High energy contribution to the GDH sum rule

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Abstract

The Gerasimov-Drell-Hearn sum rule, which has never been tested experimentally, relates the difference in the total photo-absorption cross section on nucleon for photon-nucleon helicity 1/2 and 3/2 to the anomalous magnetic moment of the target nucleon. A generalized GDH sum rule is derived for virtual photons deduced from inclusive inelastic scattering of longitudinally polarized electrons off polarized nucleons. The DIS contribution to the generalized GDH integral for the nucleons obtained at HERMES is shown. For Q^2 above 1 GeV^2 the results are at least of the same order of magnitude as expectations for the resonance part of the integral; furthermore the data are consistent with a QCD NLO fit based on a previous DIS data, indicating that higher twist effects are not significant even at the lowest measured Q^2.

1 The GDH for real photons

The GDH sum rule [1] is derived starting from a general dispersive relation for the forward Compton scattering, which follows causality, crossing symmetry and unitarity principles:

$$\int_0^\infty \frac{d\nu}{\nu}\Big[\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)\Big] = -\frac{2\pi^2\alpha}{M^2}\kappa^2 \qquad (1)$$

where ν is the photon energy, $\sigma_{1/2}$ and $\sigma_{3/2}$ are the absorption cross sections for total helicity 1/2 and 3/2, and κ is the anomalous magnetic moment of the target nucleon. The sum rule defined in eq. (1) is based on the additional assumptions of the low energy theorems and the no-subtraction hypothesis for the spin-flip part of the Compton scattering amplitude.

The sum rule is evaluated for the proton, the neutron and for the proton-neutron difference. Alternatively one can decompose the left and the right part of the sum rule (1) into components of different isospin characters (isovector, isoscalar and isovector-isoscalar interference terms). Recently these integrals have

	$GDH_p(\mu b)$	$GDH_n(\mu b)$	$GDH_{p-n}(\mu b)$
GDH sum rule	-204.5	-232.8	+28.3
Karliner	-261	-183	-78
Workman - Arndt	-260	-157	-103
Burkert - Li	-203	-125	-78
Sandorfi *et al.*	-289	-160	-130
Drechsel - Krein	-261	-180	-81
ECA	-294	-185	-109

Table 1: Expected GDH sum rule data compared with several multipoles analysis and with the ECA prediction.

been evaluated from single pion photo-production amplitude analysis plus some estimate of the double pion contribution [2]; the results are reported in Tab. 1. These multi-poles analysis suggested the possible violation of the sum rule, so that recently further arguments have been proposed in order to modify the original GDH expression. Among these the Extended Current Algebra approach (ECA), which introduced a symmetric term in the charge density commutator added to the anomalous magnetic moment term [3]. A detailed description can be found in [4]. The contribution above the resonance region to the GDH sum rule for real photon account for a large fraction [5] of the discrepancy between the sum rule expectations and the single pion photo-production analysis estimates.

Due to the availability of high quality circularly polarised photon beams, obtained from backscattered Compton laser processes or from bremsstrahlung of longitudinally polarised electrons, and due to the developments of new polarised targets, made of frozen butanol or solid HD, a big effort is now being performed in several laboratories, in order to test the GDH sum rule [6]. Even if a rather fast convergence of the sum rule is expected, due to the $1/\nu$ weighting factor, no experiment alone will be able to explore the whole needed energy range. At present it can be said that only putting together data of all forthcoming experiments, it will be possible to verify the sum rule.

2 The GDH extension for virtual photons

The GDH integral can be naturally extended to the case of the absorption of circularly polarised virtual photons with energy ν and squared four-momentum $-Q^2$ deduced from inclusive inelastic scattering of longitudinally polarised electrons off polarised nucleons:

$$I(Q^2) = \int_{Q^2/2M}^{\infty} \Big[\sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2)\Big] \frac{d\nu}{\nu} \tag{2}$$

$$= \frac{8\pi^2\alpha}{M}\int_0^1 \frac{g_1(x,Q^2)-\gamma^2 g_2(x,Q^2)}{K}\frac{dx}{x},$$

where g_1 and g_2 are the polarised structure functions of the nucleon, x is the Bjorken variable and K is the flux factor of virtual photons, defined as $\nu\sqrt{1+\gamma^2}$, with $\gamma^2 = Q^2/\nu^2$.

For the real photon case, when $Q^2 = 0$ (GDH value), the integral reduce to Eq. 1, while for the high energy limit (DIS), when $\gamma \ll 1$, the integral becomes

$$I(Q^2) = \frac{16\pi^2\alpha}{Q^2}\int_0^1 g_1(x)dx = \frac{16\pi^2\alpha}{Q^2}\Gamma \tag{3}$$

where Γ is the first momentum of $g_1(x)$. Thus the integral $I(Q^2)$ connects the GDH expectation for real photon with the Ellis-Jaffe integral Γ (for p and n) and the Bjorken integral (for the p-n difference) in the asymptotic region, with a $1/Q^2$ scaling factor. In order to be connected with the GDH-expectation at $Q^2 = 0$, a dramatic Q^2-dependence of I_p with a change of sign is required below 2 GeV^2. Moreover if the ECA predictions are correct, the Q^2-dependence of I_p should be even stronger and also the Bjorken integral I_{p-n} should change sign in the low Q^2 region.

Following the VMD model [7], several models were developed in order to predict the low-Q^2 behavior of the integral $I_p(Q^2)$ [8]. Very different Q^2-dependences are expected: the crossover point is predicted in the wide range from ~ 0.15 to ~ 1 GeV^2, while also the sign of the slope of the integral close to photon point is very different from model to model, being negative when including resonances while positive for other parameterizations.

3 High energy contributions to the GDH sum rule

In the transition region, from Q^2=0 to DIS regime, Eq. 3 is only an approximation of Eq. 2, due to the effects of the flux factor K and of the second polarised structure function g_2. Therefore a direct measurement of $\sigma_{1/2} - \sigma_{3/2}$ or A_1 is needed for a precise evaluation of the Q^2 dependence of the generalised GDH integral. It has been shown that at Q^2 around 1 GeV^2 the resonance contribution is only a fraction of the total [9]. Furthermore, while the GDH sum rule for Q^2=0 is almost saturated by the resonance excitation processes and the Ellis-Jaffe integral and Bjorken sum rule for high-Q^2 are largely dominated by DIS processes, in the low-Q^2 region the contributions of resonance and DIS regions are of the same order of magnitude.

The combination of the data sets from TJNAF [10], HERMES and E143 will provide a good estimate of the sum rule around $Q^2 = 1$ GeV^2. Moreover, the high resolution TJNAF data will also provide a deeper insight of polarised

structure functions in the resonance region, while high energy data of HERMES check the asymptotic behavior of $\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)$ and the related no-subtraction hypothesis.

Details on the HERMES experiment can be found elsewhere [11]. Details on this analysis are in [12]. The cross section difference $[\sigma_{1/2} - \sigma_{3/2}]$ can be expressed in terms of the virtual photo-absorption asymmetry A_1 and the unpolarised structure function F_1:

$$\sigma_{1/2} - \sigma_{3/2} = \frac{8\pi^2\alpha}{M} \frac{A_1 F_1}{K}. \tag{4}$$

The structure function $F_1 = F_2(1+\gamma^2)/(2x(1+R))$ was calculated from published parameterisations [13] of the unpolarised structure function F_2 and of $R = \sigma_L/\sigma_T$, the ratio of the absorption cross sections for longitudinally and transversely polarised virtual photons. The asymmetry A_1 was extracted from the measured longitudinal asymmetry $A_{\|}$ by means of the formula $A_1 = A_{\|}/D - \eta A_2$, where D is the virtual photon depolarisation factor and η is a kinematical factor. The asymmetry A_2 is bounded by the positivity limit $A_2 \leq \sqrt{R}$ and has been measured to be small; moreover, the factor η is less than 0.5 in the kinematic range covered by this analysis. The contribution of A_2 for the proton was evaluated with a parameterization $A_2^p = 0.5x/\sqrt{Q^2}$ based on existing data [14, 15], while the contribution of A_2 for the neutron was neglected and the uncertainty was included in the systematic error. The kinematic plane covered by the HERMES experiment is shown in Fig. 1. Above the minimum angle of 40 mrad the events are continously distributed allowing – without interpolation – the integration over ν for several bins in Q^2. The cross section differences $[\sigma_{1/2} - \sigma_{3/2}]$ calculated from the extracted values of A_1^p and A_1^n by means of Eq. 4 are presented in Fig. 2. They are compared to the values of $\left[\sigma_{1/2} - \sigma_{3/2}\right]$ determined from the published data from other experiments for A_1^p using polarised proton targets [16, 14, 15] and for A_1^n using polarised helium-3 targets [17, 18]. There is good agreement between different experiments in the overlapping kinematic regions.

In Fig. 2 are also shown fits to HERMES data, assuming the $a_1(1260)$ Regge trajectory with intercept $\alpha = 0$ (dashed curves). The fit extrapolations seem to slightly underestimate the absolute values of most of the data at higher energy. Recent predictions of the QCD evolution of polarised parton densities show a larger contribution from the high energy region. The dashed-dotted curves show the next-to-leading order QCD parameterization [19].

In each Q^2-bin the GDH integral have been evaluated over the range $\nu_0 \leq \nu \leq 23.4$ GeV, being $\nu_0 = (W_0^2 - M^2 + Q^2)/2M$ where $W_0 = 1.8$ (2.0) GeV for the proton (neutron). The HERMES values with different assumptions of extrapolations are shown in Fig. 3a (proton) and Fig. 3b (neutron): as it is shown, the contribution from high energy is relevant. The HERMES data represent the first direct determination of the DIS contribution to $I(Q^2)$. In Fig. 4 the data are compared with estimates of the integrals that do not include contributions

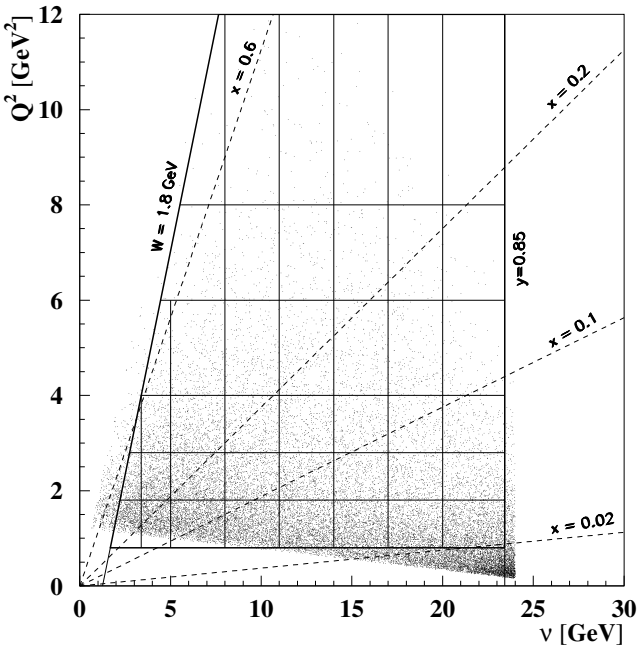

Figure 1: Kinematic plane covered by the HERMES experiment showing the boundaries and the binning in Q^2 and ν used for the evaluation of the GDH integral for the proton data above the resonance region.

from nucleon resonances or possible higher twist effects. These estimates were derived using a parameterization [19] for the asymmetry A_1 given by a next-to-leading order (NLO) QCD analysis. The dashed curves in Fig. 4 show the integrals for this parameterisation over the x range measured at HERMES for proton and neutron. They are in good agreement with all data, indicating that higher twist effects do not contribute significantly in the deep inelastic region, even at the lowest measured Q^2. The dash-dotted curves show the integrals of the NLO parameterisation from ν_0 to infinity.

Up to now the resonance part ($W < 2$ GeV) of Γ has been determined only for two Q^2 values [14, 22]. Their values are of the same order of magnitude of the HERMES determination of DIS part of the integral at the lowest Q^2; therefore the HERMES results together with the precise measurements of the resonance region contribution planned at TJNAF and with a more refined estimate of the high-energy extrapolation, will provide the Q^2 evolution of the entire GDH integral. In addition, several experiments are planned at different facilities to measure the spin-dependent photo-production cross sections to test the GDH sum rule for real photons.

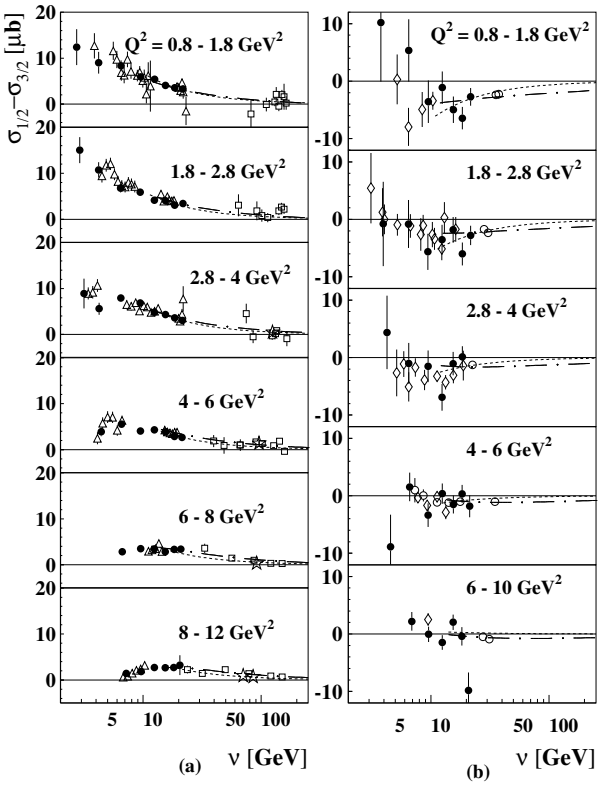

Figure 2: Cross section differences as a function of ν measured in different bins of Q^2 for the proton (a) and the neutron (b). Filled circles are data from this experiment. Open symbols are values derived from other experiments: stars [16], triangles [14], squares [15], diamonds [17], and circles [18]. Only statistical uncertainties are given. The dashed curves are ν^{-1} Regge fits to the HERMES data with $W > 4.5$ GeV; the dash-dotted curves show the next-to-leading order QCD parameterization [19].

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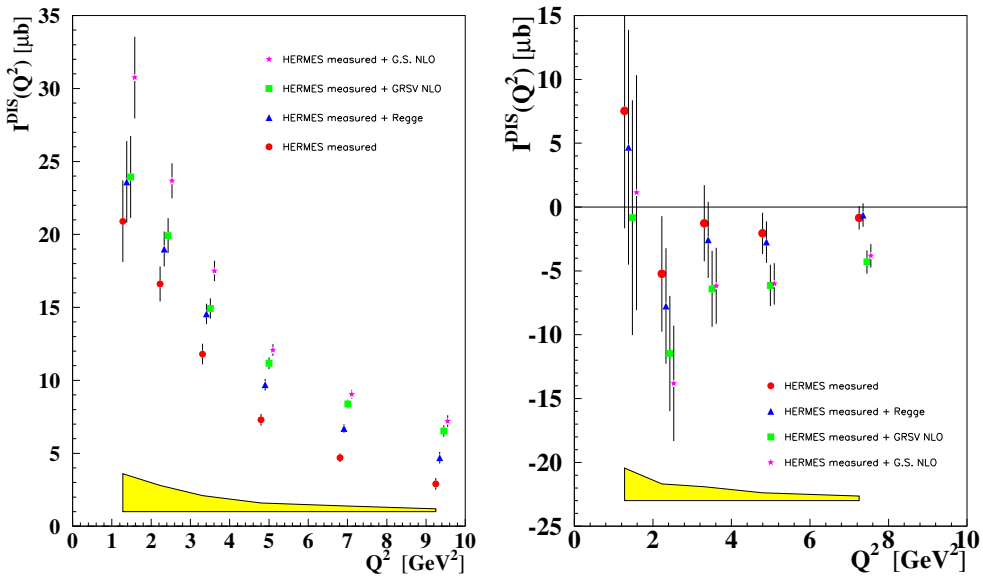

Figure 3: HERMES measured values as a function of Q^2; also shown are the same values with different high energy extrapolations; the biggest extrapolation is derived using a parameterization [20] for the spin structure function $g_1(x)$ given by next-to-leading order QCD analysis [21] and $g_2^{WW}(x)$; all other extrapolations are described in the text.

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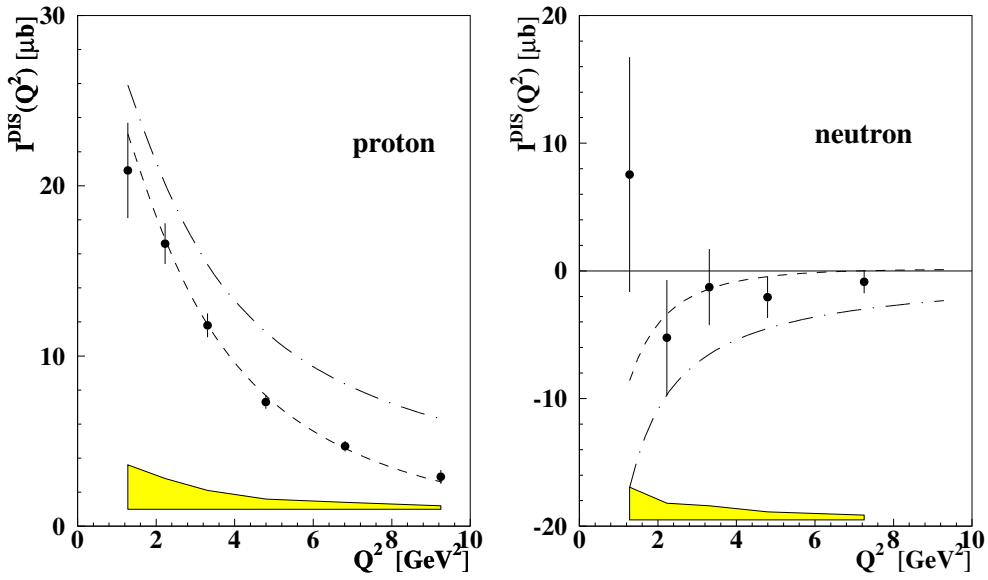

(a) (b)

Figure 4: The generalised GDH integral as a function of Q^2 in the deep inelastic region. The points are $I^{\rm DIS}(Q^2)$ as measured for HERMES data in the range $\nu_0 \leq \nu \leq 23.5$ GeV for the proton (a) and for the neutron (b). See text for the explanation of the curves.

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