$Sin\phi$ azimuthal asymmetry in semi-inclusive electroproduction on longitudinally polarized nucleon

K.A. Oganessyan[a,b], **H.R. Avakian**[b] **N. Bianchi**
LNF-INFN, I-00040, Enrico Fermi 40, Frascati, Italy
A.M. Kotzinian[b,c]
CERN, CH-1211, Geneva 23, Switzerland

We investigate the $sin\phi$ azimuthal asymmetry in the semi-inclusive deep-inelastic lepton scattering off longitudinally polarized nucleon target arising from the time reversal odd structures. The order $1/Q$ contributions of the leading twist and twist-three distribution and fragmentation functions to that asymmetry for the certain kinematical conditions are numerically estimated.

a) *corresponding author, e-mail: kogan@lnf.infn.it*
b) *On leave of absence from Yerevan Physics Institute, Alikhanian Br.2, AM-375036 Yerevan, Armenia*
c) *and JINR, RU-141980 Dubna, Russia*

I. INTRODUCTION

The combination of transverse momentum and polarization effects in the parton distribution and fragmentation functions (DF's and FF's) results in a rich variety of information on the hadronic structure and interactions [1-3]. One specific aspect of the information is the appearance of "time-reversal-odd" (T-odd) FF's [1,3].

These T-odd FF's are experimentally accessible via the measurements of azimuthal asymmetries in polarized semi-inclusive deep-inelastic scattering (SIDIS). This kind of measurements are currently studied at HERMES experiment at HERA [4] and planed in the near future at COMPASS experiment [5] at CERN and the proposed Electron Laboratory for Europe (ELFE) [6]. First leading twist numerical estimations for azimuthal asymmetries in SIDIS have been done in Ref. [7]. Since the typical values of Q^2 in the above mentioned experiments are not too high it is appropriate to evaluate the $1/Q$ effects in the azimuthal asymmetries.

The general expression for azimuthal dependence of polarized SIDIS is rather complicated [2]. The complete tree-level description expression containing contributions from twist-two and twist-three DF's and FF's has been given in [3]. The purpose of this paper is to investigate the specific $1/Q$ order $sin\,\phi$ azimuthal asymmetry in the SIDIS on a polarized target.

Let k_1 (k_2) be the initial (final) momentum of the incoming (outgoing) charged lepton, $Q^2=-q^2$, $q=k_1-k_2$ the momentum of the virtual photon , P_1 (P) the target (observed final-state hadron) momentum. We focus on interactions with non-zero outgoing hadron transverse momentum P_T, perpendicular to the virtual photon momentum, and denote by ϕ the azimuthal angle between P_T and k_{1T} around the virtual photon direction (see Fig.1).

There are three different types of contributions to $sin\,\phi$ asymmetry with the combinations of different leading and subleading DF's and FF's.

- The first, often referred to as the fifth structure function, contains the twist-three DF $\tilde{e}$ and the time-reversal-odd FF $H_1^\perp$ and depends on the lepton helicity λ_e [8]. This target spin independent term has been estimated in the Ref. [9] and shown to be negligible. For this reason we will not discuss it here.

- The second depends on the longitudinal (with respect to virtual photon momentum) component of the target polarization and contains twist-two and twist-three DF's and FF's [3].

- Finally, the third containing only twist-two DF and FF is proportional to $\sin(\phi+\phi_S)$ (ϕ_S is the azimuthal angle of the target spin vector) and to the transverse (with respect to the virtual photon momentum) target polarization, is known as the "Collins effect" [1]. In case of target longitudinal polarization it becomes proportional to the transverse component of the target polarization:

$$S_{T\,1}=S_{lab}\sin\theta_\gamma=S_{lab}\sqrt{\frac{4M^2x_B^2}{Q^2+4M^2x_B^2}(1-y-\frac{M^2x_B^2y^2}{Q^2})}, \tag{1}$$

  where M is a nucleon mass, θ_γ is a virtual photon emission angle and S_{lab} is target polarization (parallel to the incoming lepton momentum). In this case the azimuthal angle (ϕ_S) of the transverse spin ($S_{T\,1}$) in the virtual photon frame takes the values 0 and π (see Fig.2) and leads to $sin\,\phi$ azimuthal asymmetry as well.

II. THE POLARIZED SEMI-INCLUSIVE CROSS SECTION

The cross section for one-particle inclusive deep inelastic scattering is given by

$$\frac{d\sigma^{\ell+N\to\ell'+h+X}}{dx\,dy\,dz_h\,d^2P_{h\perp}}=\frac{\pi\alpha^2y}{2Q^4z_h}L_{\mu\nu}2M\mathcal{W}^{\mu\nu}, \tag{2}$$

where the kinematical (scale) variables are defined as:

$$x_B=\frac{Q^2}{2(P_1q)},\quad y=\frac{(P_1q)}{(P_1k_1)},\quad z_h=\frac{(P_1P)}{(P_1q)}.$$

The quantity $L_{\mu\nu}$ is the well-known lepton tensor. The full expression for the symmetric and antisymmetric parts of the hadronic tensor $\mathcal{W}^{\mu\nu}$ at leading $1/Q$ order are given by Eqs. (77), (78) of Ref. [3]. In order to investigate the $\sin\phi$ azimuthal asymmetry we keep only the terms giving the ϕ-independent and proportional to $\sin\phi$ contributions in the cross section[1]:

$$\begin{aligned}
2M\,\mathcal{W}^{\mu\nu} = 2z_h \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^2(\boldsymbol{p}_T - \frac{\boldsymbol{P}_{h\perp}}{z_h} - \boldsymbol{k}_T)\\
\times \Bigg\{ -g_\perp^{\mu\nu} f_1 D_1 + i\,\epsilon_\perp^{\mu\nu}\, g_{1s} D_1\\
+ i\frac{2\,\hat{t}^{[\mu}k_\perp^{\nu]}}{Q}\left[-\frac{M}{M_h}\,x_B\, e H_1^\perp + \frac{m}{M_h}\, f_1 H_1^\perp\right]\\
- \frac{\left(k_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho} S_{\perp\rho} + S_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho} k_{\perp\rho}\right)}{2M_h}\, h_{1T} H_1^\perp\\
+ \frac{2\,\hat{t}^{\{\mu}\epsilon_\perp^{\nu\}\rho} k_{\perp\rho}}{Q}\left[\frac{M}{M_h}\,x_B\, h_s H_1^\perp - \frac{m}{M_h}\, g_{1s} H_1^\perp\right]\\
+ \frac{2\,\hat{t}^{\{\mu}\epsilon_\perp^{\nu\}\rho} p_{\perp\rho}}{Q}\left[\frac{M_h}{M}\, h_{1s}^\perp \frac{H}{z_h} + \frac{\boldsymbol{k}_\perp^2}{M M_h}\, h_{1s}^\perp H_1^\perp\right]\Bigg\}
\end{aligned} \tag{3}$$

where $\{\mu\,\nu\}$ indicates symmetrization of indices and $[\mu\,\nu]$ indicates antisymmetrization. In the above expression we have used the shorthand notation h_s

$$h_s(x_B, \boldsymbol{p}_T) = \left[S_L\, h_L\,(x_B, \boldsymbol{p}_T^2)\; + h_T\,(x_B, \boldsymbol{p}_T^2)\,\frac{(\boldsymbol{p}_T\cdot\boldsymbol{S}_T)}{M}\right]. \tag{4}$$

and similarly for $h_{1s}^\perp$ and g_{1s}. We focus on the distributions in a longitudinally polarized nucleon and consequently the terms proportional to $(p_T\cdot S_T)$ of Eq.(4) vanish upon the integration of cross section over $\vec{\boldsymbol{p}_T}$ ($p_{T\,1}$ appears only linearly in the cross section). In this respect, we will omit them in further.

The contraction of leptonic and hadronic tensors leads to the cross section with the following terms

$$\frac{d\sigma^{\ell+N\to\ell'+h+X}}{dxdydz_h d^2P_{h\perp}} = \frac{\pi\alpha^2}{Q^2 y}\sum_q e_q^2\sigma_q, \tag{5}$$

where

$$\begin{aligned}
\sigma_q = \int d^2p_T\, d^2k_T\, {z_h}^2\delta^2(\vec{P}_{h\perp} - z_H(\vec{\boldsymbol{p}_T} - \vec{\boldsymbol{k}_T}))\\
\times\Bigg\{ 2\left[1+(1-y)^2\right]\; f_1^q(x_B, p_T^2) D_1^q(z_h, z_h^2 k_T^2)\\
+ 2\lambda_e S_L\; y(2-y)\, g_1^q(x_B, p_T^2) D_1^q(z_h, z_h^2 k_T^2)\\
+ 4\lambda_e\, y\sqrt{1-y}\,\frac{1}{Q} x_B\,\frac{2M}{M_h} k_{T\,2}\,\tilde{e}^q(x_B, p_T^2) H_1^{\perp q}(z_h, z_h^2 k_T^2)\\
- 4(1-y)\,\frac{k_{T\,2} S_{T\,1}}{M_h}\, h_{1T}^q(x_B, p_T^2) H_1^{\perp q}(z_h, z_h^2 \boldsymbol{k}_T^2)\\
+ \frac{S_L}{Q}\, 8(2-y)\sqrt{1-y}\left[\frac{M}{M_h} k_{T\,2}\left(x_B h_L^q(x_B, p_T^2) - \frac{m}{M} g_1^q(x_B, p_T^2)\right) H_1^{\perp q}(z_h, z_h^2 k_T^2)\right.\\
\left. + \frac{M_h}{M}\, p_{T\,2} h_{1L}^{\perp q}(x_B, p_T^2)\frac{\tilde{H}^q(z_h, z_h^2 k_T^2)}{z_h}\right]\Bigg\},
\end{aligned} \tag{6}$$

[1]to get rid of ambiguities, we will use the same notations as in Refs. [3,10].

where by $k_{T\,2}$ ($p_{T\,2}$) we denote the y component of the final (initial) parton transverse momentum vectors, and by $S_{T\,1}$-the transverse component of the target polarization (see Eq.(1)).

III. $P_{h\perp}$-INTEGRATED WEIGHTED CROSS SECTION

Let us consider the differential cross section for one quark flavour, σ_q , integrated with different weights depending on the final hadron transverse momenta $w_i(P_{h\perp})$:

$$I_i = \int d^2 P_{h\perp}\, w_i(P_{h\perp})\, \sigma_q. \tag{7}$$

Taking into account that $I_i = \int d^2 k_T\, d^2 p_T\, w_i\,(z(p_T - k_T))\,\{...\}$ and that the odd powers of $k_{T\,1}$, $p_{T\,1}$, $k_{T\,2}$, $p_{T\,2}$ give zero contribution to I_i, we get cross sections involving the transverse momentum-integrated distribution and fragmentation functions.

1. $w_1(P_{h\perp}) = 1$.
$$I_1 = 2[1 + (1-y)^2]\, f_1(x) D_1(z_h) + 2\lambda_e S_L\, y(2-y)\, g_1(x) D_1(z_h), \tag{8}$$

2. $w_2(P_{h\perp}) = -\boldsymbol{P}^2_{h\perp}/MM_h z_h = |\boldsymbol{P}_{h\perp}| \sin\phi/MM_h z_h$.
The surviving terms upon integration are
$$\begin{aligned} I_2 &= I_{2L} + I_{2T} \\ &= \frac{S_L}{Q}\, 8(2-y)\sqrt{1-y}\left[\left(x_B h_L(x_B) - \frac{m}{M} g_1(x_B)\right) H_1^{\perp(1)}(z_h) - h_{1L}^{\perp(1)}(x_B) \frac{\tilde{H}(z_h)}{z_h}\right] \\ &\quad + \frac{|S_{T\,1}|}{M}\, 4(1-y)\, h_1(x_B) H_1^{\perp(1)}(z_h), \end{aligned} \tag{9}$$
where I_{2L} corresponds to the higher twist effects and I_{2T} is representing the leading twist Collins effect. Note, that the $S_{T\,1}$ itself is $\sim 1/Q$ for longitudinally polarized target, thus the two contributions are of the same order.

Here we have straightforwardly $\boldsymbol{p}_T$-($\boldsymbol{k}_T$)-integrated distribution (fragmentation) functions and $\boldsymbol{p}_T^2/2M^2$-($\boldsymbol{k}_T^2/2M_h^2$)- weighted distribution (fragmentation) functions indicated with superscript (1) [3,11]:

$$\begin{aligned} h_{1L}^{\perp(1)}(x_B) &= \int d^2 p_T \left(\frac{p_T^2}{2M^2}\right) h_{1L}^{\perp}(x_B, p_T^2), \\ H_1^{\perp(1)}(z_h) &= z_h^2 \int d^2 k_T \left(\frac{k_T^2}{2M_h^2}\right) H_1^{\perp}(z_h, z_h^2 k_T^2). \end{aligned} \tag{10}$$

A. Gaussian parametrizations of DF's and FF's

The $\boldsymbol{P}_{h\perp}$-integration of cross section can also be performed analytically if one supposes that the transverse momentum dependence in the DF's and FF's be written in factorized exponential form:

$$f(\vec{\boldsymbol{p}_T}) = \frac{1}{a^2\pi} e^{-\boldsymbol{p}_T^2/a^2}, \qquad d(\vec{\boldsymbol{k}_T}) = \frac{1}{b^2\pi} e^{-z_h^2 \boldsymbol{k}_T^2/b^2}. \tag{11}$$

The explicit expression[2] for the hadronic cross sections up to subleading order $1/Q$ (for more details see Appendix D of Ref. [3]) looks as[3]

[2]We have omitted the charge-square weighted sum over quark flavors $\sum_i Q_i^2$.

[3]Note that this expression is identical with results in [3] where the authors have used the following relation among DF's

$$h_L(x_B, k_T^2) = \frac{m}{M} \frac{g_{1L}(x_B, k_T^2)}{x_B} - \frac{k_T^2}{M^2} \frac{h_{1L}^{\perp}(x_B, k_T^2)}{x_B} + \tilde{h}_L(x_B, k_T^2).$$

$$
\int d\sigma d^2 P_T = C(P_C)(1+(1-y)^2) f_1(x_B) D_1(z_h) + \lambda_e S_L y (1-\frac{y}{2}) g_1(x_B) D_1(z_h), \tag{12}
$$

$$
\begin{aligned}
\int d\sigma \sin\phi d\phi = C(P_T) \Big\{ & S_L 2(2-y)\sqrt{1-y}\frac{\boldsymbol{P}_{h\perp}}{z_h Q}\Big[R_1 x_B h_L(x_B) H_1^{\perp}(z_h) \\
& - R_2 g_1(x_B) H_1^{\perp}(z_h) - R_3 h_{1L}^{\perp}(x_B)\frac{\tilde{H}(z_h)}{z_h}\Big] \\
& + S_{T\,1}(1-y)\frac{\boldsymbol{P}_{h\perp}}{z_h} R_4 h_1(x_B) H_1^{\perp}(z_h)\Big\}.
\end{aligned} \tag{13}
$$

Here

$$
C(P) = \frac{4\pi^2\alpha^2}{Q^2 y}\exp(-\frac{P^2}{B}), \quad B = b^2 + a^2 z_h^2
$$

and

$$
R_1 = \frac{Mb^2}{M_h B^2}, \quad R_2 = \frac{mb^2}{M_h B^2}, \quad R_3 = \frac{M_h a^2 z_h^2}{MB^2}, \quad R_4 = \frac{b^2}{M_h B^2}.
$$

where M_h is the final hadron mass and m is a current quark mass, and $S_{T\,1}$ is the transverse component of the target polarization defined by Eq.(1).

IV. LEADING AND SUBLEADING DF'S AND FF'S

In considered SIDIS cross section together with the well-known chiral even twist-two DF's and FF's $f_1(x_B)$ and $D_1(z_h)$ also enter the combinations of different leading and subleading DF's and FF's. Their physical interpretation has been discussed in Refs. [1,7,11].

In our numerical calculations we will use the twist-two $h_{1L}^{\perp}(x_B)$, $h_{1L}^{\perp(1)}(x_B)$, $h_1(x_B)$, $g_1(x_B)$ and twist-three $h_L(x_B)$ distribution functions obtained within the framework of a diquark spectator model of Jacob, Mulders and Rodrigues [12] (JMR model).

To obtain the time-odd twist-2 ($H_1^{\perp}(z_h)$) and the interaction dependent part of the twist-three $H(z_h)$ ($\tilde{H}(z_h)$) fragmentation functions we take from the JMR model the $\boldsymbol{k}_T$ depending twist-two fragmentation function $D_1^{\pi}(z_h, z_h\vec{\boldsymbol{k}_T})$:

$$
D_1^{\pi}(z_h, z_h\vec{\boldsymbol{k}_T}) = N_{\pi}^2 \frac{(1-z_h)^{2\alpha-1}}{z_H^{2\alpha}}\frac{(m_q + M/z_h)^2 + |\boldsymbol{k}_T^2|}{(|\boldsymbol{k}_T^2| + \lambda^2(1/z_h))^{2\alpha}}, \tag{14}
$$

where $\lambda^2 = \Lambda^2(1-1/z_h) + m_q^2/z_h - (1-1/z_h)M_h^2/z_h$, m_q is the constituent quark mass, Λ and α are model parameters. N_{π}^2 is the normalization factor which drops from the ratios of polarized to unpolarized FF's defining the $sin\phi$ asymmetry.

Then we combine that function with the "guess" of Collins [1], e.g

$$
H_1^{\perp}(z_h, z_h^2\boldsymbol{k}_T^2) = \frac{M_C M_h}{M_C^2 + |\boldsymbol{k}_T^2|} D_1(z_h, z_h^2\boldsymbol{k}_T^2), \tag{15}
$$

where $M_C \simeq 0.3 \div 1.0$ GeV is a typical hadronic mass. This parameterization exhibits the leading twist asymmetry when $\boldsymbol{k}_T = O(M)$. In our numerical calculations we use $M_C = 2M_{\pi}$. Using the relation [13]

$$
H(z_h) = z_h^3 \frac{d}{dz_h}\frac{H_1^{\perp(1)}(z_h)}{z_h}
$$

and Eq.(C.33) of [3], one can derive the following expression for the interaction dependent part

$$
\frac{\tilde{H}(z_h)}{z_h} = \frac{d}{dz_h}[z_h H_1^{\perp(1)}(z_h)], \tag{16}
$$

where the function with upper index (1) indicate $\boldsymbol{k}_T^2/2M_h^2$-weighted function defined as in Eq.(10).

The plots a, and b of the Fig.3 show $h_L(x_B)$, $h_1(x_B)$, and $h_{1L}^{\perp}(x_B)$, $h_{1L}^{\perp(1)}(x_B)$ distribution functions obtained within JMR model. On the plots c, d of the Fig.3 the $H_1^{\perp}(z_h)/D_1(z_h)$ and $H_1^{\perp(1)}(z_h)/D_1(z_h)$ using the JMR model for $\Lambda = 0.4GeV$, $m_q = 0.36GeV$ and $\alpha = 1.05$ are presented (labeled by 1). The predictions for the same quantities [7] using the Binnewies, Kniehl, Kramer, (BKK) parametrization [15] $D_1(z_h)$ for the quark fragmentation function, are labeled by 2. In that approach Collins "guess" combined with the BKK $D_1(z_h)$ of a Gaussian $\boldsymbol{k}_T$ dependence.

V. NUMERICAL RESULTS

We start with the consideration of specific spin-dependent azimuthal asymmetry in polarized $\boldsymbol{P}_{h\perp}$-integrated semi-inclusive charged pion leptoproduction. Note that in our calculations we make an approximation in which we do not take into account the sea-quark contributions, as well as unfavourite fragmentation functions ($D_1^{d\to\pi^+}(z_h) = D_1^{u\to\pi^-}(z_h)$) have been set to zero (consequently, unfavourite time-odd ones as well). In view of these our numerical results for the $\sin\phi$ asymmetry should be regarded as a order-of magnitude estimates.

Consider the weighted $\sin\phi$ asymmetry defined as

$$\langle \frac{|P_{h\perp}|}{MM_h z_h} \sin\phi_h \rangle \equiv \frac{\int d^2P_{h\perp} \frac{|P_{h\perp}|}{MM_h z_h} \sin\phi_h \frac{d\sigma}{dx\,dy\,dz_h\,d^2P_{h\perp}}}{\int d^2P_{h\perp} \frac{d\sigma}{dx\,dy\,dz_h\,d^2P_{h\perp}}} = \frac{I_2}{I_1}, \tag{17}$$

where I_1, and I_2 are given by Eq.(8) and Eq.(9). Note, that this expression define the target longitudinal spin azimuthal asymmetry for unpolarized lepton beam ($\lambda_e = 0$) as well.

For numerical estimation of asymmetry, defined by Eq.(17), we use the following two sets of DFs and FFs:

- *set* **A** - the $f_1(x_B)$, $h_{1L}^{\perp(1)}(x_B), h_1(x_B), g_1(x_B)$, $h_L(x_B)$ distribution and $D_1(z_h)$ fragmentation functions obtained within the JMR model [12]. The time-reversal odd fragmentation functions defined by Eqs. (14) - (16) and Eq.(10).
- *set* **B** - the Glück, Reya, Vogt, (GRV) parton distribution functions [14] for $f_1(x_B)$ and the $h_{1L}^{\perp(1)}(x_B), h_1(x_B), g_1(x_B)$, $h_L(x_B)$ distribution functions obtained within the JMR model [12]. BKK parametrization of the $D_1(z_h)$ and the time-reversal odd fragmentation functions obtained by combining Collins "guess" with the BKK $D_1(z_h)$ of a Gaussian $\boldsymbol{k}_T$ dependence.

In order to make an average over the range of Q^2, we use the relation $Q^2 = 2ME_l x_H y$, where M is the proton mass. After integrating over the x_B, y, z_h at HERMES experiment kinematical ranges, e.g., $E_l = 27.5GeV$, $Q^2 > 1GeV^2$, $0.1 < y < 0.85$, $0.02 < x_H < 0.4$ and $0.1 < z_h < 1$, we get[4]

$$\langle \frac{|P_{h\perp}|}{MM_h z_h} \sin\phi_h \rangle = \begin{cases} 0.15 & : \quad A \\ 0.17 & : \quad B \end{cases} \tag{18}$$

It is important to mention that the contribution of I_{2L} is about 11% in case of using the *set* **A** and about 12% when using the *set* **B**.

In the same approach using the expression for I_{2T} defined in Eq.(9) we estimate also the magnitude of the weighted asymmetry in the cross section (Eq.17 with $S_L = 0$) related directly to so-called "Collins effect" for transversely polarized target. For *set* **A** we get it 0.17 and for *set* **B** it is 0.22.

Now let us define the quantity $\langle \sin\phi \rangle$ as

$$\langle \sin\phi \rangle = \frac{\int d\sigma \sin\phi d\phi}{\int d\sigma d\phi}, \tag{19}$$

[4]Note, that in our numerical calculations we neglect the contribution of the term $\sim g_1(x_B)H_1^{\perp(1)}(z_h)$ suppressed by the factor m/M, where m is the current quark mass and M the proton mass.

where $d\sigma$ is a SIDIS cross section and the integrations are over P_T (with the lower limit of the observed hadrons transverse momentum cutoff equal to P_C), ϕ, x_B, y and z_h.

Consider how $\langle \sin\phi \rangle$ as defined above with P_T cutoff P_C (only hadrons with transverse momenta above the cutoff will be included), behaves numerically. We use the Gaussian transverse momentum parametrizations of DF's and FF's defined by Eqs.(11) of the same leading and subleading DF's and FF's mentioned above.

Our numerical results at HERMES kinematics mentioned above are presented in Fig.4. We take $a = 0.5GeV, b = 0.7GeV$, which correspond to an average intrinsic transverse momenta of $\langle k_T \rangle = 0.44GeV, \langle p_T \rangle = 0.62GeV$. This choice of the average transverse momenta used to estimate the smearing effects are conditioned by the good agreement with existing data on the azimuthal structure of the hadronic final state in unpolarized deep inelastic lp scattering [16,17]. The curves labeled by 1 and 2 on the Fig.4 are corresponded to the *set* **A** and *set* **B**, respectively.

From Fig.4 one can see that the magnitude of such single asymmetry is about a few percents (while at the same kinematical range the asymmetry of the semi-inclusive pion production in the deep-inelastic scattering of a polarized lepton beam off an unpolarized nucleon target shows up also as a $\langle \sin\phi \rangle$, is around half-percent [9]). We point out that the big asymmetry predicted using the *set* **A** is mainly conditioned by the strong dependence (especially at small $\boldsymbol{k}_T$) of the FFs, based on the JMR model, on $\boldsymbol{k}_T$, whereas the $\boldsymbol{k}_T^2$ factor in the weighted T-odd FFs washed out that dependence.

It is important to emphasize that the azimuthal structure in the quark transverse momenta plays the key role in this asymmetry. At HERMES kinematical conditions we deal with the low transverse momentum range, where the effects of intrinsic transverse momentum are dominated. In this respect we assume the estimations within the Gaussian transverse parametrizations of DF's and FF's are reasonable. Another argument is that they are in a good agreement with the numerical results of the $\boldsymbol{P}_{h\perp}$-integrated weighted $\sin\phi$ asymmetry, which are valid for *any* transverse momentum dependence of DF's and FF's.

VI. SUMMARY

We have investigated the specific $1/Q$ order $sin\,\phi$ azimuthal asymmetry of single inclusive charged pion production in the deep-inelastic lepton scattering off longitudinally polarized nucleon target related to the time reversal odd structure, arising from nonperturbative final-state interactions. The contributions of both leading twist and twist-three effects to that asymmetry are taken into account.

The order $1/Q$ $\boldsymbol{P}_{h\perp}$-integrated weighted azimuthal asymmetry in terms of the leading twist and twist-three distribution and fragmentation functions is numerically estimated at HERMES kinematical conditions.

We have also analyzed the dependence of the azimuthal asymmetry parameter $\langle \sin\phi \rangle$ on the transverse momentum cutoff P_C in kinematical ranges at HERMES.

The measurement of such a single spin asymmetry can allow to determine the naively-time-odd quark fragmentation functions, which appears due to the non-applicability of time-reversal invariance for the hadronization of a quark. It is important, however, to have good particle identification and sufficient azimuthal resolution in the forward direction.

VII. ACKNOWLEDGMENTS

The authors would like to thank E. DeSanctis for helpful comments. One of the authors (K.O) wish to thank D. Boer, R. Jakob and P. Mulders for useful discussions. This work of (K.O), (H.A) and (N.B) was in part supported by the INTAS contribution (contract number 93-1827) from the European Community.

---

[1] J. Collins, Nucl. Phys. B **396** (1993) 161.
[2] A. Kotzinian, Nucl. Phys. B **441** (1995) 234.
[3] P. J. Mulders, R. D. Tangerman, Nucl. Phys. B **461** (1995) 197;
[4] The HERMES experiment, DESY HERMES-95-02, 1995.

[5] The *COMPASS* Proposal, CERN/SPSLC 96-14, SPSC/P297, 1996;
[6] J. Arvieux and E. DeSanctis, The ELFE project conf. proc., Vol. **44**, Bologna, (1993); J. Arvieux, B. Pire, Progress in Particle and Nucl. Phys. 30, (1995) 299.
[7] A. Kotzinian, P. J. Mulders, Phys. Lett B **406** (1997) 373; Phys. Rev. D (1997) .
[8] J. Levelt, P. J. Mulders, Phys. Lett. B **338** (1994) 357.
[9] K. A. Oganessyan, hep-ph/9806420.
[10] D. Boer, R. Jakob, P.J. Mulders, Nucl. Phys. B **504** (1997) 345.
[11] P. J. Mulders, Nucl. Phys. A **622** (1997) 239c.
[12] R. Jakob, P.J. Mulders, J. Rodrigues, Nucl. Phys. A **626** (1997) 937.
[13] D. Boer, P.J. Mulders, Phys. Rev. D **57** (1998) 5780.
[14] M. Glück, E. Reya, A. Vogt, Z. Phys. C **67** (1995) 433.
[15] J. Binnewies, B.A. Kniehl, G. Kramer, Phys. Rev. D **52** (1995) 4947.
[16] J.J. Aubert, et. al., EMC; Phys. Lett. B **130** (1983) 118.
[17] M.R. Adams, et. al., E665; Phys. Rev. D **48** (1993) 5057.

VIII. FIGURE CAPTIONS

Fig.1. The definition of the final hadron azimuthal angle ϕ.

Fig.2. Definition of the transverse spin azimuthal angle in the virtual photons frame.

Fig.3. a - $h_L(x_B)$, $h_1(x_B)$, b - $h_{1L}^{\perp}(x_B)$, $h_{1L}^{\perp(1)}(x_B)10^{-1}$ distribution functions obtained within JMR model. On c the $H_1^{\perp}(z_h)/D_1(z_h)$, and on d the $H_1^{\perp(1)}(z_h)/D_1(z_h)$ are ploted. The curves denoted by 1 and 2 correspond to ratio of polarized and unpolarized fragmentation functions of the *sets* **A** and **B** (see text), respectively.

Fig.4. $\langle \sin\phi \rangle$ at HERMES kinematics.