}{V}}$
- $g \equiv \frac{\epsilon_{ME}}{\epsilon_{TA}} = \sqrt{YV} \frac{\sqrt{E_{ME}}}{E_{abs}^{TA}}$
- physically senseless, numerical comparison

  $\cdot W \equiv E_{ME} = E_{abs}^{TA}$

  $\cdot$Nautilus: $V = 1$ m$^3$, $E_{abs} = 1.3 * 10^{-5}$ J,

  $Y = 10^{11}$ N/m$^2$

  $\cdot g \approx 10^8$ at equal energy value

- $T_{mode} = \frac{M*f^2*L^2}{k} * \epsilon^2 = 1.2 * 10^{33}\epsilon^2$
- holes source term
  - $\cdot H = \int_V \overrightarrow{\nabla} p \cdot \overrightarrow{u} dV/p$ in $\epsilon_{ME}$.
  - ·Partial integration, $u(x) = sin(\pi x/L)$,
  - $\cdot \rightarrow H = \int_V \cos(\pi x/L) dV =$

    $= sin(\pi x_h/L) - sin(\pi(x_h + d)/L)$

    hit position $x_h$ , cluster diameter $d$
  - $\cdot \epsilon_{ME} = 2\pi(d/L)\sqrt{E_{ME}/(VY)} \cos(\pi x_h/L)$,

    $T^{ME} = 1.2 * 10^{33} * (2\pi(d/L))^2 *$

    $* (E_{ME}/(VY)) * \cos^2(\pi x_h/L)$.
- use $d^{eff}$ in stead of $d$
  - ·to express as yet unknown effects
- remaining interstitial
  - ·may exert compensational force

- net effect

  ·maybe dipole term:

  $\epsilon_{ME} = 2\pi(d^{eff}/L)^2\sqrt{E_{ME}/(VY}\sin(\pi x_h/L)$,

  and higher order

  ·$\longrightarrow$ fluctuating between $cos^2$ and $sin^2$

  ·$T^{ME} \propto \alpha sin^2(\pi x_h/L){+}(1{-}\alpha)cos^2(\pi x_h/L)$,

  with $\alpha = 0$ or 1 at random for an event,

  ·mean value over the measurements of:

  ·$\overline{\alpha} = \beta_0(d^{eff}/L) + \beta_1 * (d^{eff}/L)^2 + ...$,

  ·unknown coefficients $\beta$

- stick to monopole & drop x-dependence

  ·$\rightarrow \epsilon_{ME} = (2\pi d^{eff}/L)\sqrt{E_{ME}/(VY}$

  ·$\rightarrow T_{mode} = 2 * 10^{22} E_{ME} * (2\pi d^{eff}/L)^2$

- $E_{ME} = N_{rec} * N_{holes/cluster} * E_{hole}$

  ·For $N_{rec} = 50$, $N_{holes/cluster} = 2100$,

  $E_{hole} = 3$ eV$= 5 * 10^{-19}$ J,

  $T_{mode} = 10^{-2}$ K,

  $\rightarrow d^{eff} = 2 * 10^{-6}$ m.

  ·rather arbitrary 'gauge'

  ·$d^{eff}$ adaptive term for a theory

- maybe recombination loss

  within $\tau_{mode} = 1$ ms

  ·$N_h(t) = N_h(0)e^{-\tau_{mode}/\tau_{rec}}$

  ·or slow diffusion?

# sound excitation

- thermo-acoustic: heating
- mechanical: DeWaele effect

  ·a particle hits the bar "like a hammer"

  ·momentum transfer → deformation

  ·Colleague from former GRAIL
  first suggested mechanical excitations
- focus on basic longitudinal mode,

  ·and excitation along the x-axis.

  ·radial forces
  → similar effect:
  see Babusci & Giordano

# reaction rate

- hadrons $E \gg 1$ GeV on nucleons in Al,

  ·hadronic cross section

  $\sigma \approx 50$ mb $= 5 * 10^{-30}$ m$^2$

- $\rightarrow$ effective mass traversed $M_{eff} = \sigma \rho d$

  ·density $\rho = 2700$ kg/m$^3$, diameter $d = 0.6$ m

- effectively hit nucleons $N_{eff} = \frac{\sigma \rho d}{m_p} = 8d$

  ·nucleon mass $m_p = 1.6 * 10^{-27}$ kg

- Nautilus: $N_{eff}^{Naut} = 5$ per incident hadron

  adding to the measured signal amplitude.

- hadrons around 1 GeV, on Al-nucleus

  ·divide by $M_{Al}$, not $m_p$

  about same $\sigma$

  ·$\rightarrow$ on average 20% interact

- hadrons coming with TA process:

  ·correlated ME contribution

- also, proton cosmic ray flux $N_p = 0.9/\mathrm{m}^2/\mathrm{sr/s}$

  ·neutron flux 1/3 of protons

  ·$\rightarrow$ Nautilus:

  $\frac{dN_p}{dt} = \pi * 2 * 1.2 = 8/\mathrm{s}$ ME signals

  $\rightarrow$ background in the mode

## multiplication

- by repeated nuclear interactions
- **For** $E_{hadron} \gg 1$ **GeV**
- for each impinging hadron

  6 protons/s
- for each of $\approx 5$ nuclear interactions

  $\rightarrow$ high momentum nucleon/pions

  contributing to thermo-acoustics

  + one recoiling nucleus
- secondary particles

  more nuclear interactions

  + more recoiling nuclei
- average 5 interactions per impinging hadron per traversal

  1 every 0.12 m, I assume each to produce 2 secondaries $\rightarrow 2^5 or \approx 50$ recoiling nuclei.

- **For $E_{hadron}$ around 1 GeV**
- 5-10 nucleons launched from the Al-nucleus
- the neutrons react further n+Al→xn+Al

  producing more recoiling nuclei,

  I assume the same total of $\approx 50$ recoiling nuclei.

## coincidence rate

- each impinging hadron → recoil nucleus

  ·→ sound signal

- $N_p = 8$/s impinging hadrons

  ·1.2/(m$^2$/s/sr) cosmic rays, Nautilus 2 m$^2$

- chance coincidence probability:

  $\cdot P_{chance} = 2 * \tau_{filter} * N_p = 16 * \tau_{filter}$

- 16% coincidences → $\tau_{filter} > 0.01$ s

- → 100% coincidences for $\tau_{filter} > 0.06$ s

- ME amplitude of cosmic ray hadrons

  independent of trigger multiplicity,

# other experiments

- e-beam (Amsterdam 1998)
  ·expect no recoils
  ·result $\longrightarrow$ no mechanical signals measured

- p-beam (Milan 1980)
  ·expect recoils, $\epsilon^{ME} \approx 2 * \epsilon^{TA}$
  ·no mechanical signals reported
  ·$\longrightarrow$ calculation needed
  ·measured $B_0 = L * \epsilon^{TA} = 5.9 * 10^{-13}$ m,
  at $E^{TA}_{abs} = 1 * 10^{-4}$ J, $L = 0.2$ m
  nuclear cross section: $\sigma(p + Al \rightarrow recoil) = 1$ barn
  cluster radius 2*$10^{-7}$ m, 3700 holes per cluster
  $\overline{d^{eff}_{cluster}} = 2 * 10^{-6}$ m
  $6 * 10^8$ protons per burst, I=10 $\mu$A, $\tau = 10$ $\mu$s
  $E_p$ =29 MeV, $E_{rec} = 1$ MeV,

  $\rightarrow \epsilon^{ME} = 1.4 * 10^{-12}$

## sensitivity

- **if** most of noise events
  - ·around $T^{now} \approx 5$ mK
  - ·would be ME from cosmic ray hadrons,
- **then** 5 m concrete shielding
  - ·$N_p^{shielded} = N_p/10 \rightarrow$

    a factor of $\approx 10$ smaller amplitude

    $T^{ME}_{shielded} = T^{ME}_{now}/10 \leq 0.5$ mK.

# Some symbols, values

| description | symbol | dimension | Nautilus | e-beam | p-beam | general |
|---|---|---|---|---|---|---|
| cylinder x-length | $L$ | m | 3.0 | 0.2 | 0.2 | |
| cylinder diameter | $d_{cyl}$ | m | 0.60 | 0.035 | 0.03 | |
| cylinder volume | $V$ | $m^3$ | 0.85 | $2*10^{-4}$ | $2*10^{-4}$ | |
| cylinder Al-mass | $M$ | kg | 2300 | 0.4 | 0.4 | |
| frequency 1st long. ac. mode | $f_0$ | Hz | 900 | 12500 | 12500 | |
| used absorbed energy | $\varepsilon$ | J | $1.6 * 10^{-5}$ | 0.01 | $1 * 10^{-4}$ | |
| velocity of sound | $c_s$ | m/s | | | | 5000 |
| Young modulus Al | $Y_{Al}$ | $Nm^{-2}$ | | | | $7 * 10^{10}$ |
| density Al | $\rho$ | $kg/m^3$ | | | | 2700 |
| velocity of light | $c$ | m/s | | | | $3 * 10^8$ |
| Avogadro's number | $N_{Avog}$ | $mol^{-1}$ | | | | $6 * 10^{23}$ |
| Boltzmann | $k$ | J/K | | | | $1.4 * 10^{-23}$ |
| proton mass | $m_p$ | kg | | | | $1.7 * 10^{-27}$ |
| nucleon binding | $B_N$ | eV | | | | $5 * 10^7$ |
| Al-atom total binding | $B_A$ | eV | | | | $10^4$ |
| lattice binding | $B_L$ | eV | | | | 3 |
| Cooper binding | $B_C$ | eV | | | | 0.003 |

Table 1: Some symbols and values used in the text

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