GDH SUM RULE AND Q^2 EVOLUTION

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Abstract

The Gerasimov-Drell-Hearn sum rule, which has never been tested experimentally, relates the difference in the total photo-absorption cross section on nucleon for photon-nucleon helicity 1/2 and 3/2 to the anomalous magnetic moment of the target nucleon. A generalized GDH sum rule can be also derived for virtual photons. It will be shown that the connection of the sum rule expression from the asymptotic DIS region to the real photon one is not trivial and the measurement at low Q^2 should be performed in a wide range of energy transferred, from the resonance region deeply into the DIS region.

I. THE GDH FOR REAL PHOTONS

The GDH sum rule[1] is derived starting from a general dispersive relation for the forward Compton scattering, which follows causality, crossing symmetry and unitarity principles:

$$\int_0^\infty \frac{d\nu}{\nu}\left[\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)\right] = -\frac{2\pi^2\alpha}{m_p^2}\kappa^2 \tag{1}$$

where ν is the photon energy, $\sigma_{1/2}$ and $\sigma_{3/2}$ are the absorption cross sections for total helicity 1/2 and 3/2, and κ is the anomalous magnetic moment of the target nucleon. The sum rule defined in eq. (1)is based on two additional assumptions:

- the low energy theorem (LET) which is derived from gauge and relativistic invariance; it states that not only the 0-frequency limit of the photon scattering (i.e. the Thomson scattering) is independent of the structure of the system, but also the first derivative is determined only by the static properties like charge, mass and magnetic moment.

- the no-subtraction hypothesis for the spin-flip part of the Compton scattering amplitude which is required to go to 0 at infinite energy without any poles.

The sum rule can be evaluated for the proton, the neutron and for the proton-neutron difference. Alternatively one can decompose the left and the right part of the sum rule (1) into components of different isospin characters (isovector, isoscalar and isovector-isoscalar interference terms). Recently these integrals have been evaluated from single pion photoproduction amplitude analysis plus some estimate of the double pion contribution (see table 1).[2] For instance, in fig. 1 it is shown the analysis of the isovector part (which is the dominant one) by Sandorfi et al.

These multipoles analysis suggested the possible violation of the sum rule, so that recently further arguments have been proposed in order to modify the original GDH expression. Among these:

- The Extended Current Algebra approach (ECA), which introduced a symmetric term in the charge density commutator added to the anomalous magnetic moment term.[3]

- a localized spontaneous breaking down of the e.m. gauge symmetry inside a nucleon.[4]

Due to the availability of high quality circularly polarized photon beams, obtained from backscattered Compton laser processes or from bremsstrahlung off longitudinally polarized electrons, and due to the developments of new polarized targets, made of frozen butanol or solid HD, a big effort is now being performed in several laboratories, in order to test the GDH sum rule. In fig. 1 are also shown the energy ranges that will be covered by the five proposed experiments.[5] As it is seen, even if a rather fast convergence of the sum rule is expected, due to the $1/\nu$ weighting factor, no experiment alone will be able to explore the whole needed energy range. At present it can be said that only putting together data of all forthcoming experiments, it will be possible to verified the sum rule.

II. THE GDH EXTENSION FOR VIRTUAL PHOTONS

The GDH integral can be naturally extended to the case of the absorption of circularly polarized virtual photons deduced from inclusive inelastic scattering of longitudinally polarized electrons off polarized nucleons:

$$I(Q^2) = \frac{m_p^2}{2\pi e^2} \int_{\frac{Q^2}{2m_p}}^{\infty} \frac{d\nu}{\nu} \left[\sigma_{1/2}(Q^2) - \sigma_{3/2}(Q^2)\right] \tag{2}$$

which is related to the experimental asymmetries A_1 and A_2 and to the polarized structure functions g_1 and g_2:

$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{2\sigma_T} = \frac{g_1 - \gamma^2 g_2}{F_1} \tag{3}$$

$$A_2 = \frac{\sigma_{TL} m}{\sigma_T} = \gamma \frac{g_1 + g_2}{F_1} \tag{4}$$

where $\gamma = \sqrt{Q^2}/\nu$, F_1 is the unpolarized structure function, σ_T is the total transverse photo-absorption cross section and σ_{TL} is a term arising from the interference between transverse and longitudinal amplitude .

It is easy to show that for the high energy limit (DIS), when $\gamma \ll 1$, or for the real photon case, when $Q^2 = 0$, the asymmetries simplify as $A_1 = g_1/F_1$ and $A_2 = 0$. Thus, only for these two limits, the integral is:

$$I(Q^2) = -\frac{\kappa}{4} \tag{5}$$

for $Q^2 = 0$ [GDH value]

$$I(Q^2) = \frac{2m_p^2}{Q^2} \int_0^1 g_1(x)dx = \frac{2m_p^2}{Q^2}\Gamma \tag{6}$$

for $\gamma \ll 1$, where Γ is the first momentum of $g_1(x)$.

Thus the integral $I(Q^2)$ connects the GDH expectation for real photon with the Ellis-Jaffe integral Γ (for p and n) and the Bjorken integral (for the p-n difference) in the asymptotic region, with an $1/Q^2$ scaling factor.

In fig. 2 are shown the $1/Q^2$-scaling of the integrals measured in the $< Q^2 >\sim 2 \div 10$ GeV^2 range by different DIS experiments[6] and the real photon point predictions: it is shown that, in order to be connected with the GDH-expectation at $Q^2 = 0$, a dramatic Q^2-dependence of I_p with a change of sign is required below $2GeV^2$. Moreover if the ECA predictions are correct, the Q^2-dependence of I_p should be even stronger and also the Bjorken integral I_{p-n} should change sign in the low Q^2 region.

The first phenomenological attempt to connect the two different regions was performed by Anselmino et al.[7] which tried to explain the spin crises which follows the EMC results on the spin structure of the proton by assuming large higher twist effects in the Q^2 range of the experiment. They assumed the simple expression

$$I_p(Q^2) = 2m_p^2\Gamma_p(\frac{1}{Q^2+\mu^2} - \frac{c\mu^2}{(Q^2+\mu^2)^2}) \tag{7}$$

where

$$c = 1 + \frac{1}{8}(\frac{\mu^2}{m_p^2})\frac{\kappa^2}{\Gamma_p} \tag{8}$$

and μ is a mass parameter which characterized the scale variation of Q^2. Vector meson considerations (VMD) suggested that μ should be close to the ρ mass.

Following the VMD model,[7] in the last few years, several models were developed in order to predict the low-Q^2 behavior of the integral $I_p(Q^2)$:[8]

- Burkert and Joffe added the Q^2-parameterization of the Δ-resonance excitation to the VMD one, in order to take into account the large and negative contribution of the Δ close to the photon point.

- Li et al. showed how the Q^2-dependence of the helicities amplitudes and of the sum rule change when assuming the Roper resonance as a radially excited q^3 state or as a q^3G hybrid baryon.

- Burkert and Li studied the contribution from all the main baryon resonance excitations, claiming that the sum rule at small Q^2 is largely affected by contributions from the lower mass resonant states.

- Soffer and Teryaev showed that the strong Q^2-dependence of the GDH sum rule is naturally given by using the Schwinger sum rule and the contribution of the g_2 structure function, which describe the difference between longitudinally and transversely polarized nucleon.
- Bernard et al. derived the small momentum evolution of the sum rule from chiral perturbation theory (χ_{PT}). A one-loop approximation within a non-relativistic formulation with the nucleon considered as a very heavy and static source was used; moreover also higher-order contributions arising from the Δ-resonance and relativistic corrections were evaluated.

Fig. 3 shows some of these model predictions for I_p compared with the $1/Q^2$ scaling behavior: as it is seen very different Q^2-dependences are expected: the crossover point is predicted in the wide range from ~ 0.15 to $\sim 1\ GeV^2$, while also the sign of the slope of the integral close to photon point is very different from model to model, being negative when including resonances while positive for other parameterizations.

III. EXPERIMENTAL

In the measurement of GDH in the Q^2-region below $2 \div 3\ GeV^2$, three main non-trivial experimental questions arise:

- what observables should be measured?
- which is the required Q^2-resolution and -binning?
- which is the transferred energy range needed to saturate the sum rule?

The first question follows the fact that eq. (6) is not valid in or close to the resonance region where the integral (2) should start. Therefore a direct measurement of $\sigma_{1/2} - \sigma_{3/2}$ or A_1 is needed. A_1 is related to A_2 by:

$$A_{||} = D[A_1 + \eta A_2] \tag{9}$$

$$A_{\perp} = d[A_2 - \xi A_1] \tag{10}$$

where D, η, d and ξ are kinematic factors, and $A_{||}$ and $A_{\perp}$ are the asymmetry yields corrected by the polarization and dilution factors of the beam and the target. Therefore A_1 can be extracted through a Rosenbluth plot obtained running the machine at different energies (TJNAF experiments) or by comparing measurements with parallel and perpendicular polarization targets (HERMES experiment). It is to point out that A_2, which is bounded by the positivity limit $A_2 \leq \sqrt{R} \sim 0.4$, is about $10 \div 20\%$ of A_1 in the DIS measured region, while is actually completely unknown at low Q^2.

The second question arises from the experimental evidence that A_1 seems to be Q^2-dependent at low Q^2. This is not the case at high Q^2 where the asymmetry yields are summed up in x-bins and the Q^2-dependence is considered only in the unpolarized structure function F_1 evolution:

$$g_1(x, Q^2) \approx A_1(x) F_1(x, Q^2) \tag{11}$$

A precise measurement of A_1 at low Q^2 will then require fine Q^2-binning and therefore high statistics and a wide and continuous kinematical plane.

The third question is strictly related to the dynamical behavior of the Δ-resonance, which form factor decreases much faster than Q^{-4} and its contribution to the sum rule, which is dominant for real photon, becomes negligible at $Q^2 \simeq 2 GeV^2$.

It is also interesting to notice how the relative weighting factor $1/\nu$ computed at the Δ-resonance invariant mass respect to the non-resonant invariant mass lower limit $W \approx 2$ GeV, also decreases from ~ 5 to ~ 1.5 when going from $Q^2 = 0$ to $Q^2 \approx 2$ GeV^2.

The kinematical limitation of the resonance region was recently studied by Li and Li.[9] In fig. 4 are shown the contribution to the integral Γ_p coming from the resonance region, which is computed from helicity amplitudes analysis of electro-production, and the total integral evaluated from EMC A_1 data and from a low Q^2-extrapolation of the global fits on unpolarized structure functions F_2 and R. From the figure is possible to see how at Q^2 around 1 GeV^2 the resonance contribution is only a fraction of the total.

In fig. 5 are shown the kinematical coverage of the proposed experiments at TJNAF,[10] of the running experiment HERMES at DESY[11] and of the data of E143 at SLAC. Clearly the combination of the three data sets will provide a good estimate of the sum rule around 1 GeV^2. Moreover, the high resolution TJNAF data will also provide a deeper insight of polarized structure functions in the resonance region, while high energy data of HERMES will check the asymptotic behavior of $\sigma_{1/2}(\nu)-\sigma_{3/2}(\nu)$ and the related no-subtraction hypothesis.

IV. CONCLUSIONS AND OUTLOOK

The Q^2-evolution of the GDH sum rule is crucial for understanding the nucleon spin structure in the nonperturbative domain and complete information is needed for the proton, the neutron and for their difference as well.

In the Q^2-region around 1 GeV^2 the GDH integral measurement will required an A_1 and A_2 separation, a fine Q^2-binning and the necessary combination of data coming from different experiments at SLAC, DESY and TJNAF in order to put together the resonance and the DIS regions.

In the very low Q^2-region close to the photon point (less than about 0.5 GeV^2) the TJNAF proposed experiments seem to be not sufficient to fulfill the sum rule due to their kinematical restriction just above the resonance region. Thus the study in this interesting region will be completed only by the measurements that will performed with a higher energy upgrade of TJNAF or with the proposed new European electron machine ELFE.

REFERENCES

[1] S.B.Gerasimov, Sov. J. Nucl. Phys. **2**, 430 (1966); S.D.Drell and A.C.Hearn, Phys. Rev. Lett. **6**, 908 (1966).

[2] I.Karliner, Phys. Rev. **D7**, 2717 (1973); R.L.Workman and R.A.Arndt, Phys. Rev. **D45**, 1789 (1992); A.M.Sandorfi et al., Phys. Rev. **D50**, R6681 (1994).

[3] L.N.Chang et al., Phys. Lett. **B329**, 514 (1994).

[4] S. Ying, J.Phys. **G22**, 293 (1996).

[5] D. I. Sober et al., CEBAF PR-91-15 (1991); G. Anton et al., ELSA proposal (1992); J.Ahrens et al., MAMI proposal (1993); D. Babusci et al., LEGs exp. L18/L19 (1994); J.P.Didelez et al., Proc. 1st ELFE summer school, p. 319, Edition Frontieres 1996.

[6] EMC, J.Ashman et al., Phys. Lett. **B206**, 364 (1988); E142, P.L.Anthony et al., Phys. Rev. Lett. **71**, 959 (1993); SMC B.Adeva et al., Phys. Lett. **B302**, 533 (1993); E143, K.Abe et al., Phys. Rev. Lett. **74**, 346 (1995) ; HERMES (preliminary) D. de Schepper, Proc. DIS Workshop, Rome, April 15-20 1996.

[7] M.Anselmino et al., Sov. J. Nucl. Phys. **49**, 136 (1989);

[8] V.D.Burkert and B.L.Joffe, Phys. Lett. **B296**, 223 (1992); Z.Li et al., Phys. Rev. **D46**, 70 (1992); V.Burkert and Z.Li, Phys. Rev **D47**, 46 (1993); J.Soffer and O.Teryaev, Phys. Rev. Lett. **70**, 3373 (1993); V.Bernard et al., Phys. Rev. **D48**, 3062 (1993)

[9] Z.Li and Z.Li, Phys. Rev. **D50**, 3119 (1994).

[10] V. Burkert et al., CEBAF PR-91-23; S. Kuhn et al., CEBAF PR-93-09; Z. E. Meziani et al., CEBAF PR-94-10.

[11] HERMES Technical Design Report, DESY-PRC 93/06 (1993).

FIGURES

FIG. 1. Difference of the isovector part of the spin-dependent cross sections $(\sigma^{3/2} - \sigma^{1/2})_V$ for single-pion photoproduction (short dashed curve); the long dashed and the solid curve are the expected contributions to the GDH integral coming from $N\pi$ and $N\pi + N\pi\pi$ channels respectively. Also shown are the energy ranges which will be covered by the proposed experiments.

FIG. 2. $1/Q^2$ evolution of the integrals $I(Q^2)$ for the proton (long dashed line and open triangles), the neutron (short dashed line and open diamonds) and the proton-neutron (solid line and open circles) derived from DIS experiments. Dotted lines are visual connections with the GDH values (open symbols at $Q^2 = 0$); also shown are the ECA predictions (close symbols).

FIG. 3. $I_p(Q^2)$ predictions of different models described in the text, compared with the $1/Q^2$-dependence

FIG. 4. The Q^2-dependence of $I_p(Q^2)$, the dashed line corresponds to the contribution from the resonance region only, and the solid line is the total result when including also the DIS region

FIG. 5. Kinematical plane covered by the TJNAF proposed measurements of GDH, by the HERMES experiment and by E143 data. The HERMES data region is delimited by virtual photon depolarization at low ν, by radiative corrections at high ν, by minimal angular acceptance at low Q^2 and by low statistic at high Q^2.

TABLES

TABLE I. Expected GDH sum rule data compared with several multipoles analysis and with the ECA prediction

	$GDH_p(\mu b)$	$GDH_n(\mu b)$	$GDH_{p-n}(\mu b)$
$GDH sumrule$	-204.5	-232.8	28.3
$Karliner$	-261	-183	-78
$Workman - Arndt$	-257	-189	-68
$Burkert - Li$	-203	-125	-78
$Sandorfi$	-289	-160	-130
$Extended - Current - Algebra$	-294	-185	-109